AN ENHANCED APPROXIMATION MATHEMATICAL MODEL INVENTORYING ITEMS IN A MULTI-ECHELON SYSTEM UNDER A CONTINUOUS REVIEW POLICY WITH PROBABILISTIC DEMAND AND LEAD-TIME

KARZAN MAHDI GHAFOUR

DOCTOR OF PHILOSOPHY
UNIVERSITI UTARA MALAYSIA
2016

Awang Had Salleh
Graduate School
of Arts And Sciences

Universiti Utara Malaysia

PERAKUAN KERJA TESIS / DISERTASI
(Certification of thesis / dissertation)

Kami, yang bertandatangan, memperakukan bahawa
(We, the undersigned, certify that)

KARZAN MAHDI GHAFOUR - 94083

calon untuk Ijazah PhD
(candidate for the degree of)

telah mengemukakan tesis / disertasi yang bertajuk:
(has presented his/her thesis / dissertation of the following title):

"AN ENHANCED APPROXIMATION MATHEMATICAL MODEL INVENTORYING ITEMS IN A MULTI-ECHELON SYSTEM UNDER A CONTINUOUS REVIEW POLICY WITH PROBABILISTIC DEMAND AND LEAD-TIME "

seperti yang tercatat di muka surat tajuk dan kulit tesis / disertasi.
(as it appears on the title page and front cover of the thesis / dissertation).

Bahawa tesis/disertasi tersebut boleh diterima dari segi bentuk serta kandungan dan meliputi bidang ilmu dengan memuaskan, sebagaimana yang ditunjukkan oleh calon dalam ujian lisan yang diadakan pada : *16 Jun 2016.*
That the said thesis/dissertation is acceptable in form and content and displays a satisfactory knowledge of the field of study as demonstrated by the candidate through an oral examination held on:
June 16, 2016.

Pengerusi Viva: *(Chairman for VIVA)*	**Prof. Dr. Najib Ahmad Marzuki**	Tandatangan *(Signature)*
Pemeriksa Luar: *(External Examiner)*	**Prof. Dr. Anton Abdulbasah Kamil**	Tandatangan *(Signature)*
Pemeriksa Dalam: *(Internal Examiner)*	**Dr. Ruzelan Khalid**	Tandatangan *(Signature)*
Nama Penyelia/Penyelia-penyelia: *(Name of Supervisor/Supervisors)*	**Assoc. Prof. Dr. Razamin Ramli**	Tandatangan *(Signature)*
Nama Penyelia/Penyelia-penyelia: *(Name of Supervisor/Supervisors)*	**Dr. Nerda Zura Zaibidi**	Tandatangan *(Signature)*

Tarikh:
(Date) ***June 16, 2016***

Permission to Use

In presenting this thesis in fulfilment of the requirements for a postgraduate degree from Universiti Utara Malaysia, I agree that the Universiti Library may make it freely available for inspection. I further agree that permission for the copying of this thesis in any manner, in whole or in part, for scholarly purpose may be granted by my supervisor(s) or, in their absence, by the Dean of Awang Had Salleh Graduate School of Arts and Sciences. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall be given to me and to Universiti Utara Malaysia for any scholarly use which may be made of any material from my thesis.

Requests for permission to copy or to make other use of materials in this thesis, in whole or in part should be addressed to:

Dean of Awang Had Salleh Graduate School of Arts and Sciences
UUM College of Arts and Sciences
Universiti Utara Malaysia
06010 UUM Sintok

Abstrak

Suatu sistem inventori berusaha mengimbangi antara lebihan stok and kekurangan stok bagi mengurangkan jumlah kos dan mencapai permintaan pengguna dalam masa yang tepat. Sistem inventori adalah seperti entiti yang tersembunyi dalam rantaian bekalan, yang mana rangkaian lengkap yang besar menyelaraskan satu siri proses yang saling berkaitan untuk sesuatu pengeluar, bagi mengubah bahan mentah kepada produk akhir dan mengagihkannya kepada pelanggan. Inventori optimum dan peruntukan dasar dalam rantaian bekalan untuk industri simen bagi kebanyakan jenis sistem pelbagai lapisan masih tidak diketahui. Dalam rangkaian pelbagai lapisan, kerumitan wujud apabila berbagai isu inventori timbul dalam pelbagai peringkat yang mana prestasi mereka dipengaruhi secara signifikan oleh permintaan dan masa-pendulu. Oleh itu, objektif kajian ini adalah untuk membangunkan satu model matematik teranggar yang ditambahbaik dalam satu sistem inventori pelbagai lapisan melalui dasar ulasan berterusan yang tertakluk kepada permintaan berkebarangkalian dan masa-pendulu. Fungsi taburan kebarangkalian permintaan semasa masa-pendulu dijana dengan membangunkan satu model simulasi baru berkaitan permintaan semasa masa-pendulu (*SMDDL*) menggunakan prosedur simulasi. Model ini berupaya meramal permintaan dan permintaan semasa masa-pendulu untuk masa hadapan. Permintaan semasa masa-pendulu untuk masa hadapan yang diperoleh digunakan untuk membangun satu model inventori pelbagai lapisan bersiri (*SMEI*) dengan menerbitkan fungsi kos inventori untuk mengira ukuran prestasi bagi sistem inventori industri simen. Berdasarkan ukuran prestasi tersebut, satu model inventori pelbagai lapisan taburan yang diubahsuai dengan aturan tiba dahulu layan dahulu (*FCFS*) (*DMEI-FCFS*) diterbitkan untuk menentukan jangka masa menunggu terbaik dan jangkaan bilangan peruncit dalam sistem berdasarkan min kadar ketibaan dan min kadar perkhidmatan. Kajian ini menghasilkan lima fungsi taburan baharu bagi permintaan semasa masa-pendulu. Semua fungsi taburan mampu menambahbaik ukuran prestasi yang mana ianya menyumbang kepada pengurangan dalam jangka masa menungu dalam sistem. Keseluruhannya, model teranggar ini dapat mencadangkan tempoh masa yang tepat bagi mengatasi masalah kekurangan inventori simen yang mana seterusnya memenuhi kepuasan pelanggan.

Kata kunci: Model inventori pelbagai lapisan, dasar ulasan berterusan, permintaan dan masa-pendulu berkebarangkalian , prosedur simulasi, aturan FCFS

.

Abstract

An inventory system attempts to balance between overstock and understock to reduce the total cost and achieve customer demand in a timely manner. The inventory system is like a hidden entity in a supply chain, where a large complete network synchronizes a series of interrelated processes for a manufacturer, in order to transform raw materials into final products and distribute them to customers. The optimality of inventory and allocation policies in a supply chain for a cement industry is still unknown for many types of multi-echelon inventory systems. In multi-echelon networks, complexity exists when the inventory issues appear in multiple tiers and whose performances are significantly affected by the demand and lead-time. Hence, the objective of this research is to develop an enhanced approximation mathematical model in a multi-echelon inventory system under a continuous review policy subject to probabilistic demand and lead-time. The probability distribution function of demand during lead-time is established by developing a new Simulation Model of Demand During Lead-Time (SMDDL) using simulation procedures. The model is able to forecast future demand and demand during lead-time. The obtained demand during lead-time is used to develop a Serial Multi-echelon Inventory (SMEI) model by deriving the inventory cost function to compute performance measures of the cement inventory system. Based on the performance measures, a modified distribution multi-echelon inventory (DMEI) model with the First Come First Serve (FCFS) rule (DMEI-FCFS) is derived to determine the best expected waiting time and expected number of retailers in the system based on a mean arrival rate and a mean service rate. This research established five new distribution functions for the demand during lead-time. The distribution functions improve the performance measures, which contribute in reducing the expected waiting time in the system. Overall, the approximation model provides accurate time span to overcome shortage of cement inventory, which in turn fulfil customer satisfaction.

Keywords: Approximation multi-echelon inventory model, continuous review policy, probabilistic demand and lead-time, simulation procedures, FCFS rule

Acknowledgement

First and foremost, I would like to thank my mother may God prolong her life and my father God's mercy him for their unconditional love, support, and encouragement throughout my entire life, especially during my academic career.

My hearty thanks and gratitude to my great supervisors Associate Professor Dr. Razamin Ramli and Dr. Nerda Zura Zaibidi for their guidance, sharing their expertise, offering constructive remarks, patience and endurance during the research period; in addition to the support that I received is invaluable to the start of my career and completes my thesis.

I also extend my sincere thanks and gratitude to the examiner committee for provides their precious time to enrich this thesis by their valuable observations and suggestions.

I would also like to thank all lecturers and management staff of the School of Quantitative Science, SQS especially and UUM generally, where, they are very friendly, helpful, assistance proactive and more importantly you are not feeling strange, conversely feeling in your home.

Finally, I would like to convey my special credit to my wife that she persevered, endured and encouraged to complete this thesis. Also to my great brothers and friends for their invaluable moral support and prayers in making this vision came true.

Table of Contents

List of Tables

List of Figures

List of Appendices

CHAPTER ONE

INTRODUCTION

Over time, there have been an increasing number of studies in the area of a multi-echelon inventory system, which is popularly called Supply Chain Management (SCM). The reason behind this continued interest is not only because of the complexity that arises from the interaction between the different stages (echelons) but also due to its immensely practical application in the real world.

A supply chain is a complete system or network that synchronizes a series of interrelated processes (businesses, works, or jobs) in order to transform raw materials into final products or semi-finished goods, and distribute these final products from a distribution center to retailers or to customers directly (Min & Zhou, 2002). The primary objective of a supply chain is to maximize the profitability for all partners involved. The partners can be a single firm or more than one firm. The objective can be met if all partners think 'win-win' and are not worried about their individual performance optimization (Chopra & Meindl, 2001).

Traditionally, inventories at various stocks in a supply chain were managed independently and stored with high inventories (Chen and Mushaluk 2014, Yvan 2011). Market globalization and competitive pressures have increasingly forced companies to make more efforts to optimally control their inventories while improving customer service (Yang and Geunes 2007, Agudelo 2009). As a result, industrial practitioners and academic researchers have begun to pay extra attention to multi-echelon inventory management, which takes the interactions between different stocks in a supply chain into consideration.

Usually, manufacturing processes go through several stages until the final product is reached (Axsater, 2010; Beamon, 1998). This is an example of what multi-echelons mean in a supply chain. The massively tangible practical application makes the interaction through the stages very complex. Because of this complexity, many researchers focus on the system of multi-stage (echelon) inventory control under the name of Supply Chain Management (SCM). A SCM requires that all parties concerned, directly or indirectly, organize and coordinate the flow of materials from suppliers to end consumers for satisfying consumers request (Chopra & Meindi, 2010).

Most of the previous studies on multi-echelon inventory systems such as Axsäter and Marklund (2008), Axsäter (1984), Clark and Scarf (1960), Graves (1998), Hausman and Erkip (1994) Hosoda and Disney (2006) and Muckstadt (1986) assume fixed lead-time or ignore the lead-time with probabilistic or constant demand. The probabilistic demand and probabilistic lead-time make the models of a multi-stage inventory system increasingly difficult than the deterministic models. Even most studies that considered probabilistic demand and lead-time adopt the Poisson distribution (Axsäter, 2011; Axsäter & Marklund, 2008; Bookbinder, Cakanyildrim, 1999; Hosoda & Disney, 2006; Simchi-Levi & Zhao, 2005), which assumed discrete distribution or slow moving items (Deng, Song, Ji, & Zhang, 2010; Ghafour, 2007; Zhao et al., 2006).

However, it is crucial also to consider and focus on probabilistic demand and probabilistic lead-time with uncertainty in high demand and lead-time, such as in the cement manufacturing sector. The rationale for investigating this manufacturing sector was triggered by various situations (Greenstone & Syverson, 2012). The manufacturing process which passes through several stages (echelons), starting with storing raw materials, then going through the process of product manufacturing and storing in warehouses, and finally distributing to retailers through distribution centers (Karaman, 2007; You & Grossmann, 2011) . All of these stages need processors according to a multi-echelon inventory system regarding how much and when to order based on a probabilistic inventory system.

1.1 Challenges of Supply chain management

Previous studies in operations management, e.g., Hesse and Rodrigue (2004) and Stadtler (2005) focused on the analysis of a single company with suitable tools to develop efficiency in the firm through optimal solutions. At present, the globalization of the market and increased competition dominate business decisions between companies (Li, 2013; Pal, Sankar, & Chaudhuri, 2012). Moreover, more items and products reach customers through a supply chain, which consists of independent companies. A longer supply chain often involves a longer delivery lead time. As a result, the chain will often be expected to be less reliable because a longer chain may have low production flexibility. In addition there increasing difficulties to adapt to changes in the system because of a higher level of inventory. The answer to the problem of a longer lead-time is to accelerate the supply chain. In other words, there are many challenges faced by manufacturing industries, such as the cement manufacturers, infrastructure, and power plants. These challenges, if not addressed, may affect the economic growth and investment opportunities. The current challenges can be classified into factors which are human resource, logistics, infrastructure, politics, and security.

1.1.1 Human resource factor

Human resource management is critical in the manufacturing world. Accordingly, human resource management practitioners have to develop new competencies in areas, such as changing the management and technology (Crouse, Doyle, & Young, 2011). Learning is a critical element and an important aspect of institution life because it helps individuals of the institutions adapt to changing environments (Doyle & Young, 2007), assists in growth and innovation, and helps develop competitiveness (Kock, 2007; Warring, Döös, Wilhelmson, Backlund, & Dixon, 2005). Furthermore, learning was positively associated with organizational performance (Olsen & Eikebrokk, 2010). Subsequently, interest in ergonomics learning has also increased in recent years (Ellinger & Cseh, 2007; Ouweneel, Taris, van Zolingen, & Schreurs, 2009).

However, the challenges to work in learning environment are factors that prevent learning from starting, impede or interrupt learning, or terminate learning earlier than it might be ordinarily happen (Hicks, Bagg, Doyle, & Young, 2007). Past literature suggested 45 learning challenges. However, Doyle, Reid and Young (2008), Lohman (2000; 2005; 2009), Paige (2002), White et al. (2000) and Crouse et al. (2011) reduced them to nine in view of the commonalities of the learning challenges. They include taking programs and courses, doing new tasks, working with others, implementing e-learning, observing, trying trial and error, reading/researching, reflecting on the action and doing feedback or replication/vision.

Other challenges appear to be more common for some groups than for others. For example, the cost of learning was found to be a bigger challenge for managers in small companies and factories than for managers in big companies and factories (Doyle et al., 2008). Partners thought that it was more difficult for trainees and directors to learn because there were too few knowledgeable people to help them. Shared and unique educational challenges exist within and through different professional strategies.

The human resource factor plays a significant role in developing the capability of the organizations. In contrast, if the organization lacks of the human resource capability, it will not be able to cope with future challenges.

1.1.2 Logistics factor

The concept of logistics or reverse logistics is an answer not only for technical recovery, customer requirements, and technological innovations in the economy, but also for environmental pollution recovery that causes conflict between the economy and the environment (Abed, Alimi, Ghédira, Hsairi, & Benabdelhafid, 2011). Reverse logistics is the tool for creating and restoring economics and environmental balance (Popa, 2009). Reverse logistics permit the operation of goods, items, or products to move from a destination of their typical final point to the source of origin/recovery, which means that the purpose of conceivable reuse for all returned items or their parts and re-empowering these materials to forward logistics can be ascertained (Lee, 2005). Therefore, reverse logistics focus on the possibilities of value recovery from the used and returned goods (Stock, 1992). During the past years, the concept of reverse logistics has involved creating a practical usage by client companies with flows of returns, such as end-of-use returns, product recall, and warranty service returns. In highlighting the importance of the market, competition, the environment and the economy, it is also necessary to define the challenging elements of reverse logistics, such as economic barriers, organizational barriers, barriers related to market, and barriers related to government (Starostka-Patyk, Zawada, Pabian, & Abed, 2013).

Sharma, Panda, Mahapatra, & Sahu (2011) classified the challenges in reverse logistics as absence of awareness about reverse logistics, management inattention, financial constraints, personal resources, problems with product quality, lack of appropriate performance management systems, inadequate information and technological systems, company policies, legal issues, administrative and financial burden of taxes, and limited forecasting and planning.

For example, the current issues of logistics factor in the cement industry in Iraq are due to the security situations. Trucks have to go through many checkpoints before they can enter the Iraq-Kurdistan region to ensure the validity of the arrivals' information. Moreover, most truck drivers that transport materials do not have the necessary documents, such as a general driving license and truck documents to present. Also, a lower level of the transportation sector is a challenge in these cases. The Iraq-Kurdistan region solely depends on road transportation because a sea port is not available due to its geographical location and because air transport is expensive.

As a result, transportation delivery takes a long time, which has a negative impact on the supply chain performance of the companies (Curtis, 2013). Therefore, logistics plays an important role to supply materials to the designated destinations.

1.1.3 Infrastructure factor

The prevailing construction element used in power plants, wharves, bridges, buildings and other infrastructures in the world is concrete (Stewart, Wang, & Nguyen, 2012). In Australia, more than $140 billion is spent yearly on houses, ports, buildings, dams, bridges and many other physical infrastructures (Zhao et al., 2006). In the United States, there are over five million commercial buildings, more than 500,000 highway bridges, over 400 huge airports, and many other physical infrastructures. Therefore, infrastructure performance is vital to provide the nation with the essential services and keep its economic activities alive (Cook, 2006).

However, infrastructure often deteriorates with age, and the worldwide annual cost is estimated to exceed $1.8 trillion, which represents 3-4% of gross domestic product (GDP) of industrialized countries (Stewart, Wang, & Nguyen, 2012). Concrete is the biggest volume material used by human and is indispensable for innumerate big infrastructure development (Agudelo, 2009).

Infrastructures play an important role to rebuild the foundations and pillars of a state. For example, the recent history of Iraq is full of tragic events, and Kurdistan is not an exception to these events. Back in history from 1980 to 1988, the Iraqi and Iranian war lasted for eight years, and then for the next two years, there were ethnic cleansing and genocide. Subsequently, from 1990 to 2003, there were Gulf War I, Gulf War II, and the US-led invasion. All of these wars and conflicts have wrecked the infrastructures. Rebuilding or rehabilitating infrastructures needs experience and skills. Because of these political issues and conflicts, the cement industry emerges as one of the most important industries for development (Bengio, 2012; Edwards, 2012; USAID, 2007). However, with the expectation of a huge demand, it is difficult to estimate the total demand for cement in Iraq, especially in the Kurdistan region.

1.1.4 Political factor

The political factor or political unrest is one of the challenges that face the growth of different fields, such as industry, economy, and agriculture (Boddewyn & Brewer, 2014; Duffield, 2012; Greenstone, List, & Syverson, 2012). Political behavior typically involves the securing, acquisition, development, and use of power in relation to other entities. Here, power is viewed as the capacity of social actors to overcome the resistance of other actors; e.g., related actors located in the nonmarket environment of the firm, governments and interest groups (Lux, 2013). But, when this factor becomes a handicap in the face of economic development because power is used for personal interests, or when a new political party takes office and changes an organization to be in accordance with its interests, it becomes a negative factor and one of the critical challenges (Chwastiak, 2013). For example, in the parliamentary elections of the Iraq-Kurdistan region's government in September 2013, a new political party appeared under the name, 'the Change'. The political party obtained a second place in parliamentary elections with a large number of parliamentary seats with the third party (Independent Higher Elections Commission, 2013, Kurdistan Regional Government, 2013). The political scenario has an overall impact on the economic process and creates disturbances in markets, which leads to the volatility of the market in terms of supply and demand. The target of this new political party is to fight against administrative and economic corruptions, so that organized situations in the regional government can be established. These situations led to a conflict of interests among large complacent corporations which in turn led to a disturbed market.

1.1.5 Security factor

Recently, the security factor has become one of the challenges and impediments in many countries (Dekker, 2009; Klein, 2007; Liow, 2004; Schwartz, 2007). Insecurity ergonomics lead to the absence of qualified staff, inadequate telecommunications, damaged and looted buildings, and general lack of fiscal infrastructure and policies (Zunes, 2009). For example, Iraq becomes one of the countries that has a serious security situation (Chwastiak, 2013). Moreover, because of the collapse of the security situation in the central and southern Iraq, the process of reconstruction has been slowed. Terrorism that is still besetting Iraq in general, except the Iraq-Kurdistan region, has made selling and delivering of materials and goods in this region more expensive and complex (Zunes, 2009). The insecurity situation has led to increased pressure and demand for raw materials used in infrastructure on the Iraq-Kurdistan region to meet the needs of other parts of Iraq, in addition to meeting Kurdistan's needs.

1.2 Supply chain Management in manufacturing industry

SCM is about organizing and coordinating the flow of materials from the supplier to the end consumer through the processes of production, storage, and distribution (Federgruen & Zipkin, 1984). Any company, whether producer or consumer, seeks to maximize its profits through certain procedures and practices. SCM is also defined as the mission of merging organizational units along a supply chain and coordinating the flow of financial, information, and materials to meet customer demands with the goal of improving the competitiveness of an SC as a whole (Huang and Xue, 2012). Integration of SC involves a systematic connection between internal and exogenous business operations during the management processes to control material, information, and flow of cash effectively (Agudelo, 2009; Kannan & Tan, 2005; Noche & Elhasia, 2013).

Normally, SCM plays an instrumental and operational role within the cement industry. The administration of the cement supply chain will empower industries and incorporate logistics into a consistent pipeline to maintain a nonstop stream of bonds from crude material sources to the final retailer (Atan, 2010). However, by virtue of the classical operation role and the complexity of SCM in the cement industry, very few studies have paid attention to the cement industry supply chain (Agudelo, 2009).

In general, the processes in supply chain management can include only one firm without partners with the objective of finding an optimal inventory system (single-echelon or multi-echelon), depending on the nature of the problem according to the inventory policies. The decision in this type of procedure is called a centralized decision (Min & Zhou, 2002; Sidola, Kumar, & Kumar, 2012).

However, there is more than one partner in the supply chain because there are many firms in each echelon, and each firm is supplied by one or more firms in the previous echelon, and likewise, each firm can supply to one or more firms in the succeeding echelon (Gurgur & Altiok, 2004). This type of supply chain is called a supply chain network (Humair, Ruark, Tomlin, & Willems, 2013; Sahraeian et al., 2010) and a decision in this sense is decentralized (Goh, Lim, & Meng, 2007). System processes in a decentralized framework mean that there is a decision maker in each echelon who is trying to maximize or optimize its own objectives because each echelon represents a firm. As mentioned earlier, the main aim of a supply chain is to maximize the profitability of firms that are partners in the supply chain (Best, 2009; Schwarz, Frederick, Gerald, & Hamdy, 1972).

That is why Chopra and Meindl (2007) and Beamon (1998) suggested supply chain to include all parties concerned, directly or indirectly, to achieve a customer request. The supply chain not only contains the manufacturer and suppliers, but also transportation, depot, retailers, and the customer themselves as exhibited in Figure 1.1. However, a typical supply chain includes a variety of echelons, which are suppliers of raw material, manufacturers, warehouses or depots, distributions centers, and retailers as the customers.

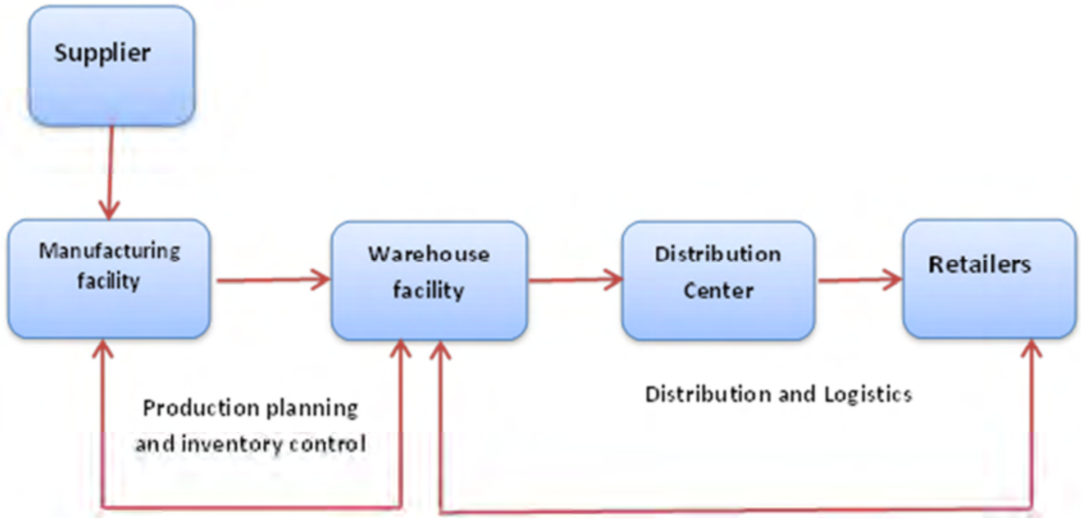


Figure 1.1. A Supply chain process life cycle

1.3 Supply chain Management in a cement industry

The supply chain of a manufacturing industry is aligned with the supply chain of a cement industry, which consists of suppliers, raw material depots, manufacturer warehouses, distribution centers, and a number of retailers to satisfy a very large number of customers (Kock, 2007; Rhee, Veen, Venugopal, & Reddy, 2010). The second most consumed material in the world after water is cement (Noche & Elhasia, 2013). It is an indispensable element in a vast majority of applications needed in our daily life. For example, civil infrastructure projects, houses, power generation stations, and many more cannot be built without it (Pyke & Cohen, 1993).

Generally, cement components are a mixture of limestone, sand, shell, clay and iron. A worldwide example is the normal Portland cement type, which is commonly used worldwide (He, Jewkes, & Buzacott, 2002).

Current demands for the improvement of construction and infrastructure, a growing consciousness of sustainable development, socially and environmentally motivated systems, resources limitation, as well as growth in some cement markets and a reduction in others, have forced cement producers to focus on supply and logistics chains (Bernstein & De Croix, 2006). Developing and executing the right strategies of SCM will lead to an improvement and increase in productivity, maximized competence, minimized costs, and reduced environmental impacts (Flatt, Roussel, & Cheeseman, 2012).

Cement, as the most important element or component of concrete, is an essential building material for society's infrastructure construction around the world. The consumption rate of cement measures the economic growth and represents a development index of several countries (Elhasia, Noche, & Zhao, 2013). According to the United Nations Environment Program (2011), 'basic construction materials serve an ever-increasing demand for the building sector, which leads to the annual growth rates of about 6% of cement and 3.8% of steel. At the same time, these industries cause about 6% of global anthropogenic greenhouse gas emissions.

The operations of the cement industry involve various stages. First, raw materials, such as limestone and clay are taken away from a quarry. Then, they are crushed in the mill and carried to the area of depot and homogenization. Next, they are ground into raw crush for softer crushing. Subsequently, the material goes during the pre-heater to the kiln where it is backed up to a temperature level of 1,500 Celsius and finally gets cooled to produce what is called, clinker. Finally, the clinker is ground with additives, like gypsum and pumice, and then ground together at a cement mill, which gives rise to what is called cement (Agudelo, 2009; Elhasia et al., 2013). Figure 1.2 illustrates the general mechanism of cement industry production.

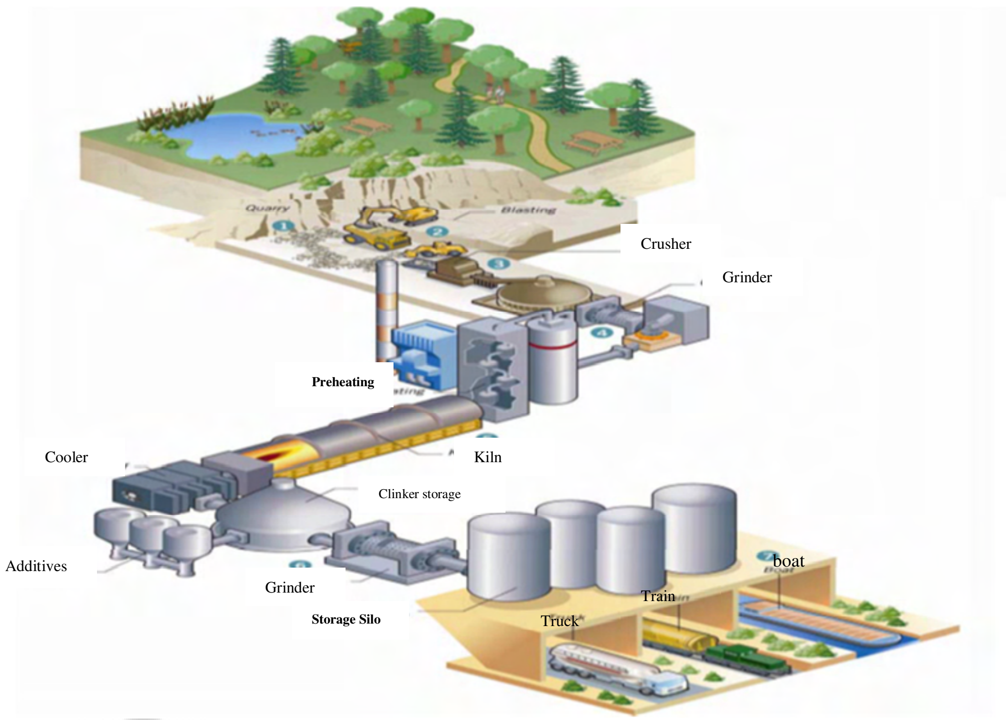


Figure 1.2. The cement production scenario (source Lafarge, (2010))

The supplier of raw materials to a cement company is divided into two types (Agudelo, 2009; Noche & Elhasia, 2013). Firstly, most of these raw materials are in the same factory site, which are stones taken from the mountain, and trucked to a raw materials depot. Secondly, external suppliers supply other materials that are used in the cement manufacturing. The main elements in the cement industry are stones. Figure 1.3 illustrates the supply chain process for a cement industry, which consists of suppliers, a raw material depot, manufacturer, three installation warehouses, distribution centers with n lines, and an unlimited number of retailers that satisfy a big number of customers (Chairman, 2012).

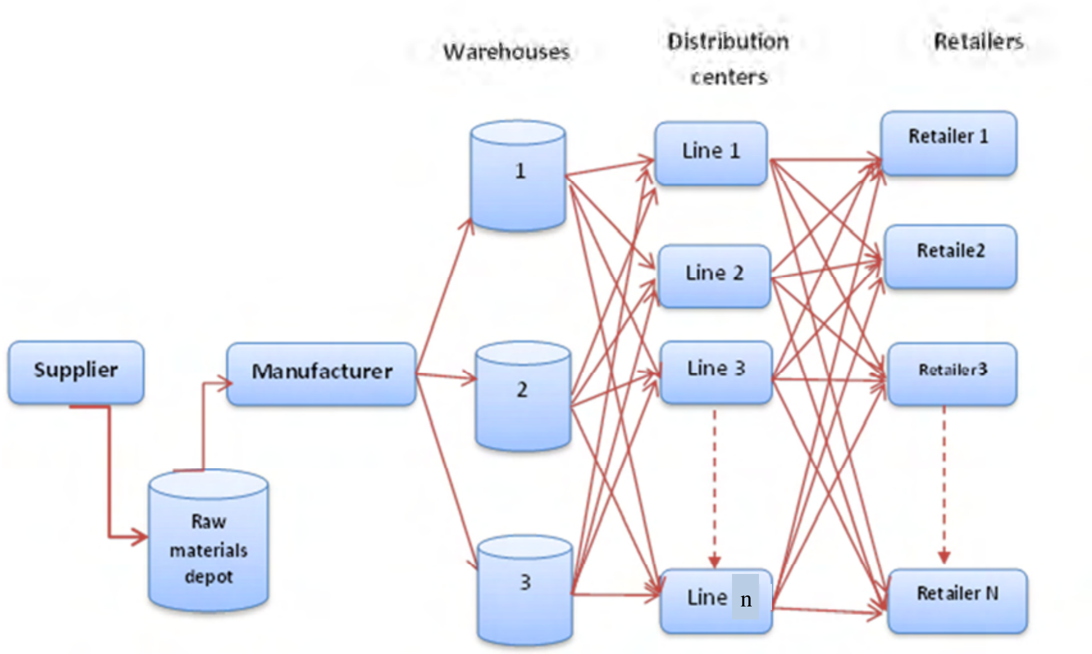


Figure 1.3. A supply chain process in a cement industry

1.4 The role of inventory in a supply chain

An inventory system plays an important role in the supply chain. That is why most studies in the field of supply chain used theories of inventory (Graves and Lesnaia, 2004; Pal et al., 2012; Wang et al., 2010; Zaojie and Guoying, 2007). Inventories try to balance between assets and reduce the total cost in order to meet consumer demands in a timely manner (Vanany, Zailani & Pujawan, 2009). Inventory policy changes by size or type of the case: single-echelon, multi-echelon, or both of them. Inventory parameters, demand, and lead-time are classified into two types: (a) deterministic, i.e., static or dynamic; and (b) probabilistic, i.e., stationary or non-stationary. All of these cases have a role in inventory policy model building. As a result, with regard to the supply chain, multi-echelon inventory control in this kind of chain is prospering rapidly. Multi-echelon inventory system policy management is an assertive section of supply chain operations (Elhasia et al., 2013). Therefore, inventory control is classified as the hidden side of a supply chain. The essential elements that have a role to develop or modify an inventory system are multi-echelon, inventory parameters demand, lead-time then demand during lead-time and cost.

1.4.1 Inventory in multiple stages

Over the last two decades, there were a large number of studies (Chung, Wee, & Yang, 2008; Gümüs & Güneri, 2007; Jie & Cong, 2009) on a multi-stage inventory system, which generally focused on SCM. A multi-stage inventory system is also known as multi-echelon. A multi-echelon inventory system looks at the inventory levels entirely across the supply chain while taking into account the effect of inventories at any given stage or echelon on other echelons (Axsater, 2006). The reason for the enchantment in this field is not only due to the complexity of the interaction through the echelons but also due to its massively practical applications in reality. Globalization, economic and trade openness in finished goods, semi-finished goods, and other types of products, overlapping activities, a multitude of competitors, and complexity of production stages require that decision maker make concerted efforts to satisfy customers' needs. Therefore, the absence of a strong inventory system, skills, and expertise in this area is likely to incur losses to the companies and manufacturers (Atan, 2010; Ganeshan, 1999; Roy, 2005). As the cement production process is subject to various supply chain stages, an effective multi-echelon inventory system to meet the market demand for cement is necessary (Elhasia et al., 2013).

1.4.2 Demand and lead-time parameters

An inventory system is a hidden side of the supply chain. The variables that have a key role in an inventory system are demand process, lead-time process, and costs (Axsäter & Viswanathan, 2012; Diks, Kok, & Lagodimos, 1996; Li, Xu, & Ye, 2007). Demand and lead-time are the keys to developing or modifying multi-echelon inventory system models (Hayya, Harrison, & Chatfield, 2009). Demand is the quantity of a particular economic item, product or service that meets a consumer or group of consumers' needs in a unit measuring time. Demand is classified into two types, which are (a) deterministic, i.e., constant or dynamic, and (b) probabilistic, i.e., stationary or non-stationary. Lead-time is the time between the order requests until received or placed in the warehouse or the customer (Axsater, 2006; Lee, 2005). Lead-time is classified as deterministic, i.e., constant or fixed, and probabilistic.

Inventory control is a wide and varied area. Generally, the essential aim of inventory control is to balance between overstock and understock (Frederick & Gerald, 2001; Min & Zhou, 2002), which depends on inventory system policies of whether a periodic review policy or continuous review policy be applied, in addition to, the variables. An inventory control problem appears when there is a need for physical storage of goods, items, and products for the purposes of meeting demand over time (deterministic and long). Meanwhile, the needs of any project in the business area are to keep inventory to ensure continued efficient operations. Usually, a project management needs to make a decision regarding the timing of the order at the order quantities of the stock. Therefore, the main objective of an inventory system is to achieve an adequate level and fewer expenses on inventory to meet future needs regarding stocking inventory (Axsater, 2006; Dolgui, Ben Ammar, Hnaien, & Louly, 2013; Funaki, 2012; Humair et al., 2013). Inventory control involves answering two conventional questions: How much should be ordered? and when should be ordered? Answering these two questions depend on the inventory policies that a company adopts. Deciding inventory policies can be very complex and risky. The purpose of these questions is to satisfy customer demands while optimizing profitability.

Typically, demand and lead-time in an inventory system can both be constant, which is the simplest, or, the demand is probabilistic, and the lead-time is constant (deterministic), or, the demand is constant or deterministic, and the lead-time is probabilistic, or alternatively, both of them are probabilistic (Frederick & Gerald, 2001; Hamdy, 2007). Therefore, the complexity comes from the behavior of the demand and lead-time. In particular, the demand and the lead-time behaviors are more complex when they are probabilistic, i.e., each of the demand and the lead-time has a probabilistic distribution function. On the basis of these two variables, the multi-echelon inventory systems policies are drawn.

1.4.3 Demand during lead-time parameter

Demand during lead-time is the mixed distribution of demand distribution and lead-time distribution for each one. However, most studies on a multi-echelon inventory system (Axsäter & Marklund, 2008; Axsäter, 1984; Clark & Scarf, 1960; Graves, 1986; Hausman & Erkip, 1994; Hosoda & Disney, 2006; Muckstadt, 1986; Ravichandran, 1995; Saffari & Haji, 2009; Sherbrooke, 1968; Zhao, Zhan, Huo, & Wu, 2006) considered demand distribution to be a Poisson, compound Poisson or normal, and mostly a constant or fixed lead-time. Generally, these assumptions are valid in supply chains that carry expensive items and face low demand, but not necessarily valid for a highly uncertain demand (Caglar, Li & Simchi-Levi, 2004; Graves, 1985; Muckstadt, 1973; Sherbrooke, 1968). This procedural treatment is about the spare part items or slow moving items (slow moving items mean the demand for the items are periodical, for example, daily, weekly or monthly) where the lead-time is equal to zero or completely ignored.

However, the treatment is different for slow and fast moving items in multi-echelon inventory systems. In spare part items or slow-moving items, demand is discrete, and subject to a discrete probabilistic distribution (Gümüs & Güneri, 2007; Schwarz, Frederick, Gerald, & Hamdy, 1972; Yang, Ding, Wang, & Dong, 2008). A fast moving item means that the demand for this item is high, very high or continuous in nature. Fast moving items are the most common inventory and the treatments of this type of items are more difficult that of the slow moving or spare part items. This is because fast moving items are subject to continuous probabilistic distribution functions (Bagchi & Hayya, 1984; Baykal-Gurosy & Erkip, 2010; Gümüs & Güneri, 2007; Mitra & Chatterjee, 2004a). Most of the studies that deal with inventory problems, whether deterministic or probabilistic models; lead-time is to be a deterministic constant or stochastic variable. Lead-time includes elements, such as order setup, transit order, supplier lead-time, delivery time and setup time (Lee, 2005).

Meanwhile, studies in fast moving items or high demand items (Chung, Wee, & Yang, 2008; Elimam & Dodin, 2013; Hosoda & Disney, 2006; Hsieh & Chou, 2010; Huang & Xue, 2012; Ignaciuk & Bartoszewicz, 2009; Mitra & Chatterjee, 2004b; Pal, Sankar, & Chaudhuri, 2012a, 2012b; Sahraeian, Bashiri, & Ramezani, 2010; Seliaman & Rahman, 2008; Seo, Jung, & Hahm, 2002; Xu, Zhang, & Liu, 2009; Yao, Yue, Mukhopadhyay, & Wang, 2009) hypothesized or proposed that demand is stochastic or probabilistic, and the lead-time is constant or they ignored the lead-time. The reason behind that is due to the stability of the market (i.e., stability of the demand and the lead-time during the long periods). When the market is not exposed to the sudden changes, the behaviors of the demand and the expected period of the lead-time may be affected (Demeter & Golini, 2014; Venkateswaran & Son, 2007). In this case, an inventory policy, whether it is a periodic review or a continuous review, is not as complicated as both of demand and lead-time are probabilistic. Based on the previous discussion, it can be concluded that the cement market has a high and fast moving demand.

1.4.4 Cost parameter

The parameter that plays a fundamental role in most studies in multi-echelon inventory system is costs, in all types and forms (Cheng, 1989; Federgruen & Zipkin, 1984; Funaki, 2012; Mehmood Khan, Jaber, & Bonney, 2011; Moslemi & Zandieh, 2011; Sheng & Wang, 2014). The studies aimed at reducing or minimizing the total cost or holding inventory cost to a minimum, for a very simple reason that these studies wished to help companies maximize their profit. Generally, costs in inventory systems are divided into four types: setup cost, holding cost, shortage cost, and purchase cost (Axsater, 2006; Bolarín, Lisec, & Esteban, 2008; Frederick & Gerald, 2001; Hamdy, 2007).

It is necessary to find the inventory levels that can minimize the costs but yet ones that can achieve the highest level of efficiency, performance, and operation. Toward these purposes, manufacturers need to determine the economic inventory levels and the number of optimal purchases accurately.

1.5 Methods for estimation of multi-echelon inventory system

There are different methods utilized in a supply chain of a multi-echelon inventory system to reach a solution, such as the exact method (Axsäter & Marklund, 2008; Cheung & Hausman, 2000; Forsberg, 1997; Seo et al., 2002), approximate method (Axsater, 1993; Axsater, 2006; Gurgur & Altiok, 2004), simulation method (Elhasia et al., 2013; Kian Ng, Piplani, & Viswanathan, 2003; Santos & Santos, 2007; Towill, Naim, & Wikner, 1992) and forecasting method (Hosoda & Disney, 2006; Snyder, Koehler, Hyndman, & Ord, 2004; Wang, 2009). These methods are considered advanced technical methods to enhance decision making by analyzing the complex situations and building a system through these methods.

A simulation method is the abstraction of the reality through the input-output relationship based on simple or complex mathematical expressions (Santos & Santos, 2007). A simulation method in a multi-echelon inventory system tries to construct the approximate reality as much as possible and provides analytical tools to study the behavior of a complex system. Simulation applied in a multi-echelon inventory system has been used as a research technique and gained approximated results

(Axsäter, 2000; Barton, 1992; Jie & Cong, 2009; Kian Ng, Piplani, & Viswanathan, 2003; Liberopoulos & Koukoumialos, 2005; Martel, 2003; Song, Li, & Garcia-Diiaz, 2008; Tee & Rossetti, 2002; Towill, Naim, & Wikner, 1992). These studies provided generalized models with *N*-echelon and examined a small example as two- or three-echelon. They assumed demand and lead times to be deterministic or constant and uncertain. However, the complexity of the system emerged when each of the demand and the lead-time was probabilistic and subject to the probabilistic distribution function.

On the other hand, a forecasting method is widely used in a multi-echelon inventory system for different purposes and in a manufacturing problem as well (Flatt et al., 2012; Huang & Xue, 2012; Snyder, Koehler, Hyndman, & Ord, 2004). Forecasting methods are often used to estimate the mean and the standard deviation of the demand (Baykal-Gurosy & Erkip, 2010; Hosoda & Disney, 2006; Snyder, Koehler, Hyndman, & Ord, 2004; Wang, Bunjira, & Lin, 2010). Therefore, the probability and uncertainty of the demand data lead to the use of a suitable forecasting method for estimation.

The most common method used in an inventory system of multi-echelon to access the exact solution are mathematical methods, such as linear programming (Chung et al., 2008; Pattnaik, 2014), integer programing (Abu Alhaj & Diabat, 2009; Elimam & Dodin, 2013; Hsieh & Chou, 2010), dynamic programing (Humair & Willems, 2011; Minner, 1997), fuzzy goal programing (Torabi & Hassini, 2009) and quadratic programing (Ignaciuk & Bartoszewicz, 2009; Manna, Chaudhuri, & Chiang, 2007). Studies that used any one of these methods considered two, three or *N*-echelon systems with deterministic or probabilistic demand. They assumed lead-time to be deterministic, constant, or zero, or they simply ignored the lead-time.

Usually, these methods lead to exact results and optimal solution. But when the behavior of the problem is probabilistic and highly uncertain, the optimality is still unknown for most types of multi-echelon inventory systems (Atan, 2010; Chan, Routroy, & Kodali, 2005; Johansen, 2005; Mitra & Chatterjee, 2004a). Hence, there has been an increase interest in developing simple procedures to obtain results that approximate the true optimal as closely as possible (Axsater, 1993, 2003; Dong, & Lee, 2003; Gurgur & Altiok, 2004; Johansen, 2005). Subsequently, these procedures are classified as the approximation mathematical methods which are summarized as follow:

Most inventory models whether single-echelon or multi-echelon adopts Probabilistic Service Approach (PSA), Chen and Zhang (2009); Chen and Lin (2009); Klosterhalfen, Dittmar and Minner (2013); Novoa and Storer (2009); Tarim and Kingsman (2004); Willemain, Smart and Schwarz (2004) and You and Grossmann (2010) as an approximation method. The PSA facilities each stock to preserve an adequate inventory level in order to meet its probabilistic demand. When the inventory level of a stock is not adequate to meet the demand coming from its downstream stocks or end customers, unsatisfied demand is fully backlogged and will be filled later when safety stock inventory becomes available. This implies that the stock may have a probabilistic delay to fill an unsatisfied demand. The lead-time for filling its demand is thus probabilistic.

Another approximation mathematical modelling method in the manufacturing industry to manage multi-echelon inventory in supply chain is the combined approaches of simulation and forecasting (Axsater, 2003; Axsäter, 2011; Bollapragada et al., 1998; Cachon & Fisher, 2000; Giannoccaro, Pontrandolfo, & Scozzi, 2003; Graves, 1996; Liberopoulos & Koukoumialos, 2005; Moinzadeh & Aggarwal, 1997; Sleptchenko, van der Heijden, & van Harten, 2002; Verrijdt & De Kok, 1996; Yoo, Kim, & Rhee, 1997). Most of these studies gained an approximate solution by combining mathematical modeling and simulation. It is considered that the approximation mathematical method is suitable when there is the existence of the probability and uncertainty environment contributing the solution of the supply chain of the multi-echelon inventory system. This method deals with the development of appropriate algorithms that is able to search for the best inventory policies of the systems. Hence, exploring the potential algorithms through this approximation method is deemed necessary.

1.6 Problem statement

The optimality of inventory and allocation policies in supply chain is still unknown for most types of multi-echelon inventory systems. The inventory control problem appears when there is a need for physical storage of goods, items, and products for the purposes of meeting the demand overtime (deterministic and long) (Elimam & Dodin, 2013; Inderfurth & Vogelgesang, 2013; Min & Zhou, 2002). An inventory system tries to balance between overstock and understock to reduce the total cost and achieve consumer demands in a timely manner. However, it is important and useful to know how much should be ordered and when should be ordered.

The stability and certainty of the processed materials and distribution to various destinations in global markets have imposed the behavior of lead-time to be constant and remained unchanged. This is the main reason that previous studies such as Demeter and Golini (2014), Hesse and Rodrigue (2004) and Venkateswaran and Son (2007) assumed the lead-time to be constant, fixed or ignored. The problem arises when the demand and the lead-time are probabilistic which involved highly uncertained lead-time (Deng et al., 2010; Humair et al., 2013; Xu et al., 2009). Therefore, this research provides a potential solution through development of an approximation mathematical model for a probabilistic situation.

The probability of demand and lead-time has imposed manufacturers to establish the demand during lead-time, which is a very critical element in an inventory system such as in a cement industry. When items, goods, or products are near completion, a decision maker starts to make a request for an order quantity to meet the needs of consumers and avoid falling into shortage as suggested by Bookbinder and Cakanyildrim (1999) and Funaki (2012) during this period, and until the required quantity arrives at the depot, customer demand is continuous. Since the processes are very nested, it is difficult to record the demand data until the items or the products reach the place. Therefore, this research explore on potential procedures such as the simulation to overcome the situation of demand during lead-time.

A demand process at any plant or institution depends on the operations and decisions of downstream locations, while the lead time process depends on the operations and decisions of upstream locations. This is the situation where it involves a continuous review policy. Studies such as Hsieh and Chou (2010), Ignaciuk and Bartoszewicz

(2009), Moussourakis and Haksever (2013) and Pal et al. (2012) adopted a stochastic or constant demand, and ignored the lead time or assumed a constant lead time. However, the stochastic environment in a multi-echelon inventory system (i.e., probability of demand, probability of lead-time) imposes an approximation model to estimate the parameters and solve the problem under a continuous review policy. Therefore, to coordinate the flows of the supply chain elements in the cement industry whose the demand and lead-time are probabilistic through inventory policy, this research attempts to developed a model in a multi-echelon inventory system under a continuous review with probabilistic demand and probabilistic lead time. This model could establish the inventory performance measures. A similar study by Axsäter (2011) has also established an inventory performance measures. However, it was for a single echelon and not for a continuous review policy. Furthermore, the demand during lead-time was not considered in that study. Hence, our research could improve the study by considering a multi-echelon inventory system under the continuous review *(R, Q)* policy, where *R* is the reorder point and *Q* is the order quantity.

In addition, Axsater (2010) developed a simple production inventory system with single-echelon and one service provider channel M/G/1 model. Subsequently, our research could be extended to introduce the multi-echelon and multi-channels service providers under the first come first serve (M/G/C-FCFS) model. By this extension the proposed model could reduce the long waiting time in the system.

1.7 Research Questions

In order to address the aforementioned issues, this research tries to answer the following questions:

1. How to establish the structure of the probability distribution function of demand during lead time?

2. How to develop a formulation for order quantity, Q in a serial multi-echelon inventory system under a continuous review system with the probability distribution of demand during lead time?

3. What is the optimal safety stock, SS that should be on hand for the warehouse, including each echelon of the three echelons in distribution with a multi-echelon inventory system under a continuous review system?

4. What is the structure to find the reorder point, R in a distribution multi-echelon inventory system under a continuous review system?

5. How to establish the formulation or model for approximating the expected total cost for the whole system?

6. How to integrate the FCFS queueing rule into the continuous review inventory system to reduce the long waiting time between the retailers and the distribution center?

1.8 Research Objectives

The primary objective of this research is to develop an approximation mathematical model in a supply chain of the multi-echelon inventory system under the continuous review policy in the cement industry that can achieve the best inventory policy to satisfy the retailer's needs, while considering the probability distribution function of demand during lead time. Specifically, this research aims at meeting the following objectives:

1. To develop the probability distribution function of demand during lead time by using a simulation procedures.

2. To develop an appropriate formulation for order quantity, Q in a serial multi-echelon inventory system under a continuous review *(R, Q)* policy with the probability distribution of the demand during the lead time.

3. To identify the optimal safety stock that should be on hand for the warehouse, including each of the three silos under a continuous review *(R, Q)* policy.

4. To determine the optimal reorder point, R in the distribution multi-echelon inventory system under a continuous review *(R, Q)* policy, this also leads to extracting the inventory position and levels at each echelon.

5. To develop the approximate total cost function for the whole system.

6. To develop the FCFS queue model in the continuous review inventory system in order to reduce the long waiting time between the distribution center and the retailers.

1.9 Scope of the research

This research was carried out based on the operation activities of a cement industry in the Iraq-Kurdistan regional government, which has a contract with the French company known as Lafarge. The secondary data were collected for the periods of 2011 to 2013 because during this time the region has just opened to the world and international companies. The involved parties in this research are the decision makers and strategic planners from the government and industrial sectors.

In order to develop an approximate mathematical model of a multi-echelon inventory system in a supply chain, demand and lead-time processes need to be identified. The demand process at any plant or institution depends on the operations and decisions of downstream locations, and lead time depends on the operations and decisions of upstream locations. The performance of individual plants depends on both demand and lead-time processes. Therefore, this requires an assumption of the probability theory.

1.10 Research contributions

In this research, we described and studied a combination of serial and distribution multi-echelon inventory supply chain and its operational rules. Our aim is to develop an approximation mathematical model in order to analyze the impacts of the probability of demand and lead time behaviors in the supply chain using statistical methods, forecasting, simulation, and algorithms developments. The contributions of this research can be divided into two parts, theoretical and practical.

1.10.1 Theoretical contribution

The theoretical contribution of this research relates to the development of an approximation mathematical model in a multi-echelon inventory system under the continuous review *(R, Q)* policy with probabilistic demand and probabilistic lead-time. The development contributes to three models: (a) simulation procedures to establish demand during lead-time probability distribution function, SMDDL model; (b) a serial multi-echelon inventory system under continuous review *(R, Q)* model, SMEI (*R, Q*) model; and (c) a distribution multi-echelon inventory system under the FCFS queue model which is known as DMEI-FCFS model. All the three models with the overall approximation model reviewed new knowledge, and thus enrich the body of knowledge for the field of multi-echelon inventory system.

1.10.2 Practical contribution

The multi-echelon inventory system under the continuous review *(R, Q)* policy coordinate the processes between the elements of the supply chain for directors, managers, officers, and retailers. However, the knowledge of demand and lead time, which is important in producing the finished goods, semi-finished goods and other types of products offers the decision makers to draw the best inventory system policy or to meet and satisfy the needs of markets, customers, and retailers.

The establishment of the three sub models in the whole approximation mathematical model can be expected to have an impact on the reorder point, lot size, and safety stock. In addition reduce the long waiting time in the whole system in order to satisfy the greatest extent of possible service seekers. By reducing the long waiting time, companies can save effort, time, and money.

1.11 Organization of the thesis

Overall, this thesis consists of six chapters, which are organized as follows Chapter One introduces background of a multi-echelon inventory system in the SCM. Emphases on the challenges faced and motivation are discussed. The issues raised and research gap or the problem statement is well defined. This chapter also delineates the objectives that need to be achieved in the research.

Chapter Two presents three main aspects. Firstly, the benefits and the issues in supply chain management. Secondly, the types and policies of inventory systems in multi-echelon inventory systems and thirdly, the fundamental and concept approaches in the multi-echelon inventory system. These aspects are important for highlighting the scientific gap between the previous studies and this research.

