# QR FACTORIZATION EQUALISATION SCHEME FOR MODE DIVISION MULTIPLEXING TRANSMISSION IN FIBRE OPTICS

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## Abstrak

Sistem komunikasi optik memainkan peranan besar dalam pengendalian trafik internet di seluruh dunia. Trafik internet telah meningkat pada kadar yang dramatik dan infrastruktur rangkaian optik semasa mungkin tidak dapat menampung pertumbuhan trafik di beberapa dekad. Pemultipleks pembahagian mod diperkenalkan sebagai satu teknik baru untuk meningkatkan kapasiti rangkaian optik dengan menggunakan mod cahaya sebagai saluran individu. Salah satu isu utama dalam MDM adalah gandingan mod yang merupakan satu fenomena fizikal apabila mod cahaya bertukar tenaga antara satu sama lain semasa pembiakan melalui gentian optik menyebabkan gangguan antara simbol (ISI).

Banyak kajian berdasarkan min kuasa dua terkecil (LMS) dan kuasa dua terkecil rekursif (RLS) telah diambil bagi mengurangkan kesan mod gandingan. Walaubagaimanapun, kebanyakan pendekatan mempunyai kerumitan pengkomputan yang tinggi dan menghalang operasi sistem komunikasi berkelajuan tinggi. Pendekatan penyamaan lenyap tidak memerlukan isyarat latihan, justeru itu akan mengurangkan muatan kekal. Sebaliknya, pemfaktoran QR menunjukkan kerumitan pengkomputan yang rendah dalam penyelidikan terdahulu, dalam domain radio. Gabungan kedua-dua konsep ini menunjukkan hasil yang ketara, kerana penggunaan algoritma yang lebih ringkas mengurangkan pemprosesan yang perlu dilakukan oleh peralatan komunikasi, manakala ketiadaan latihan isyarat menjimatkan jalur lebar dan meningkatkan prestasi keseluruhan sistem. Berdasar kepada penulisan-penulisan yang ada, penyamaan lenyap berdasarkan pemfaktoran QR belum pernah digunakan dalam penyamaan MDM.

Penyelidikan ini melalui Metodologi Penyelidikan Reka Bentuk (DRM) empat-tahap untuk mencapai objektif kajian. Peringkat perlaksanaan melibatkan dua simulator yang berbeza. Simulator pertama merupakan sebuah simulator optic yang digunakan untuk mengumpul data optik awal. Kemudiannya, MATLAB digunakan untuk membangunkan skim penyamaan. Pembangunan bermula dengan penghasilan fungsi pemindahan sistem (H) untuk digunakan sebagai input kepada penyamaan yang dibangunkan. Penyamaan lenyap berdasarkan pemfaktoran QR dipilih sebagai satu cara untuk memperkenalkan penyamaan yang cekap untuk mengurangkan ISI dengan merapatkan lebar denyut. Peringkat pembangunan termasuk peringkat di mana anggaran saluran itu berlaku. Ciri-ciri statistik berdasarkan sisihan piawaian (STD) kuasa isyarat input dan output telah digunakan untuk bahagian saluran anggaran penyamaan lenyap.

Kaedah penganggaran saluran yang dicadangkan mampu membuat anggaran saluran dengan keseluruhan ralat min persegi (MSE) 0.176588301 dari isyarat awal yang dihantar. Kajian ini mendapati bahawa saluran yang paling lemah mempunyai MSE 0.771365 berbanding isyarat awal, manakala saluran terbaik mempunyai MSE 0.000185 berbanding isyarat awal. Ia telah dilakukan dengan mengelak isu-isu berkaitan pembangunan algoritma-algoritma terdahulu yang dibangunkan bagi tujuan yang sama. Algoritma dalam kajian ini terbukti mengurangkan masalah kerumitan pengkomputan, yang merupakan salah satu isu utama yang sedang dihadapi, melalui LMS dan RLS. Hasil ujian menunjukkan skema penyamaan yang dibangunkan mempunyai kerumitan 0(N) berbanding $0(N^2)$ bagi RLS. Pada masa sama, ia adalah lebih laju dari LMS kerana masa pengiraannya adalah 0.005242 saat berbanding 0.0077814 saat bagi LMS. Keputusan ini sah bagi matrik songsang dan matrik sama.

# Abstract

Optical communication systems play a major role in handling worldwide Internet traffic. Internet traffic has been increasing at a dramatic rate and the current optical network infrastructure may not be able to support the traffic growth in a few decades. Mode division multiplexing is introduced as a new emerging technique to improve the optical network capacity by the use of the light modes as individual channels. One of the main issues in MDM is mode coupling which is a physical phenomenon when light modes exchange their energy between each other during propagation through optical fiber resulting in inter-symbol interference (ISI).

Many studies based on Least Mean Square (LMS) and Recursive Least Square (RLS) have taken place to mitigate the mode coupling effect. Still, most approaches have high computational complexity and hinders high-speed communication systems. Blind equalisation approach does not need training signals, thus, will reduce the overhead payload. On the other hand, QR factorization shows low computational complexity in the previous research in the radio domain. The combination of these two concepts shows significant results, as the use of low complexity algorithms reduces the processing needed to be done by the communication equipment, resulting in more cost effective and smaller equipment, while having no training signal saves the bandwidth and enhances the overall system performance. To the best knowledge of the researcher, blind equalisation based on QR factorization technique has been not used in MDM equalisation to date.

The research goes through the four stages of the design research methodology (DRM) to achieve the purpose of the study. The implementation stage is taken two different simulators has been used, the first one which is the optical simulator is used to collect the initial optical data then, MATLAB is used to develop the equalisation scheme. The development starts with the derivation of the system's transfer function (H) to be used as the input to the developed equalizer. Blind equalisation based on QR factorization is chosen as a way to introduce an efficient equalization to mitigate ISI by narrowing the pulse width. The development stages include a stage where the channel estimation is taken place. Statistical properties based on the standard deviation (STD) of the powers of the input and output signals has been used for the blind equalisation's channel estimation part.

The proposed channel estimation way has the ability in estimating the channel with an overall mean square error (MSE) of 0.176588301 from the initial transmitted signal. It is found that the worst channel has an MSE of 0.771365 from the transmitted signal, while the best channel has and MSE of 0.000185 from the transmitted signal. This is done by trying to avoid the issues accompanied with the development of the previous algorithms that have been utilized for the same goal. The algorithm mentioned in the study reduces the computational complexity problem which is one of the main issues that accompany currently used tap filter algorithms, such as (LMS) and (RLS). The results from this study show that the developed equalisation scheme has a complexity of O(N) compared with $O(N^2)$ for RLS and at the same time, it is faster than LMS as its calculation CPU time is equal to 0.005242 seconds compared with 0.0077814 seconds of LMS. The results are only valid for invertible and square channel matrices.

## Acknowledgements

In the name of ALLAH, Most Gracious, Most Merciful:

“Work; so Allah will see your work and (so will) His messenger and the believers;”

(The Holy Quran - AtTawbah 9:105)

There are so many wonderful and talented people whom I would like to thank for their help and patience that I am loss to where to begin.

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*Dedication*

*To my eternal love and the eternal light of divine, the soul of my world, the reason I exist*

*To the light that lit my path.... My father*

*To the sun of my life, the warmth of which is overwhelming my world.... My mother*

*To the ones who are like the stars in my skies.... My brother and sisters*

*To the ones who are the breaths I take, the ones who live deep down inside my heart, my wife and children*

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# CHAPTER ONE: INTRODUCTION

## 1.1 Research Background

Currently, the volume of data traffic is growing at an estimated annual rate of more than 50%. A major portion of this traffic is linked to high-definition video streaming, multimedia file sharing, cloud computing, mobile networking, online gaming and a host of other innovative networking technologies [1]. The predicted level of global traffic over the next three years is displayed in the following Figure 1.

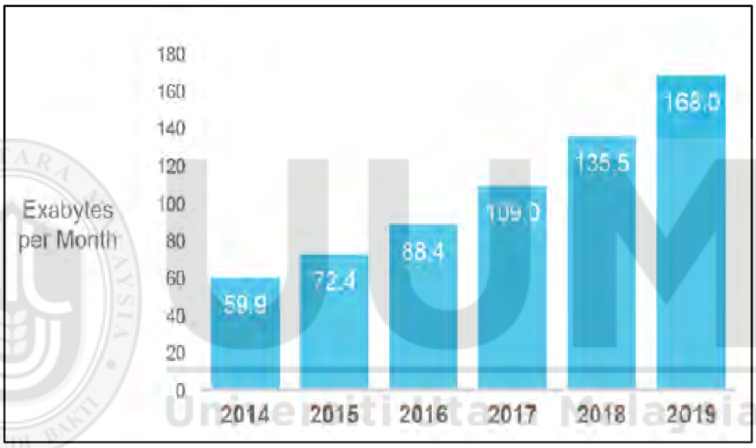


*Figure 1: Annual global internet traffic growth prediction [2]*

The rates of rising data and growth in the number of users have challenged service providers and prompted investigations on new technologies to cater to the overwhelming traffic in optical backbones. The void in this area has led to the development of innovative systems for capacity augmentation. This includes several multiplex techniques [3] and advanced modulation schemes [4] .

As a multiplexing techniques, Mode Division Multiplexing (MDM), which has the potential for spatial multiplexing, involves the use of modes as parallel channels [5].

Because of this involvement, system capacity ascends in proportion to the quantity of modes employed [6]. Mode division multiplexing has been established as the fastest developing multiplexing systems for enhancing the aggregate bandwidth of other multiplexing schemes [7]. Figure 2 portrays the effective use of mode division multiplexing, and its aim is to elevate the capacity of optical fibre networks.

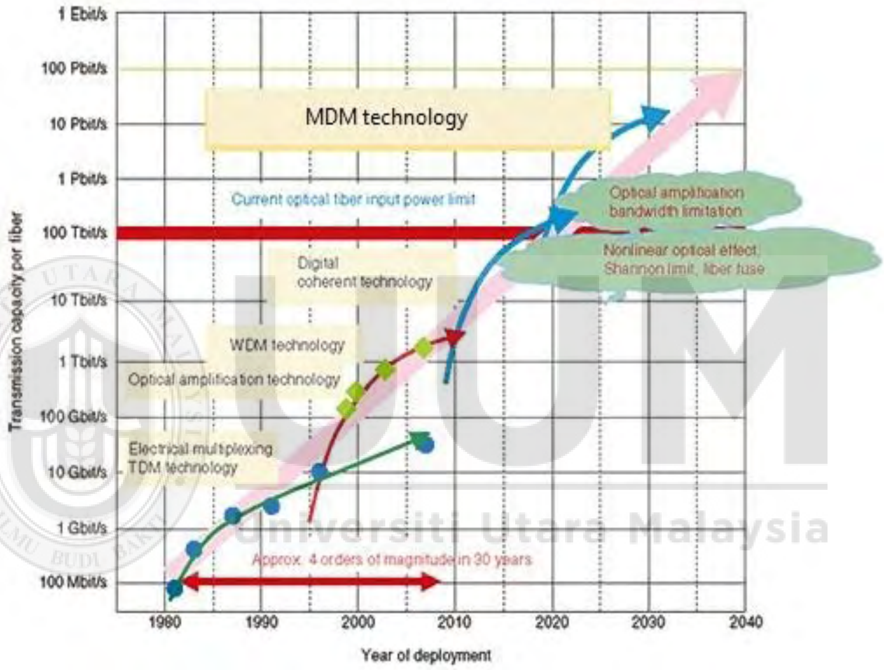


*Figure 2: Utilizing mode division multiplexing  in optical networks to significantly increase transmission capacity [8]*

This study endeavours to overcome the problem of mode coupling through a blind equalisation scheme which is based on QR factorization. This scheme is also employed for radio with similar intent.

## 1.2 Research Terminologies

For the purpose of ensuring a clear understanding of the proposal of the present study, the following subsections provide explanations on key terms that are used during the course of the investigation of the study.

### 1.2.1 Multimode Fibre

In multimode fibres (MMF), upon the entry of light into the cable, different routes are reflected differently on the cladding in accordance with the launching angle. This results in longer and shorter routes for the propagation of light. These routes are termed 'modes' [9, 10]. The most common multimode fibres available in the market come with diameters of 50µm and 62.5 µm [11].

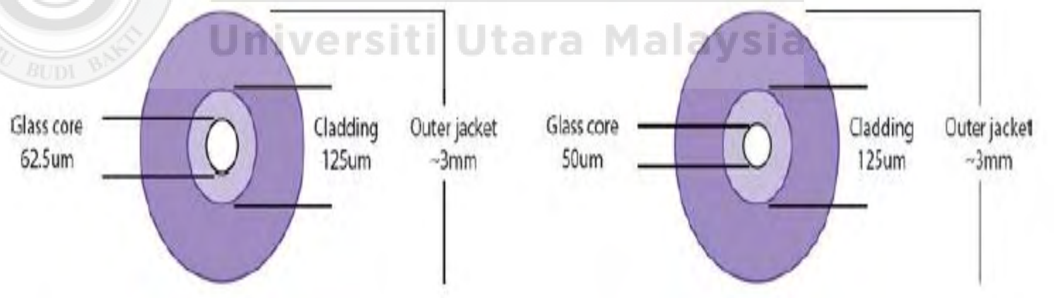


*Figure 3: Cross-section of multimode fibre optic cables*

Short-distance systems, such as local-area networks and data-centre interconnects have traditionally employed multimode fibre. For such applications, multimode fibre is frequently preferred to single mode fibre (SMF) due to its capacity for relaxed connector alignment tolerances and lower transceiver component costs [12].

### 1.2.2 Mode Division Multiplexer

In a typical multimode fibre, only a single channel is transported mainly in a fundamental mode while the other modes are mostly left unused. Mode division multiplexing (MDM) is a developing procedure in which orthogonal data channels are transmitted through different modes in a multimode fibre. Mode division multiplexing offers another approach for having multiplexing multiple data channels through a single optical fibre to raise the capacity of the channel [13]. Other approaches for this purpose include the use of polarization, wavelength, phase, intensity and time. A general mode division multiplexing system is displayed in Figure 4.

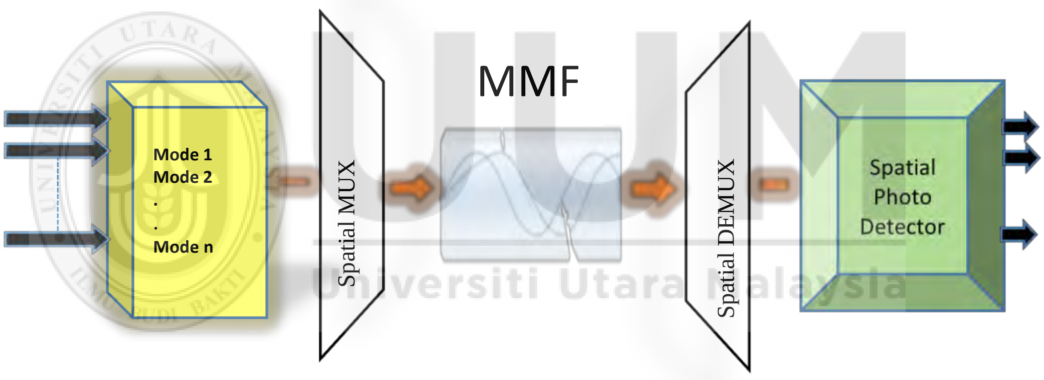


*Figure 4: General Mode division multiplexing system*

### 1.2.3 Distortion in Mode Division Multiplexing Channel

Mode coupling and modal dispersion are the two contributory factors for the occurrence of distortion in a mode division multiplexing system.

Mode coupling involves the exchange of power between modes through their propagation within the fibre. This phenomena is attributed to random perturbations, stresses and bends [14]. Mode coupling results in the fusion of independent mode signals while mode delay culminates in inter-symbol interference (ISI)[15].

Modal dispersion gets manifested itself in a multimode fibre as a superposition of several spatial modes. These modes, travelling at varying speeds, broaden the received channel impulse which leads to detection inaccuracies [16, 17].

### 1.2.4 Equalisation in a Mode Division Multiplexing Channel

An equaliser acts as a filter to overcome distortion that emanates out of the channel through the rejection or minimization of ISI. The equaliser is applied in the form of a transversal filter [18, 19]. The application of adaptive filters is achieved through inverse modelling.

Channel inverse equalisation is employed to the channel to reduce its distortion [18]. In this model, the main function of adaptive filter is to define the inverse of the channel effect on the signal in order to recover the original signal at the receiver side [18]. Given a channel of unknown impulse response, the purpose of an adaptive equaliser is to manage the channel output in such a manner that the cascade connection of the channel and the equaliser can be available with an approximation of an ideal transmission medium. In several applications, the required response is to use the input without delay [20].

On the other hand, power averaging equalisation is designed to facilitate the equal distribution of power among modes. This model is required to identify the mode that survives in an unknown channel as this is beyond the capacity of the system. The execution of this technique may lead to a situation where the most power is channeled to the weakest mode resulting in deterioration in performance [21].

### 1.2.5 QR Factorization

Matrix decomposition, which comes with both scientific and engineering significance, is an established theme in linear algebra and applied statistics [22]. In the fields of science and engineering, QR is included among the top ten most influential algorithms of the twentieth century [23]. QR factorization offers a favourable route to the solution of linear system equations. This is attributed to its exceptional error propagation properties and to its unavoidability of unitary matrices [22]. QR factorization, occasionally termed as QR decomposition, is a basic pre-processing procedure for distinguishing symbols in multi-channel systems [24].

## 1.3 Research Motivation

The escalation of network traffic in optical fibre backbones [25] calls for the need to investigate the use of mode division multiplexing as an innovative process for enhancing the system bandwidth. The urgent need for a new approach is due to the fact that the present day systems do not have the capacity to manage the imminent growth in traffic [25]. MDM is among the most rapidly developing procedures for supplementing other multiplexing schemes in their bid to enhance the aggregate bandwidth [7]. It is anticipated that mode division multiplexing will be successful in meeting the demands of the communication industry for ultra-high capacities [26].

