GEOMETRIC REPRESENTATIONS OF DISTINCT HAMILTONIAN CIRCUITS IN COMPLETE GRAPH DECOMPOSITION

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Abstrak

Penggambaran bagi perwakilan geometri untuk litar Hamiltonan berbeza dalam graf lengkap diperlukan untuk mengelakkan kemiripan struktur dalam aplikasi sebenar. Walau bagaimanapun, terdapat hanya sebilangan kecil kajian yang mempertimbangkan penggambaran graf sedangkan ramai penyelidik memberi tumpuan terhadap masa pengiraan. Oleh itu, kajian ini bertujuan untuk membina satu kaedah gambaran baharu iaitu Kaedah Rama-rama Separuh (HBM) untuk menyelesaikan senario tersebut. Bagi membina HBM, satu konsep Strategi Sayap baharu diperkenalkan untuk mendapatkan arah dari satu bucu ke bucu yang lain. Seterusnya, arah ini digunakan untuk memetakan bucu-bucu yang berbeza. Bagi mendapatkan litar Hamiltonan berbeza, konsep matriks transposisi digunakan untuk mengesan imej cermin bagi litar tersebut. Beberapa teorem dan lema baharu dibuktikan dalam penghuraian graf lengkap kepada litar Hamiltonan berbeza. Selanjutnya, hasil HBM ini digunakan untuk menyenaraikan semua pilih atur $n!$ dan beberapa teorem berkaitan dibentuk. Kesimpulannya, kajian ini berjaya menghasilkan satu kaedah baharu untuk menggambarkan litar Hamiltonan berbeza dalam penghuraian graf lengkap.

Kata kunci: Graf lengkap, Kaedah Rama-rama Separuh, Litar Hamilton, Penghuraian graf

Abstract

Visualization of geometric representations of distinct Hamiltonian circuits in complete graphs is needed to avoid structures resemblance in real application. However, there are only a few studies that consider graph visualization, whereas most researchers focus on computation time. Thus, this study aims to construct a novel picturing method called Half Butterfly Method (HBM) to address the aforementioned scenario. Towards developing HBM, the concept of Wing Strategy is introduced to create directions from one vertex to another vertex. Then, these directions are used to map distinct vertices. In order to obtain the distinct Hamiltonian circuits, the concept of matrix transpose is used to capture the mirror image of that circuit. Several new theorems and lemmas are proved in the decomposition of complete graphs into distinct Hamiltonian circuits. Furthermore, the result of HBM is applied to list $n!$ permutations and some related theorems are established. In conclusion, this study successfully produced a novel method to visualize distinct Hamiltonian circuit in complete graph decomposition.

Keywords: Complete graph, Hamiltonian circuit, Graph decomposition

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Declaration Associated with this Thesis

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List of Symbols

$\boldsymbol{G}$	A graph consists of vertices and edges
$\boldsymbol{V(G)}$	The vertices of a graph G
$\boldsymbol{E(G)}$	The edges of a graph G
$\boldsymbol{G(V,E)}$	A graph G with the set of vertices $V(G)$ and edges $E(G)$
$\boldsymbol{K_n}$	A complete graph with n vertices
$\boldsymbol{C_n}$	A cycle with n vertices
$\boldsymbol{W_n}$	A wheel with n vertices
$\boldsymbol{K_n^*}$	A complete directed graph with n vertices
$\boldsymbol{C_n^*}$	A circuit with n vertices
$\boldsymbol{P_n}$	A path with n vertices
$\boldsymbol{K_{m,n}}$	A bipartite graph with two disjoint sets, m and n
$\boldsymbol{K_{m,n}^c}$	The complement set of bipartite graph $K_{m,n}$
$\boldsymbol{CTn}$	A complete-transposition graph with n vertices
$\boldsymbol{S_k}$	A star graph
$\boldsymbol{D_{r\times c}}$	A Cartesian product with r rows and c columns
$\textbf{deg}\ (\boldsymbol{u})$	The degree of vertex u where u is a vertex in any graph
$\boldsymbol{x_i}$	A vertex in any graph, where $1 \leq i \leq n$
$\boldsymbol{n} \in \mathbb{Z}^+$	The total number of vertices of any graph, where n is positive
$\boldsymbol{\lambda}$	The block for each direction in WS
$\boldsymbol{\gamma}$	Total permutations of n elements

List of Abbreviations

WS	Wing Strategy
BS	Butterfly Strategy
S-R	Shift-and-Rotate Strategy
HBM	Half Butterfly Method

CHAPTER ONE
INTRODUCTION

Graph decomposition is an important research in graph theory because it can model several networks in our daily life such as social network, railway network and internet network (Kante, 2008). In the literature, several studies regarding graph decomposition have been focused such as the studies done by Granville, Moisiadis and Rees (1989); Adams, Bryant, Forbes and Griggs, (2012); and Yuan and Kuang, (2012); among others.

1.1 General Reviews on Graph

Let say, we are planning to visit five historic sites in Malacca without visiting each site more than once. Figure 1.1 shows the routes connecting each site. From the map, we need to manage possible route which will make sure we will never visit any site twice.

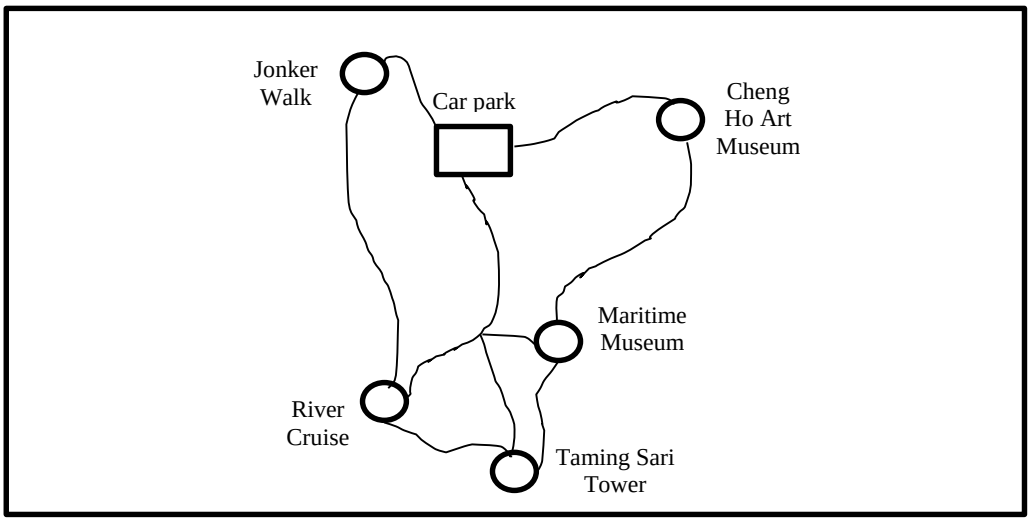

Figure 1.1. Routes for five historical sites in Malacca (this map has been modified from Google maps)

The contents of the thesis is for internal user only

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