NEW SEQUENTIAL AND PARALLEL DIVISION FREE METHODS FOR DETERMINANT OF MATRICES

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2013

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Abstrak

Penentu berperanan penting dalam kebanyakan aplikasi aljabar linear. Pencarian penentu menggunakan kaedah pembahagian bukan bebas akan menghadapi masalah sekiranya pemasukan matriks diwakili dalam ungkapan nisbah atau polinomial dan juga apabila kesilapan titik apungan wujud. Bagi mengatasi masalah ini, kaedah pembahagian bebas digunakan. Dua kaedah pembahagian bebas yang biasa digunakan dalam pencarian penentu adalah pendaraban silang dan pengembangan kofaktor. Walau bagaimanapun, pendaraban silang yang menggunakan Petua Sarrus hanya berhasil untuk matriks berperingkat kurang atau sama dengan tiga, sedangkan apabila berhadapan dengan matriks yang bersaiz besar, pengembangan kofaktor memerlukan pengiraan yang terlalu panjang dan rumit. Oleh itu, kajian ini berusaha membangunkan kaedah berjujukan dan kaedah selari yang baharu untuk mencari penentu bagi matriks. Kajian ini juga berhasrat untuk mengitlakkan Petua Sarrus bagi sebarang peringkat matriks segi empat sama berpandukan pilih atur yang diperolehi menggunakan set penjana. Dua strategi diperkenalkan bagi menjana set penjana yang berlainan iaitu operasi kitaran dan operasi saling tukar dua unsur. Beberapa hasil teori dan sifat matematik dalam penjanaan pilih atur dan penentuan penentu turut dibina bagi menyokong kajian ini. Keputusan berangka menunjukkan masa pengiraan kaedah baharu yang dicadangkan adalah lebih baik jika dibandingkan dengan kaedah sedia ada. Masa pengiraan kaedah berjujukan baharu yang dibangunkan tertakluk kepada penjanaan set penjana. Oleh demikian, dua strategi selari dibangunkan untuk menyelaraskan algoritma ini bagi mengurangkan masa pengiraan. Keputusan berangka turut menunjukkan bahawa kaedah selari berupaya mengira penentu lebih cepat berbanding kaedah berjujukan, khususnya apabila tugas diagihkan dengan sama rata. Kesimpulannya, kaedah baharu yang telah dibangunkan boleh diguna sebagai alternatif yang berdaya saing dalam pencarian penentu bagi matriks. .

Kata kunci: Penentu, Pilih atur, Set penjana, Kaedah tanpa pembahagi, Kaedah jujukan dan selari

Abstract

A determinant plays an important role in many applications of linear algebra. Finding determinants using non division free methods will encounter problems if entries of matrices are represented in rational or polynomial expressions, and also when floating point errors arise. To overcome this problem, division free methods are used instead. The two commonly used division free methods for finding determinant are cross multiplication and cofactor expansion. However, cross multiplication which uses the Sarrus Rule only works for matrices of order less or equal to three, whereas cofactor expansion requires lengthy and tedious computation when dealing with large matrices. This research, therefore, attempts to develop new sequential and parallel methods for finding determinants of matrices. The research also aims to generalise the Sarrus Rule for any order of square matrices based on permutations which are derived using starter sets. Two strategies were introduced to generate distinct starter sets namely the circular and the exchanging of two elements operations. Some theoretical works and mathematical properties for generating permutation and determining determinants were also constructed to support the research. Numerical results indicated that the new proposed methods performed better than the existing methods in term of computation times. The computation times in the newly developed sequential methods were dominated by generating starter sets. Therefore, two parallel strategies were developed to parallelise this algorithm so as to reduce the computation times. Numerical results showed that the parallel methods were able to compute determinants faster than the sequential counterparts, particularly when the tasks were equally allocated. In conclusion, the newly developed methods can be used as viable alternatives for finding determinants of matrices.

Keywords: Determinant, Permutation, Starter sets, Division free method, Sequential and parallel methods

Acknowledgement

First for all, I would like to express my deep sense of gratitude to Allah S.W.T for His Merciful and Kindness in giving me a chance to complete this study. I am highly indebted to Prof. Dr. Zurni bin Omar, Assoc. Prof. Dr. Haslinda binti Ibrahim, and Assoc. Prof. Dr. Khairil Iskandar bin Othman for their invaluable help and guidance during the course of research. I highly appreciated them for constantly encouraging me by giving their critics on my work. I am grateful to them for having given me the support and confidence.