## 1.4 Problem Statement

Although multimode fibre has lower capacities and is more cost effective, it has not been harnessed for long-haul applications [27, 28]. To overcome the existent distance limitations in multimode fibre [27, 29], mode division multiplexing has been proposed as a multiplexing technique for enhancing the distance-capacity product of multimode fibre. Under mode division multiplexing, each optical mode is exploited as an independent channel for transferring optical data; thus it allows the parallel data streams to be transferred simultaneously [27, 30].

To enable mode division multiplexing, hardware, such as mode couplers [31, 32], photonic crystal lanterns [33], spatial photodetectors [34], spatial light modulation [35, 36] have been realized for mode division multiplexing. However, most of them still suffer from mode coupling [37-40]. Mode coupling is an inevitable physical phenomena, and it arises due to multimode fibre manufacturing defects [15, 41, 42]. As a result, particular modes tend to interchange the power randomly through their propagation within the fibre, leading to inter-symbol interference (ISI) between symbols carried by these particular modes that cause higher time delay and reduce the capacity-distance product of the system [38, 39, 43, 44].

Several algorithms have been used in the equalisation scheme in mode division multiplexing, such as least mean square (LMS) and Recursive Least Squares (RLS).

Both of these two algorithms have drawbacks that prevent from being properly deployed in MDM domain. RLS converges faster than LMS, but at the same time, it has high computational complexity that can affect the overall performance of the MDM system [45, 46]. On the other hand, LMS has low computational complexity, but it is slower than RLS [47].

## 1.5 Research Questions

- How to develop a low computational complexity with equal power distribution equaliser for MDM system that is based on blind equalisation scheme by using QR factorization?
- How does one evaluate the performance of the equalisation scheme that is based on QR factorization?

## 1.6 Research Objectives

- To develop a low computational complexity blind equalisation scheme with equal channel power distribution for MDM system that is based on QR factorization with the consideration of the statistical standard deviation of the input signal.
- To evaluate the performance of the blind equalisation scheme that is based on QR factorization.



## 1.7 Research Scope

The scope of this study covers efforts to enhance the general system pulse width. This involves the implementation of blind equalisation that is based on a QR factorization inverse equalisation scheme for mode division multiplexing at the physical layer in the OSI model. This is done by relying on the techniques currently used in the radio domain and by adopting them for the use in the MDM domain.

## 1.8 Significance of the Study

A successful adaptation of low-complexity blind equalisation in MDM system will allow for the appropriate compensation of mode coupling and facilitate the utilization of a multimode fibre for future long-haul applications. On the other hand, the use of low complexity algorithms reduces the needed processing that needs to be done by the communication systems, resulting in smaller, more cost effective and more energy saving equipment.

# CHAPTER TWO: LITERATURE REVIEW

## 2.1 Introduction

This chapter provides a literature review of equalisation techniques for mode division multiplexing.

The study starts its discussion with the identification of several requirements associated with optical fibre networks, the principals governing mode division multiplexing and the limitations of mode division multiplexing in relation to the effects of mode coupling and inter-symbol interference.

In order to establish the basis of this endeavour, the following segment of this chapter focuses on the role of equalisation techniques in contrast with the negative effects of mode coupling and ISI. An explanation regarding several equaliser structures and algorithms is also provided in this segment.

In the final segment, significant previous studies in the fields of optical fibre and radio which are relevant to this proposal are discussed, and an examination on their similarities and differences is conducted. This segment also involves an investigation on the possibility of adapting some equalisation techniques for mode division multiplexing.

## 2.2 Optical Fibre Systems

The use of copper wires for data transmission declines substantially over time due to the drawbacks that include a low bandwidth, a brief transmission period and overall ineffectiveness [10]. At present, glass or plastic fibre is preferred to the transmission of data in the form of light. The information to be transferred is encoded into electrical signals which are subsequently altered into light signals. These light signals make their way through the fibre, and upon the arrival at the other end, a detector converts them into electrical signals. Ultimately, these electrical signals are decoded into decipherable information[10].

Fibre optics is supported as they have the following benefits:

1. They are electrically non-conductive.
2. They are unaffected by electromagnetism.
3. They have a sizeable bandwidth.
4. They come in small, lightweight cables.
5. They are safe to use.

## 2.3 Mode Division Multiplexing



In mode division multiplexing, one or more modes is/are propagated within the MMF with each mode expected to represent an individual data channel. An accurate launching of the modes within the fibre at the transmitter side provides the opportunity to control the pulse width of these modes as well as the volume of coupled power between them [13]. On a theoretical level, modes are orthogonal to each other. However, an equaliser inverse is still required, following the utilization of a mode de-multiplexer for signal retrieval [40]. A general concept of a mode division multiplexing system is portrayed in Figure 5 [48]. In mode division multiplexing, the system capacity ideally scales are done in proportion to the number of modes employed [6].

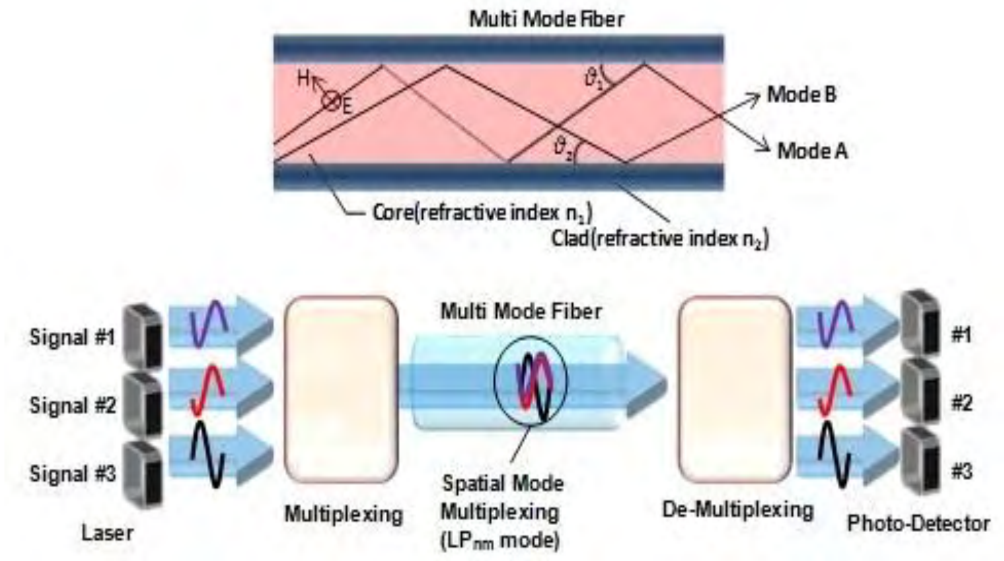


*Figure 5 Mode division multiplexing system*

## 2.4 Distortion in Mode Division Multiplexing Systems

The development of mode division multiplexing is propelled by devices, such as mode couplers designed by the use of fibre Bragg gratings [49], long period grating [50], spatial light modulators [35, 36], photonic crystal fibre [33] and phase masks [51]. However, these mode division multiplexing devices come with significant drawbacks as they have the negative influence of modal dispersion and mode coupling [39].

Generally, the different modes in a multimode fibre (MMF) get propagated with varying group delays (GDs). This effect is known as modal dispersion [52]. Due to this effect, the light pulse at the multimode fibre output spreads over time. When a multimode fibre is used to ferry a signal over numerous spatial modes, mode dispersion causes inter-symbol interference, and this circumstance restricts the usage of high bit-rate signals [16, 17]. Mode dispersion can exert a negative influence on optical fibre communication over a multimode fibre. This is illustrated in Figure 6

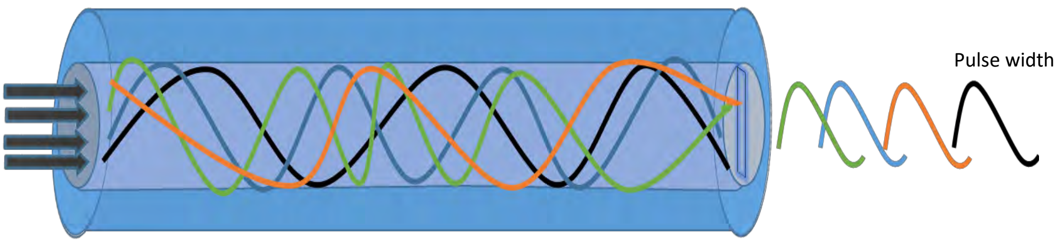


*Figure 6: Modal dispersion in multimode fibre*

Mode coupling involves the exchange of energy between light modes during propagation [14] which leads to a superposition of several pulses at the output of the multimode fibre that culminates in inter-symbol interference and a decrease in the fibre bandwidth [52]. More often than not, mode coupling stems from perturbed parameters that include microbendings or irregular refractive index distribution along the fibre link. Mode coupling leads to a mixing of the independent mode signals while mode delay results in inter-symbol interference (ISI [15], These situations can be observed in Figure 7.

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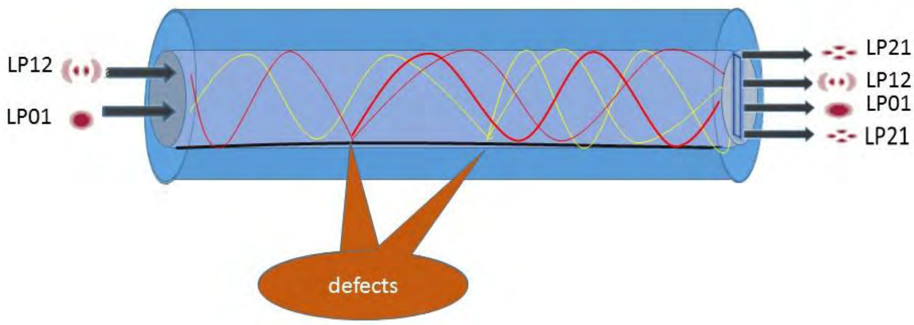



*Figure 7: Mode coupling in mode division multiplexing*

Among the many forms of communication distortions linked to data loss, ISI is undoubtedly the most prominent. The transmission of digital information is usually in a square waveform that represents the ones and zeros. In a situation where this square waveform comes together with noises and nonlinearities in the channel, it starts to spread and merge with the adjacent symbol order. This circumstance renders the data unintelligible. Such data are incorrectly decoded at the receiver end as the receiver falls short in its attempt to predict the appropriate level of the square waveform. The ultimate result of such a situation is loss of information. In general, the incidence of ISI can be ascribed to multipath propagation of the signal in a band restricted channel as well as to the non-linear frequency response of the channel. This circumstance is exhibited in Figure 8 [53].

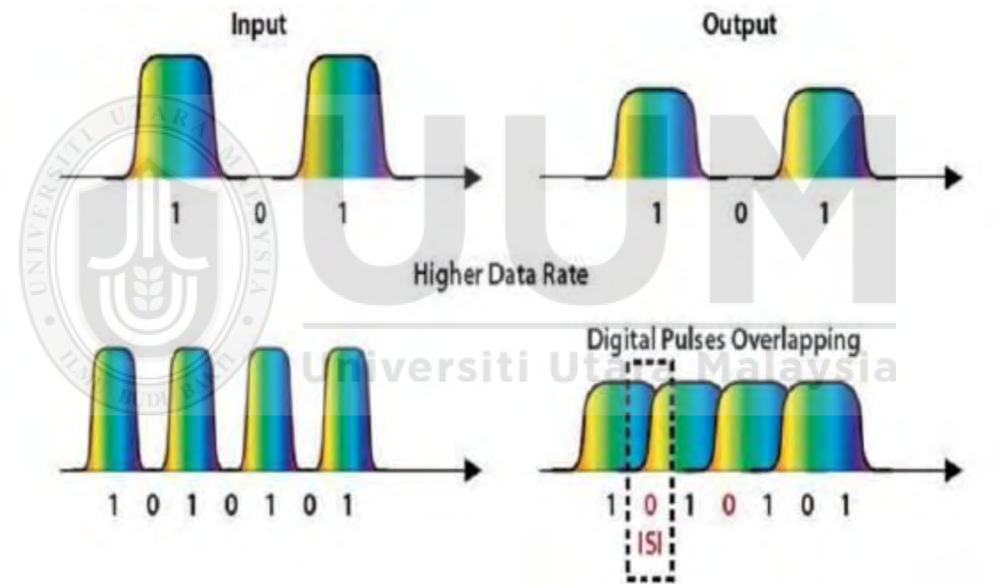
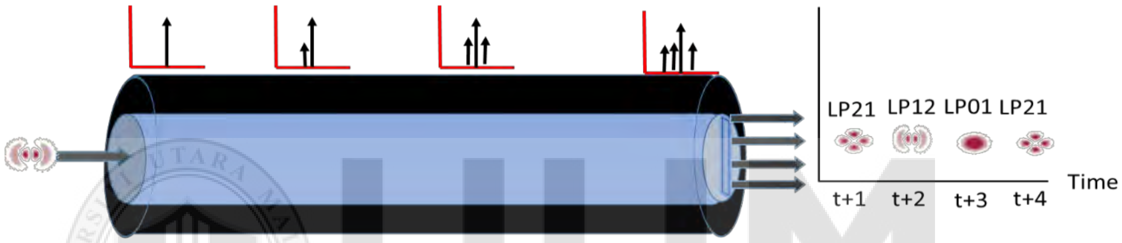



*Figure 8: Inter-Symbol Interference*

Over the past decades, research studies that emphasize on easing the harmful effects of ISI have received much attention. The designing of a filter that records observations on the ISI channel and reorganizes the input to the channel is simple, yet effective solution to this predicament. This filter is also known as an equaliser, and the restructuring technique is termed equalisation [54].

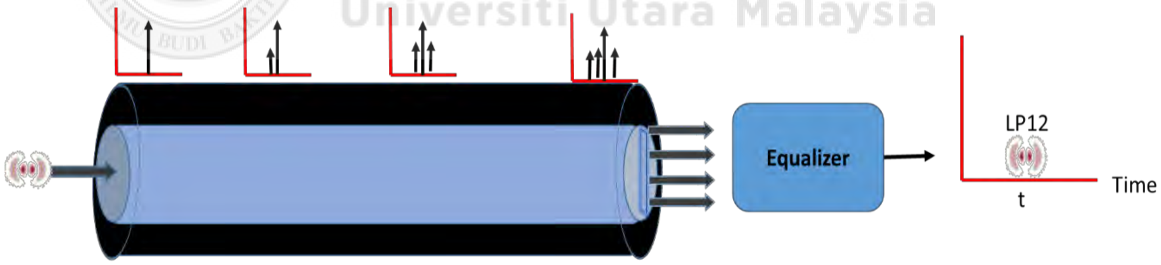
## 2.5 Equalisation in Mode Division Multiplexing

As pointed out earlier, the implementation of mode division multiplexing transmission has major problems due to the detrimental effects of mode coupling and modal dispersion [38, 40, 55] that lead to inter-symbol interference (ISI) and increase the width of the pulse of the system and this is shown in Figure 9a. The ISI predicament can be overcome through channel equalisation where the objective is to craft an effective channel equaliser [56]. This is shown in Figure 9b below, and is seen that it is possible to enhance the system pulse width by using equalisation [57].




(a) Channel Before Equalisation






(b) Channel After Equalisation

*Figure 9: Pulse width before and after Equalisation*

An adaptive equaliser comprises an adaptive filter and an adaptive algorithm that manages the filter. Initially, the transmitted signal makes its way through a transmission line and is converted into a received signal *y(n)* with distortions attributed to ISI. The received signal is evaluated against a pre-set training sequence *d(n)* and led through the adaptive filter in training mode. This is illustrated in Figure 10. Subsequently, an adaptive algorithm reduces the error signal which results in an optimization of the adaptive filter coefficients *w(n)* [58].

However, in blind equalisation, other than its probabilistic or statistical properties over several known alphabets, the desired signal is unknown to the receiver. As both the channel and its input are unidentified, the objective of blind equalisation is to retrieve the unknown input sequence that is based on exclusively on its probabilistic and statistical properties. The feasibility of blind equalisation is initially discussed in 1974 when it is analytically proven that an adjusting equaliser has the capacity to invert the channel without the requirement of a training sequence [59].

### 2.5.1 Adaptive Equaliser Structures

For contemporary digital devices, the significance of effective signal processing is inestimable. The necessity to receive reliable data as swiftly as possible is a common factor for measuring the simplest instruments to the most sophisticated communication systems. Nonetheless, these digital devices do not entirely function in a closed or protected setting, and they are predominantly get effected by accompanying superfluous signals. Noise in this form is frequently typified as some modes of periodicity. In their search for a plausible solution to this daunting dilemma, several researchers have considered the creation of a concept for filtering in the context of frequency [60].

The meaning of the terms ‘adaptive’ and ‘filter’ has very general sense. ‘Adaptive’ can be understood to mean a system which endeavours to alter itself in response to an incident occurring in its environment. Progress in terms of adaptation necessitates the involvement of an array of procedures. The technical term for the ‘system’ that executes the adaptation procedure is known as the ‘filter’. The choice of a suitable adaptation algorithm is constantly dominated by the convergence time. This is described as the time required for executing the complicated adaptation procedure, and the time is required for realizing the ultimate objective of an adaptation. The occurrence of echoes has the tendency to diminish the quality of transmission in telecommunication schemes. An echo can be described as belated and possibly distorted account of a signal that gets transmitted earlier [18].

Adaptive filtering entails two basic manoeuvres. These are (a) the filtering process which generates an output signal, stemming from an input data signal and (b) the adaptation process which modifies the filter coefficients in such a manner that cost function minimization is realized. Generally, adaptive filtering applications embrace a wide array of filter configurations and algorithms [61].



An adaptive filter can adjust itself of its transfer function to equip itself with the ability to self-learn and alter filter parameters in order to get adapted to various signal features. Two types of filters are currently accessible. These are the Finite Impulse Response filters (FIR) and the Infinite Impulse Response (IIR) adaptive filters [62].

Finite Impulse Response (FIR) digital filters are widely applied in mobile communication schemes for channelization, channel equalisation, matched filtering, pulse shaping etc. It is essential that the filters utilized for mobile schemes use up less power and function at the level of elevated velocities. Based on these features, it is widely used in the channel equalisation of mode division multiplexing systems [63].

