EFFICIENT AND HIGHLY ROBUST HOTELLING T^2 CONTROL CHARTS USING REWEIGHTED MINIMUM VECTOR VARIANCE

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Abstrak

Carta kawalan Hotelling T^2 adalah alat yang berkesan bagi kawalan proses berstatistik untuk persekitaran multivariat. Walau bagaimanapun, prestasi carta kawalan Hotelling T^2 tradisional yang menggunakan penganggar lokasi dan serakan klasik biasanya dicemari oleh kesan pelitupan dan *swamping*. Bagi mengurangkan masalah ini, penganggar teguh telah disyorkan. Penganggar teguh yang paling popular dan digunakan secara meluas dalam carta kawalan Hotelling T^2 adalah penentu kovarians minimum (MCD). Terkini, penganggar yang lebih baik dikenali sebagai varians vektor minimum (MVV) telah diperkenalkan. Penganggar ini mempunyai titik kerosakan yang tinggi, varians samaan affin dan pengiraan yang lebih cekap. Oleh kerana cirinya yang baik, kajian ini mencadangkan untuk mengganti penganggar klasik dengan penganggar lokasi dan serakan MVV dalam pembinaan carta kawalan Hotelling T^2 bagi cerapan individu pada analisis Fasa II. Walau bagaimanapun, penganggar MVV didapati mempunyai beberapa kelemahan seperti tidak tekal pada taburan normal, tidak saksama untuk sampel bersaiz kecil dan kurang cekap pada titik kerosakan yang tinggi. Bagi meningkatkan ketekalan dan kesaksamaan MVV, penganggar tersebut telah didarabkan masing-masing dengan faktor ketekalan dan faktor pembetulan. Bagi mengekalkan titik kerosakan di samping mempunyai kecekapan statistik yang tinggi, penganggar MVV berpemberat semula (RMVV) telah dicadangkan. Seterusnya, penganggar RMVV tersebut digunakan dalam pembinaan carta kawalan Hotelling T^2. Carta teguh Hotelling T^2 yang baharu ini menghasilkan kesan positif dalam mengesan titik terpencil dan pada masa yang sama mampu mengawal kadar penggera palsu. Di samping analisis terhadap data simulasi, analisis ke atas data sebenar juga mendapati carta teguh Hotelling T^2 yang baharu ini dapat mengesan cerapan luar kawalan dengan lebih baik berbanding carta lain yang diselidik dalam kajian ini. Berdasarkan prestasi yang baik terhadap analisis data simulasi dan sebenar, carta teguh Hotelling T^2 yang baharu ini adalah merupakan alternatif yang baik bagi carta Hotelling T^2 yang sedia ada.

Kata kunci: Penganggar Cekap, Kawalan Proses Berstatistik Multivariat, Varians Vektor Minimum Berpemberat Semula, Carta Hotelling T^2 Teguh, Penganggar Multivariat Teguh

Abstract

Hotelling T^2 control chart is an effective tool in statistical process control for multivariate environment. However, the performance of traditional Hotelling T^2 control chart using classical location and scatter estimators is usually marred by the masking and swamping effects. In order to alleviate the problem, robust estimators are recommended. The most popular and widely used robust estimator in the Hotelling T^2 control chart is the minimum covariance determinant (MCD). Recently, a new robust estimator known as minimum vector variance (MVV) was introduced. This estimator possesses high breakdown point, affine equivariance and is superior in terms of computational efficiency. Due to these nice properties, this study proposed to replace the classical estimators with the MVV location and scatter estimators in the construction of Hotelling T^2 control chart for individual observations in Phase II analysis. Nevertheless, some drawbacks such as inconsistency under normal distribution, biased for small sample size and low efficiency under high breakdown point were discovered. To improve the MVV estimators in terms of consistency and unbiasedness, the MVV scatter estimator was multiplied by consistency and correction factors respectively. To maintain the high breakdown point while having high statistical efficiency, a reweighted version of MVV estimator (RMVV) was proposed. Subsequently, the RMVV estimators were applied in the construction of Hotelling T^2 control chart. The new robust Hotelling T^2 chart produced positive impact in detecting outliers while simultaneously controlling false alarm rates. Apart from analysis of simulated data, analysis of real data also found that the new robust Hotelling T^2 chart was able to detect out of control observations better than the other charts investigated in this study. Based on the good performance on both simulated and real data analysis, the new robust Hotelling T^2 chart is a good alternative to the existing Hotelling T^2 charts.

