A MULTI-ATTRIBUTE DECISION MAKING PROCEDURE USING FUZZY NUMBERS AND HYBRID AGGREGATORS

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Abstrak

Proses Hierarki Analitikal (PHA) klasik mempunyai dua kelemahan utama. Pertama, ia mengabaikan aspek ketidaktentuan yang lazimnya wujud dalam kebanyakan data atau maklumat yang ditafsir oleh manusia. Kedua, ia tidak mengambil kira aspek interaksi antara atribut semasa pengagregatan. Penggunaan nombor-nombor kabur dapat membantu mengatasi isu pertama, manakala penggunaan Kamiran Choquet membantu mengatasi isu kedua. Namun, penggunaan nombor-nombor kabur dalam pembuatan keputusan berbilang atribut (PKBA) memerlukan beberapa langkah dan maklumat tambahan daripada para pembuat keputusan. Sementara itu, proses pengenalpastian nilai ukuran monoton yang perlu dilaksanakan sebelum menggunakan Kamiran Choquet juga memerlukan bilangan langkah pengiraan dan jumlah maklumat yang tinggi daripada para pembuat keputusan terutamanya dengan peningkatan bilangan atribut. Justeru, kajian ini memperkenalkan satu prosedur PKBA yang mampu mengurangkan jumlah langkah pengiraan dan maklumat yang diperlukan daripada para pembuat keputusan apabila kedua-dua aspek tersebut dipertimbangkan secara serentak. Untuk mencapai objektif utama kajian ini, sebanyak lima fasa telah dilaksanakan. Pertama, konsep set kabur dan aplikasinya dalam PHA telah dikaji. Kedua, analisa berkenaan pengagregat-pengagregat yang boleh digunakan dalam masalah PKBA telah dilaksanakan. Ketiga, fokus kajian telah dijuruskan kepada Kamiran Choquet dan konsep sekutunya, ukuran monoton. Seterusnya, prosedur yang dicadangkan dibangunkan dengan kombinasi lima komponen utama iaitu Analisis Faktor, Penganggar Kabur-Linguistik, Kamiran Choquet, PHA Kabur Mikhailov, dan Purata Berwajaran Mudah. Akhirnya, satu masalah PKBA sebenar telah diselesaikan untuk menguji kebolehfungsian prosedur tersebut di mana imej tiga buah pasaraya yang terletak di Sabak Bernam, Selangor, Malaysia telah dikaji dari perspektif suri rumah. Kajian ini berpotensi untuk mendorong lebih ramai pembuat keputusan mengambil kira aspek ketidaktentuan dalam data dan interaksi antara atribut secara serentak ketika menyelesaikan sesuatu masalah PKBA.

Kata kunci: Proses Hierarki Analitikal (PHA), Kamiran Choquet, Teori set kabur, Pembuatan Keputusan Berbilang Attribut (PKBA).

Abstract

The classical Analytical Hierarchy Process (AHP) has two limitations. Firstly, it disregards the aspect of uncertainty that usually embedded in the data or information expressed by human. Secondly, it ignores the aspect of interdependencies among attributes during aggregation. The application of fuzzy numbers aids in confronting the former issue whereas, the usage of Choquet Integral operator helps in dealing with the later issue. However, the application of fuzzy numbers into multi-attribute decision making (MADM) demands some additional steps and inputs from decision maker(s). Similarly, identification of monotone measure weights prior to employing Choquet Integral requires huge number of computational steps and amount of inputs from decision makers, especially with the increasing number of attributes. Therefore, this research proposed a MADM procedure which able to reduce the number of computational steps and amount of information required from the decision makers when dealing with these two aspects simultaneously. To attain primary goal of this research, five phases were executed. First, the concept of fuzzy set theory and its application in AHP were investigated. Second, an analysis on the aggregation operators was conducted. Third, the investigation was narrowed on Choquet Integral and its associate monotone measure. Subsequently, the proposed procedure was developed with the convergence of five major components namely Factor Analysis, Fuzzy-Linguistic Estimator, Choquet Integral, Mikhailov's Fuzzy AHP, and Simple Weighted Average. Finally, the feasibility of the proposed procedure was verified by solving a real MADM problem where the image of three stores located in Sabak Bernam, Selangor, Malaysia was analysed from the homemakers' perspective. This research has a potential in motivating more decision makers to simultaneously include uncertainties in human's data and interdependencies among attributes when solving any MADM problems.

Keywords: Analytical Hierarchy Process (AHP), Choquet Integral, Fuzzy set theory, Multi-Attribute Decision Making (MADM).

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CHAPTER ONE
INTRODUCTION

1.1 Multi-attribute Decision Making

In today's highly competitive environment, be it in profit or non-profit based organizations, it is unfeasible to make decisions by considering a single attribute or objective. As a result, multi-criteria decision making (MCDM) emerges as one of the prominent branches of decision making (Triantaphyllou, 2000) where it offers various scientific or quantitative techniques to aid decision makers in identifying, comparing, and evaluating alternatives based on varied, usually conflicting, attributes or objectives (Choo, Schoner, and Wedley, 1999; Tavares, Tavares, and Parry-Jones, 2008). Herein, decision makers are referred as an individual or a group of individuals who has the obligation to provide some critical information on the existing evaluation problem and to carry out the quantitative decision analysis by employing the developed decision-aid tools.

In general, MCDM can be split into two domains namely multi-objective decision making (MODM) and multi-attribute decision making (MADM) (Lu, Zhang, Ruan, and Wu, 2007). Chen, Kilgour, and Hipel (2009) defined MODM as a field which applies mathematical algorithms to identify alternatives that are optimal or efficient, under certain constraints, with respect to a few objectives which are expressed mathematically using decision variables. Linear programming is an example of MODM technique. On the other hand, MADM aims to assist the decision makers in making preference assessment on finite or available set of alternatives described by a set of predefined, usually conflicting, attributes. To recapitulate, the primary divergence between the two domains is MODM deals with infinite number of alternatives, whereas MADM considers choices within a finite set of alternatives (Hwang and Yoon, 1981). This research limits its concern on MADM problems.

Up to now, practitioners of decision theory have formulated various techniques to aid decision makers in surmounting MADM problems. In general, these techniques can be mainly classified into two categories namely multi-attribute utility theory (MAUT) techniques and outranking techniques (Zopounidis and Doumpos, 2002). However, applying MAUT techniques emerges as a well-accepted standard approach for modeling MADM problems (Mussi, 1999).

1.1.1 Multi-attribute Utility Theory

Triantaphyllou (2000) concluded that there are 3 fundamental phases in executing any of the MAUT techniques. In the first phase, the relevant alternatives and attributes of existing problem are identified. The basic elements of a typical MAUT model comprise a set of *m* alternatives denoted by $a_i = \{a_1, a_2, \dots, a_m\}$ and set of *n* attributes represented by $c_j = \{c_1, c_2, \dots, c_n\}$.

In following phase, the attributes' weights and local scores (performance scores of the alternatives with respect to each attribute) are derived. These weights and local scores are commonly derived by questioning decision makers (Choo and Wedley, 2008). The weight of each attribute in $c_j = \{c_1, c_2, \dots, c_n\}$ is usually represented by $w_j = \{w_1, w_2, \dots, w_n\}$ where $w_1, w_2, \dots, w_n$ are positive values. With regards to MAUT method, attributes' weights can be interpreted as the attributes' importance in achieving the goal of a MADM problem (Choo et al., 1999). In other words, these weights represent the contribution of each attribute in enhancing the performance of specific target.

On the other hand, the local scores of an alternative, i with respect to each attribute in $c_j = \{c_1, c_2, \dots, c_n\}$ can be denoted by $x_{ij} = \{x_{i1}, x_{i2}, \dots, x_{in}\}$ where $i = 1,2, \dots, m$. For instance, the set of local scores of alternative, a_1 with respect to each attribute in c_j can be represented by $x_{1n} = \{x_{11}, x_{12}, \dots, x_{1n}\}$ where $x_{11}, x_{12}, \dots, x_{1n}$ are positive values. According to Zeleny (1982), a MADM problem can be easily expressed in a matrix format, known as decision matrix. Decision matrix is a $(m \times n)$ matrix which encompasses the local scores of the alternatives with respect to each criterion under consideration. Table 1.1 presents the general form of a decision matrix for a MADM problem.

Table 1.1: General Form of Decision Matrix

Alternatives/ Attributes	$\boldsymbol{c_1}$	$\boldsymbol{c_2}$	...	$\boldsymbol{c_n}$
a_1	x_{11}	x_{12}	...	x_{1n}
a_2	x_{21}	x_{22}	...	x_{2n}
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
a_m	x_{m1}	x_{m2}	...	x_{mn}

The final phase of MAUT is known as aggregation phase. An aggregation phase uses a specific function or an aggregation operator which synthesizes the set of attributes' weights and local scores of each alternative into a single global score (Hazura, Abdul Azim, Mohd Hasan, and Ramlan, 2007; Marichal, 1999). The global score of each alternative in a_i can be denoted by $v_i = \{v_1, v_2, \dots, v_m\}$. Each global score implies the preference score of each alternative which will be helpful for DMs in selecting, ranking or sorting the alternatives. An alternative with highest global score signifies the most preferred alternative for the existing problem.

The following example would offer better understanding on the execution of MAUT techniques. Consider a best car selection problem where four cars a_1, a_2, a_3, and a_4 which are being assessed based on three attributes, comfort (c_1), speed (c_2), and design (c_3).The basic elements of this MADM problem are as shown in Table 1.2.The local scores for each car, $x_{ij} = \{x_{i1}, x_{i2}, x_{i3}\}$ where $i =$ 1,2,3,4 are derived by questioning decision makers. In addition, the set of attributes' weights, $w_i = \{w_1, w_2, w_3\}$ are also derived based on some data offered by decision makers. Finally, the global score of each car, $v_i = \{v_1, v_2, v_3\}$ can be computed by composing the attributes' weights and local scores of each car using a specific aggregation operator. Then, the performance of cars can be ranked based on these global scores where the higher is the global score, the better is the performance of the car.

Table 1.2: Example of Car Selection Problem based on MAUT

Cars/ Attributes	**Comfort (c_1)**	**Speed (c_2)**	**Design (c_3)**	**Global score, V**
a_1	x_{11}	x_{12}	x_{13}	v_1
a_2	x_{21}	x_{22}	x_{23}	v_2
a_3	x_{31}	x_{32}	x_{33}	v_3
a_4	x_{41}	x_{42}	x_{43}	v_4
Attribute's weight	w_1	w_2	w_3	

Analytical Hierarchy Process (AHP) is an instance of MADM technique which operates based on the previously-explained three fundamental MAUT phases (Fulop, 2005 and Matue, 2002).

1.1.2 Analytic Hierarchy Process

Analytical Hierarchy Process (AHP), which was developed by Saaty in early 1970's (Saaty, 1980), emerged as one of the broadly applied techniques due to its ability to simplify a complex decision analysis into a systematic and structured mode (Warren, 2004).

According to Mau-Crimmins, de Steiguer, and Dennis (2005), AHP provides a systematic approach for decision makers in comparing and weighting multiple attributes and measuring the preference on alternatives that involves in a MADM problem. The application of AHP is widespread in various domains such as telecommunication (Tam and Tummala, 2001), business marketing (Chen and Wang, 2010), resource allocation (Ramanathan and Ganesh, 1995), and project management (Zayed, Mohamad Amer, and Pan, 2008).

As mentioned beforehand, conventional AHP utilizes the 3 basic phases of MAUT and the steps involved in each of these phases can be summarized as follows (Dubois and Prade, 1980; Dagdeviren, Yavuz, and Kilinc, 2009; Bertolini, Braglia, and Carmignani, 2006).

a) Phase 1 of AHP: Identification of alternatives and attributes

   In the first phase, the existing MADM problem is defined by identifying the relevant alternatives and attributes. The complex decision problem is then decomposed into simpler hierarchy. Normally, the first level of hierarchy comprises of alternatives or choices of the problem, the subsequent levels consist of sub- attributes and attributes which are commonly determined using the experience of the experts, and the top level represents the goal of the existing MADM problem,. An example of a structured MADM hierarchy is as illustrated in Figure 1.1.

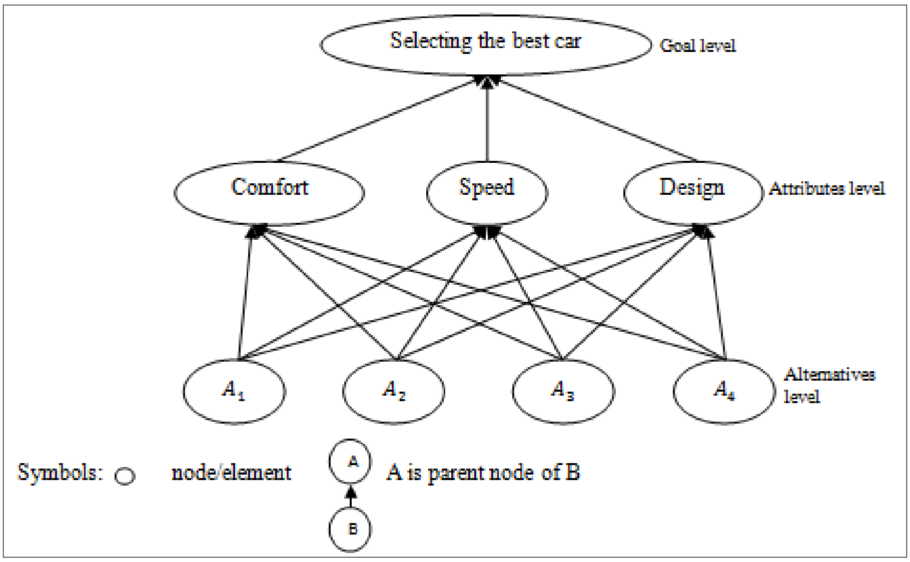


Figure 1.1: Hierarchy of Car Selection Problem (Example)

b) Phase 2 of AHP: Identifying local scores of alternatives and weights of attributes

In the second phase, the “strength” or preference of each element of a level in relation to their importance for an element in the next level needs to be determined (Bouyssou et al., 2000). The assessment of the elements may start from the bottom elements where all elements connected to the same parent element are compared pair-wise.

For instance, if one wants to apply AHP in a MADM problem consisting 3 levels as shown in Figure 1.1, firstly, pair-wise comparisons between alternatives must be performed for each attribute to derive local scores of alternatives. Then, attributes are also compared in a pair-wise manner to model their importance or weight.

During pair-wise comparison, commonly, the decision makers are required to express their preference based on Saaty's 9-point AHP scale which is actually an ordinal scaling ratio as shown in Table 1.3 (Lee, Mogi, and Kim, 2008).

Table 1.3: Saaty's AHP Scale

Preference scale	**Description**	**Reciprocal scale**
1	Two elements contribute equally	1
3	One element is slightly favored over another	$\frac{1}{3}$
5	One element is strongly favored over another	$\frac{1}{5}$
7	One element is very strongly favored over another	$\frac{1}{7}$
9	One element is most favored over another	$\frac{1}{9}$
2,4,6,8	Used to compromise between two judgments	$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

**Note, when element i is compared to j then, the preference is assigned with one of the crisp scale. Meanwhile, when element j is compared to i then, the preference is assigned with the corresponding reciprocal.*

Based to Table 1.3, each crisp number that ranges from 1 to 9 corresponds to the strength of preference for one element over another. According to Saaty (1980), generally, the 9-point scale is used because the qualitative distinctions are meaningful in practice and have an aspect of precision when the items are compared with one another. Then, by mathematically processing the data in pair-wise comparison matrices which is usually achieved through the eigenvalue method, (Bana e Costa and

Vansnick, 2008) the local scores and attributes' weights of the problem can be derived.

However, the consistency of the pair-wise judgments offered by decision makers need to be tested by computing consistency ratio (CR) (Anderson, Sweeney, Williams, Camm, and Martin, 2012). If the CR value is above 0.1 then, the decision maker has to refine the pair-wise matrix. This procedure goes on until all pair-wise matrices satisfy CR value (Arsanjani, 2012).

c) Phase 3 of AHP: Aggregation

Finally, in the aggregation phase, the attributes' weights and local scores of alternatives are combined into global scores to determine the ranking of the alternatives. Generally, in AHP, SWA operator which assumes independency between attributes is applied for the aggregation purpose.

However, despite its popularity, AHP is still being criticized for several drawbacks or issues such as rank reversal issue (Belton and Gear, 1985), lacking of consistency in pair-wise comparison (Benítez, Delgado-Galván, Gutiérrez, and Izquierdo, 2011), incompetence in dealing with uncertainty or vagueness that exists in data provided by human (Chou, Sun, and Yen, 2012), and issue of neglecting the interaction aspect between attributes during aggregation process (Buyukozkan and Ruan, 2010). The focus of this research is devoted on the last two issues.

1.1.3 Issue of Uncertainty in Human's Data

As mentioned formerly, to execute the second phase of any MAUT methods such as AHP, some data from decision makers are commonly required to derive the local scores of alternatives and attributes' weights. However, in reality, human are usually uncertain or imprecise in expressing their preference or judgment due to insufficient information on the occurring problem (Chen and He, 1997). Therefore, in many practical cases, the decision makers are reluctant or find it is burdensome to express their exact preference based on crisp numbers or scales (Torfi, Farahani, and Rezapour, 2010; Yu and Hu, 2010). They generally tend to express their preference in natural languages or linguistic terms (Onut, Kara, and Isik, 2009) such as 'unimportant', 'important', 'very important' and 'extremely important' over crisp numbers $(1,2,3,\cdots)$ as they are always uncertain about their judgment.

However, the traditional MAUT methods such as classical AHP are based on crisp numbers and not based on linguistic terms. Thus, these models are not exactly representing actual or natural human thinking style. In order to mathematically deal with uncertainty embedded in linguistic judgments (Kahraman, Cebeci, and Ulukan, 2003), fuzzy set theory which was introduced by Zadeh (1965) is usually applied into MADM models. Many fuzzy MAUT models such as fuzzy AHP were developed (Wu, Tzeng, and Chen, 2009) to deal with uncertainty in human's data with the intention to generate more practical analysis. Through fuzzy MAUT models, the decision makers are permitted to provide or express the required data in linguistic terms. Each of these linguistic terms will be then represented or quantified with appropriate fuzzy numbers which are able to mathematically capture the uncertainty embedded in linguistic estimations (Tseng, 2011).

1.1.3.1 Drawback of Applying Fuzzy Sets in MADM Environment

Unfortunately, analyzing a MADM problem under fuzzy environment requires higher computational effort from decision makers (Rao, 2007 and Zhang, 2004) than analyzing the same problem under crisp setting where, it would demand them to carry out some additional steps and to provide some extra information during the analysis.

For instance, in fuzzy analysis, the decision makers are required to translate the linguistic preference into appropriate fuzzy numbers based on a conversion scale. Normally, this scale is constructed based on the decision makers' knowledge, experience, or intuition as demonstrated in the studies conducted by Tsaur, Chang, and Yen (2002) and Chou (2007). In other words, some prior information is usually demanded from the decision makers for the fuzzification process. Besides, to determine the ordinal ranking or priorities of the attributes and alternatives, another additional process namely defuzzification (Opricovic and Tzeng, 2008) is required as the overlapping fuzzy numbers cannot be simply compared to each other.

Besides, to our knowledge, there is no any specific decision software or tools (e.g. Decision Lens, Expert Choice Professional, Logical Decision, and Criterium DecisionPlus which are designed to perform crisp analysis) have been developed to aid the decision makers in conducting MADM analysis using fuzzy MAUT models. Therefore, maintaining fuzziness or fuzzy numbers throughout an analysis may demand extra effort from the decision makers (Chen and Hwang, 1992) especially from those who are unfamiliar with quantitative analysis.

To simplify, these additional requirements could impede the real-world decision makers from evaluating the MADM problems under fuzzy environment.

1.1.4 Issue of Ignoring Interaction Aspect among Attributes

It is believed that aggregation is one of the most crucial stages in executing any of the MAUT techniques. The major issue in the aggregation phase can be described as follows. In most of the cases, even in some of the recent studies as listed in Table 1.4, the local scores of alternatives are simply aggregated with any of the additive aggregation operators such as simple weighted average (SWA) and ordered weighted average (OWA) which hypothesizes the attributes as being independent to each other (Huang, Shieh, Lee, and Wu, 2010).

Table 1.4: Recent MADM Studies which Applied Additive Aggregators

Sources	**MADM problems**	**Type of aggregators used**
Al-Yahyai, Charabi, Gastli, and Al-Badi. (2012).	Indexing wind farm land suitability	OWA
Goshal, Naskar, and Bose. (2012)	Evaluating the performance of diploma institutes	SWA
Jiang, Zhang, Hu, Wang, and Zhang. (2012).	Assessing the atmospheric environment comprehensive quality in Xi'an	SWA
Liu'an, Xiaomei and Lin, (2012)	Assessment on teaching quality	SWA
Li, Ren, and Zheng. (2013).	Risk evaluations in the spacecraft development	SWA

Prior to employing additive aggregation operator, the decision makers are only required to derive or estimate the weight for each attribute where the sum of the weights always equals to one (Ceberio and Modave, 2006). This additive property makes conventional aggregation operators fail to model the interaction information among attributes during aggregation (Zhang, Zhou, Zhu, and Li, 2006). For instance, the popular AHP simply presumes the attributes are always independent to each other as it uses SWA operator during aggregation. But, this approach is not practical in real application as in many problems the attributes are interacted to each other (Marichal, 2000).

However, it is discovered that Choquet integral operator plays an important role in capturing the interaction aspect among attributes during aggregation process (Yue, Li, and Yin, 2005). Prior to employing Choquet integral, the decision makers are not only required to define the individual weight of each attribute but also the weight of all possible combinations or subsets of attributes which are known as monotone measure (Beliakov and James, 2011). These monotone measure weights, g, not only represent the importance of each attribute but also the importance of each combination or subset of attributes (Marichal and Roubens, 2000).

Monotone measure should satisfy two key axioms namely boundary and monotonicity conditions (Angilella, Greco, Lamantia, and Matarazzo, 2004). The boundary condition interprets that an empty set, with the absence of any attributes, has no importance where $g\{\emptyset\} = 0$ and the maximal set, with the presence of all attributes, has maximal importance where $g\{c_j\} = 1$. Meanwhile, monotonicity condition implies that adding a new attribute to a combination or subset cannot decrease its importance. Monotone measure can characterize super-additive and sub-additive effect between attributes, which model the synergy support and redundancy type of interaction respectively (Grabisch, 1996a). The successful application of

Choquet integral relies on proper identification of monotone measure weights, which capture the importance of single attribute or their combination.

The total number of monotone measure weights which need to be determined in a particular MADM problem is equivalent to 2^n where n represents the number of attributes (Meyer and Roubens, 2006). This total includes the weight of empty and maximal set. For instance, consider a MADM problem with three attributes where $c_j = \{c_1, c_2, c_3\}$. Then, the monotone measure weights which need to be estimated prior to applying Choquet integral are $g\{\emptyset\}$, $g\{c_1\}$, $g\{c_2\}$, $g\{c_3\}$, $g\{c_1, c_2\}$, $g\{c_1, c_3\}$, $g\{c_2, c_3\}$, $g\{c_1, c_2, c_3\}$ and obviously the weight of empty set, $g(\emptyset)$ = 0 and weight of maximal set, $g\{c_1, c_2, c_3\} = 1$ as per the axiom.

Several approaches were proposed in studies by Tahani and Keller (1990), Chen and Wang (2001), and Takahagi (2007), to name but a few, to assist the decision makers in estimating monotone measure weights. Each approach demands different types and amount of information from decision makers. Further review on these approaches is offered in chapter three of the thesis.

1.1.4.1 Drawback of Choquet Integral

The only issue on using Choquet integral is usually, a complex computational process is required to determine the 2^n weights of monotone measure especially for a larger set of attributes (Tzeng and Huang, 2011).

The total number of monotone measure weights (2^n) which need to be identified prior to employing Choquet integral increases with the increasing number of attributes, n, for a MADM problem. Besides, the decision makers may not be able to consistently analyze and provide the information on the type of interaction shared by the attributes within each subset, in the process of determining the monotone measure weights especially when the number of attributes is large (Larbani, Huang, and Tzeng, 2011).

These shortcomings could restrict the decision makers from utilizing Choquet integral for real-world MADM problems.

1.2 Problem Statement

The identified gap or problem of this research can be recapitulated as follows: Although the classical AHP emerges as one of the broadly applied MAUT techniques for solving MADM problems, this technique is being disparaged due to two major shortcomings. Firstly, this technique is ineffectual in dealing with the aspect of uncertainty embedded in human's estimation. Secondly, it disregards the interaction between attributes during the aggregation phase.

The first issue can be elucidated as follows: Normally, in the second phase of MAUT techniques, some data or preference values from decision makers are needed to derive the local scores and attributes' weights. In reality, due to lack of knowledge, humans are usually uncertain or vague in expressing their preference (Chen et al., 2011). Therefore, it is insensible to force decision makers to precisely quantify or express their estimation or preference via crisp numbers. They actually opt to express their preference via linguistic terms or natural languages due to the factor of uncertainty or vagueness (Chou, 2007; Lee, Mogi, and Kim, 2009). Nevertheless, the classical MAUT models such as classical AHP are based on crisp numbers and not based on linguistic terms. As a result, these techniques do not exactly reflect actual human thinking pattern.

However, the application of fuzzy set theory in MAUT analysis has been found to be helpful in modeling the usual uncertainty that existed in the data offered by human (Shi et al., 2010). In fuzzy analysis, data can be provided in linguistic terms which are then quantified with fuzzy numbers. Unfortunately, performing fuzzy analysis requires higher computational effort from decision makers (Chen and Hwang, 1992, as cited in Kahraman, 2008) than the crisp MADM analysis where it would demand them to conduct some additional steps (i.e. fuzzification and defuzzification) and to offer some extra information during the analysis. This shortcoming restricts the application of fuzzy models into real MADM problems (Rao, 2007) and could be one of the reasons that compel the decision makers such as managers in an organization who are unversed in quantitative analysis to adhere on the classical MADM models.

Meanwhile, the second issue can be highlighted as follows: Generally, while implementing MAUT techniques such as classical AHP, decision makers tend to employ any of the additive aggregation operators which assumes that there is no interaction between the attributes (Bendjenna, Charre, and Zarour, 2012) (i.e. SWA operator) to aggregate the local scores. However in reality, most attributes portray inter-dependent or interactive characteristics and therefore, the aggregation should not be always carried out via conventional additive operators (Tzeng, Yang, Lin, and Chen, 2005).

However, it is proven that Choquet integral is capable to model the interaction between attributes during aggregation (Grabisch, 1996b; Marichal, 1999). Unfortunately, the process of estimating monotone measure weights prior to applying Choquet integral can turn into a complex process (Zhu, Chen, Lu, and

Zhang, 2009). The total number of monotone measure weights (2^n) which needs to be estimated prior to employing Choquet integral increases exponentially with increasing number of attributes, n. (Alavi, Jassbi, Serra, and Ribeiro, 2009). Besides, the decision makers may not be able to consistently analyze and offer the information on the type of interaction shared by the attributes within each subset in the process of estimating the monotone measure weights especially for a MADM problem which involves large set of attributes (Marichal and Roubens, 2000; Larbani, Huang, and Tzeng, 2011). These complications could limit the decision makers in utilizing the advantageous Choquet integral tool for real-world MADM problems.

With regards to abovementioned issues, it can be simplified that using fuzzy set and Choquet integral together in a MADM problem demands higher computational effort from decision maker where it would require higher number of computational steps and large amount of information from decision makers. As a result, this research discovers an opportunity or need to offer a MADM procedure which can minimize the number of computational steps and amount of information required from decision makers when simultaneously dealing with uncertainty in human's data and interaction among attributes. In other words, there is a necessity for the decision makers to have a simple and straightforward MADM procedure which concurrently captures the aspect of uncertainty in human's data and interaction among attributes. Figure 1.2 simplifies the gap identified through this research.

It is undeniable that several MADM models such as generalized Choquet fuzzy integral (GCFI), fuzzy analytical network process (FANP), and fuzzy partitioned hierarchy were developed with the intention to consider the formerly-mentioned two aspects. However, these models came with some shortcomings especially in the context of computational requirement. Table 1.5 summarizes the shortcomings of the three MADM models.

Table 1.5: Shortcomings of GCFI, FANP, and Fuzzy Partitioned Hierarchy

MADM models	**Shortcomings**
GCFI (Tsai and Lu, 2006)	a) The estimation of monotone measure weights (Demiral, Demiral, and Kahraman, 2010) and aggregation process of identifying global score of each alternative are complicated as the computation process involves fuzzy or interval values. Even, Hwang and Chen (1992) and Nijkamp, Rietveld, and Voogd (1990) affirmed that carrying out fuzzy numbers or interval values throughout computation would drag decision makers to a complicated situation. b) Requires some additional defuzzification steps to identify the ordinal ranking as the aggregated global scores exist in interval form. c) In context of data requirement, this approach needs three types of data from decision makers (importance of attributes, tolerance zone of expected local scores and local scores of each alternative)
FANP (Vinodh, Ramiya, and Gautham, 2011; Promentilla, Furuichi, Ishii, and Tanikawa, 2008)	a) As the elements (attributes and alternatives) of a MADM problem increase, the computation process of this model turns to be more complex since it will be involving larger number of pair-wise comparison matrices (Yurdakul, 2003). b) More data or judgments will be needed from decision makers with increasing number of elements as this approach requires $(n-1)/2$ judgments for n elements to compute a pair-wise comparison matrix (Hsu, Hung, and Tang, 2012).
Fuzzy partitioned hierarchy (Lin, Shiu, and Tzeng, 2011)	a) Two different clustering on attributes could be yielded after performing fuzzy factor analysis and so, twofold computation steps are required. b) During analysis, the fuzzy global scores of alternatives can be determined but not their ordinal ranking. Therefore additional defuzzification approach required. c) Uncertainty is omitted in measuring the performance of alternatives. It requires exact judgment from decision makers or expert in assigning local scores of alternatives. In other words, it should allow them to provide data in linguistic terms as usually they will be uncertain with their judgment.

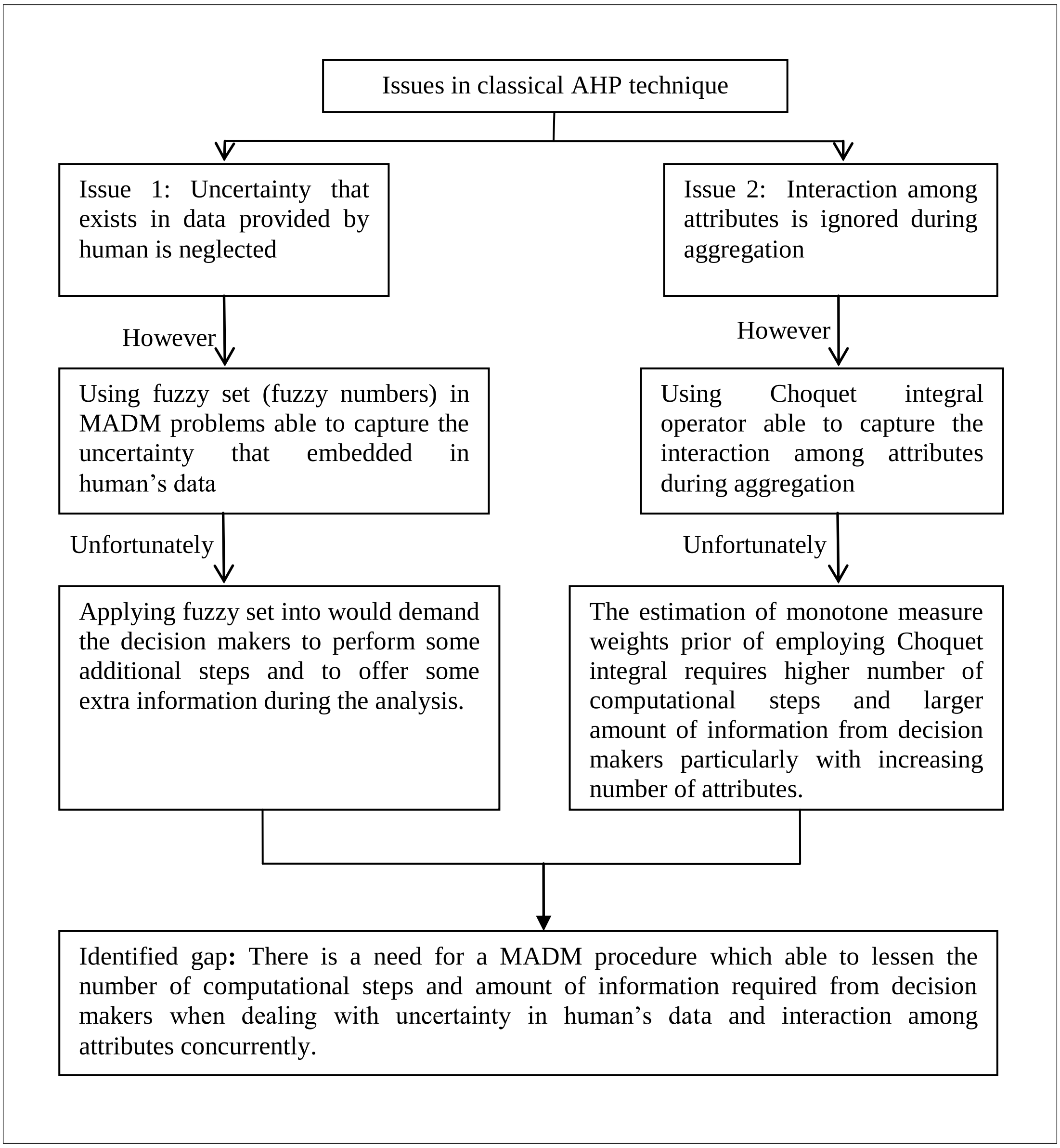


Figure 1.2: Problem Statement of the Research

In the process of discovering a resolution for the identified gap, several research questions are formulated as listed in the following section.

1.3 Research Questions

The overall quest of this study is depicted through the formulated primary and specific research questions are presented in Table 1.6.