On the other hand, Infinite Impulse Response (IIR) filters meet the given filter specifications albeit with a distinctly lower filter order; thus, they reduce the computational requirement and/or the intricacy of the hardware. IIR filters also come with the advantage of much smaller group delay [64]. Due to its capability of reducing the system complexity, it has been introduced as a key filtering solution for mode division multiplexing equalisation where the system complexity can be considered as a major challenge.

### 2.5.2 Adaptive Equalisation Algorithms

The main objective of an adaptive filter is to define the correction of the filter in accordance with the rules of adaptive algorithms [65].

When an adaptive filter is used, the relationship between the input and output signals of a filter is iteratively modeled. In this process, the filter coefficients of the adaptive filter are self-adjusted by the use of an adaptive algorithm. The coefficients are adjusted in order to minimize the power of $e(n)$. Multiple adaptive algorithms are researched and tested, and the two most promising adaptive algorithms are the Least Mean Squares (LMS) algorithm and the Recursive Least Squares (RLS) algorithm [62, 66]. In previous studies, these algorithms have been used as equalisation in mode division multiplexing devices for self-updating the coefficients of the equalisation filter that is used to compensate the effect of the mode coupling and for reducing of the system's pulse width [15].

The least-mean-square (LMS) equalisation algorithm is not only uncomplicated but also efficient as far as the functions of adaptive transversal filters are concerned. Widrow and Hoff formulate this algorithm in 1959 during their investigations on a pattern-recognition scheme which is called as the adaptive linear element. This scheme is generally known as the Adaline in related literature [67].

LMS algorithm has been used in equalisation in radio domain for equalizing the Orthogonal Frequency Division Multiplexing (OFDM) and recently in the optical domain to equalize the channel of the mode division multiplexing. The algorithm converges at the optimal interval to update the adaptive equalisation filter coefficients. This optimal solution occurs when little delay between the desired time and the output time is minimized. These coefficients are updated in each iteration in order to get the optimal solution. To update the coefficients, the LMS algorithm performs three main steps, which include calculating *y(n)* from the adaptive filter, calculating *e(n)*, and then finally updating the filter coefficients [62]. However, LMS algorithm still is unstable system with slow convergence time.

On the other hand, as per the LMS, The Recursive Least Squares (RLS) algorithm is also used in the equalisation scheme for mode division multiplexing. For OFDM channels, it recursively finds the filter coefficients that minimize weighted linear least square cost function related to the input signals. The advantage of the RLS algorithm is that it has quicker convergence than other methods, so the number of training sequence can be greatly reduced [68]. Similarly, in order to update the coefficients, the RLS algorithm performs three main steps, which include calculating *y(n)* from the adaptive filter, calculating *e(n)*, and then finally updating the filter coefficients. Although the main overall process is similar to the LMS and RLS algorithms, the method used to update the coefficients are significantly different [62]. RLS is known for its high computational complexity.

On the other hand, many other algorithms has been used in the equalisation domain, like Neural Networks (NN), Genetic Algorithm (GA) [69] and fuzzy logic [70]. These algorithms rely on training sequence or pilot signal to be sent over the channel before transmitting the data, in same way LMS and RLS algorithms function. This pilot signal is used by these algorithms to estimate the channel effect on the signal [69, 71].

### 2.5.3 Blind Equalisation

In the general context of communication systems, the transmitter side sends out the desired signal which is unknown to the receiver side, and the receiver side takes delivery of the sent out signal. The problem that exists between transmitter and receiver is putting down the signal due to the presence of an unidentified multipath channel [72].

Blind channel equalisation algorithms that do not depend on training signals are currently available. Through the utilization of these 'blind' algorithms, individual receivers can achieve self-adaptation without the support of transmitter. This capacity for blind startup also equips a blind equaliser with the ability to self-recover from system breakdowns. This faculty for self-recovery is crucial for broadcasting and multicast schemes where channel variations are common occurrences [73].

By removing the requirement for periodic training sequence, blind equalisation reduces bandwidth wastage. A blind equaliser is deemed as 'blind' because it does not call for any training sequence of predetermined symbols. Instead, it relies only on the statistical characteristics of the transmitted signal [74, 75].

## 2.6 Channel Equalisation Schemes in the Optical and Radio Domains

The next sections discusses the current researches conducted on mode division multiplexing equalisation in optical domain and the previous related researches which have been implemented in the radio domain.

### 2.6.1 Equalisation Schemes in Mode Division Multiplexing

The delving into recent studies that focus on the field of mode division multiplexing provide the opportunity to (a) evaluate the accomplishments till to date (b) weigh the pros and cons of the process and (c) investigate the prospects for enhancing performance. Table 1 below offers a portrayal of recent achievements by various researchers and the methodologies that have been employed by them.

According to [76], a mode division multiplexing transmission experiments based on Time Domain Equalisers (TDE) and Frequency Domain Equaliser (FDE) adapts the use of the Data Aided Least Mean Square (DA-LMS) algorithm. Both TDE and Frequency Domain Equalisers (FDE) are performed by the use of the same filter parameters in order to have fair comparison. However, the use of the frequency domain with LMS reduces the complexity, but on the other hand, LMS is not as stable as RLS which creates an unstable system with slow convergence time.



According to [15], the influence of mode coupling and modal delay on the transmitted signal is numerically simulated. Then it carries out an adaptive signal processing with mode division multiplexing equalisers that are based on Recursive Least Squares Constant Modulus Algorithm (RLS CMA). The simulation results shows that the distorted signals can be restored efficiently with fast convergence speed. However, the developed equaliser in this article is good in handling weak mode coupling, but not robust for strong mode coupling regime as it is designed for three light modes only.

The derived channel models for mode-division multiplexing systems is shown in [46], where they analyse their statistical properties, provide implications on system performance and show complexities that take place. They present the fundamentals architectures for mode division multiplexing equalisation. This article compares the use of LMS and RLS and shows that both algorithms have limitations. RLS is very complicated, and LMS has slow convergence time and not stable.

Based on the study of [77], it can be seen that the implementation of mode-division multiplexing over 33-km of few-mode fibre has taken place. The results shows that mode division multiplexing equalisation processing for six light modes can be used to compensate the crosstalk almost completely and to have intersymbol interference that occurs due to mode coupling in a system. As previously mentioned, it is correct that using LMS algorithm results in a system with low complexity; however, the system converges slowly and is not stable.

In [78], (LMS) algorithm is used to update the equalisation filter coefficients. The received signal is equalized for mode division multiplexing of six light modes. The equaliser consists of 36 feed forward filters. In this method, the number of filter taps increases until the performance becomes satisfactory. Using TDE increases the system complexity whereas LMS makes the system to converge slowly and not to be stable.

In [79] , the utilisation of RLS to equalize channel models for mode division multiplexing systems takes place. It is observed that in contrast to the LMS algorithm, where the convergence time is sluggish, the use of the RLS algorithm results in quick convergence without an escalated degree of complexity.

Table 1  Recent studies conducted on mode division multiplexing equalisation.

| No. | Author and Year | Equalisation Algorithm | Equaliser Structure | Findings |
|---|---|---|---|---|
| 1 | N. Bai, E. Ip, M.-j. Li, T. Wang, and G. Li, 2013 | DA-LMS | Mode division multiplexing Equaliser for six light modes using FIR Filter | LMS is not stable |
| 2 | H. Zhao, L. Zhang, B. Liu, Q. Zhang, Y. Wang, Q. Tian, *et al.*, 2014 | RLSCMA | Mode division multiplexing Equaliser for 3 light modes using FIR Filter | Not efficient for strong mode coupling. |
| 3 | J. M. Kahn and S. Ö. Arı, 2015 | LMS and RLS | Mode division multiplexing Equaliser for 6 light modes using FIR Filter | RLS is very complicated and LMS has slow convergence time and not stable. |
| 4 | S. Randel, R. Ryf, A. Sierra, P. J. Winzer, A. H. Gnauck, C. a. Bolle, *et al.*, 2011 | LMS | Mode division multiplexing Equaliser for 6 light modes using FIR Filter | The system will converge slowly and will not be stable. |

| | | | | |
|---|---|---|---|---|
| 5 | R. Ryf, S. Randel, a. H. Gnauck, C. Bolle, R.-J. Essiambre, P. J. Winzer, *et al.*, 2011 | LMS | Mode division multiplexing Equaliser for 6 light modes using FIR Filter | LMS will make the system to converge slowly and is not stable |
| 6 | S. O. Arik, D. Askarov, and J. M. Kahn, 2015 | RLS and LMS | Mode division multiplexing Equaliser for 6 light modes | RLS has high complexity and LMS is slow |

The current research in the equalisation of MDM mainly depends on RLS and LMS algorithms. These algorithms are suffering from high computational complexity in the case of RLS and slow performance in the case of LMS [46]. Having such issues prevents from widely using them to equalise MDM signals in a proper and reliable way which is highly needed for long-distance optical networks.

### 2.6.2 Equalisation Schemes in Radio

To define matrix factorization is a fundamental theme in linear algebra and applied statistics, and this has both scientific and engineering significance. Computational convenience and analytic simplicity are the two main aspects in the purpose of matrix decomposition. In the real world, it is not feasible for most of the matrix computation to be calculated in an optimal explicit way, such as matrix inversion, matrix determinant, solving linear systems and least square fitting; thus, to convert a difficult matrix computation problem into several easier task, such as solving triangular or diagonal systems greatly facilitate the calculations. Data matrices, such as proximity matrix or correlation matrix represent numerical observations, and it is often huge and hard to analyze them [22].

QR factorization is one of the most important operations in linear algebra. It can be used to find matrix inversion and to solve a set of simulations equations or in numerous applications in scientific computing. It represents one of the relatively small numbers of matrix operation primitive, and from that, a wide range of algorithms can be realized [80].

QR factorization is an elementary operation, which factorization a matrix into an orthogonal and a triangular matrix. A matrix A=QR in linear algebra is a factorization of a matrix A into an orthogonal and right triangular matrix [22].

The studying of previous works from radio domain helps to provide a more comprehensive picture of current blind equalisation techniques that are used. The knowledge is valuable for adaptation or adaption of the performance of the improvement of MDM systems. Table 2 below shows the previous equalisation research in the radio domain for equalizing an orthogonal frequency division multiplexing system in the radio domain.

Based on [81], it is seen that the use of QR factorization in combination with RLS can reduce the system complexity even if it is used in the time domain. On the other hand, the results show that the use of QR-RLS with IIR has better performance.

[82] proposes a direct use of QR factorization without the use of an ordinary FIR or IIR filters with inverse equalisation algorithm. This shows significant results in the

radio domain; however, further studies are needed to check its performance in the optical domain.

According to [83], it is seen that with a smart and precise processing of the training data, it is possible to decrease the system complexity up to 75%, and this can be done with the aid of QR factorization as the matrix inversion tool that inverses the training symbols on a progressive way (symbol by symbol) instead of waiting for the whole training stream.

In [84], QR factorization is implemented as an inversion method, and the use of this with the RLS algorithm shows significant improvement in the system complexity.

According to [85], a comparison of using LMS and RLS algorithms in the time domain with FIR filters has been taken place, and the results shows that the combining of QR factorization with the RLS can reduce the system complexity.

[86] credits the first equalisation process that relies on repeated transmissions of training sequences that are known to the receiver. However, it should be noted that for many digital communication systems, the transmission of a bandwidth consumption training sequence is deemed unsuitable. In order to evade training, blind equalisation techniques are developed to recover symbols transmitted through an unknown channel. These techniques rely solely on received data and some information on the statistics of the original sequence.

In the opinion of [87], as the desired signal or input to the channels is unknown to the receiver except for its statistical properties, blind equalisation can be harnessed to recover the unknown input sequence with the support of information on its statistical properties.

The study of [88] shows a comparison between Neural Fuzzy, LMS and Neural Networks that take place. It shows that the use of NN only in the equalisation domain has some drawbacks as it may need long training sequence for proper convergence; otherwise, higher computational complexity can be the result of using less training sequence.

The study of [89] proposes the use of Genetic Algorithm (GA) in equalisation domains that take place. It shows that in order to let GA to work with its best performance, it is mandatory to choose its parameters in an accurate way as wrong choices may lead to slow convergence or high overhead data which are undesirable in high speed communication systems.

Table 2 Comparison of previous research equalisation schemes in radio.

| No. | Author and Year | Equalisation Algorithm | Equaliser Structure | Findings |
|---|---|---|---|---|
| 1 | D. Rawal, C. Vijaykumar, and K. K. Arya, 2007 | QR-RLS | Radio channel equalisation using IIR Filter | The QR-RLS with IIR filter method gives better performance than LMS. |
| 2 | M. a. L. Davies, Sangarapillai and Foster, Joanne and Chambers, | Applying PMQRD based QR | Radio channel equalisation | System performance is significantly enhanced. |

| | | | | |
|---|---|---|---|---|
| | Jonathon and McWhirter, John, 2009 | | | |
| 3 | P. Xue, K. Bae, K. Kim, and H. Yang 2013 | upper-triangular matrix inversion for QR factorization | Radio channel equalisation by using 4 antennas radio system | The proposed method significantly reduces the computational complexity |
| 4 | A. Ragheb, M. Shoaib, S. Alshebeili, and H. Fathallah, 2012 | The main contribution of the paper is to apply the inverse QR factorization (IQR) | Apply the inverse QR factorization (IQR) for equalisation radio channel | The algorithm speed and the computational complexity are totally improved. |
| 5 | D. Rawal and C. Vijaykumar, 2008 | QR-RLS | Radio channel equalisation using FIR filter | Standard RLS based on inverse FIR filter requires a large number of computation than QR-RLS method for comparable performance. |

| | | | | |
|---|---|---|---|---|
| 6 | Fki, S., Messai, M., El Bey, A. A., & Chonavel, T. 2013 | Blind equalisation | Radio channel equalisation using statistics | blind equalisation techniques have retrieved symbols transmitted through an unknown channel to recover signal |
| 7 | Randhawa, N. S 2013 | Blind equalisation | Wireless system based on statistical properties | The objective of blind equalisation is to recover the unknown input sequence based solely on its statistical properties |
| 8 | Amira A. Elbibas, Issmail M. Ellabib, Yousef Hwegy (2013) | Neural Networks | Wireless system with training sequence | Long training sequence is needed to have proper convergence which will slow down its speed. Having less training sequence will increase the computational complexity |

| | | | | |
|---|---|---|---|---|
| 9 | Nazmat Surajudeen-Bakinde, Xu Zhu, Jingbo Gao, Asoke K. Nandi (2011) | Genetic Algorithm (GA) | Wireless system And use training to adaptive equalisation | Wrong choice of the algorithm's settings will result in slow performance or high overhead payload. |

The high similarity is fashioned in such a way that both of the optical and radio networks are affected by ISI, and how both of the two signals are distorted in almost the same manner motivates researchers to look for a technique that can efficiently handle the nature of the optical networks. This can be done by adapting this technique from the previous successful works in the radio domain.

GA, NN and fuzzy logic algorithms has shown their ability to successfully recover the received signals and to mitigate ISI; however, as these algorithms rely on training signals, they intend to be slow and increase the computational complexity. On the other hand, having training signals will increase the overhead payload on their related systems which are undesirable in high speed communication systems [75].

On the other hand, there are other techniques that are used techniques in radio domain and solved the issues related to complexity and the increment in the overhead payload. QR factorization and blind equalisation part of these techniques. The previous works on QR factorization show its simplicity and speed when it is used as a standalone technique or as a secondary one that supports other techniques (like using it with RLS algorithm), while blind equalisation does not need training signals and uses the statistical properties to estimate the channel [87]. The significant results from these approaches in the radio domain motivates the work of this study to adapt them in the equalisation of optical networks.

## 2.7 Summary

This chapter provides the background on mode division multiplexing, equaliser structure and algorithms. Previous studies highlight the major issues with current inverse equaliser implementation. Based on the performance of QR factorization in radio OFDM systems, QR factorization is deemed as a promising approach for reducing the system complexity in mode division multiplexing equalisers.

# CHAPTER THREE: RESEARCH METHODOLOGY

## 3.1 Introduction

This study aims to propose a QR-based blind equaliser model to be used in the compensation of modal dispersion and mode coupling effects in mode-division multiplexing. QR-based blind equalisation is a promising approach aimed at reducing the mode coupling effect and enhancing the capacity while reducing the pulse width of the system. As previously mentioned in Chapter 2, this will be done by investigating the previous studies on equalisation in both radio and fibre optics domain, and by comparing their outputs and identifying issues and gaps.

To realise the above- mentioned goals requires a rigorous methodology, and this is the focus point of this chapter. For this purpose, this research employs the Design Research Methodology (DRM) and emphasizes its fundamental phases [90]. This chapter begins with an account of a general study methodology as exhibited in Section 3.2. In Section 3.3, the initial phase of DRM is termed as Research Clarification (RC). This entails a deliberation on the objectives of the RC phase, the supporting procedures involved and the primary deliverables. Section 3.4 is a description of the next phase known as Descriptive Study-I (DS-I). This phase involves an elaboration on the stages required for the attainment of an adequate comprehension of the contemporary circumstance. The emphasis of Section 3.5 is on the techniques employed for developing the needed QR-based mode-division multiplexing equaliser. This is termed as the Perspective Study (PS) phase. Section 3.6 touches on the procedures employed for performance metric evaluation which is known as Descriptive Study-II (DS-II) and closes this chapter with a summary.

## 3.2 Research Approach

The primary aim of this study is to find a low complexity mode division multiplexing equaliser for mode division multiplexing. Till today, many studies study the mode division multiplexing in few-mode fibres, where only six modes are propagated at the same time in comparison with the hundreds of modes in the case of utilizing mode division multiplexing in multimode fibre [91]. Mode division multiplexing equaliser complexity has proportionally increased with the increment of the propagating modes [92]. An enhanced technique is proposed in this study which is based on QR factorization in an effort to reduce the high equaliser complexity that would occur if systems depend on currently used techniques with few-mode fibres and radio domains. These requirements are congruent with the design research definition proposed by Blessing in [93] and Adib in [90], and under their study, design research integrates the development of understanding and the development of mechanism. To achieve the goal of this study, this demands a solid understanding of how channel blind equalisers work and their related main issues. This will lead us to find out the proper equaliser in order to avoid and mitigate these issues. The DRM framework features the connections between phases, and the procedures employed during each phase and the main deliverables are portrayed in Figure 10. The main course of the process is denoted by light arrows flanked by the stages while the bold arrows point to the stages that identify the procedures utilised and the deliverables linked to each specific stage. The following sections will go through the four stages of DRM in detail.