Keywords: Efficient Estimators, Multivariate Statistical Process Control, Reweighted Minimum Vector Variance, Robust Hotelling T^2 Chart, Robust Multivariate Estimator

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List of Abbreviations

ARE	Asymptotic relative efficiency
BP	Breakdown point
CD	Covariance determinant
Fast MCD	Fast minimum covariance determinant
HDS	Historical data set
MCD	Minimum covariance determinant
MLE	Maximum likelihood estimators
MSPC	Multivariate statistical process control
MSD	Mahalanobis squared distances
MSD_{MCD}	Mahalanobis squared distances based on MCD estimators
MSD_{MVV}	Mahalanobis squared distances based on MVV estimators
MSE	Mean squared error
MVN	Multivariate normal distribution
MVE	Minimum volume ellipsoid
MVV	Minimum vector variance
$MVV_{0.25}$	MVV estimators with breakdown point of 0.25
$MVV_{0.5}$	MVV estimators with breakdown point of 0.5
n	Sample size
p	Number of dimension
PDS	Positive definite and symmetric matrix
RMCD	Reweighted minimum covariance determinant
RMVV	Reweighted minimum vector variance
$RMVV_{0.25}$	RMVV estimators with breakdown point of 0.25
$RMVV_{0.5}$	RMVV estimators with breakdown point of 0.5
SPC	Statistical process control
T_0^2	Traditional Hotelling T^2 chart without cleaning the outliers

T_S^2	Traditional Hotelling T^2 chart with standard approach, cleans the outliers once
T_{MCD}^2	Hotelling T^2 chart based on MCD estimators
T_{RMCD}^2	Hotelling T^2 chart based on RMCD estimators
T_{MVV}^2	Hotelling T^2 based on MVV estimators
$\mathrm{T}_{\mathrm{MVV(o)}}^2$	Hotelling T^2 based on the original T_{MVV}^2
$\mathrm{T}_{\mathrm{MVV(I)}}^2$	Hotelling T^2 based on the improved MVV estimators in terms of consistency and unbiased
$T_{RMVV_{0.25}}^2$	Hotelling T^2 chart based on RMVV estimators with breakdown point of 0.25
$T_{RMVV_{0.5}}^2$	Hotelling T^2 chart based on RMVV estimators with breakdown point of 0.5
UCL	Upper control limit
VV	Vector variance

Declaration Associated with this Thesis

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CHAPTER ONE
INTRODUCTION

1.1 Introduction

Success of a firm very much depends on the quality of its product. Be it goods or services, the firm has little chance of success if its core product is of inferior quality (Ferrel & Hartline, 2008). To ensure that the quality of a product is always up to a certain level, the process behavior needs be monitored and the quality of the process has to be improved. This will consequently lead to business success, growth and enhanced competitiveness. To better meet customers' expectations, many manufacturing industries have reviewed their processes and improve specifications with acceptable standards by reducing variability in the process and product, which substantially will improve performance. Thus, identifying the cause of variation to reduce variability in a process is vital in monitoring quality.

There are two distinct causes of variations in a process namely the common and special cause variations. While common cause variation can be reduced by management intervention, the special cause is hard to gauge as this variation affects the process in unpredictable ways. However, special cause can be detected by some statistical techniques. It can be eliminated from the process by the worker or process control team in charge of the particular segment of the process, which is referred as local action. When all the special-cause variation is eliminated, the process is said to be in-statistical control. The second type of variation, known as common-cause

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