Table 1.6: Research Questions

Items	Research questions
Primary research question	How to develop a MADM procedure which requires minimal number of computational steps and amount of information from decision makers when dealing with the aspect of uncertainty in human's data and interaction between attributes simultaneously?
First specific research question	What are the key elements or ideas in fuzzy set theory which are significant to the field of MADM?
Second specific research question 2	What are the pros and cons of the existing fuzzy AHP methods?
Third specific research question	What are the types of aggregation operators which can be used in solving MADM problems?
Fourth specific research question	What are approaches that have been proposed with the intention to lessen the amount of information and/or numbers of computational steps required from decision makers in the process of estimating monotone measure weights?
Fifth specific research question	How to illustrate the feasibility of the proposed procedure?

1.4 Objectives

Based on the identified research questions, the objectives of the study can be stated as follows.

1.4.1 Main Objective

The primary goal of this research is to propose a MADM procedure which requires minimal number of computational steps and amount of information from the decision makers when modeling the aspect of uncertainty in human's data and interaction between attributes simultaneously.

1.4.2 Specific Objectives

There are a few specific objectives which need to be accomplished in order to meet the main goal of this research. Firstly, this research aims to explore the crucial elements or concepts in fuzzy set theory which are applicable to the field of MADM. Secondly, this research targets to conduct a pros and cons analysis on the existing fuzzy AHP models.

Thirdly, this research intends to identify types of aggregation operators which are applicable in MADM problems. The fourth objective of this research is to identify the approaches that have been suggested in reducing the amount of information and numbers of computational steps required from decision makers in estimating monotone measure weights.

Finally, this research aims to demonstrate the feasibility of the proposed procedure by solving a real-world MADM problem.

1.5 Significance of the Research

It is projected that this research would render several positive implications to the field of MADM and its practitioners. The major contributions expected from this research can be simplified as follows. First of all, this research would offer a MADM procedure which could minimize the number of computational steps and amount of information required from the decision makers when dealing with uncertainty in human's data and interaction among attributes concurrently.

Secondly, this research could be an endeavor to inspire or encourage more decision makers (specifically the real managers from an organization who are lacking of exposure on quantitative analysis) to consider the aspect of uncertainty in human's judgment and interaction among attributes while resolving real-world MADM problems in order to assure more practical results. It was mentioned earlier that most of the decision makers are reluctant to deal with the former two aspects as they are usually dragged into a cumbersome or complicated computational requirement by doing so.

Thirdly, since the aggregation phase is being one of the primary focuses of this research, the thesis of this research would comprise a satisfactory appraisal on the characteristics and types of aggregation operators. Consequently, the thesis could be a good reference for decision makers in choosing an appropriate aggregation operator based on the problem's needs or for formulating novel operators.

Furthermore, it is hoped that this research would be helpful in stimulating some ideas or hints for practitioners of MADM to further or gradually reducing the number of computational steps and amount of information demanded from the decision makers when dealing with the aspect of uncertainty in human's judgment and interaction among attributes simultaneously.

Finally, via this research, a real-world MADM problem will be identified and solved by applying the proposed procedure in the process of proving that the proposed procedure is applicable in surmounting real-world MADM problems.

1.6 Scope of Research

The theoretical and geographical scopes of this research are as elucidated follows.

1.6.1 Theoretical Scope

As mentioned formerly, MCDM comprises of two branches namely MODM and MADM. This research limits its focus into field of MADM which concerns on decision making process involving a set of finite choices that described by a set of evaluation attributes.

However, there are various MADM techniques which can be classified into three families known as multi-attribute utility theory (MAUT) techniques, outranking techniques and some other multi-attribute decision making (OMADM) techniques. This research focuses on dealing with several issues that arise in MAUT techniques such as classical AHP.

Three fundamental phases in implementing any MAUT techniques are as follows. Identification of alternatives and relevant attributes (Phase 1), estimation of local scores of alternatives and attributes' weights (Phase 2), and aggregation phase (Phase 3). As the two current issues in MAUT techniques, issue of uncertainty in human's judgment and ignorance of interaction aspect among attributes commonly occur at Phase 2 and Phase 3 respectively then, the major concern of this research is diverted to explore these two phases in order to deal with the formerly mentioned issues. Figure 1.3 offers a clear understanding on how this research travels to its major focal points of research.

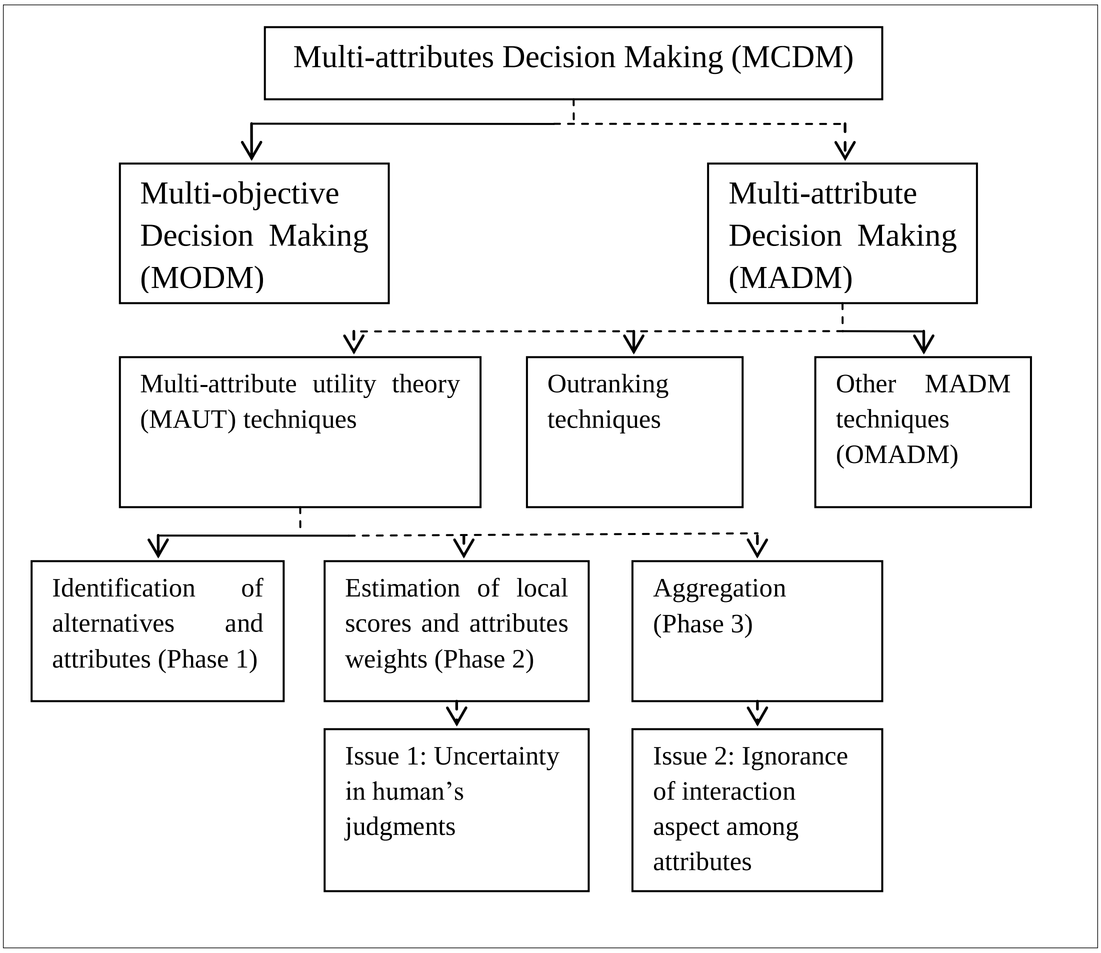


Figure 1.3: Scope of the Research

1.6.2 Geographical Scope

This research will identify and seek to find solution for a real-world MADM problem that exists within the state of Selangor, Malaysia in order to illustrate the feasibility of the proposed procedure.

1.7 Organization of the Thesis

Overall, this thesis comprises of 6 main chapters which are organized as follows. Chapter one introduces some fundamental concepts in MADM where the discussion is primarily concentrated on one of the well-accepted MAUT techniques namely classical AHP. By exploring conventional AHP, the gap or problem of the research is spotted and well-defined. Besides, this chapter identifies the objectives that need to be achieved in the process of solving the identified gap and reveals the significance of carrying out this research.

In chapter two, the literature review on the aspect of uncertainty in humans' data is presented. The chapter begins by defining the issue of uncertainty in MADM. Then, the appraisal concentrates on some key notions of fuzzy set theory and how they are being useful in modeling the usual uncertainty embedded in humans' data while conducting a MADM analysis. The chapter is ended by presenting a pros and cons analysis on the existing fuzzy AHP approaches.

Meanwhile, the aspect of interaction between attributes is examined in chapter three. The chapter kicks off by describing the aggregation phase in MADM and listing some of the essential properties expected from a good aggregation operator. Subsequently, the review is narrowed on Choquet integral and its associated monotone measure which can capture the interaction between attributes during aggregation. The following section of the chapter probes into the approaches which have been recommended so far in reducing the complexity of identifying monotone measure weights. The chapter ends with a summary on application of Choquet integral into real problems.

In chapter four, a new MADM procedure is proposed where this procedure is developed accordingly to ensure it able to reduce the number of computational steps and amount of information required from decision makers when dealing with aspect of uncertainty and interaction between attributes. The steps of implementing the proposed procedure are detailed either. At the end of chapter, a simple toy example is presented to offer better understanding on the usage proposed procedure.

In the following chapter, the image of three stores located at Pekan Sabak, Selangor, Malaysia from the perception of homemakers is assessed via proposed procedure to authenticate it's practicability in solving the real-world MADM problems.

Lastly, through chapter six, the contributions gained by accomplishing this research, limitations of the research, and the opportunities formed by this research for future studies are summarized.

1.8 Summary of Chapter One

This chapter was commenced by presenting a brief survey on the field of MADM which then focused into one of the widely applied MAUT techniques namely classical AHP. This research discovers its problem by exploring classical AHP.

It was found that classical AHP has two inabilities. Firstly, it fails to capture the usual uncertainty embedded in data provided by humans. Secondly, it neglects the interaction between attributes during aggregation. But, it was learnt that these two issues can be solved by utilizing the concept of fuzzy set theory and Choquet integral respectively. Unfortunately, by doing so, the decision makers are usually dragged into a tremendous or complicated computational requirement where higher number of computational steps and large amount of information would be required from decision makers. Therefore, this research believes that there is a need for a MADM procedure which requires minimal number of computational steps and amount of information from decision makers when dealing with these two issues simultaneously.

As a result, this research has set its primary goal to develop a MADM procedure which reduces the number of computational steps and amount of information demanded from the decision makers when modeling the aspect of uncertainty in human's data and interaction between attributes simultaneously. In achieving the main goal, several specific objectives need to be accomplished as identified in this chapter. By accomplishing all its objectives, it is believed that this research could generate some significant contributions to the field of MADM and its practitioners.

CHAPTER TWO
ON THE ASPECT OF UNCERTAINTY IN HUMAN'S DATA

2.1 Introduction

There are two major issues hooked with the conventional AHP as clarified in chapter one. Firstly, the conventional MAUT is incapable to cope with uncertainty in data offered by human. Secondly, it ignores the aspect of interdependencies among attributes during aggregation phase. This chapter is devoted to compile some significant information pertaining to the former issue by reviewing past literature that would be helpful in constructing the proposed procedure of the research.

This chapter begins with the discussion on the uncertainty phenomena in decision making process. Then, the origin of fuzzy set theory and its applicability in capturing the uncertainty embedded in data provided by human are probed. The crucial elements of fuzzy set theory such as linguistic variables, linguistic terms, fuzzy numbers, fuzzification, and defuzzification procedures that are applicable in MADM environment are detailed as well. In the subsequent section, a review on application of fuzzy set theory into MADM models is offered where a major appraisal will focus on fuzzy AHPs. A pros and cons analysis on several fuzzy AHP approaches which are frequently underlined in past studies is conducted at end of this chapter.

2.2 Defining Uncertainty in MADM

Most of the MADM problems engaged with issue of uncertainty. Uncertainty and vagueness always exist in the human decision making process (Kahraman, 2008) due to the presence of either incomplete information or abundance of information, conflicting evidence, ambiguous information or subjective information (Zimmermann, 2000). Ribeiro (1996) classified the uncertainty phenomena in MADM into three categories namely 'incompleteness', 'fuzziness' and 'illusion of validity' (Tversky and Kahneman, 1990). This research focuses on 'fuzziness' type of uncertainty where according to him, 'fuzziness' occurs when there are difficulties in assigning precise assessment for qualitative features or attributes such as how to assess 'comfort' which is an attribute for buying a car.

As stated above, it is common for people to be uncertain about their preference or judgment (Kangas, Kangas, and Kurttila, 2008) since they rarely have ample level of information about the problem. For instance, they may not know exactly how much they prefer a particular alternative over another with respect to a criterion and there may be uncertainty while comparing the relative importance among attributes. In addition, it is a cumbersome task for them to express their exact preference via crisp numbers or scales (Lee, Mogi, Kim, and Gim, 2008). Hence, they may need to rely on experts' knowledge in solving the MADM problem at hand (Bozbura, Beskese, and Kahraman, 2007).

Ribeiro (1996) highlighted that one way to reflect such uncertainty is to present the uncertain preference or judgment using linguistic terms. Human actually tend to offer the information in natural languages or linguistic scales such as 'poor', 'fair', and 'good' performance rather than using exact numbers as they are usually uncertain on their preference. An advantageous MADM model should reflect the actual human thinking style and therefore, it must cope with imprecise, vague or uncertain information such as ‘poor’, ‘fair’, and ‘good’ performance (Ribeiro, 1996) in order to generate more trustworthy result. One commonly used approach for dealing with uncertainty embedded in linguistic terms is by employing the fuzzy set theory (Lai, Chang, and Chou, 2008; Zimmermann, 2001).

2.3 Fuzzy Set Theory

Fuzzy set theory was proposed by Zadeh (1965) in order to deal with vagueness in human thought. Since its introduction by Zadeh, the theory has been widely applied to mathematically reflect the ambiguities in human’s judgments and effectively resolve the uncertainties in the available information in an ill-defined MADM environment (Chu and Lin, 2009). The theory has been successfully applied to problems in engineering, business, health sciences, and the natural sciences (Kahraman, Gulbay, and Kabak, 2006).

According to Ertugrul and Karakasoglu (2008), fuzzy set theory is a generalization of crisp or classical set theory. In crisp set theory, the membership of elements in a set is assessed based on binary terms (1 = yes and 0 = no). It is used to determine either an element belongs or does not belong to a specific set (Liou, Yen, and Tzeng, 2007). In other words, it only permits only either full membership or non-membership.

On the contrary, fuzzy sets allow partial membership where an element may partially belong to a fuzzy set (Ertugrul and Karakasoglu, 2007). This is described with the aid of a membership function, with the range encompassing the interval [0, 1], operating on the domain of all possible values. In a nutshell, the fuzzy sets are defined by the membership functions. Mathematically, fuzzy sets can be defined as follows (Onut et al., 2009; Wibowo, 2011).

The fuzzy sets represent the grade of any element x of X that have the partial membership to A. The degree to which an element, x belongs to a set, A is characterized by $\mu_A(x)$ where it usually ranges from 0 to 1. If an element x really belongs to A then, $\mu_A(x)$= 1 and if it is clearly does not belong to A then, $\mu_A(x)$= 0. The higher is the membership value, $\mu_A(x)$, the greater is the belongingness of an element x to the set A. Ertugrul and Tus (2007) affirmed that fuzzy sets theory which provides a more extensive frame than classic sets theory, has been contributing in reflecting real world scenario.

The following example, altered from the study conducted by Kangas, Kangas, and Kurttila (2008), could enlighten the distinctions between crisp set theory and fuzzy set theory. For instance, a statement 'an individual is young' can be more or less true. Therefore, fuzzy sets would be needed where a membership function to define set 'young' can be illustrated as in Figure 2.1.

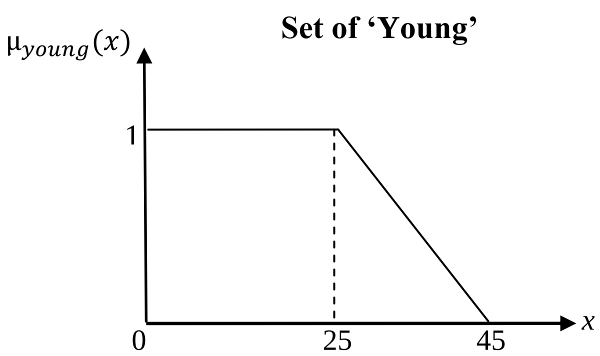


Figure 2.1: Membership Function for Set 'Young' (Example)

Based on Figure 2.1, the degree of membership, $\mu(0 \leq x \leq 25) = 1$ implies that an individual clearly belongs to set 'young' when he or she aged between 0 and 25. The belongingness of an individual to set 'young' decreases when x or age increases, i.e. larger than 25. In addition, an individual clearly does not belong to set 'young' when he or she reached 45, $\mu(x = 45) = 0$.

If a crisp definition is available, such as 'an individual is young if the age of the individual does not exceed 12' then, crisp set is applicable where the membership function would only have 0 (No) and 1 (Yes) values.

Fuzzy sets are being applied into MADM atmosphere as most of the MADM problems involve linguistic variables. Further details on linguistic variables are presented in the following section.

2.3.1 Linguistic Variables

A linguistic variable is a variable whose values are expressed in linguistic terms or words in a natural or artificial language (Zadeh, 1975). The linguistic variable is a very helpful concept for dealing with situations which are too complex or not so well-defined to be sensibly described using exact or crisp numbers. Several examples for better illustration on the concept of linguistic variables are as follows: 'Age' is a linguistic variable if its values are expressed or defined linguistically such as young, mature, old and so on, rather than using crisp numbers$(0,1,2,\cdots,100)$ (Bellman and Zadeh, 1977).

With regards to MADM setting, 'relative importance between two attributes' could be a linguistic variable whose values can be expressed in natural languages as equally important, moderately important, important, strongly important, extremely important, and so on. These linguistic terms can be further represented by specific fuzzy numbers (Chu and Lin, 2009). Fuzzy number is an extension of basic number which usually comprises of lower, upper and probable values that best represents a linguistic preference or judgement. It used to mathematically capture or represent the usual uncertainty embedded in linguistic preferences. By quantifying the preference in linguistic terms into fuzzy numbers, the decision analysis can be carried out quantitatively without losing the aspect of uncertainty. More details on fuzzy numbers are offered in subsequent sections.

2.3.2 Fuzzy Numbers

Fuzzy numbers are fuzzy sets which are both convex and normal (Chen and Niou, 2011). A fuzzy number, $\tilde{A}$ is a convex fuzzy set characterized by a given interval of real numbers, each with range of membership between 0 and 1. Its membership function is piecewise continuous and satisfies the following conditions (Hadi-Vencheh and Mokhtarian, 2011):

a) $\mu_A(x)$= 0 outside of some interval [l, u],

b) $\mu_A(x)$ is non-decreasing (monotonic increasing) on [l, m_1] and non-increasing (monotonic decreasing) on [m_2, u],

c) $\mu_A(x)$= 1 for each x ∈ [m_1, m_2].

where $l \leq m_1 \leq m_2 \leq u$ are real numbers in the real line $\mathbb{R}$. To describe further, l represents lower value, m_1 and m_2 denote middle or most probable values and u signifies upper value of the membership function that defines set $\tilde{A}$.

2.3.3 Types of Fuzzy Numbers

Among the various types of fuzzy numbers, triangular and trapezoidal fuzzy numbers are the most commonly used fuzzy numbers (Hadi-Vencheh and Mokhtarian, 2011). A triangular fuzzy number (TFN), $\widetilde{A_1}$ can be defined by a triplet (l, m_1, u) where $l \leq m_1 \leq u$ as portrayed in Figure 2.2. The membership function of TFN (2.1) is characterised as follows (Mansur, 1995):

$$\mu_{\widetilde{A_1}}(x) \begin{cases} 0 & x < l, \\ \dfrac{x - l}{m_1 - l} & l \leq x \leq m_1, \\ \dfrac{u - x}{u - m_1} & m_1 \leq x \leq l, \\ 0 & x > u \end{cases} \tag{2.1}$$

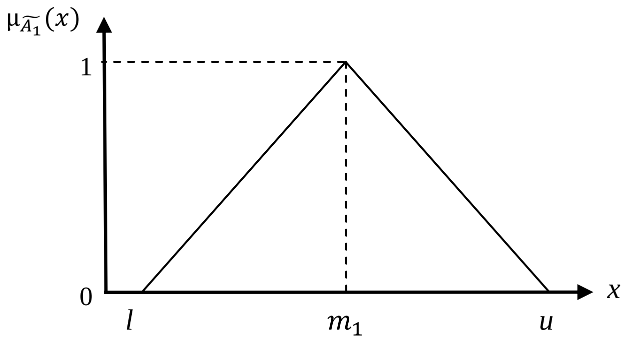


Figure 2.2: Triangular Fuzzy Number, $\widetilde{A_1}$ *= (l,* m_1*, u) (Liao, 2009)*

According to Lu and Zhang (2008), in a TFN such as $\widetilde{A_1} = (l, m_1, u)$, m_1 denotes the maximal degree of $\mu_{\widetilde{A_1}}(x)$ and it is the most possible or optimum value of the set, whereas l and u represent the lower and upper value of the same set. They added that the narrower is the interval [l, u], the lower is the fuzziness of the set.

Meanwhile, a trapezoidal fuzzy number (TrFN) $\widetilde{A_2}$ can be denoted by a quadruplet (l, m_1, m_2, u) where $l \leq m_1 \leq m_2 \leq u$ as illustrated in Figure 2.3. Mathematically, the membership function of TrFN (2.2) is defined as follows (Mansur, 1995):

$$\mu_{\widetilde{A_2}}(x) \begin{cases} 0 & x < l, \\ \dfrac{x-l}{m_1-l} & l \leq x \leq m_1, \\ 1 & m_1 \leq x \leq m_2, \\ \dfrac{u-x}{u-m_1} & m_1 \leq x \leq u, \\ 0 & x > u \end{cases} \tag{2.2}$$

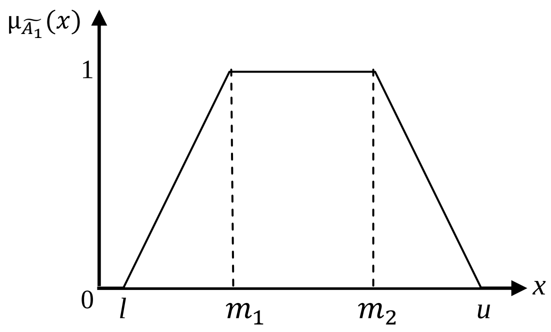


Figure 2.3: Trapezoidal fuzzy number, $\widetilde{A_1} = (l, m_1, m_2, u)$ *(Lee, 2005)*

It is apparent that TFNs are special cases of trapezoidal fuzzy numbers when $m_1 = m_2$. TFN is more advantageous over other types of fuzzy numbers in decision making environment (Moon and Kang, 2001) due to the following three reasons (Ramik, 2009). Firstly, the membership function of TFN is piecewise linear and comparatively simple. Secondly, arithmetic operations such as addition and subtraction can be performed easily in comparison to other types of fuzzy numbers.

Thirdly, crisp (or non-fuzzy) numbers which are the most practical values can be represented as triangular ones.

Figure 2.4 shows an illustration on how fuzzy set theory can be utilized to evaluate the age of people (Bellman and Zadeh, 1977) to offer clear perception on the concept of linguistic variable, linguistic terms and fuzzy numbers. Based on Figure 2.4, 'age' is a linguistic variable which can be defined or assessed using 3 linguistic terms namely 'young', 'mature' and 'old'. Each of these linguistic terms is represented or described by using TrFN consisting lower age, most optimum or probable ages and upper age of each linguistic term. For instance, the TrFN for 'mature' is (25, 45, 55, 75).

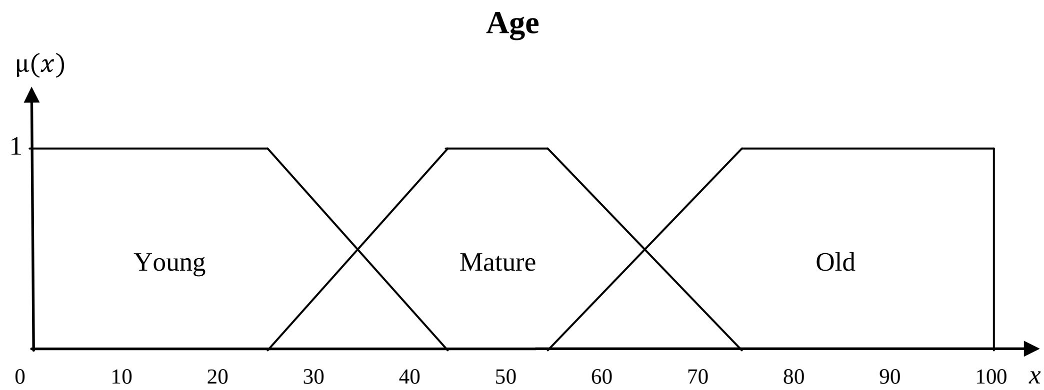


Figure 2.4: Fuzzy Numbers Used to Define Age

2.3.4 Arithmetic Operations on Triangular Fuzzy Numbers

This section only reviews the arithmetic operations involving triangular fuzzy numbers which are considered to be necessary for this research. Let $\widetilde{A_1} = (l_1, m_1, u_1)$ and $\widetilde{A_2} = (l_2, m_2, u_2)$ be two positive triangular fuzzy numbers. Then, the basic fuzzy arithmetic operations on these fuzzy numbers can be defined as follows (Kaufmann and Gupta, 1991; Hanss, 2005):

a) Inverse: $(\widetilde{A_1})^{-1} = (\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1})$

b) Addition: $\widetilde{A_1} + \widetilde{A_2} = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$

c) Subtraction: $\widetilde{A_1} - \widetilde{A_2} = (l_1 - u_2, m_1 - m_2, u_1 - l_2)$

d) Scalar multiplication: $k \times \widetilde{A_1} = (kl_1, km_1, ku_1)$ for $k > 0$

e) Multiplication: $\widetilde{A_1} \times \widetilde{A_2} = (l_1 l_2, m_1 m_2, u_1 u_2)$

f) Division: $\widetilde{A_1} \div \widetilde{A_2} = (\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2})$

It has to be noted here that the above arithmetic operations involving trapezoidal fuzzy numbers are conducted in a similar manner.

2.3.5 Fuzzification

Fuzzification refers to a process of converting each linguistic term used for evaluation into its corresponding fuzzy number. Through several studies, some fuzzification approaches are identified. The most common practise or approach for this conversion is for the decision makers to identify the lower, probable, and upper values of a fuzzy number corresponding to each linguistic preference based on their background knowledge, experience or with the aid of experts, and finally, construct the fuzzy conversion scale. This approach has been applied in many MADM studies such as in Royes and Bastos (2001) and Hung, Li, and Chiang (2007).

On the other hand, Chen and Hwang (1992) have proposed 8 conversion scales for fuzzification purpose which are applicable in MADM. In the proposed conversion scales both the score, x and membership function $\mu(x)$ are in the range from 0 to 1. All the recommended scales are portrayed in Figure 2.5.

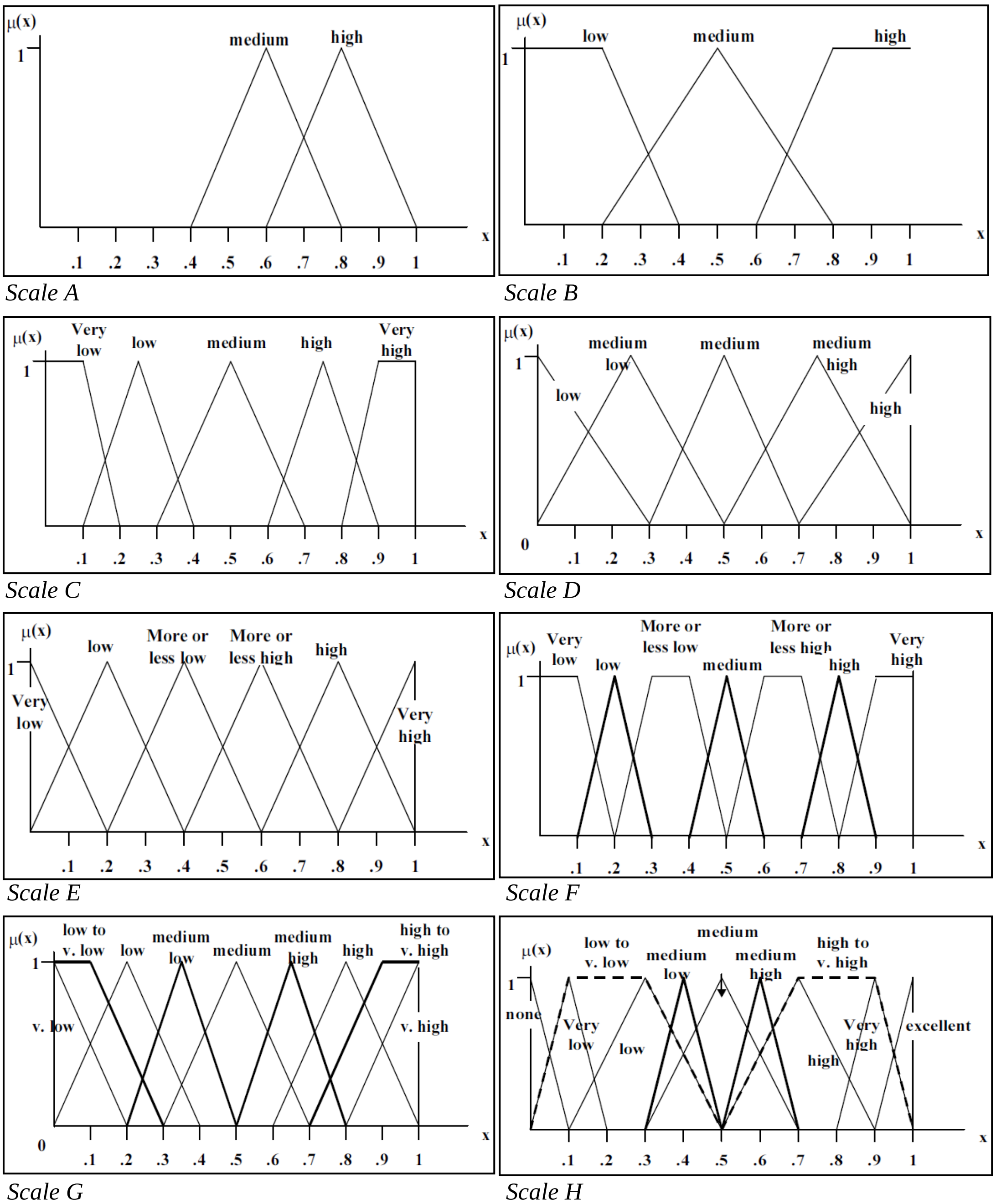


Figure 2.5: Eight Conversion Scales Proposed by Chen and Hwang (1992)

The selection and usage of each conversion scale rely on the total number of linguistic terms used to assess the linguistic variable. The principle of this approach is simply to identify a scale which contains all the linguistic terms used by decision makers and then, transform these linguistic terms into corresponding fuzzy numbers as determined in the chosen scale (Cheng, 2000). If the provided linguistic terms exist in more than one scale, the simplest one should be chosen. An example illustrated in the study conducted by Aldian and Taylor (2003) could offer better enlightenment on this approach. Let's assume five linguistic terms, very low, low, medium, high and very high, are used to represent the score of an alternative with respect to a criterion. Then, scale C, F, G, and H would be the nominee scales. However, scale C is selected as it contains all the terms besides being the simplest among the others.

As carrying fuzzy numbers could demand extra computational effort from decision makers, Chen and Hwang (1992) suggested that the converted fuzzy numbers based on any of these scales, need to be directly transformed into corresponding crisp values before applying them into any of the basic MADM models.

Meanwhile, in a MADM analysis conducted by Zhu (2010), the fuzzy conversion scale was constructed by identifying the triangular fuzzy number (TFN) that corresponds to each linguistic term by simply applying equation (2.3).

$$\widetilde{A_i} = (l_i, m_i, u_i) = \left(max\left\{\frac{i-1}{T-1}, 0\right\}, \frac{i}{T-1}, min\left\{\frac{i+1}{T-1}, 1\right\} \right) \tag{2.3}$$

where $\widetilde{A_i}$ represents triangular fuzzy number consisting l_i (lower value), m_i (most probable value), u_i (upper value) that corresponds to each linguistic term, s_i and T denotes the ranking of final linguistic preference.

For instance, let's assume total of seven linguistic terms are fixed to evaluate the performance of a set of alternatives with respect to a criterion. The set of linguistic terms, s_i comprised of $s_0 =$ absolutely poor (AP), $s_1 =$ very poor (VP), $s_2 =$ poor (P), $s_3 =$fair (F), $s_4 =$ good (G),$s_5 =$ very good (VG), and $s_6 =$ absolutely good (AG) and so, $T = 7$. Then, the triangular fuzzy number corresponds to each linguistic term can be obtained by applying equation (2.3) and the fuzzy conversion scale as shown in Figure 2.6 can be constructed.

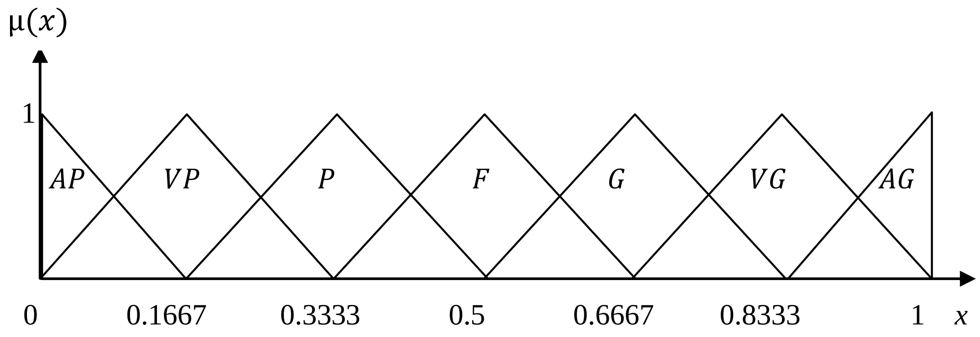


Figure 2.6: 7- point Linguistic Scale based on Zhu's Fuzzification Approach

Zhu's approach which requires simple execution is recommended for the decision makers who are unable to clearly define the fuzzy number corresponding to each linguistic term, due to lack of information or experience. They can merely convert each of the linguistic terms into triangular fuzzy number by employing equation (2.3). The conversion scale developed from this approach is homogenous where it only consists of triangular fuzzy numbers, and arithmetic operation can be carried out without any complication.

