

**THE ALL PAY. COMMON VALUE AUCTION
AS A MODEL OF CONTESTS**

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ABSTRACT

Radhi. Nungsari Ahmad. Ph.D., Purdue University, December 1994. The All Pay. Common Value Auction As a Model of Contests. Major Professors: Marie Thursby and Daniel Kovenock.

An all pay, common value auction is proposed as a model of contests. The common value of the prize is ex-ante unknown, but each contestant has private information about its true value. These private information are affiliated. Unlike in the symmetric, winner pay auction model of Milgrom and Weber (1982), the affiliation assumption is not sufficient to ensure the existence of an increasing equilibrium bid. The all pay auction is shown to require a stronger condition; a joint restriction on the expected valuation of the prize and the beliefs bidders have of each other's estimates of the prize. Equilibrium bids are increasing if there is not too strong a degree of affiliation and the expected value of the prize is increasing sufficiently rapidly in each bidder's type. When there exists an increasing equilibrium, the equilibrium is unique even in the asymmetric bidder case. The results extend to the all pay, private values auction which is obtained as a special case. The all pay and winner pay auctions are then compared at the symmetric equilibrium. With increasing equilibrium bids, it is possible to order the expected revenues from the two auctions. This was first proposed by Amann and Leininger (1994). It is shown by way of an example that although sufficient, monotonicity of equilibrium bids is not a necessary condition for the ordering. Without, monotonicity, however, comparing the two auctions can lead to conclusions which are not robust. Two examples are provided to compare the role of affiliation and the effects of nonmonotonicity in the all pay and winner pay auctions.

1. INTRODUCTION

The all pay auction has long been used to model economic contests. Among the contests modeled in the literature are rent-seeking and lobbying (Tullock, 1980; Hillman and Samet, 1987; Hillman and Riley, 1989; Baye, Kovenock and De Vries, 1993). patent races (Fudenberg, Gilbert and Tirole, 1983; Harris and Vickers, 1985). bribery games (Lien, 1990). and corporate control (Harris and Raviv, 1988). Other interesting examples of contests include the arms races (Shubik, 1971; Schelling, 1980). and the various contests for mates, territories and survival in the animal kingdom so aptly described in Maynard Smith (1982). While the auction literature itself is large, there has not been a systematic treatment of the all pay version until recently, despite its widespread use in modeling contests.

The defining characteristic of contests is that participants have to expend some resources to participate. Depending on the contest, the resources expended may be effort levels, bribes, investment or expenditure levels, or even the amount of time. The determination of the winner also varies with the particular type of contest. Generally, whether one wins the contest or not depends on the amount of resources expended relative to other contestants. There are contests where the outcome is stochastic but the probability of winning is increasing in a contestant's level of commitment. These are termed imperfectly *discriminating contests* in the literature.¹ The classic models of rent seeking (Tullock, 1980) and R&D races

¹ Hillman and Riley (1989) used the term to describe their model where "...the process cannot discriminate among contenders to designate the winner with certainty, but rather the outcome of the contest is an assignment to each agent of a probability- that he will be the winner." This is not to be confused with the term.

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REFERENCES

- Amann, E. and W. Leininger. "The All-Pay Auction with Incomplete Information," mimeo.. June 1993.
- Amann, E. and W. Leininger, "Expected Revenue of All-Pay and First Price Sealed Bid Auctions with Affiliated Signals." mimeo., July 1994.
- Baye, M. R., D. Iiovenock and C. G. de Vries, "The All Pay Auction with Complete Information." *CentER Working Paper* No. 9051, Tilburg University. 1990.
- Baye, M. R., D. Iiovenock and C. G. de Vries, "Auctions with All Pay and Winner Pay Components." mimeo.. September. 1991.
- Baye, M. R., D. Kovenock and C. G. de Vries, "Rigging the Lobbying Process: An Application of the All Pay Auction," *American Economic Review*, 83(1) 1993: 289-294.
- Baye, M. R., D. Iiovenock and C. G. de Vries, "The Solution to the Tullock Rent-Seeking Game when $R > 2$: Mixed Strategies and Mean Dissipation Rates," mimeo. 1993b.
- Beck, P. J. and M. W. Maher, "A Comparison of Bribery and Bidding in Thin Markets." *Economic Letters*, 20. 1986:1-5.
- Bikhchandani, S and J. G. Riley, "Equilibria in Open Common Value Auctions." *Journal of Economic Theory*, 53, 1991: 101-130.
- Cadot, O. "Corruption as a Gamble," *Journal of Public Economics*, 33. 1987: 223-244.
- Engelbrecht-Wiggans, M., P. Milgrom and R. J. Weber, "Competitive Bidding and Propriety Information," *Journal of Mathematical Economics*, 11, 1983 : 161-169.
- Fudenberg, D. and J. Tirole, *Game Theory*, The MIT Press, 1991.
- Fudenberg, D. and J. Tirole, "Understanding Rent Dissipation: On the Use of Game Theory in Industrial Organization." *American Economic Review*, 1987: 176-183.
- Fudenberg, D., R. Gilbert, J. Stiglitz and J. Tirole, "Preemption, Leapfrogging and Competition in Patent Races:." *European Economic Review*. 22. 1983: 3-32.
- Harris, C. and J. Vickers, "Patent Races and the Persistence of Monopoly." *Journal of Industrial Economics*. 23, 1985: 461-481.