Chapter Three is devoted to discussing the theoretical and conceptual methods used by previous studies. This chapter discusses three key issues. Firstly, the fundamentals of forecasting method in the multi-echelon inventory system; secondly, the fundamentals of simulation procedures, and thirdly, the fundamentals and concepts of multi-echelon inventory system policy. The aim of this chapter is to show the related theories of the multi-echelon inventory systems which are considered in this research.

In Chapter Four, we present the methodology of how to build and develop an approximation mathematical model based on a real word problem. This chapter is divided into two parts. The first part explains the simulation procedures to extract the probability distribution function of demand during lead time. An exponential smoothing method is also discussed. This first part or step is very important because it is the essential step to proceed to the second step, which in turn depends on the results of the first step. Moreover, step one represents a novel sub-step in this work. Secondly, the approximation mathematical model for a multi-echelon inventory system with the continuous review *(R,Q)* policy is developed.

Chapter Five presents the key results of the new alternative method which contributes to the multi-echelon inventory system in supply chain. The performances of the proposed models are evaluated. The best solution of a multi-echelon inventory system in the cement industry based on the approximation mathematical model is discussed. Chapter Six concludes the research, while emphasizing how all objectives of the research are achieved. In addition, the chapter discusses the limitations of the research and the directions of future work

CHAPTER TWO
LITERATURE REVIEW

There is a large body of literature on the importance of supply chain management (SCM). Previously, researchers focused on the different processes of manufacturing supply chain. Presently, more attention is given to the performance, design, and analysis of supply chain as a whole because of such factors increase costs of manufacturing, decrease resources of manufacturing bases, shorten product life cycle, and market globalization. This research focuses on the role of inventory control which has practical significance.

2.1 Supply Chain Management

The terms supplier and customer has been used ever since commerce started. However, the supply chain concept began in the late 1950s after Jay Forrester and his colleagues from Massachusetts Institute of Technology began to study the relationship between suppliers and customers (Bolarín et al., 2008), and the concept of bullwhip effect came into existence to explain the changes in inventory as a result of the changes in customer demands. From a functional approach to managing units, the process-oriented term of SCM was also discussed by Oliver (1982). Then, Porter (1985) developed the value chain processes, which have, until now, been the interface implementation (Blanchard, 2010).

Even though the SCM concept emerged in the early 1980s, its operation was somewhat unstructured. This is reflected in long lead-times, inclusion with functional silos, and the lack of coordination, ending with extravagant inventory and higher costs of production. Currently, the system of global planning is integrated with the supply chain members authorizing them to think and act as a team (Childerhouse & Towill, 2000).

SCM is defined by various practitioners and researchers based on the background where they come from. Generally, the following definitions can be used to be a working definition of SC and SCM.

2.2 Objectives and benefits of SCM

The aim of SCM is to satisfy the end-customer requirement (Childerhouse & Towill, 2000). Satisfied customers could be defined if they are basically a part and parcel of the system that delivers the item or product and services, giving direct input concerning their expectation (Fawcett, Ellram, & Ogden, 2007). As the core of SCM, there is a withdrawal system that starts with the customer, where the role and participation level would lead in the end to customer satisfaction.

Concentration on customer satisfaction builds customer allegiance (Greenstone & Syverson, 2012). If companies are customer concentrated, they will grasp their key competitors well and the corresponding competitive forces (Min & Zhou, 2002), such as the level of pricing, quality of product, product availability, quality of service, and customer satisfaction (Yvan, 2011). A higher level of customer satisfaction leads to a higher level of customer allegiance, a high level of revenue and market share. In the end, it drives toward a high level of profitability (Best, 2009)

Presently, customers are demanding sufficiently by virtue of the level of the consciousness created (Chan et al., 2005; Chopra & Meindl, 2007; Vanany et al., 2009). They anticipate lower prices, better quality, shorter lead times on deliveries, and high reliability (Verwaal & Hesselmans, 2004). Moreover, they are continuously searching for lower prices and comparing things with the level of technology the world availed for them. The case formed by globalization may give an opportunity for multinationals to supply products at their neighborhood gates by supplying from countries that are specified to be having a low cost of production (Duffield, 2012). Quality can be seen to be congruent to the standardization. However, this affirmation shall be clear enough to be easily grasped and found to the level of customer anticipations. Quality and prices are not the only factors that identify the level of customer gratification. Creativity, timely delivery, and service availability are the major requirements of the customers (Fawcett et al., 2007).

2.3 Supply Chain Management Issues

SCM includes the design of smooth value added operations through boundaries of an organization to enable the organization to cope with the real need of a customer (Fawcett et al., 2007). The design and execution impose many complex problems and challenges in implementing SCM. These main problems must be first well located so that the organization can come up with problem-solving mechanisms proactively.

Fawcett et al. (2007) listed the SC design and management problems to include the following:

- Weak coordination of effort
- Inconsistent information systems
- Long cycle times
- Problem of communication
- Issues of customer service
- Environmental declination and excessive waste
- Inventory of high relativity for the level of customer service achieved
- Below or less than optimal profits

Public problems are those problems that cross multiple specific problems whereas private problems are those that happen in the vertical direction of problem disintegration and deal with one particular issue (Chandra & Grabis, 2007; Greenstone & Syverson, 2012; Vanany et al., 2009). Accordingly, Chandra and Grabis (2007) classified SCM issues as follows:

- Configuration of the distribution network
- Inventory control
- Strategies for distribution
- SC integration and strategic partnering
- Procurement and outsourcing strategies
- Decision support systems and information technology
- Customer value

Some researchers (Axsäter & Zhang, 1999; Jie & Cong, 2009; Song et al., 2008; Yang & Geunes, 2007) are more interested in the SCM issues mentioned as follows:

- Inventory control
- Multi-Echelon inventory system in SCM
- Customer demand behaviors
- Lead-time behaviors

2.4 Supply chain in a multi-echelon inventory system

Previously, researchers and practitioners (Chan et al., 2005; Graves & Lesnaia, 2004; Kim, Jun, Baek, Smith, & Kim, 2005) investigated the different processes within manufacturing supply chain individually. However, there has been a growing interest in the performance, design, and analysis of supply chain as a whole (Beamon, 1998; Karaman, 2007; Tan & Xu, 2008). This interest is generally the result of the rising cost, globalization of market, a long list of products that are virtually endless, and the need for coordination (Beamon, 1998). Supply chain is an important element of business operations. Understanding its probability behaviors is a significant part of risk analysis and performance assessment in supply chain design and management (Chen, Federgruen, & Zheng, 2001; Lee & Whang, 1999; Tan & Xu, 2008). Therefore, supply chain can be defined as a complete or full manufacturing process wherein raw materials are mutated into final products, and then delivered to customers. At the highest level, a supply chain consists of two basic integrated processes: (a) the planning of the production and the inventory control, and (b) the logistic process and the distribution process (refer to Figure 1.1 in Chapter One).

Beamon (1998) analyzed 38 previous works published from 1980 to 1998 in supply chain. He classified each article according to the kinds of supply chain models used. The classification shows that the measures used were based on cost, customer responsiveness, and a combination of them (Cohen & Moon, 1990; Hammel & Kopczak, 1993; Lee & Billington, 1993). He analyzed the model types that were used (deterministic analysis, stochastic analysis, economic or simulation), decision variables (production/distribution scheduling, inventory levels/ordering batch size, number of stages, plant-product assignment, buyer-supplier relationship and number of product type help in inventory), and performance measures (cost, activity time, flexibility and customer responsiveness/backorder) (Chen, 1999; Cohen & Lee, 1989; Towill et al., 1992).

A review of inventory modeling in supply chain management literature shows three new directions in SCM: materials procurement globalization, manufacture globalization, and product globalization (Roy, 2005; Vanany et al., 2009). Consistently, three areas in inventory modeling appear: (a) multi-supplier and multi-product inventory models from upstream of the supply chain, (b) multi-echelon inventory control including manufacturers, and (c) stochastic multi-product demand inventory models from downstream of the supply chain (Zhao, Zhan, Huo, & Wu, 2006).

A typical supply chain includes a variety of echelons. An echelon means the main warehouse to manage branches or sub-stores to distribute items (Tan & Xu, 2008). In supply chain, there is more than one echelon, for example, between suppliers and manufacturers there is an echelon, and between the main warehouse and retailers there is also an echelon which includes suppliers of raw materials, manufacturer, warehouses/depots, distribution centers, and retailers (Axsater, 2001).

A supply chain is a general framework for coordinating workflow from the suppliers to the final stage, which is the customer (Hwan Lee & Rhee, 2010). Alternatively stated, the material flows downstream from suppliers to customers (Osman & Demirli, 2012). A multi-echelon system plays a significant role in supply chain because the flow of material passes through stages (echelons) before it becomes a product, stored and distributed to its final customers (Lee & Whang, 1999; Muckstadt, 1986; Rhee et al., 2010; Roy, 2005).

2.5 Inventory control systems

Multiple inventory control usually represents 45% to 90% of all expenses of the business, and there is a need to ensure that the company has the right items or products on hand in order to avert stockout to prohibit shrinkage (Cook, 2006). Inventory control involves the procurement, care, and disposition of materials (Sharma et al., 2011). There are three kinds of inventory managers have to pay attention to raw materials, semi-finished goods, and finished goods (Agudelo, 2009; Karaman, 2007). Hence, the purposes for inventory control system are to:

- avoid overstock and understock
- reduce increasing costs
- help secure the best average of inventory turnover for each item, product, or goods
- help decision makers regarding when and how much to order

2.5.1 Single-Echelon inventory system

An inventory control system can be divided into two parts: single-echelon, and multi-echelon. A single-echelon is simpler than multi-echelon because of its properties and composition (Hausman & Erkip, 1994; Shang, 2012). However, there is an interest among researchers in a multi-echelon inventory system because it reflects the real nature of the problem that needs a fact-finding of the parameters (Axsäter, 2011; Axsater, 2006a; Baten & Kamil, 2009; Deng et al., 2010; Johansen, 2005; Wu et al., 2007).

2.5.2 Multi-Echelon inventory system

Multi-echelon inventory system policy management is an assertive section of supply chain operations (Kalchschmidt, Zotteri, & Verganti, 2003; Kian et al., 2003; Rhee et al., 2010; Song et al., 2008). The probabilities for effective control of a multi-echelon inventory system increase drastically during the last two decades. One of the main reasons is the advance in research, which has resulted in new methods and approaches that are more general and effective (Axsater, 2006).

The terms multi-echelon or multilevel production distribution are also equivalent to such networks or supply chain when an item moves during more than one step before reaching the final customer (Gümüs & Güneri, 2007). There is a considerable interest by researchers in this field of study as a result of the expansion and openness of international markets, leading to intense competition. Hence, there is tremendous amount of literature on multi-echelon inventory control (Abu Alhaj & Diabat, 2009; Clark & Scarf, 1960; Graves, 1986; Hosoda & Disney, 2006; Ignaciuk &

Bartoszewicz, 2009; Jie & Cong, 2009; Liang & Huang, 2006; Mitra & Chatterjee, 2004a; Pal, Sankar, & Chaudhuri, 2012; Reyes, 2005; Sahraeian, Bashiri, & Ramezani, 2010; Song, Li & Garcia-Diiaz, 2008; Zaojie & Guoying, 2007).

In the early 1950s, many articles appeared discussing optimal policies of inventory problems (Chen, 2000; Clark & Scarf, 1960; Reyes, 2005; Tan, 1974; van Houtum, 2006). These articles were dedicated to determining the optimal purchasing quantities at a single-echelon encountering with some pattern demand. Traditionally, to make the supposition that when the installation in question requests a shipment of inventory, a shipment will be delivered in a constant or probable stochastic time length, but at any average with a time lag, which is independent of the order placed size (Axsäter & Juntti, 1996; Axsäter & Rosling, 1993).

A major example that arises is when there are a number of installations (1, 2, …, N). Installation one receives stock from installation two, and installation two receives inventory from installation three and so on. If an order is located by installation one for inventory from installation two, the period of time for delivery of this inventory is determined not only by normal lead-time between these two installations, but also by the availability of inventory at installation number two (Clark & Scarf, 1960; Hosoda & Disney, 2006; Seo et al., 2002). The model remarked about parameters that were specified (lead-time, demand distribution, purchasing cost, holding cost, shortage cost, etc.). Theoretically, the purchase quantities were optimally determined. A clear way forward would be a cost function for each arrangement of inventory at different installations, and the transit for the installations (from one to another). The type of functional equation will then be satisfied by cost function, which mostly appear in inventory theory, and from which the optimally provided policies can be determined by a recursive computation (Clark & Scarf, 1960; Diks. et al., 1996; Tan, 1974).

Clark and Scarf (1960) were the first researchers who wrote a multi-echelon inventory problem consisting of a two-echelon inventory system with a periodic review and computed the optimality of ordering policy for each echelon separately. They showed the optimality of a base-stock policy for a pure serial inventory system and developed an effective decomposing method to compute the optimal base-stock ordering policy (Federgruen & Zipkin, 1984a; Schmidt & Nahmias, 1985).

Undoubtedly, many modifications and updates have been done on the approach to the present day, but these modifications or updates depend on the formulation of the problem under the study (Bessler & Veinott, 1966; Dong & Lee, 2003; Sinha, Sobel, & Babich, 2010; Tang & Grubbström, 2003). Alternatively stated, prior studies in this area that adopted a multi-echelon inventory system in a supply chain chose the variables and formulations according to the requirements of the study. Therefore, a large proportion of these studies focused on the type of costs in multi-echelon inventory systems, such as holding cost, shortage cost, setup cost, and backorder costs (Axsater, 2003; Bolarín et al., 2008; Cohen & Moon, 1990). The objective was to reduce or minimize the total cost in the supply chain (Axsäter & Rosling, 1999; Axsäter, 2010, 2011; Axsäter & Marklund, 2008; Diks, 1996; Muckstadt, 1986; Tan, 1974). Others adopted in their studies the effects of information sharing in the supply chain with multi-echelon inventory systems (Axsater, 2006b).

Another set of studies in the multi-echelon inventory system adopted risks in their studies (Goh, Lim, & Meng, 2007; Qi-feng, Xiao-shen, & Wei, 2012; Vanany, Zailani & Pujawan, 2009). In addition to the aforementioned, simulation and forecasting have a significant share in this area. Supply chain management usually is complex, as we mentioned earlier. Therefore, it needs an effective or powerful method, such as simulation for modeling the probabilistic demand and lead-time.

The extensions of Clark and Scarf's (1960) model contain a general arborescent structure. The derivation of an expression of a closed form for the 'order-up-to-level' is in accordance with the equal fractal allocation proposition (Bessler & Veinott, 1966). Several authors also considered this problem in different ways, due to the intractability and complexity of the multi-echelon problem (Bollapragada, Akella, & Srinivasan, 1998; Dong & Lee, 2003; Federgruen & Zipkin, 1984a; Moinzadeh & Aggarwal, 1997; Rosenbaum, 1981; Schwarz, Deuermeyer, & Badinelli, 1985; Tee & Rossetti, 2002; van der Heijden, 1999; van der Vorst, Beulens, & van Beek, 2000).

Sherbrooke (1968) was the first researcher who modeled a multi-echelon inventory for managing an inventory of service parts under the name of the multi-echelon technique for recoverable item control (METRIC) which adopted a two-echelon model of an order policy for warehouses and retailers, which determine the inventory level, which reduces the expected number of backorders. Later, a wide a group of models, which were generally aimed at determining the optimal order quantity and safety stock in a framework of multi-echelon, were contributed by several researchers (Aggrwal & Moinzadeh, 1994; Moinzadeh & Lee, 1986; Nahmias & Smith, 1994; Svoronos & Zipkin, 1988; van der Vorst et al., 2000). Federgruen and Zipkin (1984c) extended the approach of Clark and Scarf (1960) to an infinite horizon and showed a new methodology.

2.5.3 Multi-Echelon Inventory Management

Research of multi-echelon inventory systems was raised by the pioneering work of Clark and Scarf (1960). Since 1960, a tremendous amount of research was performed to extend the work of Clark and Scarf. Federgruen and Zipkin (1984c) generalized the Clark-Scarf model to the case of the infinite horizon. Chen and Zheng (1994) gave new evidence to Clark and Scarf by deriving the lower bounds on the costs for the long-run of their model while Zipkin (2000) discussed these results in more detail. Inderfurth (1991) and Minner (1997) suggested different algorithms of dynamic programming to find the optimal echelon base-stock policy from the Clark-Scarf model. Zangwill (1966, 1969) and Love (1972) showed dynamic programming models with discrete time for a periodic review, as well as finite horizon serial systems with time-varying demand. Bessler and Veinott (1966) examined a general multi-echelon inventory system and tested the near-optimality of 'myopic' policies for one period of the system. Recently, Sinha, Sobel and Babich (2010) presented a unified approach and simple computations to the finite and infinite horizon of the Clark-Scarf model. For these additions, serial and assembly systems without setup costs of stage base-stock policies were indicated to be optimal. For distribution systems without setup costs, stage base-stock policies are optimal under the so-called assumption of balance, and vice versa if it is not optimal (Van Houtum, 2006). Due to the complex framework of multi-echelon systems with setup costs at each echelon, the vast majority of researchers focused on optimizing and evaluating simple batch ordering policies, such as *(R, Q)* policies.

2.6 Types of Multi-Echelon inventory system

A multi-echelon inventory system can be observed in various environments, which include:

i) Serial system. In this type, each location supplies raw materials from upstream location to downstream location, and only the downstream location meets customers demand (Dong, and Lee, 2003) see Figure 2.1.

ii) Assembly system. In this type, there is more than one supplier supplying the raw material. Each supplier is from a different location. Arguably, the upstream location receives raw materials from different suppliers and goes through the downstream location to have a single product and single location to satisfy customers' demands (Gümüs & Güneri, 2007).

iii) Distribution system. This system has a single supplier of raw materials and multiple finished goods or products (Atan, 2010) as exhibited in Figure 2.2.

In addition, there are other types of multi-echelon inventory systems, called mixed systems, which combine serial and assembly systems (Atan,2010; Axsäter & Marklund, 2008), serial and distribution systems Axsäter (2007), or assembly and distribution systems (Jie & Cong, 2009; Li & Sheng, 2008).

Work in a multi-echelon model faces difficulties and challenges (Duffield, 2012). One of the greatest challenges is that there is not enough information on demand and lead-time in the main warehouse and sub-stores because of the independency of the sub-stores in work (Hosoda & Disney, 2006). The availability of such information and data will be useful for the main warehouse to meet the needs of sub-stores' products and items. Other challenges include how to formulate a multi-echelon inventory model with the shape of complexity for the purpose of balancing between the assets in order to avoid an increase in inventory as well as a shortage of products and items (Huang & Xue, 2012).

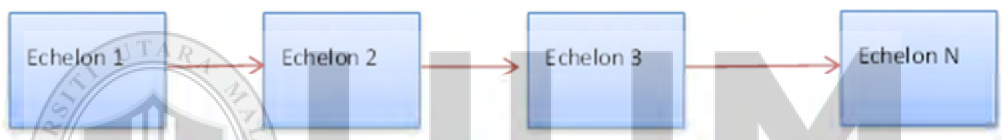


Figure 2.1. Serial Multi-Echelon inventory systems

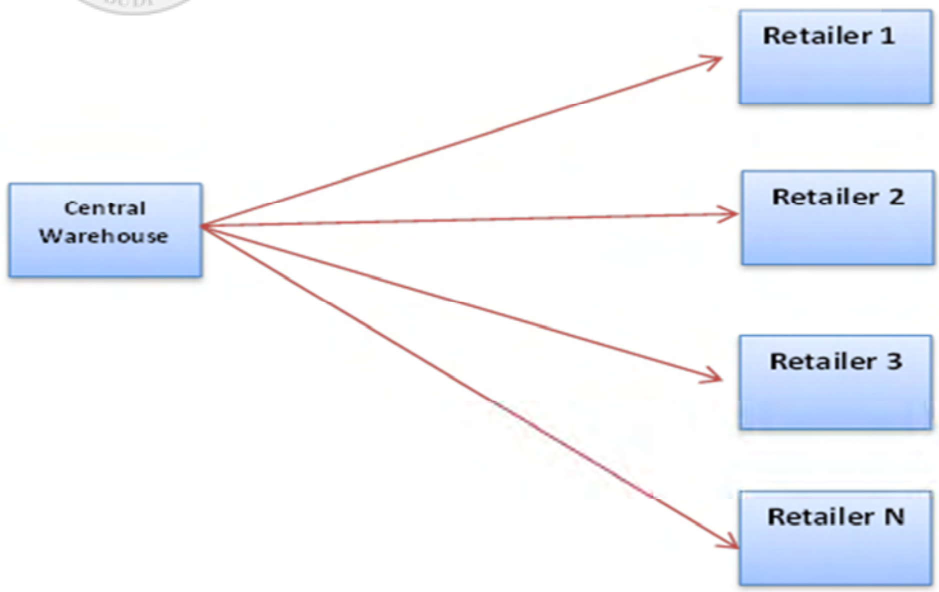


Figure 2.2. Distribution of Multi-Echelon inventory systems

2.6.1 Serial system in Multi-Echelon inventory

A significant amount of attention has been concentrated on one certain structure of a multi-echelon inventory system. This structure is known as the serial system (Atan, 2010). Initially, such structure was necessary because it effectively modeled and assembled a system (Chopra & Meindi, 2010). A serial system requires that each location has at most one predecessor and successor (Beamon, 1998). A production system is distinguished from a serial system in two ways (Wagner, 1974). Firstly, it is the value-added function of the two environments. In a production system, some operations are implemented in the product, which normally adjust both the form and value of the product in a distribution system, where the value of the product is often unchanged through the system (Gurgur & Altiok, 2004). Secondly, the serial system structure is unattractive as a distribution system (He et al., 2002; Van Houtum, 2006). It is more reasonable to simplify the connecting exit node with the entrance node and get rid of the transient utilities.

Given the former discussion about serial distribution, only a quick review of the literature in the area of the serial system of multi-echelon is presented. Clark and Scarf (1960) were the first researchers who wrote about a serial system as we previously mentioned. Santos & Santos (2007) provided a review of the extensions to the research performed by Clark and Scarf. Extending the work to a system with multiple utilities on the same level requires a supposition that all utilities on the same level are in balance. In a distribution multi-echelon system, this supposition is inappropriate.

Zangwill (1969) provided a solution to deterministic demand in a multi-echelon problem. The system was modeled to be a single network. The characteristics of this network were exploited to develop a dynamic programming solution similar to that of Wagner (1959). An important property of this procedure was that the optimal solution was a vector included in the set of extreme points. The idea of not initiating a replenishment order until a location inventory position reaches zero was essential and fundamental to all of the above research linked with deterministic demand.

Crowston, Wagner, and Williams (1973) solved the order quantity problem under the assumption that the demand is constant and instantaneously replenished. The model assumes that the optimal solution is the familiar integer multiple types, which means that the order quantity at high levels is an integer multiple of the order quantity at the next lowest level. Williams (1982) showed through an example that the integer order quantity restriction does not always guarantee optimality.

Schwarz (1973) developed a heuristic solution to the assembly problem using a 'myopic policy', which includes the optimal solution to all sequential two level problems. By considering two levels at a time, the solution procedure is simplified mathematically. He showed that the procedure of a myopic policy was optimal or a near-optimal. Furthermore, an approach of the branch and bound to the integer multiple solutions of Crowston et al. (1973) was provided, in which the result confirmed other outcomes, e.g., Clark and Scarf (1960) that decomposed the problem into a set of two-echelon problems, and gave near-optimal results.

2.6.2 Distribution system in Multi-echelon inventory

There are two direction models in a multi-echelon inventory system: the 'probabilistic service model' and 'guaranteed service model (Funaki, 2012; Humair et al., 2013; Humair & Willems, 2011). These two models can be applied to any inventory system, although for computational reasons, they are usually studied for a cyclic network. The first model supposes that the service times between the echelons can differ depending on the availability of the predecessors (You & Grossmann, 2011) while the second model supposes that each location has a guaranteed service time (Klosterhalfen et al., 2013; Li, 2013). This guaranteed service becomes possible by having external contingency resources.

Zhao (2007) also considered the same suppositions about the lead-times and studied the distribution system with external compound Poisson processes demand. The author described the waiting times by virtue of backorders for each unit of demand at each location in the supply chain and showed the approximation for assessment at the base stock levels. The guaranteed service model was considered by Graves and Willems (2000) where the decision variables are schemed to be lead-time for all of the locations. They supposed that each location guarantees a delivery at the end of these lead-times. A dynamic programming algorithm was assumed to find the optimal base-stock levels.

In a distribution system, few research has been done in one warehouse and multiple retailers (Graves, 1985; Lee & Moinzadeh, 1987a; Lee & Moinzadeh, 1987b; Sherbrooke, 1968; Svoronos & Zipkin, 1988, 1991). However, due to the expanded complexity of distribution systems caused by models with installation policies, the management of inventory in distribution systems becomes more complex than that of serial and assembly systems. In previous studies, one common installation policy considered was FCFS, which fills retailers' orders according to their arrival time. The adoption of this installation policy can be simplified to be an analysis of the distribution systems; however, it is generally not optimal (Axsäter, 2007). Due to the priority of FCFS, it is always given to the earliest backlogged order. Chen and Samroengraja (2000) also pointed out that this policy was the past priority allocation (PPA) policy. Alternatively, they presented another installation policy, called the current priority allocation (CPA) policy. The policy is used in a situation when a depot is unable to satisfy a retailer's order immediately, but at the same time has inventories dedicated to another.

Howard and Marklund (2011) considered a 'state-dependent myopic policy' rather than the FCFS, which allows an installation's decision to be delayed at a later point in time and is based on the state of the system. With these installation policies, models of inventory with single-warehouse, multi-retailers have spurred a great interest in the literature. The majority of the models assume independent demands through retailers and the use of base inventory policies or continuous review *(R, Q)* policies. Studies on distribution system based inventory policies include the following: Axsäter (2007), Axsater (1990), Caglar and Simchi-Levi (2000), Gallego, Ozer, and Zipkin (2007), and Graves (1985). Forsberg (1997) and Axsäter (1998) introduced varying cost evaluation methods for a system with demand and general distribution of inter-arrival times for customer orders. Cheung and Hausman (2000) introduced an accurate method for the steady-state performance evaluation of a distribution system with a warehouse in a two-level. Cachon (2001) considered an accurate evaluation method for mean inventory, backorders, and fill rates for a distribution system with two levels.

2.6.3 Production inventory system

Much literature is available in the design and analysis of production/inventory systems. Altiok (1989) focused on production facilities and finished product depots in a single production system. They considered the continuous review policy (*R, r*) to control the level of inventory at the depot and presented a procedure to calculate the minimum values of cost by *R* and *r* for each backorder and lost sales. Altiok and Ranjan (1995) discussed a series of multi-echelon production inventory systems.

Gurgur and Altiok (2004) extended Altiok and Ranjan's study by considering that each echelon has its own input and output inventory activities in a multi-echelon production/inventory system. They used the *(R, r)* policy to control production within an echelon and also used the (*Q, R*) policy to control procurement between echelons. Ishii, Takahashi, and Muramatsu (1988) and So and Pinault (1988) considered a production/distribution system of a pull type. The base-stock level and lead-time method were determined in Ishii et al. (1988), while the safety stock method estimation was presented in So and Pinault (1988).

Pykea and Cohen (1993) developed and analyzed the method of the flow material in an integrated production distribution by considering a single product system that includes manufacturers, finished goods, and a retailer. The *(Q, R)* policy base-stock was used for both retailers and finished goods with a constant transportation assumption and set-up times. The analyzed methodology showed the isolation of finished goods and evaluated the distribution of the inventory on hand (safety stock). The probabilities were then used to link finished goods to the manufacturer and to the retailer in order to find the inventory distribution in each echelon.

He et al. (2002) used a Markov decision approach to examine several inventory policies regarding replenishment for a make-to-order inventory production system and derived an optimal replenishment policy. Later, Bernstein and DeCroix (2006) considered an assembly system by using a stock policy. Then, Boute, Lambrecht, and Van Houdt (2007) presented a matrix-analytic procedure to calculate the replenishment lead-time distribution.

2.6.4 Deterministic inventory system

Generally, an inventory system is divided into two parts, deterministic and probabilistic (Pentico & Drake, 2011; Vrat, 2014). Whether the behaviors of an inventory system are deterministic or probabilistic, depend on the demand process (Pentico & Drake, 2011) . In practice, it is unrealistic to assume that the demand is deterministic or constant because the demand will change over time, e.g., dynamic, the demand rate changes with certain periods of time after, e.g., one week, month, season, etc. (Bookbinder & Cakanyildrim, 1999; Yao et al., 2009). But this does not mean that studies could not be done on deterministic demand. On contrary, many studies focused on this because of privacy, market stabilities, and other conditions (Fangruo Chen, 1999; Sana & Chaudhuri, 2008; Wee, 1995). However, the deterministic inventory model is much simpler than the probabilistic inventory system because the demand and, in particular, the lead-time is constant. In general, there are four types of deterministic inventory models: the purchase without a shortage model, purchase with a shortage model, production without a shortage model, and production with shortage model with a single item or multi-item inventories. There is a scientific basis for each model.

2.6.5 Probabilistic inventory system

The most realistic, comprehensive, and complex inventory system is when the demand and the lead-time are probabilistic or stochastic (Marc & Graves, 1985; Saharidis, Kouikoglou, & Dallery, 2009; Sahraeian et al., 2010; Tan & Xu, 2008) and where it is not easy to control the unexpected and uncertain demand; the procedure is a subordination to the theories of probabilistic and inference in statistics. However, the demand fluctuation can be stationary or un-stationary; the historical data shows this fluctuation in the demand. Figure 2.3 displays the fluctuation of probabilistic demand in a probabilistic inventory system.

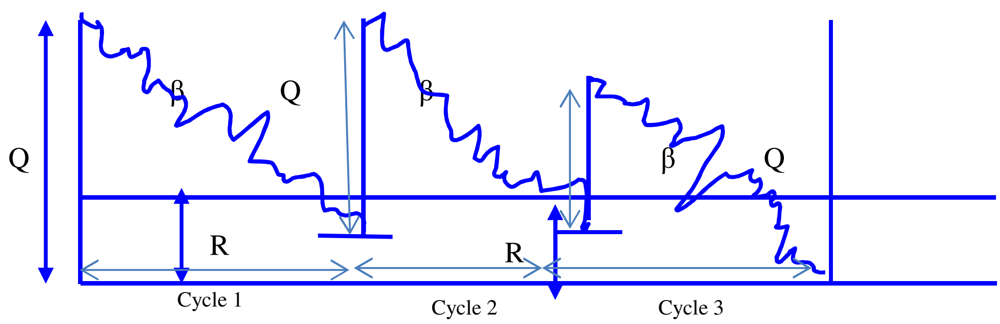


Figure 2.3. Probabilistic inventory system

2.6.5.1 Probabilistic Demand

Demand plays a critical role in an inventory system, whether it is a single-echelon or multi-echelon. Demand for certain items, goods, or products can be deterministic or probabilistic (Bagchi & Hayya, 1984; Inderfurth & Vogelgesang, 2013; Willemain et al., 2004). In deterministic cases, it is easy to determine the system state at any specific amount of time (Sana & Chaudhuri, 2008). However, in a probabilistic state, since the demand is a stochastic variable, in a discrete case it assumed to have a known probability mass function, *p.m.f* and for continuous case it assume to have probability density function, *p.d.f* (Marc & Graves, 1985; Hayya, Harrison, & Chatfield, 2009). Since most companies review historical data, they use forecasting techniques to determine the probability distribution of future demand (Starr & Miller, 1962).

Most literature in a multi-echelon inventory system supposed the demand distribution to be a Poisson or compound Poisson and lead-time to be constant or fixed (Axsäter & Marklund, 2008; Axsäter, 1984; Clark & Scarf, 1960; Graves, 1986; Hausman & Erkip, 1994; Hosoda & Disney, 2006; Muckstadt, 1986; Ravichandran, 1995; Saffari & Haji, 2009; Sherbrooke,1968; Zhao et al., 2006). The reason behind this, is that the stability of the markets (i.e., stability of the demand and the lead-time during the long periods) where, the market is not exposed to the sudden changes which may affect the behaviors of the demand and the expected period of the lead-time (Demeter & Golini, 2014; Venkateswaran & Son, 2007). Therefore, the period of the lead-time is not probabilistic. Generally, these assumptions are valid in supply chains that carry expensive items and face a low demand but highly uncertain demand (Caglar, Li, &

Simchi-Levi, 2004; Graves, 1985; Muckstadt, 1973; Sherbrooke, 1968). We can, therefore, conclude that the procedural treatment is suitable for spare part items or slow moving items. Of course, the treatment is different for slow and fast moving items in multi-echelon inventory systems. In spare part items, or as it is called slow-moving items, the demand is discrete and subject to the discrete probability distribution mass function, *p.m.f.* But the fast moving items are most common because the classification of the majority interference within fast moving items is subject to continuous probabilistic distribution density function, *p.d.f*, which is the most difficult.

2.6.5.2 Probabilistic Lead-time

Lead-time has been as an object of study for many researchers in inventory control and has been assumed to be fixed. However, lead-time can be deterministic (constant, zero, or ignored) or probabilistic. Lead-time is more realistic when it is probabilistic as it is more complex. The lead-time length impacts a customer service level, inventory on hand (safety stock) level, and efficiency of enterprises (Hariga & Ben-Daya, 1999; Li, Xu, & Ye, 2007). Lead-time is the time between the order requests until they are received or placed in the warehouse or the customer. In fact, lead-time includes the following elements: order setup, transit order, supplier lead-time, delivery time, and setup time (Lee, 2005).

Probabilistic lead-time is described as two different ways as follows:

1. Sequential deliveries are independent of demand during lead-time. Sequential deliveries mean that the order cannot cross in time. This type of probabilistic

lead-time is most common in practice (Hariga & Ben-Daya, 1999; Kim & Tang, 1997). For example, in a situation when ordering from a plant or external supplier, and the order placed with the outside suppliers are delivered according to FCFS basis. In this situation, the order cannot cross each other (Jain & Raghavan, 2008). The probabilistic lead-time for a certain order may depend on the previous demand by virtue of the earlier order that caused overcrowding in the supply system. However, the demand after the order will not affect the lead-time.

2. Independent lead-time. The probabilistic lead-time may happen if the order is rendered by many independent servers (Axsater, 2006). If the lead-time for a confirmed order is significant, it may then occur that later demands will trigger orders that will be delivered earlier than the other consideration (Axsater, 2010). In many cases, it is complex to model lead-time of type one. For example, orders for a plant and probabilistic lead-time at a confirmed time rely on the queues in front of several machines. These queues are by virtue of former orders for the time originally specified (Schwarz, Sauer, Daduna, Kulik, & Szekli, 2006). To model such lead-times, we may need to contain both the inventory and queues in an integrated model (Song, 1994; Zipkin, 2000; Zipkin, 1986). The same case may happen in connection with a multi-echelon inventory system. In reality, it is normally more complex to appraise probabilistic variations in lead-time than variations of demand. This is true except when the lead-time variations are large, where it may be reasonable to ignore them and change a probabilistic lead-time by its mean (Roy, 2005; Snyder et al., 2004).

2.6.6 Order quantity

The order quantity model with different mixed bags has surpassed the interests of various experts since it was first researched a century ago by Ford Whitman Harris in 1913. Many authors, e.g., (Clark, 1972 ; Urgeletti Tinarelli, 1983) gave far-reaching audit for utilizing an order quantity model. Cheng (1989) explained order quantity for a solitary item of interest identified with a unit value by utilizing a geometric programming strategy. This model assumes that demand decreases exponentially over time. Wee (1995) inspected an order quantity with shortage. Sana and Chaudhuri (2008) considered an order quantity model for different sorts of deterministic demand when postponement in installment is allowed by the retailer to the supplier.

Teng, Yang and Ouyang (2003) augmented Chang and Dye's work by including a non-steady purchase cost into the model. Salameh and Jaber (2000) developed models on order quantity when secured products are of poor quality, and Khan, Jaber, Guiffrida and Zolfaghari (2011) outlined the momentum collection of research that augmented Salameh and Jaber’s (2000) order quantity model for defective items.

Taleizadeh, Pentico, Jabalameli and Aryanezhad (2013a) considered an order quantity issue under fractional deferred installment, and Taleizadeh, Pentico, Saeed Jabalameli and Aryanezhad (2013b) developed order quantity models with various prepayments under no shortage, full back ordering, and fractional delay purchasing. As the opposition increases and more choices in selecting dispersion channels are accessible, numerous organizations understand that the execution of their business is very dependent upon the level of joint effort and coordination over the supply chain.

Extensive studies on supply chain in which a producer supplies an item to a retailer were also attempted (e.g., Wang & Liu 2007; Lee & Rhee 2010).

Many studies additionally examined coordination schemes on order quantity models. For example, Xia, Chen and Kouvelis (2008) inspected the supply chain coordination issue for an supply chain with different purchasers and numerous suppliers and found that matching purchasers' request profiles to suppliers' expense structures are the primary wellspring of supply chain coordination. Chen and Chen (2005) considered a multi-product inventory and production issue with joint setup costs for a solitary maker and a solitary retailer where the retailer confronts deterministic demand and offers various items in the marketplace. Based on order quantity, they identified the optimal replenishment approaches for the retailer's end-products and for the producer's crude materials to minimize the aggregate expense of the supply chain. They assumed a profit sharing instrument through a rebate plan to attain Pareto upgrades among the participants of a coordinated supply chain.

Chen and Mushaluk (2014) considered an SC coordination plan and issues in which a producer supplies an item to a retailer. The retailer chooses his optimal order quantity utilizing an economic order quantity (EOQ) model that contemplates the shipment expenses charged by the maker (Mehmood-Khan et al., 2011; Rezaei, 2014).

They demonstrated that under a few circumstances, the producer can offer an agreement that incorporates a markdown shipment charge for every conveyance and a shipment expense for every unit to organize the supply chain and improve the benefits of both the maker and the retailer. They likewise recognized under which condition the producer cannot organize the supply chain with shipment charges.

2.6.7 Probabilistic demand and lead-time

In the following, we present studies that considered the probability of demand and lead-time from 1984 to 2012, focusing on the research techniques that were used and the objectives of the study. However, these studies did not necessarily adopt the probability distribution of demand during lead-time because of the nature of the study and complexity of the structure and algorithm to extract the demand during lead-time probability distribution. This is done for the purpose of showing the scientific gap that has been considered in this research.

In fast moving items, the normal distribution was often used as a demand model (Gümüs & Güneri, 2007; Mitra & Chatterjee, 2004a). Bagchi and Hayya (1984) considered an expression for the probability density function of demand during lead time when the demand distributed normal and the lead-time distributed Erlang, a special case of the gamma distribution. In this case it was appropriate to use this probability density function of each distribution to modify the expressions for a protection level and potential lost sales mathematically.

Dekker, Kleijn and De Kok (1998) used a mathematical modeling and simulation for a two-echelon inventory system (one warehouse and *N*-retailers) with normal demand and lead-time distribution to provide insight into the effect of the break quantity rule on the inventory holding costs without focusing on the demand during lead-time probability distribution for the nature of the study . Mohebbi & Posner (1998) studied a two-echelon inventory system with the continuous review *(s, Q)* policy, compound Poisson demand, and exponentially lead-time distribution to introduce an accurate remedy of the sole versus dual sourcing problem under an inventory system of lost sales. They adopted slow moving items. Van der Heijden, Diks and De Kok (1999) developed an algorithm to analyze two-echelon divergent networks with the integral periodic review order-up-to *(R, S)* inventory control policy in the context of both probabilistic demand and lead-time, then extended to *N*-echelons. Ganeshan (1999) studied a three-echelons inventory system with multiple suppliers, one warehouse, and multiple retailers. He used a near-optimal *(s, Q)* type inventory policy for a production/distribution network. His study focused on the inventory analysis at the retailers, the demand operation at the warehouse, and the inventory analysis at the warehouse. The contribution of the model in Chen and Zhang (2009) is the smooth integration of the three components (inventory, transportation, and transit) to analyzes simple supply chains. The decisions focused on the cost framework.

Bookbinder and Cakanyildrim (1999) considered two-echelon inventory models of the lot size/reorder point supply chain. They considered two-stage supply chain, which consists of a storage location, such as a warehouse order product from a manufacturer or supplier follow the (*Q, r*) inventory policy, where *Q* is the lot size or economic order quantity and *r* is the reorder point. The demand is constant and known, but the lead time is variable (stochastic). The study emphasized the importance of the practice of stochastic lead-time using mathematical expressions. Dong and Lee (2003) prolonged the approximation to the demand process of time and correlated it according to Clark and Scarf (1960). They assumed an autoregressive demand model and variable lead-time to see the effect of autocorrelation and lead-time on the system's performance. The study adopted an inventory system periodic review with serial M-echelon.

Tang and Grubbström (2003) studied a two-level assembly system with a lot-for-lot policy. Their objective was to minimize the total stockout and inventory holding cost depending on constant demand and stochastic lead-time. Kiesmüller et al. (2004) considered a decentralized inventory control in a divergent multi-echelon network. They also used a simulation approach for the *N*-echelons continuous review *(s, nQ)* stock installation policy with stochastic compound renewal demand and stochastic lead-time. The objective was to derive an analytical approximation for performance characteristics of a divergent multi-echelon distribution network."

Chiang and Monahan (2005) adopted a stochastic independent exponential random variable lead-time and stochastic demand. The technique involves the Markov decision process and scenario analysis with a one-for-one replenishment inventory policy. The objective was to analyze the effect of customers' search rates on the performance of the channel. Johansen (2005) also used the Markov decision process research technique with simulation and Erlang loss formula. He adopted an inventory system of a single item and sequential supply chain with the Poisson demand distribution (slow-moving item) and stochastic Erlangian lead-time. The aim of the study was how to compute the optimal base stock for a lost sales inventory model with a sequential supply system and Erlangian lead-time.

Lee (2005) considered lead-time and order quantity as decision variables of a mixture of lost sales inventory model and backorders. The lead-time includes the following elements: order setup, transit order, supplier lead-time, delivery time, and setup time. The study assumed that the demand during lead-time follows a mixture of normal distribution because the demand of the different customers is not identical in the lead-time. For that, the use of a single distribution for lead-time could not be used. The study developed a mathematical algorithm procedure to find the optimal order quantity and the optimal lead-time.

Simchi-Levi and Zhao (2005) studied the problem of safety stock position single product multi-echelon supply chain with a tree network structure. Each echelon controls its inventory with a continuous review base stock policy. The study assumed that the external demands follow independent Poison processes, and the backorder at each stage came from unsatisfied demands. Which also assumed the transportation lead-time is stochastically, sequentially, and exogenously determined? Their study took only the transportation lead-time as a stochastic variable, and ignores the lead-times in the system (from upstream to downstream). The study focused on a stochastic service model approach and derived recursive equations for the backorder delays at all echelons.

Axsater (2006) considered a single-echelon inventory system under a continuous review (R, Q) policy, where, R is the reorder point and Q is the order quantity. The demand during lead-time is a normal distribution. The objective of the study was to minimize the holding and order costs underfill rate constraint depending on R and Q values. The problem is the basis for many industrial inventory control systems.

Wu, Lee and Tsai (2007) studied the situation when the demands of the various customers are not identical in the lead-time. Where, a single distribution cannot be used. Hence, the study proposed an inventory model considering a mixture of normal distribution and mixture of demand during lead-time distribution. The aim of the study was to minimize the total cost involving a negative exponential crashing cost and variable demand during lead-time using a mathematical derivation to develop two algorithmic procedures to find the optimal inventory policy.

Li and Sheng (2008), in examining China's industries of low technology, argued that the retailer's inventory system lead-time may be changed with various suppliers. This study focused on inventory strategies in various classification of supply chain, assuming that all suppliers have the same product and the competition between them are on the price and lead-time. The considered inventory control policy is (S, s) for the retailers. The retailer cannot dominate his stock under the lead-time assumption that follows some distributions and the retailer also has various classifications of suppliers. They also suggested two actuation mechanisms for the retailer in addition to using a simulation for the multi-agent system and the company environment. The objective of the study was to elucidate that s can significantly change and that a financial benefit can be achieved while the retailer cooperates with various suppliers under supply chain.

Baten and Kamil (2009) described the production inventory system with two Weibull distribution parameters for deteriorating items using the Pontryagin maximum principle and the dynamic programming principle. The aim of the study was to minimize the objective function of total cost.

Deng, Song, Ji, and Zhang (2010) considered a stochastic periodic review model with both stochastic demand and lead-time. They proved the convexity of the cost function and global optimal solution by a clear form when the lead-time deteriorates to zero. Otherwise, the simulated annealing algorithm for Monte Carlo methods was used to find the global optimal solution of the model.

Axsäter (2011) considered a single-echelon inventory system with continuous review and Poisson demand with standard linear holding cost and backorder cost, in addition to the change in the lead-time. He applied the philosophy of just-in-time (JIT) to increase the efficiency of supply chain, and the focus included the steady state situation before and after the change of the lead-time. In other words, the model discusses how to bring the system from its original steady state to the new steady state using a transient problem as in Axsater (2011). While considering that the lead-time reduction occurs at some time. The objective of this study dealt with transient inventory problems caused by lead-time changes to minimize the holding and backorder costs and the total cost.

Axsäter & Viswanathan (2012) dealt with an independent supplier inventory control problem with a continuous review system. The supplier faces demand from a single customer who in turn faces Poisson demand and follows the continuous review *(R, Q)* policy. The supplier, which is an independent entity and not part of an integrated supply chain, faces lot size demand of constant size *Q*, whose inter-arrival times are an Erlangian distribution. The aim was to reduce the average of long run inventory costs for the supplier.

Table 2.1 summarizes previous works on probabilistic demand and lead-time, inventory system policy, the number of echelons, demand, and lead-time assumptions.

Table 2.1

Multi-echelon parameters

Source	No. of Echelons	Inventory system policy	Demand assumption	Lead-time assumption
Bagchi and Hayya (1984)	2	-	Stochastic, normal	Erlang
Dekker et.al, (1998)	2	-	Stochastic, normal	Stochastic, Normal
Mohebbi and Posner (1998)	2	*(S, Q)* policy	Compound Poisson	Exponential
Van der Heijden, Diks and De Kok (1999)	2	Periodic review Order-up-to *(R, S)*	Stochastic	Stochastic
Ganeshan (1999)	3	Near-optimal *(s, Q)* type inventory policy	Stochastic	Stochastic
Bookbinder Cakanyildrim (1999)	2	*(Q, r)* policy	Constant	Stochastic
Dong and Lee (2003)	*N*	Order-up-to *S* policy	Autoregressive demand	stochastic to see the effect of autocorrelation on the system performance
Tang and Grubbström (2003)	2	Lot-for-Lot policy	Deterministic	Stochastic
Kiesmüller et al., (2004)	*N*	Continuous review *(s, nQ)* stock installation policy	Stochastic, compound renewal demand	Stochastic
Chiang and Monahan (2005)	2	Decentralized safety inventory policy	Deterministic and stochastic	Not specified
Johansen (2005)	Single	Base stock policy	Stochastic Poisson	Stochastic Erlang
Simchi-Levi and Zhao (2005)	*N*	Continuous review base stock policy	Stochastic Poisson	Stochastic transportation
Lee (2005)	2	Continuous review	Stochastic mixture normal	Stochastic mixture of normal dist.
Axsater (2006)	Single	Continuous review *(R,Q)* policy	Normal distribution	Normal distribution
Wu and Tsai (2007)	Single	-	Stochastic mixture normal	Stochastic mixture of normal distribution
Li and Sheng (2008)	2	Retailer *(S, s)* policy	Stochastic	Deterministic
Baten and Kamil (2009)	Single	2 Weibull distribution parameters	Stochastic	Deterministic
Deng, Song and Zhang(2010)	Single	Periodic review	Stochastic	Stochastic
Axsäter (2011)	Single	Continuous review system	Stochastic Poisson	Deterministic
Axsäter and Viswanathan (2012)	Single	Continuous review *(R, Q)* system	Stochastic Poisson	Stochastic Erlangian

Table 2.1 summarizes previous studies based on the demand and lead-time assumptions. Table 2.2 shows the technique/method adopted and the objective of each of these studies.

Table 2.2

Multi-echelon methods and objectives

Source	Technique	Objective
Bagchi and Hayya (1984)	Mathematical model	Modify mathematical expressions for potential lost seals
Dekker, Kleijn and De Kok (1998)	Mathematical model and simulation	Provide insight into the effect of the break quantity rule on the inventory holding cost
Mohebbi & Posner (1998)	A system-point (SP) method of level crossings	Formulate long-run average cost function with/without a service level constraint
Van der Heijden, Diks and De Kok (1999)	Derived a computational method	Obtain the order-up-to level and the allocation fractions required to achieve given target fill rates
Ganeshan (1999)	Analysis	Cost framework
Bookbinder and Cakanyildrim (1999)	Mathematical model	Obtain global minimization that depends on convexity of expected cost per unit time
Dong and Lee (2003)	Mathematical model	Extend the time correlated demand process of Clark and Scarf (1960)
Tang and Grubbström (2003)	Mathematical model and simulation	Minimize the total stockout and holding cost of inventory
Kiesmüller. et al (2004)	Simulation	Performance properties of a divergent multi-echelon distribution network to derive analytical approximations
Chiang and Monahan (2005)	Markov decision process and scenario analysis	Impact of customers search rates on channel performance analysis and present a two-echelon dual channel inventory model
Johansen (2005)	Markov decision process, simulation and Erlang loss formulation	Calculate the optimal base stock for lost sales inventory model
Simchi-Levi and Zhao (2005)	Mathematical model	Derive recursive equations for the backorder delays at all echelons
Lee (2005)	Mathematical algorithm procedure	Find the optimal order quantity and the optimal lead-time
Axsater (2006)	Mathematical expressions	Minimize the holding order cost underfill rate constraint
Wu, Lee and and Tsai (2007)	Mathematical algorithm	Minimize the total cost involving a negative exponential crashing cost and finding optimal inventory policy
Li and Sheng (2008)	Simulation and four strategic scenarios	Demonstrate that *s* can significantly change and financial benefit can be a achieve while the retailer cooperates with different suppliers under four supply chains
Baten and Kamil(2009)	Dynamic programming and quadratic control theory	Minimize the objective function of total cost
Deng, Song, Ji and Zhang (2010)	Simulation	Prove the convexity of cost function and global optimal solution
Axsäter (2011)	Mathematical model	Solve transient inventory problems caused by lead-time changes to minimize the holding and backorder costs and minimize the total cost
Axsäter and Viswanathan (2012)	Simulation and scenarios	Minimize the long run average inventory costs for the supplier

In Table 2.2, only 15 studies adopted mathematical or simulation approaches. The remaining studies relied on other approaches, which are not related to the present research. Table 2.3 summarizes the relevant literature that adopted the mathematical approaches or/and simulation approach, the number of echelons, with probabilistic demand, lead-time, and demand during lead-time.

Table 2.3

Summary of Multi-Echelon Problem with Mathematical and/or Simulation Methods

Source	No. echelons	Demand distribution assumption	Lead-time dist. assumption	Demand during lead-time dist.	Technique
Bagchi and Hayya (1984)	2	Normal	Erlang	The parameter of each distribution used separately to achieve the study objective	Mathematical model
Dekker, et al,. (1998)	2	Normal	Normal	The parameter of each distribution used separately to achieve the study objective	Mathematical model and simulation
Bookbinder and Cakanyildrim (1999)	2	Constant	Stochastic	——	Mathematical model
Dong and Lee (2003)	*N*	Autoregressive demand	Variable	Measure the effect of autocorrelation on Lead-time	Mathematical model
Tang, and Grubbström (2003)	2	Deterministic	Stochastic	——	Mathematical model and simulation
Kiesmüller, et al (2004)	*N*	Compound Poisson	Stochastic	The parameter of each distribution used separately to achieve the study objective for slow moving items	Simulation
Simchi-Levi and Zhao (2005)	*N*	Poisson	Stochastic only for transportation	The parameter of each distribution used separately to achieve the study objective for slow moving item	Mathematical model
Lee (2005)	2	Mixture normal	Mixture normal	Assumed to be mixture of normal distribution	Mathematical algorithm procedure
Axsater (2006)	Single	Normal	Normal	Assumed to be normal distribution	Mathematical expression
Wu, et al. (2007)	Single	Mixture normal	Mixture Normal	Didn't adopt	Mathematical algorithm
Li and Sheng (2008)	2	Stochastic	Deterministic	——	Mathematical model
Baten and Kamil (2009)	Single	Stochastic	Deterministic	——	Simulation
Deng et al,.(2010)	Single	Stochastic	Stochastic	Monte Carlo methods was used to find the global optimal solutions of the model	Simulation
Axsäter (2011)	Single	Stochastic Poisson	Deterministic	——	Mathematical model
Axsäter and Viswanathan (2012)	Single	Poisson	Erlang	——	Simulation and scenarios

From Table 2.3, we conclude that most of the studies did not adopt the probability of demand during lead-time, even with probabilistic demand and probabilistic lead-time and implemented either a single-echelon or two-echelon system. The problem with demand during lead-time is that this type of variable is not available in the reality because of the nested of the activities through the echelons of supply chain. They used only the parameters of each distribution of the demand and lead-time separately to achieve the objectives of the studies.

There are only three studies adopted *N*-echelon. Most of the studies focused on reducing costs as shown in Table 2.2. This research focused on multi-echelon, because the procedure is different for multi-echelon and two types of echelons. Whenever the number of echelons increases, the complexity and the procedures of the mathematical model will increase, too. Table 2.4 summarizes the studies that adopted *N*-echelon inventory system, demand, and lead-time assumption, and the methods used.