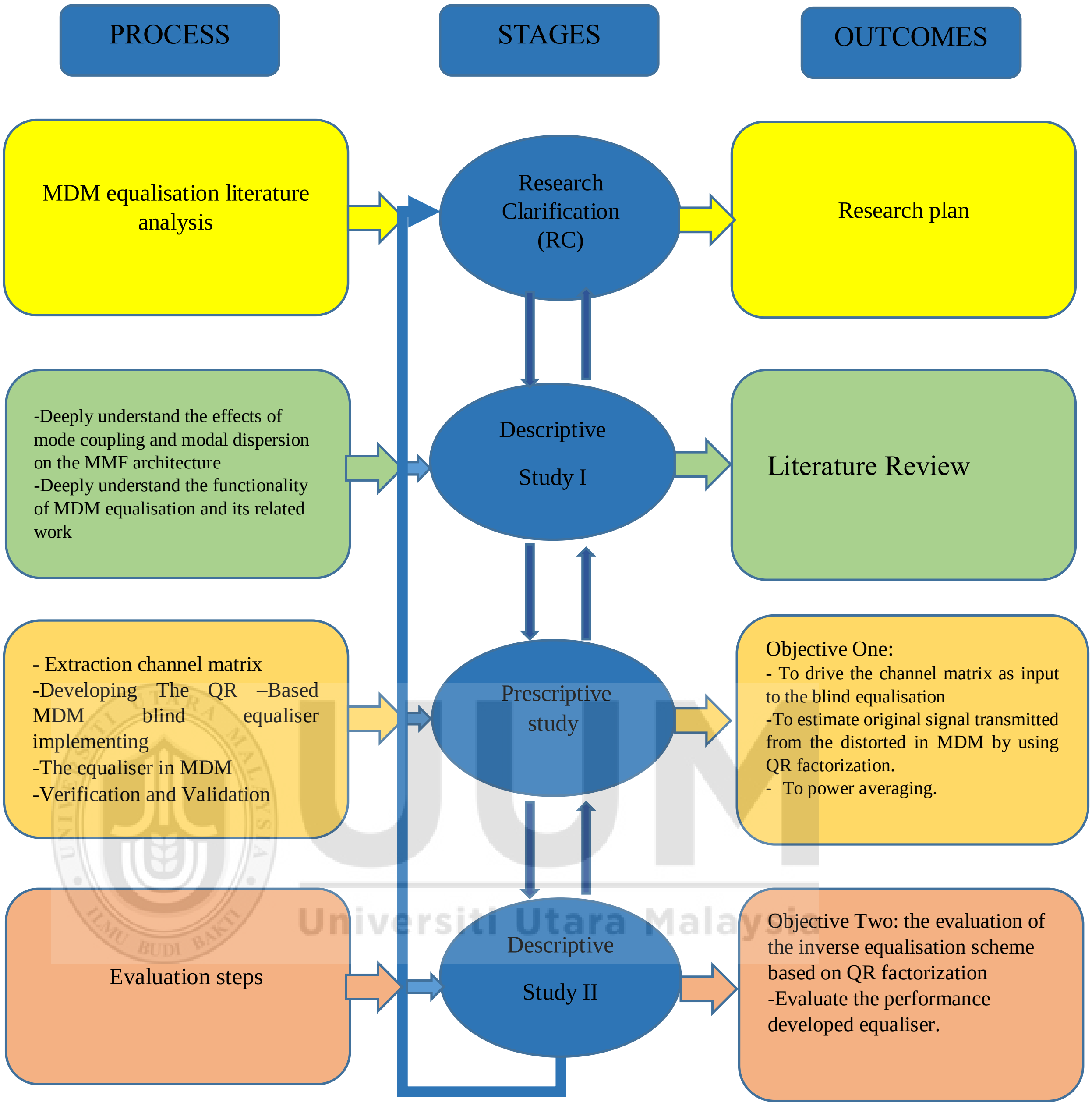



*Figure 10: Research Approach*

## 3.3 Research Clarification

Research Clarification (RC), which is the initial stage of DRM, "helps making design research more rigorous, effective and efficient and its outcomes academically and practically more worthwhile" [90, 93]. This decision is made through information derived from literature and relevant past assessments. This stage puts the study on a firm footing and paves the way for the next stage. The six iterative steps of RC are exhibited in Figure 11.

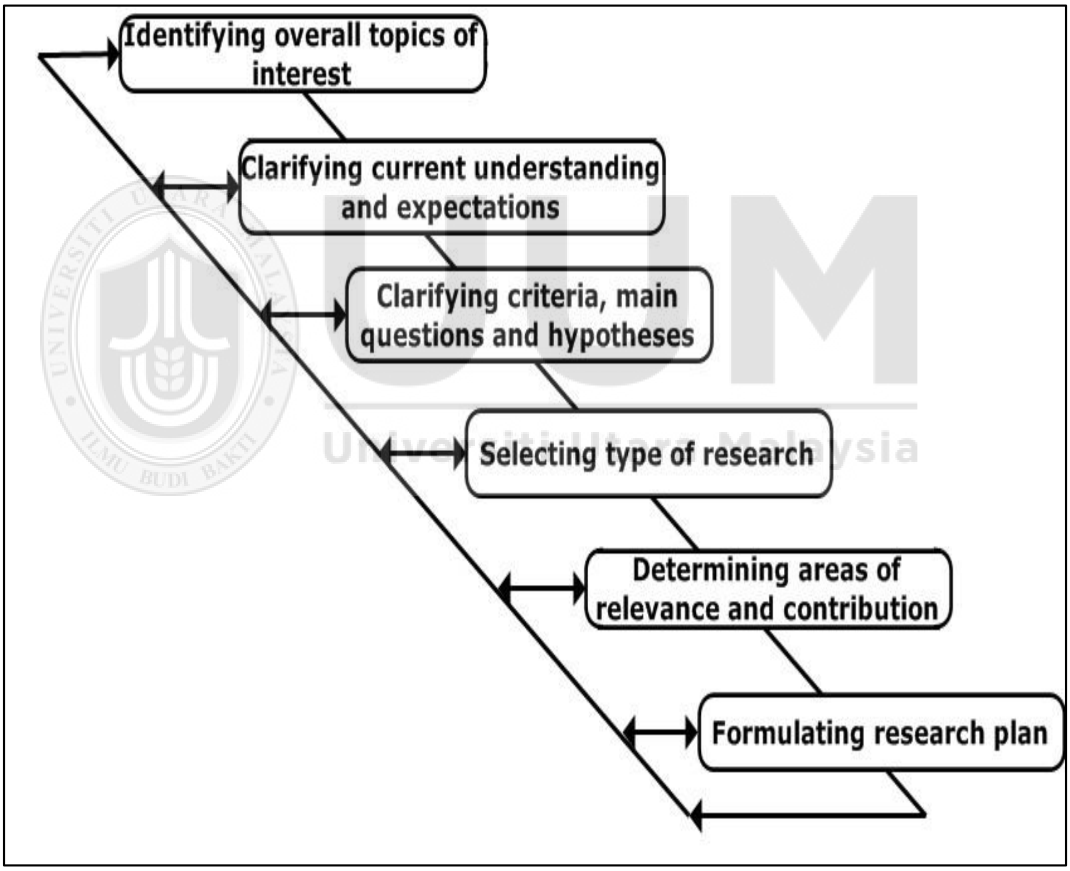



*Figure 11: Main steps in the research clarification stage*

The outcomes of the RC stage are:

a. The research motivation and the research focus on mode division multiplexing.

b. The research problem along with research questions is related with research objectives for mode division multiplexing blind equalisation in multimode fibre.

c. The research scope, type of research and research methods for mode division multiplexing blind equalisation in multimode fibre are related.

d. The expected research contribution and deliverables for mode division multiplexing equalisation in multimode fibre are given.

## 3.4 Descriptive Study-I

Once the RC stage which establishes the general study outline is completed, it is followed by the Descriptive Study-1 (DS-1) stage, which is shown in Figure 12. This stage serves to provide an improved comprehension of the present state of this domain. An impartial assessment of investigations and the empirical research are conducted through information gleaned from relevant literature. Efforts are also directed towards imparting a better understanding of mode-division multiplexing blind equalization, QR factorization as well as the issues that come with their utilisation that is based on previous mode-division multiplexing papers.

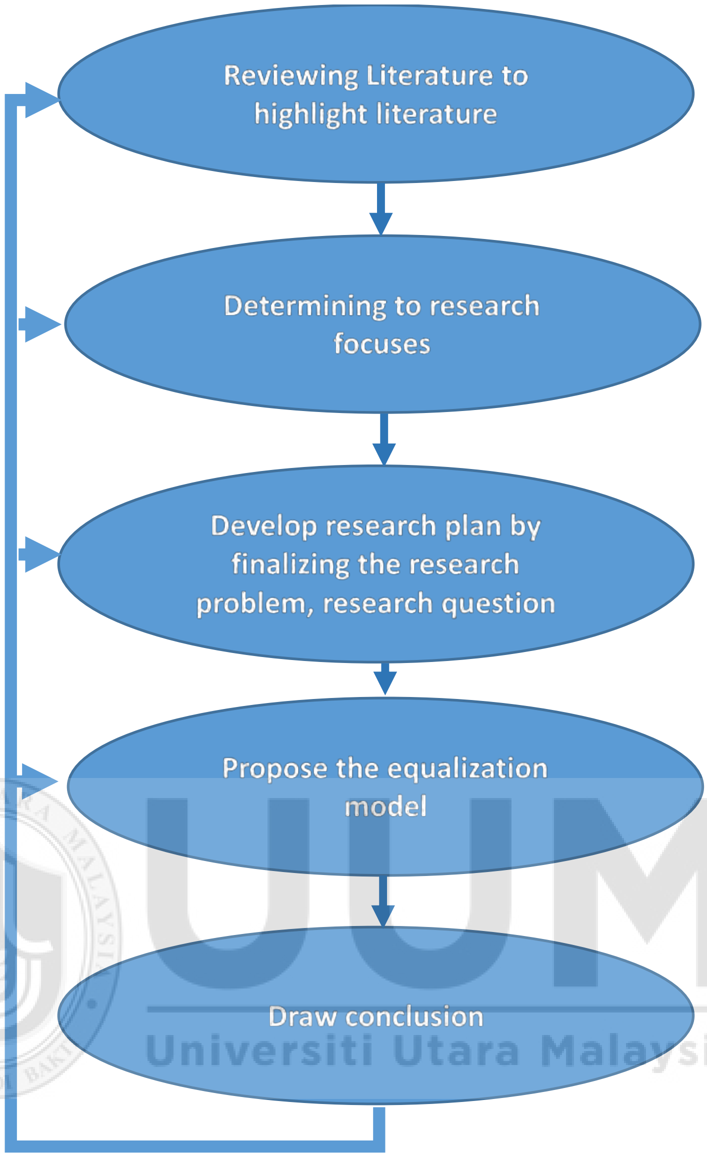



*Figure 12: Main steps in the Descriptive Study I*

To compensate the mode coupling and modal dispersion that occurred in multimode fibres in mode division multiplexing, an efficient equalisation technique is imperative. The problem with current equalisation techniques is its high complexity in terms of the needed convergence time and the number of the mathematical computations in the algorithms used in these techniques; those techniques are implemented in mode division multiplexing in multimode fibre due to the high number in the modes expected to propagate in this type of fibre optics. A proposal for developing a QR factorization that is based blind equaliser is being studied in this research which gives the premise that QR factorization has shown promising results in the radio domain for communications.

## 3.5 Prescriptive Study

In DRM, the main stage is the Prescriptive Study (PS) as it shows the design of the proposed mechanisms. Figure 13 shows a block diagram that highlights the main procedures of PS. This research methodology is based on the procedures of PS.

### 3.5.1 Experimental Design



The first step of PS involving the building the mode-division multiplexing model in the optical simulator sets the system's independent variables with the help of setting the system parameters and extracting the primary dependent variables which are the power coupling coefficients (PCC). This is can be seen in the second and third blocks of the following Figure 13.

The second step starts with the derivation of the secondary dependent variable which is the system transfer function (H), and this is done by performing the statistical properties on the dependent variables previously extracted in step one.

The third step of this stage is the implementation step in which the QR factorisation is performed on H. It also includes another part of performing the averaging power equalisation on the channel matrix. Both are implemented by compiling the needed MATLAB code.

### 3.5.2 Verification and Validation

The fourth step is the Verification and Validation (V&V) step. Model verification is the endorsement, and the model can be altered from one shape into another, as required, for the needed accuracy.

In this study, the Verification stage takes place with different scenarios that are based on differences in independent variables. This is done by changing the optical simulator parameters and checking the outcomes after each change.

The validation of the study takes place by implementing the mode-division multiplexing model in two different optical simulators. The two scenarios have exactly the same independent variables, and the work will be validated if the same dependent variables are obtained.

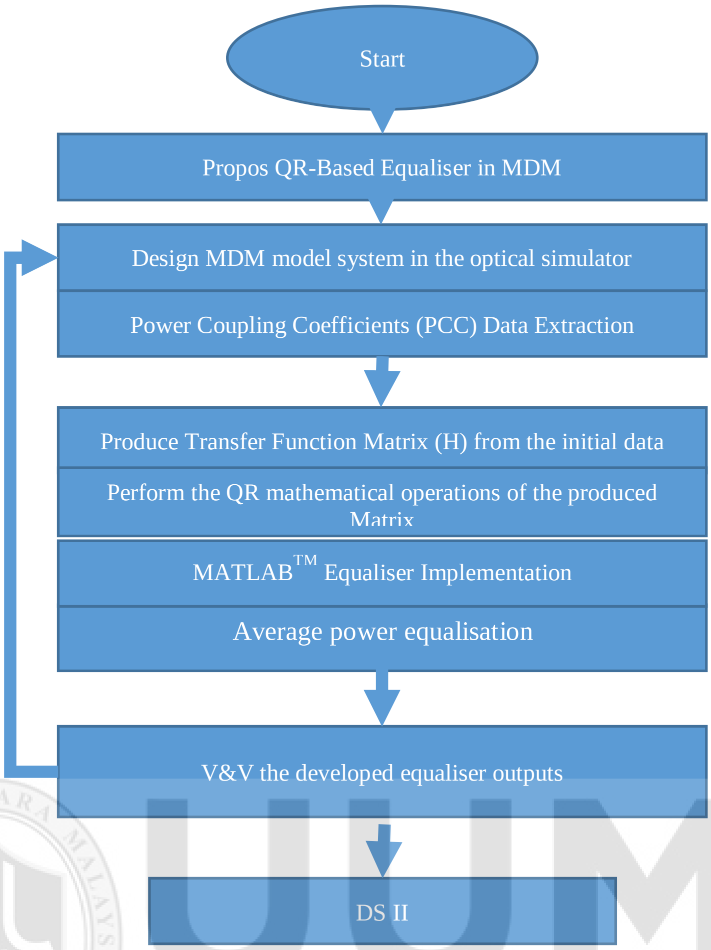



*Figure 13: Perspective Study steps*

## 3.6 Descriptive Study-II

DS-II highlights the implementation and the evaluation of the developed equaliser. This accomplishes the second objective of this research. Performance evaluation is vital step in the evaluation of any research [93].

### 3.6.1 Implementation and Evaluation Techniques

Implementation and performance evaluation of any developed or designed network communication system are essential phases and may be carried out by using any of the following three approaches individually or in combination. These three approaches are analytical modelling, measurement and simulation [94].

The analytical model is considered a representation approach that outlines how the system operates by several mathematical equations. This approach can be used to analyse very simple systems, but if it is used for a complicated system, additional major work will be required in order to simplify and make the correct assumptions [95].

Measurement is the second approach which utilises a special laboratory system to implement research experiments instead of using real systems due to the limited resources and the issues regarding the risk of using a real communication system. Researchers prefer utilising a prototype that may be examined in laboratory circumstances. This approach is suited to medium-complex communication systems, and it is difficult to apply it to large and complicated communication systems because of the enormous requirements and the processes of complicated communication systems [96].

Simulation is the third approach, and this work depends on in modelling the optical system. This is done by using a special kind of software which is called as Simulator. Communication network systems are complicated to model in the real world, and it makes simulation modelling the preferable option. The use of particular simulation software models that can be developed is analysed and evaluated in a simple way due to flexible simulation attributes that permit modifications and editing of any part of the designed model if that is needed. Simulation modelling has proven its credibility and given evidence that it can become a researcher's primary research methodology [97, 98] .

### 3.6.2 Proposed Technique for Design and Evaluation of QR-Based Mode Division Multiplexing Equaliser

Network simulators are utilized all over the world by the researchers for different purposes, like education, commerce and Industry to simulate any section or division of the network. There are many network simulators that are used in every day. It can be used to generate an approximation outcome of the network, which paves the foundation for real time enquiry or performance [99]. The simulation tools in this research are explained in the following subsections.

#### 3.6.2.1 OptSim and OptiSystem



OptSim [100] and OptiSystem [101] are a comprehensive software design suite that allows users to design, test, and simulate optical links in the transmission layer of advanced optical networks. Optsim and Optisystem are standalone products that do not rely on other simulation frameworks. They are physical layer simulators that are based on the actual modelling of fibre optic communication network. These software have powerful new simulation circumstances and also have a real hierarchical definition of components and systems. A copy of these software are installed in the optical network laboratory's computer, and they are used to design and simulate the proposed work.

OptiSystem is the main optical simulator to be used during the design and implementation, while OptSim works as a supporting environment during the Verification and Validation (V&V) stage.