On the other hand, while utilizing AHP method in fuzzy environment, Saaty's fuzzy AHP scale as presented in Figure 2.7 which is an extension of Saaty's crisp AHP scale, is commonly referred to convert linguistic terms in pair-wise comparison matrix into corresponding fuzzy numbers, as applied in studies conducted by Duran and Aguilo (2008), and Kwong and Bai (2003).

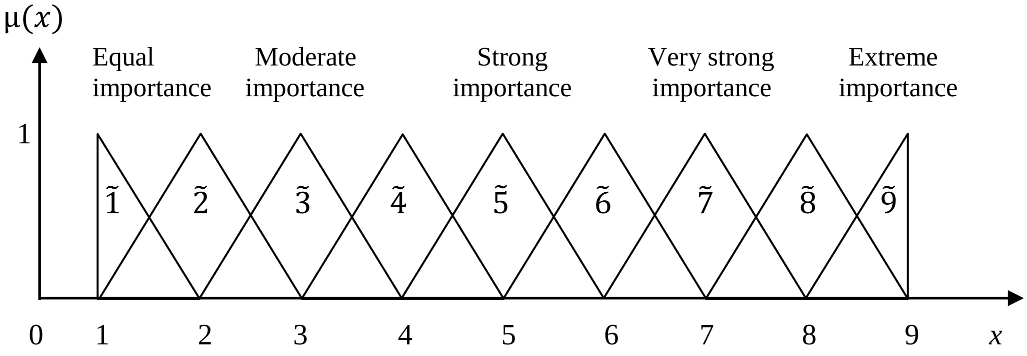


Figure 2.7: Saaty's Fuzzy AHP Conversion Scale

Table 2.1: Saaty's Fuzzy AHP Conversion Scale

Linguistic preference	**Corresponding TFN**	**Explanation**	**Reciprocal of TFN (TFN^{-1})**
Equal importance	$\tilde{1} = (1,1,2)$	Two elements contribute equally	$\tilde{1}^{-1} = (\frac{1}{2}, 1,1)$
Moderate importance	$\tilde{3} = (2,3,5)$	One element is slightly favoured over another	$\tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$
Strong importance	$\tilde{5} = (4,5,6)$	One element is strongly favoured over another	$\tilde{5}^{-1} = (\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$
Very strong importance	$\tilde{7} = (6,7,8)$	One element is very strongly favoured over another	$\tilde{7}^{-1} = (\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$
Extreme importance	$\tilde{9} = (8,9,9)$	One element is most favoured over another	$\tilde{9}^{-1} = (\frac{1}{9}, \frac{1}{9}, \frac{1}{8})$
The intermediate values	$\tilde{2} = (1,2,3)$, $\tilde{4} = (3,4,5)$, $\tilde{6} = (5,6,7)$, $\tilde{8} = (7,8,9)$	Used to compromise between two judgments	$\tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1)$, $\tilde{4}^{-1} = (\frac{1}{5}, \frac{1}{4}, \frac{1}{3})$, $\tilde{6}^{-1} = (\frac{1}{7}, \frac{1}{6}, \frac{1}{5})$, $\tilde{8}^{-1} = (\frac{1}{9}, \frac{1}{8}, \frac{1}{7})$

**Note, when element i is compared to j then, the preference is assigned with one of the TFN scale. Meanwhile, when element j is compared to i then, the preference is assigned with the corresponding reciprocal.*

2.3.6 Defuzzification

Deng and Chan (2011) considered defuzzification as a crucial step in fuzzy MADM. Defuzzification is actually a process of converting the fuzzy values into crisp values (Tseng, 2011; Kahraman and Cebi, 2009) which would be helpful in determining the ordinal ranking of MADM elements such as attributes or alternatives (Opricovic and Tzeng, 2003).

However, it has to be mentioned here that a fuzzy set cannot be exactly represented by a singleton and thus, the defuzzification can only be undertaken with the loss of information (Harris, 2006) but different defuzzification techniques extract different level of information (Deng and Chan, 2011). If a technique is capable to minimize this loss then, it tends to yield a more accurate or precise result (Luukka, 2010).

Some of the common defuzzification techniques are mean of maximum (MOM), center of area (COA), last of maximum (LOM) and α- cut method (Wang, Lu, and Chen, 2010). There is no systematic way for selecting a defuzzification technique (Lee, 1990) but this study puts its major attention on COA technique due to the following reasons:

a) This technique has been frequently applied in many MADM problems (Chu and Velásquez, 2009; Jeng, 2012; Kabak and Burmaoğlu, 2013; Yang, Chiu, Tzeng, and Yeh, 2008) due to its simple and practical calculation requirement especially in defuzzyfying TFNs (Chen and Tzeng, 2004) and also it does not require any additional information from decision makers such as the preferred α value (Chen, Tzeng, and Ding, 2008; Hsieh, Lu, and Tzeng, 2004; Opricovic and Tzeng, 2003). Therefore, this technique should be suitable and helpful in developing the intended MADM procedure as specified through the primary objective of the study.

b) Besides, in many cases, this technique yields more accurate result than other well-known techniques such as MOM and LOM, as reported in (Thammano, 1999 and Nurcahyo, Shamsuddin, Alias, and Sap, 2003).

The defuzzified value, A_i of a triangular fuzzy number (TFN), $\tilde{A}_i = (l_i, m_i, u_i)$ can be computed using COA equation (2.4) (Chou and Cheng, 2012).

$$\boldsymbol{A_i = l_i + [(u_i - l_i) + (m_i - l_i)]/3} \tag{2.4}$$

where l_i, m_i, and u_i represent lower, middle, and upper value respectively.

2.4 Fuzzy MADM Models

One of the domains where fuzzy set theory has made a noteworthy contribution is in the field of MADM (Kahraman, 2008). A fuzzy MADM model is used to assess alternatives where the local scores of alternatives and importance of attributes can be described in linguistic terms which will be later quantified into fuzzy numbers, in order to mathematically capture the uncertainty embedded human’s perception (Zhang, Ma, and Xu, 2005). Fuzzy TOPSIS (Aiello, Enea, Galante, and Scalia, 2009), fuzzy outranking (Aouam, Chang, and Lee, 2003), and fuzzy AHP are examples of MADM models which were formulated by using the idea of fuzzy set theory. The following section is devoted to investigate the fuzzy AHP approaches which earn widespread applications in MADM (Zhu, Shang, and Yang, 2012).

2.4.1 Application of Fuzzy Sets in AHP

Fuzziness or uncertainty issue in conventional AHP arises during pair-wise comparison process. In classical AHP, the decision makers need to compare the relative importance between attributes and between alternatives with respect to each attribute based on a nine-point AHP scale where each exact value represents a linguistic preference such as equally, slightly, strongly, very strongly or extremely preferred (Jiang, Feng, and Shi, 2009) as presented in Table 2.1. The crisp AHP scale is usually favoured by decision makers due its simplicity and ease of use feature (Kwong and Bai, 2003).

However, it is not rational to replace the linguistic preferences or data expressed by decision makers with crisp values as these linguistic preferences usually present some degree of uncertainty (Jiang, Feng, Feng, and Shi, 2010; Kwong and Bai, 2002). Indeed, it is more reasonable to represent them with fuzzy numbers which are capable to mathematically model the usual uncertainty embedded in the provided linguistic preferences.

As a result, many fuzzy AHP approaches were proposed by various authors where, the pair-wise comparison process in those methods is carried out based on linguistic terms scales that are associated with fuzzy numbers, with the intention of generating more reliable solutions for MADM problems.

2.4.2 Types of Fuzzy AHP Approaches

The available fuzzy AHP approaches can be classified into two groups, Group I and Group II (Zhu, Shang, and Yang, 2012). The former group comprises approaches derive a set of fuzzy priorities of elements from fuzzy pair-wise comparison matrices. Meanwhile, the latter group of approaches derive crisp priorities from fuzzy pair-wise comparison matrices. Note, in this section, priorities refer to local scores of alternatives and weights of attributes.

Firstly, the four fuzzy AHP approaches which belong to Group I are analysed by mainly focusing on their pros and cons as follows. One the of earliest fuzzy AHP approaches was proposed by Van Laarhoven and Pedrycs (1983). It is a direct extension of Saaty's AHP method which uses triangular fuzzy numbers for fuzzification of pair-wise comparison matrices and applies logarithmic least square method to derive fuzzy priorities from the fuzzy comparison matrices (Jaskowski, Biruk, and Bucon, 2010). One of the advantages of this approach is it able to model the opinions of multiple decision makers in the reciprocal matrix (Buyukozkan, Kahraman, and Ruan, 2004).

However, it comes with some disadvantages too. For example, the systems of TFNs' linear equations are linear dependent and do not always have a unique solution (Chiang and Tzeng, 2009). Besides, it demands high computational process even for small problem and only allows the usage of triangular fuzzy numbers (Buyukozkan et al., 2004). In addition, Wang, Elhag and Hua (2006) have criticized this approach for its incorrectness in the normalization of fuzzy priorities, infeasibility in generating the priorities, uncertainty of fuzzy priorities for incomplete fuzzy comparison matrices, and unrealistic global fuzzy scores.

In 1985, Buckley proposed another fuzzy AHP approach namely, geometric mean method which determines fuzzy priorities from pair-wise comparisons described by trapezoidal fuzzy numbers (as cited in Kahraman, Cebeci, and Ruan, 2004). Tzeng and Huang (2011) pointed out that even though the geometric mean method simplifies the application of AHP in fuzzy atmosphere, but the main shortcoming of this method is the problem of the irrational fuzzy interval as it does not consider the condition, such that the sum of the priorities equals 1. Meanwhile, from the study conducted by Buyukozkan et al. (2004), it is found that although this method assures a unique solution to the reciprocal comparison matrix yet the computational requirement is still tremendous.

Besides, Boender, de Grann and Lootsma (1989) has amended Laarhoven's fuzzy AHP with the intention to include a more robust approach for normalization of priorities. However, Wang et al. (2006) proved that the normalization method in Boender's approach is inappropriate. Besides, although it has the ability to capture the opinions of multiple decision makers, it still requires complex computation steps.

Therefore, Wang et al. (2006) proposed an enhanced version of Laarhoven's fuzzy AHP namely modified fuzzy logarithmic least squares method (MF-LLSM) which derives fuzzy priorities. MF-LLSM is designed as a constrained nonlinear optimization model and can directly derive normalized triangular fuzzy priorities for both complete and incomplete triangular fuzzy comparison matrices. However, the priorities obtained by MF-LLSM can change and even lead to rank reversal. Furthermore, it involves arduous numerical computations (Zhu, 2012). Besides, this method is only applicable with triangular fuzzy numbers.

It is apparent that additional defuzzification process needs to be carried out in the abovementioned four fuzzy AHP approaches if the decision makers insist to discover the ordinal ranking of attributes and alternatives involved in a MADM problem.

Now, the two fuzzy AHP approaches that belong to group II are reviewed in the same contexts. Chang (1996) introduced a new approach for dealing with fuzzy AHP, with the use of pair-wise comparison described by triangular fuzzy numbers and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. It derives crisp priorities from the fuzzy pair-wise comparison matrices. The extent analysis method perhaps has been frequently utilized in many applications due to its computational simplicity (Wang, Luo, and Hua, 2008). However, according to Zhu et al. (2012), fuzzy extent method came with several issues such as inappropriate use of normalization, inappropriate use of arithmetic mean to synthesize group's judgments, the robustness of the global scores derived by the extent analysis is weak, sometimes derives zero priority (known as zero priority dilemma), could lead to information loss and wrong rank and does not always derive reasonable priorities. Besides, this method is only valid for triangular fuzzy numbers (Tiryaki and Ahlatcioglu, 2009).

On the other hand, Mikhailov (2003) proposed a non-linear fuzzy preference programming method where by simply constructing and solving the recommended non-linear optimization model, the inconsistency value and crisp priorities of fuzzy pair-wise comparison matrices can be derived concurrently. Besides, due to the non-linearity of the Saaty $\tilde{1}$–$\tilde{9}$ scale in the region of values between $\tilde{9}^{-1}$ and $\tilde{1}^{-1}$, this method does not require the decision makers to construct fuzzy reciprocal matrices which could lead to some issues such as rank reversal. Although the method generates appropriate priorities, it is unable to show these priorities in the form of fuzzy numbers that could offer some information for decision makers in understanding the variant degree of the uncertainty (Tzeng and Huang, 2011).

Table 2.2 gives the summary of the appraised fuzzy AHP models and their corresponding advantages and disadvantages.

Table 2.2: Analysis on Fuzzy AHP Approaches

Group	Source	Advantages/ Disadvantages
I	Van Laarhoven and Pedrycs (1983)	**(A)**Able to model the opinions of multiple decision-makers (Buyukozkan et al., 2004) **(D)** Tremendous computational requirement even for small problem (Buyukozkan et al., 2004) **(D)** There is no unique solution for the linear equation (Chiang and Tzeng, 2009) **(D)** Inappropriate normalization method (Boender et al., 1989) **(D)** Infeasibility in generating the fuzzy priorities (Wang et al., 2006) **(D)** Uncertainty of fuzzy priorities for incomplete fuzzy comparison matrices(Wang et al., 2006) **(D)** Unreality of global fuzzy scores (Wang et al., 2006). **(D)** Additional defuzzification process needed for understanding the ordinal ranking of attributes or alternatives **(D)** Only applicable with triangular fuzzy numbers (Buyukozkan et al., 2004)
	Buckley (1985)	**(A)** Easy to extend the AHP into fuzzy case (Tzeng and Huang, 2011) **(A)** Assures a unique solution to the reciprocal comparison matrix (Buyukozkan et al., 2004) **(D)** Tremendous computational requirement (Buyukozkan et al., 2004) **(D)**Ignores the condition, such that the sum of the priorities equals 1 (Tzeng and Huang, 2011) **(D)** Additional defuzzification process needed for determining the ordinal ranking of attributes or alternatives
	Boender et al. (1989)	**(A)** Able to model the opinions of multiple decision-makers (Buyukozkan et al., 2004) **(D)**Tremendous computational requirement(Buyukozkan et al., 2004) **(D)**Inappropriate normalization method (Wang et al., 2006) **(D)**Defuzzification process is required to identify the ranking of attributes or alternatives **(D)**Only applicable with triangular fuzzy numbers (Boender et al., 1989)
	Wang et al. (2006)	**(A)**Can directly derive normalized triangular fuzzy priorities for both complete and incomplete triangular fuzzy comparison matrices (Wang et al. 2006) **(D)**High computational requirement (Zhu, 2012) **(D)** Priorities could change and lead to rank reversal issue(Zhu, 2012) **(D)**Additional defuzzification process needed for understanding the ordinal ranking of attributes or alternatives **(D)**Only applicable with triangular fuzzy numbers (Wang et al., 2006)
II	Chang (1996)	**(A)**Low computational requirement (Wang et al., 2008) **(A)**No need any additional defuzzification process needed to identify the ordinal ranking of attributes or alternatives **(D)**Inappropriate normalization method and use of arithmetic mean to synthesize group's judgments (Zhu et al., 2012) **(D)** Poor robustness (Zhu et al., 2012) **(D)**Zero priority dilemma(Zhu et al., 2012) **(D)**Could lead to information loss and wrong rank (Zhu et al., 2012) **(D)** Derive unreasonable priorities (Zhu et al., 2012). **(D)**Only applicable with triangular fuzzy numbers (Tiryaki and Ahlatcioglu, 2009)
	Mikhailov (2003)	**(A)**Low computational requirement **(A)**Derive crisp priorities and consistently value of a fuzzy pair-wise matrix simultaneously by simply solving a non-linear optimization model (Tzeng and Huang, 2011) **(A)**Avoid using reciprocal judgements which can cause rank reversal issue (Mikhailov, 2003) **(A)**No need any additional defuzzification process needed to identify the ordinal ranking of attributes or alternatives **(D)** Unable to show priorities in fuzzy numbers which could offer some information for decision makers in understanding the variant degree of the uncertainty (Tzeng and Huang, 2011)

From the analysis of the types of fuzzy AHP models, Mikhailov's fuzzy AHP is recommendable for decision makers relatively to other methods as the computational requirement for this approach is low where the crisp priorities and consistency values of fuzzy pair-wise comparison matrices can be derived simultaneously by simply constructing and solving the proposed non-linear optimization model. Besides, the drawback associated with this approach can be tolerated as this drawback would not render to unreliable priorities in any senses and thus, would not lead to inaccurate ranking on alternatives. Meanwhile, the drawbacks integrated with other approaches would lead to identification of infeasible priorities which could lead to some undesirable consequences in MADM such as wrong selection, ranking, and classification of alternatives.

2.5 Summary of Chapter Two

Some crucial findings from this chapter which would be helpful in achieving the primary goal of this research can be recapped as follows.

It was learnt that people usually uncertain or vague in expressing their preference or judgments as they may not have satisfactory level of information about the existing problem. As a result, it is burdensome for them to express their exact or precise preference based on crisp numbers or scales. In practice, people tend to express their preference via natural languages or linguistic terms due to uncertainty. Unfortunately, most of the MADM tools are based on numbers and not based on linguistic terms. Therefore, fuzzy sets are usually applied into MADM analysis in order to mathematically deal with the usual uncertainly embedded in linguistic preferences.

Fuzzy sets are used in MADM environment as it involves many linguistic variables. A linguistic variable is a variable whose values can be expressed in natural languages or linguistic terms. The linguistic term is a very helpful concept for dealing with situations which are too complex to be precisely or sensibly described via exact or crisp numbers. These linguistic terms can be further quantified or transformed into corresponding fuzzy numbers which mathematically capture the uncertainty in each linguistic preference so the decision analysis can be conducted quantitatively.

A fuzzy number can be defined as an interval which comprises of lower, most probable and upper values that best represents a linguistic preference. It was also found that among various types of fuzzy numbers, triangular fuzzy numbers (TFNs) have more advantages especially the arithmetic operations involving TFNs is simpler and easier to be manipulated.

It was discovered that fuzzification and defuzzification are two vital processes in adapting fuzzy set theory into MADM environment. Fuzzification refers to the process of transforming or quantifying the linguistic data into corresponding fuzzy numbers. Several fuzzification approaches were discussed in this chapter. Among them, Zhu's approach which requires simple execution is recommended for ill-informed decision makers who are unable to clearly define the fuzzy number corresponding to each linguistic preference. On the other hand, while utilizing AHP in fuzzy environment, Saaty's fuzzy AHP scale can be used to transform linguistic preferences in pair-wise comparison matrix into their corresponding fuzzy numbers.

On the contrary, defuzzification is a process of converting the fuzzy value into a crisp value which would be commonly helpful in determining the ordinal ranking of MADM elements such as attributes or alternatives. It was found that centre of area (COA) emerges as one of the frequently applied defuzzification techniques as it only demands simple calculation process for defuzzyfying TFNs, does not require any prior information from decision makers, and performs better in the aspect of accurateness than some other familiar techniques.

. The application of fuzzy set theory into MADM is gaining hiking attention from decision theorist as various fuzzy MADM tools have been formulated to this date. These fuzzy models are used to make assessment on a set of finite alternatives where required data from decision makers to derive the local scores and attributes' weights, can be provided in linguistic terms which will be then described by fuzzy numbers, in order to mathematically capture the usual uncertainty that consists in human's perception. The review on this chapter concentrated on fuzzy AHP methods which gain pervasive applications in MADM.

The uncertainty issue in conventional AHP arises during pair-wise comparison process where decision makers are required to express their preference between elements based on a crisp 9-point scale. Each crisp number in the scale represents a linguistic preference such as equally, moderately, strongly, very strongly or extremely preferred. Actually, it is more rational to represent these linguistic terms with fuzzy numbers which are capable to mathematically model the usually uncertainty embedded in the linguistic preferences offered by human. As a result, various fuzzy AHP approaches were developed with the purpose of generating more reliable solutions for MADM problems. In this chapter, the pros and cons analysis is conducted on six fuzzy AHP approaches.

From the analysis, it can be concluded that Mikhailov's fuzzy AHP method is recommendable for the decision makers relatively to other methods as the computational requirement of this approach is not enormous where the crisp priorities and consistency values of fuzzy pair-wise comparison matrices can be derived concurrently by simply resolving the proposed non-linear optimization model.

CHAPTER THREE
ON THE ASPECT OF INTERDEPENDENCIES AMONG ATTRIBUTES

3.1 Introduction

Aggregation is one of the crucial phases in implementing MAUT techniques such as AHP. Aggregation refers to a process of synthesizing the local scores of an alternative to obtain a single global score which would be helpful in selecting, ranking or classifying the alternatives under evaluation. A function which composes the local scores into a single score is commonly known as aggregation operator, $Aggre$. As emphasized earlier in chapter one, the primary issue in the aggregation phase is usually the decision makers tend to use any traditional operators which disregard the interaction aspect among attributes during aggregation.

This chapter is organized as follows. Firstly, the features or properties of a good aggregation operator are reviewed. Then, the review is extended to explore the types of aggregation operators which are applicable in MADM problems. Thirdly, the appraisal is narrowed on Choquet integral and its associated monotone measure which can model the interaction among attributes. Next, several attempts by researchers in dealing with the complexity of identifying monotone measure weights are investigated. These weights are actually demanded for the application of Choquet integral. The chapter ends with a summary of the past researches which focused on solving real MADM problems by employing Choquet integral.

3.2 Properties of an Aggregation Operator

A good aggregation operator is expected to satisfy several properties. These properties can be perceived and analyzed based on two different dimensions namely mathematical properties and behavioral properties (Clavo, Kolesarova, Komornikova, and Mesiar, 2002).

3.2.1 Mathematical Properties of an Aggregation Operator

Even though there is a long list of mathematical properties expected from a good aggregation operator, the 3 fundamental properties demonstrated by any aggregation operators are identity when unary, boundary conditions, and monotonicity (Mesiar and Komornikova, 1997). In this section, the mainly highlighted mathematical properties are identified from the study conducted by Marichal (1999), Saminger-Platz, Mesiar, and Dubois (2007), and Torra and Narukawa (2007) and summarized as presented in Table 3.1.

Table 3.1: Some Mathematical Properties Expected from an Aggregation Operator

Mathematical property	**Description**
Identity when unary	$Aggre(x) = x$ where $Aggre$ represents aggregation function or operator
Boundary conditions	The expectation is that an aggregation operator, $Aggre$, satisfies: $Aggre(0, \ldots, 0, \ldots, 0) = 0$ and $Aggre(1, \ldots, 1, \ldots, 1) = 1$
Monotonicity	It is expected that if a local score increases then the final aggregation increases or at least does not decrease. $Aggre(x_1, \ldots, y_i, \ldots, x_n) \geq Aggre(x_1, \ldots, x_i, \ldots, x_n)$ when $y_i \geq x_i$
Idempotence	If the same value is aggregated for *n* times, it is expected the result of the aggregation is equal to the initial value: $Aggre(x, x, \ldots, x) = x$
Associativity	This property demonstrates the ability to aggregate by packages. For three local scores, the property can be written as follows. $Aggre(x_1, x_2, x_3) = Aggre(Aggre(x_1, x_2), x_3) = Aggre(x_1, Aggre(x_2, x_3))$
Compensation	The aggregation operator with this property is expected to produce a 'middle value' where the result of the aggregation is lower than the maximum performance value and higher than the minimum one: $\min_i(x_i) \leq Aggre\,(x_1, x_2, \ldots, x_n) \leq \max_i(x_i)$

Based on Table 3.1, $Aggre$ represents the aggregation function or operator which combines the scores within the parentheses, $(\cdots)$ into a single score.

3.2.2 Behavioral Properties of an Aggregation Operator

Grabisch (1996b) stated that having the ability to take into account the interaction between attributes such as redundancy and synergy support is one of the salient behavioral properties expected from a good aggregation operator. With regards to MADM atmosphere, two attributes are considered redundant if they express more or less the same thing. On the other hand, synergy support refers to the phenomenon where two attributes with little importance when taken separately, become very important when considered jointly.

However, it has to be notified that if the decision makers are seeking for an aggregation operator which satisfies the entire mathematical and behavioral properties then, none of the existing aggregation operators is applicable in solving a MADM problem (Grabisch, 1996b).

3.3 Types of Aggregation Operators

The existing aggregation operators can be classified into two different clusters namely additive and non-additive operators (Wagholikar and Deer, 2007; Modave and Eklund, 2001). This taxonomy structured based on the ability of the aggregation operators in modeling the interaction between attributes during aggregation.

3.3.1 Additive Aggregation Operators

Additive aggregation operators are functions which aggregate local scores of an alternative with the presumption that attributes are independent to each other. Some crucial additive operators are identified from the studies carried out by Detyniecki (2000) and Torra and Narukawa (2007) and presented in the following sections. In following sections, to express the mathematical model of each additive operator $x_j = (x_1, x_2, \ldots, x_n)$ is used to represent the local scores of an alternative with respect to n attributes and $w_j = (w_1, w_2, \ldots, w_n)$to denote the set of attributes' weights.

3.3.1.1 Arithmetic Mean

Arithmetic mean, commonly known as the average, is the simplest approach for aggregation (Grasbich, 1998) since it merely combines the local score with the absence of attributes' weights(w_j).

$$\boldsymbol{Arithmetic\ mean(x_1, x_2, \ldots, x_n) = \frac{1}{n}\sum_{j=1}^{n} x_j} \tag{3.1}$$

3.3.1.2 Quasi- arithmetic Means

There are various means namely geometric mean, harmonic mean, quadratic mean, root-power mean, and exponential mean which can be assembled into the family of quasi-arithmetic (Liu, 2006). These means are actually the derivation of simple arithmetic mean. The mathematical models of some quasi-arithmetic means are as follows (Smolikova and Wachowiak, 2002).

$$\boldsymbol{Geometric\ mean(x_1, x_2, \ldots, x_n) = \left(\prod_{j=1}^{n} x_j\right)^{\frac{1}{n}}} \tag{3.2}$$

$$\boldsymbol{Harmonic\ mean(x_1, x_2, \ldots, x_n) = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}} \tag{3.3}$$

3.3.1.3 Simple Weighted Average

Simple weighted average (SWA), which rooted from arithmetic means, permits positioning of weights on the attributes. It is commonly preferred by decision makers since it stands out as the simplest weight-based aggregator (Lopez Orriols and de la Rosa, 2004). Mathematically, it can be expressed via formula (3.4).

$$\boldsymbol{SWA(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} (w_j . x_j)} \tag{3.4}$$

where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

3.3.1.4 Median

Median is an operator that engages to the idea of acquiring "a middle value". The median is a typical ordinal operator, taking into account the ordering of the local scores. Then, the median value can be identified using formula (3.5) (Grasbich, Marichal, Mesiar, and Pap, 2010).

$$\boldsymbol{Median(x_1, x_2, \dots x_n) = \begin{cases} x_{\left(\frac{(n+1)}{2}\right)} & if\ n\ is\ odd \\ \frac{1}{2}\left(x_{\left(\frac{n}{2}\right)} + x_{\left(\left(\frac{n}{2}\right)+1\right)}\right) & if\ n\ is\ even \end{cases}} \tag{3.5}$$

where parentheses ( ) around the index show that the scores are arranged in ascending order such that $x_{(1)} \leq x_{(2)} \leq \cdots x_{(n)}$. Order statistics are operators which function similarly to median but they produce the k^{th} value of the ordered scores as the final output (Domingo- Ferrer and Torra, 2003).

3.3.1.5 Minimum and Maximum

The minimum, $Min(x_1, x_2, \dots, x_n)$, and the maximum,$Max(x_1, x_2, \dots, x_n)$ are basic aggregation operators where the minimum produces the smallest value of a set of local scores, while the maximum gives the greatest one. From the perspective of decision making, the usage of minimum operator expresses conjunctive behavior whereas the maximum operator reflects disjunctive behavior (Sousa and Kaymak, 2002).

3.3.1.6 Weighted Minimum and Weighted Maximum

These operators were adapted from minimum and maximum operators by Dubois and Prade (1985). Some of the characteristics of these operators are as follows. If a weight w_j equals zero, then the local score x_j will be discarded from the aggregation. Furthermore, if all weights are equal, then the minimum and maximum are obtained concurrently. The weighted minimum and weighted maximum model are defined as follows (Fodor and Roubens, 1995).

$$\boldsymbol{Wmin}(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}) = \underset{\boldsymbol{j=1}}{\overset{\boldsymbol{n}}{\boldsymbol{Min}}}\left[\boldsymbol{max}\left(\boldsymbol{1} - \boldsymbol{w_j}, \boldsymbol{x_j}\right)\right] \tag{3.6}$$

$$W\boldsymbol{max}(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}) = \underset{\boldsymbol{j=1}}{\overset{\boldsymbol{n}}{\boldsymbol{Max}}}\left[\boldsymbol{min}\left(\boldsymbol{w_j}, \boldsymbol{x_j}\right)\right] \tag{3.7}$$

where weights are normalized so that $\underset{j=1}{\overset{n}{Max}}(w_j) = 1$.

3.3.1.7 Ordered Weighted Average

Ordered Weighted Average (OWA) is a generalization of minimum, maximum and arithmetic mean operator (Lin and Jiang, 2011), proposed by Yager (1988). The application of OWA operator can be summarized in three basic steps (Liu, 2011). Firstly, the local scores are reordered in descending manner. Then, the weights associated with the OWA models are estimated. Finally, the local scores and weights are precisely substituted into OWA model to obtain the single score (3.8) (Grabisch, 2011).

$$\boldsymbol{OWA(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} w_j x_{(j)}} \tag{3.8}$$

where parentheses ( ) around the index show that the scores are arranged in ascending order such that $x_{(1)} \le x_{(2)} \le \cdots x_{(n)}$, $w_j \ge 0$, and $\sum_{j=1}^{n} w_j = 1$.

3.3.2 Non-additive Aggregation Operators

Grabisch (1996b) and Detyniecki (2000) stated the one of the major drawbacks of additive operators is that they are abortive in modeling the interaction among attributes during aggregation. In other words, these operators are improper for real world phenomenon since usually the attributes present some interdependent features (Buyukozhan, 2010). As a result, non-additive or fuzzy integrals (Mesiar and Mesiarova, 2008) such as Sugeno and Choquet integral are recommended as the resolution to overcome this defect (Narukawa and Torra, 2007).

Although both integrals have the potential in capturing interaction between attributes during aggregation, this study allocates its primary attention on the usage of Choquet integral on several basis. Firstly, as affirmed by Iourinski and Modave (2003), Choquet integral is better suited for numerical or quantitative based problems whereas the Sugeno integral is ideal for qualitative problems. In other words, it can be stated that the application of Choquet integral can generate more practical outcomes (Wang and Wang, 1997) as most of the MADM problems involve numbers which have a real meaning (interval or ratio level of measurement) where cardinal aggregation is required, unlike Sugeno integral which is more suitable for ordinal aggregation where only the order of the elements is important (Detyniecki, 2000 and Grabisch, 1998). Secondly, Choquet integral has the merit in producing unique solution in comparison to the other integral (Chen and Wang, 2001).

3.4 Choquet Integral based Aggregation

The execution of Choquet integral into MADM problems comprises of three basic steps as follows (Carter, Flores, Kassin, and Pajaro, 2008). Firstly, the occurring problem need to be well-defined and the relevant attributes must be determined. Subsequently, the monotone measure weights are estimated. Finally, the local scores of each alternative and identified monotone measure weights are applied precisely into Choquet integral model to compute the global scores.

3.4.1 Monotone Measure

Choquet integral is able to represent interaction among attributes due to the fact that it utilizes the idea of monotone measure which able to model the interaction between attributes (Saad, Hammadi, Benrejeb, and Borne, 2008). Monotone measure is a generalization of classical measure (Yang, 2005) where the additivity is removed and replaced with weaker monotoniciy property (Mikenina and and Zimmerman, 1999).

Monotone measure can be defined as follows. Let $c_j = (c_1, c_2, \dots, c_n)$ be a finite set. A set function $g\{.\}$ defined on the subsets of c_j, $P(c_j)$, is called a monotone measure if it satisfies following conditions (Sugeno, 1974):

a) $g: P(c_j) \rightarrow [0,1]$, and $g\{\emptyset\} = 0, g\{c_j\} = 1$(satisfies boundary condition whereby an empty set has no importance,$g\{\emptyset\} = 0$, and the maximal set has a maximal importance, $g\{c_j\} = 1$.

b) $\forall A, B \in P(c_j)$, if $A \subseteq B$, then implies $g\{A\} \leq g\{B\}$ (satisfies monotonicity condition which means that adding a new element to a combination cannot decrease its importance).

From the study conducted by Angilella, Greco, and Matarazzo (2010), it can be concluded that the weights of monotone measure, g do not only imply the individual importance of each attribute but also denote the importance of all possible combinations or subsets of attributes. Therefore, the decision makers are generally obliged to estimate 2^n weights of monotone measure if they intend to apply Choquet integral (Alavi et al, 2009). For instance, consider a MADM problem involving three attributes, $c_j = (c_1, c_2, c_3)$. Then, the weights on eight($2^3 = 8$) subsets of attributes comprising of $g\{\emptyset\}$, $g\{c_1\}$, $g\{c_2\}$, $g\{c_3\}$, $g\{c_1, c_2\}$, $g\{c_1, c_3\}$, $g\{c_2, c_3\}$, and $g\{c_1, c_2, c_3\}$ need to be assigned where $g\{\emptyset\} = 0$ and $g\{c_1, c_2, c_3\} = 1$ as per the axiom.

Further clarification on how these weights able to express the interaction effects between the attributes is offered in the following section.