Table 2.4
Summary of inventory system with N-echelon

Source	**No. of echelons**	**Demand distribution assumption**	**Lead-time dist. Assumption**	**Demand during lead-time dist.**	**Technique**
Dong and Lee (2003)	N	Autoregressive demand	Variable	Measure the effect of autocorrelation on lead-time	Mathematical model
Kiesmüller et al., (2004)	N	Compound Poisson	Stochastic	The parameter of each distribution used separately to achieve the study objective for slow moving items	Simulation
Simchi-Levi and Zhao (2005)	N	Poisson	Stochastic only for transportation	The parameter of each distribution used separately to achieve the study objective for slow moving items	Mathematical model

From Table 2.4, Dong and Lee (2003) considered *N*-echelon with a mathematical model to measure the effect of autocorrelation on lead-time by extending the time correlation demand process of Clark and Scarf (1960). Kiesmüller et al., (2004) considered *N*-echelon but the adopted method was only simulation and the demand distribution assumption was Compound Poisson, which was suitable for slow moving items. They also did not adopt demand during lead-time distribution. Simchi-Levi & Zhao (2005) also considered *N*-echelon with a mathematical method, but they did not adopt demand during lead-time, and the demand distribution was the Poisson process.

As a conclusion, these studies focused on stochastic demand and lead-time. However, some of them did not address the importance of the probability distribution of demand during lead-time, despite using the probability distribution of demand and lead-time because of the nature of the study. Others considered the parameters of the demand and lead-time distribution separately and then extracted the mean and the standard deviation of demand during lead-time based on theoretical equations to solve the problems. The methods that used to solve the problem are the Markov process, mathematical model, simulation, information sharing, costs, transit time, and demand correlation. Most of these studies include low demand items and in both cases used either the periodic review system or the continuous review system, and a centralized or decentralized system.

2.7 Multi-Echelon inventory system policies

Any inventory models, whether single echelon or multi-echelon are based on ordering policy, followed by a location that can be categorized as a continuous review policy or periodic review policy (Axsäter & Zhang, 1999; Chen & Samroengraja, 2000; Chung et al., 2008). In a continuous review policy, a location may put an order of *Q* quantities when a reorder point of *R* is reached, called a *(Q, r)* policy (Bookbinder & Cakanyildrim, 1999; Dolgui et al., 2013). Concurrently, the time between orders is a stochastic variable. In a periodic review policy, a location puts an order as each *T* periods increases the level of inventory up to *S* quantities; for that, a periodic review policy is called *(T, S)* policy (Zaojie & Guoying, 2007). Here, the lot size or order quantity is classified as a random variable. The other more general type of inventory policy is the *(S, s)* policy (Federgruen & Zipkin, 1984a; Li & Sheng, 2008; Yang et al., 2008), where the location takes an order up to *S* quantities when the reorder point is less than or equal to *s* quantities. This policy can be either a periodic review or a continuous review policy.

The *(s, S)* policy is the optimal solution of a single-echelon under very general conditions where some results of optimality exist for a serial system (Axsater, 2006; Chen, 2000; Clark & Scarf, 1960; Muharremoglu & Tsitsiklis, 2003). But, in the structure of multi-echelon inventory systems, we can generally expect the optimal policy to be still unknown and quite complex (Axsater, 2006). Nevertheless, the policies in the inventory system can be classified as the (*s, S*) policy or base stock policy where *s* is the reorder point and *S* is the maximum level of inventory. In the (*R, Q*) policy (Matheus & Gelders, 2000; Mitra & Chatterjee, 2004), *R* is the reorder point and Q is the batch size or order quantity. The (R, nQ) policy is the same as (R, Q) but Q is multiplied by the number of orders n (Axsäter & Marklund, 2008). Usually, each one of these policies changes with the variables of the study and is linked to other scientific techniques, like statistical theory, queueing theory, stochastic programming, linear programming, integer linear programming, etc.

Yang, Ding, Wang, and Dong (2008) examined an optimal non-stationary of the *(s, S)* policy for multiple period replenishment lead-time and non-stationary stochastic demand in spare part items by analyzing the relationship of inventory between sequential periods and the stochastic dynamic programming that was established. Hence, the optimal solution algorithm was developed, and the result was a non-stationary *(s, S)* policy form. Axsäter and Viswanathan (2012) considered the (R, Q) policy. Chen and Xu (2010) considered the (s, S) policy for an inventory system with two demand categories. Li and Sheng (2008) regarded a multi-agent system under the (S, s) policy for a controlled stochastic system in various supply chains.

2.7.1 Service Level

When the demand and lead-time are probabilistic or stochastic variables, the demand during lead-time is also a probabilistic variable (Bagchi & Hayya, 1984; Inderfurth & Vogelgesang, 2013). Therefore, the inventory of hand or the safety stock of an order that arrives is not known. However, when the demand during lead-time overrides the reorder point, stock-outs occur (Graves & Willems, 2003). A clear way of forbidding stock-out is to increase the availability of the product, but increasing the availability of the product will increase the safety stock (Yang & Geunes, 2007), thereby increasing the inventory holding cost (Simchi-Levi & Zhao, 2005). As a result, there should be a trade-off between availability and cost. There are two kinds of objectives at the service level (Axsater, 2010; Axsater, 2006):

1. Cycle service level, which is the part of order cycle that meets all the requirements of customers.
2. Fill rate (FR), which is the ratio of demand that is filled from the current inventory.

For guaranteed service, GS, deterministic service time of each inventory is set for satisfying any demand from its downstream inventory and guarantees that the demand can always be satisfied in the given service time (Humair et al., 2013). This approach supposes that overuses retailers demand superior to a bound is treated by some unprecedented measures, such as expediting and overtime. With this supposition, each inventory can forecast its maximum demand to fill and ensure given service time to its downstream inventory. Subsequently, the service time of each inventory in guaranteed service and GS is deterministic (Klosterhalfen et al., 2013).

The first researcher who examined guaranteed service, GS, was Kimball (1955). He focused on a single inventory with random but deterministic demand, which was controlled by a base-inventory policy. He demonstrated that the bound of the demand during given service time of the inventory could be used to set its base-inventory level. Simpson (1958) broadened Kimball's model to a serial inventory system and demonstrated that the optimal inventory policy of the system is an "all or nothing" policy. Depending on this so-called the extreme point property, Graves, Kletter and

Hetzel (1998) stated that the optimization problem considered by Simpson can be formulated using a dynamic programming algorithm. In the ensuing years, this method was broadened to other network structures. Augmentations to assembly and distribution systems, spanning trees, or even general structures of the acyclic network can now be found (Graves & Willems, 2000; Humair & Willems, 2006, 2011; Inderfurth & Minner, 1998; Inderfurth, 1991; Minner, 2001).

2.7.2 Queueing system in multi-echelon inventory

A queueing system has a significant role in organizing and coordinating jobs in a supply chain (Jain & Raghavan, 2003). In a supply chain, each stage or echelon can be described to be a queue system with inventory, particularly when the process is subject to long waiting time for retailers to meet the product, which is due to the probability of uncertain demand and probability of lead-time, and a vast number of retailers and the limited number of distribution centers (Sahraeian et al., 2010). As a result, it is important to create a mechanism to help facilitate and reduce the waiting time in the system.

The first-come-first-serve, also known as the FCFS system, is a procedure to reduce waiting time in the system related to the inventory system (Axsäter & Marklund, 2008; Schwarz, Sauer, Daduna, Kulik, & Szekli, 2006). The disadvantage of using of the FCFS system is no limit to the proportion of increasing the cost (Schwarz et al., 2006). The complexity of the system happens when the demand during lead-time is unknown, and it has a continuous distribution. In the literature, it is often assumed that the demand was distributed as a Poisson process (Ravichandran, 1995).

Therefore, they usually assumed that backorders at the warehouse were satisfied according to FCFS.

Axsäter (2007) used FCFS in a two-echelon distribution systems with one warehouse and a number of retailers under the *(S-1, S)* policy, where the retailers faced stochastic demand distributed Poisson. He used a simple heuristic that always leads to fewer costs than an FCFS system, and the proposed heuristic was evaluated by using a simulation study. Furthermore, Saffari and Haji (2009) and Sahraeian et al. (2010) tried to reduce the waiting time in their considered system by depending on a stochastic demand distributed be a Poisson process. However, in a continuous statistical distribution (fast moving items or products) the procedure is different because the objective is not about how to deal with backorders subject to FCFS; the problem is that the distribution centers cannot satisfy a large number of retailers that leads to long waiting time in the overall system (Sahraeian et al., 2010).

Schwarz, et. al., (2006) solved stationary distributions of joint inventory operations and queue length in an explicit product form for different M/M/1-systems with a continuous review inventory system and various inventory management policies as well as with lost sales. The first *M* in the queueing discipline M/M/1 represents the statistical distribution for arrival process; second *M* represents service time distribution, and number 1 represents a number of service stations. They assumed that the demand is distributed Poisson, lead-time and service time were distributed exponentially. The Poisson and exponential distributions were used to find performance measures of the competent systems. In the case of an infinite waiting status, the basic result was that the abridging distributions of the queue length operations are the same as in a traditional M/M/1/∞-system, where ∞ means the community size is infinite.

Matheus & Gelders (2000) studied an inventory model subjected to a stochastic non-unit sized demand style, and suggested an accurate and an approximate reorder point method of calculation for the *(R, Q)* inventory policy. Furthermore, they used simulation to present the results for different distributions under various service levels. Karmakar (1987) tested the effect of order quantity on work-in-operation inventory and lead-time. The models of queueing were used to capture the impact of density and size. Furthermore, the model was extended to a multiplicity of products.

Kim and Tang (1997) concentrated on the inventory system in systems of pull production with a single warehouse and a single production facility. The form of a PA (Production Authorization system) card was used to control the inventory in the system. There was a trade-off between lead-time of manufacturing, response time. The optimal inventory at the warehouse is calculated using heuristics. A single echelon $E_K/M/1$ queueing model (continuous time) was used for analyzing the system, where E_K is the arrival rate distributed Erlang (Bhat, 2015). For the first time an inventory supply chain performance analysis was presented using queueing models with a discrete time by Jain and Raghavan (2003). Their work was an extension of the work by Kim & Tang (1997), albeit using a different approach for optimization and analysis.

Jain and Raghavan (2008) showed stylized models for a procedure performance analysis of the manufacturing supply chain network (SCN) in a probabilistic setting for order quantity. Queueing models were used to capture the flow of SCN. The analysis was restricted to an inventory optimization model that can be used for designing inventory policies. They used the three cases model. Firstly, the model showed a manufacturer with one warehouse, which supplies to different retailers. They identified the optimal warehouse inventory level, which minimizes the total expected cost of holding inventory, backorder cost connected with serving orders in the backlog queue, and setup cost. Secondly, they imposed a constraint on service level in terms of fill rate, supposing that customers do not joist from the system. Thirdly, the model was expanded to a three-echelon inventory model, which explicitly considers the logistics operation.

2.7.3 Costs

Any companies or firms that aim to gain profit must engage in economic decision making. Companies need to find ways to balance the two features, costs and production, and also an accumulation of material costs and increasing investment (Bolarín et al., 2008; Hwan Lee & Rhee, 2010). Therefore, it is necessary to find the inventory levels that can minimize costs yet achieve the highest level of efficiency, performance, and operations. In order to achieve that, firms need to be able to determine the economic inventory levels and the quantity of optimal purchases accurately. The following is a brief explanation of the most important types of costs (Axsater, 2006; Bolarín, Lisec, & Esteban, 2008; Frederick & Gerald, 2001; Hamdy, 2007).

1. Setup cost, which is independent of the size of the orders of quantities for purchases or productions. Setup cost is a fixed cost (Karaman, 2007), denoted by A. It includes the following costs:
   - The issuance of document request and follow-up.
   - Putting the items or products in the depots.
   - Production setup.
   - Arranging the place.
   - The closing of buildings.
   - Inspection of bad or shoddy items or inspection quality.
   - Transportation.
2. Holding cost, which is the cost that is carried by the firm or the project when storing materials in the warehouse or depots (Axsater, 2001) and denoted by h. It includes the following elements:
   - Interest on capital
   - The rent of the storage place including electricity, water, cooling, etc.
   - The insurance cost against unexpected accidents.
   - The damage that infects the products.

There is an extrusive relationship (direct relationship) between holding cost and inventory level, i.e., the increase of the inventory level will increase the holding cost. Figure 2.4 explains this relationship.

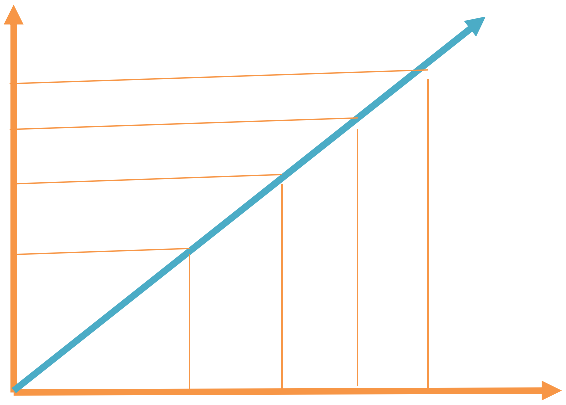


Figure 2.4. The relationship between holding cost and inventory level

Figure 2.4 shows that when the inventory level increases, the holding cost will increase, and this increase is proportional to the rate of increase in inventories i.e. any increase in inventory level will increase in holding cost and there are no limits to the increment of holding cost.

3. Shortage cost. It is divided into two types (Axsater, 2006)
   - Unavailability of items or products in the depot.
   - The lost profit due to the lack of sales.
4. Purchase cost, which is the unit cost and usually is fixed (Chang & Chun-Tao, 2004). This cost changes with purchasing large amounts of products or items; therefore, a discount price is likely to be determined.

Most studies in a multi-echelon inventory system discussed mechanisms of reducing cost to the minimum level in any companies, firms or organizations (Axsäter & Marklund, 2008; Axsater & Rosling, 1999; Axsater, 2010; Axsäter, 2011; Diks, 1996; Muckstadt, 1986; Tan, 1974).

Nevertheless, this study considers both holding, *A*, and setup costs, *h*. Axsäter (1998) regarded a serial system with two echelons under continuous review installation policies *(R, Q)* and suggested an approach to evaluate holding and shortage costs accurately. De Bodt and Graves (1985), introduced the serial system with multi-echelon echelon *(R, Q)* policies and showed an approximate model for the cost evaluation of the system. Axsäter and Rosling (1993) presented that the *(R, Q)* policies of the multi-echelon dominate installation *(R, Q)* policies for serial and assembly systems. For distribution systems in multi-echelon inventory installation and echelon under continuous review *(R, Q)* policies, however, they outperform each other in different situations. Chen and Zheng (1994) developed a procedure for exact performance evaluation of serial multi-echelon inventory systems under continuous review *(R, nQ)* policies. Axsäter (1997) suggested an alternative structure for the cost evaluation of a multi-echelon inventory system *(R, Q)* policy. The structure applies the idea of matching supply units with demand that was initially utilized for the assessment of stock installation policies.

2.8 Approaches in a multi-echelon inventory system

In a multi-echelon inventory system, there are many approaches utilized to assess the solution or objectives of the studies. These approaches can be classified as specific methods, such as linear programming, integer programming, dynamic programming, network, and quadratic programming, or general approaches that include simulation, forecasting, and information sharing. Some of the literature considered two of these approaches, for example, forecasting and simulation, linear programming and simulation, etc. The following highlights the differences in some aspects of these approaches related to the problem of the multi-echelon inventory system.

Chen (1998) and Ding, Guo, and Liu (2011) considered the effects of different information sharing and/or coordination instruments on the performance of serial inventory systems controlled by installation/echelon the *(R, nQ)* policies. Rezg, Xie and Mati (2004) showed an integrated approach for production of inventory control in a line made up of *N* machines. They suggested a combined methodology for the simulation and genetic algorithms to optimize inventory control policies. Sahin, Buzacott and Dallery (2008) and Sahin and Dallery (2009) considered a three-echelon system where implementation errors results in a contradiction between the physical inventory and information system. They provided a new cost element for the traditional Newsvendor model, capturing the cost of not satisfying an initial commitment due to inventory imprecision. Shang (2012) suggested a simple heuristic for determining inventory levels in a serial system with the non-stationary behavior of demand and no setup costs depending on single-echelon approximations. Gallego and Ozer (2003) and Huh and Janakiraman (2008) presented new evidence and new heuristic of the optimality of serial multi-echelon inventory systems without setup costs in the procedure of the Clark-Scarf (1960) model. Janakiraman (2010) considered a serial inventory system with periodic review lost sales and derived initial characteristics of the vector of optimal order quantity in this system. He showed that the optimal order quantity at each echelon was a decreasing function of the inventory at any downstream inventory and an increasing function of the inventory at any upstream inventory.

Dong and Lee (2003) reconsidered the serial multi-echelon inventory system of Clark and Scarf (1960) and offered three main outcomes depending on a mathematical modelling technique, which are (a) An approximate simple lower-bound to the echelon inventory levels and a cost upper bound of the total system for the primary model, (b) the optimal inventory policy framework of Clark and Scarf (1960) bearing in accordance with time-correlated demand processing using 'Martingale model' of forecast development, and (c) an extension of the demand process of time correlated and studied specifically for an autoregressive demand model, lead-times effect, and autocorrelation on serial inventory system performance.

Graves and Willems (2003) made a comparison between a safety stock placement model and the approach of stochastic service model. Lee and Billington (1993) developed approximate expressions for the replenishment lead-times of all the locations in the supply chain. A similar model was studied by Ettl, Feigin, Lin, and Yao (2000). The latter discriminates between nominal and real lead-times, where replenishment represents the nominal lead-time plus the extra waiting time by the virtue of stockout at the supplier. By rapprochement, the expected backorder delays determine the base-stock level in order to guarantee the provided service level targets. Glasserman & Tayur (1995) studied the same model with production capacity limits. Their formulation permits a gradient-based search to find the optimal based stock levels. A very asymptotic model with different suppositions concerning the lead-times was considered by Simchi-Levi and Zhao (2005) with outwardly distributed lead-times. They derived frequent equations for waiting times by virtue of backorders and developed approximations to coordinate the base stock levels through the supply chain. Ettl et al., (2000) assumed that the lead-times are identical and independent distribution random variables.

Most literature on installation *(R, Q)* policies highlight approximate and exact cost evaluation of such systems, such as in Svoronos and Zipkin (1988) and Axsater (1993). A general overview of these studies was given by Axsater (2003). Moreover, only a few researchers studied an optimization policy parameter of the distribution systems with installation/echelon *(R, Q)* policies. Early works on approximate optimization were completed by Deuermeyer and Schwarz (1979), Lee and Moinzadeh (1987), Moinzadeh and Lee (1986).

More recently, Axsäter and Rosling (1993) showed that installation inventory and echelon *(R, Q)* policies may outperform each other in different positions for distribution systems. Axsäter and Juntti (1996) analyzed distribution systems with a two-level, stochastic demand by simulation. The outputs showed that echelon *(R, Q)* policies seem to control installation (r, Q) policies for long lead times of the warehouse. On the other hand, the converse is true for short lead-times of the warehouse. Axsäter (2005) introduced a simple method for identifying the cost of backorder to decide its order point in order for the sum of the expected costs to be minimized.

2.8.1 Mathematical approaches

The most common methods used in an inventory system of multi-echelon, according to the literature, are mathematical approaches (Abu Alhaj & Diabat, 2009; Elimam & Dodin, 2013; Hsieh & Chou, 2010; Ignaciuk & Bartoszewicz, 2009; Manna et al., 2007; Torabi & Hassini, 2009). In fact, variables formulation and translation to a mathematical model is a complex step, whose aim is to manage a multi-echelon inventory system in a supply chain. Diks and de Kok (1998) addressed a "divergent multi-echelon", as a system of distribution or a production, and assumed constant lead-time arrival for orders.

Hariga (1998) showed a probabilistic model for a single-period production system that contains many assemblies/processing and adopts facilities in series. Chen (1999), Axsäter and Zhang (1999) and Nozick and Turnquist (2001) assumed that the retailer faces stationary independent Poisson demand. Mitra & Chatterjee (2004b) developed continuous review policies for a multi-echelon inventory problem with stochastic demand for fast moving items from the execution point of the review.

Rau, Wu, and Wee's (2003) model assumed that negligible lead-time, the rate of demand and production is deterministic and fixed, and shortage is not allowed. So and Zheng (2003) used an analytical model to analyze two significant elements that can enhance the high degree of variability of batch quantity experienced by semiconductor manufacturers: lead-time of supplier's and prediction demand updating. They assumed that the two sequential periods of time of retailer demands are correlated (Das & Tyagi, 1999).

As a conclusion, it can be said that most of the literature in mathematical approaches consider a two-, three-, or N-echelon system with probabilistic or deterministic demand. More importantly, they assumed that the lead-time is constant, fixed, deterministic, negligible, ignore, or zero (Axsäter & Marklund, 2008; Axsäter, 1984; Clark & Scarf, 1960; Graves, 1986; Hausman & Erkip, 1994; Hosoda & Disney, 2006; Muckstadt, 1986; Ravichandran, 1995; Saffari & Haji, 2009; Sherbrooke, 1968; Zhao, Zhan, Huo, & Wu, 2006) .

2.8.2 Simulation Approach

Simulation is an abstraction of reality through the input-output relationship based on a simple or complex mathematical expression (Santos & Santos, 2007). Apart of the intricacy, most systems can be shown as a diversion function between input variables and response variables. Simulation models try to build or construct the approximate reality as much as possible and provide analytical tools to study the behavior of a complex system (Axsäter, 2000; Barton, 1992; Jie & Cong, 2009; Kian, Piplani, & Viswanathan, 2003; Liberopoulos & Koukoumialos, 2005; Martel, 2003; Song, Li, & Garcia-Diiaz, 2008; Tee & Rossetti, 2002; Towill, Naim, & Wikner, 1992). One of these complex systems is the multi-echelon inventory system.

It is difficult, if not impossible, to build close-form analytical solutions due to the complexity of a multi-echelon inventory system and potential uncertainty (Song et al., 2008). A simulation model will be massive and hard to understand for a complex system, in addition to the constraints, such as costs, resources, and burdens of the model development (Tee & Rossetti, 2002). Simulation approaches can be divided into two parts in inventory systems, the general simulation approach that can be used by the companies as system software to organize the processes from upstream to downstream. METAMODEL (Barton, 1992; Santos & Santos, 2007; Song et al., 2008) is one of these approaches. METAMODEL is a simulation abstraction to expose the system's input-output relationship according to a simple mathematical expression. Another approach for supply chain warehousing is the discrete event simulation which can be analyzed using Arena software (Altiok & Melamed, 2001). This system is widely used by companies (Jie & Cong, 2009).

The second part of the simulation is designed for the problems under study. In other words, the simulation approaches designed according to the accredited variables in the problems and also by the problem assumptions. In sum, we can say that simulation can be a very powerful tool that helps model the problem that does not really exist to see the effects that occur in the future based on a scientific theory. The most important foundation of simulation is a forecasting theory.

2.8.3 Forecasting Approach

Many studies dealt with demand forecasting for different purposes and in various problem settings. Efforts has been directed to solving a production problem or inventory control (Bradford & Sugrue, 1990; Chang & Fyffe, 1971; Eppen & Iyer, 1997; Fisher & Raman, 1996; Hausman & Peterson, 1972; Hertz & Schaffir, 1960; Weng & Parlar, 1999).

Lau & Hing-Ling Lau (1996) divided the estimation of demand work into two parts or groups. The first type takes data of sales including what is eliminated by stock out and tries to estimate the parameters of demand distribution (mean and standard deviation). The procedure of maximum likelihood can be found in the literature Harter (1967) for mean and standard deviation estimation of several assumed distribution. The second type was for non-stationary demand fluctuation by providing a procedure to update the demand distribution parameters. The second type of problems appears to be the most suitable to the wholesaler or retailer. Bell (1978) achieved methods for magazine distribution to retail outlets and developed forecast demand procedure using an exponential smoothing method to estimate mean and standard deviation of demand.

2.8.4 Other Approaches

In the studies of a multi-echelon inventory system, other approaches were used, such as heuristics (Gallego, Ozer, & Zipkin, 2007; van Houtum, 2006), multi-echelon technique for recoverable item control (METRIC), (Graves, 1985; Sherbrooke, 1968), Vari-METRIC method (Sleptchenko et al., 2002), Markov process (He et al., 2002; Willemain et al., 2004), scenario analysis (Elhasia et al., 2013; Tan & Xu, 2008), statistical analysis, and model predictive control (MPC) (Braun, Rivera, Flores, Carlyle, & Kempf, 2003). These approaches, however, were seldom used and only found in a limited number of studies.

Yoo, Kim, and Rhee (1997) and Abdul-Jalbar, Gutiérrez, and Sicilia (2005) considered multi-echelon inventory heuristics in the supply chain. They considered Raundy procedure and *N log N* heuristic in their study and proposed a fixed rate of customer demand that arrives at each retailer with negligible lead-time. Yoo, Kim, and Rhee (1997) benefited from the heuristic approach in their study and did their experiment with different demand distribution, lead time, and forecast error distribution. Sleptchenko, van der Heijden and van Harten (2002) used the Vari-METRIC approach in a multi-echelon, multi-indenture supply chain system for repairable service parts. They supposed that demand happens through stationary Poisson processes.

Giannoccaro, Pontrandolfo and Scozzi (2003) showed a methodology to determine a supply chain inventory management policy, which depends on the idea of echelon-stock and Fuzzy set method. They assumed that lead-times are constant and deterministic. Kalchschmidt, Zotteri and Verganti (2003) characterized a complete system for a multi-echelon spare parts inventory management in which customers of various sizes place at the same level of supply chain. A solution of an algorithmic was presented according to inventory management and probabilistic forecasting.

Finally, we can say that the studies that adopted "other approaches" method obtained approximation solution with the assumption that demand are fixed, probabilistic, fuzzy and deterministic and lead-time is mostly fixed.

2.9 Discussion

In this Chapter, some crucial findings related to a multi-echelon inventory system which would be helpful in achieving the primary goal of this research were recapped. The methods mostly used in a multi-echelon inventory system were a mathematical method to obtain exact solution because they involve two essential elements, demand process, and lead-time process assumptions. Demand process was mostly assumed to be deterministic or probabilistic. In contrast, lead-time was mostly assumed to be deterministic, fixed, zero or ignored. The behavior of these two variables plays an essential role in drawing the inventory policy, which means that when the lead-time is deterministic, fixed, zero or ignored and the demand is even deterministic or probabilistic, an exact solution is reached. On the other hand, when the demand process and the lead-time process have a separate probability distribution function, the optimality for multi-echelon inventory system is still unknown.

The different of this research with the previous studies, the previous studies probability of demand and lead-time were taken separately without extracting and establishing the probability distribution of demand during lead-time. They also do not reformulate the inventory function of total cost based on the probability distribution of the demand during lead-time to establish and develop the inventory performance measures. They used only the mean and standard deviation of the demand and lead-time separately based on the distributions parameters instead of demand during lead-time probability distribution to achieve the objectives of the studies.

This research will establish the demand during lead-time probability distribution function using simulation procedures. In this case, an approximation mathematical method is the best solution to solve these types of problems based on this demand during lead-time.

CHAPTER THREE

THEORIES AND CONCEPTS IN A MULTI-ECHELON INVENTORY SYSTEM

This chapter focuses on the underlying concepts and methods that are the bases for the development of a multi-echelon inventory model, specifically on the continuous review system *(R, Q)* policy. The discussion is classified into two parts. First, the methods that are used to determine the demand during lead-time probability distribution function are discussed. The methods are forecasting technique for the demand data as well as the probability distribution function of the lead-time, and the simulation model. Second, the measures of a multi-echelon inventory system that are being studied in this research are presented.

3.1 Elements in forecasting technique

Forecasting estimates the values of the variables for cases that do not fall within the available observation units. Forecasting is not only intuitive or conjectures; but also the statistical treatment of past data in order to give any estimation of the variables stated in the future (Hausman & Peterson, 1972; Hertz & Schaffir, 1960). A predictive study may indicate, for example, a significant unexpected rise in consumption, which means the possibility of increasing the size of the stock, leading to a rise in items or products' inventory to face an increase in consumption, or indicate the possibility of an economic crisis occurring (French, 1986; Rentz & Reynolds, 1991). Therefore, it is necessary to follow the procedures and policies to avert a crisis and ward off risks. However, the interest in forecasting inventory policies is due to change in requirements, length of lead-time, and sudden changes in the volume of consumption (Syntetos, Boylan, & Disney, 2009).

1. A change in requirements is when the variable rate is not revolving around a constant average. Otherwise, it can extract the reserve to face this fluctuation.
2. The length of waiting time, i.e., the required period to access the orders and the possibility of achieving a balance between consumption and compensation. When the orders take a long time to arrive, changes in consumption may occur. This means that if there are no forecasts of consumption during this period, the balance will not be achieved by compensation and consumption.
3. The sudden changes in the volume of consumption, which are mostly due to unusual circumstances, allow exact estimates. The changes could be a sudden increase in income or similar or more provision of services.

In most literature, forecasting estimates the model parameters or determines the demand probability distribution (Baykal-Gurosy & Erkip, 2010; Choi, Chiu, & Fu, 2011; Wang, 2009; Wang & Lin, 2010). The most often forecasting method used to estimate the means and the standard deviation is exponential smoothing methods (Snyder et al., 2004). These two parameters, means and standard deviation, depend on the effects of data trends and seasonal trends. That is, fluctuations in behaviors of demand data can be stationary over time, increase trends over time, decrease trends over time, or have seasonal trends. Usually, a long period of data has a stationary fluctuation. Therefore, the exponential smoothing method is the best method for estimation (Brown, 1959).

Wang (2009) used the third order exponential smoothing forecast to reduce the effect of bullwhip and costs in SC. A new technique for forecasting inventory policy was presented by using multi-regression based forecasting models for predicting for a supplier the total profit in a two-echelon SC. The assumed model was built by weighing the elements method and transformation of data, conferring a higher predictive precision than traditional regression models (Wang et al., 2010).

3.2 Exponential smoothing method

The exponential smoothing method (ESM) is the mostly used forecasting technique in the field of inventory control (Baykal-Gurosy & Erkip, 2010; Wang et al., 2010). This method is used because; it is simple of calculation, sensitive to the variables at any time and no need to store a large amount of information.

Statistically, in order to forecast a variable based on the patterns, two parts are involved. First, an inevitable uniform variable without fluctuations can be expressed in an equation. Second, a stochastic variable has a certain distribution, with a mean of zero and standard deviation equal to σ_{ε}^{2}. If the first part denotes to be U_t, and the second part ε_t, then the expression becomes:

$$X_t = U_t + \varepsilon_t \, , t = 1,2,3, \quad . \; . \; . \qquad (3.1)$$

If U_t reflects the fixed amount ($U_t = \alpha$), this means that the value of the variable consists of a constant plus random variable.

$$X_t = \alpha + \varepsilon_t \qquad (3.2)$$

Brown (1959) was the first who presented the ESM technique of series values of a variable with a fixed average fluctuation.

$$F_{t+1} = \alpha\, X_t + (1 - \alpha)F_t \tag{3.3}$$

where,

F_{t+1}: the forecast observation value for the next period.

X_t : current observation value.

F_t : the forecasted observation value for the current period.

α: constant smoothing, ranging between 0 and 1, $0 \leq \alpha \leq 1$.

This method is used when the data is stationary, and when there is no seasonal or periodic pattern. From Equation (3.3), we can note that the new forecasting value of F_{t+1} is subject to:

1. the current observation with a weighed α.
2. the current period predicted with a weighed (1- α).

This method is called exponential smoothing, ES because the meaning of F_t is clearer after degenerating to its compounds as seen in Equation (3.3).

$$F_t = \alpha X_{t-1} + (1 - \alpha)F_{t-1} \tag{3.4}$$

$$F_{t+1} = \alpha X_t + (1 - \alpha)\{\alpha X_{t-1} + (1 - \alpha)F_{t-1}\} \tag{3.5}$$

$$F_{t+1} = \alpha X_t + \alpha(1 - \alpha)X_{t-1} + (1 - \alpha)^2 F_{t-1} \tag{3.6}$$

Similarly, we degenerate F_{t-1} to its compounds.

$$F_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + (1-\alpha)^2 X_{t-2}$$
$$+ (1-\alpha)^3 X_{t-3} + \ldots + (1-\alpha)^{N-1} X_{t-(N-1)} \quad (3.7)$$

Note from Equation (3.7) that the effect of the previous observation was less exponential with the time. That is, the given weighting for each observation from the last observation would be less exponential.

The problem now is how to calculate the first forecasted value, F_1. This is because, in Equation (3.3), the value of F_{t+1} depends on X_t and F_t. For example, $F_2 = \alpha X_1 + (1-\alpha) F_1$. In order to calculate the value of F_1, there is more than a direction or idea (Bates and Granger, 1969; Ghafour, 2007).

1. The initial value of F_1 is the average of real observation or historical data of X_t.

   $$F_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (3.8)$$

   This technique is used in situations when exponential smoothing needs to be done quickly and not used for future research.

2. The initial value of F_1 is the same value for the real observation, X_1. This technique is used when the data is not adequate and convergent.

   $$F_1 = X_1 \quad (3.9)$$

3. The initial value of F_1 is the average of the first quarter of the real observation, X_t.

$$F_1 = \frac{1}{\left(\frac{n}{4}\right)} \sum_{i=1}^{Int\left(\frac{n}{4}\right)} X_i \tag{3.10}$$

Therefore, this research adopts Equation (3.10) to find the first forecasted value, F_1 because this equation used for long-term of future studies (Koehler, Snyder, Ord & Beaumont, 2012; Snyder et al., 2004).

3.2.1 Smoothing constant

A smoothing constant is a number used in the exponential smoothing technique to give the most recent period a different weight than the previous periods (Hillier & Lieberman, 2010). The value of a smoothing constant has an effective impact to successfully forecast a model (Snyder et al., 2004). The value of a smoothing constant, α which ranges between 0 and 1, $0 \leq \alpha \leq 1$ depends on the amount of weight that would give to a new value. The highest value maximizes the weight, which is a reference to Equation (3.3). If α = 1, that we give all weight to the last value and ignore the old average. If α = 0.5, it indicates great importance to the last real consumption. If α = 0, the last value is ignored (Chen, Ryan, & Simchi-Levi, 2000).

As a conclusion, a high smoothing constant leads to the forecasting model having a fast response but is non-stationary. Otherwise, the low smoothing constant leads to a slow response, but a stationary forecasting model. In order to determine the smoothing constant value, we can use forecast error to be a measurement to choose the smoothing constant. The forecast error has a zero average and standard deviation σ_e.

$$e_t = X_t - F_t \tag{3.11}$$

The forecast error can be negative or positive. So, in order to solve this problem, we take the square value of forecast error. For the purpose of obtaining a uniform scale of other data, 'mean sum square error' is calculated. Since its unit measurement is the square units of original values, it is possible to calculate a new measure by finding the square root of Mean Square Error (MSE) (Forbes, Evans, Hastings, & Peacock, 2011; Hosoda & Disney, 2006).

$$MSE = \sqrt{\frac{1}{n}\sum e_i^2} \tag{3.12}$$

Based on Equation (3.12), the smoothing constant is altered within a reasonable range. Then, the constant that leads to less MSE is chosen.

3.2.2 Forecasting error

A forecasting error is a difference between the real value and the predicted value of a time series phenomenon (Croux, Gelper, & Mahieu, 2010). Irrespective of which method is used to get the forecasting value of *X*, for a single period or more periods, the real value will remain different from the predicted value (Liang & Huang, 2006).

In order to make an effort to estimate this error, forecasting error is used, where forecasting error is the difference between the real value X_t and the forecasted value F_t as shown in Equation (3.11), (Good & Hardin, 2006). In order to ensure that the predictive values is equal to the real values, the differences between these two values must be equal to zero (Baykal-Gurosy & Erkip, 2010). But the reality is not like this because the value of e_t is unknown unless the value of F_t is already known. It was found that the forecasting error normally distributes (Brown, 1959). In order to determine the parameters of this distribution, the mean and variance must be known, and, the standard deviation, too. The variance error for the forecasting model exponential smoothing (ES) (Bates & Granger, 1969) is

$$\sigma_e^2 = \frac{2}{2-\sigma}\sigma_\varepsilon^2 \tag{3.13}$$

where, σ_ε^2 is the variance of the random variable. In order to calculate the variance error (σ_e^2) from Equation (3.13), it must be specified that σ_ε^2 of the latter is also unknown. Therefore, we use the numerical relationship between mean absolute deviation (MAD) and standard deviation in order to estimate the value of σ_ε^2 (Hertz & Schaffir, 1960; Snyder et al., 2004).The original reason that MAD is estimated instead of σ or σ^2 was that this simplified the computations. There is no problem to estimate σor σ^2 directly, but still most forecasting systems first evaluate MAD. It is obvious that MAD and σ in most cases give a very similar picture of the variations around the mean. It is also possible to relate them to each other. A common assumption is that the forecast errors are normally distributed (Axsater, 2006). In that case it is easy to show that

$$\sigma_t = \sqrt{\pi/2}\ MAD_t = 1.25 MAD_t \tag{3.14}$$

where, $\sqrt{\pi/2} \approx 1.25$, therefore

MAD = sum of absolute error/number of error (3.15)

Most facilities, manufacturers, and stakeholders prefer to use mean absolute deviation to obtain the standard deviation (Forbes et al., 2011; Huang & Huang, 2006; Schwarz et al., 1972) for the following reasons:

1. It is a simple method to calculate.
2. The mean absolute deviation can fit the exponential smoothing method because it can help obtain the estimate of mean absolute deviation by using an exponential smoothing method.

$$MAD_{t+1} = \alpha|e_t| + (1-\alpha)MAD_t \tag{3.16}$$

3. It can calculate mean absolute deviation, which is the forecasting error estimating for the next period.
4. It can calculate mean absolute deviation, which is the average amount of stockout, without using the safety stock.
5. It can calculate mean absolute deviation is the median value of convergence or divergence, when using a suitable forecasting model.

From Equation (3.16), the problem is how to find the initial value of MAD. In order to determine the initial value of MAD_1, the same steps for finding F_1 as shown in Equation (3.10) are used. However, here we take the absolute error.

3.3 Lead-time distribution

Lead time is the amount of time that elapses between when a process starts and when it is completed and these times follow a statistical probability distribution (Wild, 2007). A statistical distribution has an instrumental used to draw the behaviors of the data. During these behaviors, distribution parameters that reflect the studied phenomenon can be found (Noufaily, Jones, & Mk, 2013). However, there are two statistical ways to generalize any set of data that represents a certain phenomenon, i.e., the draw method and statistical analysis (Forbes et al., 2011).

The statistical analysis includes several tests to find the probability distribution of the data. They are Kolmogorov-Smirnov *(K-S)* test, Anderson-Darling test, and Chi-Squared test (Balakrishnan & Nevzorov, 2003; Zong, 2011). These tests depend on the goodness of fit hypothesis. The goodness of fit of a statistical model portrays how well it fits with an observation's set. The indices of the goodness of fit summarize the contrast between the observed values and the values expected under a statistical analysis. The statistics of the goodness of fit are the indices of the goodness of fit with known sampling distributions, normally acquired using asymptotic methods that are used in statistical hypothesis testing (Maydeu-Olivares and Garcia-Forero (2010).

A goodness of fit (GOF) can be defined as the extent to which the observed data matches the values expected by theory (Maydeu-Olivares & Garcia-Forero, 2010). However, a GOF test; normally includes inspecting a random sample from some distributions that are unknown in order to test the null hypothesis for determine the distributions function which indeed is known (Ghosh, Delampady, & Tapas, 2006). Usually, a Kolmogorov-Smirnov *(*K-S*)* test is used to check the assumption of normality in variance analysis.

Let X be a continuous random variable drawn from some populations and is compared with $F^{*}(x)$ in some statistical ways to see in the event that it is sensible to say that $F^{*}(x)$ is the correct distribution function of the random variable (Zong, 2011). There is one sensible method of comparing the random variable with $F^{*}(x)$, which is by the mean of the empirical distribution function, $S(x)$ (Noufaily et al., 2013).

Definition of hypothesis: Let $X = (X_1, X_2, . . ., X_n)$ be a random variable. The empirical distribution function $S(x)$ is a function of *x,* which is the equivalent to the fraction of X_{iS}, which is equal to or less than *x* for each *X*, $-\infty < X < \infty$ (Boddewyn & Brewer, 2014; Zong, 2011). Therefore,

$$S(x) = \frac{\sum_{i=1}^{n} I_{\{x_i \leq x\}}}{n}$$

S(x) is useful to be an estimator of *F(x)*. X_{iS} unknown distribution function. In order to see if there is a good agreement between empirical distribution function $S(x)$ and the correct distribution function, $F^{*}(x)$ as a comparison needs $S(x)$ with the hypothesized distribution function of $F^{*}(x)$. One of the simplest measures is the biggest distance between the *S(x)* functions and $F^{*}(x)$ functions, measured in a vertical direction (Maydeu-Olivares & Garcia-Forero, 2010; Stephens, 1992).

If $F(x)$ is the data that includes a random variable $X_1, X_2, . . ., X_n$ accompanied with some unknown distribution functions, and $S(x)$ was based on the random variable $X_1, X_2, . . ., X_n$, then $F^{*}(x)$ can be a totally specified hypothesised distribution function (Ghosh et al., 2006). Suppose T has a statistic test to be the greatest and is symbolized by 'sup' for the vertical distance between $S(x)$ and $F^*(x)$. Mathematically, it is stated that

$$T = sup_x |\, F^*(x) - S(x)|$$

Therefore, the hypothesized distribution function test will be as follows:

$$H_0: F(x) = F^*(x) \text{ for } \forall\, x\ (-\infty < x < \infty)$$

$$H_1: F(x) \neq F^*(x) \text{ for at least one value of } x$$

If T overrides the quantile of $(1 - \alpha)$ as given by the statistical table, then we reject H_0 at the level of significance α. The approximate *p*-value can be obtained through interpolation in the statistical table (Conover, 1999). In another meaning, H_0: the data follow the specified distribution; H_1: the data do not follow the specified distribution. The hypothesis regarding the distributional form is rejected at the chosen significance level α if the test Kolmogorov-Smirnov (K-S) is greater than the critical value obtained from a table (Balakrishnan & Nevzorov, 2003).

The fixed values of α (0.01, 0.02, 0.05 etc.) are generally used to evaluate the null hypothesis (H_0) at various significance levels. A value of 0.05 is typically used for most applications, however, in some critical industries; a lower value may be applied (Zong, 2011).

The standard tables of critical values used for this test are only valid when testing whether a data set is from a completely specified distribution. If one or more distribution parameters are estimated, the results will be conservative: the actual significance level will be smaller than that given by the standard tables and the probability that the fit will be rejected in error will be lower (Good & Hardin, 2006).

3.4 Simulation technique

Simulation is a reflection of the truth through the information, yielding a relationship that focuses around basic or complex numerical outflow Santos and Santos (2007). Separate from its many-sided nature; most frameworks can be demonstrated to have a redirection capacity between information variables and reaction variables (Altiok & Melamed, 2001).

A simulation model attempts to construct or build an inexact reality. However, only a limited amount could reasonably be expected and give expository instruments to study the conduct of a complex framework (Jie & Cong, 2009).

Towill (1991) and Towill, Naim, and Wikner (1992) used simulation to assess the impacts of different inventory network systems on interest intensification. The techniques examined were as follows:

- Eliminating the distribution stage of SC, by embedding the function of the distribution in the assembling stage.
- Incorporating the stream of data throughout the chain.
- Actualizing a Just-In-Time (JIT) stock approach to lessen time delays.
- Enhancing the development of intermediate items and materials by altering the requested amounts strategies.
- Altering the parameters of the current order quantity strategies.

The aim of a simulation model is to figure out which methodologies are the most compelling in smoothing the varieties in the interest example demand pattern (Elhasia et al., 2013; van der Vorst et al., 2000). As explained earlier in Chapter Two, there are two types of simulation approaches in SC, general package and specific or special package in which the design depends on the study (Barton, 1992; Song et al., 2008).

3.5 Probability of demand during lead-time

Demand during lead-time is the joint distribution of a demand distribution and a lead-time distribution, which depends on the parameters (mean and standard deviation) for each (Bagchi & Hayya, 1984). The question that arises is why the demand during lead-time is critical in inventory control? The answer to this question is that when items, goods, or products are near completion, a decision maker starts to make a request for an order quantity to meet the needs of consumers and not to fall into shortage (Bookbinder and Cakanyildrim, 1999; Ravichandran, 1995). During this period, and until the required quantity arrives at the depot, customer demand is continuous, and since the processes are very nested, it is difficult to record these data on demand until the items or the products reach the place (Funaki, 2012). If the items arrive late to the warehouse, the warehouse covers or satisfies the customer demands or the markets' demands from its safety stock (inventory on hand) (Jung, Blau, Pekny, Reklaitis, & Eversdyk, 2008; Tallon, 1993). Here, the idea is the demand is probabilistic and the lead-time of the items until placed in the warehouse is also probabilistic. Therefore, we need to know the probability distribution function of demand during lead-time in order to deal with it according to the requirements of the concerned party. A mathematical approach to finding the mean and standard deviation regardless of the distribution of demand during lead-time can be calculated from the following two equations (Fishman, 1973).

$$\mu_L = E(X)E(L) \tag{3.17}$$

$$\sigma_L = \sqrt{E(X)Var(X) + [E(X)]^2 Var(L)} \tag{3.18}$$

where, *E(X)* is the expected demand, *E(L)* is the expected lead-time , *Var(X)* is the variance of demand and *Var(L)* is the variance of lead-time, μ_L, σ_L are the mean and standard deviation of demand during lead-time, respectively. However, these two equations give only the value of mean and standard deviation without knowing the probability distribution function *(pdf)*, which is of greatest importance (Fishman, 1973; Lee, 2005).

3.6 Generalized Gamma distribution

The generalized gamma distribution is a continuous probability distribution with three parameters. It is a generalization of the two-parameter gamma distribution (Stacy & Mihram, 1965). One specific well-known model is Gamma distribution, $G(\alpha, \beta)$ (Khodabin & Ahmadabadi, 2010). This distribution may help explain a wider variety of phenomenon, especially in the area of queuing theory, reliability, inventory system, etc. (Axsater, 2010). In statistics, especially in probabilistic theory, gamma distribution, $G(\alpha, \beta)$, is a family of a continuous probability distribution with two parameters, the shape, α and the scale parameter, β (Stacy, 1962). The probability density function *p.d.f* of $G(\alpha, \beta)$ distribution is:

$$G(\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad \text{for } x \geq 0 \text{ and } \alpha, \beta > 0 \tag{3.19}$$

Where,

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} \, dx \tag{3.20}$$

The generalized gamma distribution, *GG(α, β, k)* three parameters is the general form of gamma distribution, *G(α, β)* which was introduced by Stacy (1962) and Stacy and Mihram (1965). The probability density function, *p.d.f* of *GG(α, β, k)* is

$$GG(\alpha, \beta, k) = \frac{k x^{k\alpha-1}}{\beta^{k\alpha} \Gamma(\alpha)} e^{-\left(\frac{x}{\beta}\right)^{k}} \quad x \geq 0 \text{ and } \alpha, \beta, k > 0 \tag{3.21}$$

where α and k are the shape parameters, β is the scale parameter, and

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} \, \mathrm{dx} \tag{3.22}$$

GG(α, β, k) is a desirable distribution because of its properties. It involves the exponential distribution, the Weibull distribution, the lognormal distribution, the Maxwell-Boltzmann distribution, and the Chi-square distribution, which are the special cases of generalized gamma distribution subfamilies with some conditions in the general form of GG distribution (Hiros, 2000; Khodabin & Ahmadabadi, 2010; Mihe, 2010; Stacy & Mihram, 1965).

The generalized gamma distribution, *GG(α, β, k, γ)* four parameters is the general form of generalized gamma three parameters distribution.

Let X be a random variable and follow a generalized gamma distribution with four parameters (α, β, k, and γ). where α and k are the shape parameters, β is the scale parameter and γ is the location parameter. The probability density function, *p.d.f*, was given by (Stacy & Mihram, 1965):

$$GG(\alpha,\beta,k,\gamma) = f(x) =$$

$$\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}\,\Gamma(\alpha)} e^{-\left(\frac{(x-\gamma)}{\beta}\right)^{k\alpha}} \quad x \geq 0 \text{ and } \quad \alpha,\beta,k,\gamma > 0 \tag{3.23}$$

And the cumulative distribution function, *c.d.f* is:

$$GG(x;\alpha,\beta,k,\gamma) = F(X) = \frac{\Gamma(x-\gamma)/\beta^{k\alpha}}{\Gamma(\alpha)} \quad x \geq 0 \text{ and } \alpha,\beta,k,\gamma > 0 \tag{3.24}$$

The domain of x $is\, \gamma \leq x < \infty$. If the value of $\gamma \rightarrow 0$, it gives $GG(\alpha, \beta, k)$ three parameters.

Many studies examined how to find the mean and standard deviation of GG by using a Maximum Likelihood Estimation (MLE) (Hiros, 2000; Huang & Huang, 2006; Khodabin & Ahmadabadi, 2010; Mihe, 2010; Noufaily, Jones, & Mk, 2013). The mean and the variance of $GG(\alpha, \beta, k)$ can be defined as follows:

$$E(x) = \frac{k\,\Gamma\left(\frac{\alpha+1}{\beta}\right)}{\Gamma(\alpha)} \tag{3.25}$$

$$Var(x) = k^2 \left(\frac{\Gamma(\alpha + \frac{2}{\beta})}{\Gamma(\alpha)} - \left(\frac{k\Gamma(\alpha + \frac{1}{\beta})}{\Gamma(\alpha)} \right)^2 \right) \tag{3.26}$$

The estimation of the mean and the variance of GG, especially for four parameters, is very complex and subject to probabilistic theory. However, in Forbes, Evans, Hastings, and Peacok (2011), the expression of mean and the variance of GG four parameters are solved as follows:

$$E(x) = \frac{\gamma + k\, \Gamma\left(\frac{\alpha + 1}{\beta}\right)}{\Gamma(\alpha)} \tag{3.27}$$

$$Var(x) = k^2 \left(\frac{\Gamma\left(\frac{\alpha + \frac{2}{\beta}}{\gamma}\right)}{\Gamma(\alpha)} - \left(\frac{k\Gamma\left(\frac{\alpha + \frac{1}{\beta}}{\gamma}\right)}{\Gamma(\alpha)} \right)^2 \right) \tag{3.28}$$

This research dose not estimate the mean and standard deviation of *GG(α, β, k, γ)* four-parameter. These two measures are used in this research to be a sub-process in order to reach the objectives. However, any of the statistical package, e.g., (IBM, 2011; Mathwave, 2015; StatSoft.Inc, 2007) gives the value of the mean and the standard deviation depending on the value of α, β, k and γ the two shape, scale and location parameters, respectively, as shown in Equation (3.23).

3.7 Continuous Review *(R, Q)* policy

The first mathematical model of a multi-echelon inventory system adopts each echelon separately to determine the purchasing quantities, which lead to minimizing the value of total cost based on a periodic review system, which was developed by Clark & Scarf (1960).

$$F(x) = \begin{cases} hx + p \int_0^{\infty} (t - x) f(t) dt & x > 0 \\ p \int_0^{\infty} (t - x)\ f(t) dt & x \leq 0 \end{cases} \tag{3.29}$$

where *x* is the 'stock on hand' at the starting of the period, *p* is the shortage cost. However, when the shortage cost is not allowed in the total cost function of the inventory model, it can compensate the holding cost instead of the shortage cost. Therefore, *h* and *A* are the marginal holding and setup costs, respectively. Equation (3.29) will be:

$$F(x) = \begin{cases} hx + A \int_0^{\infty} (t - x) f(t) dt & x > 0 \\ A \int_0^{\infty} (t - x)\ f(t) dt & x \leq 0 \end{cases} \tag{3.30}$$

The model by Clark and Scarf (1960) was adopted and extended by many researchers. However, an extension of the model by Clark and Scarf (1960) was done by Schmidt and Nahmias (1985), who characterized a simple assembly system of an optimal inventory system. Federgruen and Zipkin (1984c) extended Clark and Scarf's model to the infinite horizon and showed a new computational model.

3.8 Order quantity policy in the multi-echelon system

In order to determine the order quantity for a multi-echelon inventory system, it was observed that the order quantity Q for a multi-echelon inventory system is not optimal to deal with each stage individually (Schwarz, Frederick, Gerald, & Hamdy, 1972). The choice of Q at a certain stage will affect the demand structure primarily at the next upstream stage (Gümüs & Güneri, 2007). This accreditation makes the Q determination more complex even though it is not optimal to determine Q for each stage individually. This procedure of order quantity is very common in practice because it is very easy and leads to the ordering of very small quantities (Chang & Chun-Tao, 2004). When dealing with order quantity even in single-echelon or multi-echelon, most of the literatures, e.g., Clark and Scarf (1960); Graves (1986); Hausman and Erkip (1994); Hosoda and Disney (2006); Sana and Chaudhuri (2008) Marc and Graves, (1985) and Hayya et al. (2009) assume that the customer demand is known. Furthermore, previous studies made a standard assumption that all lead-times are constant, when, in fact, they are equal to zero (Graves, 1985; Pal et al., 2012b). In the case of a probabilistic demand, it is normally reasonable to replace the probability demand by its mean (the mean of demand probability distribution function) and use a deterministic model when determining order quantity (Axsater, 2006). However, when the demand and the lead-time are subject to probability distribution functions separately, the procedure will be much more difficult.

