ON GAMMA-P_S-OPERATIONS IN TOPOLOGICAL SPACES

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Abstrak

Topologi merupakan salah satu bidang tumpuan matematik. Kebelakangan ini, topologi menjadi komponen penting matematik gunaan kerana pelbagai kegunaannya dalam memahami masalah kehidupan sebenar. Konsep asas ruang topologi (X, τ) berkait rapat dengan set terbuka. Operasi pada τ telah dikaji oleh ramai penyelidik. Antara operasi yang telah dikaji ialah terbuka-γ, pra-terbuka-γ, semi-terbuka-γ, terbuka-γ-b, terbuka-γ-β dan terbuka-α-γ. Walau bagaimanapun, pentakrifan kelas set baharu operasi γ pada topologi τ dengan menggabungkan operasi sedia ada belum pernah diteroka. Sehubungan itu, kajian ini bertujuan mentakrif beberapa kelas set baharu, membina kelas fungsi baharu serta memperkenal jenis aksiom pemisahan dan ruang pemisahan baharu berasaskan set terbuka-γ. Kelas baharu yang terbina ialah set sekata-terbuka-γ dan terbuka-γ-P_S. Set peluaran-τ_γ-P_S, pedalaman-τ_γ-P_S, terbitan-τ_γ-P_S dan sempadan-τ_γ-P_S turut terbentuk hasil dari takrifan set terbuka-γ-P_S dan pelengkapnya. Set terbuka-γ-P_S dan pedalaman-τ_γ-P_S seterusnya digunakan untuk mentakrif kelas set baharu bagi set terbuka-γ-P_S yang dinamakan set tertutup teritlak-γ-P_S. Seterusnya, beberapa kelas fungsi baharu yang dikenali sebagai selanjar-γ-P_S, selanjar-(γ, β)-P_S dan tak terlerai-(γ, β)-P_S yang berasaskan set terbuka-γ-P_S diperkenalkan. Selanjutnya, beberapa jenis fungsi-γ-P_S yang lain seperti terbuka-β-P_S dan terbuka-(γ, β)-P_S dibina. Di samping itu, beberapa kelas aksiom pemisahan-γ-P_S baharu diperkenal dengan menggunakan set terbuka-γ-P_S dan pelengkapnya serta set tertutup teritlak-γ-P_S. Perhubungan dan sifat bagi setiap kelas set, fungsi-γ-P_S dan aksiom pemisahan-γ-P_S turut terbentuk. Kesimpulannya, kajian ini telah berjaya mentakrif beberapa kelas set baharu dengan menggunakan operasi γ pada topologi τ.

Kata kunci: Set terbuka sekata-γ, set -γ-P_S, fungsi -γ-P_S, aksiom pemisahan -γ-P_S, ruang-γ.

Abstract

Topology is one of the focus areas in mathematics. Recently, topology has become an important component in applied mathematics due to its vast applications in understanding real life problems. The basic concept of topological space (X, τ) deals with open sets. Operations on τ have been investigated by numerous researchers. Among these operations are γ-open, γ-preopen, γ-semiopen, γ-b-open, γ-β-open and α-γ-open which involve τ_γ-P_S-interior and τ_γ-P_S-closure. However, no one has attempted to define new class of set using operation γ on the topology τ by combining the existing operations. This study, therefore, aims to define new classes of sets, construct new classes of functions, and introduce new types of separation axioms and spaces using the concept of γ-open sets. The new classes developed are γ-regular-open and γ-P_S-open sets. By applying γ-P_S-open sets and their complements, the notions of τ_γ-P_S-closure, τ_γ-P_S-interior, τ_γ-P_S-derived set and τ_γ-P_S-boundary of a set are established. The notions of γ-P_S-open and τ_γ-P_S-closure sets are then used to define a new class of γ-P_S-open sets called γ-P_S-generalised closed sets. Moreover, several new classes of functions called γ-P_S-continuous, (γ, β)-P_S-continuous and (γ, β)-P_S-irresolute functions in term of γ-P_S-open sets are introduced. Furthermore, other types of γ-P_S -functions such as β-P_S-open and (γ, β)-P_S-open are constructed. In addition, some new classes of γ-P_S-separation axioms are established by using γ-P_S-open and its complement as well as γ-P_S-generalised closed sets. The relationships and properties of each class of sets, γ-P_S-functions and γ-P_S- separation axioms are also established. In conclusion, this study has succeeded in defining new classes of sets using operation γ on the topology τ.

Keywords: γ-regular-open set, γ-P_S- sets, γ-P_S- functions, γ-P_S- separation axioms, γ- spaces.