3.4.1.1 Representing Interaction via Monotone Measure

A monotone measure can express three types of interaction that could be shared by the attributes. Suppose A and B are two subsets of attributes where $A \cap B = \emptyset$ then, the interaction phenomena between these subsets can be described as follows (Bonetti, Bortot, Fedrizzi, Pereira, and Molinari, 2012):

a) If the weight of the combination of A and B is equal to the sum of weight of$A$ and B such that $g\{A \cup B\} = g\{A\} + g\{B\}$, then it can be regarded that A and B are sharing additive effect or in other words, being independent to each other.

b) If the weight of the combination of A and B is less than or equal to the sum of weight of A and B such that $g\{A \cup B\} \leq g\{A\} + g\{B\}$ then, it can be regarded that A and B are sharing sub-additive effect or being redundant to each other.

c) If the weight of the combination of A and B is greater than or equal to the sum of weight of$A$ and B such that $g\{A \cup B\} \geq g\{A\} + g\{B\}$ then, it can be regarded that A and B are expressing super-additive effect or synergy support.

For instance, consider a MADM problem comprising three attributes, $c_j = (c_1, c_2, c_3)$. The importance of each individual attribute in enhancing the performance of target is as follows. $g\{c_1\} = 0.3$, $g\{c_2\} = 0.2$, and $g\{c_3\} = 0.1$. Then, weight on monotone measure consisting attributes c_1 and c_2, $g\{c_1, c_2\}$ can be estimated as follows:

a) If c_1 and c_2 are being redundant, the presence or combination of both attributes does not contribute to a significant enhancement on the performance of the target as both of them share some similar information. Therefore, too much weight should not be given on the combination of these attributes. Thus, the weight assigned on the combination of these two attributes should be less than or equal to 0.5 where $g\{c_1, c_2\} \leq 0.3 + 0.2$ (sub-additive effect).

b) If the synergy between c_1 and c_2 can significantly enhance the performance of the target then, more weight should be given on the combination of these attributes when considered jointly. Therefore, the weight assigned on combination of these two attributes should be greater than or equal to 0.5, where $g\{c_1, c_2\} \geq 0.3 + 0.2$ (super-additive effect).

c) If c_1 and $c_2$are independent to each other, then the weight assigned on the combination of these two attributes should be equal to 0.5, where $g\{c_1, c_2\} = 0.3 + 0.2$ (additive effect).

This example could have offered better illustration on how monotone measure weights able to represent the interaction effects between attributes in a MADM problem.

3.4.2 Choquet Integral Model

With the complete estimation of the monotone measure weights and available local scores, the Choquet integral model can then be applied to compute the aggregated score. Let g be a monotone measure on $c_j = (c_1, c_2, \ldots, c_n)$ and $x_j = (x_1, x_2, \ldots, x_n)$

be the local score of an alternative with respect to each attribute in c_j. Suppose $x_1 \geq x_2 \geq \cdots \geq x_n$. Then, T_n = $(c_1, c_2, \ldots, c_n)$ and the aggregated score using Choquet integral can be identified using (3.9) (Feng, Wu and Chia, 2010).

$$\boldsymbol{Choquet_g(x_1, x_2, \ldots, x_n)}$$
$$\boldsymbol{= x_n . g\{T_n\} + [x_{n-1} - x_n] . g\{T_{n-1}\} + \ldots + [x_1 - x_2] . g\{T_1\}} \quad (3.9)$$
$$\boldsymbol{= x_n . g\{c_1, c_2, \ldots, c_n\} + [x_{n-1} - x_n] . g\{c_1, c_2, \ldots, c_{n-1}\} + \cdots + [x_1 - x_2] . g\{c_1\}}$$

where the arrangement of attributes in T_n parallel with the descending order of the performance scores.

For better understanding, presume that the scores of a student, x in three subjects (attributes), Mathematics (x_M), Physic (x_P), Biology (x_B) are 75, 80, and 50 respectively. Hence, $x_P \geq x_M \geq x_B$. Then, $T_n = \{ P, M, B \}$and the aggregated score of the student using Choquet integral, $Choquet_g(x_M, x_P, x_B) = x_B . g\{ P, M, B \} + (x_M - x_B) . g\{ P, M\} + (x_P - x_M) . g\{ P\}$. Figure 3 illustrates the idea of aggregation via Choquet integral.

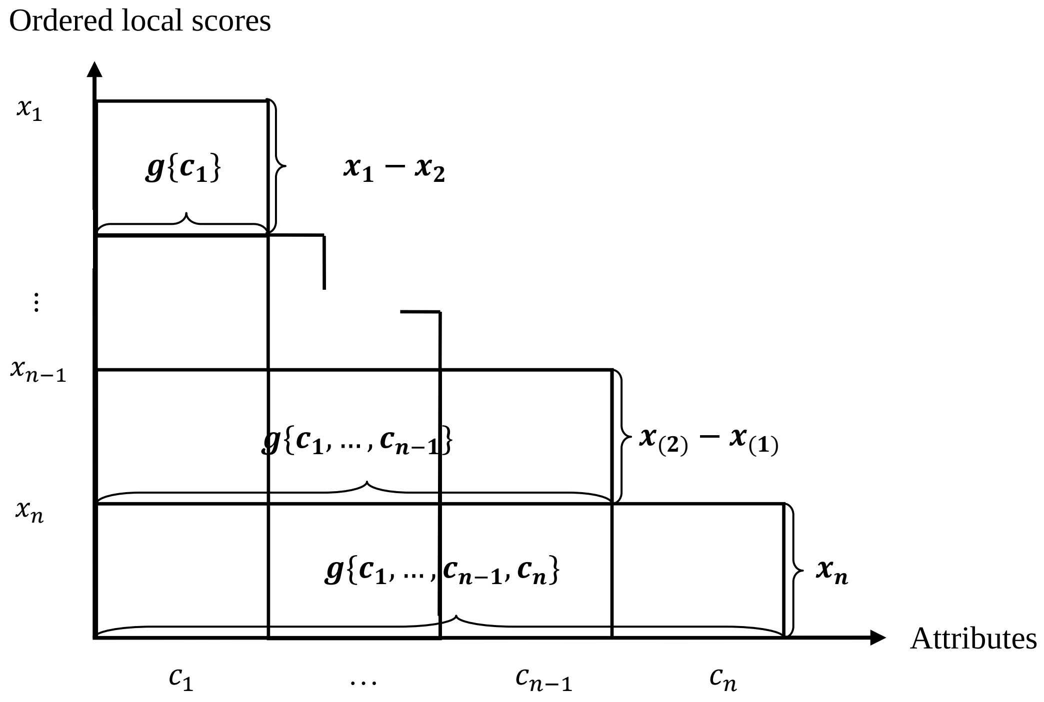


Figure 3.1: The Concept of Choquet Integral

The discrete Choquet integral captures the interaction between attributes by means of the monotone measure weights, g (Shieh, Wu, and Liu, 2009). An interesting property of Choqeut integral is that it simplifies as SWA operator if the monotone measure is additive or in other words, when attributes are being independent (Marques Pereira, Ribeiro, and Serra, 2008).

3.4.3 Significance of Considering Interaction among Attributes

In this section, two simple MADM instances have been extracted from two different studies to illustrate how the usage of additive aggregation operator deviates from norm of rational decision making, to analyze interaction effects between attributes, and to understand the necessity to consider the interaction among attributes during aggregation which can be achieved by using Choquet integral.

3.4.3.1 Television (TV) Evaluation Problem

Consider a TV evaluation problem amended from the study conducted by Wang, Yang, and Leung (2010). Assume there are three used TVs (A, B, C) in sales and a customer wants to identify the best TV which has no any weak points to be purchased. Further presume that the global qualities of the TVs are evaluated based on two attributes namely, 'picture' and 'sound'. The scores of the TVs with respect to each attribute, ranging from 0 to 100, are assigned as presented in Table 3.2.

Table 3.2: Decision Matrix for TV Evaluation Problem

TV/ Attributes	**Picture (p)**	**Sound (s)**
A	100	20
B	20	90
C	55	60

Suppose the costumer wants to estimate the global quality of each TV using SWA operator by allocating equal weight on each attribute where $w_p = 0.5$ and $w_s = 0.5$. Then, the quality score of each TV measured using SWA operator is as summarized in Table 3.3.

Table 3.3: Result of TV Evaluation Problem via SWA

Attributes/ TV	**Picture (p)**	**Sound (s)**	**Global score**	**Ranking**
A	100	20	$(100 \times 0.5) + (20 \times 0.5) = 60$	1
B	20	90	55	3
C	55	60	57.5	2

Based on the result in Table 3.3, it can be interpreted that A is identified as the worth-to-buy TV. But logically, A and B should not be identified as the best TV because they possess some weak points in the aspect of 'sound' and 'picture' respectively. To our perception, C should be identified as the best TV because it has no weak points with respect to any attributes. The root cause to this problem is due to the usage of SWA operator which assumes that the attributes independently contribute to the global quality. In other words, they are assumed to express additive effect.

In order to capture the interaction that exists between these attributes, appropriate monotone measure and Choquet integral can be used as described herein. By adhering to initial ratio (1:1), assume the weights assigned on subsets consisting single attribute are $g\{p\} = 0.4$ and $g\{s\} = 0.4$. Besides, usually, the synergy or combination of good 'picture' or 'sound' quality will significantly boost the global quality of a TV. In other words, both attributes are sharing super-additive or synergy support effect. Therefore, the joint importance of 'picture' and 'sound' should be higher than the sum of their individual importance; $g\{p, s\} > g\{p\} + g\{s\}$. Then, it is set $g\{p, s\} = 1$ (at the same time, as per the axiom of monotone measure, the maximal subset should be equal to 1). Finally, the identified monotone measure weights and local scores are replaced into Choquet integral model (3.9) to aggregate the global quality of each TV. The final result is as presented in Table 3.4.

Table 3.4: Result of TV Evaluation Problem via Choquet Integral

Student	Global score	Ranking
A	a) Firstly, rank the local scores in ascending order where $x_p \geq x_s$. Thus, $T_n = \{p, s\}$ b) Secondly, Choquet model (3.9) is applied to compute global quality: $Choquet_\mu(x_p, x_s)$ $= x_s * g\{p, s\} + (x_p - x_s) * g\{p\}$ $= 20 * 1 + (100 - 20) * 0.4 = 52$	2
B	48	3
C	57	1

Based on the result in Table 3.4, it can be concluded that the expected TV, (TVC) which has no any weak points is identified as the finest TV, via the usage of Choquet integral.

3.4.3.2 Student Evaluation Problem

Consider a student evaluation problem borrowed from the study conducted by Grabisch (1996b). Assume there are three students (A, B, and C) and we want to identify the best student who has no any weak points. Further presume that the overall performances of the students are assessed based on three subjects namely Mathematics (M), Statistics (S) and Literature (L). The scores of the students with respect to each subject, ranging from 0 to 20, are as presented in Table 3.5.

Table 3.5: Decision Matrix for Student Evaluation Problem

Student	**M**	**S**	**L**
A	18	17	10
B	10	12	18
C	14	15	15

Suppose the global performance of each student is assessed based on SWA operator with equal weight is assigned on each subject as follows; $w_M = \frac{1}{3}$, $w_S = \frac{1}{3}$, and $w_L = \frac{1}{3}$. Then, the computed global performances and final ranking of the students are as summarized in Table 3.6.

Table 3.6: Result for Student Evaluation Problem Using SWA

Student	**M**	**S**	**L**	**Global score**	**Ranking**
A	18	17	10	$\frac{1}{3}(18) + \frac{1}{3}(17) + \frac{1}{3}(10) = 15$	1
B	10	12	18	13.33	3
C	14	15	15	14.67	2

The result derived using SWA shows that student A has the highest rank followed by student C and B. However, logically, if the school is looking for well-balanced students without any weak points, student C should be considered better than student A.

The cause of this problem is that SWA operates based on the assumption that there are no interactions between attributes during aggregation. To overcome this issue, an appropriate monotone measure and Choquet integral operator can be used. Prior to applying Choquet integral, monotone measure weights are identified as follows. First of all, the weights of subsets consisting single subject (individual weights) are assigned by adhering to initial ratio (1:1:1) where $g\{M\} = g\{S\} = g\{L\} = 0.4$. Since mathematics and statistics are redundant to each other, the importance or weight on the combination of two subjects should be less than the sum of their individual weights. Therefore, it is estimated that $g\{M, S\} = 0.5 \leq g\{M\} + g\{S\}$.

Besides, the global performances of students would increase drastically if they are being good at mathematics and literature (or statistics and literature). In other words, mathematics (or statistics) shares super-additive effect with literature (Grabisch and Labreuche, 2005). Then, the importance given on the combination of L, M and L, S should be greater than the sum of their individual weights. Hence, it is assigned $g\{L, M\} = 0.9 \geq g\{L\} + g\{M\}$ and $g\{L, S\} = 0.9 \geq g\{L\} + g\{S\}$. Not to forget, as per the monotone measure axioms, $g\{\emptyset\}$ and $g\{M, S, L\} = 1$.

The estimated monotone measure weights and scores are then precisely substituted into Choqeut integral model to compute the global performance of each student. The global scores of students and their respective ranking are summarized in Table 3.7.

Table 3.7: Result for Student Evaluation Problem Using Choquet Integral

Student	Global score	Ranking
A	a) Firstly, rank the local scores in descending order where $x_M \geq x_S \geq x_L$. Thus, $T_n = \{M, S, L\}$ b) Secondly, Choquet model (3.9) is applied to compute global performance: $Choquet_\mu(x_M, x_S, x_L)$ $= x_L * g\{M, S, L\} + (x_S - x_L) * g\{M, S\} + (x_M - x_S) * g\{M\}$ $= 10 * 1 + (17 - 10) * 0.5 + (18 - 17) * 0.4 = 13.9$	2
B	14.2	3
C	14.9	1

Based on the result in Table 3.7, it can be concluded that by applying Choquet integral which captures the interaction between the subjects, the expected student (student C), who has no any weak points is identified as the best student.

3.4.2.3 Individual Weights of Attributes

Based on the presented two problems, it can be noticed that each problem involves two types of individual weights, additive and non-additive individual weights. The similarity and differences between these two weights are highlighted through Table 3.8.

Table 3.8: Differences between Additive and Non-additive Individual Weights

	Type of individual weights /Aspects	**Additive individual weights**	**Non-additive individual weights**
Similarity	Common interpretation	Marginal contribution or importance of each attribute in achieving the overall goal (Choo et al., 1999)	
Differences	Sum of weights	Sum of additive individual weights which are usually represented by $w_j = w_1, w_2, \ldots, w_n$ is equal to one (Detyniecki, 2000)	Sum of non-additive individual weights which are normally denoted by $g(c_j) = g(c_1), g(c_2), \ldots, g(c_n)$ could be sub-additive or super-additive, not necessarily equal to one (Grabisch, 1996b)
	Application	Required by additive aggregators such as SWA and OWA (Detyniecki, 2000; Grabisch, 1996b)	Required for the identification of weights of all possible combinations of attributes (monotone measure) which will be then applied into fuzzy integrals such as Choquet integral (Angilella et al., 2010)

3.4.4 Attempts on Reducing the Complexity of Identifying Monotone Measure

Prior to utilizing Choquet integral as an aggregation operator in a MADM problem, it is essential to identify the importance of all subsets of attributes or in other words, the weights of monotone measure. However, it is rather unrealistic and burdensome for the decision makers to subjectively estimate 2^n weights of monotone measure when the number of attributes, *n* is sufficiently large (Kojadinovic, 2004; Mikenina, and Zimmermann, 1999; Yager, 2000). As a result, some identification procedures such as minimization of squared error based approaches and constraint satisfaction based approaches were introduced (Grabisch and Roubens, 2000) to assist the decision makers in estimating these weights.

However, both approaches came with several inconveniences. Firstly, both approaches were developed based on optimization problems. Therefore, finding the solution via these approaches becomes more complex as the number of variables involved in the approaches increases exponentially with n (Kojadinovic, 2008). Secondly, the main shortcoming of the former approach is that it requires the information on the desired global score on each alternative which cannot always be acquired from the decision makers (Grabisch, Kojadinovic, and Meyer, 2008). Meanwhile, the later approach requires various types of initial data such as partial ranking of the alternatives, partial ranking of the attributes, intuitions about the importance of the attributes, and interaction among attributes (Marichal and Roubens, 2000) which couldn't be easily offered by the decision makers especially when they are ill-informed on the existing MADM problem.

In order to assist decision makers who are unknowledgeable on the existing problem and facing difficulties providing the necessary initial data, Kojadinovic (2004) has formulated an unsupervised identification approach. Via this approach, the weights of monotone measure can be estimated based on available local scores by means information-theoretic functions. However, the major weakness of this approach is that usually a large number of local scores is demanded to obtain precise weights of monotone measure.

With the intention to further reduce the complexity involved in estimating general monotone measure, several patterns or subfamilies of monotone measures were proposed. In comparison to other monotone measure patterns, λ- measure which was introduced by Sugeno (Sugeno, 1974) emerges as one of the widely applied monotone measures due to its ease of usage, mathematical soundness and modest degree of freedom (Ishii and Sugeno, 1985). According to Young (2008), mathematical soundness is a property which gives us the confidence that we have correctly find solutions for a system (Young, 2008). Meanwhile, modest degree of freedom refers to number of variables needed to specify completely the solution for a system (Tanton, 2005). This can be further elaborated as follows.

In identifying general monotone measure, the decision makers have to determine the type interaction shared by the attributes within each and every subset. Besides, they have to ensure the identified weights fulfill the axioms of monotone measure. But for identifying the weights of λ- monotone measure, the decision makers just need a set of individual weights of attributes and a single interaction parameter (implies the modest of degree property). Besides, these weights can be simply estimated using the available Sugeno equation which will always ensure all subsets satisfy the two axioms of monotone measure (shows the mathematical soundness property).

λ- measure can be defined as follows. Let $c_j = (c_1, c_2, \dots, c_n)$ be a finite set. A set function $g_\lambda(.)$ defined on the set of the subsets of c_j, $P(c_j)$, is called a λ-measure if it satisfies the following conditions (Liu, Jheng, Lin, and Chen, 2007):

a) $g_\lambda : P(c_j) \rightarrow [0,1]$, and $g_\lambda(\emptyset) = 0, g_\lambda(c_j) = 1$ (boundary condition)
b) $\forall A, B \in P(c_j)$, if $A \subseteq B$, then implies $g_\lambda(A) \leq g_\lambda(B)$ (monotonic condition)
c) $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B)$, for all $A, B \in P(c_j)$ where $A \cap B = \emptyset$ and $\lambda \in [-1, +\infty]$.

Note that properties (a) and (b) are fundamental for any families of fuzzy measure and (c) is the additional property of λ- fuzzy measure.

The λ- measure is constrained by a parameter λ, which describes the degree of additivity the attributes hold. In other words, the interaction phenomenon between attributes can be interpreted based on the value, λ as follows (Chu, Shyu, Tzeng, and Khosla, 2007; Hu and Chen, 2010):

a) If $\lambda < 0$ then, it interprets that the attributes, $c_j = \{c_1, c_2, \dots, c_n\}$ are sharing sub-additive effects. This means that the overall performance of a target can be increased if some attributes in c_j which have higher individual weights are enhanced simultaneously.
b) If $\lambda > 0$ then, it implies that the attributes, $c_j = \{c_1, c_2, \dots, c_n\}$ are sharing super-additive effects. This means that the overall performance of a target can be increased if all the attributes in c_j are enhanced simultaneously regardless of their individual weights.
c) If $\lambda = 0$ then, it reflects that the attributes, $c_j = \{c_1, c_2, \dots, c_n\}$ are non-interactive. This means that the overall performance of a target can be increased by simply enhancing the attribute(s) with higher individual weights.

As $c_j = \{c_1, c_2, \dots, c_n\}$ is finite, then the entire λ- measure weights can be identified by the following formula (3.10).

$$\boldsymbol{g_\lambda(c_1, c_2, \dots, c_n)}$$

$$= \sum_{i=1}^{n} \boldsymbol{g_j} + \boldsymbol{\lambda} \sum_{j_1=1}^{n-1} \sum_{j_2=j_1+1}^{n} \boldsymbol{g_{j_1} \cdot g_{j_2}} + \cdots + \boldsymbol{\lambda^{n-1} \cdot g_1 \cdot g_2 \cdots g_n} \tag{3.10}$$

$$= \frac{\mathbf{1}}{\boldsymbol{\lambda}} \left| \ \prod_{j=1}^{n} (\mathbf{1} + \boldsymbol{\lambda g_j}) - \mathbf{1}, \right| \ for - \mathbf{1} < \boldsymbol{\lambda} < +\infty$$

where$g_j = g_\lambda(c_j), j = 1, \dots, n$ denotes the fuzzy density or weights of individual attribute. If the weights of individual attribute, g_j are given, then in the case $\sum_{j=1}^{n} g_j = 1$, $\lambda = 0$. Whereas if $\sum_{j=1}^{n} g_j \neq 1$, the parameter λ can be calculated by solving the following equation (Klir, Wang, and Harmanec, 1997).

$$\mathbf{1} + \boldsymbol{\lambda} = \prod_{i=1}^{n} (\mathbf{1} + \boldsymbol{\lambda g_i}) \tag{3.11}$$

Due to the well-acceptance of λ- measure, various approaches were formulated with the aim of assisting and gradually diminishing the burden of decision makers in estimating λ- measure weights.

In the early years, LeszczySnski, Penczek, and Grochulski (1985), Sekita and Tabata (1977), Tahani and Keller (1990), and Wierzchon (1983) developed several λ-measure identification methods. However, these methods still require high number of data (2^n) from decision makers for subjective weight estimation on subset of attributes or demand complicated computations (Chen and Wang, 2001).

As a result, Lee and Leekwang (1995) developed a genetic algorithm (GA) based identification approach which is computationally simpler. This approach does not require complete subjective estimation for all subsets of attributes in order to identify λ- measure. A few years later, Chen and Wang (2001) proposed a method based on sampling design and GA which is also simple, fast, easily programmable, and most vitally, it only requires a few data to run the solution procedure. Nevertheless, these two methods still have a few drawbacks (Yue, Li, and Yin, 2005; Larbani et al., 2011).

Firstly, more subjective data on weight of subsets of attributes are expected for better solution although it is cumbersome for decision makers to offer such data. In other words, these two methods fail to offer a scheme to control the amount of lost information on the basis of generating a satisfactory solution. Secondly, these two identification approaches are based on GA. Although GA as a random searching method is feasible for most of non-linear programming models, it has many intrinsic flaws such as its slow convergence speed and uncertainty of extreme position.

Then, Takahagi (2007) proposed a λ-measure identification method based on diamond pair-wise comparisons which requires two types. The horizontal axis of the diamond is used to express the relative importance of attributes and the vertical axis is used to express the interaction between the pair of attributes. Therefore, this method only requires $n(n-1)$ data from decision makers. However, this method possesses some gaps. Firstly, in order to ensure the decision makers are able to answer the diamond pair-wise comparisons without any complications, an understandable instruction is needed. Secondly, suitable interpretations and theatrical supports of axis, especially vertical axis should be provided. Finally, unlike AHP, where consistency index is defined, this method does not propose any means to measure the consistency of interaction comparisons yet.

Wang, Lu, and Chen (2010), in a study to evaluate high technology firm performance, applied an identification procedure which is much simpler comparatively to other procedures (Hereafter, for sake of simplicity, this approach will be referred as fuzzy-linguistic estimator). This procedure mainly requires the decision makers to estimate on weight or importance of individual attribute. Firstly, to facilitate uncomplicated estimation and to model the usual uncertainty in human's estimation, the decision makers are allowed to estimate the individual weights of attributes using natural languages which are then transformed into corresponding fuzzy numbers. Next, these fuzzified individual weights are converted into crisp weights. Finally, these weights are precisely replaced into equation (3.10) and (3.11) in order to identify the parameter, λ, and λ- measure weights of the existing problem respectively. However, this approach may need some extra information from decision makers during fuzzification where they need to determine the boundary values that best represent each of the linguistic estimation. It could turn as a troublesome task for them if they are unfamiliar with the existing problem and thus, forces them to seek for experts' opinion.

Meanwhile, Lin, Shiu and Tzeng (2011) have introduced fuzzy partitioned hierarchy model which reduces the number of λ- measure weights which need to be identified prior of applying Choquet integral from 2^n to $\sum_{p=1}^{q} 2^{|f_p^-|} + \sum_{p=1}^{q} 2^{|f_p^+|}$ where $f_p^- = (f_1^-, f_2^-, \ldots, f_q^-)$ represents set of factors extracted based on left values and $f_p^+ = (f_1^+, f_2^+, \ldots, f_q^+)$ represents set of factors extracted based on right values. However, analyzing a MADM problem using this model requires huge computational effort from decision makers as enlighten in chapter one (Table 1.5).

Table 3.9 summarizes the approaches developed to help the decision makers in determining general monotone measure weights whereas Table 3.10 recaps the approaches or attempts established to assist the decision makers in estimating λ-measure weights.

Table 3.9: Reducing the Complexity of Identifying General Monotone Measure

Approaches	**Advantages (A)/ Disadvantages(D)**
Minimization of squared error and constraint satisfaction based approach	(D) become more complex as the variable involved in the approaches increases exponentially with n (Kojadinovic, 2008) (D) former types of approaches require information on the desired global score on each alternative which cannot always be acquired from the decision makers (Grabisch, Kojadinovic, and Meyer, 2008) (D) Later approaches commonly require various types of initial data which are not easy to be expressed by the decision makers (Marichal and Roubens, 2000)
Unsupervised identification approach	(A) helpful for decision makers who are unknowledgeable on the existing problem and have the complication in providing necessary initial data (Kojadinovic, 2004) (D) large number of local scores is demanded to obtain precise weights of monotone measures (Kojadinovic, 2004)

Table 3.10: Reducing the Complexity of Identifying λ-measure

Approaches	**Advantages (A)/ Disadvantages (D)**
LeszczySnski, Penczek, and Grochulski (1985), Sekita and Tabata (1977), Tahani and Keller (1990), and Wierzchon (1983)	(D) still require high number of data (2^n) from decision makers/ complicated computational process (Chen and Wang, 2001)
Genetic algorithm (GA) based approach (Lee and Leekwang, 1995)/ Sampling design and GA based approach (Chen and Wang, 2001)	(A) simple , fast , and easy to be programmed (Chen and Wang, 2001) (A) only requires a few data to run the solution procedure (Chen and Wang, 2001) (D) failed to have a scheme to control the amount of lost information(Yue, et al., 2005; Larbani et al., 2011) (D) GA has many intrinsic flaws such as its slow convergence speed, and uncertainty of extreme position (Yue, et al., 2005; Larbani et al., 2011)
Diamond pair-wise comparisons approach (Takahagi, 2007)	(D) to assure the decision makers able to answer the diamond pair-wise comparisons without any complications, an understandable instruction is needed (Takahagi, 2007) (D) still requires suitable interpretations and theatrical supports of axis, especially vertical axis (Takahagi, 2007) (D) this method has not proposed any means to measure the consistency of interaction comparisons (Takahagi, 2007)
Fuzzy-linguistic estimator (Wang et al., 2010)	(A) allows the decision makers to provide data in linguistic terms (A) only requires one type of data from decision makers (A) understandable and simple computational steps (D) need some extra data from decision makers during fuzzification
Fuzzy partitioned hierarchy model (Lin et al., 2011)	(A) reduces the number of λ- measure weights which need to be identified prior of applying Choquet integral from 2^n to $\sum_{p=1}^{q} 2^{\lvert f_p^- \rvert} + \sum_{p=1}^{q} 2^{\lvert f_p^+ \rvert}$ (D) solving MADM problem using this model still requires numerous computational steps

3.4.5 Real Applications of Choquet integral

Choquet integral is currently gaining increasing attention and frequently used by researchers. It is a flexible tool that models interaction among attributes (Grasbich, 1996b) and has been applied successfully in various domains such as logistic, education, and hospitality, as shown in Table 3.11.

Table 3.11: Real Applications of Choquet Integral

Source	Country	Domain	Problem definition
Peters and Ramanna (1996)	Canada	Software engineering	Estimating cost of software
Berrah, Mauris, and Montmain (2008)	France	Manufacturing	Monitoring the improvement of an overall industrial performance
Shieh et al. (2009)	Taiwan	Education	Evaluating of students' overall performance
Buyukozkan and Ruan (2010)	Turkey	Software engineering	Software development risk assessment
Demiral et al. (2010)	Turkey	Logistic	Selecting warehouse location for a big Turkish logistic firm
Hu and Chen (2010)	Taiwan	Hospitality	Evaluating customer service perceptions on fast food stores
Huang and Wang (2011)	China	Health	Evaluating competition ability of private hospital
Yoo, Cho, and Kim (2011)	Korea	Artificial Intelligent(AI)	Generating composite facial expressions for a robotic head

3.5 Summary of Chapter Three

The review presented in chapter three was primarily revolves on the aggregation process in MADM. Aggregation is a process of synthesizing local scores of alternatives into single scores. The function which aggregates these local scores is familiarly known as aggregation operator. The single scores obtained via aggregation will be useful in ranking or classifying the alternatives.

To be labeled as a good aggregation operator, it was found that the aggregation operator should satisfy several mathematical properties including three fundamental properties namely, identity when unary, boundary condition, and monotonicity. In addition, a good aggregation operator is also expected to portray certain behavioral properties especially the ability to express the interaction among attributes. Nevertheless, none of the available aggregation operators is applicable in resolving MADM problem if the decision makers are looking forward for an operator that fulfills the extensive list of expected properties.

The aggregation operators can be attributed into two clusters namely additive and non-additive operator. The former group comprises arithmetic means, quasi-arithmetic means, simple weighted average, minimum, maximum, weighted minimum, weighted maximum and ordered weighted average operators. The major drawback of these operators is they fail to model some understanding way of interaction between attributes during aggregation. However, it was discovered that the latter group, comprising Sugeno and Choquet integral operator have the potential to handle this gap.

Between Sugeno and Choquet integral, Choquet integral gains more attention for real applications as it is better suited for quantitative based problems, has the merit in producing unique result and can derive more feasible outcomes than using the other integrals in many cases.

Choquet integral able to capture the interaction among attributes as it utilizes the idea of monotone measure. Monotone measure is capable to characterize the interaction among attributes. Besides providing the weight of each attribute, monotone measure offers the weights of all possible combination of attributes. To say the least, the identification of monotone measure is essential prior to applying Choquet integral.

However, the common obstruction of employing Choquet integral is the necessity to estimate 2^n weights of monotone measure where n represents number of attributes. The number of weights which needs to be identified increases exponentially as *n* increases.

With regards to this issue, several means were proposed to assist and minimize the burden of the decision makers in identifying monotone measure weights. But, it was learnt that the attempt to reduce the complexity of estimating these weights is endless. Further simpler approaches will be well-welcomed as it will motivate more decision makers to utilize the beneficial aggregation tool, Choquet integral.

Finally, it was discovered that Choquet integral is earning rising fame among researchers from various domains in solving the occurring MADM problems.

CHAPTER FOUR
METHODOLOGY

4.1 Introduction

In order to attain the main objective of this research, a series of phases are structured. The execution of each phase is synopsized in this chapter. The vital 5 phases in accomplishing this research are presented in Figure 4.1.

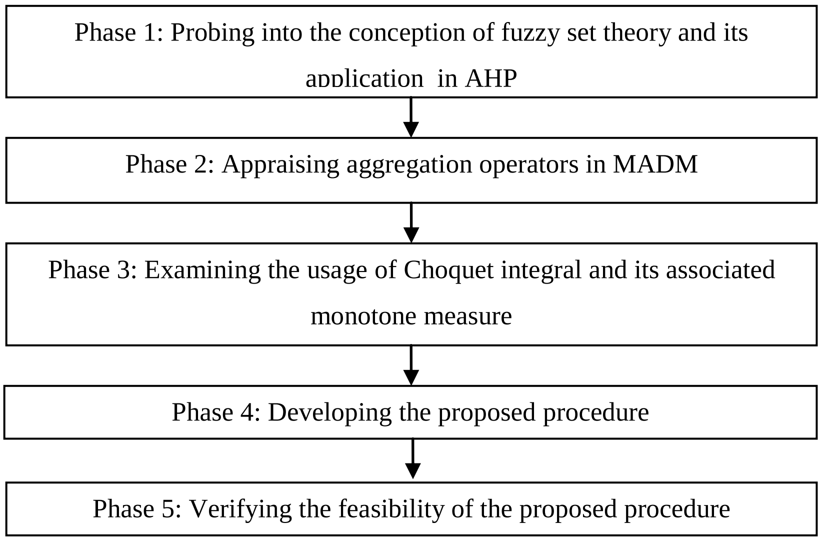


Figure 4.1: Phases to Attain the Objective of the Study

4.2 Probing Fuzzy Set Theory and Its Application in AHP

In the first phase, this research stretched its investigation into the issue of vagueness that usually exists in data or judgment offered by humans during decision making process and its linkage to fuzzy set theory. First and foremost, an analysis on preliminary studies pertaining fuzzy set theory was carried out to comprehend on some significant notions such as linguistic variables, types of fuzzy numbers, arithmetic operations involving fuzzy numbers, fuzzification, and defuzzification process that allied to the former theory. In addition, the usage of fuzzy sets in AHP analysis was reviewed as well. Finally, a pros and cons assessment among various types of fuzzy AHP methods was conducted.

4.3 Appraising the Aggregation Operators in MADM

A satisfactory level of review on aggregation phase in MADM would be helpful in developing the proposed procedure. Thus via this phase, firstly, the concept of aggregation was defined in relevant to MADM system. Then, the types of aggregation operators which are relevant to MADM problems were identified. Finally, the appraisal was centralized on aggregation operators which are able to consider the interaction aspects among attributes.

4.4 Delving into Choquet Integral and Its Associated Monotone Measure

This phase is devoted to delve into the execution of Choquet integral, which is capable to capture the interaction between attributes during aggregation. The concept of monotone measure and the hurdles in identifying monotone measure weights prior of employing Choquet integral were explored thoroughly. Moreover, the available monotone measure identification approaches were analyzed from the aspects of computational requirement, advantages and disadvantages.