Determining the order quantity Q will be as follows: when the inventory level, IL, reaches the reorder point, R. where, R is the point that indicates when to order an order quantity to promote the inventory. The aim of this procedure is to find the optimal values of Q and R, which, in turn, minimize or achieve the expected total inventory cost in the unit time De Bodt and Graves (1985).

$$F(x) = \int_0^\infty r(\mu_L/t) f(t)\, dt \tag{3.31}$$

where, $F(x)$ is the probability function of demand during lead-time, $r(\mu_L/t)$ is a conditional probability distribution function of demand during lead-time and $f(t)$ is a probability distribution function of lead-time.

To apply the equation of total inventory cost, we need the average of inventory and average of shortage for each inventory cycle (Axsäter, 2011). The inventory cost depends on the pure inventory quantity located in the store at the beginning and end of the inventory cycle (Chiu, Ting, & Chiu, 2005), where the inventory level at the end of the inventory cycle, $Cycle(IL_{end})$ is the expected value of reorder point minus the demand during lead-time.

$$Cycle(IL_{end}) = E(R - D_L) \tag{3.32}$$

and the inventory level at the beginning of the inventory cycle, $Cycle(IL_{beg})$ is

$$Cycle(IL_{beg}) = Q + E(R - D_L) \tag{3.33}$$

Therefore, the inventory average, $\overline{H}$ for each cycle is

$$\overline{H} = \frac{[Q + E(R - D_L) + E(R - D_L]}{2} \tag{3.34}$$

$$\overline{H} = \frac{Q}{2} + E(R - D_L) \tag{3.35}$$

Equation (3.35) is based on the average of the beginning and ending expected in cycle inventories of cycle $Q + E(R - D_L)$, respectively. As approximation, the exression ignores the case where $E(R - D_L)$ may be negative (Hamdy, 2007). Therefore,

$$E(R - D_L) = \int_0^{\infty} (R - D_L)\, f(x)dx \;\; = R - E(D_L)$$

$$\therefore\; R - E(D_L) = R - \;\mu_L \tag{3.36}$$

where, μ_L is the mean of demand during lead-time. Accordingly, the total cost function, *C(R, Q)* = setup cost per order + holding cost per unit time + shortage cost per unit time. Since shortage is not allowed in this research, we delete this part from the total cost function so that it becomes normal (Hadley & Whitin, 1975).

$$C(R, Q) = A\frac{D_L}{Q} + h\left(\frac{Qb}{2} + \int_0^{\infty} (R - D_L)\, f(x)dx\right) \tag{3.37}$$

where, $b = 1 - \frac{D_L}{\alpha_1}$ and α_1 is the production rate, then by substituting Equation (3.36) into Equation (3.37), we obtain

$$C(R, Q) = A\frac{D_L}{Q} + h\left(\frac{Qb}{2} + R - \mu_L\right) \tag{3.38}$$

Meanwhile, the total cost function, $C(R, Q)$ for the whole system can be extract from the following equation (Frederick & Gerald, 2001; Hamdy, 2007).

$$C(R,Q) = A\frac{D_L}{Q} + hb\left(\frac{Q}{2}\right) \tag{3.39}$$

Now we derive the corresponding relationship for demand distributed normal. When working with a normal distribution of demand, we also propose that the continuous inventory position, *IP*, is distributed uniformly on the interval $(R, R+Q)$. This is a very rigorous rounding on condition that the potential of negative demand can be ignored (Axsäter, 2011). From this assumption, the mean of demand during lead-time, μ_L and standard deviation of demand during lead-time, σ_L established and extracted according to a simulation procedures (Lee, 2005; Snyder et al., 2004). These procedures of simulation will be discussed in Chapter Four section (4.5) later. Therefore, these two variables are novel in the model to reach the research questions.

However, to determine the distribution function of the inventory level, an arbitrary time (t) when the system is in the steady state, assume the inventory position at the time t is $IP(t)$, then consider the $t+L$ and the inventory level, $IL(t+L)$. Everything was on order at time t has been delivered at time $t+L$. Orders that have been triggered between t and $t+L$ have not reached the inventory at time $t+L$ (Axsater, 2006). Accordingly,

$$IL(t+L) = IP(t) - D(t, t+L) \tag{3.40}$$

$D(t, t+\tau) = D(\tau)$ = stochastic demand in the interval $(t, t+\tau]$.

where the demand during lead-time, *D(t, t+L)* is independent of the *IP(t)*. Ignoring the probability of negative demand, the *IP* in the steady state is uniformly distributed on the interval *(R, R+Q)*. The demand during lead-time is assumed to be normally distributed. Consequently, the inventory level, *IL* and given the inventory position (*u)* at time *(t),* the interval level of time *t+L* is less than or equal to *x.* If demand during lead-time is at least *u-x*, and assume *f(x)* and *F(x)* denote the density and the distribution function of the inventory level, *IL* in the steady state, then,

$$F(x) = P(IL \leq x) = \frac{1}{Q}\int_{R}^{R+Q} \left[1 - \Phi(\frac{u - x - \mu_L}{\sigma_L})\right] du \tag{3.41}$$

where , $\Phi(x)$ is the distribution function of the standardized normal distribution with mean equal to 0 and standard deviation equal to 1 (Good & Hardin, 2006). The loss function *G(x)* of the normal distribution is introduced as follows:

$$G(x) = \int_{x}^{\infty} (v - x)\varphi(v)dv = \varphi(x) - x(1 - \Phi(x)) \tag{3.42}$$

where, v is the differentiable functions and $\varphi(x)$ is the density of standardized normal distribution. Note that,

$$G'(x) = \Phi(x) - 1 \tag{3.43}$$

which means that $G'(x)$ is increasing and negative. Accordingly, *G(x)* is convex and decreasing. By using Equation (3.43), we can reformulate Equation (3.41).

$$F(x) = \frac{1}{Q} \int_{R}^{R+Q} \left[-G' \left(\frac{u - x - \mu_L}{\sigma_L} \right) \right] du$$

$$= \frac{\sigma_L}{Q} \left[G\left(\frac{R - x - \mu_L}{\sigma_L} \right) - G\left(\frac{R + Q - x - \mu_L}{\sigma_L} \right) \right] \qquad (3.44)$$

To verify that *G(x)* is decreasing and non-negative, $G(x) \to 0$ as $x \to \infty$ and $G(-x) = G(x) + x$, i.e., $G(x) \to -x$ as $x \to -\infty$. Using that $G(x) \to 0$ as $x \to \infty$, we have

$$\int_{-\infty}^{0} F(x)dx = \int_{-\infty}^{0} \left(\frac{1}{Q} \int_{R}^{R+Q} \left[-G' \left(\frac{u - x - \mu_L}{\sigma_L} \right) \right] du \right) dx$$

$$= \frac{1}{Q} \int_{R}^{R+Q} \left(\int_{-\infty}^{0} \left[-G' \left(\frac{u - x - \mu_L}{\sigma_L} \right) \right] dx \right) du$$

$$\therefore \int_{-\infty}^{0} F(x)dx = \frac{\sigma_L}{Q} \int_{R}^{R+Q} G\left(\frac{u - \mu_L}{\sigma_L} \right) du \qquad (3.45)$$

From Equation (3.44) we can obtain the density function *f(x)* as

$$f(x) = \frac{1}{Q} \int_{R}^{R+Q} \frac{1}{\sigma_L} \varphi\left(\frac{u - x - \mu_L}{\sigma_L} \right) du$$

$$= \frac{(\frac{1}{\sigma_L})}{Q} \left[\Phi\left(\frac{R + Q - x - \mu_L}{\sigma_L} \right) - \Phi\left(\frac{R - x - \mu_L}{\sigma_L} \right) \right] \qquad (3.46)$$

while, $G'(x) = \Phi(x) - 1$ from Equation (3.43), by replacing $G'(x)$ From Equation (3.44) into Equation (3.46) and assume $Q \to 0$, we obtain

$$F(x) = G'\left(\frac{R - x - \mu_L}{\sigma_L}\right) = 1 - \Phi\left(\frac{R - x - \mu_L}{\sigma_L}\right)$$

$$= \Phi\left(\frac{x - (R - \mu_L)}{\sigma_L}\right) \qquad (3.47)$$

$$f(x) = \frac{1}{\sigma_L}\,\varphi\left(\frac{x - (R - \mu_L)}{\sigma_L}\right) \qquad (3.48)$$

when $Q \rightarrow 0$, this simply means that when applying an *S* policy to $S = R$, *IP* is kept at *R* all the time. The inventory level distribution is then normal distribution (Axsater, 2006; Matheus & Gelders, 2000).

3.9 Service Level

In order to identify an appropriate reorder point or safety stock, the reorder point can be based on safety stock whether a planned service constraint or has a certain cost of shortage. Practically, it is often easier to identify a service level (Axsater, 2006b). Therefore, the service level is identified as:

SL_1 = probability of no stockout per cycle order.

SL_2 = fill rate portion of demand that can be satisfied instantly from safety stock (stock on hand).

SL_3 = ready rate portion of time with non-negative safety stock.

SL_1 definition can be seen as the probability that an order arrives on time, i.e., before the safety stock is finished. SL_1 is very easy to use; however, it also has some important defects (Tarim & Kingsman, 2004). The problem is that SL_1 does not take the order quantity into consideration. If the order size is big and meets the demand during a long time, it is not important how much of SL_1 is low (Axsater, 2006).

In most cases, there is still plenty of safety stock due to a large order quantity. If the order quantity is small, the real service can similarly be low even if SL_1 is high (Axsäter, 1984; Tyagi & Das, 1998). As a result, SL_1 cannot be recommended for inventory control in practice. The fill rate and ready rate make the determination of the corresponding reorder points to be something more complex (Osman & Demirli, 2012; Schwarz et al., 1985).

SL_2 and SL_3 are equivalent in continuous demand. This is not the situation even though a retailer or customer may order several quantities at the same time. Even if the safety stock is non-negative, the inventory may not be enough to cover a large number of retailers or customers' order (Abu Alhaj & Diabat, 2009). If the inventory includes a small number of orders in hand, the ready rate still increases most of the time and the fill rate may still or may be very low if there are a large number of retailers and customers who order large quantities.

3.10 Echelons and installations reorder point, *R*

The differences between echelon inventory and installation inventory depend on the type of inventory system, whether it is serial or distribution as shown in Figures 2.1 and 2.2 in Chapter Two (Axsäter & Juntti, 1996). However, echelon *N* (the final stage) faces customer demand. Echelon 1 is the raw material gained from outside suppliers. In order to analyze the relationship between echelon inventory policy and installation inventory policy, the order quantity, Q_n is equal to quantity of installation

1 (Mitra & Chatterjee, 2004a). Subsequently, in order to guarantee that the policies are stationary, the Q at each echelon should be an integer multiplied by Q of the predecessor echelon (Axsater, 2006a; De Bodt & Graves, 1985). Therefore, some additional assumptions made the order quantity at installation 2 to be an integer multiplied by order quantity at installation *n-1* (it is suitable to define $Q_0 = 1$). Then,

$Q_2 = j_1 Q_1$, Therefore, the expression will be:

$$Q_n = j_{n-1} Q_{n-1} \tag{3.49}$$

where j_n is an integer and nonnegative. These suppositions are natural if the policy of rationing is to satisfy either all or nothing of an order, in which the installation inventory at installation *n* includes a number that must always be of an integer of the next downstream order quantity (Q_{n-1}). The aim of j_n is to guarantee that the policy are stationary, therefore, the order quantity at each echelon should be an integer multiplied by previous echelon order quantity. The extracting value of j_n will be discussed in Chapter Four. Now, we present some additional notations:

IP_n^i = Installation inventory position at installation *n.*

$IP_n^e = IP_n^i + IP_{n-1}^i + \ldots + IP_1^i$ = "Echelon stock of inventory position at installation n.

R_n^i = Reorder point at installation stock *n.*

R_n^e = Reorder point at echelon stock *n.*

Now, suppose that the system starts with initial inventory position IP_n^{i0} and IP_n^{e0} that satisfies:

$$R_n^i < IP_n^{i0} \leq R_n^i + Q_n \tag{3.50}$$

$$R_n^e < IP_n^{e0} \leq R_n^e + Q_n \tag{3.51}$$

An inventory installation policy is always overlapping, e.g., installation *n* will not request unless its inventory position is recently decreased by a request from installation $n-1$. But, if installation $n-1$ orders, this must then be the situation for installation $n-2$, etc., and consequently for all installations downstream (Axsater, 2001). The IP_n^e at installation *n* is not influenced by orders at installation 1, 2, . . ., *n*-1. If, for example, installation $n-1$ orders a quantity from installation *n*, IP_{n-1}^i is expanded by Q_{n-1} and IP_n^i is diminished by Q_{n-1}. This will unmistakably not influence IP_n^e. The IP_n^e at installation *n* is only changed by the final demand at installation 1 and by replenishing orders at installation *n* (Fangruo Chen, 1998). We now demonstrate Propositions (3.1) and (3.2) for the special situation of unit demand and continuous review. We might also, without any absence of an all-inclusive statement, suppose that $IP_n^{i0} - R_n^i$ is an integer multiple of Q_{n-1}. All demands at installation *n* are multiples of the order quantity at installation *n-1*, Q_{n-1}, and all replenishments are also multiples Q_{n-1} because of Equation (3.50).

Consequently, this supposition just implies that we will hit the *R* exactly when ordering. The reorder point $R_n^i + y$ where $1 \leq y < Q_{n-1}$ will trigger orders at the same times of *IP*. The main distinction is that the *IP* will be *y* units below the *R* when ordering because the *R* is *y* unit higher.

Proposition 3.1: Reorder point of an installation inventory can constantly be replaced by a tantamount reorder point policy of echelon inventory (Axsäter, 2011).

Proof: Suppose the policy of installation inventory is given. Consider installation *n*, where the policy of installation inventory is interlaced, and since we select the reorder points in which all inventory positions will reach the reorder points precisely when orders are raised, the inventory position of echelon inventory after ordering must be:

$$IP_n^e = \sum_{j=1}^{n}\left(R_j^i + Q_j\right) \tag{3.52}$$

Then

$$R_n^e = R_{n-1}^i + \sum_{j=1}^{n-1}\left(R_j^i + Q_j\right) \tag{3.53}$$

Proposition 3.2: Reorder point policy of an echelon inventory that is interlaced can always be substituted by a tantamount reorder point policy of installation inventory (Hausman & Erkip, 1994).

Proof: Suppose that the policy of an echelon inventory is given. For installation 1, there is no variance between the policy of an echelon inventory and installation inventory. Accordingly, $R_1^i = R_1^e$, will raise orders at the same time as the policy of echelon stock. Consider installation $n \geq 2$. By virtue of the unit demand, all installations will always reach their order points when ordering.

$$IP_n^i = IP_n^e - IP_{n-1}^e = R_n^e + Q_n - R_{n-1}^e - Q_{n-1} \tag{3.54}$$

Therefore, the policy of installation stock with reorder points is:

$$R_1^i = R_1^e\,, \qquad R_n^i = R_n^e - R_{n-1}^e - Q_{n-1}\,, \quad n > 1 \tag{3.55}$$

3.11 Policies of Order quantity in Serial system

This section shows the order quantity policies in a multi-echelon inventory system for a serial system. In Section (3.10), we explained the differences between echelon inventory and installation inventory. However, the order quantity policies in a serial or distribution system depend on the comparison of echelon inventory and installation inventory (Axsäter & Juntti, 1996; Zhao et al., 2006). We might now assume up to the model in Section 3.9 by considering more general echelon stock *(R, Q)* policies. As in Section 3.6, we might manage with demand distributed normal, continuous review.

We make suppositions that Q_2 and the beginning of installation 2 are integer multiples by Q_1. This implies that the stock inventory level installation at installation 2 are multiples by Q_1 all the time (Atan, 2010; Axsäter, 1998). Recall from Section 3.8 that given these suppositions, the class of echelon inventory reorder point policies includes the class of installation inventory reorder point policies to be a subset. Furthermore, Chen (2000) showed that the echelon inventory *(R, Q)* policies are optimal under truly general conditions for a serial system. We shall derive the probability distribution of the echelon stock inventory level. The derivation basically follows Chen and Zheng (1994). Note first that the statuses of echelon inventory at installation 2 are completely parallel to the status of a single-stage inventory. Consider a discretionary period, *t*.

The position of echelon stock inventory just after a potential order in period *t* (before the demand), $IP_2^e(t)$, is a steady state uniformly distributed on the interval *(R_2, R_2+Q_2)* (Inderfurth & Vogelgesang, 2013). The echelon inventory level in period *t*+L_2 after the period demand $IP_2^e(t + L_2)$ is acquired by subtracting the demands in period *(t, t+1, . . ., t+L_2)*, i.e., the demand during L_2+*1* period *D(L_2+1)*, $IP_2^e(t + L_2) = IP_2^e(t) - D(L_2 + 1)$ (Li, 2013). The demand during lead-time has a mean of μ_L and a standard deviation of σ_L as we proved earlier, and has a complete analogy with Equation (3.43).

$$F_2(x) = \frac{\frac{1}{\sigma_L}}{Q}\left[G\left(\frac{R_2 - x - \mu_L}{\sigma_L}\right) - G\left(\frac{R_2 + Q_2 - x - \mu_L}{\sigma_L}\right)\right] \tag{3.56}$$

The corresponding density is (recall that $G'(x) = \Phi(x) - 1$) from Equation (3.43)).

$$f_2(x) = \frac{1}{\sigma_L}\left[\varphi\left(\frac{R_2 - x - \mu_L}{\sigma_L}\right) - \varphi\left(\frac{R_2 + Q_2 - x - \mu_L}{\sigma_L}\right)\right] \tag{3.57}$$

The inventory position at installation 1, IP_1, after the review with plausibility to order is constantly in the interval *(R_1, R_1+Q_1)* (Axsäter & Marklund, 2008). Consequently, if $IL_2^e < R_1$, we know that there is no installation inventory at installation 2 after the review at the inventory position of installation 1 (Axsater, 2001). All inventory in IL_2^e must be en route to, or already at installation 1. The other probability is that $IL_2^e > R_1$. Recall now that the installation inventory level at installation 2 after the review of installation 1 is always a multiple of Q_1, say jQ_1. We have $jQ_1 = IL_2^e - IP_1 > R_1 -$

$(R_1 + Q_1) = -Q_1$ since $IP_1 \leq R_1 + Q_1$. However, this implies that the $j \geq 0$. Consequently, there are no backorders at installation 2 and all stock in IP_1 must be en route to, or already at installation 1 (Karaman, 2007). Note that given IL_2^e, we can obtain IP_1 uniquely from the condition $IP_1 = IL_2^e - jQ_1$ and $R_1 < IP_1 \leq R_1 + Q_1$ since the second condition determines j uniquely. Let us symbolize the resulting j by $\hat{j}$. We can infer that the acknowledged inventory position, e.g., the inventory, is on its way to, or already at installation 1, which can be expressed as:

$$O(IL_2^{e-}) = \begin{cases} IL_2^e, & IL_2^e \leq R_1 \\ IL_2^e - \hat{j}Q_1, & \text{Otherwise} \end{cases} \tag{3.58}$$

3.12 First come first serve queuing discipline in inventory system

A queueing framework has a huge part to arrange and direct the works in the inventory network (Saffari & Haji, 2009). In a production network, each stage or echelon can be portrayed with a lining framework with stock (Jain & Raghavan, 2008), especially when the procedure is liable to a long sitting tight time for retailers to meet the item.

Axsäter (2007) utilized FCFS to be a part of a two-echelon dispersion framework with one stockroom and a number of retailers under (S-1,s) approach, where the retailers confront stochastic interest disseminated Poisson. Furthermore, Saffari and Haji (2009) and Sahraeian et al. (2010) attempted to diminish the holding up time in their considered framework by relying upon stochastic interest dispersed a Poisson process. Schwarz et al. (2006) tackled stationary conveyances of joint stock operations and line length in an unequivocal item structure for distinctive 'M/M/1-frameworks' with a ceaseless audit stock framework and different stock management approaches, as well as with lost deals. They assumed that the demand was Poisson and lead-time and service time was exponential. They used these two distributions to discover the measures of capable performance frameworks.

The discipline for any system in queueing can be generalized as *(a/b/c).(d/e/f)* (Boucherie & van Dijk, 2010). Where,

a: the statistical distribution for arrival process or the arrival distribution.

b: service time distribution or departure distribution.

c: number of parallel sources or number of service stations.

d: distribution service discipline, and can be:

- FCFS: first come first serve.
- LCFS: last come first serve.
- SIRO: service in random order.
- GD: general service discipline.
- PR: priority.

e: maximum number of queue in the system (finite or infinite).

f: customer source (community size) that needs service (finite or infinite).

The standard symbols *a* and *b* used to express the arrival and departures are as follows:

M: Poisson or Markovian arrival or departure distribution, or equivalently exponential inter-arrival or service time distribution.

D: constant or deterministic inter-arrival, or service time.

E_k : Erlangian of gamma distribution of inter-arrival or service time distribution with parameter.

GI: general independent distribution of arrival.

G: general distribution of departure or service time.

Therefore, to adopt a queue system, the arrival distribution and service distribution should be known from the data of the system (Schwarz et al., 2006). Furthermore, it is necessary to know whether the queue numbers and community size are finite or infinite. Depending on these variables, we can formulate or build the queueing discipline to reach the desired goals of the system.

Remark 3.1: order-up-to-level represents a private situation of a *(R, Q)* policy while demand distributed Poisson represents a private situation of a compound Poisson demand (Axsater, 2010; Axsater, 2001).. From Remark 3.1, we can use an order-up-to-level in the continuous review *(R, Q)* policy with some changes in the model upon on the variables. This leads to additional notation to be:

λ = mean arrival rate of retailers.

μ = mean service rate of retailers.

$P_n(t)$ = the probability of arrival (n) units to the system for a period (t),

L_s= the expected number of retailers in the system.

L_q= the expected number of retailers in the queue.

W_s= expected waiting time in the system.

W_q= expected waiting time in the queue.

Under the FCFS policy, the optimal 'order-up-to-level' satisfies

$S_i \geq 0, i = 0, 1, 2, \ldots, N.$

We consider a supply chain that includes one warehouse (with three installations), one distribution center (with six lines) and *N*-retailers. Furthermore, all retailers having identical requirements are assumed. Here, identical requirement means that different retailers require similar items (cement), but in a different configuration and different quantities.

Now, the arrival process, which is the time between two consecutive arrivals of service place (Krakowski, 1974) is defined. These periods of time are probabilistic and have a probability distribution function *(p.d.f)*. Assume that the arrival process to the service process stochastically with mean arrival rate, λ. The probability of arrival one unit during period *(t)* is (Δt). Furthermore, the probability of this unit during this period depends on the duration of (Δt) or $(t,\ t + \Delta t\ )$ (Saffari & Haji, 2009). The probability of arriving more than two units during (Δt) is $O(\Delta t)$. $\frac{1}{\lambda}$ represents the arrival rate i.e. the mean time of two consecutive arrivals and distributed to be a negative exponential distribution (Wu et al., 2007). By using probabilistic arrival hypotheses and the probability distribution of mean arrival rate *(λ)* during a period time, the probability of arrival (*n*) units to the system for period *(t)*, $P_n(t)$ will be:

$$P_n(t) = g(t) \int_0^{\infty} f(t)\, dt \tag{3.59}$$

where, $g(t) = \lambda e^{-\lambda t}$, for an arrival process that has a negative exponential distribution, $t > 0$. Therefore,

$$P_n(t) = \lambda e^{-\lambda t} \int_0^{\infty} f(t)\, dt \tag{3.60}$$

In the service process, the period of service level is independent of each unit. Additionally, it assumes the homogeneity of the units (Jain & Raghavan, 2003). The probability to finish a service unit in a short period of time (Δt) depends on the previous period of $O - t$, t which follows $t + \Delta t$. Now we will assume μ = mean service rate, the probability to finish a service for one unit in period ($t + \Delta t$) is $[\mu \, \Delta t + O(\Delta t)]$, and the probability to not finish the service unit through the same period ($t + \Delta t$) is [1 - ($\mu \, \Delta t + O(\Delta t)$)]. Then, $\frac{1}{\mu}$ is the service rate, in which the service unit takes in a period time (Krakowski, 1974).

$$P_n(t) = \mu e^{-\mu t} \int_0^{\infty} f(t) \, dt \tag{3.61}$$

To find the probability of arrival n units to the system for period λt in $(t + \Delta t)$ is,

$$\begin{aligned} P_n(t + \Delta t) = {} & P_n(t)(1 - \mu_n \Delta t)(1 - \lambda_n \Delta t) + P_{n-1}(t)\lambda_{n-1}\Delta t(1 - \mu_{n-1}\Delta t) \\ & + P_{n+1}(t)\, \mu_{n+1}\Delta t(1 - \lambda_{n+1}\Delta t) \\ & + O(\Delta t) \end{aligned} \tag{3.62}$$

$$\begin{aligned} P_n(t + \Delta t) = {} & P_n(t)[1 - (\lambda_n + \mu_n)\Delta t] + \lambda_{n-1}P_{n-1}(t)\Delta t + \mu_{n+1}P_{n+1}(t)\, \Delta t \\ & + O\Delta t \qquad\qquad n \geq 1 \end{aligned}$$

$$\begin{aligned} & P_n(t + \Delta t) - P_n(t) \\ & \qquad = -[\lambda_n \Delta t + \mu_n \Delta t] + \lambda_{n-1}P_{n-1}(t)(\Delta t) + \mu_{n+1}P_{n+1}(t)\, (\Delta t) \\ & \qquad + O\Delta t \end{aligned} \tag{3.63}$$

Dividing Equation (3.63) by Δt, we obtain:

$$\frac{P_n(t+\Delta t)-P_n(t)}{\Delta t}=-[\lambda_n+\mu_n]+\lambda_{n-1}P_{n-1}(t)+\mu_{n+1}P_{n+1}(t)\ +\frac{O\Delta t}{\Delta t}$$

And taking the limit for both sides, $\Delta t \to 0$

$$P_n'(t) = -[\lambda_n+\mu_n]P_n(t)+\lambda_{n-1}P_{n-1}(t)+\mu_{n+1}P_{n+1}(t) \tag{3.64}$$

3.12.1 Steady state

In order to study any system according to a queueing theory, we need to apply the steady state. This is by calculating the steady state probability distribution and computing the most vital execution measures (Jain & Raghavan, 2008). The probability of the steady state of an item structure seems to be an extraordinary gimmick of the above model. This implies that an asymptotic and a stationary distribution of the joint (queue length/inventory size) procedure are factored into the stationary line length and stock size appropriation (Saffari & Haji, 2009). In other words, in the long run, and in balance, the length line methodology and the stock procedure carry on as though they are free (Schwarz et al., 2006).

However, studying any queueing system needs a long period time in order to get a clear understanding of steady. Therefore, t approaches a very big number, and is supposed of approaching ∞, which means $t \to \infty$. When, $t \to \infty$, then $P_n(t)$ will be independent of the time. Therefore, $P_n(t) = P_n$ is the steady state (Bhat, 2008).

By taking the first derivative of Equation (3.64) we obtain

$$P_n'(t) = -[\lambda+\mu]P_n(t)+\lambda P_{n-1}(t)+\mu P_{n+1}(t) \qquad n \geq 0$$

when $n = 0$

$$P_0'(t) = -[\lambda + \mu]P_0(t) + \mu P_1(t)$$

$$P_0'(t) = -\lambda P_0(t) - \mu P_0(t) + \mu P_1(t) \tag{3.65}$$

-$\mu P_0(t)$ can be ignored from the Equation (3.65) because there is no negative service time, which means $P_0(t) = 0$ (already there is no service) (Bhat, 2008).

Let $P_n' = 0$

$$\therefore \lambda P_0(t) = \mu P_1(t) \tag{3.66}$$

From Equation (3.66) we can reformulate P_1 to be,

$$P_1(t) = \frac{\lambda}{\mu} P_0(t) \tag{3.67}$$

If $n=1$

$$P_1'(t) = -[\lambda + \mu]P_1(t) + \lambda P_0(t) + \mu P_2(t)$$

$$P_1'(t) = -[\lambda + \mu]\frac{\lambda}{\mu} P_0(t) + \lambda P_0(t) + \mu P_2(t)$$

$$P_1'(t) = -\frac{\lambda^2}{\mu} P_0(t) - \lambda P_0(t) + \lambda P_0(t) + \mu P_2(t)$$

$$P_1'(t) = -\frac{\lambda^2}{\mu} P_0(t) + \mu P_2(t)$$

$$\therefore P_2(t) = \frac{\lambda^2}{\mu^2} P_0(t) \tag{3.68}$$

By mathematical induction, $P_n(t)$ is given by

$$P_n(t) = \left(\frac{\lambda}{\mu}\right)^n P_0(t)$$

Let $\frac{\lambda}{\mu} = \rho$ and $\rho = \frac{\lambda}{\mu} < 1$, where ρ is the proportion units' arrival rate in the unit time to the performance of the service rate in the unit time while $\frac{\lambda}{\mu}$ is the proportion of traffic density in the system. Therefore,

$$P_n(t) = \rho^n P_0(t) \tag{3.69}$$

Now we are interested in knowing the value of $P_0(t)$. The sum of probabilities values is equal to 1 (Balakrishnan & Nevzorov, 2003). Therefore,

$$\sum_{n=0}^{\infty} P_n(t) = 1 \tag{3.70}$$

By substituting Equation (3.69) into Equation (3.70), we obtain

$$\sum_{n=0}^{\infty} \rho^n P_0(t) = 1 \tag{3.71}$$

Equation (3.71) can be rewritten as:

$$P_0(t) \sum_{n=0}^{\infty} \rho^n = 1$$

$$P_0(t)(1 + \rho + \rho^2 + \rho^3 + . \quad . \quad .) = 1 \tag{3.72}$$

We can define $(1+\rho+\rho^2+\rho^3+ . \quad . \quad .)$ as infinite geometric series, which the general limit = first limit / (1- the base). Therefore, Equation (3.72) will be

$$P_0(t)\left(\frac{1}{1-\rho}\right)=1$$

$$P_0(t)=1-\rho \tag{3.73}$$

Now, we can reformulate Equation (3.69) based on Equation (3.73)

$$P_n(t)=\rho^n(1-\rho) \tag{3.74}$$

Now we calculate the performance measures in a steady state (L_S, L_q, W_S, W_q) from Bhat (2008) as follows:

Firstly, we find the expected number of retailers in the system.

$$L_s=\sum_{n=0}^{\infty} nP_n(t)=\sum_{n=0}^{\infty} n\,\rho^n(1-\rho)=$$

$$\sum_{n=0}^{\infty} n\,\rho^n-\sum_{n=0}^{\infty} n\,\rho^{n+1}$$

$$L_s=\sum_{n=0}^{\infty} n\,\rho^n-\sum_{n=1}^{\infty}(n-1)\rho^n=$$

$$\sum_{n=0}^{\infty} n\,\rho^n-\sum_{n=1}^{\infty} n\,\rho^n+\sum_{n=1}^{\infty}\rho^n$$

$$L_s = \sum_{n=0}^{\infty} \rho^n = \rho + \rho^2 + \rho^3 + \rho^4 + . . .$$

$$L_s = \frac{\rho}{(1-\rho)} \tag{3.75}$$

where, $\rho = \frac{\lambda}{\mu}$ and the expected waiting time in the system depends on the expected number of retailers in the system. Therefore,

$$W_s = \frac{L_s}{\text{arrival rate}} = \frac{\frac{\rho}{(1-\rho)}}{\lambda}$$

$$\therefore\ W_s = \frac{\rho}{\lambda(1-\rho)} \tag{3.76}$$

Secondly, the expected number of retailers in the queue (L_q) is

$$L_q = \sum_{n=1}^{\infty} (n-1) P_n\,(t) =$$

$$\sum_{n=1}^{\infty} n P_n(t) - \sum_{n=1}^{\infty} P_n\,(t)$$

$$L_q = L_S - (1 - P_0(t))$$

$$L_q = L_S - (1 - (1-\rho))$$

$$L_q = L_S - 1 + (1-\rho)$$

$$L_q = L_S - \rho \quad = \frac{\rho - \rho + \rho^2}{(1-\rho)}$$

$$\therefore L_q = \frac{\rho^2}{(1-\rho)} \tag{3.77}$$

The expected waiting time in the queue (W_q) will be

$$W_q = \frac{L_q}{\text{arrival rate}}$$

$$W_q = \frac{\rho^2}{\lambda(1-\rho)} \tag{3.78}$$

In the case of an infinite population, the restricting distribution of the queue length techniques harmonize with that of the M/M/1/∞ framework with the arrival rate λ and service rate μ (Jain & Sigman, 1996). This demonstrates a surprising and vital invariance property for the queueing frameworks regarding inventory systems and lost sales. In fact, we can see that for the effective arrival rate, $\lambda_{eff} \neq \lambda$ holds, and that for the effective service rate $\mu_{eff} \neq \mu$ holds (Schwarz et al., 2006). The surprising conclusion is that the framework without any other input manages to be a powerful service and has arrival rates in response to the lead time qualities, as well as the inventory system policy in a manner that the service system dependably encounters a traffic intensity $\rho = \frac{\lambda}{\mu} = \frac{\lambda_{eff}}{\mu_{eff}}$. The main side condition is $\rho = \frac{\lambda}{\mu} < 1$ (Boucherie & van Dijk, 2010; Karaman, 2007).

There is a strong relationship between L_s & W_s and between L_q & W_q. By determining one of these measures, the other can be determined (Axsäter, 2007; Diks et al., 1996). Suppose another measure is λ_{eff}, where λ_{eff} is the effective average arrival rate (independent of the number of the system). Therefore, the previous relation can be resolved as:

$$L_s = \lambda_{eff} W_s \tag{3.79}$$

$$L_q = \lambda_{eff} W_q \tag{3.80}$$

and

$$\lambda_{eff} = \sum_{n=0}^{\infty} \lambda_n P_n$$

While there is a direct relationship between W_s *and* W_q, this relationship is logical and can be determined by the definition: expected waiting time in the system = expected waiting time in the queue + expected service time (Bhat, 2008). Therefore,

$$W_s = W_q + \frac{1}{\mu} \tag{3.81}$$

where, μ is the rate of service provider for each station or line, and the expected service time is $\frac{1}{\mu}$. Multiply Equation (3.81) by λ_{eff}, we obtain

$$L_s = L_q + \frac{\lambda_{ef}}{\mu} \tag{3.82}$$

3.12.2 Waiting time distribution based on FCFS service discipline

The expected waiting time which has a probability density function is independent of the service discipline (Sahraeian et al., 2010).

Let τ be the amount of time retailers are arriving and must wait in the system until the service is complete. Based on the FCFS service discipline, if there are n retailers in the system ahead of an arriving retailer, then

$$\tau = \tau' + \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_{n-1} + \tau_n + \tau_{n+1}$$

where τ' is the time needed for a retailer to actually be in service to complete the service while $(\tau_1 + \tau_2 + \tau_3 + \ \ldots + \tau_n)$ is the service time for $n - 1$ retailer in the queue (Saffari & Haji, 2009). τ_{n-1} is the time represented, i.e., the service time arriving to a retailer. Assume $w(\tau/n + 1)$ is the condition's probability density function of τ given n retailers in the system ahead of the arriving retailer (Axsäter, 2007). Since τ_i for all i is exponentially distributed to be a forgetfulness property (see Remark 3.2), τ' also has the same exponential distribution as $\tau_1 + \tau_2 + \tau_3 + \ \ldots + \tau_{n-1}$ consequently, τ is the sum of $n + 1$ which is independent and identically distributed (i.i.d) of exponential distribution. From probability theory, $w(\tau/n + 1)$ must be a Gamma distribution with parameter μ. Therefore,

$$w(\tau) = \sum_{n=0}^{\infty} w\left(\frac{\tau}{n} + 1\right) P_n = \sum_{n=0}^{\infty} \frac{\mu(\mu\tau)^n \, e^{-\mu\tau}}{n!} (1 - \rho)\rho^n$$

$$w(\tau) = (1 - \rho)\mu e^{-\mu\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} = \mu(1 - \rho)e^{-\mu\tau + \lambda\tau} \qquad \tau > 0$$

$$w(\tau) = \mu(1 - \rho)e^{-\mu(1-\rho)\tau} \qquad \tau > 0 \tag{3.83}$$

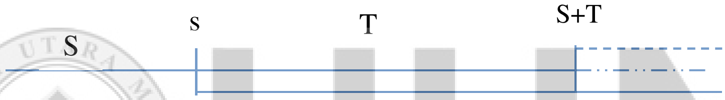
where Equation (3.83) is a distributed negative exponential with a means of

$$E[\tau] = \frac{1}{\mu(1-\rho)} \tag{3.84}$$

In fact, the mean $E[\tau]$ is equal to be an expected waiting time in the system, W_s.

Remark 3.2: Forgetfulness property

Poisson is a completely random operation because it contains the property in which the time interval remaining until the occurrence of the next event is totally independent of the time interval that elapsed since the occurrence of the last event (Bhat, 2008; Krakowski, 1974).



This property is equivalent to proving the following probability statement $P[t > T + S/t > s]$

$$P\left[t > T + \frac{S}{t} > s\right] = \frac{P[t > T + S \cap t > s]}{P[t > s]} = \frac{P[t > T + S]}{P[t > s]} =$$

$$= \frac{\left[1 - e^{-\lambda(T+S)}\right] - \left[1 - e^{-\lambda t}\right]}{e^{-\lambda t}} = 1 - e^{-\lambda T}$$

$$\therefore P[t > T] = 1 - e^{-\lambda T} \tag{3.85}$$

3.12.3 Performance measures for more than one parallel service station

When the parallel service stations, *C,* in the queue discipline are more than one (i.e. 6 stations in the distribution centre), the measures of performance (L_S, L_q, W_S, W_q) will change (Bhat, 2008). This change or this effect will be as follows:

$$\sum_{n=0}^{\infty} P_n = 1 \quad \Rightarrow \qquad P(n \ < c) + P(n \geq c)$$

$$= \sum_{n=0}^{c-1} \frac{1}{n!} \rho^n P_0 + \sum_{n=c}^{\infty} \frac{\rho^n}{c! \ c^{n-c}} P_0$$

$$1 = \sum_{n=0}^{c-1} \frac{1}{n!} \rho^n P_0 + \sum_{n=c}^{\infty} \frac{1}{c! \ c^{n-c}} \ \rho^n P_0$$

$$1 = P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \sum_{n=c}^{\infty} \frac{1}{c! \ c^{n-c}} \ \rho^n \right]$$

$$1 = P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \sum_{n=c}^{\infty} \frac{1}{c^{n-c}} \ \rho^{n-c} \right]$$

$$1 = P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \sum_{n=c}^{\infty} \left(\frac{\rho}{c}\right)^{n-c} \right]$$

$$1 = P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \ (1 + \frac{\rho}{c} + \frac{\rho^2}{c} + \frac{\rho^3}{c} + \ . \ \ . \ \ .) \right]$$

$$1 = P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1 - \frac{\rho}{c}} \right) \right]$$

$$\therefore \ P_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c! \, (1 - \frac{\rho}{c})} \right]^{-1} \tag{3.86}$$

$$L_q = \sum_{n=c}^{\infty} (n - c) P_n \, , \qquad\qquad c > 1$$

$$= 0P_c + 1P_{c+1} + 2P_{c+2} + 3P_{c+3} + \ . \ . \ . \ + kP_{c+k}$$

$$= \sum_{k=0}^{\infty} kP_{k+c}$$

$$= \sum_{k=0}^{\infty} k \frac{\rho^{k+c}}{c!\, c^k} P_0$$

$$L_q = P_0 \frac{\rho^c}{c!} . \frac{\rho}{c} \sum_{k=0}^{\infty} k \left(\frac{\rho}{c}\right)^{k-1}$$

Let $\frac{\rho}{c} = \theta$ then,

$$L_q = P_0 \frac{\rho^{c+1}}{c!\, c} \sum_{k=0}^{\infty} k\theta^{k-1}$$

which mean,

$$L_q = P_0 \frac{\rho^{c+1}}{c!\, c} \left[1 + 2\theta + 3\theta^2 + 4\theta^3 + \ . \ . \ . \ \right]$$

$$1 + 2\theta + 3\theta^2 + 43\theta^3 + . \ . \ . = \sum_{n=1}^{\infty} n\theta^{n-1}$$

$$= \frac{d}{d\theta} \left\{ \sum_{n=1}^{\infty} \theta^n \right\}$$

$$= \frac{\theta}{d\theta} \left\{ \frac{1}{1-\theta} \right\}$$

$$= \frac{1}{(1-\theta)^2}$$

$$\therefore L_q = P_0 \frac{\rho^c \rho}{c!\ c} * \frac{1}{\left(1-\frac{\rho}{c}\right)^2}$$

$$\therefore L_q = P_0 \frac{\rho^{c+1}}{c!\left(1-\frac{\rho}{c}\right)^2} \tag{3.87}$$

In order to determine or extract the rest of the formulas (L_s, W_s, W_q), we use the little formula (Bhat, 2008; Krakowski, 1974) which links between the averages of number units in the system and the average of queuing time in the system. The relationship is as follows:

$$L_s = \lambda W_s \tag{3.88}$$

Moreover, there is a relationship linking the average of number units in the queue with the average of queue time in the queue line and the expected queue time in the system (Jain & Raghavan, 2008), which is given by,

$$\lambda W_s = \lambda W_q + \frac{\lambda}{\mu}$$

$$\therefore L_s = L_q \rho \tag{3.89}$$

As previously mentioned in Section 3.13.1, there is a strong relationship between L_s, L_q W_s and Wq. Therefore, when we know the value of one of them, we can find the values of others.

3.12.4 Gamma distribution

The completeness of the gamma distribution has the following density functions (Hiros, 2000; Khodabin & Ahmadabadi, 2010; Mihe, 2010).

$$g(x) = \frac{\beta(\beta x)^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta x} , \qquad x \geq 0 \tag{3.90}$$

The two parameters α and β are both nonnegative and $\Gamma(\alpha)$ is the gamma function:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx. \tag{3.91}$$

Given t, the mean μ, and the standard deviation σ, the parameters α and β are uniquely determined to be:

$$\alpha = \left(\frac{\mu}{\sigma}\right)^2 \tag{3.92}$$

$$\beta = \frac{\mu}{\sigma^2} \tag{3.93}$$

It is helpful to note that $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ and that:

$$\int_0^\infty x^{\alpha} e^{-\beta x} dx = \frac{\Gamma(\alpha + 1)}{\beta^{(\alpha+1)}} \tag{3.94}$$

3.13 Discussion and summary

In this Chapter, some crucial methods related to a multi-echelon inventory system, which would be helpful in meeting the objectives of this research, were recapped. The review presented revolves around the approximation mathematical model in a multi-echelon inventory system under the continuous review policy. So, these methods are linked to the simulation and forecasting methods in addition to the role of the first-come-first-serve (FCFS) queue model in a multi-echelon inventory system. In addition, the continuous review *(R, Q)* inventory policy was selected compare with other policies in an inventory system. When the inventory position, *IP* deteriorates to be below the reorder point, *R* an order quantity of size *Q* is ordered. If the *IP* is adequately low it may be necessary to order more than one orders to get above *R* (Axsater, 2006). If the demand is continuous or one unit at a time, we will always reach the reorder point exactly in the case of continuous review.

In fast moving items with probabilistic and high uncertain of demand and lead-time, the appropriate inventory policy is *(R, Q)* compared with other policies (Moslemi & Zandieh, 2011) because the properties of the *(R, Q)* policy with regard to lead-time, (Li, 2013; Mitra & Chatterjee, 2004). Although management of the lead time is enormously important in competitive ergonomics and is conceptually stressed by operations management strategies (e.g. quick response, just in times and time based competition) (Elhasia et al., 2013).

However, for the multi-echelon inventory system under the continuous review policy to be labeled as a good approximation mathematical model, it should satisfy several mathematical properties. They include a simulation procedures to extract the demand during lead-time probability distribution function, a suitable forecasting method which is often the exponential smoothing method, a suitable policy in the multi-echelon inventory system, such as an *(R, Q)* policy, and an inventory systems, such as serial and distribution inventory system.

While the optimality is still unknown for most types of multi-echelon inventory systems because of the probability behavior of the demand and the lead-time, an approximation mathematical model offers the best solution by considering demand during lead-time probability distribution.

In the next step, this research will develop an approximation mathematical model based on a simulation procedure to establish the demand during lead-time probability distribution. The model includes an exponential smoothing model and a lead-time probability distribution function by proposing two models, serial multi-echelon inventory system under the continuous review *(R, Q)* policy and distribution multi-echelon inventory system under the FCFS queue model. The next chapter explains the model development in detail.

CHAPTER FOUR
RESEARCH METHODOLOGY

This chapter details the development of an approximation mathematical model in a multi-echelon inventory system under a continuous review *(R, Q)* policy depending on the novelty of the probability distribution function of the demand during lead-time. It starts with the design of the research, data source, collection and types, simulation procedures to establish the demand during lead-time data, model formulation, and evaluation of the proposed models.

4.1 Research Design

In this research, we consider a supply chain of multi-echelon inventory system under a continuous review *(R, Q)* policy with probabilistic demand and lead-time, which includes four echelons. The data involve demand, lead-time, retailer's arrival rates, service rate, and costs (i.e. holding cost and setup cost). The type of this data is secondary and obtained from the database of the cement company directly. Overall, this research involves a model development of a multi-echelon inventory system.

The first part of this research is establishing the demand during lead-time probability distribution function data by using simulation procedures, which is called the SMDDL model. This model develops a structure and an algorithm to establish the demand during lead-time probability which is not available in the reality. In the previous studies, probability of demand and lead-time were taken separately without extracting and establishing the probability distribution of demand during lead-time and integrate the distribution of demand during lead-time in the total cost function of inventory to develop the inventory performance measures (i.e., the equation of order quantity, safety stock or reorder point) as explained in Chapter Two. They e.g., Dekker, et al.(1998), Van der Heijden et al, (1999), Tang and Grubbström (2003), Chiang and Monahan (2005), Wu et al., (2007), Baten and Kamil (2009) and Axsäter and Viswanathan (2012) used only the mean and standard deviation of the demand and lead-time separately based on the distributions parameters to achieve the objective of the study

The SMDDL model depends on two types of data (demand and lead-time) as inputs, then a structure and an algorithm of simulation procedures. However, the extractions of these two types of data rely on, the parameters of the lead-time data distribution and the suitable demand forecasting method to establish the mean and standard deviation of the demand instead of using the available data of the demand. This is because demand forecasting estimates the values of the variables do not fall within the available data, and since it is demand, it is necessary to extract the expected future quantity of the demand based on the historical data (i.e., the mean and standard deviation). Additionally, the importance of demand forecasting can be short term, midrange, or long term. Typically, firms would use all three types of forecasting.

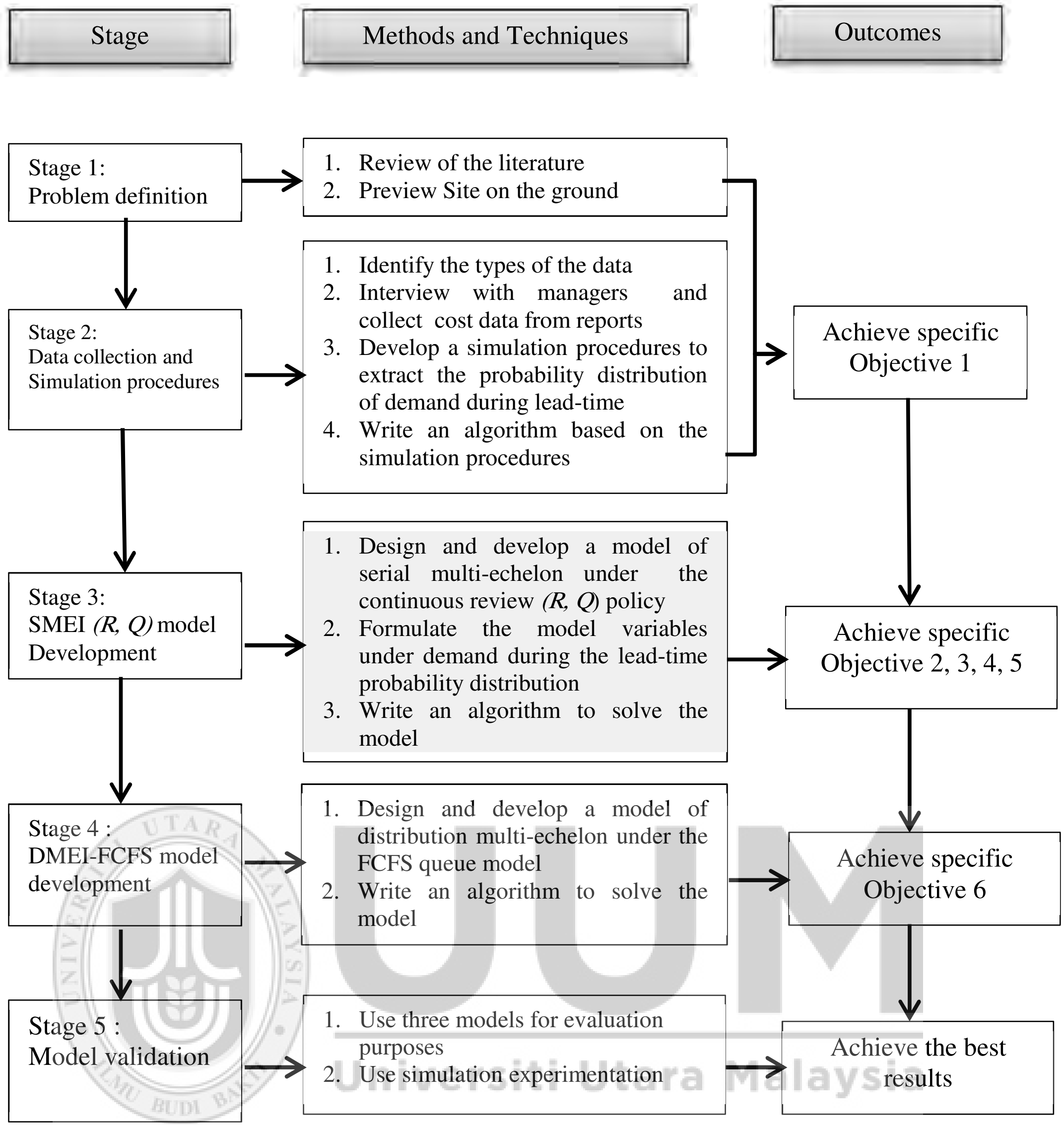
The second part of this research attempts to develop an approximation mathematical model in a multi-echelon inventory system under a continuous review *(R, Q)* policy. At this stage, two sub-models are proposed. The first one is called the SMEI *(R, Q)* model and the other one is the DMEI-FCFS model. The first model is developed for a serial multi-echelon inventory system under a continuous review *(R, Q)* policy while the second model is developed for a distribution multi-echelon inventory system under a first-come-first-serve queue model.

In the SMEI *(R, Q)* model, the approximate order quantity Q for a serial multi-echelon inventory model is developed. In addition, the reorder point R in a distribution system for each installation and echelon under a continuous review *(R, Q)* policy is established, the safety stock, *SS,* is optimized, and the approximate total cost equation based on the developed approximate order quantity Q is developed.

In the DMEI-FCFS model, performance measures to reduce the long waiting time in the supply chain between the distribution centre and the retailers in the system by using a first-come-first-serve (FCFS) queue model with a finite production rate is developed. Finally, the validation and evaluation of the proposed models are compared with previous models based on different criteria.

4.2 Research Process

The present research activities are implemented in five stages as shown in Figure 4.1, starting with defining the problem and ending with validating the models developed.



The Objectives:

1. To develop the probability distribution function of demand during lead time by using a simulation procedures.
2. To develop an appropriate formulation for order quantity, Q in a serial multi-echelon inventory system under a continuous review (R, Q) policy with the probability distribution of the demand during the lead time.
3. To identify the optimal safety stock that should be on hand for the warehouse, including each of the three silos under a continuous review (R, Q) policy.
4. To determine the optimal reorder point, R in the distribution multi-echelon inventory system under a continuous review (R, Q) policy, this also leads to extracting the inventory position and levels at each echelon.
5. To develop the approximate total cost function for the whole system.
6. To develop the FCFS queue model in the continuous review inventory system in order to reduce the long waiting time between the distribution center and the retailers.

Figure 4.1. Structure of research activities

A detailed research process framework is elaborated based on stages two, three, and four which describe the objectives to be achieved in each stage. The research process shows the types of data used to develop a simulation procedures, SMEI *(R, Q)* model, and DMEI-FCFS model. The process flow is exhibited in Figure 4.2.

