

**RECONSTRUCTION OF BINARY IMAGE USING
TECHNIQUES OF DISCRETE TOMOGRAPHY**

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RECONSTRUCTION OF BINARY IMAGE USING TECHNIQUES OF DISCRETE TOMOGRAPHY

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of Arts and Sciences in partial
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by

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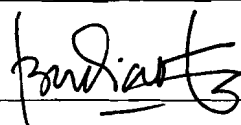
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PEMBINAAN SEMULA IMEJ BINARI MENGUNAKAN TEKNIK TOMOGRAFI DISKRET

ABSTRAK

Tomografi diskret melibatkan pembinaan semula imej-imej, terutamanya imej binari dari unjuran mereka. Beberapa masalah pembinaan semula binari telah dipertimbangkan dalam kesusasteraan, dengan menggunakan model unjuran yang berbeza atau kekangan tambahan. Di sini, kami akan mempertimbangkan pembinaan semula imej binari dengan memberikan beberapa maklumat berangka tambahan pada baris imej binari dengan menganggapnya sebagai matriks binari yang mengandungi 0 dan 1. Masalah ini melibatkan maklumat, yang disebut sebagai unjuran baris, pada bilangan 1's dan bilangan subword 01 dalam baris imej binari untuk dibina. Algoritma yang dicadangkan membina salah satu antara imej binari yang mempunyai maklumat yang sama berangka pada nombor 1 dan nombor subword 01. Sebaliknya bagi jenis khas imej binari dengan baris dalam bentuk tertentu algoritma akan membina imej yang unik.

RECONSTRUCTION OF BINARY IMAGE USING TECHNIQUES OF DISCRETE TOMOGRAPHY

ABSTRACT

Discrete tomography deals with the reconstruction of images, in particular binary images, from their projections. A number of binary image reconstruction methods have been considered in the literature, using different projection models or additional constraints. Here, we will consider reconstruction of a binary image with some prescribed numerical information on the rows of the binary image treated as a binary matrix of 0's and 1's. The problem involves information, referred to as row projection, on the number of 1's and the number of subword 01's in the rows of the binary image to be constructed. The algorithm proposed constructs one among the many binary images having the same numerical information on the number of 1's and the number of subword 01. This proposed algorithm will also construct the image uniquely for a special kind of a binary image with its rows in some specific form.

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“ the one exclusive sign of thorough knowledge is the power of teaching “

Aristotle

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LIST OF ABBREVIATIONS

ART algebraic reconstruction technique

BT binary tomography

CT computed tomography

DT discrete tomography

NDT non-destructive testing

VLSI very-large-scale integration

CHAPTER 1

INTRODUCTION

In Greek, 'Tomos' means 'cut' or 'section', while the suffix -graphy means 'field of study', and tomography is a technique for digitally cutting/ sectioning a specimen open using X-rays to reveal its inner details. It deals with techniques to reconstruct images in a non-destructive way from their projections, especially, to reconstruct some physical object, by acquiring projection images (e.g., X-ray images) using a tomographic scanner. A common problem in image processing is how to obtain information about the inside of an object in a non-destructive way (non-destructive testing, NDT); that is, without damaging it in any way. Several kinds of methods have been developed for this reason like X-ray, gamma-ray and neutron imaging. In industrial metallic material examinations, neutron and gamma-ray sources are mainly used, while X-ray sources are applied for non-metallic objects or living specimen.

Discrete tomography (DT) is a special field of tomography that focuses on the problem of reconstruction of images that contain only a few different grey levels, and in particular, is concerned with the problem of recovering binary images from their projections. Normally algorithms that are proposed for computing such reconstructions are called tomographic reconstruction algorithms. These algorithms are used to compute accurate reconstructions, with a drawback that it might require a large number of projections. A binary image can be considered as an m by n array of pixels or cells that are coloured either black or white, with the black pixel denoted as 1 and the white as 0. A row projection of a binary picture is an ordered set of sums over the rows with each sum being the number of 1's in a row. Likewise a column projection is an ordered set of sums over the columns with each sum being the number of 1's in

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