#### 3.6.2.2 MATLAB

MATLAB is a computer simulation environment for performing mathematical calculations. Its name comes from "Matrix Laboratory" which gives a hint that MATLAB is initially developed in handling the mathematical calculations related with matrices in a more comfortable way of that C and FORTRAN. However, because of its initial purposes, MATLAB developers add many functions to the software that makes it a very powerful tool, and it becomes one of the best mathematical simulation environment which is being used by many researcher, students and designers [102].

Numerical and graphical computations can be done by MATLAB. On the other hand, MATLAB can be considered as a procedural programming language that combines an efficient programming structure with pre-defined mathematical commands [102].

In this research, MATLAB is used in the derivation of the secondary dependent variables, data analysis and development of the QR-based mode division multiplexing blind equaliser.

#### 3.6.2.3 Evaluation metrics

The evaluation of this study takes place by comparing the pulse width and capacity before and after implementing the developed equaliser. Having a narrower pulse width reduces the effect of the mode coupling and increases the overall system performance.

Additionally, the developed algorithm's complexity is compared with other similar algorithms.

Finally, the evaluation of the power averaging output takes place by measuring the mean square error (MSE) of the signal before and after the equaliser.

## 3.7 Summary

This chapter puts forward a comprehensive account of the research methodology which focuses on the recommended QR-based blind equaliser and its application in mode- division multiplexing. Four activities related to DRM are described. Completing each one of these stages is mandatory to ensure that the study is going as planned and matching the intended objectives

# CHAPTER FOUR: MATHEMATICAL MODEL AND SIMULATION OF QR BLIND EQULIZER AND POWER AVERAGING EQULIZER

## 4.1 Introduction

This chapter proposes the implementation of QR factorisation to get rid of mode coupling in mode-division multiplexing that is based on the previously described studies. It also illustrates the setup of the simulation model used in generating the initial output, and it also discusses the final results after implementing the proposed equalisation scheme.

Section 4.2 provides a model for mode division multiplexing in OptiSystem. In Section 4.3, the modelling of blind Equalisation that is based on QR Factorization is discussed.



## 4.2 Mode Division Multiplexing Model in OptiSystem

This section describes the design of mode-division multiplexing. In this section, the parameters and scenarios of simulation are shown and described. The purpose is to illustrate the behaviour of links of optical fibre when the signal goes through all the elements, including optical fibre, splitters, and multiplexers. Also, the goal is to find a good quality of signal in all receivers. Figure 14 describes the quality of the received signal with the presence of mode coupling.

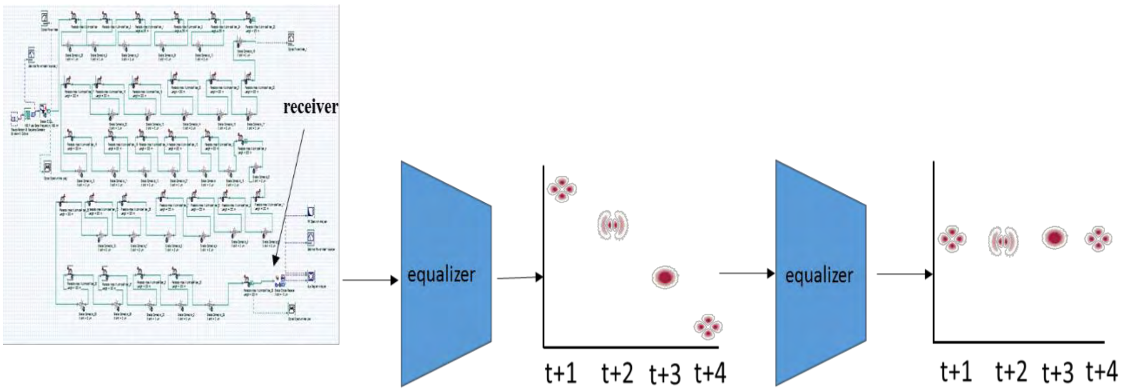



*Figure 14: Model for mode division multiplexing*

The overall system configuration is shown in the figure above. The first part is the light source which carries the data through the optical fibres. In this project, the signal is divided into six different modes; each having different power in the multimode fibre. However, in this research, the researcher simulates the distance of the multimode fibre to be 6 km.

This is the complete design for mode-division multiplexing. The design length, 6 km, prescribes each of the elements with their most significant parameters. The most important global parameters of the network can be seen in the use of transmitter of one type of mode, whose LG with random power ratio is equal to one [101] each light source that produces light beams of unequal power intensity [103]. Transmission in multimode fibre for each segmentation measures 200m, and each parameter has standard that is only used in this system refractive index step 0.1 [104, 105]. For the spatial connector component, the horizontal offset parameter is set to 1.7 µm [101].

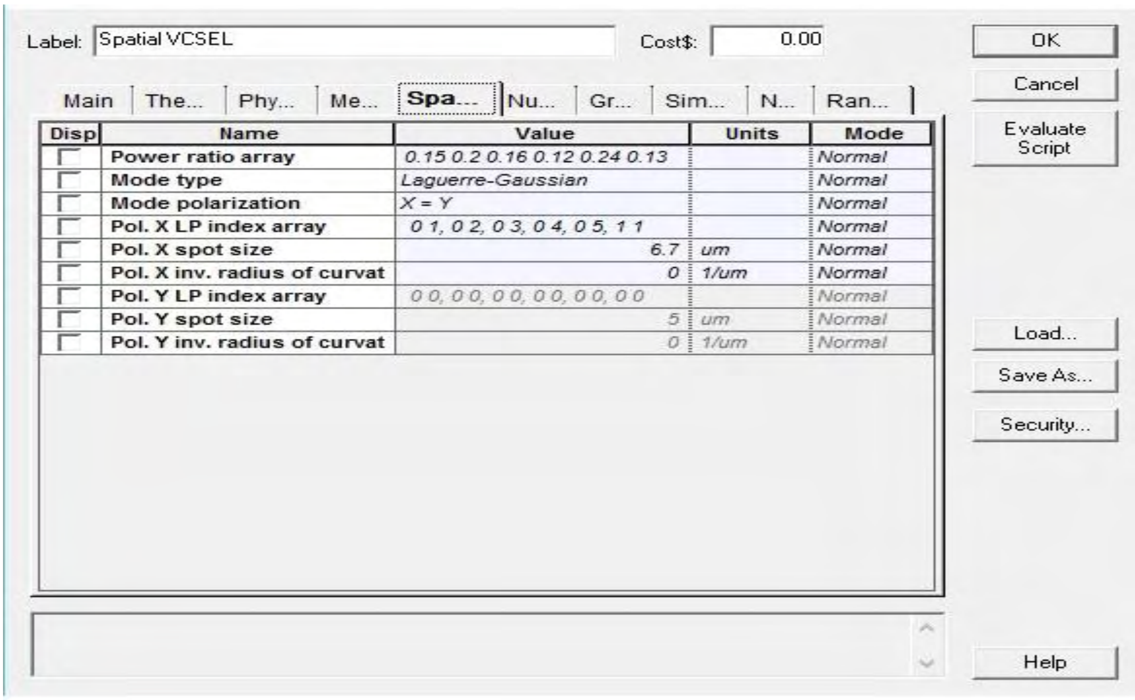



*Figure 15 Simulator's parameters used in the spatial VCSEL*

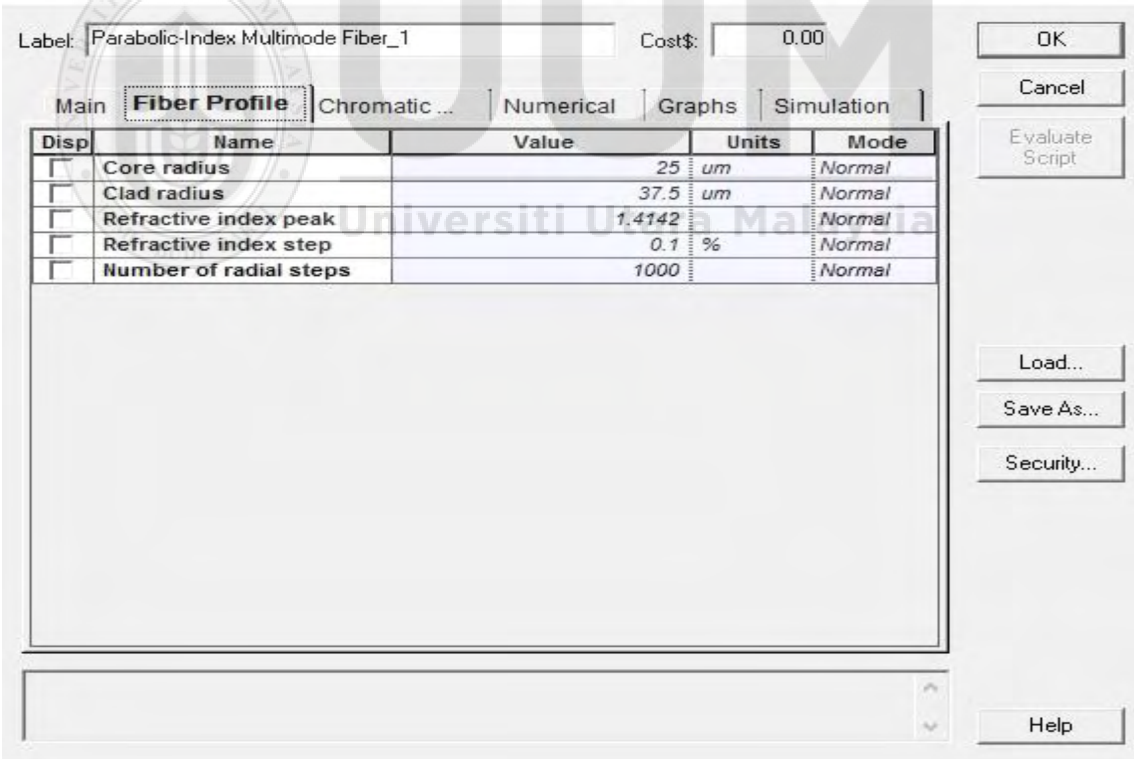



*Figure 16 Simulator's parameters used in the Multimode fibre cable*

The overall system configuration is shown in Figure 14. Meanwhile, this system mode couples this according to the following results:

Table 3 channel matrix before blind equalisation

| | | | | | | |
|---|---|---|---|---|---|---|
| LP [0,1] | 0.194091954 | 0.780356502 | 0.012588645 | 0.000000026 | 0.011939800 | 0.000000012 |
| LP [0,2] | 0.147983648 | 0.012439996 | 0.466190601 | 0.000000986 | 0.036916309 | 0.000001071 |
| LP [1,1] | 0.000000011 | 0.000000026 | 0.000000987 | 0.779111506 | 0.000001006 | 0.194384218 |
| LP [2,1] | 0.148566599 | 0.011799594 | 0.036910051 | 0.000001005 | 0.475877020 | 0.000000352 |
| LP [-1,1] | 0.445922141 | 0.194027031 | 0.148000893 | 0.000000011 | 0.148585865 | 0.000000024 |
| LP [-2,1] | 0.000000024 | 0.000000012 | 0.000001072 | 0.194315483 | 0.000000351 | 0.435263929 |

According to Table 3, in the transmitter, when the researcher gives input 6 modes through the fibre, the researcher has 36 modes on the other side (receiver). That means when the researcher enters the mode through the fibre, it will be reflected within the fibre and it will spread. Afterwards, those particular modes tend to interchange the power randomly through their propagation within the fibre which is the input channel matrix as input to the blind equalisation.

## 4.3 Blind Equalisation based on QR Factorization

Figure 17 describes the implementation of the inverse equaliser from the simulation that results in generated in this project. The first step involves the mode-division multiplexing model in the optical simulator. The second step is the establishment of the secondary dependent variable, i.e., the system transfer function (H) is done by performing statistical properties on the primary dependent variables (PCCs) that are previously extracted from the simulated optical model in step one.

The third step of this stage is to perform the QR factorisation on H by compiling the needed MATLAB code. Fourthly, the researcher takes the inverse model by linear algebra, which is programmed by MATLAB.

Finally, in terms of the energy distributed equally between the modes, the researcher takes the average energy algorithm to get equal signals.

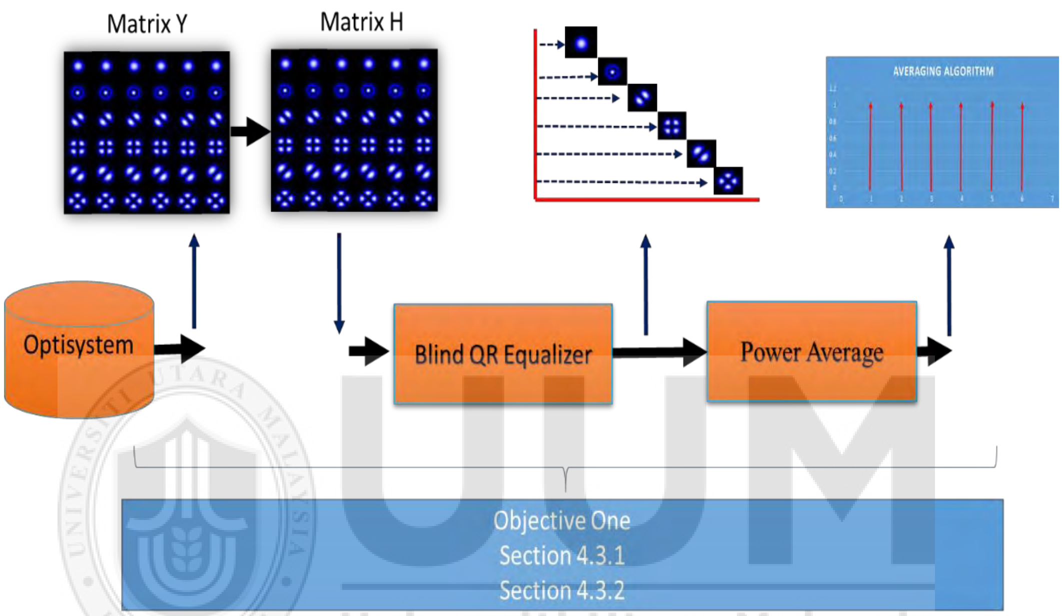




*Figure 17: Input and output for Blind QR Equaliser*

However, due to the complexity and long sequences required for building simulations with adaptive filtering, the current project equalizes the MDM channel by estimating the pulse width for every possible path between modes in the fibre. This study uses that estimation blind equalisation to recover the original input by inverting the effect of the channel.

Once the estimation of the channel matrix has been acquired with the existence of mode coupling according to Table 3, there are different ways to recover the original signals. One of them requires the calculation of the inverse of this matrix, and also the multiplication of the input signal vector with it [106] is needed.

### 4.3.1 Channel Matrix as Input to Blind Equaliser

In the general case of communication systems, the transmitter side transmits the desired signal which is unknown to the receiver side, and the receiver side then receives the transmitted signal. The problem is that between the transmitter and receiver, there is an unknown multipath channels too. Because of this, the received signal from the transmitter side is distorted by the multipath channels. As both the channel and its input are unknown, the objective of blind equalisation is to recover the unknown input sequence that is based solely on its statistical properties [72, 73], by using standard deviation according to the equation (4.1). To obtain the channel matrix which is considered as the input to equaliser, the derivation of the channel matrix equation goes through the below:

$$y_{rec_i} = H_i X_{rec_i} \tag{4.1}$$

Where $X$ and $y$ represent the transmitted and received vectors, respectively, and $H$ is the matrix channel

then,

$$H_i = \frac{X_{rec_i}}{y_{rec_i}} \tag{4.2}$$

and as,

$$X = PR \times TP \tag{4.3}$$

where *PR* is the Power Ratio and *TP* is the total power

then,

$$X_{esst} = \frac{PR}{STi} \times TP \qquad (4.4)$$

where *STi* is the Standard deviation of the input, then:

$$H_{est} = Y * \frac{PR}{STi} * TP * \mathrm{STo} \qquad (4.5)$$

Where Y is output, PR is power ratio, TP is total power and STi, STo is standard deviation to input and output respectively for each channel and average.

In this research, the pulse magnitude's range is from 0-0.12 with a duration of 0.5ps compared with almost the same range in [107] which has a value between 0-0.1 during the first 0.5ps; however, the transfer function has shorter pulse with only 0.5ps compared with 50ps in [107]. On the other hand, the derived transfer function in this research has a short period pulse of about $5.5 \times 10^{-13}$ seconds with hill's shape which is similar to the pulse obtained in [108]. In [77], a 6X6 signal is obtained, and the transfer function of the obtained signal consists of 36 guided modes which are similar to the case of this research with the difference in the pulse duration.

On the other hand, the mean MSE of each channel between the estimated and the actual transfer functions has been calculated and as shown in Table 4 below.

Table 4 MSE values of the estimated transfer function with respect to the actual transfer function

| Channel 1 | Channel 2 | Channel 3 | Channel 4 | Channel 5 | Channel 6 |
|---|---|---|---|---|---|
| 0.013366 | 0.203722 | 0.771365 | 0.049104 | 0.000185 | 0.021784 |

It is found that the $5^{th}$ channel is the best estimated one with error of 0.000185 and the third one is the worst one with error of 0.771365. The overall average MSE for the whole 6X6 channel matrix is equal to 0.176588.

### 4.3.2 Implementation of Blind Equaliser by Using QR Factorization

In this section, the aim of first step is to discuss the general mathematical model of the inverse QR factorisation. The aim of second step is to discuss the general mathematical model of the channel inverse equalisation process that is based on QR factorisation.

QR is used because there are several algorithms that employ QR decomposition of the channel matrix and decode the symbol's iterative in order to improve the performance and reduce the massive matrix inversion operations [109]. QR factorisation is an efficient and frequently used methodology when matrix inversion is needed. A typical application area is fibre communication systems that use mode division multiplexing. The QR decomposition factorises a matrix into an orthogonal and an upper triangular matrix. Then one needs to execute the following algorithm for QR factorisation[110].

