BALL SURFACE REPRESENTATIONS USING PARTIAL DIFFERENTIAL EQUATIONS

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Abstrak

Sejak dua dekad lalu, pemodelan geometri menggunakan pendekatan persamaan pembezaan separa (PPS) telah dikaji secara meluas dalam Rekabentuk Geometri Bantuan Komputer (RGBK). Pendekatan ini pada mulanya diperkenalkan oleh beberapa orang penyelidik berdasarkan kepada permukaan Bézier yang berkaitan dengan luas permukaan minimum ditentukan oleh lengkung sempadan yang ditetapkan. Walau bagaimanapun, perwakilan permukaan Bézier boleh diperbaiki dari segi masa pengiraan dan luas permukaan minimum dengan menggunakan perwakilan permukaan Ball. Sehubungan itu, kajian ini membangunkan satu algoritma untuk mengitlak permukaan Ball dari lengkung sempadan menggunakan PPS eliptik. Dua permukaan Ball khusus iaitu harmonik dan dwiharmonik pertamanya dibina dalam membangunkan algoritma yang dicadangkan. Permukaan terdahulu dan kemudian masing-masing memerlukan dua dan empat syarat sempadan. Bagi mengitlak permukaan Ball dalam penyelesaian polinomial untuk sebarang PPS peringkat empat, kaedah Dirichlet digunakan. Keputusan berangka diperolehi keatas contoh titik data yang diketahui umum menunjukkan algoritma permukaan Ball teritlak yang dicadangkan mempamerkan keputusan lebih baik daripada perwakilan permukaan Bézier dari segi masa pengiraan dan luas permukaan minimum. Tambahan pula, algoritma yang baharu dibina juga memenuhi sebarang permukaan dalam RGBK termasuk permukaan Bézier. Algoritma ini kemudiannya diuji dalam permasalahan pengekalan kepositifan permukaan dan pembesaran imej. Keputusan menunjukkan algoritma yang dicadangkan adalah setanding dengan kaedah yang sedia ada dari segi kejituan. Justeru, algoritma ini adalah satu alternatif berdaya maju untuk membina permukaan Ball teritlak. Dapatan daripada kajian ini menyumbang kearah bidang pengetahuan untuk pembinaan semula permukaan berdasarkan pendekatan PPS dalam bidang pemodelan geometri dan grafik komputer.

Kata kunci: Permukaan Ball, Persamaan pembezaan separa, Kaedah Dirichlet, Pengekalan kepositifan, Pembesaran imej.

Abstract

Over two decades ago, geometric modelling using partial differential equations (PDEs) approach was widely studied in Computer Aided Geometric Design (CAGD). This approach was initially introduced by some researchers to deal with Bézier surface related to the minimal surface area determined by prescribed boundary curves. However, Bézier surface representation can be improved in terms of computation time and minimal surface area by employing Ball surface representation. Thus, this research develops an algorithm to generalise Ball surfaces from boundary curves using elliptic PDEs. Two specific Ball surfaces, namely harmonic and biharmonic, are first constructed in developing the proposed algorithm. The former and later surfaces require two and four boundary conditions respectively. In order to generalise Ball surfaces in the polynomial solution of any fourth order PDEs, the Dirichlet method is then employed. The numerical results obtained on well-known example of data points show that the proposed generalised Ball surfaces algorithm performs better than Bézier surface representation in terms of computation time and minimal surface area. Moreover, the new constructed algorithm also holds for any surfaces in CAGD including the Bezier surface. This algorithm is then tested in positivity preserving of surface and image enlargement problems. The results show that the proposed algorithm is comparable with the existing methods in terms of accuracy. Hence, this new algorithm is a viable alternative for constructing generalized Ball surfaces. The findings of this study contribute towards the body of knowledge for surface reconstruction based on PDEs approach in the area of geometric modelling and computer graphics.

Keywords: Ball surface, Partial differential equation, Dirichlet method, Positivity preserving, Image enlargement.

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$B_i^n(t)$	Bernstein polynomial.
$S_i^n(t)$	Said-Ball basis function.
$D_i^n(t)$	DP-Ball basis function.
$A_i^n(t)$	Wang-Ball basis function.
P_{ij}	Bézier control points.
v_{ij}	Said-Ball control points.
d_{ij}	DP-Ball control points.
w_{ij}	Wang-Ball control points.
$B(u,v)$	Bézier rectangular surface.
$S(u,v)$	Said-Ball rectangular surface.
$D(u,v)$	DP-Ball rectangular surface.
$W(u,v)$	Wang-Ball rectangular surface.
$\mathcal{B}$	Bézier monomial matrix.
$\mathcal{S}$	Said-Ball monomial matrix.
$\mathcal{C}$	DP-Ball monomial matrix.
$\mathcal{A}$	Wang-Ball monomial matrix.
$\frac{\partial X(u,v)}{\partial u}$	Partial derivatives.
$\sum_{i=0}^{n} x_i$	The sum $x_0 + x_1 + \cdots + x_n$.
$\binom{n}{i}$	Binomial coefficients.
$\bigtriangledown^2$	Laplace operator.
$\bigtriangledown^4$	Bilaplace operator.
$B^n(t)$	Bézier curve on monomial matrix form.
$A^n(t)$	Wang-Ball curve on monomial matrix form.
$D^n(t)$	DP-Ball curve on monomial matrix form.
$S^n(t)$	Said-Ball curve on monomial matrix form.
$<,>$	Inner product.
$\|\vec{X}\|$	Norm of vector $\vec{X}$
$\triangle$	The usual difference operators.

CHAPTER ONE
INTRODUCTION

1.1 Research Background

Partial differential equations (PDEs) is a large subject with a history that goes back to Newton and Leibnitz. Many mathematical models involve functions which have the property that the value at a point depends on its value in a neighborhood. Dependencies like these can be modeled with a PDE. Famous examples are Newton's second law, Laplaces equation, Schrödinger's equation and Einstein's equations.

Geometric modeling using PDEs have been widely studied in computer graphics for over two decades and was first introduced in blend surface generation by Arnal, Monterde and Ugail (2011), Du and Qin (2004), Monterde (2004), Zhang and You (2004).

Advantages of the PDE methods have been gradually recognized by researchers. A principle advantage comes from the ability that the differential operator of PDEs can ensure the generation of smooth surfaces, where the smoothness is strictly governed by the order of the PDE used. A second advantage of using the PDE method is that the PDE surface can be generated by intuitively manipulating a relative small set of boundary curves. Moreover, the behavior of PDE surfaces has been proven to be compatible with underlying tensor product surfaces, such as Bézier surface (Monterde & Ugail, 2006), B-spline (Bloor & Wilson, 1990) and etc. These advantages have contributed to the widespread adoption of the PDE methods in a wide range of disciplines, such as free-form surface design, solid modeling computer aided manufacturing, shape morphing, web visualization, mesh reconstruct and facial geometry parameterization (Sheng, Sourin, Castro & Ugail, 2010).

The contents of the thesis is for internal user only

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