STATISTICAL PROCESS CONTROL USING MODIFIED ROBUST HOTELLING’S $T^2$ CONTROL CHARTS

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Abstrak


Katakunci: Hotelling’s $T^2$, Carta kawalan, Penganggar teguh
Abstract

Hotelling’s $T^2$ chart is a popular tool for monitoring statistical process control. However, this chart is sensitive to outliers. To alleviate the problem, three approaches to the robust Hotelling’s $T^2$ chart namely trimming, Winsorizing and median based were proposed. These approaches used robust location and scale estimators to substitute for the usual mean and covariance matrix, respectively. For each approach, three robust scale estimators: $MAD_n$, $S_n$ and $T_n$ were introduced, and these estimators functioned accordingly to the approach. The first approach, denoted as $T^2_k$, applied the concept of trimming via Mahalanobis distance. The robust scale estimator was used to replace the covariance matrix in Mahalanobis distance. The trimmed mean and trimmed covariance matrix were the location and scale estimators for the $T^2_k$ chart. The second approach, $T^2_{W}$, employed each scale estimator as the Winsorized criterion. This approach applied Winsorized modified one step M-estimator and its corresponding Winsorized covariance as the location and the scale matrix for $T^2_{W}$ chart, respectively. Meanwhile, in the third approach, $T^2_{HL}$, the robust scale estimator took the role of the scale matrix with Hodges-Lehman as the location estimator. This approach worked with the original data without any trimming or Winsorizing. Altogether, nine robust control charts were proposed. The performance of each robust control chart was assessed based on false alarm rates and probability of detection. To investigate on the strengths and weaknesses of the proposed charts, various conditions were created by manipulating four variables, namely number of quality characteristics, proportion of outliers, degree of mean shifts, and nature of quality characteristics (independent and dependent). In general, the proposed charts performed well in terms of false alarm rates. With respect to probability of detection, all the proposed charts outperformed the traditional Hotelling’s $T^2$ charts. The overall findings showed that, the proposed robust Hotelling’s $T^2$ control charts are viable alternatives to the disputed traditional charts.

Keywords: Hotelling $T^2$, Control chart, Robust estimator
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# List of Abbreviations

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<th>Description</th>
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<tbody>
<tr>
<td>MOM</td>
<td>Modified One-step $M$-estimator</td>
</tr>
<tr>
<td>HL</td>
<td>Hodges and Lehmann estimator</td>
</tr>
<tr>
<td>Med</td>
<td>Median</td>
</tr>
<tr>
<td>MAD$_n$</td>
<td>Median absolute deviation</td>
</tr>
<tr>
<td>$S_n$</td>
<td>A scale estimator</td>
</tr>
<tr>
<td>$T_n$</td>
<td>A scale estimator</td>
</tr>
<tr>
<td>FA</td>
<td>False Alarms</td>
</tr>
<tr>
<td>POD</td>
<td>Probability of Detection</td>
</tr>
<tr>
<td>ARE</td>
<td>Asymptotic Relative Efficiency</td>
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<tr>
<td>MD</td>
<td>Mahalonobis Distance</td>
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CHAPTER ONE
MULTIVARIATE QUALITY CONTROL CHARTS

1.1 Introduction

The invention of Statistical Process Control (SPC) chart was pioneered by Dr. Walter Shewhart while he was working for Bell Labs in 1920. He aimed to monitor the quality of a process mathematically. Since then, this tool has received tremendous attention and interest from many researchers and practitioners from various fields including statistics, engineering and education to name just a few. There are some definitions of SPC charts tool. We refer to Montgomery (2005), who defined the SPC charts as tool for optimizing the amount of information needed for decision-making purposes. In addition, Nedumaran and Pignatiello (2000) defined the charts as tools to monitor performance or state of the process.

In general, SPC charts are graphical presentations that display the stability of a process. Unlike other common charts, such as bar chart, line chart or pie charts, SPC charts have some main features such as the following:

(i) The upper limit and lower limit’s lines that create a range to where a process output is considered “in control”

(ii) A center line which located in the middle of the lower and upper limits that reflects the average state of the process.
The contents of the thesis is for internal user only
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