The algorithm calculates the above matrix QR factorisation of a real square matrix. A is a factorisation of A as $A = QR$, where Q is an orthogonal matrix and R is an upper triangular matrix [111].

$$A = QR \tag{4.6}$$

$$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\mathbf{Q}=\begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix}$$

Where *($Q^T \times Q = I$)* and the $Q^T$ is a transpose of a matrix, $Q$ is another matrix which is created by writing the rows of $Q$ as the columns of a transpose matrix, or writing the columns of $Q$ as the rows of a transpose matrix. The transpose of matrix $Q$ is written as $Q^T$ [80]. And I, an identity matrix, is the diagonal square matrix with 1s down its diagonal. That means:

$$(Q^T \times Q = I) \tag{4.7}$$

$$\begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix} * \begin{pmatrix} q_{11} & q_{21} & q_{31} \\ q_{12} & q_{22} & q_{32} \\ q_{13} & q_{23} & q_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Matrix decomposition is a fundamental theme in linear algebra and applied statistics, and it has both scientific and engineering significance. Computational convenience and analytic simplicity are the two main aspects in the purpose of matrix decomposition. In the real world, it is not feasible for most of the matrix computation to be calculated in an optimally explicit way, such as matrix inversion. Therefore, it is necessary to convert a difficult matrix computation problem into several easier tasks, such as solving triangular or diagonal systems, which would greatly facilitate the calculations.

In (4.8) the inverse of $A$ is derived. For that, the inverse of $R$ is needed, which is a straightforward operation, since $R$ is upper triangular. The transpose of $Q$ is basically done with orthogonal [110].

$A = Q * R$ According to (4.5)

$$A^{-1} = (Q * R)^{-1}$$
$$A^{-1} = Q^{-1} * R^{-1}$$

When $Q$ is orthogonal that mean $Q^{-1} = Q^{T}$ [112]

$$A^{-1} = R^{-1} * Q^{T}$$

$$A^{-1} = R^{-1} * Q^{T} \qquad (4.8)$$

The second step of using QR factorisation has become one of the most important operations in linear algebra. It can be used to find matrix inversion, to solve a set of simulations equations, or in numerous applications in scientific computing.

At this point, the QR decomposition can function as a method to find the inverse. After decomposition in terms of matrix A into Q and R matrices.



**Algorithm 4.1: Blind equalizer based on QR factorization**

Step 1. Decompose the estimated channel matrix ($H_{est}$) obtained from Equation $H_{est} = Y * \frac{PR}{STi} * TP * \text{STo}$ .

Step 2. $H_{est}$ =QR Decomposition of matrix into Q and R.

Step 3. Find the inverse of $H_{est}$ based on the decomposed Q and R matrices as follow:

$QRX_{rec} = b$ . Letting $c = RX_{rec}$ , we can find c by solving Qc = b, Q is orthogonal, so c = $Q^{T}$b. We can then obtain x by solving RX = c by back substitution. The process follows [80]

Where $H_{est} = QR$

By linear algebra

$AX_{rec} = b$ b is Identity Matrix (4.9)

$$QRX_{rec} = b$$
$$RX_{rec} = bQ^{-1}$$

Where $Q^{-1} = Q^{T}$

Is obtained which equivalent is to

$$RX_{rec} = bQ^{T}$$
$$c = bQ^{T}$$ Solve 'c=$Q^T$ b' and finds c (4.10)

Now we use this c to solve RX = c

$$RX_{rec} = c$$
$$X_{rec} = cR^{-1} \qquad (4.11)$$

Use the back-substitution method to solve the upper-triangular matrix. For this mathematical model, the inverse QR can be explored through the application of the algorithm by using MATLAB.



Step 4. Use the $H_{est}^{-1}$ to recover the original signal:

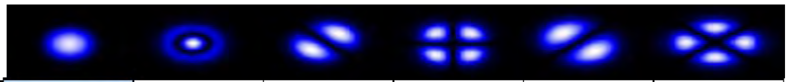
as:

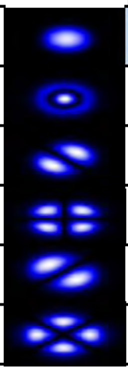
$$Y_{rec} = H * X_{rec}$$

then,

$$X_{est_i} = H_{est_i}^{-1} * Y_{rec_i} \qquad (4.12)$$

Table 5  channel after blind equaliser





| | | | | | | |
|---|---|---|---|---|---|---|
| LP [0,1] | 0.52515134 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| LP [0,2] | 0.00000000 | 0.41636807 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| LP [1,1] | 0.00000000 | 0.00000000 | 0.17641847 | 0.00000000 | 0.00000000 | 0.00000000 |
| LP [2,1] | 0.00000000 | 0.00000000 | 0.00000000 | 0.96536245 | 0.00000000 | 0.00000000 |
| LP [-1,1] | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 1.26699666 | 0.00000000 |
| LP [-2,1] | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 1.21690141 |

According to Table 5, each one of the channels has one mode in the receiver side that is based on power.

For the mode-division multiplexing, over 6 km multimode fibre is investigated. It is shown that 6×6 MDM processing can be used to compensate entirely for mode coupling as shown to Table 3.

For the signal after the blind equalizer, the symbols have no overlapping, and this has better results compared to the results in [113-115] where some overlapping still exist. In [114, 115], the pulse has scattered symbols while in [113], the symbol under observation still interferes with the next symbol as the equalized pulse is still not complete, like a Gaussian shape pulse. Also, as per [116], with good propagation channel where no or weak mode coupling occurs, the signal can be described by matrix values only at its diagonal and zeros elsewhere. In fact, this is similar to the obtained signal after the blind equalizer, and this is shown in Table 5.

This situation is portrayed in Figure 18, and each one of the mode is spread power to the six modes before the blind equaliser. After the implementation of the blind equaliser, Figure 19 shows to have only one mode in one time.  It has the same number of mode in the transmitter side.

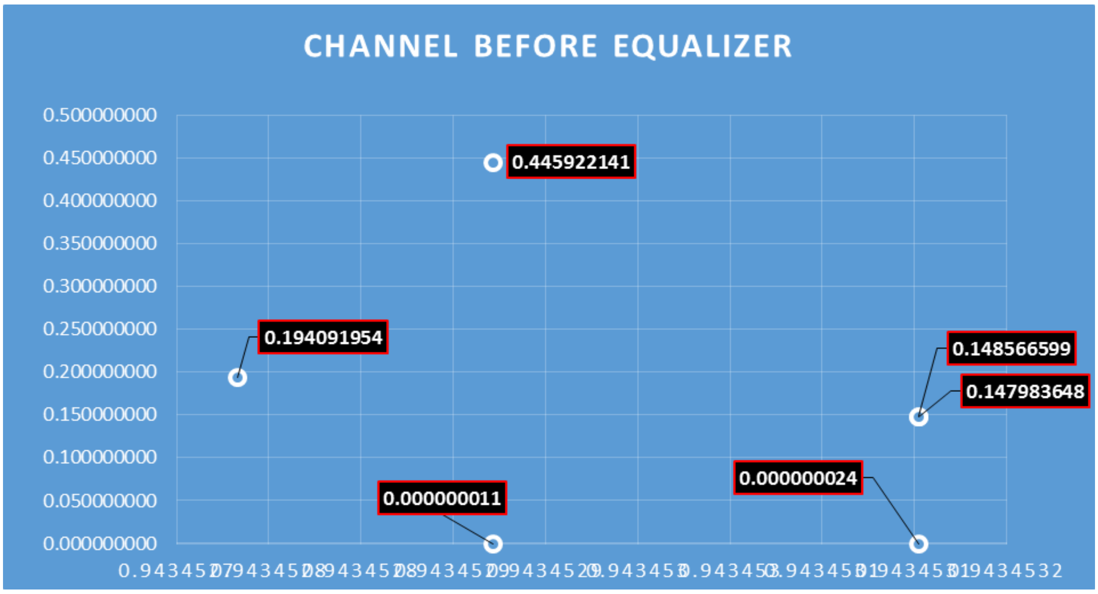



*Figure 18 : Six mode before blind equaliser*

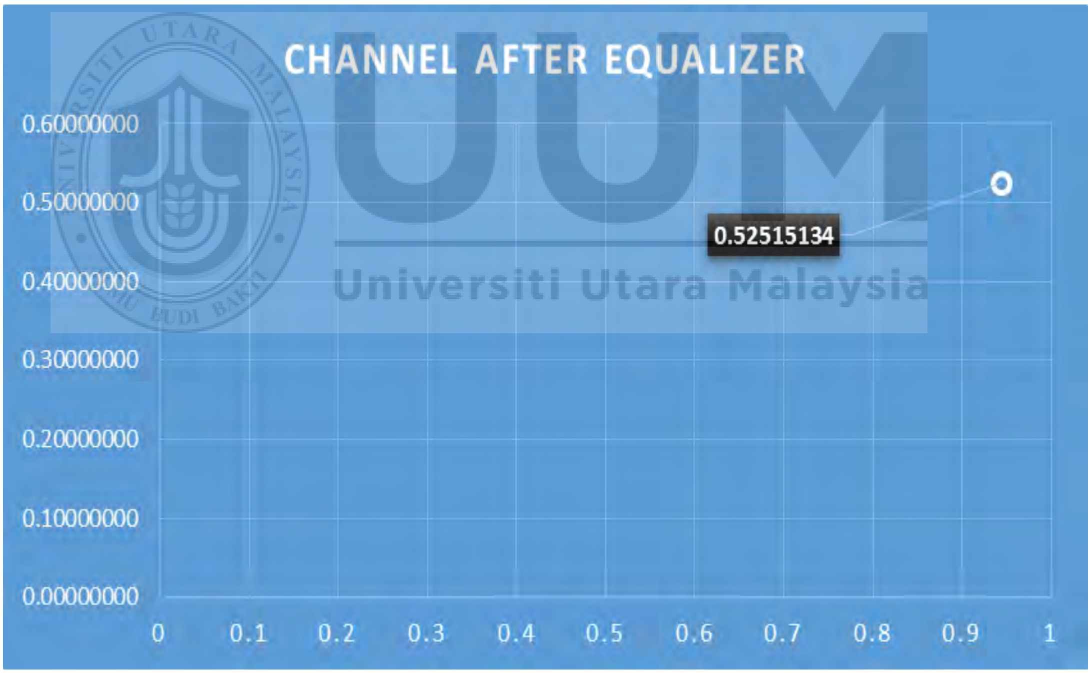





*Figure 19 : One mode after blind equaliser*

In spite of the above results, the six channels still have different powers, so another algorithms is needed to perform this task.

When channel state information is available, the power may be allocated to each mode in an optimal way [12]. Otherwise, in long-haul systems, channel state information is likely to be unavailable, so equal power must be allocated to each mode [117]. To compensate lost power, the use of amplifier EDFA is also used for optical amplifier in order to boost the optical signal at any point in transmission line[21]. This can be achieved by following the steps in Algorithm 4.2 below.

**Algorithm 4.2: Power Averaging**

Step 1. Obtained the equalized signal from the blind equalisation scheme

Step 2. Find the average power value based on the below:

$$Av = \frac{\sum_{i=1}^{n} X_i}{D} \tag{4.13}$$

where $X_i$ is the transmitter total power and $D$ is the number of the transmitter modes.

Step 3. Amplify or attenuate the power of each channel accordingly.

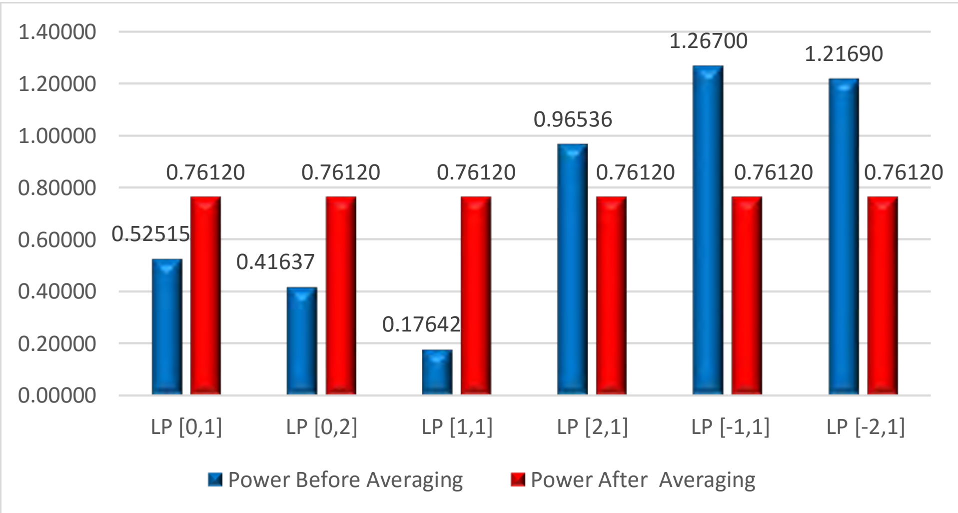



*Figure 20 : Before and After Averaging Equalisation*

#### 4.3.2.1 Verification

The verification of the equaliser mechanism is done by the use of the QR factorization method. A portion of the equaliser mechanism after the verification process is presented in Figure 21, and this is confirmed in the following:

- Equalisation was programmed correctly, and
- Equalisation implementation did not contain any errors or bugs.

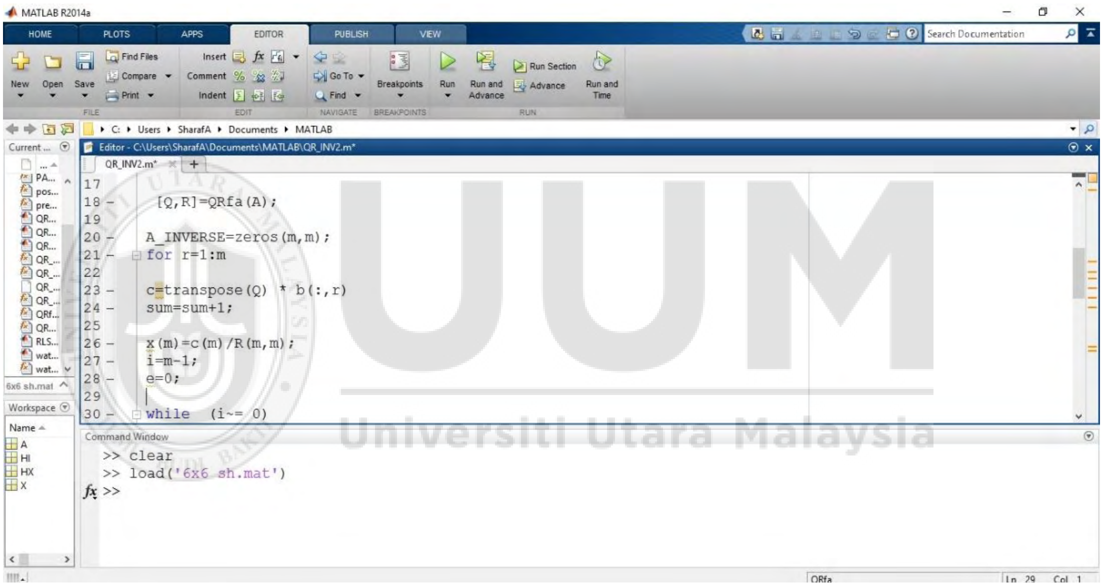



*Figure 21: Inverse code for QR factorization*

#### 4.3.2.2 Validation

Model validation is usually defined as the "substantiation that the model, within its domain of applicability, behaves satisfactorily and is consistent with the study objectives" [118].

To validate the optical model, the same scenario has been taken place in two different optical simulators by setting the same input parameters to the simulator and measuring the simulator's output. After applying this, it is found that the two models still produce the same output. This output is used as the input signal to the developed equaliser.

### 4.4 Summary

This chapter introduces the proposed inverse equaliser along with its implementation. After defining the problem in chapter one, the researcher starts taking initial data from simulators, after which the researcher applies this data by mathematical model to inverse equalisation implementation by the use of the MATLAB software. This model is used to solve the problem and to compare the actual results with simulation results. After that, the evaluation best suited for this model is known.

# CHAPTER FIVE: EVALUATION AND DISCUSSION OF RESULTS

## 5.1 Introduction

Once the problem and its solution are defined, the implementation of the same equalisation process by using the MATLAB software gets started. The software is used in solving the problem and comparing the actual outcomes with simulation results that are based on Tables 3 and 4, which are found in Chapter 4. This chapter assesses the study.

The performance assessment of QR factorisation through pulse width, capacity, and complexity in Sections 5.2. And 5.3, using Equal Power with MSE, is presented in this chapter.

## 5.2 Performance Evaluation of QR factorization

Testing the ability of QR factorisation that is based on the implementation of blind equaliser in mode division multiplexing is the main goal of these experimental evaluations. The aim of this is to measure the QR factorisation performance on a specific workload in order to gather good results. Achieving this goal needs the focus on three types of performance metrics: complexity, capacity, and pulse width.

### 5.2.1 Pulse Width

Propagation of multiple spatial modes that possess different mode delays and different potential losses is supported by MMF. Coupling between different modes is a result of manufacturing variations, mechanical stresses, bends, thermal gradients, and other effects.

Intersymbol interference (ISI) becomes the result of this mode coupling effect. The solution of this problem is solved by the developing of the needed equalizer, and as mentioned in chapter four, the evaluation for this part takes place by measuring the enhancement in the pulse width.

The pulse width for each channel is calculated on the basis of equation 5.1 which is given below.

$$\text{Pulse width} = (T_D - T_1) \qquad (5.1)$$

where $T_D$ is the last arrived mode and Tl is the first one.

Based on the above equation, the pulse width of the received signal is measured before and after the adding the blind equaliser. It is found that as a result to the reduction of the number of the received modes, the pulse width of each channel is also reduced mitigating the ISI effect on the received signal.

Table 6 shows that due to the mode coupling, each channel has six modes that arrives at different times. Based on this, the pulse width is calculated and becomes equal to $3.7 \times 10^{-13}$ seconds.

On the other hand, Table 7 shows the channel signal after the equalization; thus, it is clear that the pulse width for each channel is reduced to an effect of equalizing the signal.

