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**WINSORIZED MODIFIED ONE STEP M-ESTIMATOR  
IN ALEXANDER-GOVERN TEST**



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**UUM**  

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**Universiti Utara Malaysia**

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**WINSORIZED MODIFIED ONE STEP M-ESTIMATOR  
IN ALEXANDER-GOVERN TEST**

**A Thesis Submitted to College of Arts & Sciences in partial  
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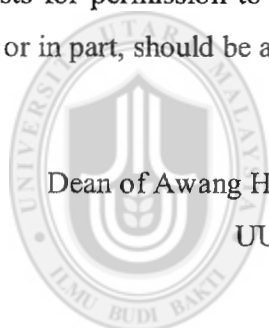
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## Abstrak

Kajian ini tertumpu kepada ujian kumpulan bebas bagi membandingkan dua atau lebih min menggunakan kaedah berparameter iaitu ujian Alexander-Govern (*AG*). Ujian ini menggunakan min sebagai sukatan kecenderungan memusat dan dianggap sebagai alternatif yang lebih baik berbanding *ANOVA*, ujian Welch dan ujian James. Walaupun ujian *AG* mempunyai kawalan yang baik terhadap kadar ralat Jenis I dan menghasilkan kuasa yang tinggi pada varians heterogen, ujian ini tidak teguh pada data yang tidak normal. Justeru, min terpangkas telah dicadangkan dalam ujian tersebut untuk menangani masalah ketaknormalan dan kemudiannya, satu penganggar yang lebih teguh dikenali sebagai penganggar *M* satu langkah terubahsuai telah diperkenalkan. Penganggar berkenaan adalah tidak dipengaruhi oleh bilangan kumpulan, namun telah gagal untuk menghasilkan kawalan yang baik terhadap kawalan ralat Jenis I, dalam keadaan kepencongan dan kurtosis yang ekstrim. Kajian ini mencadangkan penganggar *MOM* terWinsor (*WMOM*) sebagai sukatan kecenderungan memusat dalam usaha untuk meneguhkan ujian *AG*. Ujian *AG* yang ditambah baik ini, *AGWMOM* mampu menyingkirkan kewujudan data terpencil daripada taburan data. Satu kajian simulasi terhadap 5,000 set data telah dilaksanakan untuk membandingkan prestasi ujian: *AG*, *AGMOM* (ujian *AG* menggunakan penganggar *MOM*), *AGWMOM*, ujian-*t* dan *ANOVA*. Keputusan menunjukkan bahawa ujian *AGWMOM* telah meningkatkan bilangan kondisi teguh pada taburan terpencong dengan hujung normal dan taburan terpencong dengan hujung berat berbanding ujian yang lain.

Sebagai tambahan, ujian ini telah menghasilkan kuasa yang tinggi dalam kebanyakan kondisi pada empat kumpulan dengan saiz sampel tidak seimbang. Dapatan kajian mendorong untuk ujian ini menjadi paling sesuai apabila taburan data adalah berhujung berat.

**Kata kunci:** ujian Alexander-Govern, penganggar *MOM*, kadar ralat Jenis I, Kuasa ujian, ujian *AGWMOM*

## Abstract

This research centres on independent group test of comparing two or more means by using the parametric method, namely the Alexander-Govern (*AG*) test. It uses mean as its central tendency measure and is considered as a better alternative to the *ANOVA*, the Welch test and the James test. Although the *AG* test has a good control of Type I error rate and produces a high power under variance heterogeneity, it is not robust to non-normal data. Thus, trimmed mean was proposed in the test to handle the problem of non-normality and later, a more robust estimator called modified one step *M* (*MOM*) estimator was introduced. These estimators are not influenced by the number of groups, but failed to give a good control of Type I error rate, under extreme conditions of skewness and kurtosis. This research proposes the Winsorized *MOM* (*WMOM*) estimator as a measure of central tendency in attempt to robustify the *AG* test. This enhanced *AG* test, *AGWMOM* is able to remove the appearance of outliers from the data distribution. A simulation study of 5,000 data sets was conducted to compare the performance of the tests: *AG*, *AGMOM* (*AG* test using *MOM* estimator), *AGWMOM*, *t*-test and *ANOVA*. The results show that the *AGWMOM* test has improved the number of robust conditions under skewed normal tailed and skewed heavy tailed distributions compared to the other tests. Additionally, the test produced high power in most conditions under four groups with unbalanced sample size. It leads that this test is convenient specifically when the data distribution is heavy tailed.