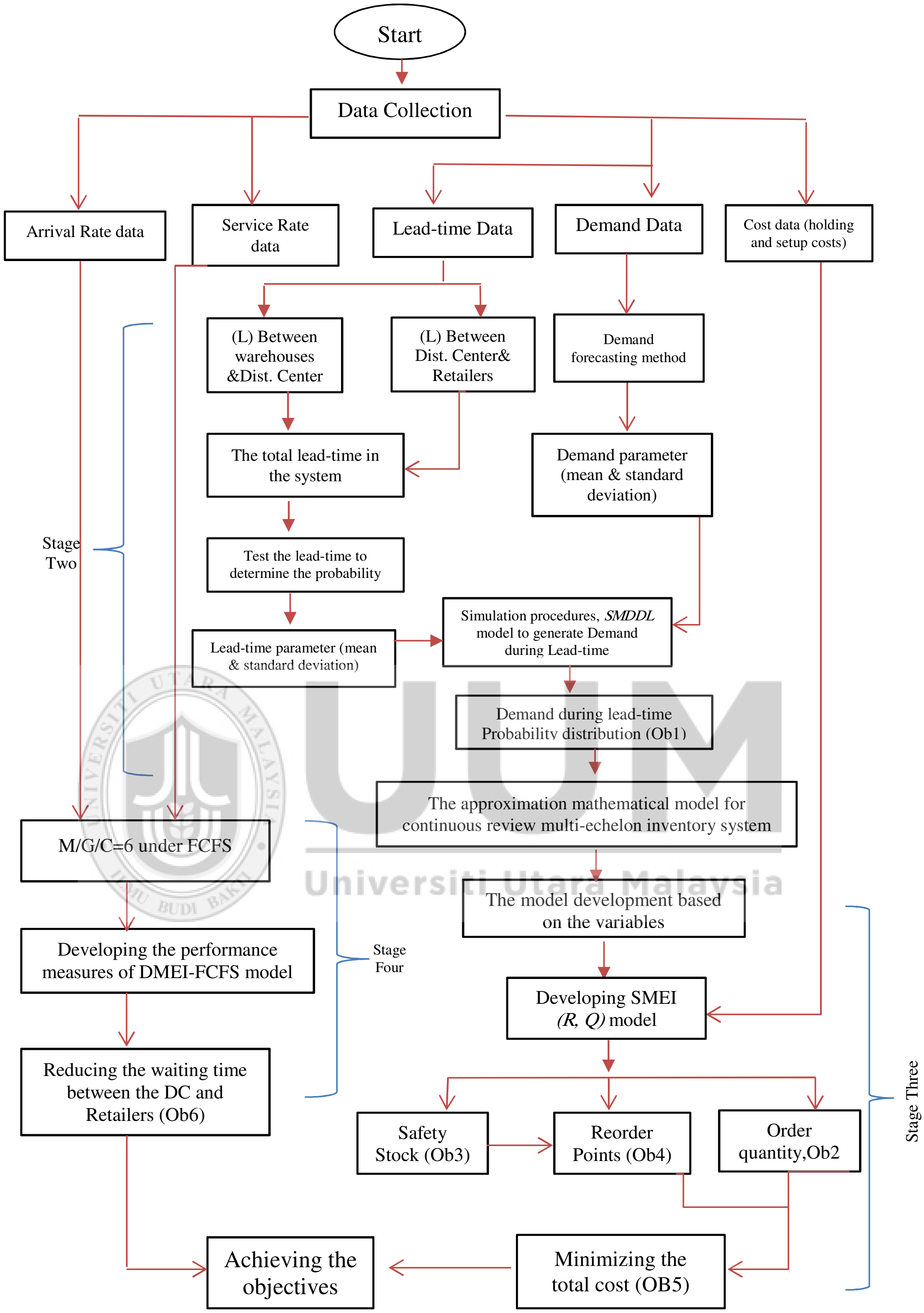


Figure 4.2. Details of the research process

Furthermore, we build the structure of the work and show in which part the objectives are met, as indicated in Figure 4.3.

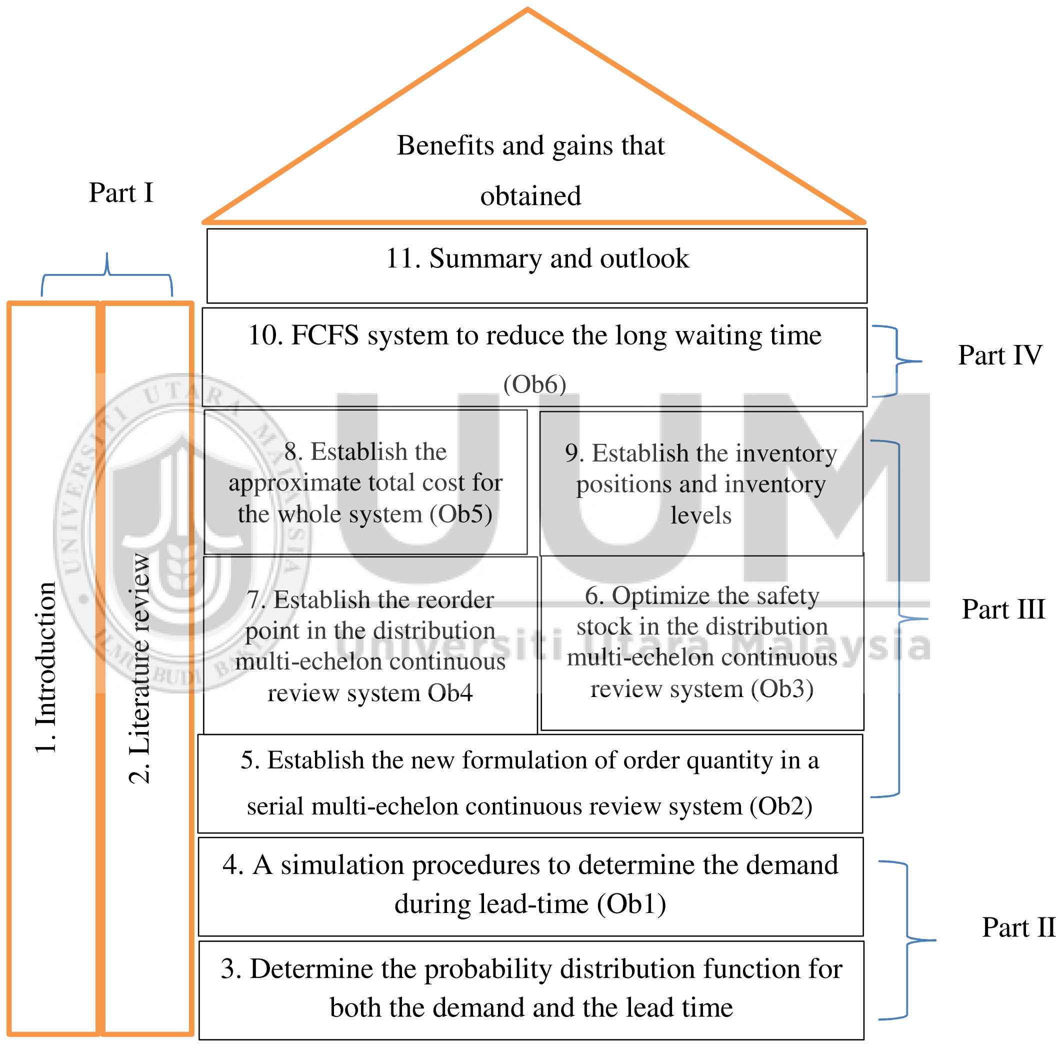


Figure 4.3. The research work structure

4.3 Data source

Lafarge is one of the leading global companies in cement production that has invested in the Iraq-Kurdistan region since 2008 through signing a contract joint venture with the Iraq-Kurdistan government to operate Tasluja United Cement Company (UCC), which is located in Sulaimanyia, the second biggest city in Kurdistan. Lafarge has improved the production capacity to 2.3 million tonne of cement per day by investing USD200 million (Lafarge, 2012; Lafont, 2011).

4.4 Data types and collection

This research involves five main data types. They are demand data, lead-time data, costs (i.e., holding cost and setup cost), arrival rate data (the number of retailers who arrive at the system), and service rate data. The data were secondary data obtained from the database of the cement company directly for the three-year periods (2011-2013). The database has a lot of information and data about the work procedures, and most of the data sets were profile data.

The data collected were preliminary, and they had to be processed to meet the requirements of the research. From this vast amount of data, loaded quantity (demand), packing, parking, total waiting time (lead-time or service time), and the number of arrivals had to be extracted. The following sections illustrate each of the data used in this research.

4.4.1 Demand Data

Demand means the amount of cement that meets the retailers' needs in tonnes per unit time. The demand data include the quantity required for load time, quantity in tonne, type of loading (bagged or bulk), the number of channels that provide service, and destination of loading, among others. The most important data are the net loaded quantity in tonne per unit time, for example, during a day.

The aim of collecting demand data is to construct the distribution and then estimate the mean and the standard of deviation of the demand based on the appropriate forecasting method instead of using the mean and standard deviation of the available data. The reason behind that is to extract the expected future quantity of the demand and then its standard deviation which is the estimate for future quantities that do not fall within the available data.

Additionally, the importance of demand forecasting in an inventory system is to increase customer satisfactions, reduce inventory stockout, schedule production more effectively, lower safety stock requirement, manage shipping better and plan sales strategies. Therefore, the suitable forecasting method depends on the pattern fluctuations of the demand data, increasing fluctuations, decreasing fluctuations, seasonal fluctuations or stationary fluctuations during the selected periods.

4.4.2 Lead-time Data

Lead-time means the periods confined between the preparation of orders until they are obtained and placed in the warehouse for each order of time. These periods are subject to a probabilistic distribution. The extraction of lead-time data is divided into several axes, including the total period, which starts from when the raw materials are received, passed through the production process, and then loaded until they arrive at a retailer or customers. The aim of extracting lead-time data is to determine the probability distribution function parameters which are later used as input for the SMDDL model.

However, the most difficult part of the data extraction process, which makes lead-time long, is the loading period. The entry of trucks for the purpose of getting the required quantities takes a long time due to the following reasons:

1. The huge number of retailers and the limited number of distribution centers.
2. Loading period which includes the following elements: dispatch for an entry truck, booking, truck movement to the scale, the parking area which is the customer's attainment and driver's satisfaction, and packing house area. These elements are categorized in three conditions:
    a) Loading performance in terms of the efficiency of the mechanical status of the packers and the workers at loading.
    b) Contractor's ability and commitment to providing enough workers for increasing demand and his treatment of the workers and drivers which constantly caused problem by the workers and drivers.

c) Shift change. Because there two shifts that occur on a daily basis, the loading has to stop for three hours, which means a loss of 600 tons from daily loading performance.

Therefore, the mathematical formula of the total lead-time (waiting time) can be expressed as follows:

Total waiting time = Parking Area + Packing House.

On this basis, the system is subject to first-come-first-serve (FCFS), especially in the case of probabilistic lead-time.

4.4.3 Demand during lead-time data

Demand during lead-time is the joint distribution of lead-time distribution data and demand distribution data depending on the lead-time distribution parameters and the mean and standard deviation of the demand which extracted from the forecasting method. This type of data is not available directly from the source and is not recorded in reality due to the nesting of the activities and the complexity of the procedures.

When the products are near completion, a decision maker starts to make a request for an order quantity to meet the needs of the retailers so that not the products to fall in shortage. During this period, and until the required quantity is placed at the warehouse the retailers' demands are continuous. Since the processes are very nested, it is hard to record these data on demands until the products are placed. If the products arrive late, the warehouse covers or satisfies the retailers' demands from its safety stock (inventory on hand).

Therefore, simulation procedures will impose itself to generate demand during lead-time probability distribution data, which named a SMDDL model. The procedure of the SMDDL model is by generating lead-time data based on the parameters of the original lead-time distribution (lead-time collected data) and integrating the demand data probability distribution parameters (i.e., mean and standard deviation) which is extracted from a demand forecasting method inside the generated lead-time data probability distribution. Afterward, the structure and the algorithm of the integrating of demand during lead-time probability distribution data to extract the mean and the standard deviation and the parameters of the distributions. This procedure is repeated for any adopting times, such as 1000 or 1500 times.

The differ of this model with previous studies, in the previous studies, e.g., Dekker, et al.(1998), Van der Heijden et al, (1999), Tang and Grubbström (2003), Chiang and Monahan (2005), Lee, (2005), Axsater (2006), Li and Sheng (2008), Baten and Kamil (2009) and Axsäter and Viswanathan (2012) the probability of demand and lead-time were taken separately without extracting and establishing the probability distribution of demand during lead-time for the purposes of their study.

In addition, the complexity of the mathematical derivations of integrating the distribution of demand during lead-time in the total cost function of inventory to develop the inventory performance measures. Subsequently, these two parameters (i.e., mean and standard deviation) of demand during lead-time play an essential role in the approximation mathematical model. The SMDDL model is discussed in details in Section 4.5.

4.4.4 Arrival rate data

Arrival rate is the number of customers or retailers that arrive at the system during a certain period of time. For example, between 10 am and 11 am, 15 customers or retailers enter the system to receive the service and this process is repeated during the day for all of the periods mentioned above. This type of data is called arrival rate distribution, which is represented by λ and distributed Poisson. The extraction of this type of data takes time and effort. In order to obtain the arrival rate data, we need to know how many retailers enter the system for each hour during the selected periods of the three years to obtain the product. Later this type of data is used as input to the DMEI-FCFS model.

4.4.5 Service rate data

Service rate is the mean of retailers that can be served at 100% utilization by each individual server per unit time (normally per hour or day) or the time spent to having the service. This type of data is called service rate distribution or departure distribution. The extraction of this type of data is similar to that of lead-time data. The difference is that the service rate data is to be the input of the DMEI-FCFS model as exhibited in Figure 4.2.

4.4.6 Holding and setup costs

This research includes two types of costs, holding cost, h, and setup cost, A. A holding cost is the cost that is incurred by the firm or the project when materials are stored in the warehouse or the manufacturer cost when products are stored in warehouses. A setup cost is the cost incurred to obtain equipment ready to process a various batch of items. Hence, the setup cost is considered a batch-level cost in activity-based costing. It is not easy to obtain data on holding and setup costs directly because of the sensitivity and complexity of the data as well as the overlapping of the components of these costs with each other.

The data of holding cost and setup cost were obtained in two ways. First, they were obtained from the managers and accountants in the company concerned (UCC). Second, the data were obtained from the periodical or yearly reports, which have information about the capacity, total investment cost, selling price, and cash cost per tonne (Brokerage, 2010; Lafarge, 2012). The value of holding cost, h is;

$$h = (I)(C) \tag{4.1}$$

where, I is the holding cost that is always represented in the percentage of inventory levels and includes the investment cost; insurance cost; and opportunity cost while C is the unit price cost. The value of I can be extracted by the investment cost per tonne, IC divided by the expected daily demand, D_L.

$$I = \frac{IC}{D_L} \tag{4.2}$$

using the values of holding cost h, the expected daily demand, D_L and the price per tonne, we can extract the value of setup cost, A, as follows

$$A + h = \frac{(D_L)(SP)}{IC}$$

where, SP is the selling price per tonne. Therefore,

$$A = \frac{(D_L)(SP)}{IC} - h \tag{4.3}$$

4.5 SMDDL model

After the first stage, this research develops a structure and an algorithm to establish the probability distribution function of demand during lead-time data by a set of simulation procedures known as the SMDDL model. The aim of the SMDDL model is to generate data of demand during lead-time because this type of data is not recorded in reality due to the nesting of the activities and complexity of the procedures. There are two key purposes of the SMDDL model: (a) to establish the probability distribution function of the demand during lead-time, which, in turn, is used to develop the approximation mathematical model; and (b) to extract the mean and the standard deviation of the demand during lead-time which, in turn, is used to meet the remaining research objectives. Therefore, the implementation structure and algorithm of the SMDDL model are as follow:

1. Analyze the historical data of the demand and then extract the mean and standard deviation using a suitable forecasting method based on the probability distribution function of the demand data. Often in previous studies, e.g., (Baykal-Gurosy & Erkip, 2010; Brown, 1959; Snyder et al.,

2004; wang et al. 2010; Wang, 2009) as mentioned in Chapter Three, data for long periods were distributed as a normal distribution. Moreover, the forecasting method relies on the pattern and fluctuations of the demand data. For example, increasing or decreasing fluctuation trends, seasonal fluctuation trends or stationary fluctuation trends. Each type of these fluctuation trends and patterns have its own method for forecasting (Baykal-Gurosy & Erkip, 2010; Choi e al., 2011; Wang, 2009; Wang & Lin, 2010). The importance of demand forecasting is much higher in Made-to-Stock (MTO), Assemble-to-Order (ATO) or just in time (JIT) Supply Business. In order to keep customer's satisfaction, we need to provide them with the product they want when they need it. This advantage of forecasting in production or business will help predict product demand so that enough product is available to fulfill customer orders with short lead time and on-time.

2. Analyze and test the lead-time data to extract the parameters of the lead-time probability distribution using the Kolmogorov-Smirnov (K-S) test.

In generating demand during lead-time, the SMDDL model prepares an algorithm for this purpose as follows: Simulation algorithm generates a random number no less than 1,500 observations of lead-time depending on the original lead-time parameters that have been analyzed (step two). Next, 1,500 random numbers are generated inside each random number of the generated lead-time based on the demand mean and standard deviation which is extracted from the forecasting method. In order to show the demand during lead-time, we integrate these two distributions of lead-time and demand. In other words, when generating lead-time data for 1,500 random numbers, we generate data according to the forecasted demand mean and standard deviation of each value of the generated lead-time data. The following example illustrates the idea better.

Suppose the initially generated value of lead-times is equal to 6.57. Then, generate a value inside the initial value of the lead-time according to the forecasted demand parameters (i.e., mean and standard deviation) as much as the initial value which is 6.57 times and calculate their sum. Repeat these procedures for the second lead-time generated value, third lead-time generated value, and as much as the lead-time generated data (1,500 times). The result of this process is the SMDDL model that represents the demand during lead-time data and has 1,500 observations of demand during lead-time. To show which probability distributions are identical with the generated demand during lead-time, the *p*-value, in contrast to fixed values, is calculated based on the test statistic, and denotes the threshold value of the significance level in the sense that the null hypothesis H_0 will be accepted for all values of α less than the *p*-value. For example, if $p = 0.025$, the null hypothesis will be accepted at all significance levels less than *p* (i.e., 0.01 and 0.02), and rejected at higher levels, including 0.05 and 0.1. The *p*-value can be useful; in particular, when the null hypothesis is rejected at all predefined significance levels, and it need to know at which level it could be accepted. The *p*-values depend on the Kolmogorov-Smirnov, (K-S) test calculated for each fitted distribution.

Figure 4.4 shows a flow chart that describes the steps of generating data of demand during lead-time.

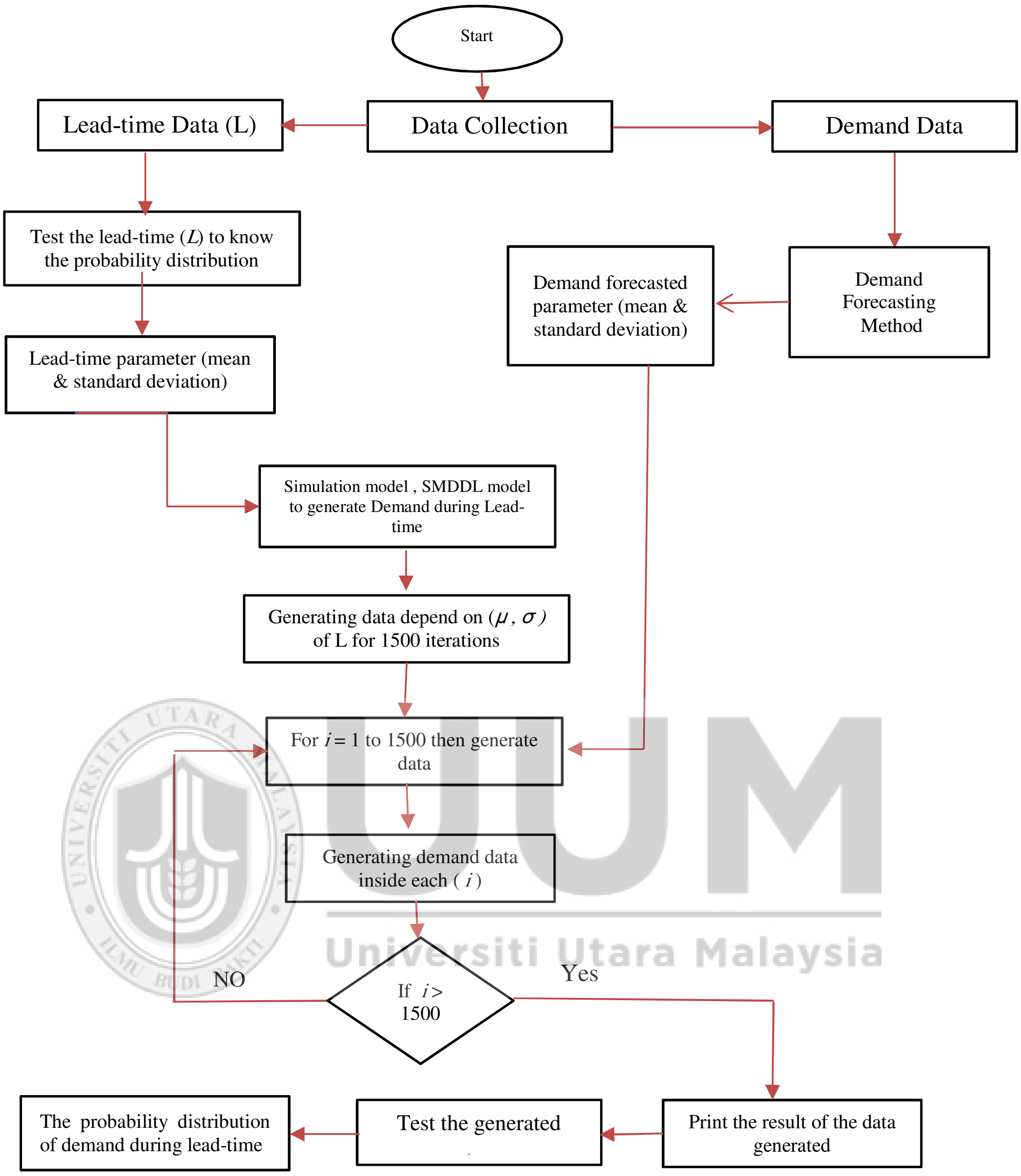


Figure 4.4. Framework of SMDDL for generating demand during lead-time data

The question that appears here is why we generate data 1,500 times? Why not for more or less than this number? The answer to this question is that typically when generating data based on a certain algorithm in the simulation procedures, there is a seed number that controls the generation. Therefore, when generating data 1,500 times, there must be a compatible seed number with the generated numbers.

Furthermore, when generating more or less than 1,500 times, there must be a compatible seed number for each generation. As a result, to ensure the validity of the considered simulation procedures, the standard error, *SE,* of the generated data and the original lead-time distribution must be similar as recommended by several researchers (Fishman, 1973; Forbes et al., 2011; Good & Hardin, 2006).

The next step is the algorithm of generating lead-time data. The procedures of the SMDDL are as follows:

Let X be a continuous random variable drawn from a population with a non-integer α. We may consider X to be the sum of $K+1$ independent variants, all with a scale parameter of β, but the first K has a unit shape parameter of α, while $K+1$ has a shape parameter of $\gamma = \alpha - [x]$. Assume Y and Z have independent variants from beta $(\gamma, 1-\gamma)$ and $G(1, 1)$, respectively. Then, $W = \beta YZ$ has variants with $G(\gamma, \beta)$. To see this, we note that:

$$f_{y,Z}(y,z) = \begin{cases} \dfrac{1}{\Gamma(\gamma)\Gamma(1-\gamma)} y^{\gamma-1}(1-y)^{-\gamma}e^{-z} & 0 \le y \le 1, \le z \le \infty \\ 0 & \text{otherwise} \end{cases}$$

Then

$$f_{W,Z}(w,z) = \begin{cases} \dfrac{\beta^{-\gamma}}{\Gamma(\gamma)\Gamma(1-\gamma)} w^{\gamma-1}\left(z - \dfrac{w}{\beta}\right)^{-\gamma} e^{-z} & 0 \le w, z \le \infty \\ 0 & \text{otherwise} \end{cases}$$

$$f_W(w) = \int_0^{\infty} f_{W,Z}(w,z)dz = \begin{cases} \dfrac{\beta^{-\gamma}}{\Gamma(\gamma)} w^{\gamma-1} e^{-w/\beta} & 0 \leq w \leq \infty \\ 0 & w \leq 0 \end{cases}$$

which is $G(\gamma, \beta)$. Therefore, the generation algorithm will be as follows:

Algorithm for the program generation to simulate from the distribution with shape parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, $\boldsymbol{G(\alpha, \beta)}$

Set $K \leftarrow$ Int (α), $\gamma \leftarrow \alpha - K$

$K \leftarrow G(1)$

Generate K from $G(1)$ and compute V as this sum

Generate Y from Beta $(\gamma, 1 - \gamma)$ and Z from G(1) as follows:

    Compute K_1 as Int (a) and K_2 as Int (b)

    Set $\gamma_1 \leftarrow \alpha - K_1,\ \gamma_2 \leftarrow b - K_2$

    Generate K_1 from $G(1)$ and compute X_1^* as their sum

    Generate K_2 from $G(1)$ and compute X_2^* as their sum

    Generate Y_1, Z_1, Y_2 and Z_2 from beta $(\gamma_1, 1 - \gamma_1)$

    Compute $X = (X_1^* + Y_1Z_1)/(X_1^* + X_2^* + Y_1Z_1 + Y_2Z_2)$ from Beta (a,b)

Compute $X = \beta(V + YZ)$ from $G(\alpha, \beta)$.

$G(\alpha, \beta) \leftarrow U_j$ # U_j denotes the random number generator

---

The flow chart in Figure 4.5 shows these steps in detail for generation in a flow chart.

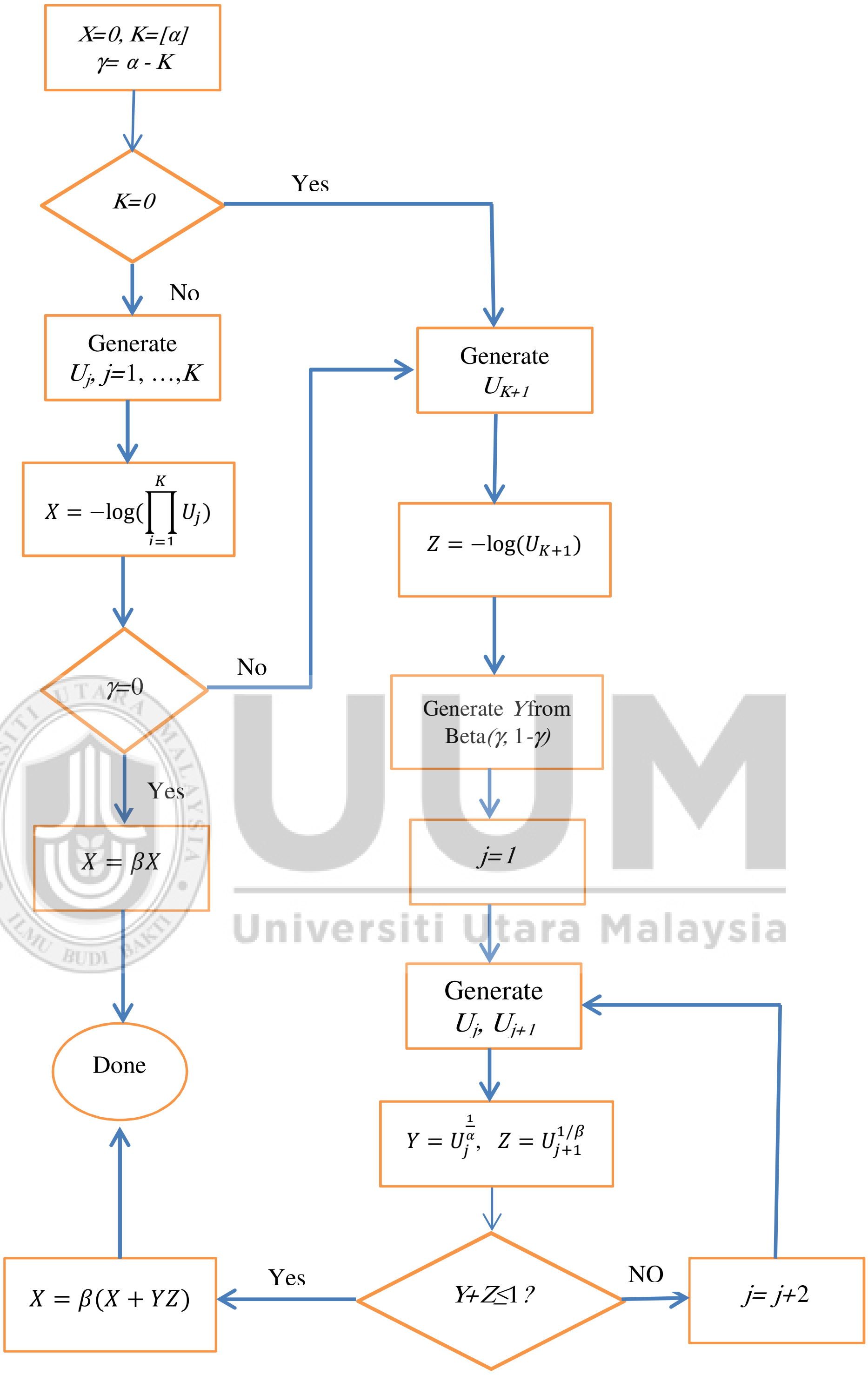


Figure 4.5. Lead-time flow chart generation

4.6 Proposed models

This research considers a multi-echelon inventory system under a continuous review *(R, Q)* policy in a supply chain that includes four echelons (the manufacturer that produces one type of product (cement), warehouses with three installations, distribution centers with limited lines or installations equal to six and *N* retailers). In this multi-echelon model, we develop an approximation mathematical model by dividing it into two sub-models: (a) the serial multi-echelon inventory system under a continuous review *(R, Q),* SMEI *(R,Q)* policy to achieve research objectives two to five, which is an extension of (Axsäter, 2011), and (b) the distribution multi-echelon inventory system under the first-come-first-serve FCFS queue model, called the DMEI-FCFS model to achieve objective six, which is an extension of (Axsater, 2010). In Axsater (2010) model, a simple production inventory system with single-echelon and one service provider channel M/G/1 model was developed. However, in DMEI-FCFS model, multi-echelon and multi-channels service providers under the first come first serve M/G/C-FCFS model is proposed.

The first sub-model, which is a serial multi-echelon inventory system under a continuous review *(R,Q),* SMEI *(R,Q)* policy aims to: (a) formulate the order quantity, *Q* (Objective 2), reorder point, *R,* at each installation and echelon (Objective 3) based on the probability distribution function of demand during lead-time, and SMDDL model (Objective 1); (b) optimize the safety stock, *SS*, which is a part of the reorder point (Objective 4); and (c) extract the approximate total cost (Objective 5) and the inventory position at each installation and echelon. A discussion of this model is detailed in Section 4.6.1.

The second sub-model, which is a distribution multi-echelon inventory system under the first-come-first-serve FCFS queue model, the DMEI-FCFS model aims to develop the performance measures in the considered model to reduce the long waiting time in the system between the distribution centre and the retailers by applying an inventory FCFS queue model (Objective 6). A discussion of this model is detailed in Section 4.6.2.

In order to start developing the approximation mathematical model for the probabilistic multi-echelon inventory system, some assumptions and notations are imposed.

- **The assumptions**
  1. Demand occurs only at the final echelon (the retailers).
  2. Each of the demand and the lead-time is probabilistic for the whole system.
  3. The long lead-time appears between the distribution centre and the retailers because of the huge number of retailers and a limited number of distribution centers. Therefore, this leads us to use the first-come-first-serve (FCFS) condition.
  4. The demand is probabilistic and subject to continuous probability density function.
  5. The lead-time is probabilistic and subject to continuous probability density function.
  6. Only holding cost and setup cost are adopted. Shortage and backorders costs are not allowed.

- **The notations**

Q: Order quantity or the lot size

R: Reorder point

A: Setup cost per order.

h: Holding cost per unit.

$f(x)$: Probability density function of demand.

$F(x)$: probability density function of demand during lead-time

D_L: Expected demand / per unit time (the mean of demand probability distribution)

L: Total lead-time in the system

L_1: Lead-time between the distribution central and retailers

SS: Safety stock

μ_L: Mean of demand during lead-time

σ_L: Standard deviation of demand during lead-time

μ : Mean of service rate

σ: Standard deviation of the service rate

CV: Coefficient of variation

SE: Standard Error

λ: Intensity arrival distribution that is Poisson

ρ: Traffic density

S: Order-up-to level

α_1: Production rate/per unit per unit time and equal to uniform replenishment rate. Also, $\alpha_1 > D_L$ (the production rate should be greater than the demand rate in order to not fall into a shortage rate)

W_s: Expected waiting time of demand at the retailer

W_q: Expected waiting time in the queue

L_S: Expected number of the arrival in the system

L_q: Expected number of the arrival in the queue

C (Q, R): Expected cost per unit time to be a function parameter of order quantity *Q* and reorder point *R*

IL_j^i: Inventory level of installation stock at installation j before period demand

IL_j^e: Inventory level of echelon stock at installation j before period demand.

IP_j^e : Inventory position of echelon stock at installation *N*

IP_{j-1}^e : Inventory position of echelon stock at installation *N-1*, in period $(t+L_{j+1})$.

4.6.1 Development of SMEI *(R, Q)* model

The aim of this section is to develop an approximate mathematical model by reformulating the order quantity, *Q*, the reorder point, *R* and optimizing the safety stock, *SS*. Moreover, the inventory positions, *IP,* and inventory level, *IL*, are developed to achieve the expected minimum total cost *C(R, Q)* function under a serial multi-echelon inventory continuous review *(R, Q),* known as the SMEI *(R, Q)* model. To achieve the new function of the total cost function, the variables that play a role in the SMEI *(R, Q)* model should developed: the order quantity, *Q*, the reorder point, *R*, and the safety stock, *SS*, which is a part of *R*.

The extraction of these variables depends on the novelty of the demand during a lead-time probability, i.e. the SMDDL model. Therefore, we need to extract a new formula of order quantity, *Q*, depending on the result of the SMDDL model and the safety stock, *SS,* that leads to the reorder point at each installation R_n^i and at each echelon R_n^e.

4.6.1.1 Development of the approximation mathematical model for order quantity

The development of order quantity, Q, when the demand and the lead-time are probabilistic is subject to two constraints: (a) the demand during lead-time probability distribution which is extracted by the SMDDL model and (b) the function of total cost in the multi-echelon inventory system under continuous review *(R, Q)* policy. The first mathematical model of a multi-echelon inventory system adopts each echelon separately in order to determine the purchasing quantities, which lead to minimizing the value of total cost based on the periodic review system (Clark & Scarf, 1960). Many researchers, e.g., (Bessler & Veinott, 1966; Federgruen & Zipkin, 1984; Schmidt & Nahmias, 1985) adopted Clark and Scarf's (1960) model and extended it to different policies. However, Axsäter's (2011) work is mostly related to this study as the former considered a single-echelon inventory system with continuous review and Poisson demand.

In order to determine the order quantity for a multi-echelon inventory system, it should be observed that the order quantity, *Q,* for a multi-echelon inventory system is not optimal to deal with at each stage individually. The choice of *Q* at a certain stage will affect the demand structure primarily at the next upstream stage. This makes the *Q* determination more complex. Even though it is not optimal to determine *Q* for each stage individually, this is common in practice because it is easy and leads to ordering a very small quantity.

When dealing with order quantity, either in a single-echelon or multi-echelon system, most literature assumes that the customer demand is known. Furthermore, it is assumed that all the lead-time is constant, and, in fact, equal to zero. In case of probabilistic demand, it is normally reasonable to replace the probability demand by its mean (the mean of demand probability distribution function) and use a deterministic model when determining order quantity.

However, when the demand and the lead-time are subject to probability distribution function separately, the procedure is much more difficult. The determination of the order quantity, *Q*, will be as follows: when the inventory level, *IL*, reaches the reorder point, *R*, an order quantity to promote the inventory is ordered. The aim of this procedure is to reach a new formula of *Q* and *R*, which, in turn, minimizes or achieves the expected total inventory cost in the unit time. Therefore, the development process of *Q* requires is to:

Integrate the distribution of Generalised Gamma four parameters as Equation (3.23) into Equation (3.37) with the function of the total cost, where Equations (3.23) and Equation (3.37) respectively given by:

$$GG(\alpha,\beta,k,\gamma) = f(x) =$$

$$\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}\Gamma(\alpha)} e^{-\left(\frac{(x-\gamma)}{\beta}\right)^{k\alpha}} \quad x \geq 0 \text{ and } \alpha,\beta,k,\gamma > 0$$

$$C(R,Q) = A\frac{D_L}{Q} + h\left(\frac{Qb}{2} + \int_0^\infty (R - D_L)\, f(x)dx\right)$$

Therefore,

$$C(R,Q) = A\frac{D_L}{Q} + h\left(\frac{Q\left(1-\frac{D_L}{\alpha_1}\right)}{2} + \int_0^{\infty}(R-D_L)\,\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}\Gamma(\alpha)}\,e^{-\left(\frac{(x-\gamma)}{\beta}\right)^{k\alpha}}dx\right) \tag{4.4}$$

Let $b = 1 - \frac{D_L}{\alpha_1}$, thus,

$$C(R,Q) = A\frac{D_L}{Q} + h\left(\frac{Qb}{2} + \int_0^{\infty}(R-D_L)\,\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}\Gamma(\alpha)}\,e^{-\left(\frac{(x-\gamma)}{\beta}\right)^{k\alpha}}dx\right) \tag{4.5}$$

From Equation (3.36), $R - E(D_L) = R - \mu_L$. Therefore,

$$C(R,Q) = A\frac{D_L}{Q} + h\left(\frac{Qb + 2(R-\mu_L)}{2} * \;\Gamma(\alpha)\int_0^{\infty}\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}}\,e^{-\left(\frac{x-\gamma}{\beta}\right)^{k\alpha}}dx\right) \tag{4.6}$$

We simplify the integral part which is

$$y = \int_0^{\infty}\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}}\,e^{-\frac{(x-\gamma)^{k\alpha}}{\beta^{k\alpha}}}dx \tag{4.7}$$

For this, we derive the first derivative of $-\left(\frac{x-\gamma}{\beta}\right)^{k\alpha}$

Taking $u = -\frac{(x-\gamma)^{k\alpha}}{\beta^{k\alpha}}$

$$\frac{du}{dx} = \frac{-k\alpha(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}}$$

$$du = \frac{-k\alpha(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}} \quad dx$$

$$\frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}} \, du = -\frac{1}{\alpha} \, du \tag{4.8}$$

By substituting Equation (4.8) into Equation (4.7), we obtain

$$\therefore y = \int_0^\infty \frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}} \, e^{-\frac{(x-\gamma)^{k\alpha}}{\beta^{k\alpha}}} dx$$

$$= -\frac{1}{\alpha} \int_0^\infty e^u \, du$$

$$= \frac{-1}{\alpha} \; e^{-\left(\frac{x-\gamma}{\beta}\right)^{k\alpha}} \Bigg|_0^\infty$$

$$= \frac{-1}{\alpha} \left(\frac{1}{e^\infty} - \frac{1}{e^{\left(\frac{\gamma}{\beta}\right)^{k\alpha}}} \right)$$

$$= \frac{1}{\alpha} \, e^{-\left(\frac{\gamma}{\beta}\right)^{k\alpha}} \tag{4.9}$$

Then, substituting Equation (4.9) in Equation (4.6) we obtain

$$C(R,Q) = A\frac{D_L}{Q} + h\left(\frac{Qb}{2} + 2(R-\mu_L) * \Gamma(\alpha)\left(\frac{1}{\alpha}\, e^{-\left(\frac{\gamma}{\beta}\right)^{k\alpha}}\right)\right) \tag{4.10}$$

The aim of Equation (4.10) is to extract the formula of *Q*. By taking the partial derivative of *Q* and equal it to zero, $\frac{\partial C(Q,R)}{\partial Q} = 0$, we obtain

$$0 = \frac{-AD_L}{Q^2} + \frac{hb}{2} \tag{4.11}$$

Multiply Equation (4.11) by $2Q^2$, we obtain

$$2AD_L = Q^2 hb$$

Thus,

$$Q^2 = \frac{2AD_L}{hb}$$

Taking the square root for both sides to obtain the value of *Q*, thus

$$\therefore\ Q = \sqrt{\frac{2AD_L}{hb}} \tag{4.12}$$

Equation (4.12) is the formula of order quantity, *Q*, under a continuous review *(R, Q)* policy when the demands during lead-time are distributed generalised Gamma four parameters. Moreover, Equation (4.12) is used only to extract the first-echelon quantities, *j* = 1. When *j* > 1, the equation of *Q* will change based on the reorder point, R, as well as the inventory position, IP. Therefore, the extraction of the new order quantity equations for $j > 1$ will be performed after the establishment of the reorder point, R, as exhibited in Section 4.6.1.3.

4.6.1.2 Optimisation of the safety stock

Regarding the measure of service level, SL, the probability of no stockout per cycle order is normally used with the continuous review and continuous model of demand. The order quantity is supposed to be given, and the problem is to identify the safety stock, SS, in order to have a nomination identifying the probability service level for the demand during lead-time, D_L, to be lower than the reorder point, R. Consequently,

$$P(D_L \leq R) = SL = \emptyset\left(\frac{R - \mu_L}{\sigma_L}\right) = \emptyset\left(\frac{SS}{\sigma_L}\right) \quad (4.13)$$

Since $SS = R - \mu_L$, for a given value of SL, the ratio of $\frac{SS}{\sigma_L}$ is equal to K. The ratio value of K (the safety factor) depends on the probability distribution of demand during lead-time. For example, if the demand during lead-time is normally distributed, the value of K is taken from the normal distribution table under the adopted service level. However, in this research, the value of K is taken from the Gamma distribution table with two shape parameters under the considered service level. Therefore,

$$SS = K\sigma_L \quad (4.14)$$

In turn, Equation (4.14) represents a novelty in included elements because the safety factor, K, and the standard deviation, σ_L are extracted from the probability distribution by the SMDDL model.

Moreover, the safety stock, *SS*, plays an important role in determining the reorder point and/or if any delays occur in preparation or replenishment of the order quantity, Q, at the reorder point, R. Until Q is placed, the retailers' orders will be satisfied from the safety stock. Furthermore, any extra quantities of *SS* lead to an extra holding cost, and any lacking quantities lead to a shortage. Therefore, Equation (4.14) gives the optimal *SS* under a continuous review (*R, Q)* policy.

4.6.1.3 Establishment technique for the reorder point

One of the necessary measures in a multi-echelon inventory system is the reorder point, *R*, especially when the considered policy is *(R, Q)*. To establish the reorder point, regardless of which inventory policies are followed, the extracting of the reorder point consists of the safety stock, *SS*, plus the mean of demand during lead-time, μ_L, $R = SS + \mu_L$, which has to be an initial value for one echelon (single-echelon).

Therefore, the novelty of the extraction reorders point, *R* represented in the element, which are the safety stock and the mean of demand during lead-time. The problem appears when $n \geq 2$, i.e. when the number of echelons are more than one. In Section 3.11 of Chapter Three, the methods that calculate the reorder point at each echelon and installation were discussed. The initial value of the reorder point at the installation is equal to the reorder point at the echelon, which is Proposition 3.2 and is extracted from Equation (3.55).

Based on Equations (3.53) and (3.55), Table 4.1 displays how to establish the reorder point at each installation and echelon when $n \geq 2$.

Table 4.1

Establishing the Reorder Point at Installations and Echelons for N-echelons

N	$\boldsymbol{R_n^i}$	$\boldsymbol{R_n^e}$	$\boldsymbol{Q_n}$
1	$R_1^i = SS + \mu_L$	$R_1^e = SS + \mu_L$	Q_1
2	$R_2^e - R_1^e - Q_1$	$R_1^i + (R_1^i + Q_1)$	Q_2
3	$R_3^e - R_2^e - Q_2$	$R_2^i + \left[\left(R_1^i + Q_1\right) + (R_2^i + Q_2)\right]$	Q_3
4	$R_4^e - R_3^e - Q_3$	$R_3^i + \left[\left(R_1^i + Q_1\right) + \left(R_2^i + Q_2\right) + \left(R_3^i + Q_3\right)\right]$	Q_4
.			.
.			.
.			.
n	$R_n^e - R_{n-1}^e - Q_{n-1}$	$R_{n-1}^i + \sum_{j=1}^{n-1} (R_j^i + Q_j)$	Q_n

The Equations in Table 4.1 represent the values of the reached reorder point at each installation and echelon, R_n^i and R_n^e, to promote the inventory system by the order quantities, Q_n, under a continuous review *(R, Q)* policy.

The next step is to establish the order quantity at each echelon, Q_n,, which, in turn, depends on the inventory position at each echelon and installation, IP_n^e & IP_n^i.

Table 4.2 illustrates the procedure of how to establish the inventory position at the echelons and at the installations when $n \geq 2$ from Equations (3.54) and (3.55).

Table 4.2

Extracting the Inventory Position at Echelons and Installations for N-echelons

N	IP_n^e	IP_n^i
2	$(R_1^i + Q_1)$	$IP_2^e - IP_1^e = R_2^e + Q_2 - R_1^e - Q_1$
3	$(R_1^i + Q_1) + (R_2^i + Q_2)$	$IP_3^e - IP_2^e = R_3^e + Q_3 - R_2^e - Q_2$
4	$(R_1^i + Q_1) + (R_2^i + Q_2) + (R_3^i + Q_3)$	$IP_4^e - IP_3^e = R_4^e + Q_4 - R_3^e - Q_3$
.		.
.		.
.		.
n	$\sum_{j=2}^{n}(R_{j-1}^i + Q_{j-1})$	$IP_n^e - IP_{n-1}^e = R_n^e + Q_n - R_{n-1}^e - Q_{n-1}$

Equation (4.12) is used only for one echelon (single-echelon) in order to analyze the relationship between echelon inventory *(R, Q)* policies and installation inventory *(R, Q)* policies and the order quantity Q_n = order quantity of installation 1. Subsequently, note that in order to guarantee that the policies are stationary; the *Q* at each echelon should be on an integer multiplying by *Q* immediately succeeding the echelon. Therefore, $Q_n = j_{n-1}Q_{n-1}$, where j_n is an integer and nonnegative. This supposition is natural if the policy of rationing is to satisfy all or nothing of an order. Furthermore, when the installation inventory at installation *n* includes a number, it must always be of an integer of the next downstream order quantity (Q_{n-1}). Table 4.3 explains the procedure of how to establish Q_n for more than one echelon.

Table 4.3

Extracting the Value of Q_n by multiplying it with Integer j_n

N	j_n	j_nQ_{n-1}
2	$IP_2^i - R_2^i$	$(IP_2^i - R_2^i)Q_1$
3	$IP_3^i - R_3^i$	$(IP_3^i - R_3^i)Q_2$
4	$IP_4^i - R_4^i$	$(IP_4^i - R_4^i)Q_3$
.		.
.		.
.		.
n	$IP_n^i - R_n^i$	$(IP_n^i - R_n^i)Q_{n-1}$

The Equations in Table 4.3 illustrate that the order quantity at echelon two is equal to the inventory position at Installation 2 minus the reorder point at Installation 2 multiplied by the order quantity at Echelon 1. The missing content in this Equation is the order quantity at Echelon 1, Q_1. Therefore, Q_1 will be extracted from Equation (4.12), which is the first echelon and by compensating it in other equations, the order quantity for each echelon can be obtained.

4.6.1.4 Establishment of inventory level at each echelon

The best technique and original measure for identifying the inventory levels in an inventory system of a multi-echelon were introduced by Clark and Scarf (1960). This technique fits a serial system and can be considered a disintegration technique. This technique is fits for a two-echelon serial system with periodic review. Our suppositions (the continuous review [*R, Q*]*)* system with N-echelon, probability demand and lead-time) are different from those in Clark and Scarf (1960). The literature points out to those adopted this technique to either a single-echelon or two-echelon system (Diks & de Kok, 1998; Van Houtum & Zijm, 1991). The inventory level for *n* echelon IL_n^e was extracted as follows

$$IL_n^e = IP_n^e - \mu_L\,, \quad IL_n^e \geq IP_{n-1}^e \tag{4.15}$$

Table 4.4 illustrates the procedure of how to find the inventory level when $n \geq 2$. Note that the value of the reorder point at each echelon, IP_n^e, and the mean of the demand during lead-time, μ_L, was extracted in previous steps.

Table 4.4

Extracting Inventory Level for n ≥ 2-echelon

The echelons	$\boldsymbol{IL_n^e}$
2	$IP_2^e - \mu_L$
3	$IP_3^e - \mu_L$
4	$IP_3^e - \mu_L$
.	.
.	.
.	.
n	$IP_n^e - \mu_L$

4.6.1.5 Establishment of the approximate total cost

The function of a total cost is a very important measure to evaluate or compare the model with previous models as a basis to assess the proposed model. Therefore, establishing a new approximated total cost function based on the developed order quantity equation needs to be done. In order to extract the new formula of the approximated total cost for the proposed model, a serial multi-echelon inventory system under continuous review *(R, Q)* policy, the SMEI *(R, Q)* model is run. Equation (4.12) is integrated with Equation (3.39). This cost function is called an approximate total cost function since the value of setup cost, *A*, and holding cost, *h*, are approximated values.

Now by substituting Equation (4.12) into Equation (3.39), a new approximate total cost equation is obtained as follows:

$$C(R,Q) = \frac{AD_L}{\sqrt{\frac{2AD_L}{hb}}} + hb\left(\frac{\sqrt{\frac{2AD_L}{hb}}}{2}\right) \tag{4.16}$$

$$C(R,Q) = \frac{\sqrt{AD_L}}{\sqrt{\frac{2}{hb}}} + \frac{\sqrt{2AD_L hb}}{2} \tag{4.17}$$

Multiply Equation (4.17) by $\sqrt{hb}$

$$C(R,Q) = \frac{\sqrt{AD_L hb}}{\sqrt{2}} + \frac{\sqrt{2AD_L hb}}{2}$$

$$= \frac{\sqrt{2AD_L hb} + \sqrt{2AD_L hb}}{2}$$

$$\therefore\ C(R,Q) \ = \sqrt{2AD_L hb} \quad (4.18)$$

Note that, if the production price per tonne, PC is available, therefore, the quantities of the demand during lead-time, D_L multiplied by the PC will be added to Equation (4.18). Thus,

$$\therefore C(R,Q) = (PC * D_L) \ + \sqrt{2AD_L hb} \quad (4.19)$$

The Equation in (4.19) is the formula for the total cost function under a continuous review *(R, Q)* policy when the demands during lead-time are distributed to be Generalised Gamma four parameters.

4.6.2 Development of the DMEI-FCFS model

This model is developed to reduce the long waiting time in a three-echelon inventory system, which includes the main warehouse, distribution centre with six line, and unlimited *N* retailers by applying the FCFS queueing model. Thus, this model is named the DMEI-FCFS model. Figure 4.6 illustrates the structure of the model.

The DMEI-FCFS model is an extension of Axsater's (2010) model, which considered a simple production inventory system, one echelon (production system and inventory) that has Poisson demand, and stochastic production time with the Gamma distribution. This model is controlled by *S* policy and uses M/G/1.

Retailers in the DMEI-FCFS model have probabilistic demand with a Poisson distribution and a continuous review system policy, i.e., the inventory position at the installation is preserved by a certain 'order-up-to-level'. Order-up-to-level represents a private situation of *(R, Q)* policy. The inventory position is the safety stock (stock on hand) plus outstanding orders to determine the cost function.

Therefore, the DMEI-FCFS model analyzes the performance measures of the queueing model (M/G/6):(FCFS/∞/∞) with a *(R, Q)* policy in the supply chain multi-echelon inventory system, where *M* is the arrival distribution rate or the arrival time distribution of the retailers to the system, which is Poisson with an arrival rate, λ. *G* is the general distribution of departure or the service time with the rate, μ, which is distributed Gamma with the parameters α and β. Number 6 represents the parallel sources or the number of service provider stations (the distribution centre which has six lines to distribute the cement product). FCFS represents the service distribution discipline, and we impose FCFS to absorb a large number of retailers in the system. Finally, the numbers of queues in the system and customer source (community size) are infinite.

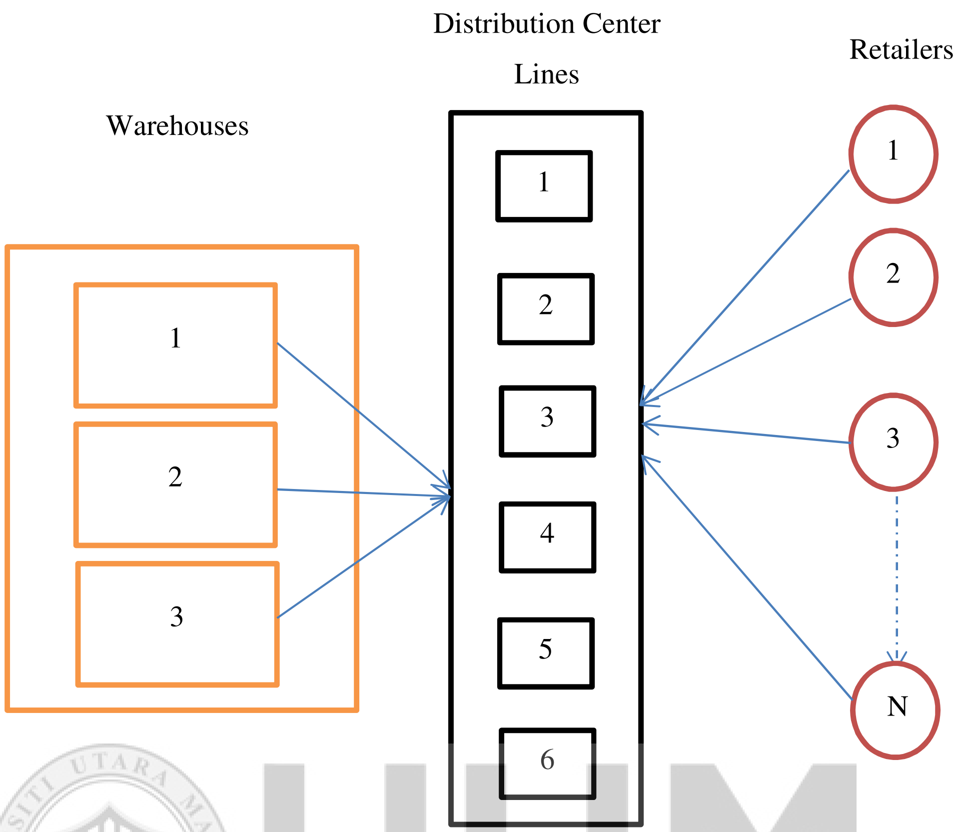


Figure 4.6. Three echelon supply chain network

The initial information about the considered model shows that there is a strong demand for cement. Also, because of a large number of retailers as well as a limited number of central distributions, a long waiting time in the queue and/or in the system until they get the product occurs. The waiting time in the whole system is very long. Therefore, the DMEI-FCFS model works to reduce the waiting time to a minimum level, in addition to extracting other performance measures in the considered model.