4.5 Formulating the Proposed Procedure

By assimilating the information gathered throughout the former three phases, this research is now all set to develop a MADM procedure which is able to reduce the number of computational steps and amount of information required from decision makers when dealing with 'uncertainty in data' and 'interaction between attributes' simultaneously. The proposed procedure is configured with the convergence of five key components namely factor analysis, a revised fuzzy-linguistic estimator, Choquet integral, Mikhailov's fuzzy AHP method, and simple weighted average (SWA) in order to assure it functions as anticipated. Overall, the steps of implementing the proposed procedure can be summarized as follows.

Step 1: Defining problem and identifying evaluation attributes
Step 2: Constructing linguistic scale for performance measurement
Step 3: Designing questionnaire and reliability test
>If the questionnaire is not reliable, revise it and go to step 3
>If it is reliable, proceed to step 4
Step 4: Data collection by means of questionnaire
Step 5: Deriving decision matrix of the problem (alternatives versus attributes)
Step 6: Transforming the collected data for factor analysis (FA)
>If the data infringed any criteria for sensible FA, then skip to step 9, 10, and 14
>If the data met all the criteria for sensible FA, proceed to step 7
Step 7: Performing factor analysis
Step 8: Decomposing problem into simpler hierarchy structure
Step 9: Identifying monotone measure weights within each factor using the revised fuzzy-linguistic approach
Step 10: Employing Choquet Integral to aggregate interactive scores within each actor
Step 11: Constructing new decision matrix (alternatives versus factors)
Step 12: Using Mikhailov's fuzzy AHP to estimate the weights of independent factors
Step 13: Compute the global score of each alternative via simple weighted average (SWA) operator
Step 14: Rank the alternatives based on the global scores

The elaborations on these 14 steps are offered in the next sections. Meanwhile, Figure 4.2 reflects the graphical illustration on the proposed procedure.

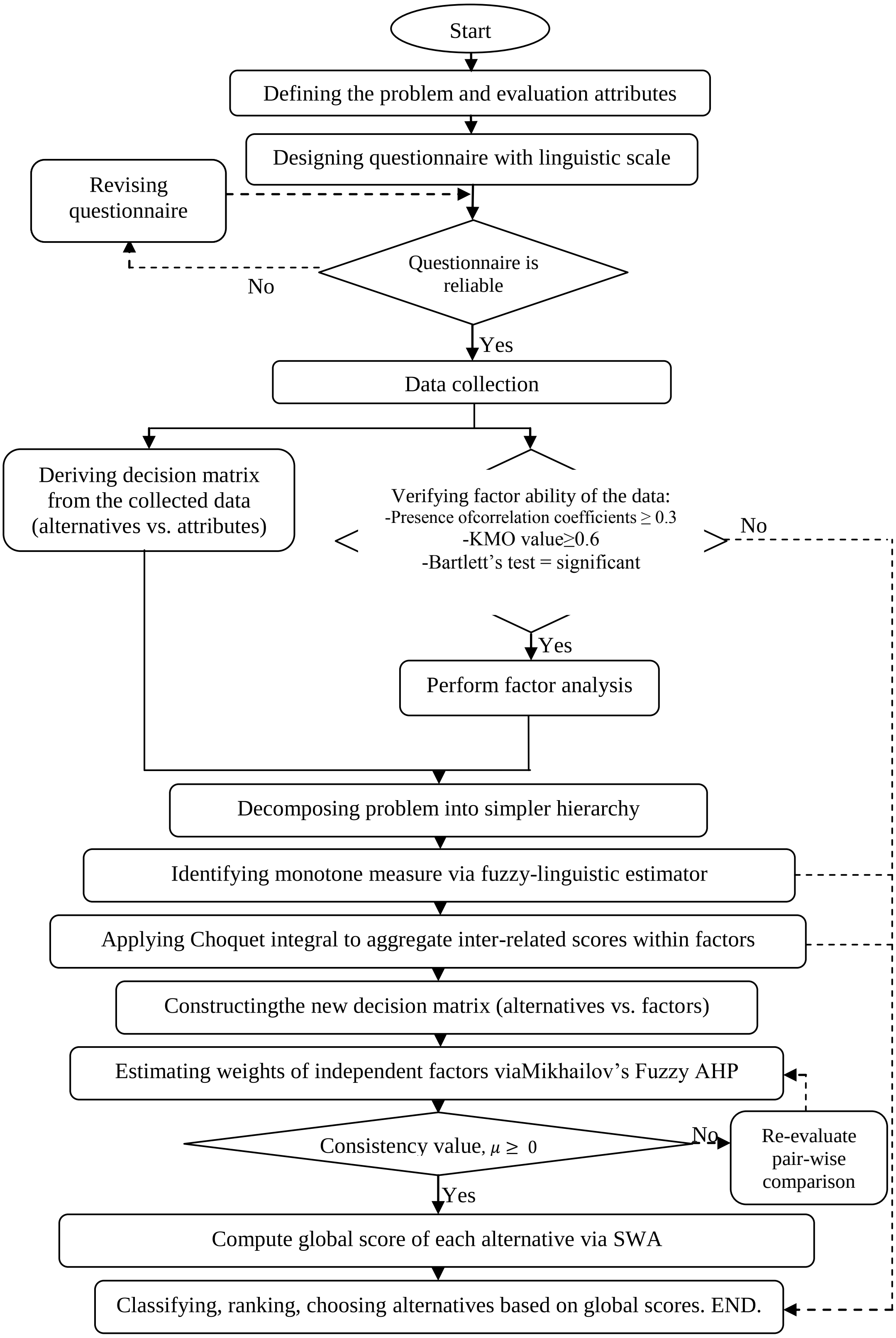


Figure 4.2: The Proposed Procedure

4.5.1 Defining Problem and Identifying Evaluation Attributes

Alike other MADM models, the first stage of the procedure requires decision makers to clearly delineate and understand the occurring problem, listing the available alternatives and deriving appropriate set of attributes to assess the performance of the pre-identified alternatives. Perhaps, this could be the most time consuming stage in employing the proposed procedure but it is necessary to invest sufficient amount of time to extract appropriate attributes as omitting any crucial attributes during evaluation could lead to a dreadful decision making. The attributes can be elicited by referring to past studies, based on decision makers' self-experience or through brainstorming with group of experts.

4.5.2 Constructing Linguistic Scale for Performance Measurement

Usually, the decision makers are required to express the performance of an alternative with respect to qualitative attributes using crisp scale. Therefore, in the proposed procedure, in order to capture the usual vagueness encompassed in the data provided by decision makers and for the sake of convenient data offering, linguistic scale is constructed. The steps to generate an appropriate linguistic scale for measuring performance of alternative can be recapitulated as follows.

Firstly, the decision makers need to determine the linguistic terms or preferences, $s_i = (s_0, s_1, \dots, s_T)$ to assess the performance of alternatives where s_0 denotes 'extremely unsatisfactory' and s_T denotes 'extremely satisfactory'.

Secondly, the triangular fuzzy number (TFN) that corresponds to each linguistic term is identified via Zhu's equation (4.1).

$$\widetilde{A_i} = (l_i, m_i, u_i) = \left(max\left\{\frac{i-1}{T}, 0\right\}, \frac{i}{T}, min\left\{\frac{i+1}{T}, 1\right\} \right) \tag{4.1}$$

In (4.1), $\widetilde{A_i}$ represents TFN consisting l_i (lower value), m_i (most probable value), u_i (upper value) that best represent each linguistic preference, s_i. Meanwhile, T represents the ranking of final linguistic preference (for instance, if a decision maker has determined nine linguistic preferences for the assessment then, $T = 8$ as the ranking on the linguistic terms starts at zero level based on the Zhu's equation). In this procedure, the Zhu's equation is used for fuzzification purpose due to its simplicity and its merit on helping inexperienced decision makers who are unable to clearly define the fuzzy number corresponding to each linguistic term as explained in chapter two. Finally, with the available linguistic preference and their corresponding TFNs, the $(\mathrm{T} + 1)$- point linguistic scale can be constructed.

4.5.3 Designing Questionnaire and Reliability Test

A questionnaire is then designed by adhering to the constructed $(T + 1)$- point linguistic scale in order to acquire the performance scores of each alternative with respect to each attribute. However, prior to carrying out the actual survey, it is advisable and obligatory for the decision makers to conduct a pilot test to authenticate the reliability of the questionnaire. The proposed procedure suggests two widely accepted reliability estimators namely test-retest reliability or internal consistency reliability for the purpose of reliability assessment on the designed questionnaire.

The former approach is used to analyze the consistency of a measure from one time to another. According to Surhone, Timpledon, and Marseken (2010), a questionnaire's test-retest reliability can be measured by conducting the same survey within the same group of respondents in two different time periods. They added that this approach is applicable if there is no drastic change is predicted in the construct being measured between the two time intervals. If the correlation between the separate surveys is equal or above 0.7, then it implies that the questionnaire has satisfactory test-retest reliability.

Meanwhile, the later approach is used to assess the consistency of results across items within a test or survey by computing Cronbach's alpha coefficient which ranges from 0 to 1 (Dornyei and Taguchi, 2010). Generally, Cronbach's alpha value, $\propto \geq 0.7$ indicates a good internal consistently of the questionnaire (George and Mallery, 2003). Research conducted by Hsu (2012) is an instance of MADM study which measures the reliability of questionnaire by computing Cronbach's alpha.

4.5.4 Data Collection by Means of Questionnaire

The reliable questionnaire is then used to gather data on performance of each alternative with respect to each attribute from a selected group of respondents. The raw data gained from the survey can be portrayed as in Table 4.1.

Table 4.1: Collected Raw Data Set by Means of Questionnaire

Alternatives	**Attributes/ Respondents**	$\boldsymbol{c_1}$	$\boldsymbol{c_2}$	$\cdots$	$\boldsymbol{c_n}$
$\boldsymbol{a_1}$	a_{1_1}	$s_{1_1 1}$	$s_{1_1 2}$	$\cdots$	$s_{1_1 n}$
	a_{1_2}	$s_{1_2 1}$	$s_{1_2 2}$	$\cdots$	$s_{1_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{1_r}	$s_{1_r 1}$	$s_{1_r 2}$	$\cdots$	$s_{1_r n}$
$\boldsymbol{a_2}$	a_{2_1}	$s_{2_1 1}$	$s_{2_1 2}$	$\cdots$	$s_{2_1 n}$
	a_{2_2}	$s_{2_2 1}$	$s_{2_2 2}$	$\cdots$	$s_{2_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{2_r}	$s_{2_r 1}$	$s_{2_r 2}$	$\cdots$	$s_{2_r n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\boldsymbol{a_m}$	a_{m_1}	$s_{m_1 1}$	$s_{m_1 2}$	$\cdots$	$s_{m_1 n}$
	a_{m_2}	$s_{m_2 1}$	$s_{m_2 2}$	$\cdots$	$s_{m_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{m_r}	$s_{m_r 1}$	$s_{m_r 2}$	$\cdots$	$s_{m_r n}$

Based on Table 4.1, $a_i = (a_1, a_2, \cdots, a_m)$ indicates the set of alternatives, $c_j = (c_1, c_2, \cdots, c_n)$ represents the set of attributes, and $a_{i_k} = (a_{i_1}, a_{i_2}, \dots, a_{i_r})$ denotes the set of respondents who evaluate alternative i where r implies number of respondents. Meanwhile, $s_{i_k j} = (s_{i_k 1}, s_{i_k 2}, \dots, s_{i_k j})$ represents the linguistic scores of alternative, i that expressed by respondent, k.

The collected raw data will be further utilized to construct the decision matrix of the existing problem as well as for the execution of factor analysis. Therefore, in order to ensure a meaningful factor analysis, it is essential to obtain an adequate total observations, N (N = number of respondents for each alternative, r $\times$ number of alternatives, m) during data collection. The proposed procedure uses the familiar ‘ten observations per attribute’ rule, (Treiblmaier and Filzmoser, 2010) to estimate the ample total observations for factor analysis. For instance, based on this rule, if there is a MADM problem involving 5 attributes then, N should be ≥ 50.

4.5.5 Deriving Decision Matrix of the Problem (Alternatives vs. Attributes)

The next stage in the proposed procedure is to derive the decision matrix (alternatives versus attributes) of the existing MADM problem from the available raw data. The process of generating decision matrix can be described as follows.

First of all, the linguistic scores in raw data need to be quantified into their corresponding TFNs based on constructed performance scale. The fuzzified raw data can be presented as shown in Table 4.2.

Table 4.2: Fuzzified Raw Data

Alternatives	**Attributes/ Observations**	$\boldsymbol{c_1}$	$\boldsymbol{c_2}$	$\cdots$	$\boldsymbol{c_n}$
$\boldsymbol{a_1}$	a_{1_1}	$\tilde{A}_{1_1 1}$	$\tilde{A}_{1_1 2}$	$\cdots$	$\tilde{A}_{1_1 n}$
	a_{1_2}	$A_{1_2 1}$	$\tilde{A}_{1_2 2}$	$\cdots$	$\tilde{A}_{1_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{1_r}	$\tilde{A}_{1_r 1}$	$\tilde{A}_{1_r 2}$	$\cdots$	$\tilde{A}_{1_r n}$
$\boldsymbol{a_2}$	a_{2_1}	$\tilde{A}_{2_1 1}$	$\tilde{A}_{2_1 2}$	$\cdots$	$\tilde{A}_{2_1 n}$
	a_{2_2}	$A_{2_2 1}$	$\tilde{A}_{2_2 2}$	$\cdots$	$\tilde{A}_{2_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{2_r}	$\tilde{A}_{2_r 1}$	$\tilde{A}_{2_r 2}$	$\cdots$	$\tilde{A}_{2_r n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\boldsymbol{a_m}$	a_{m_1}	$\tilde{A}_{m_1 1}$	$\tilde{A}_{m_1 2}$	$\cdots$	$\tilde{A}_{m_1 n}$
	a_{m_2}	$\tilde{A}_{m_2 1}$	$\tilde{A}_{m_2 2}$	$\cdots$	$\tilde{A}_{m_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{m_r}	$A_{m_r 1}$	$\tilde{A}_{m_r 2}$	$\cdots$	$\tilde{A}_{m_r n}$

According to Table 4.2, $\tilde{A}_{i_k j} = (\tilde{A}_{i_k 1}, \tilde{A}_{i_k 2}, \ldots, \tilde{A}_{i_k n})$ represents the fuzzy scores of alternative, i that derived from respondent, k. Secondly, the average fuzzy score, $\tilde{A}_{ij}$ of each alternative, i with respect to each attribute, j is computed based on equation (4.2).

$$\boldsymbol{\widetilde{A}_{ij} = \frac{1}{r}\sum_{k=1}^{r} \widetilde{A}_{i_k j}} \tag{4.2}$$

At the end of the averaging process, the fuzzified raw data as in Table 4.2 can be reduced into fuzzy decision matrix as shown in Table 4.3.

Table 4.3: Fuzzy Decision Matrix

Attributes/ Alternatives	c_1	c_2	...	c_n
a_1	$\tilde{A}_{11}$	$\tilde{A}_{12}$	...	$\tilde{A}_{1n}$
a_2	$\tilde{A}_{21}$	$\tilde{A}_{22}$	...	$\tilde{A}_{2n}$
⋮	⋮	⋮	...	⋮
a_m	$\tilde{A}_{m1}$	$\tilde{A}_{m2}$	...	$\tilde{A}_{mn}$

Based on Table 4.3, $\tilde{A}_{ij} = (\tilde{A}_{i1}, \tilde{A}_{i2}, \dots, \tilde{A}_{in})$ represents the fuzzy local scores of alternative, i.

Finally, to attain the final decision matrix for the problem, each of these fuzzy local scores, $\tilde{A}_{ij}$ is converted into crisp local scores, x_{ij} by employing the centre of area (COA) technique (4.3).

$$\boldsymbol{x_{ij} = l_{ij} + \left[\left(u_{ij} - l_{ij}\right) + \left(m_{ij} - l_{ij}\right)\right]/3} \tag{4.3}$$

Based on (4.3), l_{ij}, m_{ij}, and u_{ij} represent lower, middle, and upper value of a fuzzy local score, $\tilde{A}_{ij}$. The final decision matrix after the defuzzification process can be presented as in Table 4.4.

Table 4.4: Final Decision Matrix

Attributes/ Alternatives	c_1	c_2	...	c_n
a_1	x_{11}	x_{12}	...	x_{1n}
a_2	x_{21}	x_{22}	...	x_{2n}
⋮	⋮	⋮	...	⋮
a_m	x_{m1}	x_{m2}	...	x_{mn}

Based on Table 4.4, $x_{ij} = (x_{i1}, x_{i2}, \dots, x_{in})$ represents the crisp local scores of alternative, i.

4.5.6 Data Transformation for Factor Analysis

The same raw data from the survey is then utilized to perform factor analysis. However, since the raw data from the survey comprises scores in the form of linguistic terms, they need to be transformed into a valid form where factor analysis can be performed. Therefore, a simple data conversion approach applied in a study conducted by Senel and Senel (2011) is adapted in this proposed procedure. The data manipulation process before performing factor analysis can be summarized as follows.

Firstly, the raw data from survey need to be converted into fuzzified raw data as shown in Table 4.2. However, this step can be omitted as the fuzzified raw data should have been obtained in the process of deriving the decision matrix. Secondly, by using COA equation (4.3), the fuzzy scores are directly defuzzified into crisp scores. Finally, these crisp scores are translated into their equivalents in the $(T + 1)$-point Likert scale which can be computed using equation (4.4).

$$\boldsymbol{X_{i_k j} = x_{i_k j} \times (T + 1)} \tag{4.4}$$

Based on equation (4.4), $x_{i_k j}$ represents the crisp score of alternative, i with respect to attribute, j by respondent, k. Meanwhile, $X_{i_k j}$ denotes the equivalent value of $x_{i_k j}$ in $(T + 1)$ - point Likert scale. Table 4.5 shows the modified data which are in the valid state to be factor analyzed.

Table 4.5: Transformed Data for Factor Analysis

Alternatives	**Attributes/ Respondents**	$\boldsymbol{c_1}$	$\boldsymbol{c_2}$	$\cdots$	$\boldsymbol{c_n}$
$\boldsymbol{a_1}$	a_{1_1}	$X_{1_1 1}$	$X_{1_1 2}$	$\cdots$	$X_{1_1 n}$
	a_{1_2}	$X_{1_2 1}$	$X_{1_2 2}$	$\cdots$	$X_{1_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{1_r}	$X_{1_r 1}$	$X_{1_r 2}$	$\cdots$	$X_{1_r n}$
$\boldsymbol{a_2}$	a_{2_1}	$X_{2_1 1}$	$X_{2_1 2}$	$\cdots$	$X_{2_1 n}$
	a_{2_2}	$X_{2_2 1}$	$X_{2_2 2}$	$\cdots$	$X_{2_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{2_r}	$X_{2_r 1}$	$X_{2_r 2}$	$\cdots$	$X_{2_r n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\boldsymbol{a_m}$	a_{m_1}	$X_{m_1 1}$	$X_{m_1 2}$	$\cdots$	$X_{m_1 n}$
	a_{m_2}	$X_{m_2 1}$	$X_{m_2 2}$	$\cdots$	$X_{m_2 n}$
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
	a_{m_r}	$X_{m_r 1}$	$X_{m_r 2}$	$\cdots$	$X_{m_r n}$

4.5.7 Performing Factor Analysis Data

According to Emin Ocal, Oral, Erdis, and Vural (2007), factor analysis is a statistical tool which is capable to reduce larger set of variables or attributes into fewer numbers of underlying factors. The fundamental idea of factor analysis is based on correlation where attributes that belong to the same group are highly correlated among themselves but relatively have small correlation with attributes in different group (Dongxiao, Jie, and Ling, 2011; Zhang, Shin, and Pham, 2001). Since the evaluation attributes in MADM problems are not absolutely independent to each other, factor analysis can be exploited in order to extract the common factors where the factors are mutually independent (Feng, Wu, and Chia, 2010).

It is believed that apart from grouping a vast number of attributes into fewer and mutually independent factors, there are another three crucial reasons which motivates the inclusion of factor analysis into the proposed procedure. First of all, factor analysis will be helpful in unraveling a complex problem into a simpler hierarchy (Lin, Shiu, and Tzeng, 2011) and therefore, decision makers are able to analyze the MADM problem in a more interpretable and systematic mode. Besides, it helps to extract the main determinants of a MADM problem.

Moreover, it is expected that by conducting factor analysis, the actual number of monotone measure weights which need to be estimated prior to employing Choquet integral can be reduced from 2^n to $\sum_{p=1}^{q} 2^{|f_p|}$ where $f_p = (f_1, f_1, \dots, f_q)$ set of extracted factors, q denotes the total number of factors, and $|f_p|$ represents the number of attributes within factor, p. This phenomenon was justified in a study conducted by Lin, Shiu, and Tzeng (2011) as well.

After processing the raw data into an appropriate state as elaborated in section 4.5.6, factor analysis can be performed with the aid of SPSS software. The execution of factor analysis can be summarized as follows (DeCoster, 1998; Pallant, 2011).

Firstly, the suitability or factor-ability of obtained data for factor analysis need to be verified. The strength of inter-correlation between attributes needs to be investigated as it is not so appropriate to perform factor analysis if there is no any strong inter-correlation between attributes (Pallant, 2011).

To be considered suitable for factor analysis, the correlation matrix should show at least some correlation, $r \geq 0.3$ (Tabachnick and Fidell, 2007). Besides, another two statistical measures which can be generated via SPSS namely Barlett's test of sphericity and Kaiser- Meyer-Olkin (KMO) are also useful to determine the factor-ability of data. It is suggested the Bartlett's test of Sphercity should be statistically significant at $p < 0.05$ (Bartlett, 1954) and the KMO measure of sampling adequacy value ≥ 0.6 (Kaiser, 1974) for a sensible factor analysis. With regards to the proposed procedure, if even one of these conditions is not satisfied then, it is advisable to omit factor analysis and proceed the analysis with step 9, 10, and 14.

Secondly, the smallest number of factors that can be used to best represent the interrelations among the set of attributes is determined or extracted. As per the proposed procedure, the well-known principal component technique is employed to extract the number of underlying factors or dimensions. Then, the Kaiser's criteria (Cudeck, 2000) and scree test (Catell, 1966) rules can be further used as the guideline in identifying the final number of factors that should retained to represent the original attributes.

The third stage of factor analysis is known as factor rotation and interpretation stage. At this level, the factors are rotated for the sake of ease and meaningful interpretability on the extracted factors. In the proposed procedure, the frequently used rotation method, Varimax which yields uncorrelated factors (Treiblmaier and Filzmoser, 2010) is suggested for the rotation purpose. Then, the factors are interpreted or renamed based to the meaning of attributes. Certainly, some background knowledge on existing problem will be required for the renaming purpose.

4.5.8 Decomposing Problem into Simpler Hierarchy Structure

After performing factor analysis, in order to examine the problem in a more interpretable means and to implement the decision making process in a systematic mode, the existing problem are decomposed into simpler hierarchy diagram consisting of four major levels namely 'alternatives' , 'attributes' , 'factors' and 'goal' level.

4.5.9 Estimating Monotone Measure via Revised Fuzzy-Linguistic Estimator

The attributes within each factor are inter-dependent. Thus, the local scores within each factor will be aggregated using Choquet integral to obtain the 'factor scores' of an alternative. However, prior to employing Choquet integral, the monotone measure weights are identified. For this purpose, a trivial amendment is done on the existing fuzzy-linguistic estimator (Wang et al., 2010) to come up with an easy-to-implement identification approach for decision makers. The suggested estimation approach is as follows.

Firstly, the decision maker should determine the linguistic terms, $s_i = (s_0, s_1, \dots, s_T)$ to assess the individual contribution or importance of attributes toward their respective factor where s_0 denotes 'least important' and s_T denotes 'extremely important'. Secondly, each of these linguistic terms is quantified into corresponding TFN using Zhu's equation (4.1). With the determined linguistic terms and their corresponding TFNs, the $(T + 1)$- point linguistic scale can be constructed.

Based on the developed scale, the decision makers can express their own opinion on the importance of the attributes (in linguistic terms) which are then converted into corresponding TFNs. Subsequently, the average fuzzy importance, $\tilde{I}_{jp}$ of attribute, j corresponding to factor, p can be identified using equation (4.5).

$$\tilde{\boldsymbol{I}}_{\boldsymbol{ij}} = \frac{\mathbf{1}}{\boldsymbol{z}}\sum_{\boldsymbol{e}=\mathbf{1}}^{\boldsymbol{z}} \tilde{\boldsymbol{I}}_{\boldsymbol{j}_{\boldsymbol{d}_{\boldsymbol{e}}}\boldsymbol{p}} \qquad (4.5)$$

Suppose $d_e = \{d_1, d_2, \dots, d_z\}$ denotes the decision makers involved in the analysis. Then, based on equation (4.5), $\tilde{I}_{j_{d_e}p}$ represents the fuzzy importance of attribute, j given by decision maker, d_e with respect to factor, p and z implies the number of decision makers. The average fuzzy importance are then defuzzified into crisp importance via COA technique (4.3). These crisp importance actually represent the individual weight of attributes, $g_j = g_\lambda\{c_j\}$, $j = 1,2, \dots, n$. If the sum of individual weight of attributes within a factor, $\sum_{j=1}^{n} g_j = 1$ then, the interaction parameter λ of the specific factor is zero($\lambda = 0$). On the other hand, if $\sum_{j=1}^{n} g_j \neq 1$, the parameter λ can be calculated by solving the equation (4.6).

$$\mathbf{1} + \boldsymbol{\lambda} = \prod_{\boldsymbol{j}=\mathbf{1}}^{\boldsymbol{n}} (\mathbf{1} + \boldsymbol{\lambda}\boldsymbol{g}_{\boldsymbol{j}}) \qquad (4.6)$$

If $-1 \leq \lambda < 0$ then, it implies the attributes within the specific factor hold sub-additive effect. Meanwhile, if $\lambda = 0$ then, it implies the attributes within the factor are additive. Lastly, if $\lambda > 0$ then, it reflects the attributes possess super-additive effect.

Finally, with the available individual weights and interaction parameter, λ, the monotone measure weights can be identified using λ- measure equation (4.7).

$$\boldsymbol{g_\lambda(c_1, c_2, \ldots, c_n) = \frac{1}{\lambda}| \prod_{j=1}^{n}(1 + \lambda g_j) - 1|} \quad (4.7)$$

The identification process is executed gradually from one factor to the other.

4.5.10 Using Choquet Integral to Aggregate Interactive Scores

The available λ-measure weights and local scores will be then replaced into Choquet integral model (4.7) to compute the score for each factor. Let g_λ be a monotone measure on $c_j = (c_1, c_2, \ldots, c_n)$ and $x_j = (x_1, x_2, \ldots, x_n)$ be the performance score of an alternative with respect to each attribute in c_j. Suppose $x_1 \geq x_2 \geq \cdots \geq x_n$. Then, T_n = $\{c_1, c_2, \ldots, c_n\}$ and the aggregated score using Choquet integral can be identified using (4.8)

$$\boldsymbol{Choquet_{g_\lambda}(x_1, x_2, \ldots, x_n)}$$
$$\boldsymbol{= x_n . g_\lambda\{T_n\} + [x_{n-1} - x_n] . g_\lambda\{T_{n-1}\} + \ldots + [x_1 - x_2] . g_\lambda\{T_1\}} \quad (4.8)$$
$$\boldsymbol{= x_n . g_\lambda\{c_1, c_2, \ldots, c_n\} + [x_{n-1} - x_n] . g_\lambda\{c_1, c_2, \ldots, c_{n-1}\} + \cdots + [x_1 - x_2] . g_\lambda\{c_1\}}$$

where the arrangement of attributes in T_n parallel with the descending order of the performance scores.

4.5.11 Construction of New Decision Matrix (Alternatives vs. Factors)

At the end of foregoing stage, each alternative will have a set of factor scores (scores with respect to each factor). Therefore, a new decision matrix, alternatives versus factors, can be constructed as presented in Table 4.6. The further analysis will rely on this decision matrix.

Table 4.6: New Decision Matrix (Alternatives vs. Factors)

Factors/ Alternatives	$\boldsymbol{f_1}$	$\boldsymbol{f_2}$	...	$\boldsymbol{f_q}$
$\boldsymbol{a_1}$	y_{11}	y_{12}	...	y_{1f}
$\boldsymbol{a_2}$	y_{21}	y_{22}	...	y_{2f}
⋮	⋮	⋮	...	⋮
$\boldsymbol{a_m}$	y_{m1}	y_{m2}	...	y_{mq}

Based on Table 4.6, $y_{ip} = (y_{i1}, y_{i2}, \dots, y_{iq})$ refers to a set of factor scores of alternative, i with respect to factors, $f_p = (f_1, f_2, \dots, f_q)$.

4.5.12 Estimating Weights of Independent Factors

Since the extracted factors are completely independent to each other, the global score of an alternative can be simply obtained by integrating factor scores via SWA operator. However, the weight for each independent factor has to be identified prior to applying SWA. For this purpose, Mikhailov's Fuzzy AHP method will be employed.

Besides dealing with aspect of uncertainty by allowing the decision makers to express their comparison in linguistic terms, this approach has the capability to simultaneously derive the consistency value of pair-wise comparison and weight of factors in the form of crisp value. In addition, Mikhailov claimed that due to the non-linearity issue in Saaty's $\tilde{1}$–$\tilde{9}$ scale which lies in the region of values between $\tilde{9}^{-1}$ and $\tilde{1}^{-1}$, constructing fuzzy reciprocal matrices could lead to some other problems such as rank reversal. Thus, in order to avoid using reciprocal judgment, in this fuzzy AHP, the decision makers are only required to provide assessment whenever factor f_a is equally or more important than f_b. If it is found that f_a is less important than f_b then, the evaluation should be done oppositely where f_b is compared to f_b.

With regards to the proposed procedure, the steps to execute Mikhailov's fuzzy AHP for the estimation of factors' weights can be described as follows. Firstly, for sake of simplicity, the decision makers are required to linguistically express the relative importance of factors through a single pair-wise comparison matrix (after achieving consensus) based on Saaty's fuzzy AHP scale as shown in Table 4.7 (Cakir and Canbolat, 2008). The reciprocal judgments are not offered in the table 4.7 as they are not required in implementing Mikhailov's fuzzy AHP.

Table 4.7: Saaty's fuzzy AHP scale (Cakir and Canbolat, 2008)

Linguistic terms	**Corresponding TFN**	**Descriptions**
Equally important	$\tilde{1} = (1,1,2)$	Two elements contribute equally
Slightly important	$\tilde{3} = (2,3,4)$	One element is slightly favoured over another
Strongly important	$\tilde{5} = (4,5,6)$	One element is strongly favoured over another
Very strongly important	$\tilde{7} = (6,7,8)$	One element is very strongly favoured over another
Extremely important	$\tilde{9} = (8,9,9)$	One element is most favoured over another
The intermediate values	$\tilde{2} = (1,2,3)$, $\tilde{4} = (3,4,5)$, $\tilde{6} = (5,6,7)$, $\tilde{8} = (7,8,9)$	Used to compromise between two judgments

Secondly, the linguistic terms in the assessed pair-wise comparison matrix are then converted into their corresponding TFN. Finally, the nonlinear optimization model (4.9) as suggested by Mikhailov (2003) is developed to concurrently derive the consistency value of pair-wise comparison and the weights of factors.

$$\begin{aligned}
&\text{Maximize } \boldsymbol{\mu} \\
&\text{Subject to } (\boldsymbol{m_{ab}} - \boldsymbol{l_{ab}})\boldsymbol{\mu w_b} - \boldsymbol{w_a} + \boldsymbol{l_{ab} w_b} \leq \mathbf{0}, \\
&(\boldsymbol{u_{ab}} - \boldsymbol{m_{ab}})\boldsymbol{\mu w_b} + \boldsymbol{w_a} - \boldsymbol{u_{ab} w_b} \leq \mathbf{0}, \\
&\sum_{\boldsymbol{p}=\mathbf{1}}^{\boldsymbol{q}} \boldsymbol{w_p} = \mathbf{1}, \boldsymbol{w_p} > \mathbf{0}, \boldsymbol{p} = \mathbf{1}, \dots \boldsymbol{q}
\end{aligned} \tag{4.9}$$

With regards to the proposed procedure, l_{ab}, u_{ab}, and m_{ab} are the lower, upper and most probable values corresponding to the fuzzy judgment given by the decision makers when comparing factor, a with respect to b. Meanwhile, w_p denotes the weight for factor, p and μ is the consistency index of the pair-wise comparison.

According to Mikhailov (2003), if the consistency index is positive ($\mu \geq 0$) then, it indicates that the fuzzy pair-wise comparison matrix is being consistent where all the solution ratios completely satisfy the initial judgments such that $l_{ij} \leq \frac{w_i}{w_j} \leq u_{ij}$. Meanwhile, if the consistency index is negative ($\mu < 0$) then, it implies that the comparison matrix is being inconsistent and re-evaluation on the pair-wise comparison is required.

4.5.13 Applying Simple Average Weighted to Compute Global Score

After identifying the weights for each independent factor through Mikhailov's fuzzy AHP, SWA operator (4.10) is applied to compute the global score of each alternative.

$$\sum_{p=1}^{q}(\boldsymbol{w_p}.\boldsymbol{y}_{\mathbf{p}}) \tag{4.10}$$

In relevant to the proposed procedure, w_p denotes the weight for factor, p and y_{p} denotes the score of an alternative with respect to factor, p. Then, the alternatives can be ranked, classified or chosen based on their global scores. An alternative with the highest global score reflects the most favorable alternative whereas an alternative with the lowest global score indicates the most unfavorable alternative.

4.6 Numerical Example

The purpose of this section is to facilitate a better understanding on the computational process involved in the proposed procedure. With regards to this, a simple MADM problem is formed and the steps involved in solving the problem using the proposed procedure are presented.

Suppose a decision maker is required to perform an assessment on the reputation of three airline industries in Malaysia namely a_1, a_2, and a_3. Then, as the first step to employ the procedure, the decision maker identifies five attributes namely scheduling (*s*), promptness (*p*), comfort of seats (*c*), cabin service (*cs*), and routes (*r*) to measure the reputation of each airline.