**Keywords:** Alexander-Govern test, *MOM* estimator, Type I error rate, power of test, *AGWMOM* test

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## Table of Contents

Permission to Use .....	i
Abstrak.....	ii
Abstract.....	iii
Acknowledgement .....	iv
Table of Contents.....	v
List of Tables.....	viii
List of Figures.....	x
List of Abbreviations.....	xi
List of Appendices .....	xii
<b>CHAPTER ONE INTRODUCTION.....</b>	<b>1</b>
1.1 Background of the Study.....	1
1.2 Problem Statement.....	6
1.3 Objective of the Research .....	8
1.4 The Scope of the Research.....	9
1.5 Significance of the Research .....	9
1.6 Organization of the remaining Chapters .....	10
<b>CHAPTER TWO LITERATURE REVIEW.....</b>	<b>11</b>
2.1 Introduction .....	11
2.2 Robust Statistics .....	12
2.3 Dealing with Non-Normal Data .....	13
2.4 <i>MOM</i> Estimator.....	16
2.5 Trimming and Winsorization Methods.....	19
2.6 The Alexander-Govern Test and its Test Statistic.....	21
2.7 Summary .....	24
<b>CHAPTER THREE RESEARCH METHODOLOGY.....</b>	<b>26</b>
3.1 Introduction .....	26
3.2 The Modified Alexander-Govern Test.....	27
3.3 Variables Investigated in this Research .....	31
3.3.1 Balanced and Unbalanced Sample Size.....	32
3.3.2 Variance Ratios .....	34
3.3.3 Group Sizes .....	35

3.3.4 Types of Distribution.....	36
3.3.5 Types of Pairing .....	38
3.4 The Research Design.....	39
3.5 Statistical Power Analysis.....	43
3.5.1 The Effect Size Index... ..	45
3.5.2 The Effect Size Index for $J = 2$ .....	45
3.5.3 The Effect Size Index for $J = 4$ or More.....	46
<b>CHAPTER FOUR      RESULTS AND ANALYSIS.....</b>	<b>54</b>
4.1 Introduction.....	54
4.2 The Type I Error Rates.....	54
4.2.1 Two Groups Case .....	55
4.2.1.1 Normal Distribution ( $g = 0; h = 0$ ).....	55
4.2.1.2 Symmetric Heavy Tailed Distribution ( $g = 0$ and $h = 0.5$ ).....	56
4.2.1.3 Skewed Normal Tailed Distribution ( $g = 0.5$ and $h = 0$ ) .....	57
4.2.1.4 Skewed Heavy Tailed Distribution ( $g = 0.5$ and $h = 0.5$ ) .....	58
4.2.2 Four Groups <i>Condition</i> .....	59
4.2.2.1 Normal Distribution ( $g = 0$ and $h = 0$ ) .....	59
4.2.2.2 Symmetric Heavy Tailed Distribution ( $g = 0$ and $h = 0.5$ ).....	60
4.2.2.3 Skewed Normal Tailed Distribution ( $g = 0.5$ and $h = 0$ ) .....	61
4.2.2.4 Skewed Heavy Tailed Distribution ( $g = 0.5$ and $h = 0.5$ ) .....	62
4.2.3 Six Groups.....	63
4.2.3.1 Normal Distribution ( $g = 0$ and $h = 0$ ) .....	63
4.2.3.2 Symmetric Heavy Tailed Distribution ( $g = 0$ and $h = 0.5$ ).....	64
4.2.3.3 Skewed Normal Tailed Distribution ( $g = 0.5$ and $h = 0$ ) .....	65
4.2.3.4 Skewed Heavy Tailed Distribution ( $g = 0.5$ and $h = 0.5$ ) .....	66
4.2.4 Overall Conclusion on the Type I Error Rate .....	67
4.3 The Power Rate of the Test.....	70
4.3.1 Two Groups Condition under Normal Distribution .....	71
4.3.2 Two Groups Condition under Symmetric Heavy Tailed Distribution.....	72
4.3.3 Two Groups Condition under a Skewed Normal Tailed Distribution .....	75
4.3.4 Two Groups Condition under a Skewed Heavy Tailed Distribution.....	77
4.4 The Power of the Four Tests, For Four Groups Condition, Under Four Different Distributions.....	79