4.6.2.1 Development of the queue performance measures

Queueing models, in which the arrival and/or departure procedure do not follow the Poisson assumption, are more complex and may produce less tractable systematic results. In general, it is advisable in such cases to use simulation as a tool for analysis.

In this section, we present a non-Poisson queue model, of which analytic results are dealt with in our model *(M/G/C):(FCFS/∞/∞),* where G is the service time, described by a general probability distribution, which is Gamma distribution with a mean $E[t]$ and variance $V[t]$. In this situation, when the service time distributed Gamma we cannot get the performance measures (P_n ,L_s, L_q W_s and W_q) by the same method previously (as shown in Sections 3.13.1, 3.13.2 and 3.13.3) because the service time has a general probability distribution. Suppose λ is the arrival rate at service facilities and is given $E[t]$ and $V[t]$ to be the mean and variance of service time distribution. However, when the service time is distributed gamma, the $E[t]$ and $V[t]$ can be extracted as follows:

$$E[t] = \alpha\beta$$
$$V[t] = \alpha\beta^2$$

However, in Section 3.13.3, we proved how to determine the performance measures (L_s, L_q W_s and W_q) for more than one parallel service station. Now, when the service time has a general distribution (Gamma) and multiple service channels, C, the extraction of performance measures based on the steady state theory is as follows:

$$L_s = \lambda E[t] + \frac{\lambda^2\big[E[t]^2 + V[t]\big]}{C - \big[1 - \lambda E[t]\big]} \tag{4.20}$$

where, $\lambda E[t] < 1 \Rightarrow \rho < 1$

This expression is known to be the Pollaczek-Khintchine Formula (Krakowski, 1974). The theory of the steady state as presented in Section 3.13.1 Chapter Three explains the relationship between the performances measures (L_s, L_q W_s and W_q). From this relationship, if one of the performance measures is extracted, the rest can be obtained based on this relationship. Therefore, based on L_s as in Equation (4.20), we can obtain the rest of the queue performance measures as follows:

$$W_s = \frac{L_s}{\lambda} \tag{4.21}$$

$$L_q = L_s - \frac{\lambda}{E[t]} \tag{4.22}$$

$$W_q = \frac{L_q}{\lambda} \tag{4.23}$$

Note that the service rate, $\mu = \frac{1}{E[t]}$

Furthermore, we need to extract the value of the probability of zero numbers of retailers in the system or the system is idle, P_0, and the probability of arrival n units to the system, P_n.The values of P_0 and P_n depend on the value traffic intensity ρ. When the service time follows a general distribution and the server channel is more than one, the value of ρ will change and cannot be extracted according to the traditional methods. Therefore, we can reformulate ρ in the model and for the steady state to be:

$$\rho = \frac{\lambda}{cE[t]} \tag{4.24}$$

Therefore, the probability of zero numbers in the system or the system is idle

$$P_0(t) = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!\,(1-\rho)}\right]^{-1} \tag{4.25}$$

$P_0(t)$ represent the probability of zero numbers in the system (the system is idle). In contrast, $P_n(t)$ represent the probability of the system is busy. The probability of $P_n(t)$ depend on the server provider channels, number of retailers arrival to the system and $P_0(t)$. Therefore, the formula of $P_n(t)$ is:

$$P_n(t) = \begin{cases} \dfrac{1}{n!}\rho^n\, P_0(t) & n < c \\ \\ \dfrac{1}{c!\,c^{n-c}}\,\rho^n P_0(t) & n \geq c \end{cases} \tag{4.26}$$

4.6.2.2 Distribution of the arrival during Gamma service time

Now, the probability of arrival *n* units to the system for a period *t* as in Equation (3.67) is considered. The number of arrivals and the probability of service time are independent. Let q_j be the probability of demand *j*. Therefore,

$$q_j = \int_0^\infty \left(\frac{\beta(\beta x)^{\alpha-1}e^{-\beta x}}{\Gamma\alpha}\right)\left(\frac{(\lambda x)^j e^{-\lambda x}}{j!}\right) dx \tag{4.27}$$

It is helpful to note that $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ and using Equation (3.94) which is $\int_0^\infty x^\alpha e^{-\beta x} dx = \frac{\Gamma(\alpha+1)}{\beta^{(\alpha+1)}}$, and based on (Axsater, 2010; Axsäter, 2007) we can obtain:

$$q_j = \frac{\beta^\alpha \lambda^j \Gamma(\alpha + j)}{\Gamma(\alpha\,) j!\,(\lambda + \beta)^{\alpha + j}} \tag{4.28}$$

for $j = 0$, we have:

$$q_0 = \frac{\beta^\alpha}{(\lambda + \beta)^\alpha} \tag{4.29}$$

and for $j > 0$, we have:

$$q_j = \left(\frac{\beta^\alpha}{(\lambda + \beta)^\alpha}\right)\left(\frac{\lambda^j}{(\lambda + \beta)^j}\right)\left(\frac{\alpha(\alpha + 1).\ \ .\ \ .(\alpha + j - 1)}{j!}\right) \tag{4.30}$$

This implies that q_j has a negative binomial distribution that is not difficult to manage. At the point when μ is given and σ is equal to zero, the distribution in (4.29) and (4.30) will approach a Poisson distribution. Moreover, for $\sigma = 0$, or very small, it is then computationally much more efficient to substitute (4.29) and (4.30) by a Poisson distribution i.e.,

$$q_j = \frac{\left(\frac{\lambda\alpha}{\beta}\right)^j e^{\frac{-\lambda\alpha}{\beta}}}{j!}$$

$$\therefore\ q_j = \frac{(\lambda\mu)^j\, e^{-\lambda\mu}}{j!} \tag{4.31}$$

Recall from the steady state that was defined in Section 3.16.1 that the traffic density is equal to $\frac{\lambda}{c\mu}$, while the coefficient of variation, CV is equal to $\frac{\sigma}{\mu}$. The distribution of q_j in Equation (4.31) is expressed in parameters λ, μ, and σ. It is not difficult to note that it can be determined entirely by the two parameters ρ and CV. We refer that $\frac{\lambda}{\beta} = \rho CV^2$ and $\alpha = \frac{1}{CV^2}$.

4.7 Validation and evaluation of the models

Experimentation and verification are done on the proposed models by dividing this section into subsections as follows: (a) the validity and credibility of the SMDDL model and evaluating the whole system based on the SMEI *(R,Q)* model; (b) evaluating the DMEI-FCFS model based on the expected waiting time in the system, W_S, in addition to the effect of increasing the server channels' of service providers to the system.

4.7.1 Validation of the SMDDL model

In order to validate the generated data of the SMDDL model, some tests should be done on the generated data. We remind that, the SMDDL model generates two types of data in order to reach the demand during lead-time data. First, generating lead-time data based on the original lead-time parameters which are the basis for generating, then generating the demand during lead-time data. The validity implementations are as follows:

1. Extracting the mean and the standard deviation of the generated lead-time data distribution from the SMDDL model.
2. Calculating the mean and the standard deviation of the demand during lead-time regardless of the statistical distribution from Equations (3.17) and (3.18).

If the results of the mean and the standard deviation of the generated lead-time distribution by the SMDDL model and the mean and the standard deviation of the demand, as well as the lead-time regardless of the statistical distribution of Equation (3.17) and (3.18) are close, the statistical distribution of the generated data is correct. Otherwise, there is an error in the SMDDL model. When extracting the standard error, *SE* refers to the standard deviation of the sample statistic based on the criteria of mean (Forbes et al., 2011; Good & Hardin, 2006) . The small and similar criteria of standard error, SE, measures the accuracy that there is not a notable difference in the distribution (Ghosh et al., 2006), where the standard error, SE, is the term that measures the accuracy in which a sample represents a population or a certain phenomenon. The standard error is also inversely proportional to the sample size, in which a larger sample size means a smaller standard error because the statistic will approach the actual value. Standard error, SE, is equal to the standard deviation divided by square root of the sample size, $SE_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$.

4.7.2 Evaluation of SMEI *(R, Q)* model

The evaluation of the SMEI *(R, Q)* model is performed to investigate how good our model is by comparing it with two models and different criteria. First, the model by Elhasia, Noche, and Zhao (2013) that used sustainable supply chain management, SSCM, to analyze the cement industry operations by proposing three policies scenarios. These scenarios are 'Make-to-stock' (MTS), 'Pack-to-order' (PTO), and 'Grind-to-order' (GTO) strategies. The implementation of these three scenarios is created by using an Arena program based on a discrete event simulation (DES) model of SSCM. The comparison is made based on the total production cost in similar ergonomics, which is the cement industry.

The second model for comparison is Moslemi and Zandieh's (2011), they used a 'multi-objective practical swarm optimization' (MOPSO) algorithm, which depends on the Pareto control to address conflict goals in a continuous review probabilistic inventory *(r, Q)* system. They employed the MOPSO for the multi-objective inventory system problem to integrate the mutation factor for the conservation diversity in the swarm, in addition to exploring all of the search spaces into the MOPSO. The basis for choosing Moslemi and Zandieh's (2011) model is because their model was found to be a better model than its predecessors. In addition, they used geographically based (Grids), MOPSO (grids), and, alternatively, a crowding distance element to select the global optimal particle as leader. The criteria they used to make the comparisons were based on the coefficient of variation, *CV*.

The coefficient of variation, *CV,* is a measure or criteria used to compare between the dispersion data of two phenomena or more than one phenomenon. Meanwhile, the data that has a small *CV* means that it has less dispersion and vice versa. All experimentations are discussed in more detail in Chapter 5.

4.7.3 Evaluation of DMEI-FCFS model

The evaluation of the DMEI-FCFS model is performed to investigate how good the model by comparing it with the model by Mital (2012) which adopted an analytical approach based on real life data in accordance with the service level prescribed by the concerned authority. The queue discipline referred to either first-in and firs-out (FIFO). The study developed the performance measures of expected waiting tine in the system and the queue, W_s and W_q and the probability of zero units in the system, $P_0(t)$. The comparison is made based on the W_s and $P_0(t)$.

Moreover, a what-if analysis is conducted to be a part of comparative evaluations by using a simulation experiment on the proposed model to show the impact of adding more channels' of services providers to the system. All experimentations are discussed in more detail in Chapter 5.

4.8 Summary of the chapter

In order to accomplish the objectives of this research, five main stages were premeditated. In the first and second stages, the research explored the problem definition and the data source and types. In the third stage, the research developed the algorithm and the structures of how to generate data of the demand during lead-time probability distribution by the simulation procedures SMDDL model.

In the fourth stage, the approximate in mathematical model in a multi-echelon inventory system under a continuous review *(R, Q)* policy was developed by dividing the work into two sub-models, the SMEI *(R, Q)* and the DMEI-FCFS.

The SMEI *(R, Q)* model discusses the methods to develop the performance measures of order quantity, Q, the reorder point at each installation and echelon, R_n^i, R_n^e, the safety stock, *SS*, and the expected total cost *C (R, Q)*.

The DMEI-FCFS model discusses the methods to reduce the long waiting time in the system under an FCFS queue model when the arrival rate is distributed Poisson, and the service rate is distributed Gamma. Finally, this chapter discusses the evaluation and the comparison of the proposed procedure by certain criteria based on the previous models. In the next chapter, this research will examine the proposed models to test its feasibility.

CHAPTER FIVE

RESULTS AND DISCUSSIONS

In this chapter, the implementations of the proposed models are performed to generate the parameters based on the types of distribution obtained. From the parameters generated, the approximation models are developed to estimate the performance measures of the multi-echelon inventory system. Next, the evaluations of the proposed models are conducted by determining the selection criteria of performance measures.

5.1 Establishing demand data distribution

Demand data of the quantity amount of cement are recorded and presented in Figure 5.1. The figure shows that the demand is seen too high in the beginning period. However it reduces sharply to a normal pattern at the end of the period. Therefore, it can be concluded that demand data have a clear stationary pattern because there is no obvious an increasing and decreasing trend.

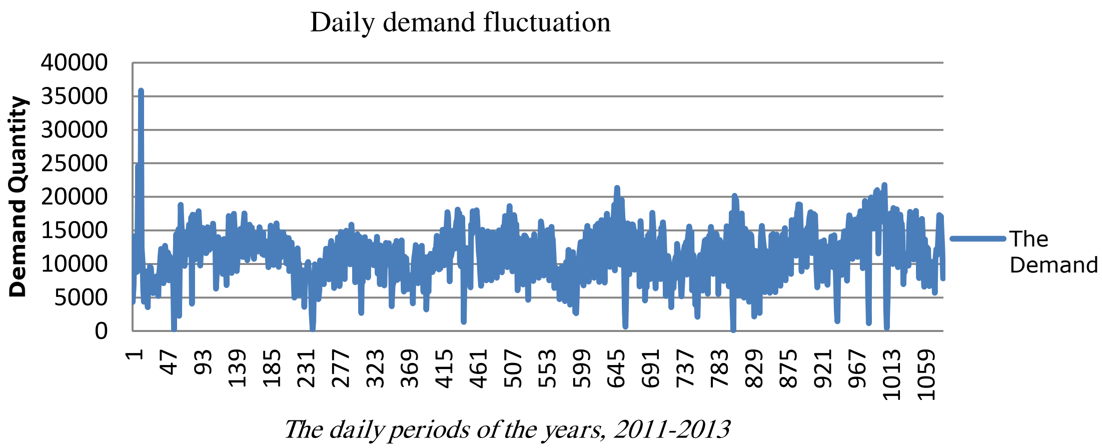


Figure 5.1. Daily demand fluctuations for 2011-2013

The demand data are analysed to examine the normality of the demand by using the P-P plot test (probability-probability plot or percent-percent plot) (IBM, 2011) as shown in Figure 5.2. Based on this plot, the demand data observed are nearly close to the straight line which reveals that the demand data is normal. This finding is also supported when the analysis of business forecasting is applied using the GMDH software version 3.6.1 (GMDH-SHELL, 2014).

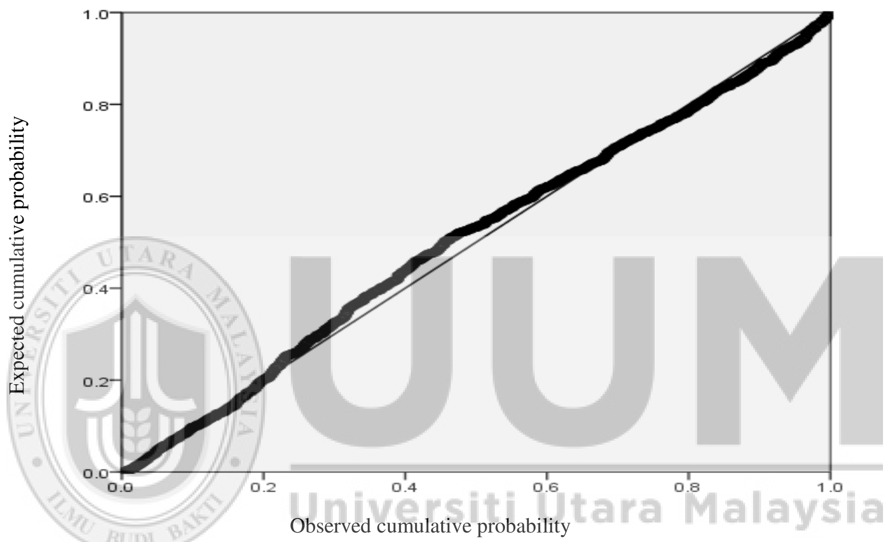


Figure 5.2. Analysis of normality base on P-P plot

The normality of demand data is also verified by plotting the histogram. The histogram in Figure 5.3 indicates the shape of a normal distribution based on demand data. The frequency of classes increasing gradually until it reaches the highest order quantity, which then was decreased gradually afterwards.

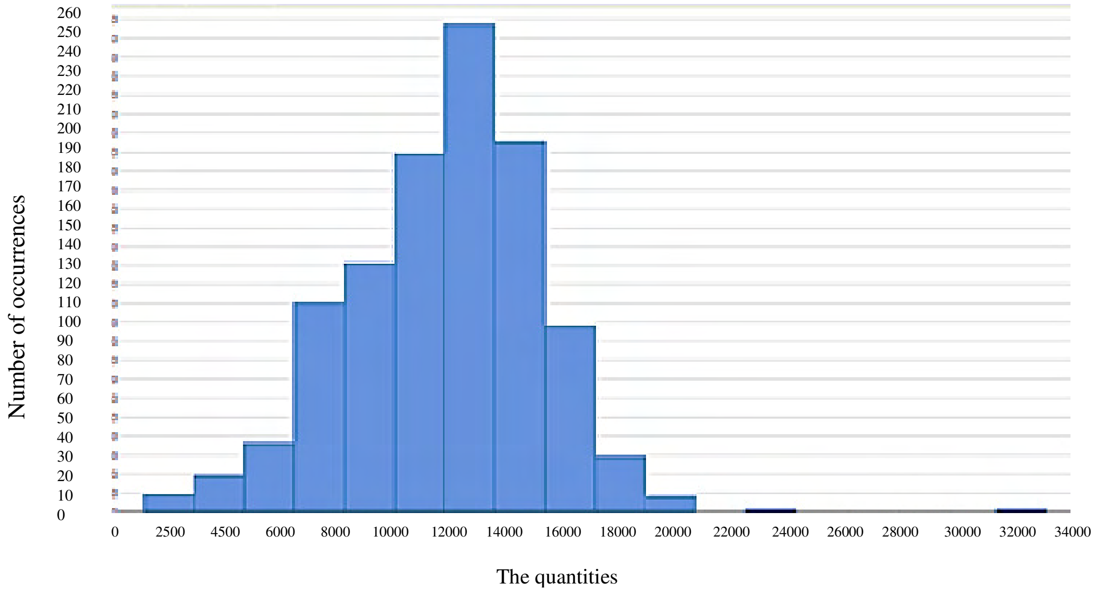


Figure 5.3. Normality analysis based on histogram

It seems that demand data are distributed normal and stationary pattern fluctuations. Therefore an exponential smoothing method (ES) is the most suitable forecasting method to extract the mean and the standard deviation as recommended by (Durbin & Koopman, 2012; Koehler, Snyder, Ord, & Beaumont, 2012). The ES method is used to forecast the demand data which shown in Figure 5.4. The mean (i.e., the expected demand per unit time) extracted for one period F_{t+1} from Equation (3.3), where, F_{t+1} is a forecasted demand value for the next period, and the standard deviation is extracted from Equation (3.14) which depend on MAD_{t+1} in Equation (3.16). MAD_{t+1} is a mean absolute deviation for the next period. The initial value of F_1 was extracted from Equation (3.10). The value of smoothing constant, α is chosen to be 0.1 based on Forbes et al. (2011), because it shows a small value of mean square error (MSE) (Bates & Granger, 1969 and Maydeu-Olivares & Garcia-Forero, 2010) as shown in Table 5.1.

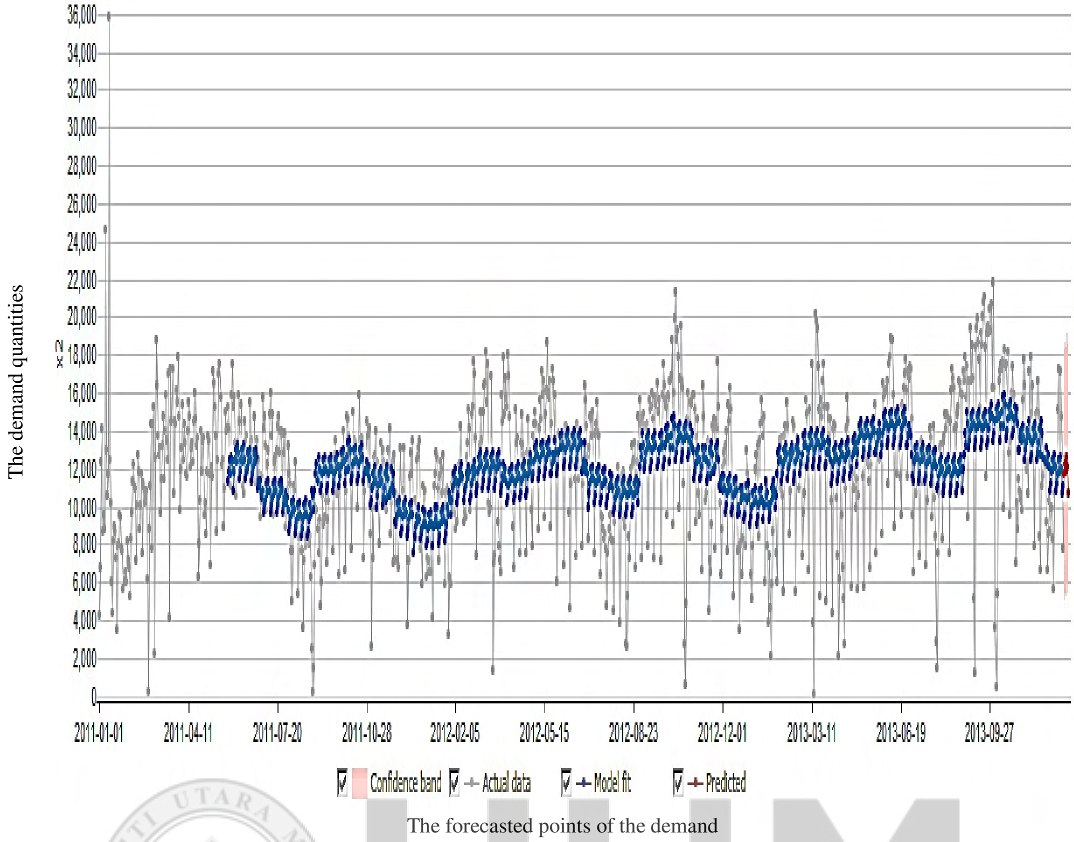


Figure 5.4. Forecasted demand during three years period

Figure 5.4 shows the forecasted values of demand data based on the ES method. Based on the figure, a clear stationary trend of the demand fluctuations are observed for both actual and forecasted demand during the period of 2011 until 2013.

Table 5.1

The Value of α based on the smallest MSE

Weights, α	**Value of MSE**
0.1	3330.07
0.2	3338.14
0.3	3334.95
0.4	3347.62
0.5	3367.91
0.6	3394.16
0.7	3428.21
0.8	3473.56
0.9	3534.23

Table 5.1 represents the values of MSE under different weights of α, where, the smallest value of MSE is when α equal to 0.1.

Table 5.2 shows the value of the expected daily demand, D_L, and the standard deviation, σ of demand distribution data based on the ES method. The detail procedures of the ES are available in Appendix A.

Table 5.2

Forecasted Mean and Standard Deviation for Demand Data

Measures	**Value (tonne)**
The mean, (D_L)	12037.75
The standard deviation, σ	3369.1413

Table 5.2 represents the expected daily demand, $D_{L,}$ of cement products in tonne and standard deviation generated using the ES method. These two parameters will be

considered as the inputs for the generation of the demand during lead-time probability distribution by implementing the SMDDL model.

5.2 Establishing Lead-time data distribution

The period of lead-time means the period confined between the preparations of the orders in tonnes until they reach the customers. These periods (days) are subject to probabilistic. The lead-time data need to be recorded and analyzed to determine its distribution. The analysis shows the daily lead-time data have more than the distributions using Easy Fit software version 5.6, (Mathwave,2015). The Kolmogorov-Smirnov test (K-S) is conducted to determine the daily lead-time data distributions. The distributions are Gamma distribution, Gamma three parameters distribution, Generalized Gamma four parameters distribution, generalized gamma, Weibull distribution and Weibull three parameters distribution. The significant and strongest p-value obtained is 0.985 which is from Gamma distribution with a shape parameter, α, and a scale parameter, β. Table 5.3 shows the p-values based on the Kolmogorov-Smirnov test (K-S) of each distribution. Figure 5.5 shows the plotted histogram of the Gamma distribution.

Table 5.3

The Distributions of the Daily Lead-time

Distributions	***p*-value based on the Kolmogorov-Smirnov test (K-S)**
Gamma	0.985
Gamma three parameters	0.812
Gen. Gamma four parameters	0.806
General Gamma	0.804
Weibull	0.570
Weibull three parameters	0.522

From Table 5.3, the strongest and highest significant *p*-value obtained is shown in Gamma Distribution with the *p*-value is 0.985.

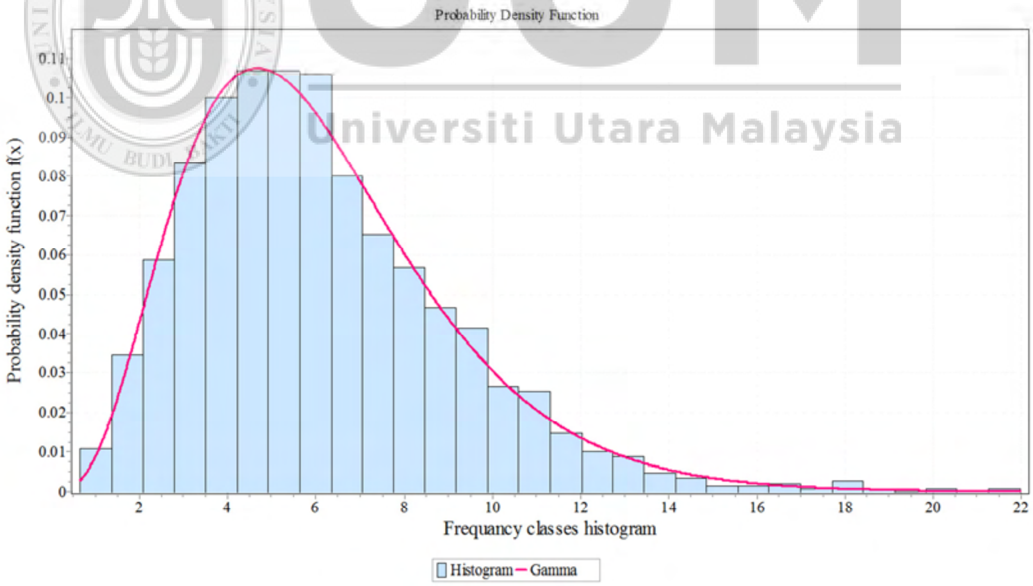


Figure 5.5 Gamma distribution as shown by the lead-time histogram

Table 5.4 shows the obtained parameter values of the daily lead-time data based on the significant *p*-value obtained which is Gamma distributed.

Table 5.4

Lead-time Parameters Based on the Gamma Distribution

Parameters	**Values**
The shape parameter, α	1.3484134
The scale parameter, β	4.4800435

5.3 The SMDDL model

SMDDL model is developed to establish a demand during lead-time distribution data by preparing an algorithm. Its implementation requires the mean and standard deviation values of the forecasted demand and the parameters of the extracted lead-time distribution.

Most of the studies Axsäter and Marklund (2008); Axsäter (1984); Graves (1986); Hausman and Erkip (1994); Hosoda and Disney (2006); Muckstadt (1986); Ravichandran (1995); Saffari and Haji (2009) and Zhao et al.(2006), treated demand data as constant or stochastic (normal, Poisson or compound Poisson) while the lead-time as constant, fixed, zero, or neglected. There are also some studies assuming probabilistic lead-time has normal, Poisson, Weibull, or Erlang distribution (Bagchi & Hayya, 1984; Baten & Kamil, 2009). These assumptions are valid in an inventory system when the market is stable and not subject to any sudden changes, such as political factors and security circumstances, as exemplified in this research. Unfortunately, these assumptions cannot be applied in this research as the scenario of cement production in Iraq-Kurdistan is unstable. Therefore, this research has established the new distributions for the demand during lead-time by implementing the SMDDL model which is realistic to represent the probabilistic scenario of the cement production.

The implementation of the SMDDL model shows five new probability distribution functions generated for demand during lead-time. The abnormal behaviors of the demand on cement and long lead-time in the system led to these new probability distributions of demand during lead-time which were not given attention in the past literature. The analyses of the SMDDL model are conducted by preparing an algorithm based on the structure of the SMDDL model as shown in Appendix E. The five distributions generated for the demand during lead-time are Generalized Gamma Four-Parameter, Pearson Type 6 Four-Parameter, Log-Pearson 3, Fatigue Life (Birnbaum-Saunders), and Inverse Gaussian Three-Parameter Distributions. The results and the details of these distributions which were obtained by the SMDDL model based on the Easy Fit software version 5.6 (Mathwave, 2015) are presented below.

5.3.1 Generalized Gamma, *GG* Four Parameters Distribution

The Generalized Gamma, *GG* distribution has four parameters, two continuous shape parameters, K and α, a continuous scale parameter, β and a continuous location parameter, γ. The distribution is also verified by plotting the histogram. The histogram in Figure 5.6 indicates the shape of a $GG(\alpha, K, \beta, \gamma)$ distribution based on the SMDDL model results. The frequency of classes starts increasing until it reaches the highest level, which then was decreased gradually afterwards.

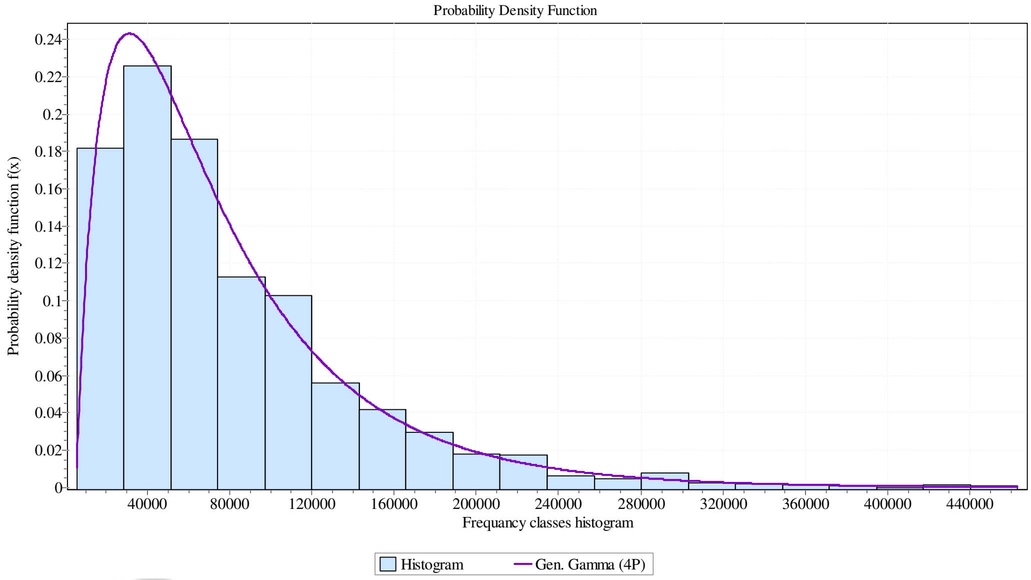


Figure 5.6. Generalized Gamma Four-Parameter distribution based on the demand during lead-time histogram

Moreover, the shape of the curve in Figure 5.7 matches with the Generalized Gamma four parameters shape and reveals the significance level of the Generalized Gamma Four-Parameter distribution with the *p*-value = 0.93584 based on the (K-S) test, which is a very strong and significant *p*-value.

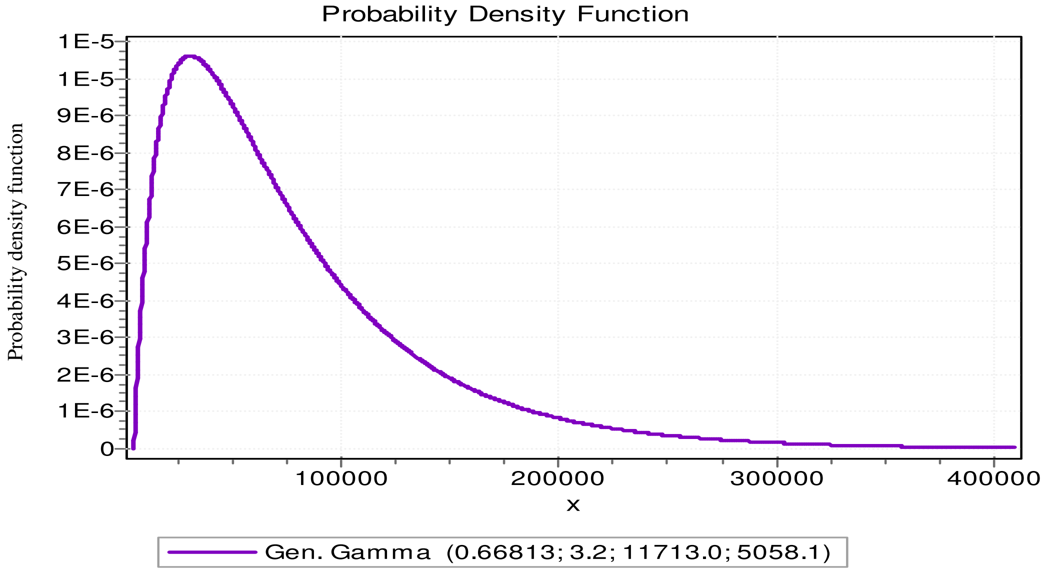


Figure 5.7 Generalized Gamma Four-Parameter Distribution of demand during lead-time

5.3.2 Pearson Type 6 Distribution four-parameters

The Pearson Type 6 distribution has four parameters, two continuous shape parameters, α_1 and α_2, a continuous scale parameter, β and a continuous location parameter, γ. The distribution is also verified by plotting the histogram. The histogram in Figure 5.8 indicates the shape of a Pearson Type 6 distribution based on the SMDDL model results. The frequency of classes starts from an increasing state until it reaches the highest level, which was then decreased gradually afterwards.

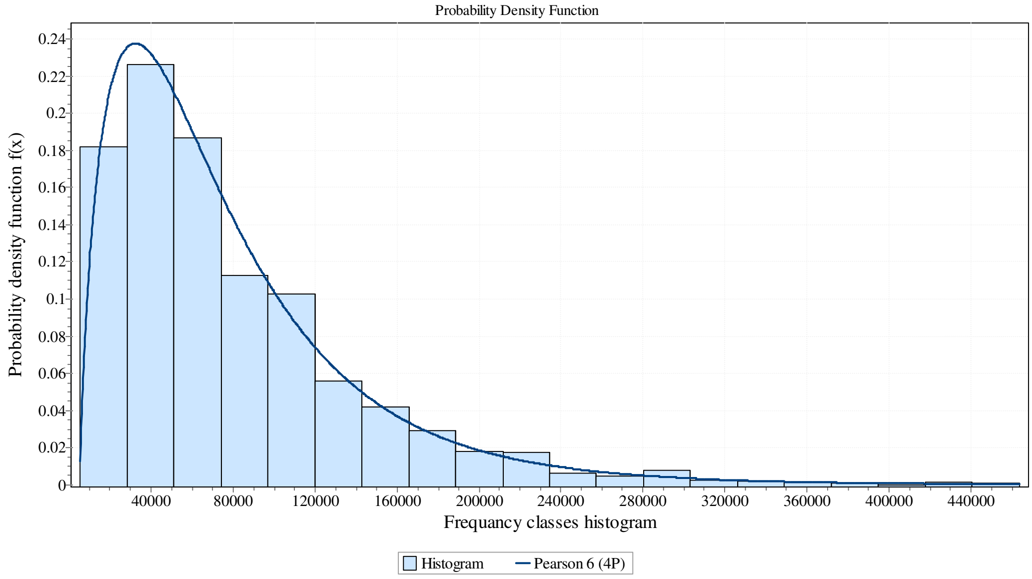


Figure 5.8. Pearson Type 6 Distribution Four-Parameter based on demand during lead-time histogram

Additionally, the shape of the curve in Figure 5.9 matches with The Pearson Type 6 distribution four-parameter shape and reveals the significance level of the Pearson Type 6 four-parameter distribution with the *p*-value = 0.85335 based on the (K-S) test, which is a very strong and significant. However, compared with the previous result, it was less accurate.

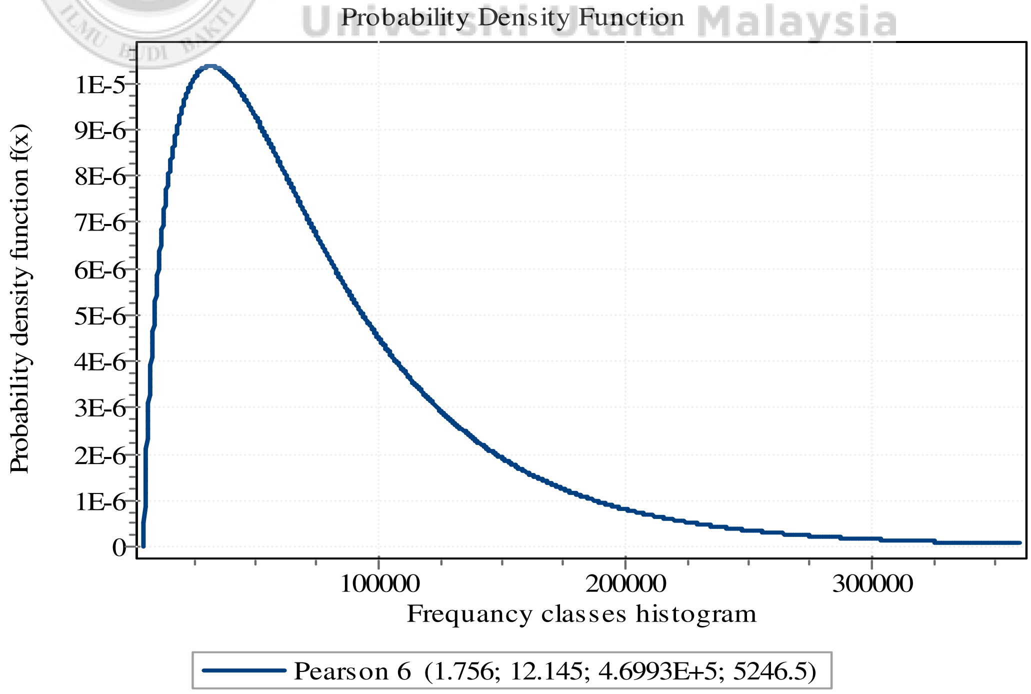


Figure 5.9. Pearson Type 6 Distribution of demand during lead-time

5.3.3 Log-Pearson 3 Distribution

The Log-Pearson 3 distribution has three parameters, a continuous shape parameters, α_1, a continuous scale, β and a continuous location parameter, γ. The distribution is also verified by plotting the histogram. The histogram in Figure 5.10 indicates the shape of a Log-Pearson 3 distribution based on the SMDDL model's results. The frequency of classes starts increasing until it reaches the highest level, which then was decreased gradually afterwards.

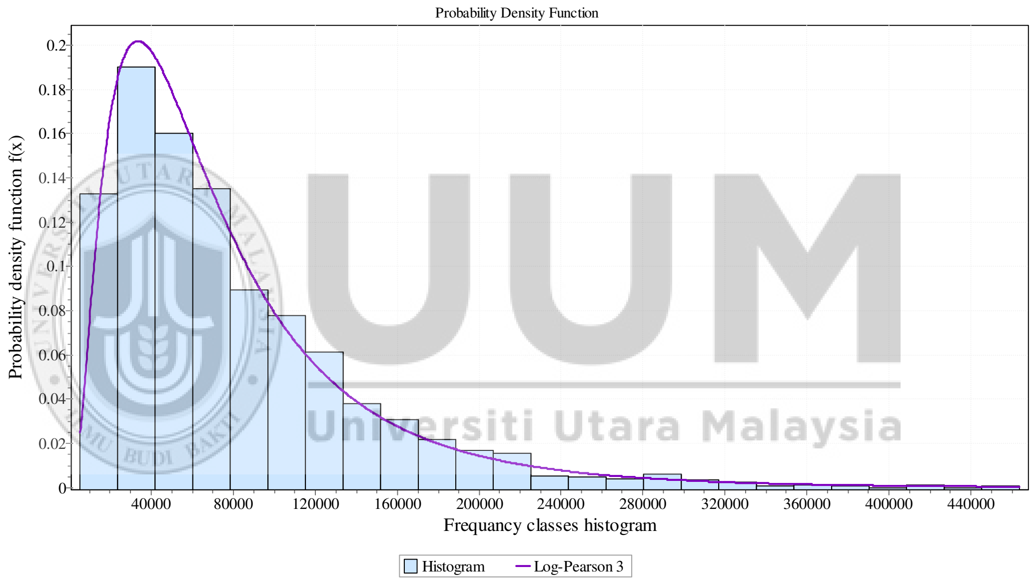


Figure 5.10. Log-Pearson 3 distribution based on the demand during lead-time histogram

Moreover, The shape of the curve in Figure 5.11 reveals the significance level of Log-Pearson 3 distribution with the *p*-value = 0.4285 based on the (K-S) test. When the *p*-value is compared with its predecessors, the value was less even though it had some significance. Even though the *p*-value was not strong, it was not rejected to be described a distribution.

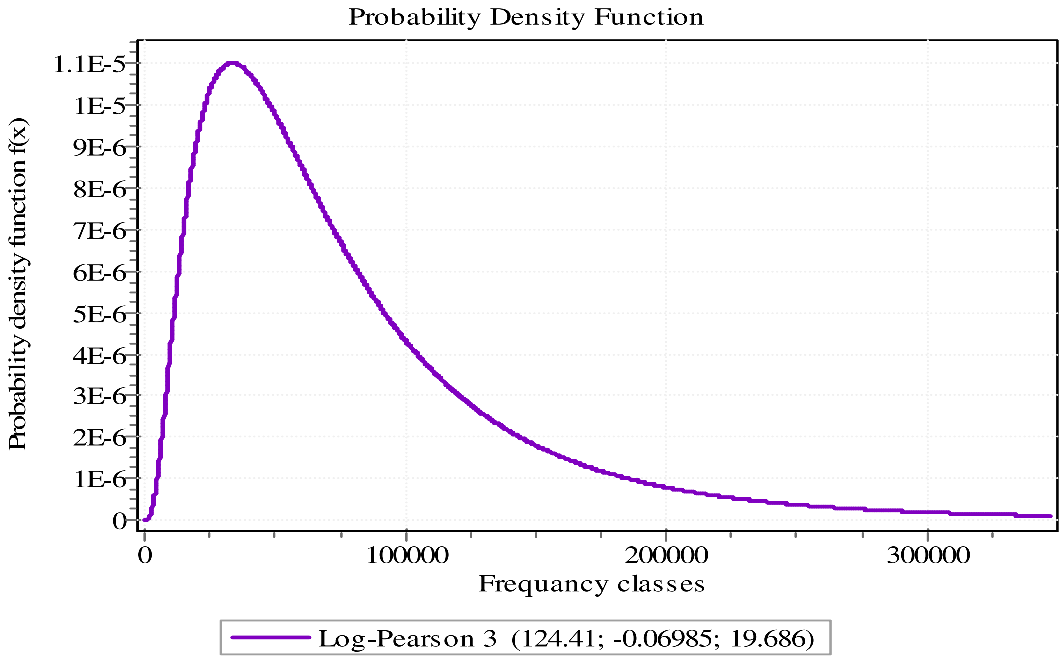


Figure 5.11. Log-Pearson 3 distribution of demand during lead-time

5.3.4 Fatigue Life (Birnbaum-Saunders) Distribution

The Fatigue Life (Birnbaum-Saunders) distribution has three parameters, a continuous shape parameter, α_1 a continuous scale parameter, β and a continuous location parameter, γ. The distribution is also proved by drawing the histogram. The histogram in Figure 5.12 indicates the shape of a Fatigue Life (Birnbaum-Saunders) distribution based on the SMDDL model's results. The frequency of classes starts increasing until it reaches the highest level, which then was decreased gradually afterwards.

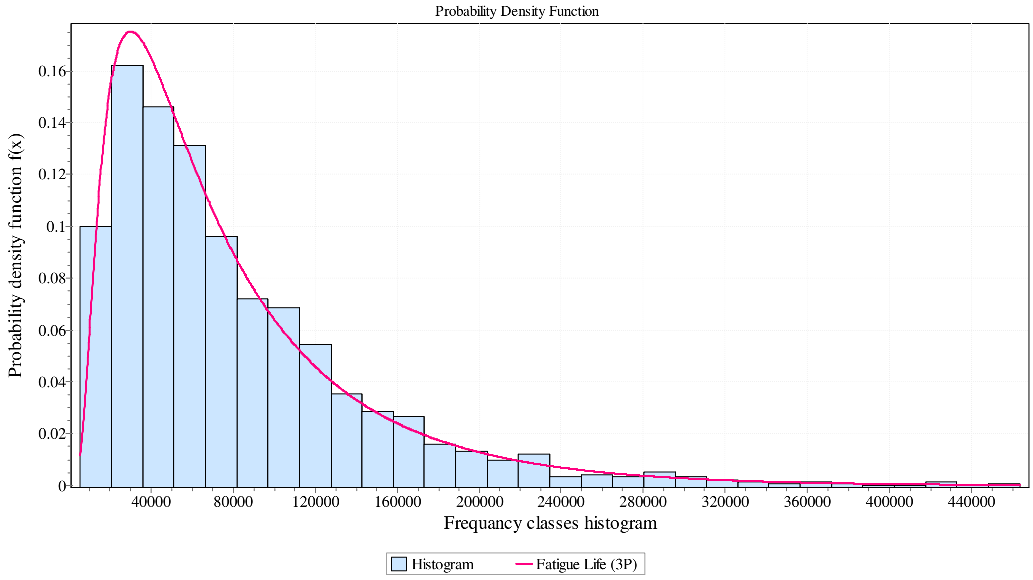


Figure 5.12. Fatigue Life (Birnbaum-Saunders) based on demand during lead-time histogram

Furthermore, the shape of the curve in Figure 5.13 reveals the significance level of the Fatigue Life distribution with the *p*-value = 0.41052 based on the (K-S) test. A similar comparison of the *p*-value can be made with its predecessors which shows less significant and not strong. However, it was acceptable and was not rejected to be a considered distribution.

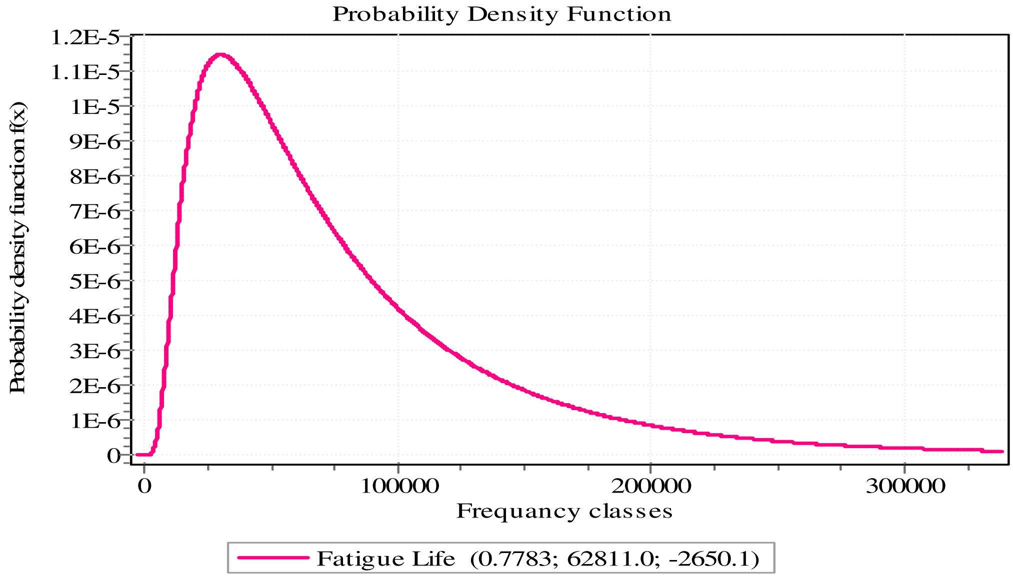


Figure 5.13. Fatigue Life (Birnbaum-Saunders) distribution of demand during lead-time

5.3.5 Inverse Gaussian distribution three parameters

The Inverse Gaussian distribution has three parameters, two continuous parameters, λ and μ and a continuous location parameter, γ. The distribution is also confirmed by plotting the histogram. The histogram in Figure 5.14 indicates the shape of an Inverse Gaussian distribution based on the SMDDL model's results. The frequency of classes starts increasing until it reaches the highest level, which was then decreased gradually subsequently.

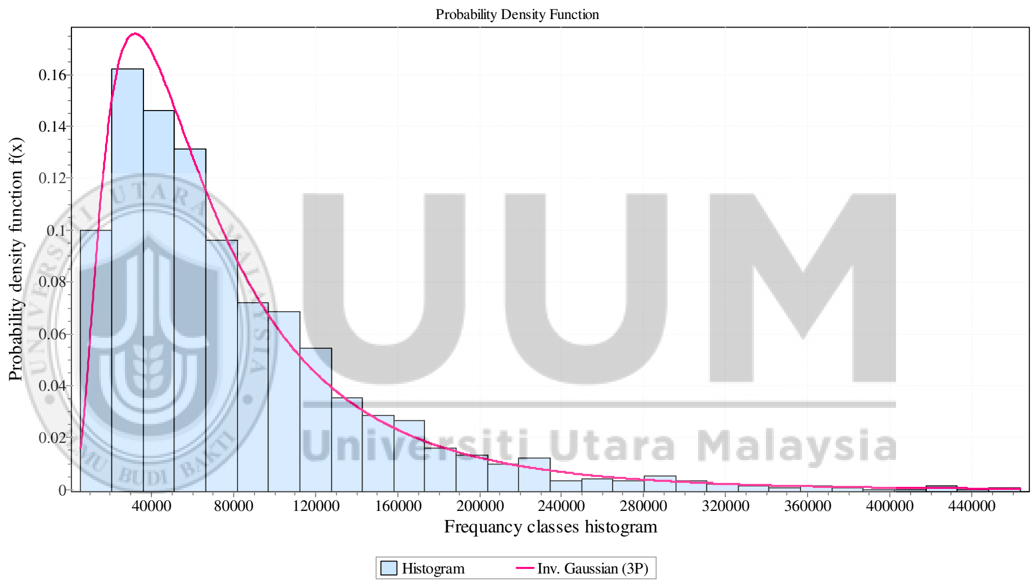


Figure 5.14. Inverse Gaussian distribution based on demand during lead-time histogram

Besides that, the shape of the curve in Figure 5.15 discloses the significance level of Inverse Gaussian distribution with the p-value = 0.312592 based on the (K-S) test. Compared with its predecessors, it was the least significant and strong. However, it was acceptable and was not rejected to be a distribution.

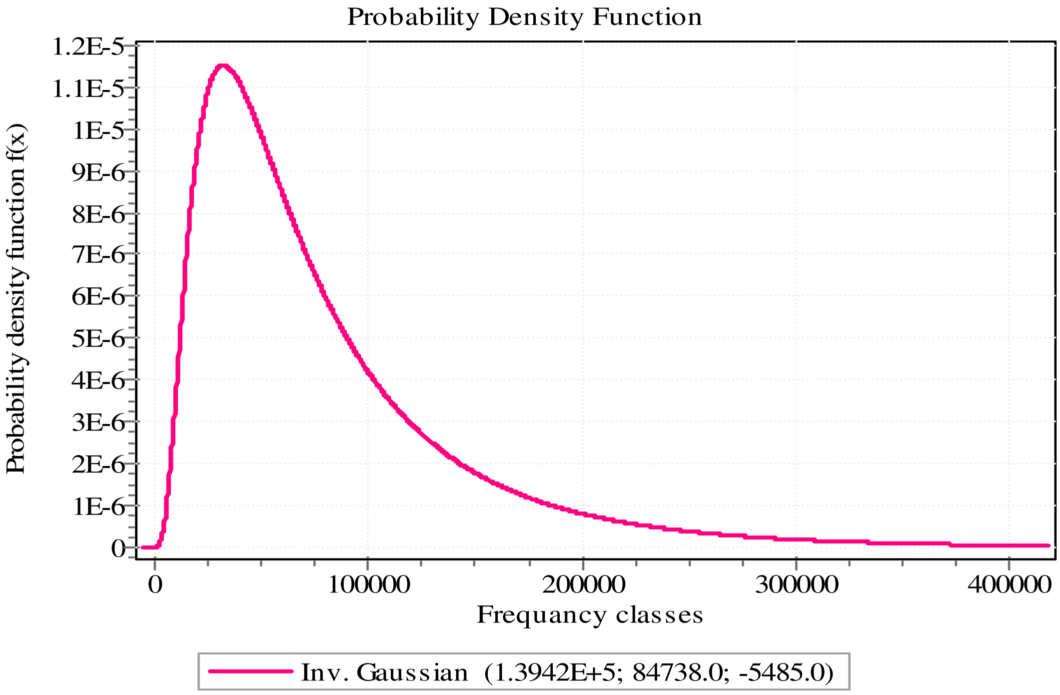


Figure 5.15. Inverse Gaussian distribution of demand during lead-time

5.3.6 Discussion of SMDDL model

This section summarized the conclusion of the SMDDL model based on the five distributions. Table 5.5 exhibits and summarizes the parameters values of the demand during lead-time distribution for each of the five distributions.

Table 5.5

Summary Values of the Demand during Lead-Time parameters with the Five Different Distributions

Distributions	Shape parameters				Scale parameter	Location parameter	continuous parameter	continuous parameter	p-Value
	K	α	α_1	α_2	β	γ	λ	μ	
GG four parameters Dist.	3.2	0.66813	–	–	11713.0	5058.1	–	–	0.93584
Pearson Type 6 Dist.	–	–	1.756	12.145	4.6993E+5	5246.5	–	–	0.85335
Log-Pearson 3 Dist.	–	124.41	–	–	-0.6985	19.686	–	–	0.4285
Fatigue Life Dist.		0.7783	–	–	62811	-2650.1	–	–	0.4105
Inverse Gaussian Dist	–	–	–	–	–	-5485	1.3942E+5	84739	0.31259

From the Table 5.5, the strongest and highest significant p-value obtained based on the (K-S) is shown in the Generalized Gamma Four Parameter Distribution with the p-value of 0.93584. Whenever the p-value increases, the accuracy of the results is more harmonious and convenient.