Table 6 Before Blind equaliser with delay

| c1 | c2 | c3 | c4 | c5 | c6 | Delay |
|---|---|---|---|---|---|---|
| 0.194091954 | 0.780356502 | 0.012588645 | 0.000000026 | 0.011939800 | 0.000000012 | 9.434527329E-07 |
| 0.147983648 | 0.012439996 | 0.466190601 | 0.000000986 | 0.036916309 | 0.000001071 | 9.434531021E-07 |
| 0.000000011 | 0.000000026 | 0.000000987 | 0.779111506 | 0.000001006 | 0.194384218 | 9.434528713E-07 |
| 0.148566599 | 0.011799594 | 0.036910051 | 0.000001005 | 0.475877020 | 0.000000352 | 9.434531021E-07 |
| 0.445922141 | 0.194027031 | 0.148000893 | 0.000000011 | 0.148585865 | 0.000000024 | 9.434528713E-07 |
| 0.000000024 | 0.000000012 | 0.000001072 | 0.194315483 | 0.000000351 | 0.435263929 | 9.434531021E-07 |

Table 7 After Blind equaliser with delay

| c1 | c2 | c3 | c4 | c5 | c6 | Delay |
|---|---|---|---|---|---|---|
| 0.52515134 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 9.434527329E-07 |
| 0.00000000 | 0.41636807 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 9.434531021E-07 |
| 0.00000000 | 0.00000000 | 0.17641847 | 0.00000000 | 0.00000000 | 0.00000000 | 9.434528713E-07 |
| 0.00000000 | 0.00000000 | 0.00000000 | 0.96536245 | 0.00000000 | 0.00000000 | 9.434531021E-07 |
| 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 1.26699666 | 0.00000000 | 9.434528713E-07 |
| 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 1.21690141 | 9.434531021E-07 |

### 5.2.2 Capacity

The MDM system's mathematical representation is performed by the use of a matrix. This matrix is dependent on the scenario, and it is considered at every time (i.e. selective or flat spatial fading). The Shannon extended capacity formula is used to study the capacity that is achieved by the MDM channel in all of the above cases.

The channel shows multiple inputs and outputs in the case of mode division multiplexing at both the receiver and the transmitter ends. The extended Shannon's capacity formula can be used to estimate its capacity [119].

The channel capacity is measured on the basis of the Equation 5.2 below[12]:

$$\mathrm{C} = \log_2(1 + p_t \, / D) \qquad (5.2)$$

where $C$ is the channel capacity, $P_t$ is the total power and $D$ is the number of modes.

Based on the above equation, the channel capacity is calculated before and after adding the blind equaliser. As the number of modes within the channel is reduced from 6 to one after equalisation, the channel capacity is increased. Table 8 and Figure 22 below show the comparison of the channel capacity before and after the equalisation.

Table 8 Capacity before and after inverse equalisation

| Capacity | | |
|---|---|---|
| **Channels** | **Capacity in bits/hertz Before Equalisation** | **Capacity in bits/hertz After Equalisation** |
| **C1** | **0.20925878** | **0.608952411** |
| **C2** | **0.22210863** | **0.502196226** |
| **C3** | **0.15135927** | **0.234401342** |
| **C4** | **0.21690574** | **0.974795397** |
| **C5** | **0.15344226** | **1.180782268** |
| **C6** | **0.14397012** | **1.148544613** |
| **Total** | **1.09704481** | **4.649672257** |



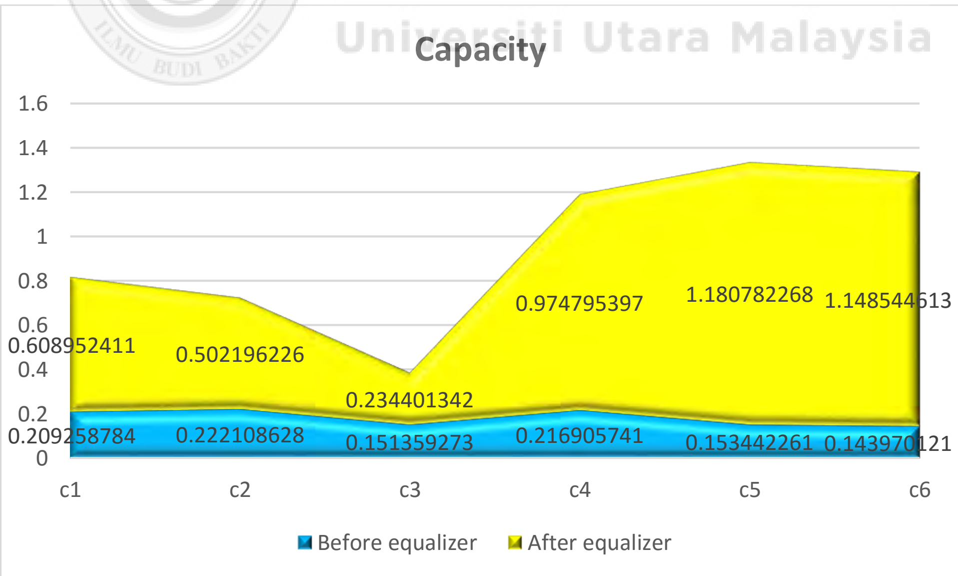



*Figure 22 : Capacity before and after blind equaliser*

### 5.2.3 Complexity and Processer Time for QR Factorization

The characterisation of the efficiency of an algorithm, in terms of time of execution and without particular program or computer, means requirement to quantify the number of operations or steps required by the algorithm. The time of algorithm's execution can be expressed as the number of steps is needed to address the problem if every step is considered as a basic unit of computation. The process of choosing an opposite basic unit of computation can be a complicated issue, and it depends on the implementation of the algorithm [120].

Generally, the interest lies in the worst case: the maximum number of operations could be done for a given problem size. For instance, inserting an element into an array involves moving the current element and all the elements after it to one place to the right of the array. The worst case in this scenario concerns inserting in the beginning of the array. This means that all of the array's elements have to be moved. Thus, in the worst case, the time of insertion is directly proportional to the number of elements in the array. In other words, the worst-case time for the insertion operation is linear in the number of elements in the array. If the problem size doubles in a linear time algorithm, the number of operations also gets doubles [121].

Another example can be taken in into consideration, and that is a summation of algorithm. A good basic unit of computation can involve counting the number of assignment statements that are done in order to calculate the sum. The function sum_of_n has the number of assignment statements set at 1 (sum=0) and a value of n (the number of times sum=sum+$i$ is performed). This can be denoted by the function $T$, where $(n)$ = n+1. The "size of the problem" is often represented by the parameter $n$ and it can be read as "$(n)$ is the time it takes to solve a problem of size $n$, namely 1+$n$ steps". This means that since the loop is executed N times, the sequence of statements are also executed N times, that is, if it is assumed that the statements are O(1), the total time for the loop is N * O(1), which is O(N) overall [120-123].

Based on the above discussion and the implemented algorithm code in MATLAB, the computational complexity of the proposed algorithm is calculated. It is found that the algorithm is intend to have complexity of O(N) because it depends on individual loops with no overlapping and the main loop of the code is changed linearly with the increment in the number of modes m and this is shown in Figure 23 below.

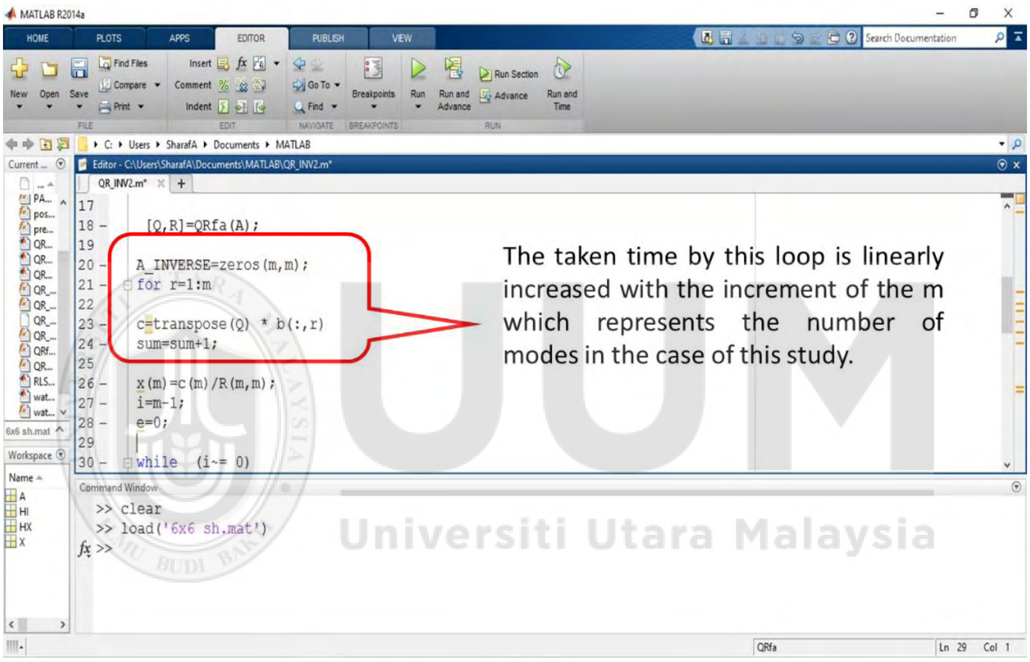



*Figure 23: Computational complexity for QR factorization*

As mentioned above, the complexity of the developed algorithm is equal to O(N).This is considered as an improvement if it is compared with the currently used algorithms. For instance, RLS that has a complexity of O($N^2$) [45, 46] while LMS is also linear with a complexity of O(N) [45, 46], but it has slower processing time. Table 9 below shows the processing time in comparison with RLS, LMS and QR factorization. Depending on using the same data, this comparison is taken place by the tic and toc commands to compare the running time of QR factorization with RLS and LMS. The comparison shows that QR factorisation needs less time in comparison with LMS and RLS.

Table 9 compares between QR factorization and (LMS, RLS)

| Algorithm | CPU Computation speed / seconds | Computation Complexity |
|---|---|---|
| QR | 0.005242 | O(n) |
| LMS | 0.0077814 | O(n) |
| RLS | 0.008747 | $O(n^2)$ |

On the other hand, the computational complexity of the QR factorization can also be measured by using the tic and toc commands. The linear result O(N) can still be obtained [124].

Five attempts have taken place in performing the above mentioned comparison, and also the average running time is taken into consideration.



## 5.3 Power Averaging Equalisation

In order to check the performance of the power averaging algorithm, MSE is measured on the basis of the following Equation (5.3) that concerns the received signal before and after performing the power averaging algorithm on it. The obtained results show that the channel has an MSE value which is equal to zero in comparison with 0.17030 before applying the power averaging algorithm.

$$MSE = \frac{1}{D}\sum_{i=1}^{D}(\overline{Y} - Y)^2 \qquad (5.3)$$

where D is the number of modes, $\overline{Y}$ is the power of $i_{th}$ mode after performing the power averaging and $Y$ is the power of $i_{th}$ mode before performing the power averaging

## 5.4 Summary

In this chapter, the researcher discusses the evaluation of the inverse equaliser by the help of QR factorisation. The evaluation is taken place by comparing the developed algorithm with the currently used algorithms in addition to the measurement of other performance metrics. The developed equaliser is less complex and faster with narrower pulse width and has higher channel capacity.

# CHAPTER SIX: CONCLUSION AND FUTURE WORKS

## 6.1 Introduction

The aim of this thesis is to develop an inverse equalisation scheme in mode division multiplexing by using QR factorisation. The conclusion of the research work is presented in this chapter. Section 6.1 discusses the research findings and talks about the significance of the QR factorisation and the possible implementation during blind equalisation on mode division multiplexing. Section 6.2 highlights the contributions made by this research. In conclusion, some suggestions related to the study are presented in Section 6.3.

## 6.2 Summary of the Study

A specific solution to attain mode division multiplexing has been reported. The correct behaviour of this technology is demonstrated with the help of the system implemented in this thesis. This reinforces the notion that enhancing the capacity output of the optical transmission can be done through multi-mode fibre singular properties. Nonetheless, the key goal of this thesis is to introduce enhancement equalisation while reducing complexity and creating a faster algorithm as well get focused.

Chapter 1 mentions the motivation behind this research work. The rising network traffic in optical fibre backbones makes it necessary to consider mode division multiplexing as a new approach to increase the system bandwidth since current systems will be incapable of sustaining future traffic growth. Mode division multiplexing is one of more popular schemes that are used to complement other multiplexing schemes in order to raise the aggregate bandwidth. However, in mode division multiplexing, mode coupling limits many applications. Improving equalisation technique for mode division multiplexing is necessary to overcome such limitations. The framework and guideline used to accomplish this research is the Design Research Methodology (DRM). Three aspects are used as basis for the design of this methodology.

The first aspect is the conduct of an impartial evaluation of the studies, and empirical research are conducted by using the information gathered from relevant literature. Attempts are also made to gain a better understanding of QR factorisation and mode division multiplexing inverse equalisation, as well as the issues which often arise from their usage. All this is based on previous papers about mode division multiplexing.

Afterwards, the inverse equalisation scheme in mode division multiplexing by using QR factorisation is implemented. The first implementation step of QR factorisation is designing the model of the mode division multiplexing as the optical simulator. Once this is accomplished, the secondary dependent variable is established. In this case, the system transfer function (H) is formulated by conducting mathematical calculations on the dependent variables that are gathered in the first step. Performing the QR factorisation on H by compiling the needed MATLAB code is the last step for this stage. Afterwards, an assessment of the investigations and empirical research which is conducted using information gathered from relevant literature is done.

An efficient equalisation technique is needed to compensate for the mode coupling and modal dispersion that takes place in multimode fibres in mode division multiplexing. Current equalisation techniques are very complex in terms of the required CPU time and speed. On the other hand, implementation blind equalisation by the use of QR factorisation in mode division is quicker and less complex. Moreover, comparisons are done before and after the implementation of this algorithm that reveals that there is lesser lag time after the algorithm is used. It is also discovered that implementation of the QR factorisation increases the absorption capacity when the inverse equalisation is finished.

## 6.3 Research Contributions

This overall contribution of this research is the development of an equalisation technique in mode division multiplexing through the use of a quicker and less complex algorithm. It is able to lessen mode coupling and modal dispersion; both of them often lead to intersymbol interference. This thesis has the following specific contributions:

- The development of an inverse equalisation process in mode division multiplexing through the use of QR factorisation. The results obtained from the equalisation are validated by comparing the model results as the results are gathered from a valid simulator. Also, the QR factorization with RLS and LMS algorithms has been compared.
- The study helps achieving equalisation by using a new algorithm and averaging the power between the modes.



## 6.4 Future Works

The target from this research is the development of an equalisation technique that is capable of reducing the mode coupling effect in the optical fibres. The developed technique is able to do so; however, there are some points that have been considered within this research and can be put as future work.

The equaliser offline is developed by taking data from the optical simulator and processing them in MATLAB environment. Future works can investigate how to enhance this practice by developing real time components to work directly in the optical simulator environment.

The accomplishment of this can further enhance the functionality of the MDM systems by providing the researchers more flexibility in dealing with MDM equalisation.

## References


[1] S. Ö. Ar, J. M. Kahn, and K.-p. Ho, "MIMO Signal Processing for Mode-Division Multiplexing," *IEEE Signal Processing Magazine,* pp. 1-23, 2014.

[2] CISCO, "The Zettabyte Era: Trends and Analysis," 2015.

[3] N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, *et al.*, "Terabit-scale orbital angular momentum mode division multiplexing in fibers," *Science,* vol. 340, pp. 1545-8, Jun 28 2013.

[4] T. Nakano and Y. Okaie, "Channel Model and Capacity Analysis of Molecular Communication with Brownian Motion," *IEEE Communications Letters,* vol. 16, pp. 797-800, 2012.

[5] S. O. Arik and J. M. Kahn, "Coupled-core multi-core fibers for spatial multiplexing," *IEEE Photonics Technology Letters,* vol. 25, pp. 2054-2057, 2013.

[6] S. Ö. Arik, D. Askarov, and J. M. Kahn, "Effect of mode coupling on signal processing complexity in mode-division multiplexing," *Journal of Lightwave Technology,* vol. 31, pp. 423-431, 2013.

[7] P. Singh, "Investigations with mode division mutipexed transmission," *Electrical and Electronics Engineering: An International Journal (ELELIJ),* vol. 3, pp. 43-51, 2014.

[8] T. Morioka, M. Jinno, H. Takara, and H. Kubota, "Innovative future optical transport network technologies," *NTT Technical Review,* vol. 9, p. 2011, 2011.

[9] F. Core, "Multimode Fiber: 50µm versus 62.5µm," pp. 3-5.

[10] N. R. Teja, M. A. Babu, T. R. S. Prasad, and T. Ravi, "Different Types of Dispersions in an Optical Fiber," *International Journal of Scientific and Research Publications,* vol. 2, pp. 1-5, 2012.

[11] K. Appaiah, S. Vishwanath, and S. R. Bank, "Impact of fiber core diameter on dispersion and multiplexing in multimode-fiber links," *Optics Express,* vol. 22, p. 17158, 2014.

[12] K.-P. Ho and J. M. Kahn, "Mode-Dependent Loss and Gain: Statistics and Effect on Mode-Division Multiplexing," *Optics express,* vol. 19, p. 22, 2011.

[13] A. Amphawan, Y. Fazea, and H. Ibrahim, "Mode division multiplexing of spiral-phased donut modes in multimode fiber," *International Conference on Optical and Photonic Engineering (icOPEN2015),* vol. 9524, p. 95240S, 2015.

[14] J. M. Kahn, K.-P. Ho, and M. Bagher Shemirani, "Mode coupling effects in multi-mode fibers," *Optical Fiber Communication Conference,* p. OW3D.3, 2012.

[15] H. Zhao, L. Zhang, B. Liu, Q. Zhang, Y. Wang, Q. Tian*, et al.*, "MIMO signil processing for mode Division multiplexing with RLSCMA algorithm," in *Optical Communications and Networks (ICOCN), 2014 13th International Conference on*, ed, 2014, pp. 1-3.

[16] M. B. Shemirani and J. M. Kahn, "Compensation of Multimode Fiber Dispersion by Optimization of Launched," *Journal of Lightwave Technology*, vol. 28, pp. 2084-2095, 2010.