4.4.1 Four Groups Condition under a Normal Distribution.....	79
4.4.2 Four Groups Condition under Symmetric Heavy Tailed Distribution.....	81
4.4.3 Four Groups Condition under a Skewed Normal Tailed Distribution.....	83
4.4.4 Four Groups Condition, Under a Skewed Heavy Tailed Distribution.....	85
4.5 The Power of the Tests, For Six Groups Condition, Under Four Different Distributions.....	87
4.5.1 Six Groups Condition, Under a Normal Distribution.....	87
4.5.2 Six Groups Condition, Under a Symmetric Heavy Tailed Distribution.....	89
4.5.3 Six Groups Condition, Under a Skewed Normal Tailed Distribution.....	91
4.5.4 Six Groups Condition, Under a Skewed Heavy Tailed Distribution.....	93
4.6 Evaluating the Capacity of the Test Using Real Data.....	95
<b>CHAPTER FIVE DISCUSSION AND CONCLUSION.....</b>	<b>99</b>
5.1 Summary.....	99
5.2 Implication and Conclusion.....	100
5.3 Suggestion and Future Research.....	102
<b>REFERENCES.....</b>	<b>103</b>



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## List of Tables

Table 3.1 The characteristics of the $g$ - and $h$ -distribution.....	38
Table 3.2 Research Design for Two Groups Condition with $N = 40$ .....	40
Table 3.3 Research Design for Four Groups Condition with $N = 80$ .....	41
Table 3.4 Research Design for Six Groups Condition with $N = 120$ .....	42
Table 3.5 Pattern of Variability of the Effect Size Index for 4 Groups and 6 Groups.	48
Table 4.1 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and $t$ -test Under Normal Distribution for Two Groups Condition .....	56
Table 4.2 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and $t$ -test Under a Skewed Normal Tailed Distribution for Two Groups Condition.....	57
Table 4.3 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and $t$ -test Under a Skewed Heavy Tailed Distribution for Two Groups Condition.....	58
Table 4.4 Comparison of the Type I error rates for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Normal Distribution for Four Groups condition.....	59
Table 4.5 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Symmetric Heavy Tailed Distribution, for Four Groups Condition.....	60
Table 4.6 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Skewed Normal Tailed Distribution for Four Groups Condition.....	62
Table 4.7 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Symmetric Heavy Tailed Distribution for Six Groups Condition.....	62
Table 4.8 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Skewed Normal Tailed Distribution for Six Groups Condition.....	63
Table 4.9 Comparison of the Type I Error Rate for the $AG$ , $AGMOM$ , $AGWMOM$ , and the $ANOVA$ Under a Skewed Heavy Tailed Distribution for Six	

Groups Condition.....64

Table 5.1 Number of Conditions of *AGWMOM* test from normal to skewed normal tailed distribution.....101



## List of Figures

Figure 4.1: Power versus Effect Size Index, for two groups condition under a normal distribution.....	72
Figure 4.2: Power versus Effect Size Index, for a symmetric heavy tailed distribution, for two groups condition.....	74
Figure 4.3: Power versus Effect Size Index, for two groups condition under a skewed normal tailed distribution.....	76
Figure 4.4: Power versus Effect Size Index, for two groups condition, for $g = 0.5$ and $h = 0.5$ .....	78
Figure 4.5: Power versus Effect Size Index, for four groups condition.....	80
Figure 4.6: Power against Effect Size Index, for four groups condition, for $g = 0$ and $h = 0.5$ .....	82
Figure 4.7: Power versus Effect Size Index, for four groups condition, Under a skewed normal tailed distribution.....	84
Figure 4.8: Power versus Effect Size Index, under a skewed heavy tailed distribution, for four groups condition.....	86
Figure 4.9: Power versus <i>Effect</i> Size Index, for six groups condition under a normal distribution.....	88
Figure 4.10: Power versus Effect Size Index, for six groups condition, under a symmetric heavy tailed distribution.....	90
Figure 4.11: Power versus Effect Size Index under a skewed normal tailed distribution, for six groups condition.....	92
Figure 4.12: Power versus Effect Size Index, for six groups condition, under a skewed heavy tailed distribution.....	94

