NEW ALGORITHM FOR DETERMINANT OF MATRICES VIA COMBINATORIAL APPROACH

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Abstrak

Kaedah pencarian penentu bagi matriks telah lama diterokai dan menarik minat ramai penyelidik. Namun begitu, kebanyakan kaedah yang sedia ada adalah rumit serta memerlukan masa pengiraan yang panjang terutamanya bila saiz matriks semakin besar. Sehubungan dengan itu, kajian ini cuba membangunkan kaedah baharu yang dapat mengurangkan masa pengiraan penentu bagi matriks sebarang peringkat. Kaedah yang dibangunkan adalah berasaskan pilih atur yang dijana dengan menggunakan set pemula. Semua set pemula bagi n elemen yang diperolehi melalui pendekatan kombinatorik akan menghasilkan pilih atur $n!$ yang lengkap dan berbeza. Strategi set pemula yang digunakan ini juga telah dibuktikan lebih cekap berbanding dengan kaedah untuk menyenaraikan semua pilih atur yang sedia ada seperti kaedah *lexicographic*. Semua pilih atur yang diperolehi seterusnya digunakan bagi membina kaedah baharu untuk mencari penentu matriks bersaiz n x n. Kajian ini juga telah menghasilkan teorem baharu untuk mencari penentu matriks bersaiz n x n dan teorem ini telah dibuktikan mempunyai persamaan dengan teorem yang sedia ada iaitu Leibniz teorem. Di samping itu, beberapa teori dan sifat matematik untuk penjanaan pilih atur dan pengiraan penentu turut dibina dan dibukti bagi mengesahkan kaedah baharu yang dibina. Keputusan berangka mendapati masa pengiraan penentu bagi kaedah baharu adalah lebih cepat jika dibandingkan dengan kaedah yang sedia ada. Pengujian kaedah baharu turut dilakukan ke atas beberapa matriks khas seperti matriks Toeplitz, Hilbert dan Hessenberg bagi membuktikan kecekapan kaedah yang telah dibangunkan. Keputusan berangka juga menunjukkan kaedah baharu dapat mencari penentu matriks khas dengan lebih pantas jika dibandingkan dengan kaedah Gauss dan Gauss Jordan.

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Katakunci: Pilih atur, Set pemula, Penentu matriks.

Abstract

Methods for finding determinants for matrices have long been explored and attracted interest of numerous researchers. However, most of the existing methods are tedious and require lengthy computation time particularly as the size of matrices becomes larger. Therefore, this study attempts to develop a new method which can reduce determinant computation time for matrices of any order. The developed method was based on permutations which were generated using starter sets. All starter sets for n elements were obtained by using combinatorial approach which then produced all $n!$ distinct permutations. This starter sets strategy was proven to be more efficient if compared to other existing methods for listing all permutations such as lexicographic method. All permutations obtained were then used to construct a new method for finding determinants of $n \times n$ matrices. This study also produced a new theorem for finding determinant of $n \times n$ matrices and this theorem was proven to be equivalent to the existing theorem i.e Leibniz theorem. Besides that, several new theoretical works and mathematical properties for generating permutation and determining determinant were also constructed to verify the new developed method. The numerical results revealed that the determinant computation time for the new method was faster if compared to the existing methods. Testing of the new method on several special matrices such as Toeplitz, Hilbert and Hessenberg matrices was also carried out to prove the efficiency of the developed method. The numerical results also indicated that the new method outperformed Gauss and Gauss Jordon methods in term of computation time..

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Keywords: Permutation, Starter sets, Determinant of matrices.

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Table of Contents

List of Tables

List of Figures

List of publications

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CHAPTER ONE

INTRODUCTION

1.1 Background

The study of determinants has a long history and appears to be essential in some geometric and algebraic computations (see e.g. Bareiss, 1968; Benno, 2008; Clarkson, 1992; Gentleman& Johnson, 1974). The literature indicates that there are various different algorithms to find the determinants of matrices (see, e.g.; Bareiss, 1968; Gentleman & Johnson, 1974; Griss, 1976; Hajrizaj, 2009; Iqbal, 1995; Sasaki & Murao, 1982; Thongchiew, 2007). The most common algorithms are Expansion algorithms, i.e. Cofactor expansion, Minor expansion (Gentleman & Johnson, 1974; Griss, 1976), Elimination algorithms, i.e. Gauss, Gauss Jordan (Bareiss, 1968; Gentleman & Johnson, 1974; Sasaki & Murao, 1982), and Algorithms of permutation, i.e. Permutation algorithm (Iqbal, 1995; Mahajan & Vinay, 1997; Thongchiew, 2007). These algorithms for the determinant of matrices can be easily computed either by hand or computer if the size of the matrices is small (Hajrizaj, 2009; Gentleman & Johnson, 1974; Griss, 1976; Li, 2007). However, when the size of the matrices is larger, computations for these algorithms will be problematic and complicated. This is because the process of finding the determinant is not a mere sequential calculation process of values but involves highly structured operations (Li, 2007). Whether the computations are carried out manually or by using a computer program, more time is needed because of the increasing level of difficulty involved (Li, 2007). In fact, according to Dingle (2005), the execution time appears

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