Harris, M. and A. Raviv, "Corporate Control Contests and Capital Structure," *Journal of Financial Economics*, 20, 1988: 55-86.

Hausch, D. B.. "An Asymmetric Common Value Auction Model:" *RAND Journal of Economics*, 18(4), 1987: 611-621.

Hendricks, K. and R. H. Porter. "An Empirical Study of an Auction with Asymmetric Information." *American Economic Review*, 78(5), 1988: 865-883.

Hillman, A. L. and D. Samet, "Dissipation of Contestable Rents by Small Numbers of Contenders." *Public Choice*. 54, 1987: 63-82.

Hillman, A. L. and J. G. Riley, "Politically Contestable Rents and Transfers," *Economics and Politics*. 1, 1989: 11-40.

Hirshleifer, J. and J. G. Riley, *The Analytics of Uncertainty and Information*, Cambridge University Press. 1992.

Kim, I-G. "Government Procurement and Competitive Bribery in Less Developed Countries," mimeo., June 1992.

Kagel, J. and D. Levin. "The Winner's Curse and Public Information in Common Value Auctions." *American Economic Review*, 76(5), 1986: 894-920.

Krishna, V.. "Revenue Equivalence of the All-Pay and First Price Sealed Bid Auctions." mimeo. March 1994.

Krueger, A.. "The Political Economy of the Rent Seeking Society." *American Economic Review*. 64. 1974: 291-303.

Lien, D-H.. "A Note on Competitive Bribery Games," *Economics Letters*, 22, 1986: 337-341.

Lien, D-H.. "Asymmetric Information in Competitive Bribery Games." *Economics Letters*, 23, 1987: 153-156.

Lien, D-H.. "Corruption and Allocation Efficiency," *Journal of Development Economics*. 33, 1990: 153-164.

Marshall, R.C.. M.J. Meurer, J-F Richard and W. Stromquist, "Numerical Analysis of Asymmetric First Price Auctions," mimeo., August 1992.

Maynard Smith, J., "The Theory of Games and Evolution of Animal Conflicts." *Journal of Theoretical Biology*, 47, 1974: 209-21.

Maynard Smith, J., *Evolution and the Theory of Games*. Cambridge University Press. 1982.

McAfee, R. P. and J. McMillan, "Auctions and Bidding." *Journal of Economic Literature*, 25, 1987: 699-738.

McAfee, R.P. and P. J. Reny, "Correlated Information and Mechanism Design." *Econometrica*, 60. 1992: 395-421.

Milgrom, P. and R. J. Weber. "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50(5), 1982: 1089-1122.

Milgrom, P. and R. J. Weber. "The Value of Information in a Sealed Bid Auction." *Journal of Mathematical Economics*, 10, 1982b: 105-114.

Milgrom, P.. "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49(4), 1981: 921-943.

Myerson, R., "Optimal Auction Design," *Mathematics of Operations Research*: 6, 1981: 58-63.

Myerson, R.. *Game Theory: Analysis of Conflicts*, Harvard University Press, 1991.

Riley, J. G.. "Ex Post Information in Auctions." *Review of Economic Studies*, 55, 1988: 409-430.

Schelling, T. C.. *The Strategy of Conflict*. 2nd Edition, Harvard University Press, 1980.

Shubik, M. "The Dollar Auction Game: A Paradox in Non-Cooperative Behavior and Escalation," *Journal of Conflict Resolution*. 15, 1971:545-547.

Thaler, R. H., *The Winner's Curse: Paradoxes and Anomalies of Economic Life*. The Free Press. 1992.

Theil, S. E.. "Multidimensional Auctions." *Economics Letters*. 28, 1988: 37-40.

Tullock, G.. "Efficient Rent Seeking," in J. M. Buchanan, R. Tollison and G. Tullock(eds.), *The Theory of the Rent Seeking Society*, Texas A&M University Press, 1980: 267-292.

Vickrey, W., "Counter-Speculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, 16, 1961: 8-37.

Weber, R. J., "Auctions and Competitive Bidding," in H.P. Young (ed.), *Fair Allocation*, American Mathematical Society. 1985: 143-170.

Weverbergh, M.. "Competitive Bidding With Asymmetric Information Reanalyzed." *Management Science*, 25, 1979: 291-294.

Wilson, R., "Competitive Bidding with Asymmetric Information," *Management Science*, 13(11), 1967: 816-820.

Wilson, R.. "A Bidding Model of Perfect Competition," *Review of Economic Studies*, 44. 1977: 511-518.

Wilson, R. *Two Surveys: Auctions and Entry Deterrence*, Stanford Institute for Theoretical Economics Technical Report No. 3. Stanford University, May 1990.