QUANTUM SYSTEM BEHAVIOUR OF JAYNES-CUMMINGS MODEL WITH KERR-LIKE MEDIUM

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Abstrak

Model Jaynes-Cummings digunakan secara meluas dalam sistem kuantum kerana kemampuannya untuk menerangkan telatah kuantum dengan lebih tepat dan mudah. Terkini, kajian tentang model Jaynes-Cummings tidak melibatkan peralihan multi-foton dan keterlibatan kuantum tri-*qubit* yang kedua-duanya digandingkan bersama medium Kerr-like. Oleh itu, tujuan utama kajian ini adalah mencari telatah baharu untuk sistem kuantum dengan kedua-dua syarat tersebut digandingkan bersama medium Kerr-like. Bagi mencapai objektif ini, model Jaynes-Cummings diubahsuai dengan menambah peralihan multi-foton dan sistem kuantum tri-*qubit* digandingkan bersama medium Kerr-like. Berdasarkan syarat peralihan multi-foton, keformalan Pegg-Barnett digunakan untuk mengukur telatah sistem kuantum dalam model Jaynes-Cummings terubah suai. Hasil kajian menunjukkan apabila kekuatan gandingan meningkat, telatah sistem kuantum menjadi lebih aktif. Walau bagaimanapun, peningkatan dalam bilangan peralihan foton akan mengurangkan pengaruh medium Kerr-like terhadap telatah sistem kuantum. Seterusnya, berdasarkan syarat sistem kuantum tri-*qubit* bersama peralihan foton-tunggal, keadaan tri-*qubit* kuantum berinteraksi dengan persekitaran Markovan dan tak-Markovan, yang keduanya diwakili oleh ketumpatan spektrum Lorenztian. Keserentakan batas bawah digunakan untuk mengukur keteguhan keterlibatan kuantum.Hasil kajian menunjukan apabila kekuatan gandingan Kerr-like ditingkatkan untuk kedua-dua persekitaran Markovan dan tak-Markovan, keterlibatan kuantum bertambah teguh. Pada masa yang sama, pengaruh keteguhan keterlibatan kuantum berkurang apabila interaksi dwikutub-dwikutub semakin kuat. Kesimpulannya, kajian ini telah menemui telatah baharu bagi sistem kuantum dengan pengaruh medium Kerr-like yang mempunyai potensi dalam aplikasi pemprosesan maklumat kuantum.

Kata kunci: Model Jaynes-Cummings, Keadaan kuantum tri-*qubit*, Peralihan multi-foton, Medium Kerr-like.

Abstract

Jaynes-Cummings model is widely used to represent a quantum system as it is able to explain quantum behaviour in a more accurate and simple way. To date, the study of Jaynes-Cummings model does not involve multi-photon transitions and also three-qubit quantum entanglement, both coupled with Kerr-like medium. Thus the main objective of this study is to discover new behaviour for quantum system under these two conditions coupled with Kerr-like medium. In achieving this objective, Jaynes-Cummings model is modified to include multi-photon transition and three-qubit quantum system coupling with Kerr-like medium. Under the multi-photon transition condition, Pegg-Barnett formalism is used to measure the quantum system behaviour in the modified Jaynes-Cummings model. The result shows that as the strength of the coupling increases, the quantum system behaviour becomes more active. However, as the number of photons transition increases, the influence from Kerr-like medium towards quantum system behaviour decreases. Next, under the three-qubit quantum system with one-photon transition condition, the three-qubit state interacts with Markovian and non-Markovian environments, both represented by Lorenztian spectral density. The lower bound concurrence is used to measure quantum entanglement robustness. Result shows that when Kerr-like medium coupling strength is increased for both Markovian and non-Markovian environments, the quantum entanglement are more robust. Concurrently, the influence of quantum entanglement robustness is reduced when dipole-dipole interaction is getting stronger. As a conclusion, this study discovered new quantum system behaviour under the influence of Kerr-like medium with potential application in quantum information processing.

Keywords: Jaynes-Cummings model, Three-qubit quantum state, Multi-photon transition, Kerr-like medium.

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List of Symbols

$|GHZ\rangle$ GHZ state for three-qubit

$|W\rangle$ W State for three-qubit

$|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |001\rangle\right)$

$|001\rangle, |010\rangle, |100\rangle$ and $|111\rangle$ Different three-qubit state

$|1\rangle$ Excited state

$|0\rangle$ Ground state

Δx Change of particle's vector

Δp Change of particle's momentum

$\hbar$ Planck constant

H Hamiltonian of total system

V Potential energy

T Kinetic energy

n Number of particles or photon

r Position vector

p Momentum of the particle

t Time

m Mass of the particle

ω_0 Atomic transition frequency

ω Cavity field frequency

σ_z Atomic pseudo spin inversion

σ^+ Raising operator

σ^- Lowering Operator

g Atom field coupling constant

χ Kerr-like medium coupling strength

a^t Annihilation operator

a Creation operator

H_0 Free Hamiltonian and

H_1 Atom-cavity field interaction

i Complex number $\sqrt{-1}$

$|\Psi(t)\rangle$ Quantum state after t

$|\Psi(0)\rangle$ Quantum state at $t = 0$

exp, ℓ Exponential

$|\alpha\rangle$ Cavity field state

φ Phase angle

$\overline{n}$ Average photon number

α Amplitude of the cavity field

$|n\rangle$ Energy eigenvector of Hamiltonian

G Markovian or non-Markovian environment

R Qubit and cavity field coupling strength

Γ Half-width at half-height of field spectrum profile

α_n Dimentionless real constant

T_q Relaxation time

T_c Cavity correlation time

E Quantity of entanglement

ρ Density operator

U Unitary Operator

$S_v(\rho)$ Von Neumann entropy

p Probabilities outcome of measurement

ρ_{Q1} Reduce density operator of quantum state

$\left|\Phi^+\right\rangle$ Bell state of two-qubit quantum system with $\left|\Phi^+\right\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

ρ_A Reduce density operator of quantum state A

ρ_B Reduce density operator of quantum state B

$\left|\Psi^+\right\rangle$ Bell state of two-qubit quantum system with $\left|\Psi^+\right\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

$\left|\Psi^-\right\rangle$ Bell state of two-qubit quantum system with $\left|\Psi^-\right\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$

$H1$ Hilbert space

$\left|\Phi\right\rangle$ Quantum state

$C(\rho)$ Concurrence

λ Square root of eigenvalue of quantum system

E_f Entanglement of formation

$\langle \,|$ adjoint of quantum state, $|\, \rangle$

d dipole-dipole interaction

K Kerr-like medium coupling strength

CHAPTER ONE

INTRODUCTION

1.1 Introduction

Quantum physics has gained a considerable interest for its potential impact on technology. One of the uses of quantum physics is quantum information processing. Quantum information processing can be divided into quantum cryptography, computation, and teleportation (Atteberry, n.d.). Quantum information processing needs a robust quantum entanglement. The behaviour of a quantum system is described by its quantum state as a function of time. A quantum state is a vector in a vector space, which can also be called a state vector that describes the quantum system. A state vector contains the position and momentum of a particle, which describe the quantum state. This study mainly focuses on quantum entanglement and quantum system behaviour, which are useful in quantum information processing application.

1.2 Qubit in Quantum System

In quantum information processing, a quantum system is used. Data is stored, processed, and transmitted digitally in terms of qubit. The term qubit is used to represent a quantum system, which has two dimensions. For a quantum system consisting of two qubits, it will be represented by a density matrix with the symbol ρ.

The contents of the thesis is for internal user only

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