## List of Abbreviations

AG	Alexander-Govern Test
MOM	Modified One Step $M$ -estimator
AGMOM	Modified One Step $M$ -estimator in the Alexander-Govern Test
WMOM	Winsorized Modified One Step $M$ -estimator
AGWMOM	Winsorized Modified One Step $M$ -estimator in the Alexander-Govern Test
ANOVA	Analysis of Variance



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## List of Appendices

APPENDIX A: SAS Code For The Winsorized <i>MOM</i> Estimator In AG Test.....	109
APPENDIX B: Table B1 The Power of the tests for $g = 0$ and $h = 0$ , Under Two groups Condition.....	114
APPENDIX C: Table C2 The Power of the tests for $g = 0$ and $h = 0.5$ , Under Two Groups Condition.....	116
APPENDIX D: Table D3 The Power of the tests for $g = 0.5$ and $h = 0$ , Under Two Groups Condition.....	117
APPENDIX E: Table E4 The Power of the tests for $g = 0.5$ and $h = 0.5$ , Under Two Groups Condition... ..	118
APPENDIX F: Table F5 The Power of the tests for $g = 0$ and $h = 0$ , Under Four Groups Condition.....	120
APPENDIX G: Table G6 The Power of the tests for $g = 0$ and $h = 0.5$ , Under Four Groups Condition.....	122
APPENDIX H: Table H7 The Power of the tests for $g = 0.5$ and $h = 0$ , Under Four Groups Condition.....	125
APPENDIX I: Table I8 The Power of the tests for $g = 0.5$ and $h = 0.5$ , Under Four Groups Condition.....	129
APPENDIX J: Table J9 The Power of the tests for $g = 0$ and $h = 0$ , Under Six groups Condition.....	134
APPENDIX K: Table K10 The Power of the tests for $g = 0$ and $h = 0.5$ , Under Six Groups Condition.....	139
APPENDIX L: Table L11 The Power of the tests for $g = 0.5$ and $h = 0$ , Under Six Groups Condition.....	144
APPENDIX M: Table M12 The Power of the tests for $g = 0.5$ and $h = 0.5$ , Under Six groups Condition.....	149
APPENDIX N: Table N13 Real Life Data of Group Young, Middle and Old respectively.....	154
APPENDIX O: Table O14 The Winsorized Data Distribution, From The Real Life Data.....	155



# CHAPTER ONE

## INTRODUCTION

### 1.1 Background of the Study

This research makes comparison of the performances of the Type I error rate and power of five different tests. These tests are (i) Alexander-Govern test (*AG test*), (ii) Modified One Step *M*-estimator (*MOM*) estimator in the Alexander-Govern test (*AGMOM test*), (iii) Winsorized Modified One Step *M*-estimator (*WMOM*) estimator in the Alexander-Govern test (*AGWMOM test*), (iv) *t-test* (v) Analysis of Variances (*ANOVA*). Each test is performed under two, four and six groups conditions, with the combination of both balanced and unbalanced sample sizes, equal and unequal variances respectively, with each of the *g*- and *h*- distributions. The *g*- and *h*-distribution is used to determine the level of skewness and kurtosis respectively in a data distribution.

The best among the five tests will produce the best control of Type I error rate and also produce high power, under skewed heavy tailed distribution. The independent group tests such as the *ANOVA* have been applied in different field of life, for example in medicine, economics, sociology and agriculture, as discussed by Pardo, Pardo, Vincente and Esteban (1997). Three main assumptions have to be fulfilled before the *ANOVA* can work effectively, namely: (i) homogeneity of the variance (ii) normality of the data and (iii) independent observations of the data distribution.

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