I am also indebted to the Government of Malaysia (Ministry of Higher Education), and Universiti Utara Malaysia for sponsoring this research.

I would like to thank my family. I could not have made it to this level without their love and prayers. Gratitude to my parents, Haji Karim bin Karon and Hajjah Besah binti Nordin, and my siblings. They always believed in me and have always supported me.

Last but not least, I would like to thanks all my friends including Hasimah binti Sapiri, Azizah binti Mohd Rohni, Idariyana binti Mohd Seran, Fatimah Bibi binti Hamzah, Teh Raihana Nazirah binti Roslan, Hashibah binti Hamid, Hartima binti Abu Talib and INSPEM, UPM especially to Miss Hazwanie Abdullah for helping me in conducting HPC machines and INSPEM's staff for giving me a convenient office to work in.

Declaration Associated With This Thesis

Karim, S., Ibrahim, H., Omar, Z., Othman. K. I.& Sulaiman, M. (2009). Generating permutation based on starter sets, Eletronics *Proceeding of 5th Asian Mathematical Conference*,1, 90-96. available url: http://www.mat.usm.my/AMC 20200920 Proceedings/Pure/P134.PDF

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Glossary of Terms

Abbreviation	Details
ATM	Across The Method
ATT	Across The Time
CO	Circular Operation
CP	Circular Permutation
D	Dimension
DFM	Division Free Method
GB	Giga Bytes
HPC	High Performance Computing
ID	IDentity
ISS	Initial Starter Sets
ISSG1	Initial Starter Sets Generator of first strategy
ISSG2	Initial Starter Sets Generator of second strategy
MPI	Message Passing Interface
nDFM	non Division Free Method
PDATM1	Permutation Determinant for Across The Method of circular strategy
PDATM2	Permutation Determinant for Across The Method of exchange strategy
PERATM1	Permutation for Across The Method of circular strategy
PERATM2	Permutation for Across The Method of exchange strategy
PERMUT1	Permutation of circular strategy
PERMUT2	Permutation of exchange strategy
PERMUTIT3	Permutation of iterative circular strategy
PERMUTDET1	Permutation determinant of circular strategy
PERMUTDET2	Permutation determinant of exchange strategy
PERMUTDETIT3	Permutation determinant of iterative circular strategy
RAM	Random Access Memory
RoCP	Reverse of Circular Permutation
SIMD	Single Instruction Multiple Data
SPU	Synergistic Processing Unit

CHAPTER ONE

INTRODUCTION TO DETERMINANT METHODS

1.1 Background of the Study

Matrices and determinants are the backbone of linear algebra (Bernstein, 2008). A determinant provides useful geometrical and algebraical information of a square matrix. Algebraically, a matrix has an inverse if and only if the determinant is not zero. This happens when the vectors are linearly independent. Meanwhile geometrically, the row entries of $n \times n$ matrix define the edges of a parallelepiped in n-dimensional space, of which the area and volume are the absolute value of the determinant of a square matrix for spaces R^2 and R^3 respectively.

The determinant has been the subject of study for over 200 years. The name determinant was introduced by Carl Friedrich Gauss (1777-1855) while discussing quadratic forms. The term determinant was used because it determined the properties of the quadratic form (O'Connor & Robertson, 1996). The theory of determinant was expanded gradually during the 18th century through the theory of equations in the work of Leibniz, Maclaurin, Cramer and Laplace (Rice & Torrence, 2006). Then it became an increasingly significant subject in the mathematical area by the 19th century.

The applications of determinant can be found in various areas for example in mathematical physics in which any solvable equation having a solution can be expressed as a determinant (Vein & Dale, 1999). The determinant is required in inverse kinematics singularity analysis of parallel manipulator which this manipulator is described as 6×6 transformation matrix (Luyang et al., 2006). Meanwhile from the statistical perspective, the determinant is used in normalizing the constant of the probability density function of the multivariate normal distribution, and is also involved in experimental design (Harville, 1997). In addition, the determinant is a beneficial tool in eigenvalue problems in which

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