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Declaration Associated With This Thesis

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5. Asaad B.A., Ahmad N. and Omar Z. (2015). (γ, β)-P_S-Irresolute and (γ, β)-P_S-Continuous Functions, *International Journal of Pure and Applied Mathematics*, *99*(1), 77-96. DOI: http://dx.doi.org/10.12732/ijpam.v99i1.7

6. Ahmad N., Omar Z. and Asaad B.A. (2015). New Separation Axioms via γ-P_S-Open Sets in Topology, *Submitted*.

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$P(X)$	Power set of the space X
τ_γ	The class of all γ-open subsets of a space X
τ_γ-$Cl(A)$	τ_γ-closure of a set A of X
τ_γ-$Int(A)$	τ_γ-interior of a set A of X
$\tau_{\alpha\text{-}\gamma}$	The family of all α-γ-open subsets of a space X
τ_γ-$PO(X)$	The class of all γ-preopen subsets of a space X
τ_γ-$SO(X)$	The class of all γ-semiopen subsets of a space X
τ_γ-$\beta O(X)$	The class of all γ-β-open subsets of a space X
τ_γ-$BO(X)$	The class of all γ-b-open subsets of a space X
τ_γ-$RO(X)$	The class of all γ-regular-open subsets of a space X
τ_γ-$PC(X)$	The class of all γ-preclosed subsets of a space X
τ_γ-$SC(X)$	The class of all γ-semiclosed subsets of a space X
τ_γ-$\beta C(X)$	The class of all γ-β-closed subsets of a space X
τ_γ-$BC(X)$	The class of all γ-b-closed subsets of a space X
τ_γ-$RC(X)$	The class of all γ-regular-closed subsets of a space X
τ_γ-$P_SO(X)$	The class of all γ-P_S-open subsets of a space X
τ_γ-$P_SC(X)$	The class of all γ-P_S-closed subsets of a space X
τ_γ-$pCl(A)$	τ_γ-preclosure of a set A of X
τ_γ-$sCl(A)$	τ_γ-semi-closure of a set A of X
$\tau_{\alpha\text{-}\gamma}$-$Cl(A)$	$\tau_{\alpha\text{-}\gamma}$-closure of a set A of X
τ_γ-$bCl(A)$	τ_γ-b-closure of a set A of X
τ_γ-$\beta Cl(A)$	τ_γ-β-closure of a set A of X
τ_γ-$P_SCl(A)$	τ_γ-P_S-closure of a set A of X
τ_γ-$pInt(A)$	τ_γ-preinterior of a set A of X
τ_γ-$sInt(A)$	τ_γ-semi-interior of a set A of X
$\tau_{\alpha\text{-}\gamma}$-$Int(A)$	$\tau_{\alpha\text{-}\gamma}$-interior of a set A of X

τ_γ-***bInt*(*A*)**	τ_γ-*b*-interior of a set *A* of *X*
τ_γ-***βInt*(*A*)**	τ_γ-*β*-interior of a set *A* of *X*
τ_γ-P_S***Int*(*A*)**	τ_γ-P_S-interior of a set *A* of *X*
τ_γ-P_S***D*(*A*)**	The set of all τ_γ-P_S-limit points of a set *A* of *X*
τ_γ-P_S***Bd*(*A*)**	τ_γ-P_S-boundary of a set *A* of *X*
τ_γ-P_S***GC*(*X*)**	The class of all γ-P_S-*g*-closed subsets of a space *X*
τ_γ-P_S***GO*(*X*)**	The class of all γ-P_S-*g*-open subsets of a space *X*

CHAPTER ONE

INTRODUCTION

1.1 Introduction

Topology is one of the major areas in mathematics. Recently, topology has become an important component of applied mathematics as many mathematicians and scientists employing concepts of topology to model and understand real-world structures and phenomena. The term topology, literally, means the study of position or location. In other words, topology is the study of shapes, including their properties, deformations applied to them, mappings between them, and configurations composed of them. The definition of a topology that used throughout this study is stated as follows.

Definition 1.1.1. (see Steen and Seebach, 1978) Let X be a nonempty set. A class τ of subsets of X is a topology on X if τ satisfies the following three conditions:

1. The empty set ϕ and the whole set X belong to τ.

2. The union of any finite or infinite number of sets in τ belongs to τ.

3. The intersection of any two (finite) number of sets in τ belongs to τ.

Members of τ are often called as open sets and the pair (X, τ) is called a topological space. A subset A of a topological space (X, τ) is closed if its complement is open. The closure of A is the intersection of all closed sets of X containing A. The interior of A

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REFERENCES

Abd El-Monsef M. E., El-Deeb S. N. & Mahmoud R. A. (1983). *β*-open sets and *β*-continuous mappings, *Bull. Fac. Sci. Assuit. Univ., 12*(1), 1-18.

Abd El-Monsef M. E., Mahmoud R. A. & Lashin E. R. (1986). *β*-closure and *β*-interior, *J.* Fac. Ed. Ain. Shams. Univ., *10*, 235-245.