Table 5.6, on the other hand, displays the extracted mean of demand during lead-time, μ_L, as well as the standard deviation of demand during lead-time, σ_L , according to each type of the distribution. The mean and the standard deviation are extracted according to the arithmetic equation for each distribution (Mathwave, 2015).

Table 5.6

Mean and Standard Deviation of the Demand during Lead-Time Based on the Five Distributions

Distributions	**Values of the mean and standard deviations (tonne)**	
	Mean, μ_L	**Standard deviation, σ_L**
Generalized gamma four parameters distribution (*GG)*	79274	63018
Pearson Type 6 Distribution with four-parameters	79289	63009
Log-Pearson 3 distribution	79692	66744
Fatigue Life (Birnbaum-Saunders) distribution	79185	64803
Inverse Gaussian three parameters distribution	79253	66062

Therefore, this research only adopts the Generalized Gamma Four-Parameter distribution and with mean of demand during lead-time, $\mu_L = 79274$ tonne/day and standard deviation of demand during lead-time, $\sigma_L = 63018$ tonne/day since it shows the most significant value. The study by Axsäter (2011) also considered only one distribution for probabilistic cases.

5.4 Arrival rate

The development of the DMEI-FCFS model requires the determination of the arrival rate. To determine the arrival rate, the (K-S) test was conducted. The (K-S) test shows the arrival rate was distributed as Poisson with the mean, λ is equal to 19.74 retailers/ hour. As it is integer, mean, λ considered is 20 retailers /hour.

5.5 Service rate

Another requirement to develop the DMEI-FCFS model is the determination of the service rate. To determine the service rate, the (K-S) test was conducted. The (K-S) test shows the service rate was distributed as Gamma with shape parameter, α equal to 1.3484 and scale parameter, β equal to 4.4800.

5.6 Establishing the costs

The costs involved in the multi-echelon inventory system under the continuous review are holding cost, h and setup cost, A. It is observed from the annual reports of Lafarge (HC, Brokerage 2010), the current capacity of production, α_1 is 2.3 million tonne per day with the total the investment cost of US$276 million, while the selling price per tonne is US$120 and the production cost per tonne is US$25. Figure 5.16 shows key indicators of cement operations: the price per tonne, cost per tonne including energy cost, labor, packaging, maintenance, other expenses and depreciation, and profit margin (%).

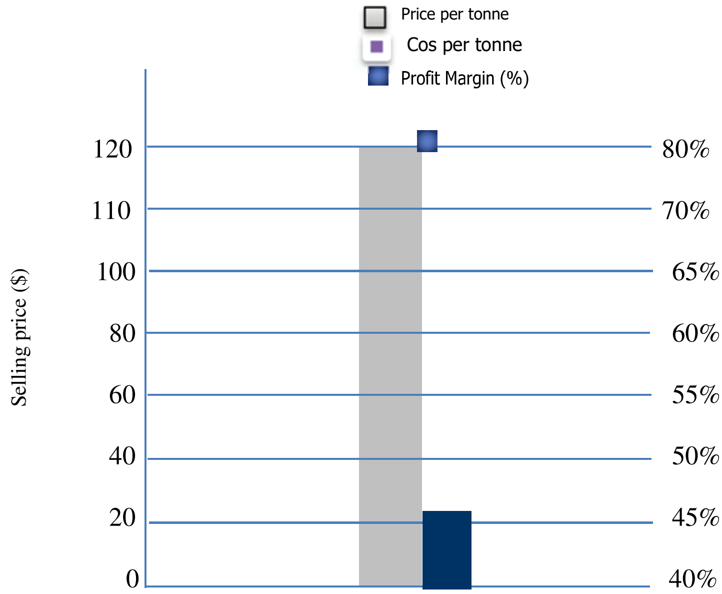


Figure 5.16. Cement Operations - key indicators. Source: HC Brokerage (2012)

5.6.1 Holding cost value

Holding cost, *h* is the cost carried by the firm, factory, or the project when store materials are in the depot. It includes the following elements: interest on capital, rent of storage place (which includes electricity, water, cooling, etc.), insurance cost against unexpected accidents, damage that infects the products, and investment cost. For some cases, the holding cost cannot be extracted directly because it depends on the percentage cost of inventory level, *I* and the cost per unit, *C*. Therefore, according to Equation (4.2),

$$I = \frac{\$120}{12037.74} = 0.01$$

The unit costs *C= $25/tonne,*

Then by using Equation (4.1),

$$h = (I)(C) = (25)(0.01) \; = \; \$0.25/\text{tonne}.$$

Therefore, the holding cost, h for the whole day:

$$h = (0.25)(12037.74) = \$\,3009.44/\text{day}.$$

5.6.2 Setup cost value

The setup cost is the fixed cost which is independent of the size of order quantities for purchases or productions. It involves the cost of issuance of document request and follow-up, putting the items or products in the depots, production setup, arranging the place, demise of buildings, and inspection of bad or shoddy items or inspection quality. The extracted value of setup cost, A depends on Equation (4.3),

where, $h \cong \$3009.44/\text{ day}$

$$A = \frac{(12037.74)(120)}{120} - 3009.44 = \$9028.3/\text{Order}$$

5.7 The initial value of order quantity, Q

The extraction of the initial value of order quantity, Q_1, is according to Equation (4.12) with production rate, $\alpha_1 = 2.3$ million tonne, $A=\$9028.3$, $h=3009.44$ and $D_L=12037.75$

$$\therefore\ Q_1 = \sqrt{\frac{2AD_L}{h(1-\frac{D_L}{\alpha_1})}} = \sqrt{\frac{(2)(9028.3)(12037.74)}{(3009.44)(0.995)}}$$

$$\therefore\ Q_1 = 270 \text{ tonnes/ order}$$

The value of order quantity, Q_n for the number of echelons more than one is equal to $j_{n-1}Q_{n-1}$. The value of j_n can be obtained from the Table 4.4 depending on the inventory position and the reorder point at each installation.

5.8 Optimal Safety stock

One of the necessary measures in the SMEI *(R, Q)* model is the safety stock, *SS*. Optimal *SS* makes a balance between overstock and understock so that it does not fall into shortage. The *SS* is extracted from Equation (4.14).

The safety factor, *K*, under any service level (confidence level), was extracted from the Generalized Gamma Distribution, *GG(α, β, k, γ)*, four parameters based on the number of shape parameters. Table 5.7 represents the value of *k* (the safety factor) from SPSS, StatSoft-8.Inc (2007) based on a different service level.

Table 5.7

Values of Safety Factor, k, Based on GG(α ,β, k, γ)

Service levels	*K*-values	Service levels	*k*-values
0.90	3.88720	0.95	4.743865
0.91	4.021718	0.96	5.012760
0.92	4.168266	0.97	5.355947
0.93	4.333214	0.98	5.833922
0.94	4.522184	0.99	6.638352

From Table 5.7, it can be stated that when the service level value increase the safety factor *k* increases. The value of *SS* is extracted by preparing source code (see Appendix F) and it is based on different safety factor.

Table 5.8

Values of Safety Stock, SS, under Different Service Levels

Service levels	**Value of *SS* in tonne**	**Service levels**	**Value of *SS* in tonne**
0.90	244963.6	0.95	298948.9
0.91	253440.6	0.96	315894.1
0.92	262675.8	0.97	337521.2
0.93	273070.5	0.98	367642.1
0.94	284979	0.99	418335.7

In fact, the best service level adopted in most similar cases or in the literature had 95% credibility and 0.05% errors, which is more realistic (Graves & Willems, 2000; Humair, Ruark, Tomlin, & Willems, 2013).

5.9 Order quantity, Q and Reorder point, R

The reorder point at the installations and echelons are extracted from the equations in Table 4.1. The reorder point at Installation One is equals to the reorder point at Echelon One Proposition (3.2), $R_1^i = R_1^e = SS + \mu_L$.

where,

$\mu_L = 79274$ and *SS*= 298948.9 under service level 0.95%

$R = 298948.9 + 79274 \cong 378223$ tonne /day.

$\therefore R_1^i = R_1^e = 378223$ tonne /day.

for $n \geq 2$,

$R_n^i = R_n^e - R_{n-1}^e - Q_{n-1}$, if $n=2$, $R_2^i = R_2^e - R_1^e - Q_1$.

In this equation, the value of reorder point at Echelon Two is missing. Therefore, based on the equations in Table 4.1, we can extract the reorder point at the echelons.

$$R_n^e = R_{n-1}^i + \sum_{j=1}^{n-1}\left(R_j^i + Q_j\right), \quad for \ n \geq 2$$

$$R_2^e = R_1^i + (R_1^i + Q_1)$$

The value of R_2^i is extracted by substituting R_2^e in R_2^i and the process is repeated until n installations and echelons. However, the problem appears in the value of Q_n, for $n \geq 2$. The value of Q_2 depends on the extracting value of R_2^e, where the value of $Q_n = j_{n-1}Q_{n-1}$, then it compensates with $j_n = IP_n^i - R_n^i$. The formula of IP_n^i is extracted from Table 4.4. For this purpose, source code was prepared to solve and find the value of the order quantity as well as the reorder point at each installation and echelon as shown in Appendix G.

Table 5.9 shows the results of the order quantity and the reorder point for each installation and echelon by the algorithm outputs at service level 95%.

Table 5.9

Reorder Point at Installation and Echelon stock with a Service Level of 95% for Three Echelons

N	Reorder point at installation (i = 3) in tonne/ day	Reorder Point at Echelon (i = 3) in tonne/ day	Order quantity for each order/tonne
1	378223	378223	270
2	378223	756716	96271
3	1058223	1231210	96877

The results in Table 5.9 shows that the reorder point at the first and second installation are identical, which means that the best reorder point at each installation is 378223tonne/day. For each echelon, the reorder point is different. The reorder point at echelon 1 is the same as in installation 1, the reorder point at echelon 2 is equal to the sum of the reorder point at installation 1 and 2, and the order quantity (i.e., 378223+378223+270). While the reorder point at echelon 3 is equal to the sum of reorder point at echelon 2, the reorder point at installation 2 and the order quantity (i.e., 756716+378223+96271). The difference between the daily production capacity which is 2.3 million tonne and the three reorder point is 66149 tonne. The difference shown is small, which means in some ways the model seems to be represent the real situation.

In order to generalise, the proposed model can be generated to any number of installations and echelons under any service levels by developing source code as in Appendix G and H.

5.10 Inventory position

Important measures that must be extracted is the inventory position, *IP* at each of the installations and echelons. The *IP* is extracted from the Equations in Table 4.3. In order to generalise the *IP* to any number of installation and echelon, source code in Appendix I is generated. Table 5.10 represents the results of the inventory position in each installation and echelon at service level 95%.

Table 5.10

Extracted Values of Inventory Positions at Service Level 95%

N	*IP* at the installations (tonne)	*IP* at the echelons (tonne)
1	378223	378223
2	379156	378534
3	379156	757068
4	379156	1135602
5	379156	1514136
6	379156	1892670
7	379156	2271204
8	379156	2649738
9	379156	3028272
10	379156	3406806

The results in Table 5.10 show the inventory position that should be at each installation and echelon under the service level of 95%. The inventory position at the echelons is determined according to the sum of the reorder point at the installations and the order quantity. While the inventory position at the installations depends on the difference between the current inventory position at the echelon and the previous one. In the SMEI *(R, Q)* model, four echelons are considered. However it can be generalized to *N* echelons at any service levels as presented in Appendix J.

5.11 Inventory level

The inventory level is extracted to help decision makers to make the right decisions at the right time in order to promote and enhance the inventory. Table 5.11 represents the results of the inventory level at each echelon based on the inventory position at service level 95%.

Table 5.11

Extracted Values of the Inventory Level at Each Echelon

N	***IL* at the echelon (tonne)**
2	299260
3	677794
4	1271328
5	1434862
6	1813396
7	2191930
8	2570464
9	2948998
10	3327532

The results in Table 5.11 show the inventory levels that should be at each echelon. As this research considered four echelons, the sum of the inventory level at echelon 2, echelon3 and echelon 4 is 2248382tonne/day. Based on the production quantity of 2.3 million tonnes per day, the difference is 51618 tonnes. The results in Appendix K are generalised for *N* echelons at any service levels.

5.12 Approximated total cost analysis

In this model, these assumptions are made; only a single partner is considered and the type of the decision is centralized. The extracted approximate overall cost is generated as in Equation (4.19) where, PC = \$25, A = \$9028.3, $D_L = 12037.75$, h = \$3009.4 and b = 0.995

Therefore,

$$C(R,Q) = (25\ X\ 12038) + \sqrt{2X9028.3\ X\ 12038\ X\ 3009.4\ X\ 0.995} =$$
$$\$\ 1107714/\text{day}.$$

It is observed that the demand is in a normal range in comparison to the production rate and the high demand. Suppose that the selling price per tonne is \$120, cost per tonne is \$25, and the mean of demand during lead-time, μ_L, is 79274 tonne per day. Therefore, the approximate profit calculated is \$7531030. Now, if the approximate cost is subtracted from the approximate profit, the net profit will be \$6423316. The amount obtained is not solely the profit value because there are some other extra costs that may not be taken into consideration, such as salaries and other additional expenses.

5.13 Performance measures of DMEI-FCFS model

It is been observed that the initial information of the expected waiting time is 13 hours and 30 minutes, which is quite a long waiting time for the retailers. This happened because of a limited number of distribution centers and a large number of retailers. The performance measures which include P_0, L_s, L_q, W_s and W_q of the DMEI-FCFS model are presented.

Initially, the traffic intensity ρ is extracted from Equation (4.24), the value of P_0 from the Equation (4.25), and P_n. from Equation (4.26) by preparing source code as in Appendix L. The result obtained for each value is shown in Table 5.12.

Table 5.12

Values of Probability Measures ρ, P_0

Measures	Value
Traffic intensity or system utilization, ρ	0.552
The probability of no retailers in the system, P_0	0.004

The result in Table 5.12 shows that the value of traffic intensity, ρ, is 0.552. It shows that the queue model is effective because $\rho < 1$. The value of ρ is not valid since, $0 \leq \rho \leq 1$. it means the queue model does not achieve the drawn objectives and it may lead to rising costs.

The value of the probability that there are no retailers in the system, P_0 is 0.004. The small value obtained indicates that the system is busy. Indirectly it shows that there is a strong demand for cement products. On the other hand, the results obtained of the probability that there are n retailers in the system, P_n based on Equation (4.26) are shown in Table 5.13.

Table 5.13

Values of Probability of that there are n retailers in the system, P_n

Number of arrivals, n	**Values of P_n**
1	0.769
2	0.943
3	0.970
4	0.861
5	0.770
6	0.715

Table 5.14 represents the obtained values of the rest of performance measures (W_s, L_s, L_q and W_q) by applying the Equations (4.20), (4.21), (4.22) and (4.23) respectively. The values for each performance measures of L_s, L_q, W_q and W_s are obtained using source code as in Appendix L.

Table 5.14

Performance Measures of the DMEI-FCFS Model

Measures	**Value**
The expected number of the retailers in the queue, L_q	118
The expected number of the retailers in the system, L_s	121
The expected waiting time in the queue, W_q / hour	5 hours
The expected waiting time in the system, W_s/hour	6 h and 2 m

The results in Table 5.14 show that the expected number of retailers in the system is 121 while the expected number of retailers in the queue is 118. It is believed the slight difference between these two numbers is because the arrangement of the procedures for obtaining the demand for each retailer is within the same area.

As we can see, the expected waiting time in the system is 6 hours and 2 minutes, which shows a considerable reduction from 13 hours and 30 minutes. Hence, the proposed DMEI-FCFS model is effective in achieving the model objectives. Lastly, the expected waiting time is 5 hours.

5.14 Evaluation of the proposed models

This section explains the validation and comparison of the most important results. This section is divided into three parts. First, the simulation model's validity and credibility are presented to test that our results are valid. Second, the comparison of the whole system based on the SMEI *(R, Q)* model using two different criteria, the coefficient of variation, *CV*, and expected total cost in the system.

Finally, the comparison of the DMEI-FCFS model, which depends on the performance measure of the expected waiting time in the system, W_s, is explained.

5.14.1 Validation of the SMDDL model

In order to ensure the SMDDL model's validity as well as the credibility of the generated data of demand during lead-time, some tests were done as follows:

We compared the mean and standard deviation of the generated lead-time distribution by SMDDL model with the mean and standard deviation of demand and lead-time regardless of the statistical distributions from Equations (3.17) and Equation (3.18). The mean and the standard deviation of the generated lead-time data by the SMDDL model which is Gamma distribution has a shape parameter, $\alpha = 1.412$, a scale parameter, $\beta = 4.2040$, mean equal to 5.953 and standard deviation equal to 25.916. The mean and the standard deviation of the demand and the lead-time regardless of the statistical distributions from Equations (3.17) and (3.18) are 6.040 and 27.063 respectively. The extracted standard error, *SE* for both techniques is calculated and is shows in Table 5.15.

Table 5.15

Comparison values of the mean, standard deviation and standard error

Measures	Mean	Standard deviation	Standard Error
Practically extracted from the SMDDL model	5.953	25.916	0.7
Theoretical values, regardless of the distributions (Fishman, 1973)	6.040	27.063	0.81

Table 5.15 shows how the results of the practical and theoretical mean and standard deviation are satisfied with each other. When the mean and standard deviation for both the practical and theoretical are similar, there is no notable difference in the distribution of this kind of satisfaction. Furthermore, a small standard error, *SE,* measures shows that there is no difference of overly between the two sample sizes (Forbes et al., 2011; Ghosh et al., 2006; Good & Hardin, 2006).

5.14.2 Evaluation of the SMEI *(R, Q)* model

This section evaluates the proposed SMEI *(R, Q)* model based on two different criteria, the expected total cost of the system (Elhasia et al., 2013) and coefficient of variation, *CV* (Moslemi & Zandieh, 2011). The first criteria is considered with three policies or scenarios in a multi-echelon inventory system: "Make-to-stock" (In_{MTS}), "Pack-to-order" (In_{PTO}), and "Grind-to-order" (In_{GTO}) strategies which are commonly used to draw the inventory policies in order to reach the lowest possible cost. In this criteria, this assessment is to demonstrate that the proposed new formula, $In_{\mathrm{SMEI}\,(R,\,Q)}$, gives less approximate total cost in the whole system in comparison with the three inventory policies. The comparison of the total production cost is made in a similar ergonomics, which is the cement industry.

The cement industry, in terms of variables, modus operandi and ergonomics, is similar all over the world. The model was run 1500 times and the results obtained are illustrated in Table 5.16.

Table 5.16

Key Performance Indicators Based on Cost Analysis

Index	Key performance indicators / cost analysis ($)
	Approximate total Cost
Current expected Cost	57500000
In_{MTS} Scenario 1	37720020
In_{PTO} Scenario 2	37690198
In_{GTO} Scenario 3	31553600
$In_{\mathrm{SMEI}(R,\,Q)}$	15003799

From Table 5.16, note that the minimum value of the total approximate cost of the proposed new formula is $15003799, compared with the three scenarios and the current expected cost of the ergonomics.

For the second criteria of evaluation, one of the important statistical metrics to measure the dispersion of the data is the coefficient of variation, *CV*, *CV* is the division of the standard deviation, *SD*, on the average μ of the data multiplied by 100. Whenever the *CV* proportion is less, there is less dispersion of data and indicates the accuracy of the results. The results of the comparison between the SMEI *(R, Q)* model and 'multi-objective practical swarm optimization MOPSO (Grids) based on *CV* are illustrated in Table 5.17.

Table 5.17

Results of Data Dispersion Based on CV

Measures	**MOPSO (Grids) Algorithm**	**SMEI *(R, Q)* model**
Average, μ	1880.09	12037.74
Standard deviation, *SD*	1380.43	3369.141
CV	0.73	0.28
CV Percentage %	73%	27%

From the results in Table 5.17, note that the proportion of the coefficient of variation, *CV*, in the SMEI *(R, Q)* model is 27%, and which that in the MOPSO (Grids) is 73%. Based on this measure, the SMEI *(R, Q)* model gives a less ratio dispersion of data in comparison with the MOPSO (Grids) algorithm, which means that accurate results are obtained from the proposed model.

5.14.3 Evaluation of the DMEI-FCFS model

This section evaluates the proposed DMEI-FCFS model based on the waiting time in the system , E_0, which presented by Mital (2012) and what-if analysis of adding extra server channels to the model. In the first criteria, this assessment is to demonstrate that the proposed new formula *Ws*-DMEI-FCFS gives less waiting time in the system compared with E_0 of Mital (2012). The results obtained are illustrated in Table 5.18.

Table 5.18

Results of Performance Measure based on Waiting Time

Measure	Expected waiting time in the system (minutes)
E_0	10
W_s-DMEI-FCFS	6

From the results in Table 5.18, the proposed W_s-DMEI-FCFS gives less waiting time in the system compared with E_0 with the difference of 4 minutes.

The second evaluation of the DMEI-FCFS model was made based on the performance measure of the expected waiting time in the system, W_s. The comparison was carried out based on two tests. The first test used the WinQSB-queueing model analysis software in order to extract W_s for the same model inputs (WinQSB-2.0, 2002). The second test performed a sensitivity analysis using simulation for 1000 hours of loops based on an FCFS model to examine the effect of adding extra server channels to the model. For instance, the changes from 6 channels to 15 channels were recorded to observe the degree of reduction of the expected waiting time in the system. The results are illustrated in Table 5.19.

Table 5.19

Impact of Simulated Time on Expected Waiting Time in the System, Ws

Simulation sensitivity analysis of servers number	
Number of servers channel	**The expected waiting time in the system W_s**
6	6.818
7	6.2867
8	6.1217
9	6.067
10	6.049
11	6.0432
12	6.0415
13	6.041
14	6.0409
15	6.0408

From the results in Table 5.19, it can be seen that when the server channel is 6, the W_s is 6 hours and 49 minutes with a difference of 47 minutes for each retailer with the proposed model, i.e., 6 hours and 2 minutes (see Section 5.13). The result of the sensitivity analysis based on the simulation shows that when three more servers are added to the system, e.g., from 6 to 9 service providers, W_s is reduced by 45.06 minutes. From 10 to 15 service providers, W_s constantly shows a very slight reduction and nearly remains the same.

The model proposed in this research reduces the expected waiting time in the system at a rate of 47 minutes. However, when adding service providers to the system to satisfy the retailers' orders from 9 channels to 15 channels, the expected waiting time in the system will not significantly change, and the service rate remains the same, taking into consideration that any extra service providers means an extra cost on the system. These interval changes are illustrated in Figure 5.17.

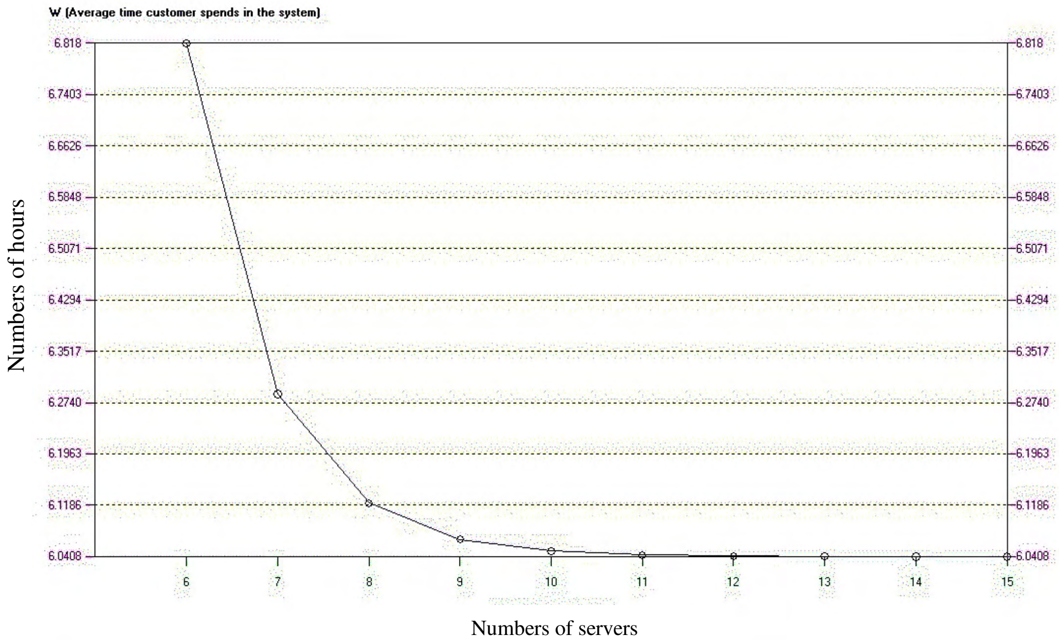


Figure 5.17. The Effect of Increasing Numbers of Servers' Channel on *Ws*

Finally, it can be stated that the proposed DEMI-FCFS model, significantly reduces the expected waiting time in the system from 13 hours and 30 minutes to 6 hours and 2 minutes with an average reduction of 7 hours in the whole system.

5.15 Discussion and summary

In this chapter, all results were clarified and discussed. With the help of the proposed procedure, the aspects of probability and uncertainty in the multi-echelon inventory data were considered. The result found in the research can be summarized as follows: Firstly, the SMDDL model was able to establish five new probability distributions of demand during lead-time. They are the Generalized Gamma Distribution Four Parameters, Pearson Type 6 Distributions Four Parameters, Log-Pearson 3 distribution, Fatigue Life (Birnbaum-Saunders) Distribution, and Inverse Gaussian Distribution.

The most previous studies, demand is often assumed to be constant or stochastic (normal, Poisson or compound Poisson) while the lead-time is constant, fixed, zero, or neglected. In reality, the probabilistic of both demand and lead-time make those assumptions are not applicable. Thus, this research has suggested new distribution to represent the demand during lead-time that considered both demand and lead-time are probabilistic.

Secondly, based on the SMEI *(R, Q)* model, some findings such as the best order quantity, Q_j, the optimal safety stock, *SS*, the best time to promote the reorder point, *R* at each installation and echelon, the inventory position, *IP* and inventory levels, *IL*, as well as, the minimum approximate total cost for the whole system are extracted.

Thirdly, the DMEI-FCFS model is able to suggest the expected waiting time to 6 hours and 2 minutes, which shows approximately seven hours reduction in comparison to actual waiting time of 13 hours and 30 minutes. Nonetheless, it has shown the effectiveness of the model proposed.

Finally, the proposed models were evaluated based on different criteria for validation of the SMDDL model, SMEI *(R, Q)* model and DMEI-FCFS model.

CHAPTER SIX
CONCLUSIONS

This chapter summarizes the research and discusses some of its contributions and limitations. Moreover, it additionally addresses a few potential works in the future.

6.1 Summary of multi-echelon inventory system in supply chain

This research addresses a multi-echelon inventory system under the continuous review *(R, Q)* policy, where *R* is the reorder point and *Q* is the order quantity with probabilistic demand and probabilistic lead-time, which plays the primary role in developing an approximation mathematical model in combining a serial and a distribution multi-echelon inventory system. An inventory system plays a significant role in supply chain, where most studies in the area of supply chain employ theories of inventory. There are a large number of studies on a multi-echelon inventory system that generally can be categorized as supply chain management.

The supply chain of a manufacturing industry, which consists of suppliers, raw materials depots, manufacturer warehouses, distribution centers, and a number of retailers that satisfy a big number of customers, is aligned with the supply chain of a cement industry. The essential aim of an inventory system is to balance between overstock and understock. This balance depends on relevant inventory system policies, i.e., whether a periodic review policy or a continuous review policy and the behaviors of other variables, i.e., demand, lead-time process and costs.

An inventory problem appears when there is a need for a physical storage of goods, items, and products for the purposes of meeting demand overtime, a need for any project in the business area to keep inventory to ensure continued efficient operations. Therefore, demand and lead-time are the keys to developing or modifying multi-echelon inventory system models.

Typically, demand and lead-time in an inventory system can both be constant which is the simplest model, or demand is probabilistic and the lead-time is constant (i.e. deterministic), or demand is constant or deterministic and the lead-time is probabilistic, or alternatively, both of them are probabilistic. In particular, the demand and lead-time behaviors are very important and complex when they are probabilistic as each one has its own probability distribution function. Therefore, the probability and uncertainty ergonomics aim is to contribute to the supply chain of a multi-echelon inventory system by implementing algorithms that approximate the optimal inventory system policies of these systems.

This research presents a new insight into an approximation mathematical model to obtain the best approximate solution of a multi-echelon inventory system under the continuous review *(R, Q)* policy in a cement industry with probabilistic demand and probabilistic lead-time. In the proposed approximation mathematical model, three sub models were developed which are the SMDDL model, the SMEI *(R, Q)* model and the DMEI-FCFS model.

In the SMEI *(R, Q)* model, we developed the formula for the order quantity, Q_j, the reorder point at each installation and echelon, R_n^i, R_n^e, and the method of optimizing the safety stock, *SS*. In addition, a new formula was obtained for extracting the approximated total cost for the whole system, the inventory positions at each installation and echelon, and the inventory level at each echelon.

In the DMEI-FCFS model, the performance measures were developed to reduce the long waiting time between the distribution center and the retailers by adopting (M/G/6):(FCFS/∞/∞) system with Poisson mean arrival rate and Gamma distribution service rate. Other performance measures in the considered model were the expected number of retailers in the system and the queue, L_s, L_q, the expected waiting time in the queue, W_q, the probability that there is no retailers in the system (the system is idle), P_0, and the probability that there are more than *n* retailers in the system (the system is busy), P_n.

6.2 Accomplishment of the research objectives

This research has fruitfully achieved all the research objectives as described in the Chapter One. The main objective of this research is to develop an approximation mathematical model in a supply chain of a multi-echelon inventory system under the continuous review policy in a cement industry that can achieve the best inventory policy to satisfy the retailer's needs while considering the probability distribution function of demand during lead time.

In order to reach the best approximate solution, we outlined six specific objectives. The first specific objective is to develop simulation procedures to extract the demand during lead-time, the SMDDL model as described in Section 4.5. After running the model for 1,500 times, the demand during lead-time probability distribution data were generated. The data were generated using an algorithm based on demand data and lead-time data as described in Appendix E. The results of its implementation presented in Section 5.3 show five new probability distribution functions of demand during lead-time, which were not considered in the previous literature of a multi-echelon inventory system. The abnormal behaviors of the demand on cement and long lead-time in the system led to these new probability distributions of demand during lead-time. They are Generalized Gamma Distribution Four Parameters, Pearson Type 6 Distributions Four Parameters, Log-Pearson 3 Distribution, Fatigue Life (Birnbaum-Saunders) Distribution, and Inverse Gaussian Distribution. The strongest and the highest significant *p*-value obtained from the results of the SMDDL model is the Generalized Gamma Four Parameter Distribution under *p*-value = 0.93584.

The second specific objective is to develop the formula for order quantity, Q_j, in a serial multi-echelon inventory system under the continuous review *(R, Q)* policy, the SMIE *(R, Q)* model, with the probability distribution of demand during lead-time. In order to obtain the new formula, the probability distribution function of demand during lead-time, which is the Generalized Gamma four-parameter distribution, was integrated with the approximate total cost function of a serial multi-echelon inventory system under the continuous review policy. This was accomplished in Section 4.6.1.1, and the result is presented in Section 5.7.

The third specific objective is to identify the optimal safety stock, *SS* that should be in hand to avoid falling into shortage. The optimal safety stock was obtained based on the mean of demand during lead-time, μ_L, which was extracted from the SMDDL model and safety factor, *K*, under different service levels explained in Sections 4.6.1.2. The results are presented in Table 5.8.

The fourth specific objective is to establish the reorder point, *R* at each installation in each echelon in a serial multi-echelon inventory system under the continuous review *(R, Q)* policy. However, the extraction of the reorder point at the first installation in each first echelon contains the safety stock, *SS* plus the mean of demand during lead-time, μ_L. Therefore, the novelty extraction of reorder point, *R*, is represented in the elements. The problem appears when $n \geq 2$, where the number of echelons are more than one. Hence, the new formula of the order quantity, *Q*, which was developed and plays a role in establishing the reorder point at each installation in each echelon when $n \geq 2$ is described in Section 4.6.1.3. The results are displayed in section 5.9. The reorder point, *R*, at the first installation in each first echelon include the safety stock, *SS*, which depends on different safety factor, *k*. satisfying the fourth objective, the inventory position and inventory level at each echelon were established.

The inventory level at the echelon depends on the difference between the inventory position in the same echelon and the mean of demand during the lead-time, μ_L. On the other hand, the inventory position at each echelon depends on the sum of the reorder point at the installation and the order quantity, Q_j as $\sum_{j=1}^{n}(R_j^i + Q_j)$. Therefore, by calculating R_j^i and Q_j, we obtained the inventory position and inventory level as described in detailed in Section 4.6.1.4. Its results are explained in Sections 5.9 and 5.10, specifically in Tables 5.10 and 5.11.

The fifth specific objective is to develop and establish the approximate total cost function for the whole system under a serial multi-echelon inventory *(R, Q)* policy. This objective was established by developing a new formula for the approximate total cost function by integrating the developed formula of the order quantity, Q with the equation of total cost function, which is explained in Section 4.6.1.5. Its result is described in Section 5.12.

The final objective is to develop and identify the first-come-first-serve, FCFS queue rule in a distribution multi-echelon inventory system under a continuous review *(R, Q)* policy. The DMEI-FCFS model is developed to reduce the long expected waiting time between the distribution center and the retailers. The DMEI-FCFS model of queueing, in which the arrival rate and/or departure rate do not follow the Poisson assumption, leads to a higher complex and possibly less tractable systematic results. Therefore, we presented a non-Poisson queue model of *(M/G/C):(FCFS/∞/∞)*, where G is the service time. It is described by a general probability distribution, which is Gamma distribution from the service time data with a mean, $E[t]$ and variance, $V[t]$. The reason to this is that, the performance measures and the system utilization (traffic intensity) cannot be extracted by the classical queue methods. The $E[t]$ and , $V[t]$ depends on the probability distribution of the service rate, which, in turn, affects the rest of the performance measures based on the FCFS discipline as described in Section 4.6.2. The results obtained are presented in Section 5.13.

Consequently, the objective of the multi-echelon inventory system under the continuous review *(R, Q)* policy which proposed three models of the SMDDL model, the SMEI *(R, Q)* model, and the DMEI-FCFS model was met with the development of an approximation mathematical model.

6.3 Contributions of the research

By achieving all of the objectives, the research has contributed to the field of the multi-echelon inventory system, particularly in the design of a multi-echelon inventory system under the continuous review policy. The contributions of this research can be explained further as follow in two aspects.

6.3.1 Theoretical contribution

The main theoretical contribution is the study on multi-echelon inventory system under the continuous review *(R, Q)* policy problem, which established an approximation mathematical model for a combination of a serial and distribution supply chain subject to probabilistic of demand and lead-time. This theoretical contribution is in the development of three sub-models, i.e., the SMDDL, the SMEI *(R, Q)* and the DMEI-FCFS models. All the three models with the overall approximation model reviewed new knowledge, and thus enrich the body of knowledge for the field of multi-echelon inventory system. The establishments of these specialized sub-models are described below.

1. The SMDDL model provides a new knowledge related to the set of structure and algorithm based on simulation procedures for obtaining the demand during lead-time probability distribution. It consists of an algorithm and structure to establish the probability distribution of demand during lead-time based on a demand probability distribution and a lead-time probability distribution. The proposed SMDDL model is able to provide five new probability distributions that were not given attention in the past literature. This model, in turn, leads to new formulation of performance measures in a multi-echelon inventory system.
2. The proposed SMEI *(R, Q)* model is a set of approximation mathematical formulations, which are reformulated in four specialized formulations to obtain the order quantity, *Q,* safety stock, *SS*, reorder point, *R* and the total cost function, *C(R, Q)*. This new formula led to the best values of *Q* and *R* under the continuous review policy problem. The specialized formulation for *Q* and *R* led to the best value of minimum total cost in the whole multi-echelon inventory system. Then, formulation for the safety stock, *SS* is able to identify the optimal quantity of the *SS*. The development of the *SS* relied on the demand during lead-time probability distribution parameters and the safety factor of the demand during lead-time probability distribution, which, in turn, is a new contribution to the inventory control variables.

   In addition, the formulation of the reorder point, *R* depends on the safety stock, *SS*, and the mean, μ_L of the demand during lead-time probability distribution. As a result, the formula for *N* installations and *N* echelons can be generalized.

Subsequently, a new formulation for the total cost function, *C(R, Q)* in the problem of a multi-echelon inventory system under the continuous review *(R, Q)* policy is obtained, which minimizes the expected total costs in the system.

3. The proposed DMEI-FCFS model can include a new formula of the expected waiting time in the system, W_s, which affects the rest of the performance measures formula to identify the expected waiting time in the queue, W_q, and the expected number of the retailers in the system and the queue, L_s and L_q, because of the relationship between all of the performance measures.

6.3.2 Practical contribution

Several suggestions can be made to the general practitioners in relation to this research. The advantage of this research is that the efficient inventory policies established are able to coordinate the flows of the elements of the supply chain in the cement industry. Each model and formulation can benefit practitioners in the supply chain for the cement industry sector.

- The proposed SMEI *(R, Q)* model gives a clear policy in a multi-echelon inventory system in order to coordinate the works in the cement supply chain industry for the directors and managers.
- The proposed SMEI *(R, Q)* model reduces effort and time as much as possible for those in charge of the production and operating systems as well as retailers.
- The SMEI *(R, Q)* model gives accurate information on the required quantities, *Q* in each order to enhance the inventory. This accurate information helps to

avoid the procurement department to fall into the possibility of not satisfying the needs of its customers.

- Knowing the right time to promote the inventory of the reorder point, *R* at each installation and echelon helps the operating system decision makers to make the right decisions at the right moment.
- Knowing the optimal quantity of safety stock, *SS* (inventory-on-hand) in order to meet the future needs can overcome shortage that may occur when processing retailers' demand for the sales department and procurement department.
- The DMEI-FCFS model is able to reduce the long expected waiting time between the distribution center and the retailers from 13:30 hours to only 6 hours. Reducing this time leads to meeting the needs of more retailers, and therefore, more profits. The benefits of this reduction time are that the retailers spend less time to obtain the orders, resulting in the increase in the number of beneficiaries, and, hence, more profit for the manufacturer.

- The DEMI-FCFS model is able to provide the expected number of the retailers in the system and the queue in addition to the expected waiting time in the queue. This information enables future policies to be drawn more clearly for the decision makers in the marketing department to be able to fill the needs of the market and face the competitors.

6.4 Limitations of the research

As other research works, the present research has some limitations. The limitation of this research lies in the type of inventory model, i.e., whether it is deterministic or probabilistic. The treatment and the procedures for these two types are different because they depend on the demand and the lead-time variables. Therefore, the main limitation of this research is the data behaviors of demand (i.e., slow moving, spare part items) which prevent us from further explore them due to the complexity of the inventory environment. Besides that, due to limited access to data, this research is able to address only three years of data. Hence, a complete generalization was not able be done in this research.

6.5 Future Research

This research has offered some potential pathways for future research as described below.

- In future, the SMDDL model can be utilized to address other multi-echelon inventory system problems that occur in different domains, such as service sector, vendor managed, food &and beverages, and just in time.
- The new contribution of the demand during lead-time probability distribution function can lead to a new investigation in multi-echelon inventory systems, especially regarding the theoretical parts, to reformulate the equations of order quantity, Q, and reorder point, R, based on the obtained probability distribution of the demand during lead-time. Each obtained distribution can lead to a new formulation in a multi-echelon inventory system.

- Further work could consider other inventory system policies instead of the *(R, Q)* policy. For example, *(Q, r)* policy where *Q* is the order quantity when a reorder point of *r* is reached or *(S, s)* policy where the location takes an order up to *S* quantities when the reorder point is less than or equal to *s* quantities.
- This research adopted the Generalized Gamma Distribution four parameters to be the demand during lead-time probability distribution function. There are four probability distribution functions of the demand during lead-time that was not considered before. These functions can be adopted to establish the impact of the proposed models or other multi-echelon inventory system policies.
- Moreover, no estimation has been done on a multi-echelon inventory system in the Iraq-Kurdistan cement sector before. Hence, it might be useful to analyze further the implications of the Iraq-Kurdistan cement sector.
- In future, if more data involving longer periods of data, for example, five or six years, can be carried out.
- The use of Gamma distribution is probably rare in the real world but useful. Therefore, the use of it can be extended to other complex situations and probably can be used in abnormal behaviours

REFERENCES

Abdul-Jalbar, B., Gutiérrez, J., & Sicilia, J. (2005). Integer-ratio policies for distribution/inventory systems. *International Journal of Production Economics*, *93–94*(0), 407–415. doi:http://dx.doi.org/10.1016/j.ijpe.2004.06.037

Abu Alhaj, M., & Diabat, A. (2009). Joint location-two-echelon-inventory supply chain model with stochastic demand. In *41st International Conference on Computers & Industrial Engineeringst* (pp. 25–30).

Aggrwal, P. K., & Moinzadeh, K. (1994). Order expedition in multi-echelon production/distribution systems. *IIE Transactions*, *26*(2), 86–96. doi:10.1080/07408179408966599

Agudelo, I. (2009). *Supply Chain Management in the Cement Industry*. Massachusetts Institute of Technology.Universidad Javeriana Cali, Colombia.

Altiok, T. (1989). (R, r) Production/Inventory Systems. *Operations Research*, *37*(2), 266–276. doi:10.1287/opre.37.2.266

Altiok, T., & Melamed, B. (2001). Simulation Modeling and Analysis with Arena. Cyber Research. *Inc. and Enterprise Technology Solutions, Inc*.

Altiok, T., & Ranjan, R. (1995). Multi-stage, pull-type production/inventory systems. *IIE Transactions*, *27*(2), 190–200. doi:10.1080/07408179508936731

Atan, Z. (2010). *Inventory Management for Multi-Echelon Supply Chains*.US: Lehigh Unversity.

Averbakh, I., & Xue, Z. (2007). On-line supply chain scheduling problems with preemption. *European Journal of Operational Research*, *181*(1), 500–504. doi:10.1016/j.ejor.2006.06.004

Axsater, S. (1993). Exact and approximate evaluation of batch ordering policies for two-level inventory systems. *Operations Research*, *41*, 777–785.

Axsater, S. (2001). A framework for decentralized multi-echelon inventory control. *IIE Transactions*, *33*, 91–97.

Axsater, S. (2003). Approximate optimization of a two-level distribution inventory system. *International Journal of Production Economics*, *81*, 545–553.

Axsater, S. (2006a). A simple procedure for determining order quantities under a fill rate constraint and normally distributed lead-time demand. *European Journal of Operational Research*, *174*, 480–491. doi:10.1016/j.ejor.2005.01.037

Axsater, S. (2006b). *Inventory control*. (2nd ed.). New York, Springer.

Axsater, S. (2010). A capacity constrained production-inventory system with stochastic demand and production times. *International Journal of Production Research*, *48*(20), 6203–6209. doi:10.1080/00207540903283808

Axsäter, S. (1984). In-Process safety stock. In *23rd conference on Decision and Control* (pp. 839–842).

Axsäter, S. (1997). Simple evaluation of echelon stock (R, Q) policies for two-level inventory systems. *IIE Transactions*, *29*(8), 661–669.

Axsäter, S. (1998). Evaluation of installation stock based (R, Q)-policies for two-level inventory systems with Poisson demand. *Operations Research*, *46*(3-supplement-3), S135–S145.

Axsäter, S. (2000). Exact analysis of continuous review (R, Q) policies in two-echelon inventory systems with compound Poisson demand. *Operations Research*, *48*(5), 686–696.

Axsäter, S. (2007). On the First Come – First Served Rule in Multi-Echelon Inventory Control. *Wiley InterScience*, *54*(4), 485–491. doi:10.1002/nav

Axsäter, S. (2011). Inventory Control when the Lead-time Changes. *Production and Operations Management Society*, *20*(1), 72–80.

Axsäter, S., & Juntti, L. (1996). Comparison of echelon stock and installation stock policies for two-level inventory systems. *International Journal of Production Economics*, *45*(1), 303–310.

Axsäter, S., & Marklund, J. (2008). Optimal Position-Based Warehouse Ordering in Divergent Two-Echelon Inventory Systems. *Operations Research*, *56*(4), 976–991. doi:10.1287/opre.1080.0560

Axsater, S., & Rosling, K. (1999). Ranking of generalised multi-stage KANBAN policies. *European Journal of Operational Research*, *113*, 560–567.

Axsäter, S., & Rosling, K. (1993). Installation vs. Echelon Stock Policies for Multilevel Inventory Control. *Management Science*, *39*(10), 1274–1280. doi:10.1287/mnsc.39.10.1274

Axsäter, S., & Viswanathan, S. (2012). On the value of customer information for an independent supplier in a continuous review inventory system. *European Journal of Operational Research*, *221*, 340–347. doi:10.1016/j.ejor.2012.03.022

Axsäter, S., & Zhang, W.F. (1999). A joint replenishment policy for multi-echelon inventory control. *International Journal of Production Economics*, *59*(1–3), 243–250. doi:http://dx.doi.org/10.1016/S0925-5273(98)00018-8

Bagchi, U., & Hayya, J. (1984). Demand during Lead Time for Normal Unit Demand and Erlang Lead Time. *Operations Research Society1*, *35*(2), 131–135.

Balakrishnan, N., & Nevzorov, V. B. (2003). *A primer on statistical distributions* Hamilton, Canada: McMaster University. doi:DOI: 10.1002/0471722227

Barton, R. R. (1992). Metamodels for Simulation Input-output Relations. In *Proceedings of the 24th Conference on Winter Simulation* (pp. 289–299). New York, NY, USA: ACM. doi:10.1145/167293.167352

Baten, M. A., & Kamil, A. (2009). An Optimal Control Approach to Inventory-Production Systems with Weibull Distributed Deterioration. *Mathematics and Statisitcs*, *5*(3), 206–214.

Bates, M. and Granger, C. (1969), "The Combination of Forecasts", *Operations Research Society*, Vol. 20 No. 4, pp. 451–468.

Baumol, W., & Vinod, H. D. (1970). An Inventory Theoretic Model of Freight Transport Demand. *Management Science*, *16*(7), 413–421. doi:10.1287/mnsc.16.7.413

Baykal-Gurosy, M., & Erkip, N. (2010). Forecasting for Inventory Planning under Correlated. *Wiley Encyclopedia of Operations Research and Management Science*, 1–11. Doi: 10.1002/9780470400531.eorms0330

Beamon, B. M. (1998). Supply Chain Design and Analysis : Models and Methods. *International Journal of Production Economics*, *55*(3), 281–249.

Bell, P. C. (1978). A new procedure for the distribution of periodicals. *Journal of Operational Research Society*, *29*, 427–434.

Bengio, O. (2012). *The Kurds of Iraq: Building a state within a state*. Lynne Rienner Publishers Boulder.

Bernstein, F., & DeCroix, G. A. (2006). Inventory Policies in a Decentralized Assembly System. *Operations Research*, *54*(2), 324–336. doi:10.1287/opre.1050.0256

Bessler, S. A., & Veinott, A. F. (1966). Optimal policy for a dynamic multi-echelon inventory model. *Naval Research Logistics Quarterly*, *13*(4), 355–389. doi:10.1002/nav.3800130402

Bhat, N. (2015). *An introduction to queueing theory: modeling and analysis in applications*. Boston :Birkhäuser.

Bhat, U. N. (2008). *An Introduction to Queueing Theory Modeling and Analysis in Applications*. *Evaluation*. Boston: Birkhäuser. doi:10.1002/masy.200350723

Boddewyn, J. J., & Brewer, T. L. (2014). International-Business Behavior : New Theoretical Direction. *Academy of Managment Review*, *19*(1), 119–143.

Bolarín, F. C., Lisec, A., & Esteban, F. C. (2008). Inventory Cost Consequences of Variability Demand Process within a Multi-Echelon Supply Chain. *Logistics and Sustainable Transport*, *1*(3), 1-12.

Bollapragada, rinivas, Akella, R., & Srinivasan, R. (1998). Centralized ordering and allocation policies in a two-echelon system with non-identical warehouses. *European Journal of Operational Research*, *106*(1), 74–81. doi:http://dx.doi.org/10.1016/S0377-2217(97)00148-3

Bookbinder, J., & Cakanyildrim, M. (1999). Random lead times and expedited orders in (Q, r) inventory systems. *European Journal of Operational Research*, *115*, 300–313.

Boucherie, R., & van Dijk, N. (2010). *Queueing networks: a fundamental approach* (Vol. 154). US: Springer Science & Business Media. doi: 10.1007/978-1-4419-6472-4

Boute, R. N., Lambrecht, M. R., & Van Houdt, B. (2007). Performance evaluation of a production/inventory system with periodic review and endogenous lead times. *Naval Research Logistics (NRL)*, *54*(4), 462–473. doi:10.1002/nav.20222

Bradford, J. W., & Sugrue, P. K. (1990). A Bayesian approach to the two-period style-goods inventory problem with single replenishment and heterogeneous Poisson demands. *Journal of Theoperational Research Society*, *41*(3),211–218.

Braun, M., Rivera, D., Flores, M., Carlyle, W., & Kempf, K. (2003). A model predictive control framework for robust management of multi-product, multi-echelon demand networks. *Annual Reviews in Control*, *27*(2), 229–245.

Brown, R. G. (1959). *Statistical forecasting for inventory control*. McGraw-Hill, New York.

Cachon, G. P., & Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management Science*, *46*(8), 1032–1048.

Caglar, D., Li, C. L., & Simchi-Levi, D. (2004). Two-echelon spare parts inventory system subject to a service constraint. *C IIE Transactions*, *36*(7), 655–666.

Chairman, S. (2012). Investment Overview of Iraq. *Republic of Iraq National Investment Commision*, 1–125.

Chan, F., Routroy, S., & Kodali, R. (2005). Differential evolution algorithm for supply chain inventory planning. *Journal of Manufacturing Technology Management*, *16*(1), 7–17.

Chandra, C., & Grabis, J. (2007). *Supply chain configuration: concepts, solutions, and applications*. New York, NY, USA: Springer US.

Chang, C., & Chun-Tao, C. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. *International Journal of Production Economics*, *88*(3), 307–316. doi:10.1016/s0925-5273(03)00192-0

Chang, S. H., & Fyffe, D. E. (1971). Estimation of Forecast Errors for Seasonal-Style-Goods Sales. *Management Science*, *18*(2), B–89–B–96. doi:10.1287/mnsc.18.2.B89

Chen, F. (1998). Echelon reorder points, installation reorder points, and the value of centralized demand information. *Management Science*, *44*(12-part-2), S221–S234.

Chen, F. (1999). Decentralized supply chains subject to information delays. *Management Science*, *45*(8), 1076–1090.

Chen, F. (1999). Worst-case analysis of (R,Q) policies in a two-stage serial inventory system with deterministic demand and backlogging. *Operations Research Letters*, *25*(2), 51–58. doi:http://dx.doi.org/10.1016/S0167-6377(99)00035-8

Chen, F. (2000). Optimal Policies for Multi-Echelon Inventory Problems with Batch Ordering. *Operations Research*, *48*(3), 376–389. doi:10.1287/opre.48.3.376.12427

Chen, F., Federgruen, A., & Zheng, Y.-S. (2001). Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers.*Management Science*, *47*(5), 693–708. doi:10.1287/mnsc.47.5.693.10484

Chen, F., Ryan, J., & Simchi-Levi, D. (2000). The impact of exponential smoothing forecasts on the bullwhip effect. *Naval Research Logistics*, *47*(4), 269–286.

Chen, F., & Samroengraja, R. (2000). A staggered ordering policy for one-warehouse, multiretailer systems,. *Operations Research*, *48*, 281–293.

Chen, J., & Mushaluk, G. (2014). Coordinating a Supply Chain with an EOQ Model. *Handbook of EOQ Inventory Problems SE - 10* (Vol. 197, pp. 201–220). US :Springer. doi:10.1007/978-1-4614-7639-9_10

Chen, J.-M., & Chen, T.-H. (2005). The multi-item replenishment problem in a two-echelon supply chain: the effect of centralization versus decentralization. *Computers & Operations Research*, *32*(12), 3191–3207. doi:http://dx.doi.org/10.1016/j.cor.2004.05.007

Chen, S., & Xu, J. (2010). Note on the optimality of (s , S) policies for inventory systems with two demand classes. *Operations Research Letters*, *38*(5), 450–453. doi:10.1016/j.orl.2010.07.005

Chen, X., & Zhang, J. (2009). A Stochastic Programming Duality Approach to Inventory Centralization Games. *Operations Research*, *57*(4), 840–851. doi:10.1287/opre.1090.0699

Chen, Y. M., & Lin, C.-T. (2009). A coordinated approach to hedge the risks in stochastic inventory-routing problem. *Computers & Industrial Engineering*, *56*(3), 1095–1112.

Cheng, T. C. E. (1989). An economic order quantity model with demand-dependent unit cost. *European Journal of Operational Research*, *40*(2), 252–256. doi:http://dx.doi.org/10.1016/0377-2217(89)90334-2

Cheung, K. L., & Hausman, W. H. (2000). An exact performance evaluation for the supplier in a two-echelon inventory system. *Operations Research*, *48*(4), 646–653.