In the second step, he determines five linguistic preferences ($s_0 =$ extremely unsatisfactory, $s_1 =$ unsatisfactory, $s_2 =$fair, $s_3 =$satisfactory, $s_4 =$ extremely unsatisfactory) to assess the performance of airlines with respect to each attribute. In this case, $T = 4$. By using equation (4.1), the TFN corresponding to each linguistic preference is identified as presented in Table 4.8. As a result, a 5- point linguistic scale for measuring airlines' performance as portrayed in Figure 4.3 is developed.

Table 4.8: Linguistic Terms and Their Corresponding TFNs (Airline Problem)

Linguistic term	**TFN**
s_0 = Extremely unsatisfactory	$\tilde{A}_0 = \left(max\left\{\frac{0-1}{(5-1)}, 0\right\}, \frac{0}{(5-1)}, min\left\{\frac{0+1}{(5-1)}, 1\right\}\right) = (0,0,0.25)$
s_1 =Unsatisfactory	$\tilde{A}_1 = (0,0.25,0.5)$
s_2 = Fair	$\tilde{A}_2 = (0.25,0.5,0.75)$
s_3 = Satisfactory	$\tilde{A}_3 = (0.5,0.75,1)$
s_4 = Extremely satisfactory	$\tilde{A}_4 = (0.75,1,1)$

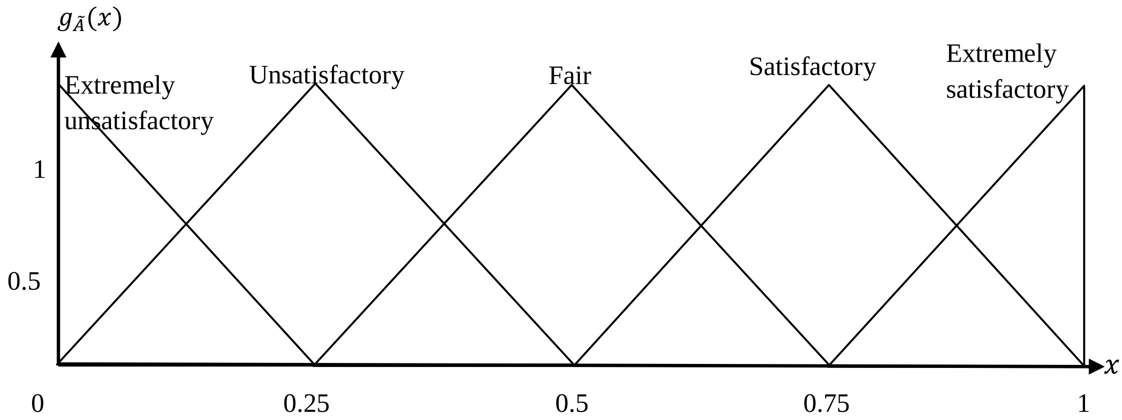


Figure 4.3: 5- point Linguistic Scale for Measuring Airlines' Performance

Based on the developed 5- point linguistic scale, a questionnaire is designed in order to assess the performance of each airline corresponding to each attribute. For this toy example, further assume that a pilot test is conducted to measure the internal consistency of the questionnaire where the computed Cronbach's alpha surpasses

0.7. This value indicates that the instrument is reliable for data collection and can be employed for actual survey.

The decision maker then disseminates the questionnaires to the particular airlines consumers and asks them to linguistically express their satisfactions towards each airline with respect to each attribute. In this example, assume that there are three respondents ($r = 3$) which renders to the total observations of nine ($N = 9$). Presume that the raw data set yielded from this survey is as presented in Table 4.9.

Table 4.9: Raw Data Set of Airline Problem

Airlines	**Attributes/ Respondents**	**s**	**p**	**c**	**cs**	**r**
$\boldsymbol{a_1}$	$\boldsymbol{a_{1_1}}$	U	EU	S	S	ES
	$\boldsymbol{a_{1_2}}$	EU	F	F	S	F
	$\boldsymbol{a_{1_3}}$	F	F	ES	ES	S
$\boldsymbol{a_2}$	$\boldsymbol{a_{2_1}}$	U	S	ES	S	S
	$\boldsymbol{a_{2_2}}$	EU	S	ES	F	F
	$\boldsymbol{a_{2_3}}$	ES	EU	EU	S	ES
$\boldsymbol{a_3}$	$\boldsymbol{a_{3_1}}$	ES	U	F	S	ES
	$\boldsymbol{a_{3_2}}$	S	U	U	S	F
	$\boldsymbol{a_{3_3}}$	F	F	S	ES	ES

**EU = extremely unsatisfactory, U = Unsatisfactory, F = fair, S = satisfactory, ES = extremely satisfactory*

In the process of deriving the decision matrix for this problem, firstly, the decision maker quantifies the linguistic scores in raw data set into their corresponding TFNs based on the 5- point linguistic scale. Hence, the fuzzified data set as presented in Table 4.10 was obtained.

Table 4.10: Fuzzified Data Set of Airline Problem

Airline	Attributes/ Respondents	*s*	*p*	*c*	*cs*	*r*
a_1	a_{1_1}	(0,0.25,0.5)	(0,0,0.25)	(0.5,0.75,1)	(0.5,0.75,1)	(0.75,1,1)
	a_{1_2}	(0,0,0.25)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)
	a_{1_3}	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)	(0.75,1,1)	(0.5,0.75,1)
a_2	a_{2_1}	(0,0.25,0.5)	(0.5,0.75,1)	(0.75,1,1)	(0.5,0.75,1)	(0.5,0.75,1)
	a_{2_2}	(0,0,0.25)	(0.5,0.75,1)	(0.75,1,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
	a_{2_3}	(0.75,1,1)	(0,0,0.25)	(0,0,0.25)	(0.5,0.75,1)	(0.75,1,1)
a_3	a_{3_1}	(0.75,1,1)	(0,0.25,0.5)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.75,1,1)
	a_{3_2}	(0.5,0.75,1)	(0,0.25,0.5)	(0,0.25,0.5)	(0.5,0.75,1)	(0.25,0.5,0.75)
	a_{3_3}	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.75,1,1)	(0.75,1,1)

Subsequently, the average fuzzy scores of each airline is computed using equation (4.2) to generate fuzzy decision matrix of the problem as presented in Table 4.11.

Table 4.11: Fuzzy Decision Matrix of Airline Problem

Attributes/ Alternatives	*s*	*p*	*c*	*cs*	*r*
a_1	$= \frac{1}{3}(0+0+0.25, 0.25+0+0.5, 0.5+0.25+0.75) = (0.083,0.25,0.5)$	(0.167,0.333,0.583)	(0.5,0.75,0.917)	(0.583,0.833,1)	(0.5,0.75,0.917)
a_2	(0.25,0.417,0.583)	(0.333,0.667,0.75)	(0.5,0.75,0.875)	(0.417,0.667,0.917)	(0.5,0.75,0.917)
a_3	(0.5,0.75,0.917)	(0.083,0.333,0.583)	(0.25,0.5,0.75)	(0.583,0.833,1)	(0.583,0.833,0.917)

Then, the final decision matrix for the further analysis is acquired by defuzzifying the average fuzzy scores in Table 4.11 into their respective crisp scores via the usage of COA technique (4.3). The final decision matrix of the problem after the defuzzification process is as shown Table 4.12.

Table 4.12: Decision Matrix of Airline Problem

Attributes/ Alternatives	***s***	***p***	***c***	***cs***	***r***
$\boldsymbol{a_1}$	0.278	0.361	0.721	0.805	0.722
$\boldsymbol{a_2}$	0.236	0.583	0.708	0.667	0.722
$\boldsymbol{a_3}$	0.722	0.333	0.5	0.805	0.778

Next, in order to prepare the data for factor analysis, the decision maker defuzzifies the fuzzy scores in Table 4.10 into crisp scores using COA equation (4.3). The crisp data set for this problem is as presented in Table 4.13.

Table 4.13: Crisp Data Set of Airline Problem

Airlines	**Attributes/ Respondents**	***s***	***p***	***c***	***cs***	***r***
	$\boldsymbol{a_{1_1}}$	0.25	0.0832	0.75	0.75	0.9168
$\boldsymbol{a_1}$	$\boldsymbol{a_{1_2}}$	0.0832	0.5	0.5	0.75	0.5
	$\boldsymbol{a_{1_3}}$	0.5	0.5	0.9168	0.9168	0.75
	$\boldsymbol{a_{2_1}}$	0.25	0.75	0.9168	0.75	0.75
$\boldsymbol{a_2}$	$\boldsymbol{a_{2_2}}$	0.0832	0.75	0.9168	0.5	0.5
	$\boldsymbol{a_{2_3}}$	0.9168	0.0832	0.0832	0.75	0.9168
	$\boldsymbol{a_{3_1}}$	0.9168	0.25	0.5	0.75	0.9168
$\boldsymbol{a_3}$	$\boldsymbol{a_{3_2}}$	0.75	0.25	0.25	0.75	0.5
	$\boldsymbol{a_{3_3}}$	0.5	0.5	0.75	0.9168	0.9168

These crisp scores are then translated into their equivalents in the 5-point Likert scale using equation (4.4) as shown in Table 4.14 where at this phase the data is all set to be factor analyzed. Based on the '10 observations per attribute' rule, the minimum total observations, N for this problem should be 50 to assure a meaningful factor analysis. However, for sake of simplicity, this study has deliberately maintained a small number of total observations, N.

Table 4.14: Data for Factor Analysis: Airline Problem

Airlines	**Attributes/ Respondents**	***s***	***p***	***c***	***cs***	***r***
$\boldsymbol{a_1}$	$\boldsymbol{a_{1_1}}$	$= 0.25 \times 5 = 1.25$	0.416	3.75	3.75	4.584
	$\boldsymbol{a_{1_2}}$	0.416	2.5	2.5	3.75	2.5
	$\boldsymbol{a_{1_3}}$	2.5	2.5	4.584	4.584	3.75
$\boldsymbol{a_2}$	$\boldsymbol{a_{2_1}}$	1.25	3.75	4.584	3.75	3.75
	$\boldsymbol{a_{2_2}}$	0.416	3.75	4.584	2.5	2.5
	$\boldsymbol{a_{2_3}}$	4.584	0.416	0.416	3.75	4.584
$\boldsymbol{a_3}$	$\boldsymbol{a_{3_1}}$	4.584	1.25	2.5	3.75	4.584
	$\boldsymbol{a_{3_2}}$	3.75	1.25	1.25	3.75	2.5
	$\boldsymbol{a_{3_3}}$	2.5	2.5	3.75	4.584	4.584

Assume that the decision maker carries out factor analysis using the data in Table 4.14 and the three statistical preconditions for a significant factor analysis are satisfied where there are some correlation coefficients, r among attributes surpassed 0.3, the Kaiser-Meyer Olken value is larger than 0.6 and Barlett's test of Sphercity is statistically significant. In addition, assume that after completing factor analysis, two common factors are extracted. The first factor which is renamed as 'planning' consists of scheduling, on time performance, and routes attributes. Meanwhile, the second factor which is labeled as 'service' comprises comfort of seats and cabin service attributes.

Next, by referring to the output of factor analysis, the decision maker decomposes the airline problem into simpler yet interpretable hierarchy structure encompassing 4 levels as illustrated in Figure 4.4.

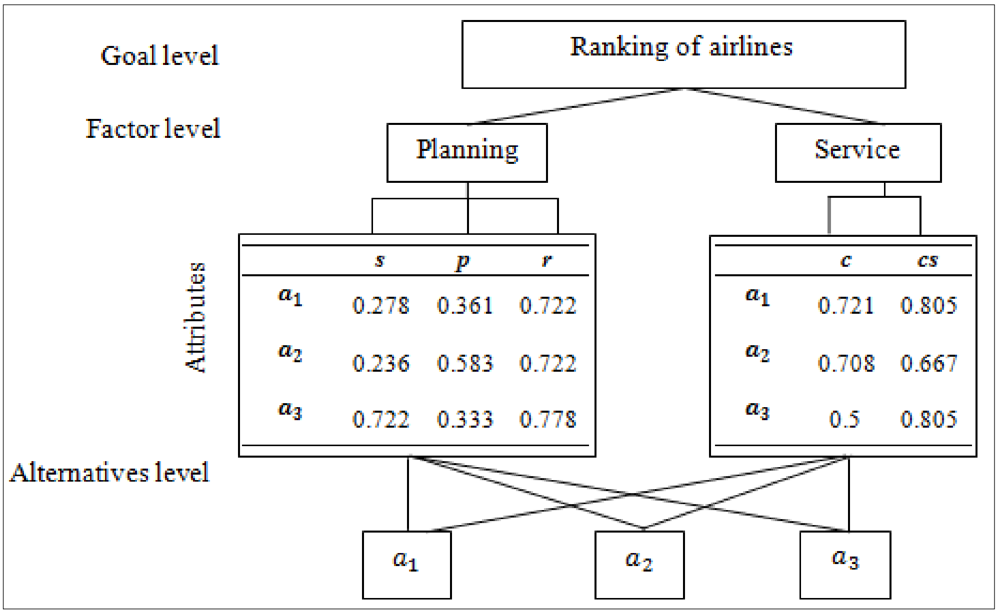


Figure 4.4: Hierarchy Structure of Airline Problem

Since the scores within each factor possess interactive characteristic, Choquet integral should be used to aggregate these scores. Therefore, before employing Choquet integral, the decision maker estimates the weights of monotone measure using the revised version of fuzzy-linguistic estimator. The identification process can be summarized as follows.

First of all, the decision maker determines five linguistic terms, $s_i = (s_0 =$ least important, $s_1 =$ important, $s_2 =$strongly important, $s_3 =$ very strongly important, $s_4 =$ extremely important) to assess the individual importance of attributes. In this case, $T = 4$. Then, the TFN corresponding to each linguistic term are identified via equation (4.1). With the determined linguistic terms and their corresponding TFNs, 5- point linguistic scale for the assessment on the importance of attributes is constructed.

Based on the developed scale, the decision maker linguistically expresses the individual importance of attributes which are then converted into corresponding TFNs and finally, defuzzifies them into crisp importance via equation (4.3). These crisp importance actually represent the individual weight of attributes. The evaluation on the importance of attributes and identification of individual weights for airline problem are summarized in Table 4.15. Note, since this problem only involve one decision maker then, the equation (4.5) is not utilized.

Table 4.15: Individual Weight of Attributes within Each Factor

Factors	**Attributes**	**Importance (in linguistic terms)**	**Corresponding TFN**	**Crisp values/ Individual weights of attributes)**
Planning	Scheduling	Very strongly important	(0.75,1,1)	0.9168
	On time performance	Strongly important	(0.5,0.75,1)	0.75
	Routes	Least important	(0,0.25,0.5)	0.25
Service	Comfort of seats	Important	(0.25,0.5,0.75)	0.5
	Cabin service	Least important	(0,0.25,0.5)	0.25

With the available individual weights, equation (4.6) is then applied to identify the interaction parameter, λ of each factor and subsequently, equation (4.7) is employed to estimate the monotone measure weights within each factor. The identified monotone measure weights for each factor for the airline problem are summarized in Table 4.16.

Table 4.16: Weights of Monotone Measure for Airline Problem

Factors	Parameter, λ	Subsets	Weights
Planning	-0.9796	$g_\lambda\{\emptyset\}$	0
		$g_\lambda\{s\}$	0.9168
		$g_\lambda\{p\}$	0.75
		$g_\lambda\{r\}$	0.25
		$g_\lambda\{s,p\}$	0.9932
		$g_\lambda\{s,r\}$	0.9422
		$g_\lambda\{p,r\}$	0.8163
		$g_\lambda\{s,p,r\}$	1
Service	2	$g_\lambda\{\emptyset\}$	0
		$g_\lambda\{c\}$	0.5
		$g_\lambda\{cs\}$	0.25
		$g_\lambda\{c,cs\}$	1

For better understanding, the computation involved in estimating $g_\lambda\{s,p\}$ is provided. Prior to identifying the weights of monotone measure, the interaction parameter, λ of attributes within the 'planning' factor is estimated using equation (4.6) as follows.

$$1+\lambda=\prod_{i=1}^{n}(1+\lambda g_i)$$

$$\lambda+1=(0.9168\lambda+1)(0.75\lambda+1)(0.25\lambda+1)$$

$$0.1719\lambda^3+1.1044\lambda^2+0.9169\lambda=0$$

By solving the above equation, following roots are obtained, $\lambda = (0,-5.445,-0.9796)$. Since $-1<\lambda<\infty$, then the value -5.445 is discarded. In addition, since $\sum_{i=1}^{n} g_i \neq 1$ then, the value 0 is also unacceptable. Therefore, for this problem, $\lambda=-0.9796$ is pertinent. This value implies that the three attributes within 'planning' factor are sharing sub-additive interaction.

Then, the identified individual weight of scheduling and promptness and λ value are replaced into equation (4.6) to estimate $g_\lambda\{s, p\}$ as follows.

$$g_\lambda\{Sh, Pr\} = \left|\frac{[(1 + (-0.9796 \times 0.9168)][1 + (-0.9796 \times 0.75)] - 1}{-0.9796}\right| = 0.9932$$

After completely identifying all the required λ- measure weights, Choquet integral model (4.8) is employed to aggregate the interactive local scores within each factor to obtain the factor scores of each alternative. Then, based on these factor scores, a new decision matrix (airlines vs. factors) is constructed as shown in Table 4.17 where the further evaluation is performed by adhering to this matrix.

Table 4.17: New Decision Matrix (Airlines vs. Factors)

Attributes/ Airlines	**Planning**	**Service**
a_1	0.436	0.742
a_2	0.554	0.688
a_3	0.714	0.714

To facilitate clearer understanding on the aggregation using Choquet integral, the computational process involved in finding the factor score of airline a_1 with respect to 'planning' is elaborated as follows. First of all, the local scores within the 'planning' factor are arranged in descending order where $x_{Ro} \geq = x_{Pr} \geq x_{Sh}$ and thus, T_n= $\{Ro, Pr, Sh\}$. Then, by adhering to the λ-measure weights estimated in Table 4.15 and based on local scores of a_1 as in Table 4.11, the factor score of $a_1$with respect to 'planning' is computed as follows by using equation (4.8).

$y_{1(Planning)}$

$= 0.278 \times g_\lambda\{Ro, Pr, Sh\} + (0.361 - 0.278) \times g_\lambda\{Ro, Pr\} + (0.722 - 0.361) \times g_\lambda\{Ro\}$

$= 0.278 \times 1 + 0.083 \times 0.8163 + 0.361 \times 0.25 = 0.436$

Later, in order to estimate the weights for independent factors, the decision maker utilizes Mikhailov's fuzzy AHP technique. For this purpose, the decision maker linguistically expresses the relative importance between factors through pair-wise comparison matrix based on Saaty's fuzzy AHP scale. The linguistic preferences in the evaluated pair-wise comparison matrix are then transformed into their corresponding TFNs. Assume the fuzzy pair-wise comparison matrix for airline problem is as presented in Table 4.18.

Table 4.18: Pair-wise Comparison for Airline Problem

Factors	**Planning**	**Service**
Planning	(1,1,1)	(2,3,4)
Service		(1,1,1)

Based on the pair-wise comparison assessment, a mathematical programming model (4.9) as suggested by Mikhailov (2000) is constructed to derive the weight for factors and consistency value for pair-wise comparison concurrently.

$$\text{Max } \mu$$

$$\text{Subject to } \mu w_2 - w_1 + 2w_2 \leq 0$$

$$\mu w_2 + w_1 - 4w_2 \leq 0$$

$$w_1 + w_2 = 1$$

$$w_1, w_2 \geq 0$$

The solution obtained using EXCEL SOLVER shows w_1= 0.6698, w_2= 0.3209, and μ = 1 where w_1 denotes the weight for 'planning' factor, and w_2 denotes the weight for 'service' factor. μ = 1 implies the pair-wise comparison matrix is consistent.

Finally, according to estimated factors' weights and available factor scores, the global score of each alternative is computed via SWA operator (4.9). The final result of this airline problem is as presented in Table 4.19.

Table 4.19: Final Result of Airline Problem

Attributes/ Airlines	**Planning** w_1**= 0.6698**	**Service** w_2**= 0.3209**	**Global score**	**Ranking**
a_1	0.436	0.742	(0.6698×0.436) + (0.3209×0.742) =0.5301	3
a_2	0.554	0.688	0.5918	2
a_3	0.714	0.714	0.7074	1

The results obtained from the proposed procedure shows that airline a_3 topped the ranking as the outstanding airline which is then followed by airline a_2 and a_1.

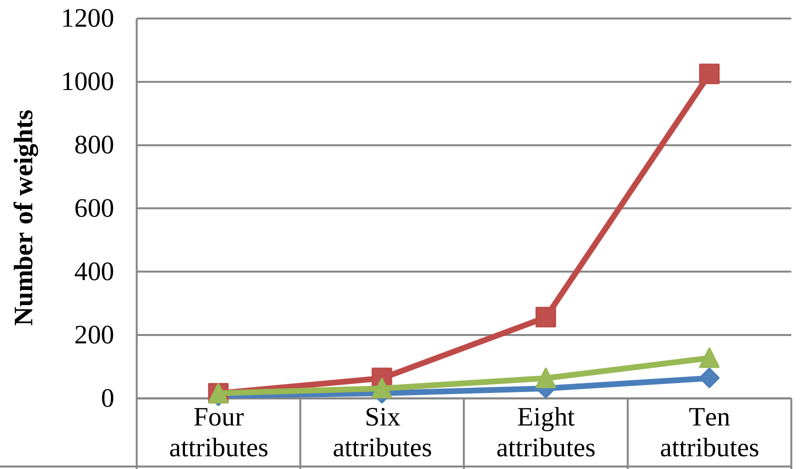
4.7 Comparing Proposed Procedure, GFCI, and Fuzzy Partitioned Hierarchy Model

In this section, the proposed procedure is compared with other MADM tools which able to deal with aspects of uncertainty in human's information and interaction between attributes namely, GFCI and fuzzy partitioned hierarchy model as highlighted in chapter one. Fuzzy ANP is discarded from this comparison as it completely involves different computational procedure (especially, it does not use Choquet integral) and thus, most of the comparison aspects are not applicable for it.

4.7.1 Comparison based on Numbers of Monotone Measure Weights Required

Firstly, the comparison is done based on the numbers of monotone measure weights required by each of the method. It should be restated here that the actual number of monotone measure weights which need to be identified by the decision makers in a particular MADM problem is equivalent to 2^n, where n represents number of attributes. However, using proposed procedure these numbers can be reduced from 2^n to $\sum_{p=1}^{q} 2^{|f_p|}$ where $f_p = (f_1, f_1, \dots, f_q)$ denotes the set of extracted factors, q denotes the total number of factors, and $|f_p|$ represents number of attributes within factor, p. Meanwhile, the number of weights required remains as 2^n when using GCFI. On the other hand, fuzzy partitioned hierarchy reduces these numbers from 2^n to $\sum_{p=1}^{q} 2^{|f_p^-|} + \sum_{p=1}^{q} 2^{|f_p^+|}$ where $f_p^- = (f_1^-, f_2^-, \dots, f_q^-)$ represents set of factors extracted based on left values and $f_p^+ = (f_1^+, f_2^+, \dots, f_q^+)$ represents set of factors extracted based on right values.

To provide better illustration on this, a simple analysis is conducted where the numbers of monotone measure weights required by each of the method across different numbers of attributes (four, six, eight, and ten attributes) are identified. The result of the analysis is portrayed via Figure 4.5. In this analysis, since the proposed procedure and fuzzy partitioned hierarchy involves factor analysis, it is assumed that after factor analyzing, n attributes are extracted into two factors where each factor comprised of $\frac{n}{2}$ attributes.



	Four attributes	Six attributes	Eight attributes	Ten attributes
Proposed procedure	8	16	32	64
GCFI	16	64	256	1024
Fuzzy partitioned hierarchy	16	32	64	128

Figure 4.5: Number of Monotone Measure Weights Required by Each of the Method

Based on figure 4.5, it can be noticed that the requirement on numbers of monotone measure weights hikes up with increasing number of attributes, regardless of any methods. However, it can be concluded that the decision makers can expect to identify lesser number of monotone measure weights using the proposed procedure over the other two methods.

4.7.2 Comparison based on Amount of Information Required

Secondly, the number of information required from decision makers in implementing each of the method is investigated. However, fuzzy partitioned hierarchy is excluded from this comparison as there is no sufficient info on how the factors' weights should be derived (to understand what kind of information will be required from decision makers for this identification process).

By using the proposed procedure, the decision makers are only required to provide the individual importance of attributes and relative importance between factors which usually can be offered by decision maker easily (in linguistic terms). In has to be reminded here that the performance scores in the proposed procedure can be derived via the data collected from the respondents. Therefore, the total amount of information required from the decision makers in using the proposed procedure is, $\theta = n + q(q-1)/2$ where n denotes the number of attributes and q implies the number of factors. Meanwhile, GCFI requires the decision makers to estimate the performance of the alternatives, importance of attributes, and tolerance zone with respect to each attributes. Thus, the total information requirement is, $\theta = n(m+2)$.

For further understanding, the amount of information required from the decision makers in each of the method under varying number of attributes (four, six, eight, and ten) and alternatives (three, four, five alternatives) is analyzed. The results of the analysis are portrayed through Figure 4.6, Figure 4.7, and Figure 4.8. Again, since the proposed procedure and fuzzy partitioned hierarchy involves factor analysis, in this analysis, it is assumed after factor analyzing, n attributes are extracted into two factors where each factor comprised of $\frac{n}{2}$ attributes.

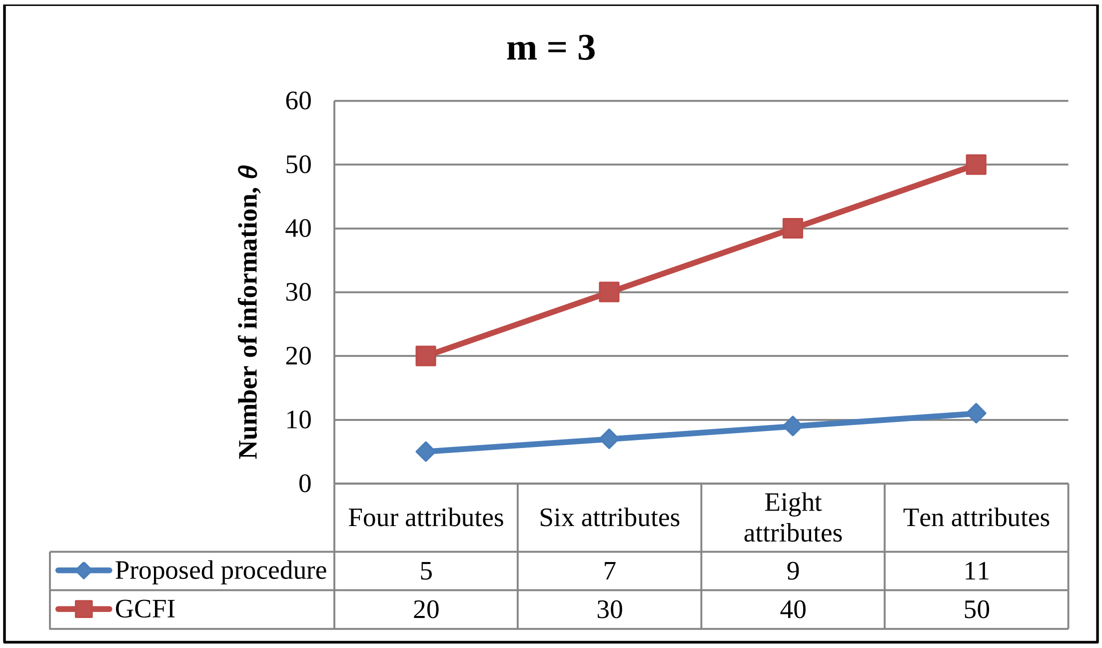


Figure 4.6: Number of Information Required From Decision Makers, (m = 3)

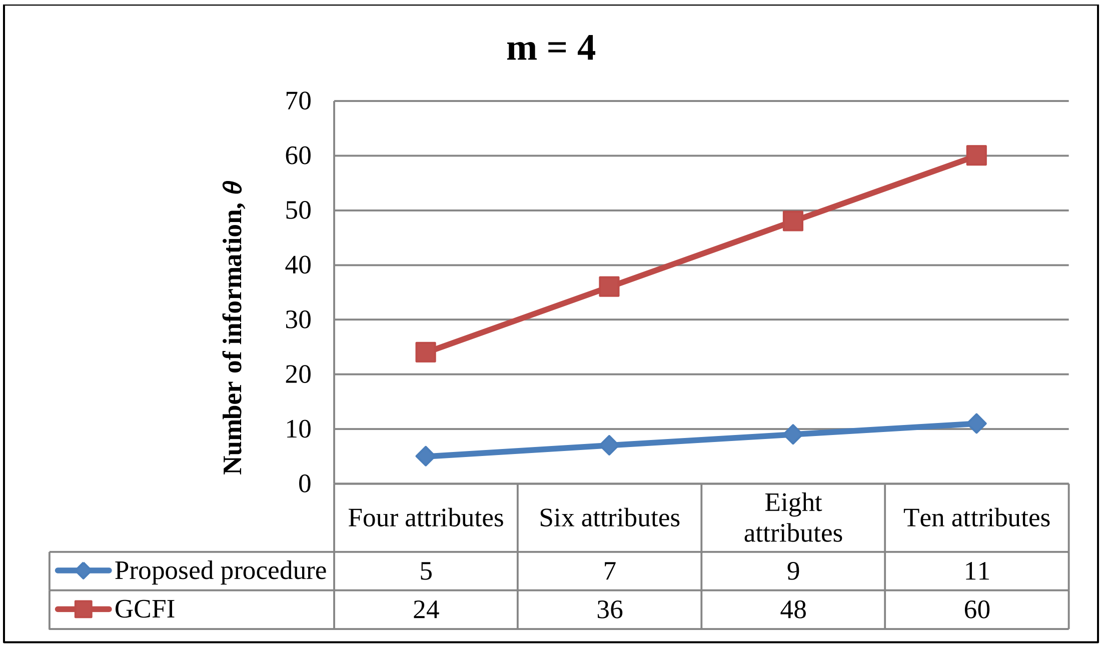


Figure 4.7: Number of Information Required From Decision Makers, (m = 4)

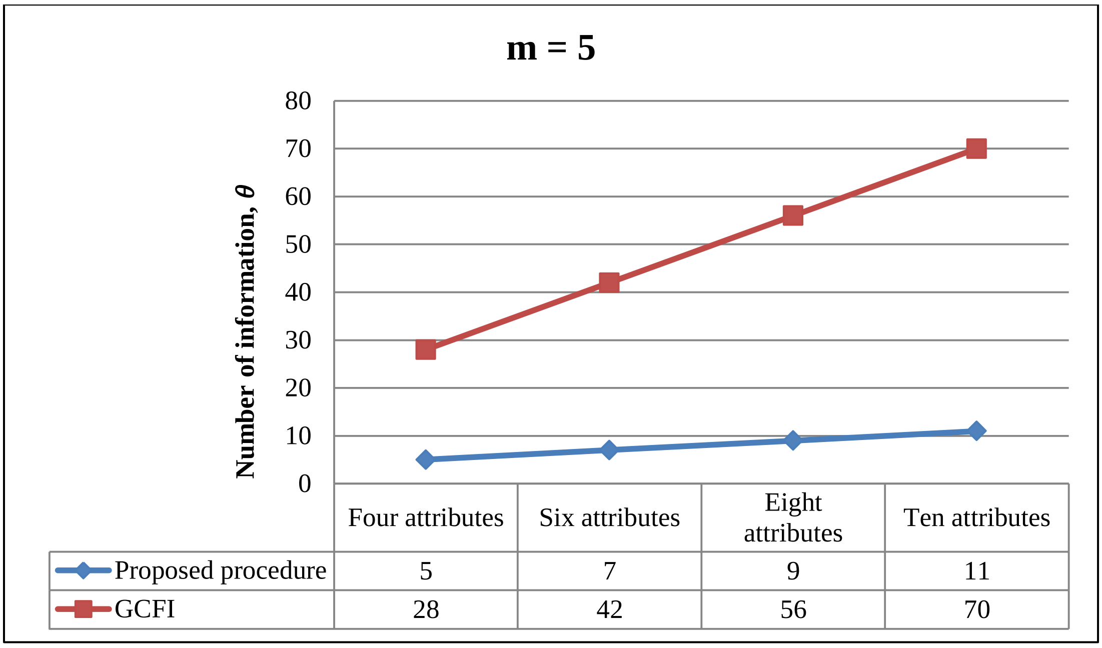


Figure 4.8: Number of Information Required From Decision Makers, $(m = 5)$

Based on Figure 4.6, Figure 4.7, and Figure 4.8, following conclusions can be drawn out. Firstly, the amount of information required from the decision makers in the proposed procedure is not influenced by the number of alternatives involved in a MADM problem. Secondly, the information requirement in both methods rises with the growing number of attributes. However, it can be noticed that the proposed procedure always requires lesser number of information from the decision makers than GCFI with respect to any scenarios in the analysis.

4.7.3 Comparison based on Other Aspects

The other differences between the proposed procedure, GFCI, and fuzzy partitioned hierarchy are summarized as portrayed in Table 4.20.