Andrijevic D. (1996). On b-open sets, *Matematicki Vesnik*, *48*(1-2), 59-64.

Andrijevic D. (1986). Semi-preopen sets, *Matematicki Vesnik*, *38*, 24-32.

Arya S. P. & Gupta R. (1974). On strongly continuous functions, *Kyungpook Mathematical Journal*, 14, 131-143.

Basu, C. K., Afsan, B. M. U. & Ghosh, M. K. (2009). A class of functions and separation axioms with respect to an operation. *Hacettepe Journal of Mathematics and Statistics*, *38*(2), 103-118.

Carpintero C., Rajesh N. & Rosas E. (2012*a*). Operation-*b*-open sets in topological spaces, *fasciculi mathematici*, *48*, 13-21.

Carpintero C., Rajesh N. & Rosas E. (2012*b*). Operation approaches on *b*-open sets and applications, *Bulletin of Parana´s Mathematical Society*, *30*(1), 21-33.

Carpintero C., Rajesh N. & Rosas E. (2012*c*). Somewhat (*γ, β*)-semicontinuous functions, *Bulletin of Parana´s Mathematical Society*, *30*(1), 45-52.

Crossley, S. G. & Hildebrand, S. K. (1971). Semi-closure, *Texas Journal of Science*, *22*, 99-112.

El-Deeb, S. N., Hasanein, I. A., Mashhour, A. S. & Noiri, T. (1983). On p-regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie*, *27*(4), 311-315.

Ghosh M. K. (2012). On γ-Preopen Sets, *International Journal of Mathematical Analysis*, *6(*53), 2633-2645.

Jankovic D. S. (1983). On functions with α-closed graphs, *Glasnik Matematicki*, *18*(38), 141-148.

Khalaf A. B. & Asaad B. A. (2009). P_S-open sets and P_S-continuity in topological spaces, *Journal of Duhok University*, *12*(2), 183-192.

Kalaivani N. & Krishnan G. S. S. (2009). On α-γ-open sets in topological spaces, *Proceedings of ICMCM*, 370-376.

Kalaivani N. & Krishnan G. S. S. (2012). Operation approaches on α-(γ, β)-open (closed) mappings and γ generalized α-open sets, *International Journal of Scientific and Engineering Research*, *3*(8), 358-373.

Kalaivani N. & Krishnan G. S. S. (2013). Operation approaches on α-γ-open sets in topological spaces, *International Journal of Mathematical Analysis*, *7*(10), 491-498.

Kalaivani N., Kumar S. D. & Krishnan G. S. S. (2012). On α-γ-irresolute functions and α-γ-continuous functions in topological spaces, *Proceedings of the Jangjeon Mathematical Society*, *15*(4), 465-476.

Kasahara, S. (1979). Operation compact spaces. *Mathematica Japonica*, *24*(1), 97-105.

Krishnan G. S. S. (2003). A new class of semi open sets in a topological space, *Proc. NCMCM, Allied Publishers, New Delhi*, 305-311.

Krishnan G. S. S. & Balachandran K. (2006*a*). On a class of γ-preopen sets in a topological space, *East Asian Math. J.*, *22*(2), 131-149.

Krishnan G. S. S. & Balachandran K. (2006*b*). On γ-semiopen sets in topological spaces, *Bulletin Calcutta Mathematical Society*, *98*(6), 517-530.

Krishnan G. S. S., Ganster M. & Balachandran K. (2007). Operation approaches on semi-open sets and applications, *Kochi Journal of Mathematics*, *2*, 21-33.

Levine, N. (1963). Semi-open sets and semi-continuity in topological spaces, *American Mathematical Monthly*, *70*(1), 36-41.

Long P. E. (1986). *An Introduction to General Topology*, Charles E. Merrill Publishing Company.

Mashhour, A. S., Abd El-Monsef, M. E. & El-Deeb, S. N. (1982). On pre-continuous and week precontinuous mappings, *Proc. Math. Phys. Soc. Egypt*, *53*, 47-53.

Njastad O. (1965). On some classes of nearly open sets, *Pacific Journal of Mathematics*, *15*(3), 961-970.

Ogata, H. (1991). Operation on topological spaces and associated topology, *Mathematica Japonica*, *36*(1), 175-184.

Reilly I. L. & Vamanmurthy M. K. (1985). On α-continuity in topological spaces, *Acta Mathematica Hungarica*, *45*(1-2), 27-32.

Steen L. A. & Seebach J. A. (1978). *Counterexamples in Topology*, New York Heidelberg Berlin: Springer Verlag.

Tahiliani S. (2011). Operation approach to β-open sets and applications, *Mathematical Communications*, *16*, 577-591.

Van An T., Cuong D. X. & Maki H. (2008). On operation-preopen sets in topological spaces, *Scientiae Mathematicae Japonicae Online*, e-2008, 241-260.