Childerhouse, P., & Towill, D. (2000). Engineering supply chains to match customer requirements. *Logistics Information Management*, *13*(6), 337–346. doi:10.1108/09576050010355635

Chiu, S. W., Ting, C., & Chiu, Y. P. (2005). Optimal order policy for EOQ model under shortage level constraint. *Engineering Modeling*, *18*(2), 41–46.

Choi, T.-M., Chiu, C.-H., & Fu, P.-L. (2011). Periodic Review Multiperiod Inventory Control Under a Mean–Variance Optimization Objective. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *41*(4), 678–682. doi:10.1109/TSMCA.2010.2089515

Chopra, S., & Meindi, P. (2010). *Supply Chain Management strategy, Planning and Operation* (Fourth.). US: Springer.

Chopra, S., & Meindl, P. (2001). *Supply Chain Management – Strategy Planning and Operation*. Prentice Hall, New Jersey.

Chopra, S., & Meindl, P. (2007). *Supply chain management. Strategy, planning & operation* (Second.). US: Springer.

Chung, S., Wee, H., & Yang, P. (2008). Optimal policy for a closed-loop supply chain inventory system with remanufacturing. *Mathematical and Computer Modelling*, *48*, 867–881. doi:10.1016/j.mcm.2007.11.014

Clark, A. J. (1972). An informal survey of multi-echelon inventory theory. *Naval Research Logistics Quarterly*, *19*(4), 621–650. doi:10.1002/nav.3800190405

Clark, A., & Scarf, H. (1960). Optimal policies for a Multi-echelon inventory problem. *Management Science*, *6*(4), 475–490.

Cohen, M., & Lee, H. (1989). Resource deployment analysis of global manufacturing and distribution networks. *Journal of Manufacturing and Operations Management*, *2*(2), 81–104.

Cohen, M., & Moon, S. (1990). Impact of production scale economies, manufacturing complexity, and transportation costs on supply chain facility networks. *Journal of Manufacturing and Operations Management*, *3*(4), 269–292.

Conover, W. (1999). *Practical Nonparametric Statistics* (3rd editio.). New York: John Wiley & Sons, Inc.

Cook, A. (2006). Logistics cost under pressure. *Logistics Management*.

Corbeit, C. (2001). Stochastic inventory systems in a supply chain with asymmetric ... *Operations Research*, *49*(4), 487–500.

Crouse, P., Doyle, W., & Young, J. (2011). Workplace learning strategies, barriers, facilitators and outcomes: a qualitative study among human resource management practitioners. *Human Resource Development International*, *14*(1), 39–55. doi:10.1080/13678868.2011.542897

Croux, C., Gelper, S., & Mahieu, K. (2010). Robust exponential smoothing of multivariate time series. *Computational Statistics & Data Analysis*, *54*(12), 2999–3006.

Das, C., & Tyagi, R. (1999). Effect of correlated demands on safety stock centralization: patterns of correlation versus degree of centralization. *Journal of Business Logistics*, *20*, 205–214.

De Bodt, M. A., & Graves, S. C. (1985). Continuous-review policies for a multi-echelon inventory problem with stochastic demand. *Management Science*, *31*(10), 1286–1299.

Dekker, R., Kleijn, M., & De Kok, A. (1998). The break quantity rule's effect on inventory costs in a 1-warehouse, N-retailers distribution system. *International Journal of Production Economics*, *56*, 61–68.

Demeter, K., & Golini, R. (2014). Inventory configurations and drivers: An international study of assembling industries. International Journal of Production Economics, 157(1), 62–73. doi:10.1016/j.ijpe.2013.10.018.

Deng, J., Song, S., Ji, P., & Zhang, B. (2010). An inventory model with stochastic lead-time and stochastic demand. In *IEEE International conference on control and Automation* (pp. 1778–1782). IEEE. doi:10.1109/ICCA.2010.5524275

Diks, E. B., & de Kok, A. G. (1998). Optimal control of a divergent multi-echelon inventory system. *European Journal of Operational Research*, *111*(1), 75–97. doi:http://dx.doi.org/10.1016/S0377-2217(97)00327-5

Diks.A, Kok.A, D., & Lagodimos.A. (1996). Multi-echelon systems : A service measure perspective. *European Journal of Operational Research*, *95*, 241–263.

Ding, H., Guo, B., & Liu, Z. (2011). Information sharing and profit allotment based on supply chain cooperation. *International Journal of Production Economics*, *133*(1), 70–79.

Dolgui, A., Ben Ammar, O., Hnaien, F., & Louly, M. A. O. (2013). A state of the art on supply planning and inventory control under lead time uncertainty. *Studies in Informatics and Control*, *22*(2), 255–268. doi:10.3182/20130619-3-RU-3018.00592

Dong, L., & Lee, H. (2003). Optimal policies and approximations for a serial multiechelon inventory system with time-correlated demand. *Operations Research*, *51*(6), 969–980.

Doyle, W., Reid, J. G., & Young, J. D. (2008). Barriers to and facilitators of managers' workplace learning in small and large knowledge-based firms. *Small Business Institute Research Review*, *35*, 79–93.

Doyle, W., & Young, J. (2007). Workplace learning strategies of managers in small and large firms in knowledge-based industries. In *Proceedings of the University Forum on Human Resource Development* (pp. 27–29).

Duffield, M. (2012). Challenging environments: Danger, resilience and the aid industry. *Security Dialogue*, *43*(5), 475–492. doi:10.1177/0967010612457975

Durbin, J., & Koopman, S. J. (2012). *Time series analysis by state space methods* (2nd ed.). UK: Oxford University Press.

Edwards, P. (2012). Iraqi cement. *Global Cemen Magazine*, (January), 54–61.

Elhasia, T., Noche, B., & Zhao, L. (2013b). Simulation of a Sustainable Cement Supply Chain ; Proposal Model Review. *Engineering and Technology*, *7*(3), 464–472.

Elimam, a. a., & Dodin, B. (2013). Project scheduling in optimizing integrated supply chain operations. *European Journal of Operational Research*, *224*(3), 530–541. doi:10.1016/j.ejor.2012.09.007

Ellinger, A., & Cseh, M. (2007). Contextual factors influencing the facilitation of others' learning through everyday work experiences. *Journal of Workplace Learning*, *19*(7), 435–452.

Eppen, G. D., & Iyer, A. V. (1997). Improved Fashion Buying with Bayesian Updates. *Operations Research*, *45*(6), 805–819. doi:10.1287/opre.45.6.805

Ettl, M., Feigin, G. E., Lin, G. Y., & Yao, D. D. (2000). A Supply Network Model with Base-Stock Control and Service Requirements. *Operations Research*, *48*(2), 216–232. doi:10.1287/opre.48.2.216.12376

Everaert, P., Bruggeman, W., Sarens, G., Anderson, S. R., & Levant, Y. (2008). Cost modeling in logistics using time-driven ABC: Experiences from a wholesaler. *International Journal of Physical Distribution & Logistics Management*, *38*(3), 172–191.

Federgruen, A. (1993). Centralized planning models for multi-echelon inventory systems under uncertainty, Handbooks in operations research and management science, *4*, (pp. 133–173).

Federgruen, A., & Zipkin, P. (1984). Allocation policies and cost approximations for multilocation inventory systems. *Naval Research Logistics Quarterly*, *31*(1), 97–129. doi:10.1002/nav.3800310112

Federgruen, A., & Zipkin, P. (1984a). An efficient of dynamic algorithm for computing optimal (s, S) policy. *Operations Research*, *32*, 818–832.

Federgruen, A., & Zipkin, P. (1984b). Approximations of Dynamic, Multilocation Production and Inventory Problems. *Management Science*, *30*(1), 69–84. doi:10.1287/mnsc.30.1.69

Federgruen, A., & Zipkin, P. (1984c). Computational Issues in an Infinite-Horizon, Multiechelon Inventory Model. *Operations Research*, *32*(4), 818–836. doi:10.1287/opre.32.4.818

Fisher, M., & Raman, A. (1996). Reducing the Cost of Demand Uncertainty Through Accurate Response to Early Sales. *Operations Research*, *44*(1), 87–99. doi:10.1287/opre.44.1.87

Fishman, G. S. (1973). *Concepts and methods in discrete event digital simulation*. New York: Wiley.

Flatt, R. J., Roussel, N., & Cheeseman, C. R. (2012). Concrete: An eco material that needs to be improved. *Journal of the European Ceramic Society*, *32*(11), 2787–2798. doi:10.1016/j.jeurceramsoc.2011.11.012

Forbes, C., Evans, M., Hastings, N., & Peacock, B. (2011). *Statistical Distributions* (4^{th} ed.). NY: Wiley Subscription Services, Inc.

Forsberg, R. (1997). Exact evaluation of (R, Q)-policies for two-level inventory systems with Poisson demand. *European Journal of Operational Research*, *96*(1), 130–138.

Frederick, S. H., & Gerald, J. L. (2001). *Introduction To Operations Research* (seven edit.). McGraw-Hill, New York.

French, K. (1986). Detecting Spot Price Forecasts In Futures Prices. *Journal of Business*, *59*(2, Part 2: Futures and Options Markets), S39–S54. doi:10.1086/296338

Funaki, K. (2012). Strategic safety stock placement in supply chain design with due-date based demand. *International Journal of Production Economics*, *135*(1), 4–13. doi:10.1016/j.ijpe.2010.11.015

Gallego, G., Ozer, O., & Zipkin, P. (2007). Bounds, heuristics, and approximations for distribution systems. *Operations Research*, *55*, 503–517.

Ganeshan, R. (1999). Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model. *International Journal of Production Economics*, *59*(1-3), 341–354. doi:10.1016/S0925-5273(98)00115-7

Ghafour, K. (2007). Inventory Control Model building for slow Moving Items in Baghdad Electricity Distribution State. *Tanmiat Al-Rafidain*, *29*(85), 137–150.

Ghosh, J., Delampady, M., & Tapas, S. (2006). *An introduction to Bayesian Analysis*. Springer US. Retrieved from http://www.library.wisc.edu/selectedtocs/bg0146.pdf

Giannoccaro, I., Pontrandolfo, P., & Scozzi, B. (2003). A fuzzy echelon approach for inventory management in supply chains. *European Journal of Operational Research*, *149*(1), 185–196. doi:http://dx.doi.org/10.1016/S0377-2217(02)00441-1

GMDH-SHELL. (2014). Business Forecasting Solution for Business. Retrieved from https://www.gmdhshell.com/

Goh, M., Lim, J. Y. S., & Meng, F. (2007). A stochastic model for risk management in global supply chain networks. *European Journal of Operational Research*, *182*, 164–173. doi:10.1016/j.ejor.2006.08.028

Good, P. I., & Hardin, J. W. (2006). *Common Error in Statistics* (2nd ed.). New Jersy: John Wiley & Sons, Inc.

Graves, C., & Willems, P. (2000). Optimizing Strategic Safety Stock Placement in Supply Chains. *Manufacturing & Service Operations Management*, *2*(1), 68–83. doi:10.1287/msom.2.1.68.23267

Graves, S. (1986). A Multi-Echelon Inventory Model for a Repairable Item with One-For-One. *Managenetn Science*, *31*(10), 1247–1256.

Graves, S. C. (1996). A multiechelon inventory model with fixed replenishment intervals. *Management Science*, *42*(1), 1–18.

Graves, S. C., Kletter, D. B., & Hetzel, W. B. (1998). A dynamic model for requirements planning with application to supply chain optimization. *Operations Research*, *46*(3-supplement-3), S35–S49.

Graves, S., & Lesnaia, E. (2004). Optimizing safety stock placement in general network supply chains. *Innovation in Manufacturing Systems and Technology (IMST)*, *1,* 7-13.

Graves, S., & Willems, S. (2003). *Supply chain design: safety stock placement and supply chain configuration*. *operations research and management science* (Vol. 11). Chicago: operations research and management science.

Greenstone, M., & Syverson, C. (2012). The Effects of Environmental Regulation on the copetitiveness of U.S. Manufacturing. UK: *MIT Center for Energy and Enviromental Policy Research*, (September).

Gümüs, a T., & Güneri, a F. (2007). Multi-echelon inventory management in supply chains with uncertain demand and lead times: literature review from an operational research perspective. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, *221*(10), 1553–1570. doi:10.1243/09544054JEM889

Gurgur, C., & Altiok, T. (2004). Approximate Analysis of Decentralized, Multi-Stage, Pull-Type Production/Inventory Systems. *Annals of Operations Research*, *125*(1-4), 95–116. doi:10.1023/B:ANOR.0000011187.52502.37

Hadley, G., & Whitin, T. M. (1975). *Analysis of inventory systems*. Englewood Cliffs: Prentice Hall.

Hamdy, A. T. (2007). *Operations research: An introduction* (7th ed.). US: Pearson Prentic Hall..

Hammel, T. R., & Kopczak, L. R. (1993). Tightening the supply chain. *Production and Inventory Management Journal*, *34*(2), 63–70.

Hariga, M. (1998). A single-period, multi-echelon stochastic model under a mix of assemble to order and assemble in advance policies. *Naval Research Logistics (NRL)*, *45*(6), 599–614. doi:10.1002/(SICI)1520-6750(199809)45:6<599::AID-NAV4>3.0.CO;2-Y

Hariga, M., & Ben-Daya, M. (1999). Some stochastic inventory models with deterministic variable lead time. *European Journal of Operational Research*, *113*, 42–51.

Harter, H. L. (1967). Maximum-Likelihood Estimation of the Parameters of a Four-Parameter Generalized Gamma Population from Complete and Censored Samples. *Technometrics*, *9*(1), 159–165. doi:10.1080/00401706.1967.10490449

Hausman, W., & Erkip, N. (1994). Multi-echelon vs . single-echelon inventory control policies. *Management Science*, *40*(5), 597–602.

Hausman, W. H., & Peterson, R. (1972). Multiproduct Production Scheduling for Style Goods with Limited Capacity, Forecast Revisions and Terminal Delivery. *Management Science*, *18*(7), 370–383. doi:10.1287/mnsc.18.7.370

Hayya, J. C., Harrison, T. P., & Chatfield, D. C. (2009). A solution for the intractable inventory model when both demand and lead time are stochastic. *International Journal of Production Economics*, *122*(2), 595–605.

HC, B. (2010). *Orascom Construction Industries (OCI) Re-Initiation of Coverage Report*. Retrieved from www.hc-si.com

He, Q., Jewkes, E., & Buzacott, J. (2002). Optimal and near-optimal inventory control policies for a make-to-order inventory–production system. *European Journal of Operational Research*, *141*(1), 113–132. doi:http://dx.doi.org/10.1016/S0377-2217(01)00257-0

Hertz, D. B., & Schaffir, K. H. (1960). A Forecasting Method for Management of Seasonal Style-Goods Inventories. *Operations Research*, *8*(1), 45–52. doi:10.1287/opre.8.1.45

Hesse, M., & Rodrigue, J. P. (2004). The transport geography of logistics and freight distribution. *Journal of Transport Geography*, *12*(3), 171–184. doi:10.1016/j.jtrangeo.2003.12.004

Hicks, E., Bagg, R., Doyle, W., & Young, J. (2007). Canadian accountants: examining workplace learning. *Journal of Workplace Learning*, *19*(2), 61–77.

Hii, Y., Rocklöv, J., Wall, S., Ng, L., Tang, C., & Ng, N. (2012). Optimal Lead Time for Dengue Forecast. *PLoS Neglected Tropical Diseases*, *6*. doi:10.1371/journal.pntd.0001848

Hillier, F. S., & Lieberman, G. J. (2010). *Introduction to operations research (pp. 195-259). Dubuque, IA: McGraw-Hill* (Dubuque, I.). Dubuque, IA: McGraw-Hill, New York.

Hiros, H. (2000). Maximum likelihood parameter estimation by model augmentation with applications to the extended four-parameter generalized gamma distribution. *Mthrmatical and Computer in Simulation*, *54*(1), 81–97.

Hosoda, T., & Disney, S. (2006). On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. *The International Journal of Management Science*, *34*, 344–358. doi:10.1016/j.omega.2004.11.005

Hsieh, C., & Chou, W. (2010). An Integrated Location-Inventory Model for Vendor-Managed Inventory in the Retail Supply Chain. In *IEEE* (pp. 1258–1262).

Huang, B., & Xue, X. (2012). An application analysis of cluster supply chain: a case study of JCH. *Kybernetes Emerald*, *41*(1), 254–280. doi:10.1108/03684921211213070

Huang, P., & Huang, T. (2006). On new Moment Estimation of Parameters of the Generalized Gamma Distribution using it's Characterization. *TAIWANESE Journal of Mathematics*, *10*(4), 1083–1093.

Humair, S., Ruark, J. D., Tomlin, B., & Willems, S. P. (2013). Incorporating stochastic lead times into the guaranteed service model of safety stock optimization. *Interfaces*, *43*(5), 421–434. doi:10.1287/inte.2013.0699

Humair, S., & Willems, S. P. (2006). Optimizing strategic safety stock placement in supply chains with clusters of commonality. *Operations Research*, *54*(4), 725–742.

Humair, S., & Willems, S. P. (2011). Technical Note-optimizing Strategic Safety Stock Placement in General Acyclic Networks. *Operations Research*, *59*(3), 781–787.

Hwan Lee, C., & Rhee, B.-D. (2010). Coordination contracts in the presence of positive inventory financing costs. *International Journal of Production Economics*, *124*(2), 331–339. doi:http://dx.doi.org/10.1016/j.ijpe.2009.11.028

IBM.(2011).SPSS Advanced Statistics 20.0. Retrieved from http://www-01.ibm.com/software/analytics/spss/

Ignaciuk, P., & Bartoszewicz, A. (2009). LQ optimal inventory control for multi-supplier periodic-review systems with disparate lead-time delay. In *48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference* (pp. 6244–6249). Ieee. doi:10.1109/CDC.2009.5399901

Inderfurth, K. (1991). Safety stock optimization in multi-stage inventory systems. *International Journal of Production Economics*, *24*(1), 103–113.

Inderfurth, K., & Minner, S. (1998). Safety stocks in multi-stage inventory systems under different service measures. *European Journal of Operational Research*, *106*(1), 57–73.

Inderfurth, K., & Vogelgesang, S. (2013). Concepts for safety stock determination under stochastic demand and different types of random production yield. *European Journal of Operational Research*, *224*(2), 293–301. doi:10.1016/j.ejor.2012.07.040

Ishii, K., Takahashi, K., & Muramatsu, R. (1988). Integrated production, inventory and distribution systems. *International Journal of Production Research*, *26*(3), 473–482. doi:10.1080/00207548808947877

Jain, G., & Sigman, K. (1996). A Pollaczek-Khintchine formula for M/G/1 queues with disasters. *Journal of Applied Probability*, *33*(4), 1191–1200.

Jain, S., & Raghavan, N. R. S. (2003). Performance analysis of make to stock supply chains using discrete-time queueing models. In U. Leopold-Wildburger, F. Rendl, & G. Wäscher (Eds.), *Operations Research Proceedings 2002 SE - 10* (Vol. 2002, pp. 65–70). Berlin: Springer Berlin Heidelberg. doi:10.1007/978-3-642-55537-4_10

Jain, S., & Raghavan, S. (2008). A queuing approach for inventory planning with batch ordering in multi-echelon supply chains. *Central European Journal of Operations Research*, *17*(1), 95–110. doi:10.1007/s10100-008-0077-8

Jie, W., & Cong, Z. (2009). Simulation Research on Multi-echelon Inventory System in Supply Chain Based on Arena. In *the 1st international conference on Information Science and Engineering* (pp. 397–400).

Johansen, S. G. (2005). Base-stock policies for the lost sales inventory system with Poisson demand and Erlangian lead times. *International Journal of Production Economics*, *93*, 429–437.

Jung, J. Y., Blau, G., Pekny, J. F., Reklaitis, G. V, & Eversdyk, D. (2008). Integrated safety stock management for multi-stage supply chains under production capacity constraints. *Computers & Chemical Engineering*, *32*(11), 2570–2581.

Kalchschmidt, M., Zotteri, G., & Verganti, R. (2003). Inventory management in a multi-echelon spare parts supply chain. *International Journal of Production Economics*, *81–82*(0), 397–413. doi:http://dx.doi.org/10.1016/S0925-5273(02)00284-0

Karaman, A. S. (2007). *Performance analysis and design of batch ordering policies in supply chains*. New Brunswick, New Jersey: The State University of New Jersey.

Karmakar, U. (1987). Lot sizes, lead times and in-process inventories. *Management Science*, *33*(3), 409–418.

Kevin Chiang, W., & Monahan, G. (2005). Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, *162*(2), 325–341.

Khan, M., Jaber, M. Y., & Bonney, M. (2011). An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, *133*(1), 113–118. doi:10.1016/j.ijpe.2010.01.023

Khan, M., Jaber, M. Y., Guiffrida, A. L., & Zolfaghari, S. (2011). A review of the extensions of a modified EOQ model for imperfect quality items. *International Journal of Production Economics*, *132*(1), 1–12. doi:http://dx.doi.org/10.1016/j.ijpe.2011.03.009

Khodabin, M., & Ahmadabadi, A. (2010). Some properties of generalized gamma distribution. *Mathematical Science*, *4*(1), 9–28.

Kian Ng, W., Piplani, R., & Viswanathan, S. (2003). Simulation workbench for analysing multi-echelon supply chains. *Integrated Manufacturing Systems*, *14*(5), 449–457.

Kiesmüller, G., de Kok, T., Smits, S., & Van Laarhoven, P. (2004). Evaluation of divergent N-echelon (s, nQ)-policies under compound renewal demand. *Operations Research-Spektrum*, *26*(4), 547–577.

Kim, C. O., Jun, J., Baek, J. K., Smith, R. L., & Kim, Y. D. (2005). Adaptive inventory control models for supply chain management. *The International Journal of Advanced Manufacturing Technology*, *26*(9-10), 1184–1192.

Kim, I., & Tang, C. S. (1997). Lead time and response time in a pull production control system. *European Journal of Operational Research*, *101*(3), 474–485. doi:http://dx.doi.org/10.1016/S0377-2217(96)00174-9

Kimball, G. E. (1988). General principles of inventory control. *Journal of Manufacturing and Operations Management*, *1*(1), 119–130.

Klosterhalfen, S., Dittmar, D., & Minner, S. (2013). An integrated guaranteed-and stochastic-service approach to inventory optimization in supply chains. *European Journal of Operational Research*, *231*(1), 109–119.

Kock, H. (2007). The team as a learning strategy: Three cases of team-based production in the Swedish manufacturing industry. *Journal of Workplace Learning*, *19*(8), 480–496.

Koehler, A., Snyder, R., Ord, J. K., & Beaumont, A. (2012). A study of outliers in the exponential smoothing approach to forecasting. *International Journal of Forecasting*, *28*(2), 477–484. doi:10.1016/j.ijforecast.2011.05.001

Krakowski, M. (1974). Arrival and departure processes in queues. Pollaczek-Khintchine formulas for bulk arrivals and bounded systems. *RAIRO-Operations Research-Recherche Opérationnelle*, *8*(V1), 45–56.

Lafarge. (2012). *Annual Report Registration Document*, 29-34.

Lafont, B. (2011). *Sustain ability, Lafarge's presence in the world. 11th report*, 1-82.

Lau, H.-S., & Hing-Ling Lau, A. (1996). Estimating the demand distributions of single-period items having frequent stockouts. *European Journal of Operational Research*, *92*(2), 254–265. doi:http://dx.doi.org/10.1016/0377-2217(95)00134-4

Lee, H., & Billington, C. (1993). Material Management in Decentralized Supply Chains. *Operations Research*, *41*(5), 835–847. doi:10.1287/opre.41.5.835

Lee, H. L., & Moinzadeh, K. (1987a). Operating characteristics of a two?echelon inventory system for repairable and consumable items under batch ordering and shipment policy. *Naval Research Logistics (NRL)*, *34*(3), 365–380.

Lee, H. L., & Moinzadeh, K. (1987b). Two-Parameter Approximations For Multi-Echelon Repairable Inventory Models With Batch Ordering Policy. *IIE Transactions*, *19*(2), 140–149. doi:10.1080/07408178708975380

Lee, H., & Whang, S. (1999). Decentralized Multi-Echelon Supply Chains: Incentives and Information. *Management Science*, *45*(5), 633–640. doi:10.1287/mnsc.45.5.633

Lee, W. (2005). Inventory model involving controllable backorder rate and variable lead time demand with the mixtures of distribution. *Applied Mathematics and Computation*, *160*, 701–717. doi:10.1016/j.amc.2003.11.039

Levy, L. D. (1995). International Sourcing and Supply Chain Stability. *Journal of International Business Studies*, *26*(2), 343–360

Li, J., & Sheng, Z. (2008). Simulating the (S, s) Controlled Stochastic System in Different Supply Chains by a Multi-Agent System. In *2008 International Conference on Computer Science and Information Technology* (pp. 785–790). Ieee. doi:10.1109/ICCSIT.2008.117

Li, P. (2013). Optimization of (R, Q) policies for serial inventory systems using the guaranteed service approach. *Computers and Industrial Engineering*, *80*, 261–273. doi:10.1016/j.cie.2014.12.003

Li, Y., Xu, X., & Ye, F. (2007). Research on Supply Chain Inventory Optimization and Benefit Coordination with Controllable Lead Time. In *International Conference on Wireless Communications, Networking and Mobile Computing* (pp. 6507–6510). Ieee. doi:10.1109/WICOM.2007.1598

Liang, W., & Huang, C. (2006). Agent-based demand forecast in multi-echelon supply chain. *Decision Support System*, *42*, 390–407. doi:10.1016/j.dss.2005.01.009

Liberopoulos, G., & Koukoumialos, S. (2005). Tradeoffs between base stock levels, numbers of kanbans, and planned supply lead times in production/inventory systems with advance demand information. *International Journal of Production Economics*, *96*(2), 213–232.

Lohman, M. (2000). Environmental inhibitors to informal learning in the workplace: A case study of public school teachers. *Adult Education Quarterly*, *50*(2), 83–101.

Lohman, M. (2005). A survey of factors influencing the engagement of two professional groups in informal workplace learning activities. *Human Resource Development Quarterly*, *16*(4), 501–527.

Lohman, M. (2009). A survey of factors influencing the engagement of information technology professionals in informal learning activities. *Information Technology, Learning, and Performance Journal*, *25*(1), 43.

Manna, S. K., Chaudhuri, K. S., & Chiang, C. (2007). Replenishment policy for EOQ models with time-dependent quadratic demand and shortages. Research. *International Journal of Operational*, *2*(3), 321–337.

Martel, A. (2003). Policies for multi-echelon supply: DRP systems with probabilistic time-varying demands1. *INFOR*, *41*(1), 71.

Matheus, P., & Gelders, L. (2000). The (R, Q) inventory policy subject to a compound Poisson demand pattern. *International Journal of Production Economics*, *68*(3), 307–317. doi:http://dx.doi.org/10.1016/S0925-5273(99)00110-3

Mathwave. (2015). EasyFit-Distribution fitting software-Benefits. Retrieved from http://www.mathwave.com/products/easyfit.html

Maydeu-Olivares, A., & Garcia-Forero, C. (2010). Goodness-of-Fit Testing. *International Encyclopedia of Education*, *7*, 190–196.

Mihe, V. (2010). On a general class of modified gamma approximating operators. *General Mathematics*, *18*(1), 71–80.

Min, H., & Zhou, G. (2002). Supply chain modeling: past, present and future. *Computers & Industrial Engineering*, *43*(1-2), 231–249. doi:10.1016/S0360-8352(02)00066-9

Minner, S. (1997). Dynamic programming algorithms for multi-stage safety stock optimization. *Operations-Research-Spektrum*, *19*(4), 261–271.

Minner, S. (2001). Strategic safety stocks in reverse logistics supply chains. *International Journal of Production Economics*, *71*(1), 417–428.

Mital, K. .(2012). Queuing analysis for outpatient and inpatient services: a case study. *Management Decision*, *48*(3), 419–439. doi:http://dx.doi.org/10.1108/09564230910978511

Mitra, S., & Chatterjee, A. (2004a). Echelon stock based continuous review (R,Q) policy for fast moving items. *Omega*, *32*(2), 161–166. doi:http://dx.doi.org/10.1016/j.omega.2003.10.003

Mitra, S., & Chatterjee, A. K. (2004b). Leveraging information in multi-echelon inventory systems. *European Journal of Operational Research*, *152*, 263–280. doi:10.1016/S0377-2217(02)00656-2

Mohebbi, E., & Posner, M. J. (1998). Sole versus dual sourcing in a continuous-review inventory system with lost sales. *Computers & Industrial Engineering*, *34*(2), 321–336.

Moinzadeh, K., & Aggarwal, P. (1997). An Information Based Multiechelon Inventory System with Emergency Orders. *Operations Research*, *45*(5), 694–701. doi:10.1287/opre.45.5.694

Moinzadeh, K., & Lee, H. L. (1986). Batch Size and Stocking Levels in Multi-Echelon Repairable Systems. *Management Science*, *32*(12), 1567–1581. doi:10.1287/mnsc.32.12.1567

Moslemi, H., & Zandieh, M. (2011). Comparisons of some improving strategies on MOPSO for multi-objective (r, Q) inventory system. *Expert Systems with Applications*, *38*(10), 12051–12057. doi:10.1016/j.eswa.2011.01.169

Moussourakis, J., & Haksever, C. (2013). Models for Ordering Multiple Products Subject to Multiple Constraints , Quantity and Freight Discounts. *American Journal of Operations Reseach*, *3*, 521–535. Retrieved from http://dx.doi.org/10.4236/ajor.2013.36051

Muckstadt, J. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science*, *20*(4 part1), 472–481.

Muckstadt, J. (1986). A Model for a Multi-Item , Multi-Echelon , Multi-Indenture Inventory System. *Management Science*, *20*(4), 472–481.

Muharremoglu, A., & Tsitsiklis, J. N. (2003). A single-unit decomposition approach to multi-echelon inventory systems. *Operations Research Center*., 59(5), 1089-1103.

Nahmias, S., & Smith, S. A. (1994). Optimizing Inventory Levels in a Two-Echelon Retailer System with Partial Lost Sales. *Management Science*, *40*(5), 582–596. doi:10.1287/mnsc.40.5.582

Noche, B., & Elhasia, T. (2013). Approach to Innovative Supply Chain Strategies in Cement Industry; Analysis and Model Simulation. *Procedia - Social and Behavioral Sciences*, *75*(0), 359–369. doi:http://dx.doi.org/10.1016/j.sbspro.2013.04.041

Noufaily, A., Jones, M. C., & Mk, M. K. (2013). On Maximisation of the Likelihood for the Generalised Gamma Distribution. *Computational Statistics*, *28*(2), 505–517.

Novoa, C., & Storer, R. (2009). An approximate dynamic programming approach for the vehicle routing problem with stochastic demands. *European Journal of Operational Research*, *196*(2), 509–515.

Nozick, L. K., & Turnquist, M. A. (2001). A two-echelon inventory allocation and distribution center location analysis. *Transportation Research Part E: Logistics and Transportation Review*, *37*(6), 425–441. doi:http://dx.doi.org/10.1016/S1366-5545(01)00007-2

Olsen, D., & Eikebrokk, T. (2010). Training, competence, and business performance: Evidence from E-business in European small and medium-sized enterprises. *International Journal of E-Business Research*, *1*, 92–116.

Osman, H., & Demirli, K. (2012). Integrated safety stock optimization for multiple sourced stockpoints facing variable demand and lead time. *International Journal of Production Economics*, *135*(1), 299–307. doi:10.1016/j.ijpe.2011.08.004

Ouweneel, A., Taris, T., van Zolingen, S., & Schreurs, P. (2009). How task characteristics and social support relate to managerial learning: empirical evidence from Dutch home care. *The Journal of Psychology*, *143*(1), 28–44.

Paige, H. (2002). An exploration of learning, the knowledge-based economy, and owner-managers of small bookselling businesses. *Journal of Workplace Learning*, *14*(6), 233–244.

Pal, B., Sankar, S., & Chaudhuri, K. (2012a). A multi-echelon supply chain model for reworkable items in multiple-markets with supply disruption. *Economic Modelling*, *29*(5), 1891–1898. doi:10.1016/j.econmod.2012.06.005

Pal, B., Sankar, S., & Chaudhuri, K. (2012b). A three layer multi-item production – inventory model for multiple suppliers and retailers. *Economic Modelling*, *29*(6), 2704–2710. doi:10.1016/j.econmod.2012.08.022

Pattnaik, M. (2014). Optimality test in fuzzy inventory model for restricted budget and space: Move forward to a non-linear programming approach. *Yugoslav Journal of Operations Research*, *201*(00), 23–34. doi:10.2298/YJOR130517023P

Pentico, D. W., & Drake, M. J. (2011). A survey of deterministic models for the EOQ and EPQ with partial backordering. *European Journal of Operational Research*, *214*(2), 179–198.

Pyke, D. F., & Cohen, M. A. (1993). Performance characteristics of stochastic integrated production-distribution systems. *European Journal of Operational Research*, *68*(1), 23–48. doi:http://dx.doi.org/10.1016/0377-2217(93)90075-X

Qi-feng, W., Xiao-shen, L., & Wei, H. (2012). Multi-source Supply Chains with Disruption Risk. In *2012 Fifth International Joint Conference on Computational Sciences and Optimization* (pp. 428–432). Ieee. doi:10.1109/CSO.2012.100

Rau, H., Wu, M.-Y., & Wee, H.-M. (2003). Integrated inventory model for deteriorating items under a multi-echelon supply chain environment. *International Journal of Production Economics*, *86*(2), 155–168. doi:http://dx.doi.org/10.1016/S0925-5273(03)00048-3

Ravichandran, N. (1995). Theory and Methodology Stochastic analysis of a continuous review perishable inventory system with positive lead time and Poisson demand. *European Journal of Operational Research*, *84*(93), 444–457.

Rentz, J., & Reynolds, F. (1991). Forecasting the effects of an aging population on product consumption: An age-period-cohort framework. *Journal of Marketing Research*, 355–360.

Reyes, P. M. (2005). A mathematical example of the two-echelon inventory model with asymmetric market information. *Applied Mathematics and Computation*, *162*, 257–264. doi:10.1016/j.amc.2003.12.095

Rezaei, J. (2014). Economic order quantity for growing items. *International Journal of Production Economics*, *155*, 109–113. doi:10.1016/j.ijpe.2013.11.026

Rezg, N., Xie, X., & Mati, Y. (2004). Joint optimization of preventive maintenance and inventory control in a production line using simulation. *International Journal of Production Research*, *42*(10), 2029–2046.

Rhee, B. Van Der, Veen, J., Venugopal, V., & Reddy, V. (2010). A new revenue sharing mechanism for coordinating multi-echelon supply chains. *Operations Research Letters*, *38*(4), 296–301. doi:10.1016/j.orl.2010.03.004

Rosenbaum, B. A. (1981). Service Level Relationships in a Multi-Echelon Inventory System. *Management Science*, *27*(8), 926–945. doi:10.1287/mnsc.27.8.926

Roy, D. (2005). A Stochastic Petri Net Approach for Inventory Rationing in Multi-Echelon Supply Chains. *Journal of Heuristics*, *11*((5-6)), 421–446.

Saffari, M., & Haji, R. (2009). Queueing system with inventory for two-echelon supply chain. In *2009 International Conference on Computers & Industrial Engineering* (Vol. 2, pp. 835–838). Ieee. doi:10.1109/ICCIE.2009.5223801

Saharidis, G. K. D., Kouikoglou, V. S., & Dallery, Y. (2009). Centralized and decentralized control polices for a two-stage stochastic supply chain with subcontracting. *International Journal of Production Economics*, *117*(1), 117–126.

Sahraeian, R., Bashiri, M., & Ramezani, M. (2010). A Stochastic Multi-Product , Multi-Stage Supply Chain Design Considering Products Waiting Time in the Queue. In *International conference on Industrial Engineering and operations management JANUARY,2010* (pp. 123–129). Dhoka, Bangladesh.

Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, *64*(3), 59–64. doi:http://dx.doi.org/10.1016/S0925-5273(99)00044-4

Sana, S. S., & Chaudhuri, K. S. (2008). A deterministic EOQ model with delays in payments and price-discount offers. *European Journal of Operational Research*, *184*(2), 509–533. doi:http://dx.doi.org/10.1016/j.ejor.2006.11.023

Santos, I., & Santos, P. (2007). Simulation metamodels for modeling output distribution parameters. In *IEEE, Simulation Conference.* (pp. 910–918). doi:10.1109/WSC.2007.4419687

Schmidt, C. P., & Nahmias, S. (1985). Optimal Policy for a Two-Stage Assembly System under Random Demand. *Operations Research*, *33*(5), 1130–1145. doi:10.1287/opre.33.5.1130

Schwarz, L. B. (1973). A Simple Continuous Review Deterministic One-Warehouse N-Retailer Inventory Problem. *Management Science*, *19*(5), 555–566. doi:10.1287/mnsc.19.5.555

Schwarz, L., Deuermeyer, B., & Badinelli, R. (1985). Fill-Rate Optimization in a One-Warehouse N-Identical Retailer Distribution System. *Management Science*, *31*(4), 488–498. doi:10.1287/mnsc.31.4.488

Schwarz, L., Frederick, S. H., Gerald, J. L., & Hamdy, A. T. (1972). Economic order quantities for products with finite demand horizons. *A I I E Transactions, 4*(3), 234-237.

Schwarz, M., Sauer, C., Daduna, H., Kulik, R., & Szekli, R. (2006). M/M/1 Queueing systems with inventory. *Queueing Systems*, *54*(1), 55–78. doi:10.1007/s11134-006-8710-5

Seliaman, M. E., & Rahman, A. (2008). Optimizing inventory decisions in a multi-stage supply chain under stochastic demands. *Applied Mathematics and Computation*, *206*(2), 538–542. doi:10.1016/j.amc.2008.05.057

Seo, Y., Jung, S., & Hahm, J. (2002). Optimal reorder decision utilizing centralized stock information in a two-echelon distribution system. *Computer and Operations Research*, *29*, 171–193.

Shang, K. H. (2012). Single-stage approximations for optimal policies in serial inventory systems with nonstationary demand. *Manufacturing & Service Operations Management*, *14*(3), 414–422.

Sharma, S., Panda, B., Mahapatra, S., & Sahu, S. (2011). Analysis of Barriers for Reverse Logistics: An Indian Perspective. *International Journal of Modeling and Optimization*, *1*(2), 101–106. doi:10.7763/IJMO.2011.V1.18

Sheng, X., & Wang, Z. (2014). Cost Model and Profit Sharing of Jointly Managed Inventory between Vendor and Retailer. *International Journal of Applied Mathematics and Statistics*^{TM}, *52*(5), 177–184.

Sherbrooke, C. (1968). METRIC: A Multi-Echelon Technique for Recoverable Item Control. *Operations Research*, *16*(1), 122–141. doi:10.1287/opre.16.1.122

Simchi-Levi, D., & Zhao, Y. (2005). Safety Stock Positioning in Supply Chains with Stochastic Lead Times. *Manufacturing & Service Operations Management*, *7*(4), 295–318. doi:10.1287/msom.1050.0087

Simpson Jr, K. F. (1958). In-process inventories. *Operations Research*, *6*(6), 863–873.

Sinha, C., Sobel, M. J., & Babich, V. (2010). Computationally simple and unified approach to finite-and infinite-horizon Clark–Scarf inventory model. *IIE Transactions*, *43*(3), 207–219.

Sleptchenko, A., van der Heijden, M., & van Harten, A. (2002). Effects of finite repair capacity in multi-echelon, multi-indenture service part supply systems. *International Journal of Production Economics*, *79*(3), 209–230. doi:http://dx.doi.org/10.1016/S0925-5273(02)00155-X

Snyder, R., Koehler, A., Hyndman, R., & Ord, J. K. (2004). Exponential smoothing models: Means and variances for lead-time demand. *European Journal of Operational Research*, *158*(2), 444–455. doi:10.1016/S0377-2217(03)00360-6

So, K. C., & Zheng, X. (2003). Impact of supplier's lead time and forecast demand updating on retailer's order quantity variability in a two-level supply chain. *International Journal of Production Economics*, *86*(2), 169–179. doi:http://dx.doi.org/10.1016/S0925-5273(03)00050-1

So, K. u t c, & Pinault, S. C. (1988). Allocating buffer storages in a pull system. *International Journal of Production Research*, *26*(12), 1959–1980. doi:10.1080/00207548808948008

Song, J. (1994). The Effect of Leadtime Uncertainty in a Simple Stochastic Inventory Model. *Management Science*, *40*(5), 603–613. doi:10.1287/mnsc.40.5.603

Song, L., Li, X., & Garcia-Diiaz, A. (2008). Multi-Echelon Supply Chain simulation using METAMODEL. In *2008 Winter Simulation Conference* (pp. 2691–2699).

Stacy, E. . (1962). A generalization of the gamma distribution. *The Annals of Mathematical Statistics*, *33*(3), 1187–1192.

Stacy, E. W., & Mihram, G. A. (1965). Parameter Estimation for a Generalized Gamma Distribution. *Technometrics*, *7*(3), 349–358. doi:10.1080/00401706.1965.10490268

Stadtler, H. (2005). Supply chain management and advanced planning - Basics, overview and challenges. *European Journal of Operational Research*, *163*(3), 575–588. doi:10.1016/j.ejor.2004.03.001

Starostka-Patyk, M., Zawada, M., Pabian, A., & Abed, M. (2013). Barriers to reverse logistics implementation in enterprises. In *2013nternational Conference on Advanced Logistics and Transport, ICALT 2013* (pp. 506–511). doi:10.1109/ICAdLT.2013.6568510

StatSoft.Inc. (2007). Statistica dta analysis software system. Retrieved from www.statsoft.com

Stephens, M. A. (1992). Introduction to Kolmogorov (1933) on the empirical determination of a distribution. *Breakthroughs in statistics* (pp. 93–105). New York: Springer. DOI: 10.1007/978-1-4612-4380-9_9

Stephens, M. A. (1992). Introduction to Kolmogorov (1933) on the empirical determination of a distribution. In *Breakthroughs in statistics* (pp. 93–105). Springer.

Stewart, M. G., Wang, X., & Nguyen, M. N. (2012). Climate change adaptation for corrosion control of concrete infrastructure. *Structural Safety*, *35*, 29–39. doi:10.1016/j.strusafe.2011.10.002

Svoronos, A., & Zipkin, P. (1988). Estimating the Performance of Multi-Level Inventory Systems. *Operations Research*, *36*(1), 57–72. doi:10.1287/opre.36.1.57

Syntetos, A., Boylan, J., & Disney, S. (2009). Forecasting for inventory planning: a 50-year review. *Journal of the Operational Research Society*, *60*, S149–S160. doi:10.1057/jors.2008.173

Taleizadeh, A. A., Pentico, D. W., Jabalameli, M. S., & Aryanezhad, M. (2013a). An economic order quantity model with multiple partial prepayments and partial backordering. *Mathematical and Computer Modelling*, *57*(3–4), 311–323. doi:http://dx.doi.org/10.1016/j.mcm.2012.07.002

Taleizadeh, A. A., Pentico, D. W., Saeed Jabalameli, M., & Aryanezhad, M. (2013b). An EOQ model with partial delayed payment and partial backordering. *Omega*, *41*(2), 354–368. doi:http://dx.doi.org/10.1016/j.omega.2012.03.008

Tallon, W. J. (1993). The impact of inventory centralization on aggregate safety stock: the variable supply lead time case. *Journal of Business Logistics,14(1), 185*.

Tan, F. (1974). Optimal Policies for a Multi-Echelon Inventory Problem with Periodic Ordering. *Management Science*, *20*(7), 1104–1111.

Tan, L., & Xu, S. (2008). Model check stochastic supply chains. In *2008 IEEE International Conference on Information Reuse and Integration* (pp. 416–421). Ieee. doi:10.1109/IRI.2008.4583067

Tang, O., & Grubbström, R. (2003). The detailed coordination problem in a two-level assembly system with stochastic lead times. *International Journal of Production Economics*, *81*, 415–429.

Tarim, A., & Kingsman, B. (2004). The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics*, *88*(1), 105–119. doi:10.1016/S0925-5273(03)00182-8

Tee, Y.S., & Rossetti, M. D. (2002). A robustness study of a multi-echelon inventory model via simulation. *International Journal of Production Economics*, *80*(3), 265–277. doi:http://dx.doi.org/10.1016/S0925-5273(02)00259-1

Teng, J.T., Yang, H.-L., & Ouyang, L.-Y. (2003). On an EOQ model for deteriorating items with time-varying demand and partial backlogging. *J Oper Res Soc*, *54*(4), 432–436. Retrieved from http://dx.doi.org/10.1057/palgrave.jors.2601490

Torabi, S. A., & Hassini, E. (2009). Multi-site production planning integrating procurement and distribution plans in multi-echelon supply chains: an interactive fuzzy goal programming approach. *International Journal of Production Research*, *47*(19), 5475–5499.

Towill, D. R. (1991). Supply chain dynamics. *International Journal of Computer Integrated Manufacturing*, *4*(4), 197–208. doi:10.1080/09511929108944496

Towill, D. R., Naim, M. M., & Wikner, J. (1992). Industrial dynamics simulation models in the design of supply chains. *Management, International Journal of Physical Distribution & Logistics*, *22*(5), 3–13.

Tyagi, R., & Das, C. (1998). Extension of the square-root law for safety stock to demands with unequal variances. *Journal of Business Logistics*, *19*, 197–203.

United Nations Environment Program. (2011). *UNEP; Division of Early Warning and Assessment "Keeping Track of Our Changing Environment: From Rio to Rio+20 (1992-2012)."* Nairobi. Retrieved from ISBN: 978-92-807-3190-3.

Urgeletti Tinarelli, G. (1983). Inventory control Models and problems. *European Journal of Operational Research*, *14*(1), 1–12.

USAID, I. (2007). Iraq private sector growth and employmaent generation. *USAID / IRAQA*, (November), 1–28.

Van der Heijden, M. C. (1999). Multi-echelon inventory control in divergent systems with shipping frequencies. *European Journal of Operational Research*, *116*(2), 331–351. doi:http://dx.doi.org/10.1016/S0377-2217(98)00048-4

Van der Heijden, M., Diks, E., & De Kok, T. (1999). Inventory control in multi-echelon divergent systems with random lead times. *Operations Research-Spektrum*, *21*(3), 331–359.

Van der Vorst, J., Beulens, A., & van Beek, P. (2000). Modelling and simulating multi-echelon food systems. *European Journal of Operational Research*, *122*(2), 354–366. doi:http://dx.doi.org/10.1016/S0377-2217(99)00238-6

Van Houtum, G. (2006). Multi-echelon production/inventory systems: Optimal policies, heuristics, and algorithms. *Tutorials in Operations Research*, 163–199.

Van Houtum, G. J., & Zijm, W. H. M. (1991). Computational procedures for stochastic multi-echelon production systems. *International Journal of Production Economics*, *23*(1–3), 223–237. doi:http://dx.doi.org/10.1016/0925-5273(91)90065-2

Vanany, I., Zailani, S., & Pujawan, N. (2009). Supply Chain Risk Management : Literature Review and Future Research. *Int Journal of Information System and Supply Chain Managment*, *2*(1), 16–33.

Verrijdt, J. H. C. M., & De Kok, A. G. (1996). Distribution planning for a divergent depotless two-echelon network under service constraints. *European Journal of Operational Research*, *89*(2), 341–354.

Venkateswaran, J., & Son, Y. J. (2007). Effect of information update frequency on the stability of production -- inventory control systems. *Int. J. Prod. Econ.*, *106*(1), 171–190.

Vrat, P. (2014). Probabilistic inventory models. *Materials management* (pp. 123–150). India: Springer. dio: 10.1007/978-81-322-1970-5_8

Wagner, H. M. (1974). The Design of Production and Inventory Systems for Multifacility and Multiwarehouse Companies. *Operations Research*, *22*(2), 278–291. doi:10.1287/opre.22.2.278

Wang, C. (2009). Using the third order exponential smoothing forecasts on reducing the bullwhip effect and inventory costs in supply chain. In *2009 IEEE International Conference on Grey Systems and Intelligent Services (GSIS 2009)* (pp. 1612–1617). Ieee. doi:10.1109/GSIS.2009.5408172

Wang, K.J., Bunjira, M., & Lin, Y. S. (2010). Forecasting based inventory management for supply chain. In *2010 Seventh International Conference on Fuzzy Systems and Knowledge Discovery* (pp. 2964–2967). Ieee. doi:10.1109/FSKD.2010.5569080

Wang, X., & Liu, L. (2007). Coordination in a retailer-led supply chain through option contract. *International Journal of Production Economics*, *110*(1–2), 115–127. doi:http://dx.doi.org/10.1016/j.ijpe.2007.02.022

Warring, N., Döös, M., Wilhelmson, L., Backlund, T., & Dixon, N. (2005). Functioning at the edge of knowledge: a study of learning processes in new product development. *Journal of Workplace Learning*, *17*(8), 481–492.

Wee, H.-M. (1995). A deterministic lot-size inventory model for deteriorating items with shortages and a declining market. *Computers & Operations Research*, *22*(3), 345–356. doi:http://dx.doi.org/10.1016/0305-0548(94)E0005-R

Weng, Z. K., & Parlar, M. (1999). Integrating early sales with production decisions: analysis and insights. *IIE Transactions*, *31*(11), 1051–1060. doi:10.1023/A:1007671407701

White, J., Armstrong, H., Armstrong, P., Bourgeault, I., Choiniere, J., & Mykhalovskiy, E. (2000). The impact of managed care on nurses' workplace learning and teaching*. *Nursing Inquiry*, *7*(2), 74–80.

Wild, T. (2007). *Best practice in inventory management*. Canada: John Wiley & Sons, Inc.

Willemain, T., Smart, C., & Schwarz, H. (2004). A new approach to forecasting intermittent demand for service parts inventories. *International Journal of Forecasting*, *20*(3), 375–387. doi:10.1016/S0169-2070(03)00013-X

Williams, J. F. (1982). Note—On the Optimality of Integer Lot Size Ratios in "Economic Lot Size Determination in Multi-Stage Assembly Systems." *Management Science*, *28*(11), 1341–1349. doi:10.1287/mnsc.28.11.1341

WinQSB, Inc. (2002). WinQSB-queueing model analysis software, 2.0.

Wu, J., Lee, W., & Tsai, H.-Y. (2007). Computational algorithmic procedure of optimal inventory policy involving a negative exponential crashing cost and variable lead time demand. *Applied Mathematics and Computation*, *184*, 798–808. doi:10.1016/j.amc.2006.05.202

Xia, Y., Chen, B., & Kouvelis, P. (2008). Market-Based Supply Chain Coordination by Matching Suppliers' Cost Structures with Buyers' Order Profiles. *Management Science*, *54*(11), 1861–1875. doi:10.1287/mnsc.1080.0900

Xu, J., Zhang, J., & Liu, Y. (2009). An adaptive inventory control for a supply chain. In *2009 Chinese Control and Decision Conference* (pp. 5714–5719). Ieee. doi:10.1109/CCDC.2009.5195218

Yang, B., & Geunes, J. (2007). Inventory and lead time planning with lead-time-sensitive demand. *IIE Transactions*, *39*, 439-452 doi:10.1080/07408170600838456

Yang, J., Ding, H., Wang, W., & Dong, J. (2008). A New Optimal Policy for Inventory Control problem with Nonstationary Stochastic Demand. In *IEEE* (pp. 1668–1673).

Yao, D., Yue, X., Mukhopadhyay, S. K., & Wang, Z. (2009). Strategic inventory deployment for retail and e-tail stores. *The International Journal of Management Science*, *37*, 646–658. doi:10.1016/j.omega.2008.04.001

Yoo, Y., Kim, W., & Rhee, J. (1997). Efficient inventory management in multi-echelon distribution systems. *Computers & Industrial Engineering*, *33*(3–4), 729–732. doi:http://dx.doi.org/10.1016/S0360-8352(97)00233-7

You, F., & Grossmann, I. (2010). Integrated multi-echelon supply chain design with inventories under uncertainty: MINLP models, computational strategies. *AIChE Journal*, *56*(2), 419–440. doi:10.1002/aic.12010

You, F., & Grossmann, I. E. (2011). Stochastic inventory management for tactical process planning under uncertainties: MINLP models and algorithms. *AIChE Journal*, *57*(5), 1250–1277. doi:10.1002/aic.12338

Yvan, N. (2011). *Supply Chain performance – The impact of interactions between flexibility enablers and uncertainty*. France: Unversity of Neuchatel.

Zangwill, W. I. (1969). A Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System—A Network Approach. *Management Science*, *15*(9), 506–527. doi:10.1287/mnsc.15.9.506

Zaojie, K., & Guoying, H. (2007). The Effect of Information Sharing on Inventory in A Two-stage Supply Chain. In *2007 IEEE International Conference on Automation and Logistics* (pp. 2883–2886). Ieee. doi:10.1109/ICAL.2007.4339073

Zhao, X., Hou, J., & Gilbert, K. (2014). Measuring the variance of customer waiting time in service operations. *Management Decision*, *52*(2), 296–312. doi:10.1108/MD-01-2013-0012

Zhao, D., Zhan, Y. A. N., Huo, Y., & Wu, X. (2006). Lot-Sizing Disposal Model in Reverse Logistics. In *the Fifith International Conference on Machine Learning and cybernetics* (pp. 13–16).

Zipkin, P. (1986). Stochastic leadtimes in continuous-time inventory models. *Naval Research Logistics Quarterly*, *33*(4), 763–774. doi:10.1002/nav.3800330419

Zipkin, P. H. (2000). *Foundations of inventory management* (vol 2.). New York: McGraw-Hill.

Zong, Z. (2011). *Information-Theoretic Methods for Estimating Complicated Probability Distribution*. (Oxford, Ed.). Amsterdam: Elsevier B.V.

Zunes, S. (2009). The US invasion of Iraq: the military side of globalization. *Globalizations*, *6*(1), 99–105.