Table 4.20: Comparison between Proposed Procedure, GFCI, and Fuzzy Partitioned Hierarchy Model

Models/ Aspects	**Proposed procedure**	**GFCI (Tsai and Lu, 2006 and Demiral, Demiral, and Kahraman, 2010)**	**Fuzzy partitioned hierarchy (Lin, Shiu, and Tzeng, 2011)**
Data collection via questionnaire	The procedure has set some preconditions before and during data collection to ensure sensible output (i.e. reliability of questionnaire, total observations)	Not applicable	No specific requisitions or additional information on data collection
Identification of local scores	The same collected linguistic data is utilized to derive local scores	Need to be provided by decision makers or experts (in linguistic terms)	Need to be provided by decision makers or experts. In addition they are obliged to provide data in crisp or exact numbers. So, uncertainty in these particular data is not captured.
Usage of fuzzy numbers	Only applies triangular fuzzy numbers (TFNs) which naturally simple to be used by decision makers	Uses trapezoidal fuzzy numbers (TrFNs) which are then converted into interval valued fuzzy numbers consisting left values and rights. Therefore, twofold computational process are required	The collected data for factor analysis are converted into interval valued fuzzy numbers consisting left values and rights. This could yield two different grouping after performing fuzzy factor analysis and so, twofold computational steps are usually needed.
Uncertainty in data	Uncertainty in every type of data from human is taken into consideration. Even at the stage of identifying weights of factors, Mikhailov's fuzzy AHP is applied.	Uncertainty in every type of data from human in taken into consideration.	The uncertainties in most of the data from human are not taken into consideration except the data from respondents which are used for factor analysis. For example, it requires the decision makers or experts to exactly express the local scores. Besides, there is no any fuzzy approach suggested to estimate the weights of extracted factors.
Fuzzification approach	Uses Zhu's approach where by simply determining the linguistic terms for assessment and utilizing Zhu's equation, the fuzzy conversion scale can be constructed	No specific approach is defined and only uses the fuzzy scale from past literature	No specific approach is defined

Models/ Aspects	Proposed procedure	GFCI	Fuzzy partitioned hierarchy
Approach to identify monotone measure weights	Revised the existing fuzzy-linguistic estimator -Easy-to-implement -It models the uncertainty that exists in the provided data. -The required data for identification (individual importance of attributes) can be simply provided in linguistic terms -Finally, Sugeno equation is utilized to identify the monotone measure weights	Uses Sugeno equation to identify the weight. - Involves interval-valued fuzzy numbers (consisting right and left values). Therefore, twofold computational process needs to be carried out to identify the weights (extra complication for decision makers)	Uses GA based approach -Easy-to-implement -But, this approach has some drawbacks (i.e. failed to control the amount of information lost) - Involves interval-valued fuzzy numbers (consisting right and left values). Therefore, twofold computational process needs to be carried out to identify the weights
Capability in identifying the key determinants of the problem	The model is helpful in identifying the main determinants of problem thus, the existing problem can be decomposed into simpler hierarchy	Usually need to be identified by decision makers based on their knowledge and experience	The model helpful in identifying the main determinants of problem thus, the existing problem can be decomposed into simpler hierarchy
Overall computational requirement	Simple and possibility to engage with generation of mistakes during the computational process computation is higher	Requires higher computational effort from decision makers and possibility to engage with generation of mistakes during the computational process computation is higher	Requires higher computational effort from decision makers and possibility to engage with generation of mistakes during the computational process computation is higher

4.8 Feasibility of the Proposed Procedure

In the final phase of this research, the workability of the proposed procedure is verified by solving a real-world MADM problem. Chapter five is devoted for this purpose.

4.9 Summary of Chapter Four

To accomplish the objectives of this research, five main phases were premeditated. Firstly, the research explored the issue of uncertainty that engaged in the human’s data, its linkage to fuzzy set theory, and application of the theory into AHP. In the second phase, the investigation was extended on the aspect of interaction among attributes where the review was mainly focused on the conception of aggregation and types of aggregation operators which are applicable for MADM problems. Meanwhile, in the subsequent phase, the usage of Choquet integral and its associated monotone measure are probed. Besides, an analysis on the past studies which are focused on reducing the complexity of identifying monotone measure was conducted.

In fourth phase, a new MADM procedure was formulated to reduce the number of steps and amount of information required from decision makers when dealing with the aspect of uncertainty in human's judgement and interaction among attributes simultaneously. The proposed procedure was constructed by assembling 5 main components namely factor analysis, revised fuzzy-linguistic estimator, Choquet integral, Mikhailov's fuzzy AHP, and Simple Average Weighted (SAW).

The comparison of the proposed procedure under certain aspects, with other MADM models which also able to deal with fuzziness in human's data and interaction between attributes (CGFI and fuzzy partitioned hierarchy model) shows that the proposed procedure is being more advantageous especially in term of computational and information requirement from decision makers.

In final phase, this research will discover and solve a real MADM problem using the proposed procedure in order to test its feasibility which will be presented in the following chapter.

CHAPTER FIVE
ASSESSING THE IMAGE OF STORES FROM HOMEMAKERS' PERSPECTIVE: A CASE STUDY

5.1 Introduction

This chapter is dedicated to discover a real-world MADM problem and resolve it by applying the proposed MADM procedure as offered in chapter four in order to verify the procedure's ability in generating practicable or feasible result. In this study, the proposed procedure will be applied in order to quantitatively measure the image of three chain stores situated in Pekan Sabak, Selangor, Malaysia from the homemakers' perception.

Store image actually defines the way a store is perceived by the customers (Boulding, 1956 and Martineau, 1958, as cited in Hansen and Solgaard, 2004) or the customers' total attitude towards a store (Baker, Grewal, and Parasuraman, 1994). Customers usually illustrate a store's overall image via their own post-purchasing experience, word-of-mouth sources, or through marketing communications such as advertisements (Normann, 1991).

Every retail store has its own image and it influences a customer whether to choose a store for purchasing (Verma and Gupta, 2005). A positive image usually leads to customer satisfaction and increases number of loyal customers (Kandampully and Suhartanto, 2000). If a store does not have a unique or favorable image than their competitors, the customers would not find a reason on why they should purchase there (Andersen, 1997). Therefore, the retailers should timely analyze and enhance the store's image because a desirable store image appears as a key determinant for long-term business success in an increasingly competitive marketplace (Grewal, Krishnan, Baker, and Borin, 1998).

An evaluation on store image is obviously a MADM problem as it involves multiple dimensions and should be measured via multiple attributes (Kim and Jin, 2001, as cited in Wong, Osman, Jamaluddin, and Yin-Fah, 2012). Unfortunately, the review on past literature reveals that there are only handful numbers of quantitative approaches which have been introduced to this date in assessing store's image. Therefore, through this study, the proposed MADM procedure which able deals with the aspect of uncertainty in human's judgment and interaction between the attributes is applied for the evaluating the image of three chain stores located at Pekan Sabak. The results of this study would be helpful for the retailers to comprehend their relative position with other stores and develop better strategies to enhance their image from the customer's point of view.

The following sections comprise the steps in utilizing the proposed procedure which begins with defining background of the problem until the stage of computing the stores' overall image and determining the ranking of stores based on the global image scores. Besides, based on the results, some potential strategies in recuperating the stores' image are discussed as well.

5.2 Background of the Case Study

Sabak is a subdivision of Sabak Bernam district, located at the northwest Selangor. It is a rural area, largely covered by traditional villages and plantation estates where most of the populace is engaged with agricultural activities. Alike other rural regions, Sabak has its own, progressing town which is locally known as 'Pekan Sabak'. The town has been experiencing a satisfying growth for the past few years.

Mushrooming of new housing and shop lots projects, the presence of new banks, fast food franchise, new budget hotels, resort, and home-stays, mini convention centre, government community college and not to forget, the emergence of chain stores are reflecting the town's development for the past 15 years.

Focusing on the chain stores, there are three chain stores operating in Sabak Bernam namely Big Shop, 99 Speedmart, and Billion. Billion is the first chain store of the town then followed by 99 Speedmart and Big Shop. Both Billion and Big Shop are running their business in a double storey building whereas, 99 Speedmart is operating in a broad, single storey building. The main selling products of these stores are household items and foodstuffs.

It can be said that these stores are competing in attracting the customers for a long survival. The main customers of these stores are the locals from the villages and plantation estates situated close to the town. Therefore, it is necessary for the each retailer to understand the behavior of rural people in choosing the retail stores. In order words, it is essential for the retailers to analyze their stores' image from the view of local people and so, proper strategies can be organized to enhance their image in order to boost up the number of repetitive and loyal customers.

However, in this study, we are only interested to measure the image of the three stores from the perception of the homemakers living in Sabak Bernam Plantation Estate, which is located three kilometers away from the town. But it is still important to analyze the image of stores from homemakers' perspective as they not only make purchase decisions for their own consumption but also influence family purchase decisions (James, 2012 and Kandoje, 2009).

5.3 Eliciting Store Attributes

As mentioned previously, the store image is usually characterized by multi-attribute construct. A summary on some of the past studies which have discussed on attributes that could influence the image of a store is presented through Table 5.1.

Table 5.1: Store Attributes Identified in Past Studies

Sources	**Attributes/Components/Elements**
Lindquist (1974)	Identified nine elements namely merchandise, service, clientele, physical facilities, comfort, promotion, store atmosphere, institutional, and post transaction satisfaction as the main determinants of a store's image
Joyce and Lambert (1996)	Used attributes such as physical condition of store, the store's selection on merchandise, and courteousness of the salesperson for measuring store image
Thompson and Chen (1998)	Provided a long list of attributes comprising elements such as uncongested environment, trendy merchandise, availability of store cards, and large layout as some pertinent criteria in gauging a store's image
Hansen and Solgaard (2004)	Employed attributes such as long opening hour, introduction of new products, advertisement in local papers, and parking facility in assessing a store's image
Yoo and Chang (2005)	Identified quality of products, price, assortment, promotion, and advertisement, convenience of shopping, convenience of location, salesperson service, and credit service as several vital components of a store's image
Chan and Chan (2008)	Proved that unique merchandise display also influences the desire of a customer to purchase at the store
Theodoridis and Chatzipanagiotou (2009)	Identified twenty attributes to define a store's image which were then clustered into six main classes namely personnel (e.g. caring and friendly service), atmosphere (e.g. temperature and cleanliness), products (e.g. variety and quality), pricing (e.g. good price), merchandising (e.g. easy to find the products and labeling), and in-store convenience (e.g. carriage)
Cornelius, Natter, and Faure (2010)	Disclosed that an innovative storefront display able to improve a store's image

With regards to this study, the two decision makers who involved in this analysis have initially extracted fifteen attributes from past literature which were believed to be significant for evaluating the image of stores located in small towns but latterly, after further consideration, two attributes ('long opening hour' and 'distance from home') were dropped out from the final list due to following reasons. The attribute ‘distance from home’ was discarded as the distance of the three stores from the estate is more or less same and the attribute ‘long opening hour’ was excluded as the three stores operate almost in a same time frame. The final list of store attributes used for this analysis was as presented in Table 5.2.

Table 5.2: Finalized Store Attributes

No.	Attributes	Description
1	Quality products (c_1)	The products sold at the store are in good quality, durable, function as expected and fresh (for foodstuffs)
2	Assortment (c_2)	The store carries different kinds or brands of products
3	Price(c_3)	The price of the products are reasonable and cheaper in comparison to other stores
4	Staff(c_4)	Store staff is neatly uniformed and always welcome the customers with friendly attitudes.
5	Fast checkouts(c_5)	I don’t have wait for so long in the queue at payment counters
6	Cleanliness(c_6)	The store is clean, neat, and tidy
7	Internal environment(c_7)	The internal atmosphere of the store always creates a pleasurable mood during purchasing activities
8	Store layout(c_8)	The design of store is spacious and makes shopping is easier and comfortable
9	Product display(c_9)	The products are displayed and arranged according to their usage and in an easy-to-find manner
10	Storefront(c_{10})	The store has attractive storefront with eye-catching decors, banners, or posters
11	In-store facilities(c_{11})	The store has satisfying level of necessary facilities within the stores such as such baskets, carriers, and fitting rooms
12	Parking facility(c_{12})	It is easy to get parking space around the store
13	Promotion(c_{13})	Good sales are offered timely

5.4 Constructing Linguistic Scale for Expressing Perception

A linguistic scale was then developed for the respondents to express their perception on each item or attribute with respect to each store. To achieve this, firstly, the decision makers determined nine linguistic terms or preferences, $\boldsymbol{s_i} =$ {absolutely disagree ($\boldsymbol{s_0}$), very disagree ($\boldsymbol{s_1}$), disagree ($\boldsymbol{s_2}$), somewhat disagree ($\boldsymbol{s_3}$), neutral ($\boldsymbol{s_4}$), somewhat agree ($\boldsymbol{s_5}$), agree ($\boldsymbol{s_6}$), very agree ($\boldsymbol{s_7}$), absolutely agree ($\boldsymbol{s_8}$)} to provide better distinction for the respondents while expressing their agreement on each attribute. In this case, the rank of last linguistic preference, $\boldsymbol{T = 8}$. Using equation (4.1), the TFN corresponding to each linguistic preference was then identified as demonstrated in Table 5.3.

Table 5.3: Linguistic Preferences and Corresponding TFNs for Expressing Agreement

Linguistic preferences	**TFNs**
$\mathbf{s_0}$ = Absoutely disagree	$\widetilde{A}_0 = \left(\max\left\{\frac{0-1}{(9-1)},0\right\},\frac{0}{(9-1)},\min\left\{\frac{0+1}{(9-1)},1\right\}\right) = (0,0,0.125)$
$\mathbf{s_1}$ = Very disagree	$\widetilde{A}_1 = (0,0.125,0.25)$
$\mathbf{s_2}$ = Disagree	$\widetilde{A}_2 = (0.125,0.25,0.375)$
$\mathbf{s_3}$ = Somewhat disagree	$\widetilde{A}_3 = (0.25,0.375,0.5)$
$\mathbf{s_4}$ = Neutral	$\widetilde{A}_4 = (0.375,0.5,0.625)$
$\mathbf{s_5}$ = Somewhat agree	$\widetilde{A}_5 = (0.5,0.625,0.75)$
$\mathbf{s_6}$ = Agree	$\widetilde{A}_6 = (0.625,0.75,0.875)$
$\mathbf{s_7}$ = Very agree	$\widetilde{A}_7 = (0.75,0.875,1)$
$\mathbf{s_8}$ = Absoutely agree	$\widetilde{A}_8 = (0.875,1,1)$

With the determined linguistic preferences and their corresponding TFNs, a 9- point linguistic scale for expressing agreement on each item was developed as portrayed in Figure 5.1.

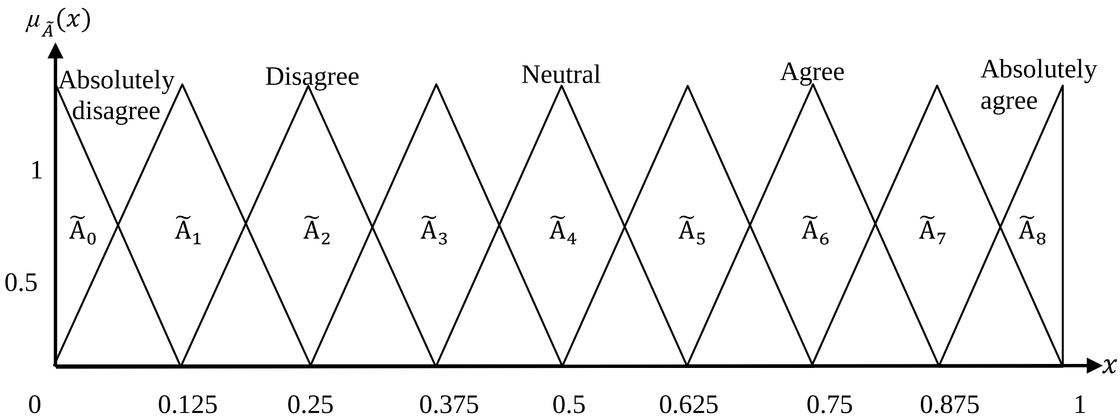


Figure 5.1: 9-point Linguistic Scale (Expressing Agreement on Each Item)

5.5 Designing Store Image Questionnaire and Reliability Test

A questionnaire was then designed based on the developed 9-point linguistic scale as an instrument to capture the perception on each store. The questionnaire was mainly prepared in Malay and Tamil versions since it was understood that most of targeted respondents are only excel in their mother tongue. The English version of the questionnaire is presented in Appendix A. The questionnaire was organized into two major sections (A and B). Section A was dedicated to obtain some profiles of respondents such as age, race, period of residing in the estate, and total household income. Meanwhile, in Section B, the respondents were requested to linguistically express their agreement on the identified attributes with respect to each of the store based on the 9-point linguistic scale, ranging from 'absolutely disagree' to 'absolutely agree'.

However, before conducting the actual survey, the questionnaire was pre-tested with a group of selected respondents in order to validate the reliability of the questionnaire and to assure the intended meaning of items in the questionnaire is understandable.

The pilot study was implemented as follows. A house-to-house survey was conducted in an old housing area where 45 homemakers who had the purchasing experience at the designated stores were identified. They were given three days to respond on the given questionnaire and also recommended to comment on the clarity of the questionnaire, puzzling terms, simplicity in answering the questionnaire, and overall format of the questionnaire. The collected raw data set from section B were then transformed into appropriate form (the transformation procedure was exactly the same as preparing data for factor analysis as enlightened in section 5.8) and analyzed using SPSS software to compute the Cronbach's alpha value. The reliability test showed that the Cronbach's alpha value was 0.891 which implied the questionnaire was internally consistency. Meanwhile, based on the respondents' feedback, some alterations were made on the questionnaire especially some rare terms were replaced with simpler and straightforward words.

5.6 Data Collection: Perception on the Stores

Before embarking the actual survey, an approval from the estate management was obtained as shown in Appendix B. The overall data collection procedure for this study can be summarized as follows.

5.6.1 Target Population

As mentioned in section 5.1, this study was intended to understand the image of the stores from the view of female homemakers who are dwelling in Sabak Bernam Estate. By interviewing the head of workers' union, it was discovered that around 51 houses in the area were occupied by Malaysian families (the remaining were occupied by few male bachelors and some foreign labors who were beyond of the study's focus). Therefore, the finalized population of this analysis was the 51 homemakers from each of these families.

5.6.2 Sampling Procedure

Using the online calculator available at http://www.surveysystem.com/sscalc.htm, as suggested by Connaway and Powell (2010), it was understood that the minimum sample size required to correctly represent the population of this study is 45 (in the case of 5% of confidence interval). However, in this analysis, no specific sampling procedure was applied as we believed that the overall population was small and thus, the perception from all the homemakers can be obtained without any difficulties.

5.6.3 Data Collection via the Questionnaire

With the help of two primary school teachers who are familiar with the local people, a house-to-house survey was conducted. For sake of caution, prior to offering the questionnaire, a screening question was asked to the respondents to ensure they had the purchasing experience at all the three stores. As expected, all of them had purchased at the three stores for at least once. In addition, in order avoid biased evaluation from the loyal customers, it was clearly explained to them that the intention of the survey is not to compare the performance of the stores. They were simply informed that the survey is being conducted to enhance the existing services and facilities within each store.

Each of these 51 homemakers was requested to express their perception on each item in the questionnaire with respect to each store. We assisted them throughout the answering process and assured that the questionnaires were fulfilled completely. The survey was scheduled and conducted after 5pm as most of the working women would be only available after this point of time. Therefore, it took almost a week to accomplish the survey.

At the end of survey, a large data set comprising a total of 153 observations [number of observations on each store (51) × number of stores (3)] were obtained. Since the store image evaluation system constructed by 13 attributes, as per the rule of '10 observations per attribute', the total observation, N for this problem should be at least 130 to perform a meaningful factor analysis. This indicated that the total observation ($N = 153$) gathered via this survey was enough to guarantee a trustworthy factor analysis result. The collected raw data set from section B were recorded in EXCEL spreadsheet.

5.7 Developing Decision Matrix of the Stores

The following steps were adopted in order to derive the decision matrix of the existing evaluation problem. Firstly, the linguistic scores in the raw data set were quantified into their corresponding TFNs based on Table 5.3. The fuzzified data set of the evaluation problem was recorded in EXCEL spreadsheet.

Later, the average fuzzy scores of each store were computed using equation (4.2) to generate fuzzy decision matrix as depicted in Appendix C. Then, the final decision matrix for the further analysis was derived by defuzzifying the fuzzy scores in Appendix C into their respective crisp scores via COA technique (4.3). The defuzzification process was accomplished with the aid of “defuzz (x, mf, ‘centroid’)” function in MATLAB software. The final decision matrix which was used for further analysis on stores’ image is as presented in Table 5.4.

Table 5.4: Decision Matrix of Store Image Problem

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}
B	0.7271	0.7753	0.6291	0.5547	0.7418	0.7435	0.7288	0.6855	0.6462	0.6871	0.7263	0.7663	0.7132
S	0.8374	0.7247	0.7770	0.8521	0.7582	0.8685	0.8268	0.7549	0.8358	0.7255	0.7631	0.3374	0.4592
BS	0.6438	0.8137	0.6977	0.7541	0.7574	0.7002	0.6087	0.8080	0.8145	0.8113	0.8015	0.8668	0.8121

**B=Billion, S=Speedmart, and BS= Big Shop*

5.8 Modifying the Available Raw Data Set for Factor Analysis

Since the raw data set obtained by means of questionnaire encompassed scores in the form of linguistic terms then, it was not applicable for performing factor analysis. Therefore, the raw data set needs to be transformed into a valid form for it to be factor analyzed.

As the first step of carrying out this transformation, the gathered raw data set should be converted into fuzzified data set. However, this step was omitted since the fuzzified data set was already attained during the decision matrix formation process. The fuzzy scores (in fuzzified data set) were then converted into crisp scores using COA equation (4.3). Finally, these crisp values were translated into their equivalent in 9-point Likert scale using equation (4.4) where at this phase the data was ready to be factor analyzed.

5.9 Factor Analyzing the Store Image Data

Prior to conducting factor analysis, the factor ability of the transformed data was investigated. For this purpose, the correlation matrix, KMO measure of sampling adequacy value and Bartlett's Test of Sphericity of the data were derived with the aid of SPSS software. The assessment on the correlation matrix, presented in Table 5.5, disclosed the presence of several coefficients of 0.3 and above.

Table 5.5: Correlation between Store Attributes

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}
c_1	1	.027	.336	.261	.151	.391	.433	.100	.206	.003	.123	-.270	-.191
c_2	.027	1	.206	.059	.186	.042	.023	.247	.060	.178	.045	.216	.147
c_3	.336	.206	1	.378	-.005	.447	.484	.260	.237	.135	.340	-.127	.069
c_4	.261	.059	.378	1	.000	.182	.114	.221	.399	.189	.094	-.235	-.104
c_5	.151	.186	-.005	.000	1	.273	.009	.117	.007	.136	.173	.034	.083
c_6	.391	.042	.447	.182	.273	1	.588	.204	.180	.221	.316	-.255	-.143
c_7	.433	.023	.484	.114	.009	.588	1	.130	.115	.115	.072	-.205	-.162
c_8	.100	.247	.260	.221	.117	.204	.130	1	.327	.281	.263	.122	-.021
c_9	.206	.060	.237	.399	.007	.180	.115	.327	1	.376	.214	-.083	.040
c_{10}	.003	.178	.135	.189	.136	.221	.115	.281	.376	1	.224	.223	.188
c_{11}	.123	.045	.340	.094	.173	.316	.072	.263	.214	.224	1	.056	.315
c_{12}	-.270	.216	-.127	-.235	.034	-.255	-.205	.122	-.083	.223	.056	1	.565
c_{13}	-.191	.147	.069	-.104	.083	-.143	-.162	-.021	.040	.188	.315	.565	1

Meanwhile, by referring to SPSS output as in Table 5.6, it was noted that the KMO value was 0.662, surpassing the recommended 0.6 and Bartlett's Test of Sphericity reached statistical significance as the *p*-value, 0 is less than 0.05. These three circumstances clearly justified that the data was appropriate to be factor analyzed.

Table 5.6: KMO and Bartlett's Test for Store Image Data

KMO and Bartlett's Test		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.662
Bartlett's Test of Sphericity	Approx. Chi-Square	496.844
	df	78
	Sig.	.000

After factor analyzing the modified data via SPSS software, the large set of thirteen attributes was reduced into five independent factors. However, it has to be understood that the attributes within each extracted factor were still inter-correlated to each other. The result of factor analysis for this study can be further detailed as follows.

Extraction through principal component analysis revealed the presence of five common factors with eigenvalues exceeding one, explaining 24.386 %, 16.679 %, 10.071 %, 8.269 %, and 7.705 % of the variance respectively as shown in Table 5.7 and Table 5.8. The total variance explained reached 67.110 %. To aid in the interpretation of these five common factors, varimax rotation was performed and the result as in Table 5.9 was obtained.

Four attributes c_7, c_6, c_3 , and c_1 had higher loading at factor 1 (refer Table 5.8) and had been renamed as 'in-store experience' factor (f_1) as it is believed pleasing internal environment, cleanliness level, price and quality of products could play significant roles in determining assenting in-store purchasing experience.

Another four attributes c_9, c_4, c_{10} , and c_8 had higher loading at factor 2 and was labeled as 'first impression' factor (f_2) as the way the products are displayed and arranged, the appearance and attitude of staff, the exterior and layout of store are the first features which can be noticed by the customers even before purchasing the products.

Meanwhile, attributes c_{13}, c_{12}, and c_{11} formed a new common factor which was then identified as 'customer care' factor (f_3) because usually, with a good sales promotion, sufficient facilities provided within the stores, and satisfactory parking facility, the customers believe the retailers are reflecting their appreciation and concern towards them.

Both c_2 and c_5 did not show any relationships with other attributes and independently had higher loading at factor 4 and factor 5 respectively. Therefore, the name of these two factors were retained as 'assortment' (f_4) and 'checkout' (f_5).

Table 5.7: Total Variance Explained

Component	Initial Eigenvalues		
	Total	**% of Variance**	**Cumulative %**
1	3.170	24.386	24.386
2	2.168	16.679	41.066
3	1.309	10.071	51.137
4	1.075	8.269	59.405
5	1.002	7.705	67.110
6	.868	6.676	73.786
7	.783	6.022	79.809
8	.658	5.062	84.871
9	.563	4.333	89.204
10	.476	3.665	92.869
11	.346	2.658	95.527
12	.302	2.321	97.848
13	.280	2.152	100.000

Extraction method: Principal component analysis

Table 5.8: Component Matrix

	Component				
	1	2	3	4	5
Environment(c_7)	.739		.374		
Clean(c_6)	.723			-.308	.336
Price(c_3)	.636		.329		
Quality(c_1)	.594	-.329			
Display(c_9)	.486	.344			
Staff(c_4)	.455	.399		-.358	-.387
Storefront(c_{10})		.761			
Layout(c_8)		.749		-.396	
Promotion(c_{13})	.400	.507			
Parking(c_{12})	.530		-.567		
Facility(c_{11})	.532		-.554		
Assortment(c_2)			.453	.582	-.454
Checkout(c_5)		.425		.484	.557

Extraction Method: Principal Component Analysis
5 components extracted.

Table 5.9: Rotated Component Matrix

	Component				
	1	**2**	**3**	**4**	**5**
Environment(c_7)	.835				
Clean(c_6)	.760				.357
Price(c_3)	.750				
Quality(c_1)	.616				
Display(c_9)		.818			
Staff(c_4)		.694			
Storefront(c_{10})		.566	.314		
Layout(c_8)		.549		.389	
Promotion(c_{13})			.865		
Parking(c_{12})			.690	.386	
Facility(c_{11})	.335		.574		
Assortment(c_2)				.863	
Checkout(c_5)					.911

**Extraction Method: Principal Component Analysis*
**Rotation Method: Varimax with Kaiser Normalization.*
**Rotation converged in 25 iterations.*

5.10 Decomposing Store Image Problem into Hierarchy System

By adhering to the result of factor analysis, the complex evaluation system of store image was decomposed into simpler and interpretable system which comprised of four levels namely goal level, factors level, attributes level, and alternatives level as illustrated in Figure 5.2. The decomposed hierarchy enabled the decision makers to have clearer picture on the main determinants (factors) and sub-determinants (attributes) of the stores' image. Besides, it was helpful in conducting further analysis gradually from one level to the others.

Based on Figure 5.2, the first level presents all the three stores under evaluation and the second level comprises of attributes that influence each of the main factors with their respective scores captured from the decision matrix. Meanwhile, the third level discloses the main determinants or factors that influence the stores' image. Finally, the fourth level reflects the goal of the existing MADM problem which was to assess the stores' image from the homemakers' viewpoint.

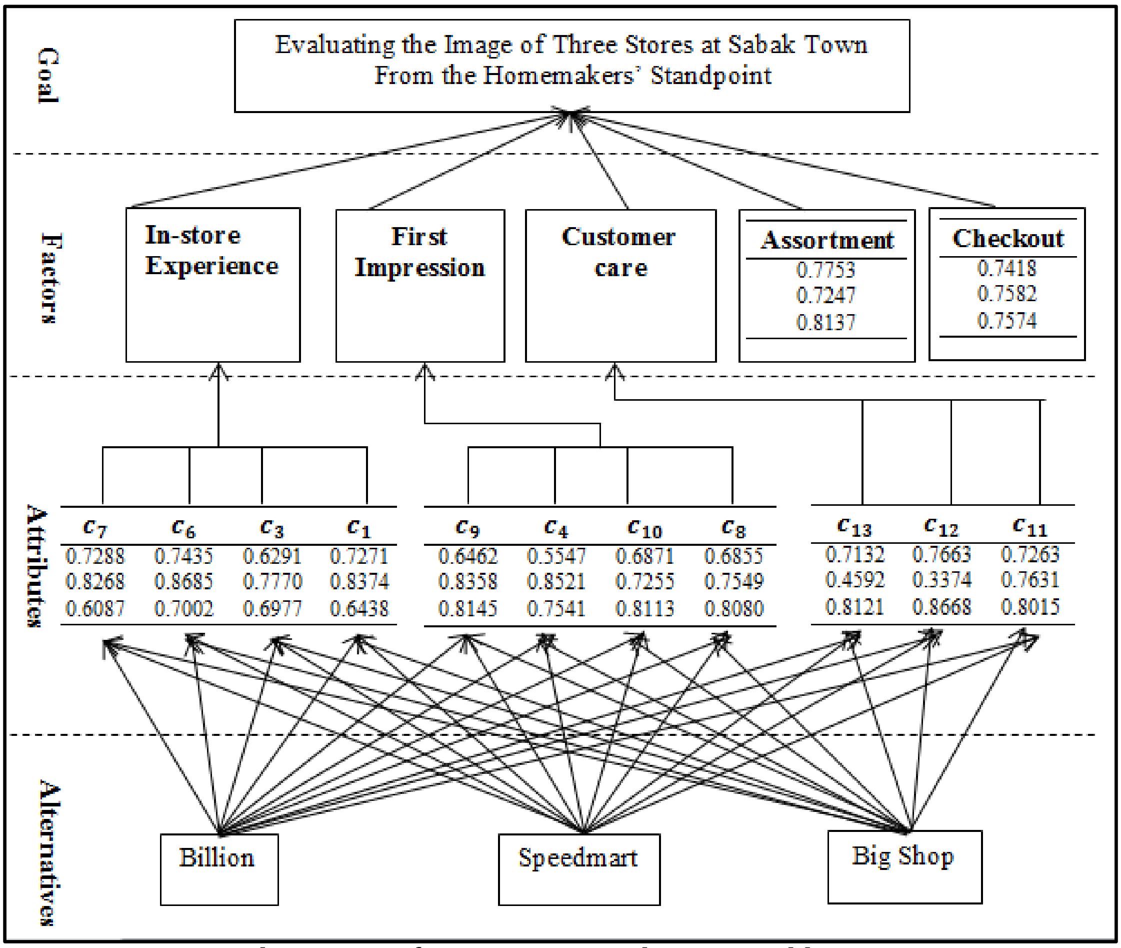


Figure 5.2: Hierarchy System of Store Image Evaluation Problem

5.11 Monotone Measure within Each Store Image Factor

As the store attributes within each factor interacted to each other, the local scores within each factor were then aggregated using Choquet integral operator. Prior to applying Choquet integral, the weights of monotone measure or combinations of attributes within each factor were estimated. The estimation process was carried out as follows.

Firstly, the decision makers determined nine linguistic terms to assess the individual contribution or importance of attributes towards their respective factor where s_0 denotes 'least important' and s_8 denotes 'extremely important'. Then, the TFN associated to each linguistic term was identified via Zhu's equation (4.1) as presented in Table 5.10. As a result, a 9-point linguistic scale for assessing the individual importance of attributes was constructed as displayed in Figure 5.3.

Table 5.10: Linguistic Terms and Corresponding TFNs for Expressing Individual Importance of Attributes

Linguistic variables	**TFNs**
S_0 = Least important	$\tilde{I}_0 = \left(\max\left\{\frac{0-1}{(9-1)},0\right\},\frac{0}{(9-1)},\min\left\{\frac{0+1}{(9-1)},1\right\}\right) = (0,0,0.125)$
S_1 = Somewhat important	$\tilde{I}_1 = (0,0.125,0.25)$
S_2 = Important	$\tilde{I}_2 = (0.125,0.25,0.375)$
S_3 = Somewhat strongly important	$\tilde{I}_3 = (0.25,0.375,0.5)$
S_4 = Strongly important	$\tilde{I}_4 = (0.375,0.5,0.625)$
S_5 = Somewhat very stongly important	$\tilde{I}_5 = (0.5,0.625,0.75)$
S_6 = Very strongly important	$\tilde{I}_6 = (0.625,0.75,0.875)$
S_7 = Somewhat extremely important	$\tilde{I}_7 = (0.75,0.875,1)$
S_8 = Extremely important	$\tilde{I}_8 = (0.875,1,1)$

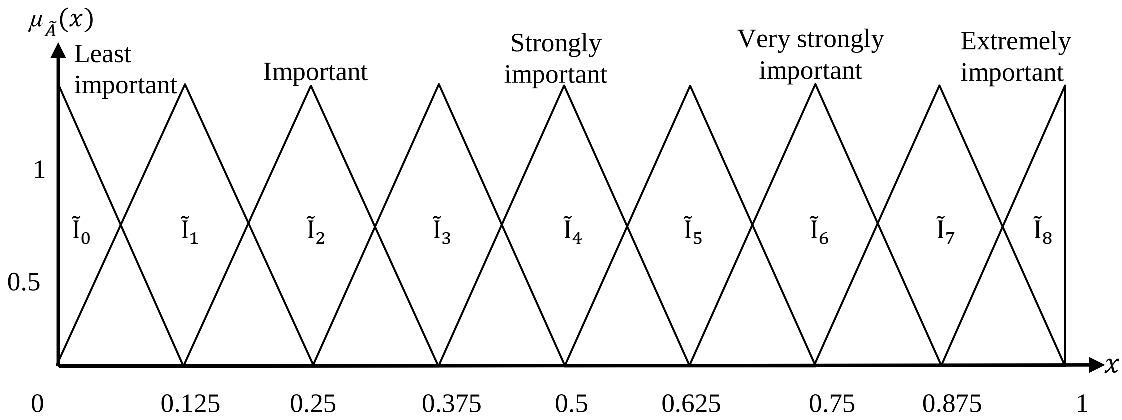


Figure 5.3: 9-point Linguistic Scale for Expressing Individual Importance of Attributes

Based on the developed scale, each decision maker linguistically expressed their own perception on the importance of each attribute towards its factor. These linguistic judgments were then converted into corresponding TFNs. Subsequently, the average fuzzy importance of each attribute was determined using equation (4.5). The average fuzzy importance were then defuzzified into crisp importance via COA technique (4.3). The importance in crisp values actually represented the individual weights of attributes which were required to identify the weights of monotone measure within each factor. The assessments on the importance of attributes and identification of individual weights within each factor are summarized in Table 5.11.

Table 5.11: Identification of Individual Weights within Each Store Image Factor

Factor	Attributes	Importance (Linguistic terms)		Fuzzy importance		Average fuzzy importance	Final individual weights
		1st DM	2nd DM	1st DM	2nd DM		
In-store experience	Environment	I	SI	(0.125,0.25,0.375)	(0,0.125,0.25)	(0.0625,0.1875,0.3125)	0.1875
	Clean	I	SI	(0.125,0.25,0.375)	(0,0.125,0.25)	(0.25,0.375,0.5)	0.3750
	Price	SI	SI	(0,0.125,0.25)	(0,0.125,0.25)	(0.1875,0.3125,0.4375)	0.3125
	Quality	STI	VSI	(0.375,0.5,0.625)	(0.625,0.75,0.875)	(0.5,0.625,0.75)	0.625
First impression	Display	I	SI	(0.125,0.25,0.375)	(0,0.125,0.25)	(0.0625,0.1875,0.3125)	0.1875
	Staff	SSI	I	(0.25,0.375,0.5)	(0.125,0.25,0.375)	(0.1875,0.3125,0.4375)	0.3125
	Storefront	SSI	I	(0.25,0.375,0.5)	(0.125,0.25,0.375)	(0.1875,0.3125,0.4375)	0.3125
	Layout	SI	SI	(0,0.125,0.25)	(0,0.125,0.25)	(0, 0.125, 0.25)	0.125
Customer care	Promotion	STI	SSI	(0.375,0.5,0.625)	(0.25,0.375,0.5)	(0.3125,0.4375,0.5625)	0.4375
	Parking	I	I	(0.125,0.25,0.375)	(0.125,0.25,0.375)	(0.125,0.25,0.375)	0.25
	Facility	SI	I	(0,0.125,0.25)	(0.125,0.25,0.375)	(0.0625,0.1875,0.3125)	0.1875

**DM= decision maker, SI= somewhat important, I= important, SSI= somewhat strongly important, SI= strongly important, VSI= very strongly important*

The identified individual weights were then replaced into equation (4.6) in order to estimate the interaction parameter, λ of each factor. Finally, with the available individual weights and interaction parameters, λ, equation (4.7) was utilized to estimate the weights of monotone measure within each factor. The identified interaction parameter, λ and monotone measure weights of each store image factor were as presented in Table 5.12, Table 5.13, and Table 5.14.

Table 5.12: Interaction Parameter and Monotone Measure of In-store Experience Factor

In-store experience (λ= -0.7470)	
Subsets	**Weights**
{}	0.0000
{Environment}	0.1875
{Clean }	0.3750
{Environment, Clean}	0.5100
{Price }	0.3125
{Environment, Price}	0.4562
{Clean, Price}	0.6000
{Environment, Clean , Price }	0.7034
{Quality}	0.6250
{Environment, Quality}	0.7250
{Clean ,Quality}	0.8249
{Environment, Clean ,Quality}	0.8969
{Price, Quality}	0.7916
{Environment, Price ,Quality}	0.8682
{Clean, Price, Quality}	0.9449
{Environment, Clean , Price ,Quality}	1.0000

Based on Table 5.12, λ= -0.7470 indicated that the attributes within the in-store experience factor shared sub-additive effect. Therefore, in order to improve a customer's in-store experience, it would be sufficient to simultaneously enhance some of the attributes which have higher individual weights (quality of product and cleanliness).

Table 5.13: Interaction Parameter and Monotone Measure of First Impression Factor

First impression (λ= 0.1922)	
Subsets	**Weights**
{}	0.0000
{Display }	0.1875
{Staff}	0.3125
{Display, Staff}	0.5113
{Storefront}	0.3125
{Display ,Storefront}	0.5113
{Staff, Storefront}	0.6438
{Display , Staff, Storefront}	0.8545
{Layout }	0.1250
{Display ,Layout }	0.3170
{Staff, Layout }	0.4450
{Display, Staff, Layout }	0.6485
{Storefront, Layout }	0.4450
{Display, Storefront, Layout }	0.6485
{Staff, Storefront, Layout }	0.7842
{Display ,Staff, Storefront, Layout }	1.0000

Based on Table 5.13, λ= 0.1922 implied that the attributes within first impression factor shared super-additive effect. Therefore, in order to enhance the customers' first impression on a store, all the attribute (display, staff, storefront, and layout) have to be improved simultaneously regardless of their individual weights.

Table 5.14: Interaction Parameter and Monotone Measure of Customer Care Factor

Customer-care attitude (λ= 0.5029)	
Subsets	**Weights**
{}	0
{Promotions }	0.4375
{Parking}	0.2500
{Promotions, Parking}	0.7425
{Facility}	0.1875
{Promotions, Facility}	0.6663
{Parking, Facility}	0.4611
{Promotions, Parking, Facility}	1.0000

Based on Table 5.14, λ= 0.5029 implied that the attributes within customer care factor shared super-additive effect. Therefore, in order for the stores to improve the customer care aspect, all the attributes (promotions, parking, and facility) need to be enhanced concurrently regardless of their individual weights.

As the proposed procedure consists of factor analysis, it is able to decrease the actual number of monotone measure weights which need to be identified by the decision makers prior to applying Choquet integral from 8192 (2^{13}) weights to 40 (2^4+2^4+2^3) weights. Therefore, there was about 99.5% computational saving achieved when determining the weights of monotone measure for this specific problem. The percentage of computational saving relies on the result of factor analysis. In general, through the proposed procedure, the actual number of monotone measure weights can be reduced from 2^n to $\sum_{p=1}^{q} 2^{|f_p|}$ where $f_p = (f_1, f_1, \dots, f_q)$set of extracted factors, q denotes the total number of factors, and $|f_p|$ represents the number of attributes within factor, p.

5.12 Using Choquet integral to Aggregate Interactive Local Scores

After identifying weights of monotone measure, Choquet integral model (4.8) was then applied to aggregate the interacted local scores within each factor to obtain factor scores. The local scores within each factor and their aggregated factor scores via Choquet integral were as presented in Table 5.15, Table 5.16, and Table 5.17 respectively.

Table 5.15: In-store Experience Score of the Stores

	Environment	**Cleanliness**	**Price**	**Quality**	**In-store experience score**
Billion	0.7288	0.7435	0.6291	0.7271	0.7234
Speedmart	0.8268	0.8685	0.7770	0.8374	0.8421
Big	0.6087	0.7002	0.6977	0.6438	0.6751

Table 5.16: First Impression Score of the Stores

	Display	Staff	Storefront	Layout	First impression score
Billion	0.6462	0.5547	0.6871	0.6855	0.6320
Speedmart	0.8358	0.8521	0.7255	0.7549	0.7910
Big	0.8145	0.7541	0.8113	0.8080	0.7913

Table 5.17: Customer Care Score of the Stores

	Promotions	Parking	Facility	Customer care score
Billion	0.7132	0.7663	0.7263	0.7292
Speedmart	0.4592	0.3374	0.7631	0.4755
Big	0.8121	0.8668	0.8015	0.8230

5.13 Construction of New Decision Matrix (Stores vs. Factors)

Then, a new decision matrix (stores versus factors) was constructed based on the computed factor scores as portrayed in Table 5.18. Note that the stores' scores with respect to assortment and checkout factor were elicited from the previous decision matrix (Table 5.4). Further evaluation on stores' image was based on this newly constructed decision matrix.

Table 5.18: New Decision Matrix (Stores vs. Factors)

	In-store experience	First impression	Customer care	Assortment	Checkout
Billion	0.7234	0.6320	0.7292	0.7753	0.7418
Speedmart	0.8421	0.7910	0.4755	0.7247	0.7582
Big	0.6751	0.7913	0.8230	0.8137	0.7574

5.14 Estimating the Weights of Independent Store Image Factors

Mikhailov's Fuzzy AHP technique was then utilized in order to estimate the weights of independent factors. As the first step to employ Mikhailov's Fuzzy AHP, the two decision makers had a detailed discussion on the relative importance between the store image factors. After achieving a consensus, a single pair-wise matrix was assessed linguistically based on Saaty's fuzzy AHP scale (refer Table 4.7) as shown in Table 5.19. It can be noticed that since the 'first impression' and 'customer care' were found to be less important than 'assortment' factor, then the evaluation was done vice versa to avoid using reciprocal values as mentioned in section 4.5.12.

Table 5.19: Linguistic Pair-wise Comparison between Store Image Factors

	In-store experience	**First impression**	**Customer care**	**Assortment**	**Checkout**
In-store experience	(1,1,1)	Slightly important	Somewhat strongly important	Somewhat slightly important	Somewhat strongly important
First impression		(1,1,1)	Somewhat slightly important		Somewhat slightly important
Customer care			(1,1,1)		Equally Important
Assortment		Somewhat slightly important	Slightly important	(1,1,1)	Slightly important
Checkout					(1,1,1)

The linguistic terms in the evaluated pair-wise comparison matrix were then quantified into their corresponding TFNs by adhering to the same Saaty's fuzzy AHP scale. The fuzzy pair-wise comparison matrix between store image factors was as presented in Table 5.20.

Table 5.20: Fuzzy Pair-wise Comparison between Store Image Factors

	In-store experience	First impression	Customer care	Assortment	Checkout
In-store experience	(1,1,1)	(2,3,4)	(3,4,5)	(1,2,3)	(3,4,5)
First impression		(1,1,1)	(1,2,3)		(1,2,3)
Customer care			(1,1,1)		(1,1,2)
Assortment		(1,2,3)	(2,3,4)	(1,1,1)	(2,3,4)
Checkout					(1,1,1)

Based on the evaluated fuzzy pair-wise comparison, Mikhailov's nonlinear optimization model (4.9) was constructed as follows to derive the consistency value of pair-wise comparison and the crisp weights of residential factors simultaneously.

$$
\begin{aligned}
&\text{Max } \mu \\
&\text{Subject to} \\
&\mu w_2 - w_1 + 2w_2 \leq 0 \\
&\mu w_3 - w_1 + 3w_3 \leq 0 \\
&\mu w_4 - w_1 + w_4 \leq 0 \\
&\mu w_5 - w_1 + 3w_5 \leq 0 \\
&\mu w_3 - w_2 + w_3 \leq 0 \\
&\mu w_5 - w_2 + w_5 \leq 0 \\
&-w_3 + w_5 \leq 0 \\
&\mu w_2 - w_4 + w_2 \leq 0 \\
&\mu w_3 - w_4 + 2w_3 \leq 0 \\
&\mu w_5 - w_4 + 2w_5 \leq 0 \\
&\mu w_2 + w_1 - 4w_2 \leq 0 \\
&\mu w_3 + w_1 - 5w_3 \leq 0 \\
&\mu w_4 + w_1 - 3w_4 \leq 0 \\
&\mu w_5 + w_1 - 5w_5 \leq 0 \\
&\mu w_3 + w_2 - 3w_3 \leq 0 \\
&\mu w_5 + w_2 - 3w_5 \leq 0 \\
&\mu w_5 + w_3 - 2w_5 \leq 0 \\
&\mu w_2 + w_4 - 3w_2 \leq 0 \\
&\mu w_3 + w_4 - 4w_3 \leq 0 \\
&\mu w_5 + w_4 - 4w_5 \leq 0 \\
&w_1 + w_2 + w_3 + w_4 + w_5 = 1 \\
&w_1, w_2, w_3, w_4, w_5 \geq 0
\end{aligned}
$$

where w_1,w_2,w_3,w_4, and w_5 denoted the weight of in-store experience, first impression, customer care, assortment, and checkout factor respectively whereas μ represented the consistency value of the pair-wise comparison matrix. By solving the constructed nonlinear optimization model using EXCEL SOLVER, the following result was obtained. The weight of in-store experience factor (w_1) was 0.4091, weight of first impression factor (w_2) was 0.1532, weight of customer care factor (w_3) was 0.0937, weight of assortment factor (w_4) was 0.2503, and finally, the weight of checkout factor (w_5) was 0.0937. Meanwhile, the pair-wise comparison value, μ =0.6340, implied that the consistency of pair-wise comparison was satisfactory.

5.15 Computing Global Image Score of Each Store

Finally, based on estimated weights of factors and available factor scores, the overall image of each store was computed via simple weighted average (SWA) operator (4.10). The image score of each store and its corresponding ranking are summarized in Table 5.21.

Table 5.21: Image Scores and Ranking of Stores

	In-store experience $(w_1 = \mathbf{0.4091})$	**First impression** $(w_2 = \mathbf{0.1532})$	**Customer care** $(w_3 = \mathbf{0.0937})$	**Assortment** $(w_4 = \mathbf{0.2503})$	**Checkout** $(w_5 = \mathbf{0.0937})$	**Global score**	**Ranking**
Billion	0.7234	0.6320	0.7292	0.7753	0.7418	0.7247	3
Speedmart	0.8421	0.7910	0.4755	0.7247	0.7582	0.7627	1
Big	0.6751	0.7913	0.8230	0.8137	0.7574	0.7492	2

5.16 Additional Analysis on the Proposed Procedure

Some further analysis were conducted on proposed procedure using the same case study presented in this chapter in order to provide some extra information on the performance of the MADM procedure.

5.16.1 Proposed Procedure versus Classical MAUT

In this section, the same stores' image problem was solved through a classical MAUT approach (to be specific, using the common SWA operator) and the obtained result was compared with the result from the proposed procedure. The reason of choosing classical SWA was to demonstrate on the consequence of disregarding the elements of uncertainty in human's data and interaction between attributes in analyzing MADM problems. As usual, the analysis was conducted by employing the basic three phases of MAUT as follows.

a) Phase 1: Identifying the alternatives and attributes of problem

The same three stores and thirteen attributes were used to carry out the analysis. The problem was then decomposed into hierarchy structure comprising of 'alternatives' (three stores), 'attributes' (store attributes), 'goal' (evaluating the stores based on their image score) levels.

b) Phase 2: Identifying local scores of alternatives and weights of attributes

To make sensible comparison on the results (outputs) from two different MADM tools, same data (inputs) should be used. Therefore, in this case, the existing store image data was utilized to derive the local scores and weights required for the application of SWA.

To obtain the performance or decision matrix of problem, firstly the linguistic scores in raw data were converted or represented with their equivalent crisp numbers as in 9- point Likert scale (instead of quantifying into fuzzy numbers as required in the proposed procedure). Then, by averaging the crisp scores corresponding to each store, the local scores were computed. As a result, a decision matrix as shown in Table 5.22 was constructed.

Table 5.22: Decision Matrix for SWA

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}
Billion	6.8431	7.2353	6.0392	5.4510	6.9608	7.0000	6.8824	6.5098	6.1961	6.5294	6.8431	7.1961	6.7451
Speedmart	7.8039	6.8235	7.2745	7.9412	7.1176	8.0588	7.7255	7.0980	7.7843	6.8431	7.1569	3.6667	4.6863
Big	6.1961	7.6078	6.6471	7.0980	7.1176	6.6471	5.8824	7.5686	7.6078	7.5686	7.4902	8.0784	7.6078

On the other hand, since SWA assumes interdependency between attributes then, it is essential to ensure the sum of weights of the 13 attributes is being additive or equal to one. To derive the weights for SWA, firstly, the individual weights of attributes within each factor were normalized to assure the sum of the weights is equal to one. These normalized weights were just implied the contribution or importance of attributes towards their respective factor. Therefore, the final weight of each attribute (contribution of attributes towards overall image of the stores) was then estimated by multiplying its normalized weight with the weight of respective factor. It has to be reminded that the weights of factors do not demand normalization as they were already in the additive state. Table 5.23 recaps the computational process of determining the additive weights of attributes for SWA.

Table 5.23: Final Additive Weights for SWA

Factors	Attributes	Individual weights	Normalized weights	Final weights
In-store experience (0.4091)	Environment	0.1875	0.1250	0.0511
	Clean	0.3750	0.2500	0.1023
	Price	0.3125	0.2083	0.0852
	Quality	0.625	0.4167	0.1705
	SUM	1.5000	1	
First impression (0.1532)	Display	0.1875	0.2000	0.0306
	Staff	0.3125	0.3333	0.0511
	Storefront	0.3125	0.3333	0.0511
	Layout	0.125	0.1333	0.0204
	SUM	0.9375	1	
Customer care (0.0937)	Promotion	0.4375	0.5000	0.0469
	Parking	0.25	0.2857	0.0268
	Facility	0.1875	0.2143	0.0201
	SUM	0.875	1	
Assortment (0.2503)	-			0.2503
Checkout (0.0937)	-			0.0937
SUM				1

`.

c) Phase 3: Aggregation

In this phase, the local scores of each store were composed into a global score using SWA operator. Based on these global scores which represented the overall image, the stores were ranked up. Table 5.24 portrays the variation on the global scores and ranking of the stores derived from the proposed procedure and classical SWA.

Table 5.24: Comparing the Result from Proposed Procedure and SWA Operator

Stores	**Proposed procedure**		**Classical AHP**	
	Global scores	**Ranking**	**Global scores**	**Ranking**
Billion	0.7247	3	6.6247	3
Speedmart	0.7627	1	6.2881	2
Big Shop	0.7492	2	7.3283	1

Based on Table 5.24, it can be concluded that there was a significant disparity between the result generated through the proposed procedure and classical SWA. For example, the proposed procedure assigned Speedmart as the store with the finest image but, based on classical SWA, Big Shop appeared as the most preferred store.

However, based on the data collected on the frequency of purchasing at each of the store (through section A of the questionnaire) which is summarized into Table 5.25, it was discovered that 82.85 % of the respondents purchase at Speedmart for at least twice in a month. Meanwhile, 72.55% of the same group of homemakers would purchase at Billion for at least twice in a month. Only 52.94% of the homemakers would purchase at Billion for at least twice a month.

Table 5.25: Frequency of Purchasing at Each of the Store

Stores	Frequency of purchasing at each of the store	Percentage (%) of respondents
Billion	Once in a month	47.06
	Twice in a month	25.49
	More than twice in a month	27.45
	SUM	100
Speedmart	Once in a month	17.65
	Twice in a month	23.53
	More than twice in a month	58.82
	SUM	100
Big Shop	Once in a month	27.45
	Twice in a month	21.57
	More than twice in a month	50.98
	SUM	100

Obviously, in actual scenario, Speedmart appeared as their first choice store then followed by Big Shop and Billion. By using this order as the benchmark ranking, it can be concluded that the proposed procedure manage to yield a ranking which is closer to the actual ranking in comparison to the classical SWA for this specific case study.

It depends on the decision makers either to choice the proposed procedure or to simply adhere to the common SWA before conducting a MADM analysis. Nevertheless, if the decision makers believe that they are unable to precisely offer the necessary information for the analysis and also consider that the attributes are interrelated to each other then, the proposed procedure is recommended.

5.16.2 Cautions on the Proposed Procedure

It is transparent that the successful and total application of the proposed procedure mainly relies on the result of factor analysis. It was found if an observed attribute had similar scores then, it is invalid to perform factor analysis. This is because the correlation coefficients between the specific attribute and other attributes cannot be computed. A complete correlation matrix cannot be obtained. Therefore, factor analysis which works based on the correlation between attributes cannot be implemented. To test the explained scenario, it was assumed that the 51 homemakers expressed that they are 'extremely disagree' towards the 'quality' attribute with respect to the three stores. By altering the existing the data as per the presumption, SPSS was failed to perform factor analysis.

If the decision makers facing this issue after the data collection stage, it is advisable for them to utilize the collected data to construct the decision matrix of the problem as explained in section 4.5.5 and carry out the further analysis with any other preferred MAUT models.

Besides, if the factor analysis yields only one component to represent the whole attributes then, the proposed procedure cannot be fully utilized. The decision makers are then need to skip to the step of estimating monotone measure weights and directly apply Choquet Integral as the attributes within the only factor are still considered to be interacted with each other. But then, the decision makers may need to estimate huge number of weights. For example, if all the thirteen store attributes in the presented case study were grouped into one factor then, the decision makers would have to identify 8192 (2^{13}) weights instead of 40 (2^4+2^4+2^3) weights prior to employing Choquet integral.

5.17 Discussion on the Result

In this empirical study, the proposed procedure was applied in order to assess the image of three chain stores located in Pekan Sabak from the viewpoints of all the homemakers who are residing at Sabak Bernam Plantation Estate. The result of the analysis can be summarized as follows.

Through the proposed procedure, the thirteen attributes which were finalized to characterize the image of the stores, were then clustered into five main factors namely in-store experience, first impression, customer care, assortment, and checkout factors. The prioritization on these five store image factors based on the proposed procedure was as follows. In-store experience (0.4091) $\geq$ assortment (0.2503) $\geq$ first impression (0.1532) $\geq$ customer care (0.0937) $\geq$ checkout (0.0937). It was understood that both in-store experience and assortment factors played major role in forming positive image on the stores from the homemakers' perception. This showed that the retailer of each store should concentrate more on preserving satisfactory in-store experience and assortment aspects.

In addition, the interaction parameter of service factor, λ = -0.7470 indicated that in order to improve the image of a store in term of in-store experience, it is sufficient to simultaneously enhance some of the attributes which had higher individual weights such as quality of product (0.6250) and cleanliness (0.3750). In general, if the customers know that the products are being in good quality, the customer would consider the prices are reasonable and acceptable where they should be willing to pay the prices (Rao and Sieben, 1992). Besides, a clean store always plays a role in creating pleasing internal atmosphere for purchasing (Akinyele, 2010 and Bäckström and Johanssan, 2006) and encourages the customers to purchase longer or revisit the store (Carpenter and Moore, 2006). In addition, the acceptance on the pricing could be high during purchasing if the internal environment of the specific store is clean and pleasurable as claimed by Grewal and Baker (1994).

Meanwhile, the interaction parameter, λ= 0.1922 implied that in order to significantly improve the customers' first impression on a store, all the attributes such as products display (0.1875), staff (0.3125), storefront (0.3125), and layout (0.1250) have to be enhanced simultaneously regardless of their individual weights. The similar approach can be applied in order to augment the customer care factor as it had a positively valued interaction parameter, λ = 0.5029.

According to the proposed procedure, the ranking of the stores based on image scores was as follows. Speedmart $\geq$ Big Shop $\geq$ Billion.

Speedmart ruled the top position as it had satisfactory scores on in-store experience factor, which was the main determinant of the stores' image. However, to retain the position and to form a greater image among the customers, the retailer could broaden the assortment of products (the second main determinant) in the store. From our observation, Speedmart does not carry much variety in food stuffs and there is no clothing section in the store in contrary to Billion and Big Shop.

Big Shop has the potential to be in top position in future if the retailer puts major efforts on creating a satisfactory in-store experience by simultaneously assuring the quality of products are in high standard and ensure the store is always clean.

Meanwhile, Billion was identified as the store with most unfavorable image due to its unsatisfactory performance with respect to in-store experience and first impression aspects. Thus, appropriate strategies should be planned to achieve perfection in those aspects. With an average score in in-store experience factor, the retailer should focus on bringing in more quality products and assure the store is being cleaned timely and flawlessly. Besides, to improve the customers' first impression on the store, the retailer should simultaneously enhance all the attributes that influence the factor (display, staff, storefront, and layout), regardless of their individual weights.

The same problem was analyzed using a classical MAUT approach to demonstrate the consequence of ignoring the element of uncertainty in human's data and interaction between attributes. As a result, dissimilar ranking as follows was obtained. Big Shop $\geq$ Speedmart $\geq$ Billion. However, it was discovered that the ranking generated by the proposed procedure was matched with the benchmark ranking. Yet, the choice of decision makers between these two approaches depends on their interest whether to deal with the previously-mentioned two elements.

It has notified here that presented study was only interested to investigate the image of the stores from the viewpoints of homemakers who are dwelling at Sabak Bernam Estate. The derived result was solely based on the perception of homemakers staying in the estate. Therefore, the result is not a total representative of all the homemakers living in Sabak.

5.18 Summary of Chapter Five

In this chapter, an evaluation on three stores' image based on the perception of homemakers living in Sabak Bernam Estate was carried out via the proposed procedure in order to validate the feasibility of the procedure in solving the real-world MADM problems.

With the help of the proposed procedure, which considers the aspect of uncertainty in human's data and interaction among attributes simultaneously, the image on each store was quantitatively measured. The evaluation from the proposed procedure could be helpful for each retailer to comprehend their actual relative position with others in term of the store's image. Besides, some possible strategies were suggested based on the result in forming a favorable store image among the homemakers.

To further understand the performance of the proposed procedure, the same evaluation problem was carried out using classical SWA operator and a different result was obtained. However, it was found the ranking yielded by the proposed procedure matched with the benchmark ranking.

CHAPTER SIX
CONCLUSION

6.1 Conclusion of the Research

Conclusively, this research has successfully actualized its primary goal. It has finally introduced a feasible MADM procedure which capable in reducing the number of computational steps and amount of information required from decision makers when dealing with aspect of uncertainty in human's data and interaction among attributes simultaneously.

In the path of achieving the main goal, several specific objectives were accomplished gradually as follows. To achieve the first specific objective, a review was carried out in order to identify the basic elements in fuzzy set theory which are being supportive in capturing the usual uncertainty embedded in human's data. It was learnt that by applying fuzzy sets in MADM environment, the decision makers are permitted to express their preference linguistically, which can be later quantified into fuzzy numbers. Fuzzy numbers mathematically represent or capture the usual uncertainty entrenched in linguistic preferences. Then, based on these fuzzy preferences, the MADM problem can be analyzed quantitatively by retaining the element of uncertainty.

Meanwhile, in fulfilling the second specific objective, a pros and cons analysis on the available fuzzy AHP approaches was conducted. It was discovered that each of these approaches demands different computational procedures. Mikhailov's fuzzy AHP was identified as a method which requires lesser computational effort from decision makers as it derives the weights of attributes and consistency value of pair-wise comparison concurrently.

The third specific objective of the study was achieved by analyzing several aggregation operators which are applicable in the field of MADM. It was understood that these operators can be classified into additive and non-additive operators where the former operators presume independency between attributes whereas later operators capture the interactions between attributes. The appraisal was then narrowed on Choquet integral operator (one of the non-additive operators) and its associated monotone measure which are able to model the interaction among interaction during aggregation.

To attain the fourth specific objective, the study has explored several approaches which were proposed with the intention to reduce the complexity of estimating monotone measure weights.

With the aid of the information gathered through the process of accomplishing the formerly-mentioned four specific objectives, the proposed procedure was then developed by congregating five main components namely factor analysis, revised fuzzy-linguistic estimator , Choquet integral, Mikhailov's fuzzy AHP, and SWA operator.

After developing the proposed procedure, to achieve the final specific objective, the feasibility of the proposed procedure was verified by solving a real MADM problem. In this study, the image of three stores located at Pekan Sabak was assessed from the viewpoint of homemakers using the proposed procedure. The same analysis was conducted via a classical MAUT approach and a dissimilar result was attained. However, it was discovered that the ranking yielded by the proposed procedure was parallel to the benchmark ranking.

6.2 Contributions of the Research

By accomplishing all its objectives, the research has made some notable contributions to the field of MADM.

First and foremost, by the end of the research, a different MADM procedure was introduced where it has the ability to reduce the number of computational steps and amount of information required from decision makers when considering the aspect of uncertainty in human's data and interaction among attributes simultaneously. The merits of the proposed procedure are as follows.

a) Deals with uncertainty in human's data

   The proposed procedure requires data from human at 3 stages. During the process of acquiring alternatives' performance via survey, identification of monotone measure weights, and identification of weights of independent factors. Therefore, in order to mathematically deal with the common uncertainty embedded in data provided by human, the proposed procedure allows them to express their preference in linguistic terms which are then converted or quantified into appropriate fuzzy numbers.

b) Deals with interaction among attributes

   The proposed procedure takes into account the interaction element among attributes within each factor during aggregation as it utilizes Choquet integral operator.

c) The proposed procedure reduces the amount of information and number of computational steps required from decision makers when handling the aspect of uncertainty in human's data and interaction among attributes concurrently. Further clarifications on how these features are achieved are as follows.

i. Through the proposed procedure, the decision makers are only required to provide the individual importance of attributes and relative importance between factors which usually can be expressed by the decision makers in linguistic terms. Meanwhile, the performance scores of alternatives can be simply derived from the data collected from the respondents. In nutshell, the total amount of information required from the decision makers in utilizing the proposed procedure can be denoted as follows; $\theta = n + q(q-1)/2$ where n denotes the number of attributes and q implies the number of factors.

ii. The proposed procedure only applies triangular type of fuzzy numbers (TFNs) as the arithmetic operations involving TFNs are naturally simpler than other types of fuzzy numbers.

iii. For the construction of linguistic scale, the procedure uses Zhu's fuzzification approach which is less complicated and helpful for decision makers who are unable to clearly define the fuzzy number corresponding to each linguistic term, due to scarce of information or experience.

iv. The procedure utilizes COA defuzzification approach in determining the ordinal ranking of attributes and alternatives as it is a simple yet accurate technique which does not demand any prior information from decision makers.

v. To estimate the weights of independent factors, the procedure uses Mikhailov's fuzzy AHP which simultaneously derives the crisp weights of factors and consistency value of pair-wise matrix by simply solving the recommended non-linear optimization model via EXCEL Solver.

vi. The proposed procedure employs a slightly amended fuzzy-linguistic estimator to identify the weights of monotone measure. This approach requires simple execution and most importantly, it models the uncertainty that exists in the provided data. The type of data required by the approach (individual importance of attributes) can be easily offered by the decision makers especially to express them in the form of linguistic terms.

vii. With the inclusion of factor analysis, the actual number of monotone measure weights is reduced from 2^n to $\sum_{p=1}^{q} 2^{|f_p|}$ where $f_p = (f_1, f_2, \ldots, f_q)$ set of extracted factors, q denotes the total number of factors, and $|f_p|$ represent number of attributes within factor, p.

viii. In the proposed procedure, with the aid of factor analysis, a complex MADM problem is decomposed into a simpler and interpretable hierarchy that would be helpful for decision makers in carrying out further analysis in more systematic and understandable manner.

d) Identifies the main determinants of a MADM problem

The usage of factor analysis in proposed procedure is being helpful in extracting an understanding the main determinants (or factors) that influence a goal of a MADM problem.

Secondly, this research has emerged as one of the attempts to encourage or motivate more real-world decision makers such as managers from an organization to simultaneously deal with the aspect of uncertainty in human's data and interaction between attributes, as the proposed procedure minimalize the number of computational steps and amount of information that usually required from the decision makers when dealing with the these two aspects.

Meanwhile, the third contribution of the research can be recapped as follows. The thesis of this research provides some details on the aggregation phase in MADM which comprised of the properties of a good aggregation operator, types of aggregation operators, and their corresponding mathematical models. Thus, the thesis can be a reference for the researchers in selecting the suitable aggregation operators for their respective problems or in formulating new aggregation operators.

Furthermore, the thesis has offered an analysis on the attempts that have been carried out to this date, in solving the complexity of identifying monotone measure weights. Therefore, it has the potential to stimulate some new ideas for continuously or gradually simplifying the complicated identification process.

Finally, in order to attest the viability of the proposed procedure, an assessment on the image of three stores located at Pekan Sabak based on the perception of homemakers, was solved via the proposed procedure. Several promising strategies were also suggested by referring to the yielded output in order to augment each of the store's image.

6.3 Limitations of the Research

This research is allied with some limitations as follows.

a) The proposed procedure only applies symmetric type of triangular fuzzy numbers (TFN) with the intention to offer simple-to-execute procedure.

b) The successful application of the proposed procedure relies on the result of factor analysis. If the data is invalid for performing factor analysis or if the analysis yields only one component to represent the whole attributes then, the proposed procedure cannot be completely utilized as explained in section 5.16.2.

c) The case study presented in chapter five was only interested to investigate image of stores among the homemakers residing at Sabak Bernam Estate. In other words, the result obtained in the analysis was solely based on the perception of homemakers of the specific estate.

d) The proposed procedure requires data collection by means of questionnaire which will be then processed to derive decision matrix of the problem and to perform factor analysis. However, the data collection through the questionnaires could be costly relying on the type of the problems.

6.4 Recommendations

It is believed that the completion of this research has instigated some potential pathways for conducting several interesting studies in near future as follows.

a) In future studies, the proposed procedure can be employed in surmounting other real MADM problems that occur in different domains.

b) The target population in the case study presented in chapter five can be extended where in future, the stores' image can be investigated based on the viewpoints of all homemakers dwelling in Sabak division. The same list of attributes can be adhered.

c) Besides, the researchers could apply the proposed MADM procedure with different types of fuzzy numbers such as non-symmetric triangular fuzzy number and trapezoidal fuzzy number.

d) Further enhancing the proposed procedure is a commendable direction for future work.

   i. As the proposed procedure sometimes demands costly data collection process to perform factor analysis, future research can focus on substituting factor analysis with some other simpler but effective approach which can function as factor analysis (grouping large set of attributes into fewer set of independent factors).

   ii. Besides, the future researches could concentrate on formulating further easy-to-implement monotone measure identification approach which can be then swapped with the suggested fuzzy-linguistic estimator.

iii. In addition, the practitioners of MADM could carry out more studies on developing further less-complicated fuzzy MADM tools which can be then replaced into the proposed procedure to estimate the weights of independent factors.

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