[17] G. Milione, D. A. Nolan, and R. R. Alfano, "Determining principal modes in a multimode optical fiber using the mode dependent signal delay method," *JOSA B,* vol. 32, pp. 143-149, 2015.

[18] M. Umer, I. Technology, and O. Lahore, "Adaptive LMS Based Channel Equalization," *International Journal of Technology and Research,* vol. 2, pp. 111-113, 2014.

[19] R. L. Ali, S. a. Khan, A. Ali, Anis-ur-Rehman, and S. a. Malik, "A Robust Least Mean Square Algorithm for Adaptive Array Signal Processing," *Wireless Personal Communications*, pp. 1449-1461, 2013.

[20] O. Gurrapu, "Adaptive filter algorithms for channel equalization," 2009.

[21] G. Le Cocq, L. Bigot, A. Le Rouge, M. Bigot-Astruc, P. Sillard, C. Koebele, *et al.*, "Modeling and characterization of a few-mode EDFA supporting four mode groups for mode division multiplexing," *Optics express*, vol. 20, pp. 27051-27061, 2012.

[22] M. Agarwal and R. Mehra, "Review of Matrix Decomposition Techniques for Signal Processing Applications," *Int. Journal of Engineering Research and Applications*, vol. 4, pp. 90-93, 2014.

[23] G. Golub and F. Uhlig, "The QR algorithm: 50 years later its genesis by John Francis and Vera Kublanovskaya and subsequent developments," *IMA Journal of Numerical Analysis*, vol. 29, pp. 467-485, 2009.

[24] T. H. Kim, "Low-Complexity Sorted QR Decomposition for MIMO Systems Based on Pairwise Column Symmetrization," *IEEE Transactions on Wireless Communications*, vol. 13, pp. 1388-1396, 2014.

[25] M. Kasahara, K. Saitoh, T. Sakamoto, N. Hanzawa, T. Matsui, K. Tsujikawa, *et al.*, "Design of few-mode fibers for mode-division multiplexing transmission," *Photonics Journal, IEEE*, vol. 5, pp. 7201207-7201207, 2013.

[26] S. Randel, P. Winzer, and a. Linecard, "DSP for Mode Division Multiplexing," presented at the 18th OptoElectronics and Communications Conference, 2013.

[27] S. Ö. Arik, J. M. Kahn, and K. P. Ho, "MIMO signal processing for mode-division multiplexing: An overview of channel models and signal processing architectures," *IEEE Signal Processing Magazine*, vol. 31, pp. 25-34, 2014.

[28] R.-j. Essiambre, R. W. Tkach, and R. Ryf, "Fiber Nonlinearity and Capacity: Single-Mode and Multimode Fibers 1," *Optical Fiber Telecommunications V1B,* pp. 1-43, 2013.

[29] A. Amphawan, Y. Fazea, and H. Ibrahim, "Mode division multiplexing of spiral-phased donut modes in multimode fiber," in *International Conference on Optical and Photonic Engineering (icOPEN2015)*, ed: International Society for Optics and Photonics, 2015, pp. 95240S-95240S.

[30] T. Uematsu, Y. Ishizaka, Y. Kawaguchi, K. Saitoh, and M. Koshiba, "Design of a compact two-mode multi/demultiplexer consisting of multimode interference waveguides and a wavelength-insensitive phase shifter for mode-division multiplexing transmission," *Journal of Lightwave Technology,* vol. 30, pp. 2421-2426, 2012.

[31] Y. Jung, R. Chen, R. Ismaeel, G. Brambilla, S.-U. Alam, I. Giles, *et al.*, "Dual mode fused optical fiber couplers suitable for mode division multiplexed transmission," *Optics express,* vol. 21, pp. 24326-24331, 2013.

[32] C. Poole, C. D. Townsend, and K. Nelson, "Helical-grating two-mode fiber spatial-mode coupler," *Lightwave Technology, Journal of,* vol. 9, pp. 598-604, 1991.

[33] A. Amphawan, B. Nedniyom, and N. M. Al Samman, "Selective excitation of LP01 mode in multimode fiber using solid-core photonic crystal fiber," *Journal of Modern Optics,* vol. 60, pp. 1675-1683, 2013.

[34] A. Amphawan, S. Chaudhary, and V. W. S. Chan, "2x20 Gbps - 40 GHz OFDM Ro-FSO transmission with mode division multiplexing," *Journal of the European Optical Society: Rapid Publications,* vol. 9, 2014.

[35] A. Amphawan, "Binary encoded computer generated holograms for temporal phase shifting," *Optics express,* vol. 19, pp. 23085-23096, 2011.

[36] A. Amphawan, "Binary spatial amplitude modulation of continuous transverse modal electric field using a single lens for mode selectivity in multimode fiber," *Journal of Modern Optics,* vol. 59, pp. 460-469, 2012.

[37] M. Cvijetic and I. B. Djordjevic, "Framework for Optimization of Characteristics of Optical Fibers Supporting Spatial Division Multiplexing," 2014.

[38] K. P. Ho and J. M. Kahn, "Linear propagation effects in mode-division multiplexing systems," *Journal of Lightwave Technology,* vol. 32, pp. 614-628, 2014.

[39] V. S. Lyubopytov, V. K. Bagmanov, and A. K. Sultanov, "Adaptive SLM-based compensation of intermodal interference in few-mode optical fibers," *International Society for Optics and Photonics,* vol. 9216, p. 92160I, 2014.

[40] X. Pan, B. Liu, H. Zhao, and Q. Tian, "Fast convergence equalization algorithm for mode division multiplexing system," *Optical Engineering,* vol. 54, p. 056108, 2015.

[41] V. S. Lyubopytov, V. K. Bagmanov, and A. K. Sultanov, "Adaptive SLM-based compensation of intermodal interference in few-mode optical fibers," in *SPIE Optical Engineering+ Applications* vol. 9216, ed, 2014, p. 92160I.

[42] L. Su, K. S. Chiang, and C. Lu, "Microbend-induced mode coupling in a graded-index multimode fiber," *Applied optics,* vol. 44, pp. 7394-7402, 2005.

[43] N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, *et al.*, "Terabit-Scale Orbital Angular Momentum Mode Division Multiplexing in Fibers," *Science,* vol. 340, pp. 1545-1548, 2013.

[44] V. S. Lyubopytov, A. R. Gizatulin, A. Z. Tlyavlin, and A. K. Sultanov, "Optical-domain mode coupling compensation for mode division multiplexing systems," *International Society for Optics and Photonics,* vol. 9156, p. 915604, 2014.

[45] P.-F. Cui, Y. Yu, Y. Liu, W.-J. Lu, and H.-B. Zhu, "Joint RLS and LMS adaptive equalization for indoor wireless communications under staircase environments," in *Wireless Communications & Signal Processing (WCSP), 2015 International Conference on*, 2015, pp. 1-5.

[46] J. M. Kahn and S. Ö. Arı, "MIMO Channel Statistics and Signal Processing in Mode-Division Multiplexing Systems," in *Signal Processing Advances in Wireless Communications (SPAWC), 2015 IEEE 16th International Workshop on*, ed, 2015, pp. 440-444.

[47] S. Liu, G. Shen, Y. Kou, and H. Tian, "Special cascade LMS equalization scheme suitable for 60-GHz RoF transmission system," *Optics Express*, vol. 24, pp. 10599-10610, 2016.

[48] A. Okamoto, K. Morita, Y. Wakayama, J. Tanaka, and K. Sato, "Mode Division multiplex communication technique based on dymnamic volume hologram and phase conjugation," vol. 7716, pp. 771627-771627-10, 2010.

[49] J. Lumeau, C. Koc, O. Mokhun, V. Smirnov, M. Lequime, and L. B. Glebov, "Single resonance monolithic Fabry-Perot filters formed by volume Bragg gratings and multilayer dielectric mirrors," *Opt Lett,* vol. 36, pp. 1773-5, May 15 2011.

[50] W. Jin and K. S. Chiang, "Mode switch based on electro-optic long-period waveguide grating in lithium niobate," *Opt Lett,* vol. 40, pp. 237-40, Jan 15 2015.

[51] J. A. Carpenter, B. C. Thomsen, and T. D. Wilkinson, "Optical vortex based mode division multiplexing over graded-index multimode fibre," in *Optical Fiber Communication Conference*, 2013, p. OTh4G. 3.

[52] M. B. Shemirani and J. M. Kahn, "Higher-order modal dispersion in graded-index multimode fiber," *Journal of Lightwave Technology,* vol. 27, pp. 5461-5468, 2009.

[53] I. Engineering, "The effects of Inter Symbol Interference ( ISI ) and FIR Pulse Shaping Filters : A survey," *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering,* vol. 3, pp. 9411-9416, 2014.

[54] H. Zhang, Y. Shi, and a. Saadat Mehr, "Robust equalisation for inter symbol interference communication channels," *IET Signal Processing,* vol. 6, p. 73, 2012.

[55] M. Cvijetic and I. B. Djordjevic, "Framework for Optimization of Characteristics of Optical Fibers Supporting Spatial Division Multiplexing," *Transparent Optical Networks (ICTON), 2014 16th International Conference on IEEE,* vol. 12, pp. 1-4, 2014.

[56] Y. P. Sree and G. V. Srıdhar, "Channel equalization of adaptive filter using LMS and RLS algorithms," *International Journal of Advance Technology in Engineering and Science,* vol. 3, pp. 71-77, 2015.

[57] J. Carpenter, B. J. Eggleton, and J. Schröder, "Observation of Eisenbud–Wigner–Smith states as principal modes in multimode fibre," *Nature Photonics,* vol. 9, pp. 751-757, 2015.

[58] T. Mori, T. Sakamoto, T. Yamamoto, and S. Tomita, "Modal dispersion compensation by using digital coherent receiver with adaptive equalization in

multi-mode fiber transmission," *Optical Fiber Technology,* vol. 19, pp. 132-138, 2013.

[59] S. Abrar, A. K. Nandi, and A. Zerguine, *Adaptive blind channel equalization*: INTECH Open Access Publisher, 2012.

[60] P. Okoniewski and J. Piskorowski, "A concept of IIR filters with time-varying coefficients and equalised group delay response," *Measurement*, vol. 60, pp. 13-24, 2015.

[61] J. Yadav, M. Kumar, R. Saxena, and a. K. Jaiswal, "P Erformance a Nalyis of Lms a Daptive Fir F Ilter a Nd Rls a Daptive Fir F Ilter," *Sipij,* vol. 4, 2013.

[62] P. Reviewed, "Electronic Thesis and Dissertations Los Angeles Utilization of Adaptive Filters for Artifact Cancellation in Electroencephalography Signals A thesis submitted in partial satisfaction of the requirements for the degree Master of Science in Electrical Engin," ed, 2015.

[63] V. Sandhiya and I. Introduction, "A survey of new reconfigurable architectures for implementing FIR filters with low complexity," in *Computer Communication and Informatics (ICCCI), 2014 International Conference on*, ed, 2014, pp. 1-9.

[64] R. C. Nongpiur, D. J. Shpak, S. Member, A. Antoniou, and L. Fellow, "Improved Design Method for Nearly Linear-Phase IIR Filters Using Constrained Optimization," *Signal Processing, IEEE Transactions on,* vol. 61, pp. 895-906, 2013.

[65] A. Islam and S. S. Nitu, "Comparing Dual Microphone System with Different Algorithms and Distances between Microphones .", 2013.

[66] J. A. Apolinário, "QRD-RLS adaptive filtering," *Springer*, pp. 1-350, 2009.

[67] A. J. Jimi, M. M. Islam, and M. F. Mridha, "A New Approach of Performance Analysis of Adaptive Filter Algorithm in Noise Elimination," *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering (IJAREEIE),* vol. 2, pp. 4571--4580, 2013.

[68] Y. Wang, X. Huang, L. Tao, J. Shi, and N. Chi, "45-Gb/s RGB-LED based WDM visible light communication system employing CAP modulation and RLS based adaptive equalization," *Optics Express,* vol. 23, p. 13626, 2015.

[69] J. Mohammed, "A study on the suitability of genetic algorithm for adaptive channel equalization," *International journal of electrical and computer engineering,* vol. 2, p. 285, 2012.

[70] Y. Zheng, M. Ouyang, L. Lu, J. Li, X. Han, and L. Xu, "On-line equalization for lithium-ion battery packs based on charging cell voltages: Part 2. Fuzzy logic equalization," *Journal of Power Sources,* vol. 247, pp. 460-466, 2014.

[71] P. Melin and O. Castillo, "A review on type-2 fuzzy logic applications in clustering, classification and pattern recognition," *Applied soft computing,* vol. 21, pp. 568-577, 2014.

[72] F. Asharif, S. Tamaki, M. R. Alsharif, and H. Ryu, "Performance Improvement of Constant Modulus Algorithm Blind Equalizer for 16 QAM Modulation," *InternationalJournal on Innovative Computing, Information and Control,* vol. 7, pp. 1377-1384, 2013.

[73] Z. Ding, "Adaptive filters for blind equalization,"" *Digital Signal Processing Handbook, VK Madisetti and D. Williams, editors,* 2000.

[74] R. Adnan, "Blind Equalization for Tomlinson-Harashima Precoded Systems," 2007.

[75] C. Papadias, "Methods For Blind Equalization and Identfication of Linear Channels," 1995.

[76] N. Bai, E. Ip, M.-j. Li, T. Wang, and G. Li, "Experimental demonstration of adaptive frequency-domain equalization for mode-division multiplexed transmission," *Journal of the American Chemical Society,* pp. 3-5, 2013.

[77] S. Randel, R. Ryf, A. Sierra, P. J. Winzer, A. H. Gnauck, C. a. Bolle*, et al.*, "6×56-Gb/s mode-division multiplexed transmission over 33-km few-mode fiber enabled by 6×6 MIMO equalization," *Optics Express,* vol. 19, p. 16697, 2011.

[78] R. Ryf, S. Randel, a. H. Gnauck, C. Bolle, R.-J. Essiambre, P. J. Winzer*, et al.*, "Space-division multiplexing over 10 km of three-mode fiber using coherent 6X6 MIMO processing," *2011 Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference,* pp. 1-3, 2011.

[79] S. O. Arik, D. Askarov, and J. M. Kahn, "MIMO DSP Complexity in Mode-Division Multiplexing," in *Optical Fiber Communication Conference*, 2015, p. Th1D. 1.

[80] A. U. Irturk, "Implementation of QR Decomposition Algorithm using FPGAs," UNIVERSITY OF CALIFORNIA, 2007.

[81] D. Rawal, C. Vijaykumar, and K. K. Arya, "QR-RLS based adaptive channel shortening IIR-TEQ for OFDM wireless LAN," *2007 3rd International Conference on Wireless Communication and Sensor Networks, WCSN,* pp. 21-26, 2007.

[82] I. Repository, "Institutional Repository A polynomial QR decomposition based turbo equalization technique for frequency selective MIMO This item was

submitted to Loughborough ’ s Institutional Repository ( https://dspace.lboro.ac.uk/) by the author and is made available " 2009.

[83] P. Xue, K. Bae, K. Kim, and H. Yang, "Progressive Equalizer Matrix Calculation using QR Decomposition in MIMO-OFDM Systems," in *Consumer Communications and Networking Conference (CCNC), 2013 IEEE*, ed, 2013, pp. 801-804.

[84] A. Ragheb, M. Shoaib, S. Alshebeili, and H. Fathallah, "Inverse QR decomposition (IQRD) blind equalizer for QAM coherent optical systems," *2012 9th International Conference on High Capacity Optical Networks and Enabling Technologies, HONET 2012*, pp. 221-225, 2012.

[85] D. Rawal and C. Vijaykumar, "QR-RLS Based Adaptive Channel TEQ for OFDM Wireless LAN," *2008 International Conference on Signal Processing, Communications and Networking*, 2008.

[86] S. Fki, M. Messai, A. A. El Bey, and T. Chonavel, "Blind equalization based on pdf distance criteria and performance analysis," 2013.

[87] N. S. Randhawa, "An Overview of Adaptive Channel Equalization Techniques and Algorithms," *signal*, vol. 8, p. 9, 2013.

[88] A. A. Elbibas, I. M. Ellabib, and Y. Hwegy, "Neuro-Fuzzy Network for Equalization of Different Channel Models," *channels*, vol. 2, p. 3.

[89] N. Surajudeen-Bakinde, X. Zhu, J. Gao, A. K. Nandi, and H. Lin, "Genetic Algorithm based Frequency Domain Equalization for DS-UWB Systems without Guard Interval," in *Communications (ICC), 2011 IEEE International Conference on*, 2011, pp. 1-5.

[90] AdibHabbal, "Tcp sintok: Transmission control protocol with delay- based loss detection contention avoidance mechanisms for mobile ad hoc networks," thesis, UUM, 2014.

[91] R. Ryf, S. Randel, A. H. Gnauck, C. Bolle, A. Sierra, S. Mumtaz*, et al.*, "Mode-division multiplexing over 96 km of few-mode fiber using coherent 6×6 MIMO processing," *Journal of Lightwave Technology*, vol. 30, pp. 521-531, 2012.

[92] D. Askarov, J. M. Kahn, and O. Sercan, "Adaptive Frequency-Domain Equalization in Mode-Division Multiplexing Systems," *Lightwave Technology, Journal of,* vol. 32, pp. 1841-1852, 2014.

[93] L. T. M. Blessing and A. Chakrabarti, *DRM: A Design Reseach Methodology*: Springer, 2009.

[94] R. Jain, *The art of computer systems performance analysis*: John Wiley & Sons, 2008.

[95] W. D. Kelton and A. M. Law, "Simulation modeling and analysis," 2000.

[96] K. Wehrle, M. Günes, and J. Gross, "Modeling and tools for network simulation," 2010.

[97] J. L. Burbank, W. Kasch, and J. Ward, *An introduction to network modeling and simulation for the practicing engineer* vol. 5: John Wiley & Sons, 2011.

[98] W. T. Kasch, J. R. Ward, and J. Andrusenko, "Wireless network modeling and simulation tools for designers and developers," *Communications magazine, IEEE,* vol. 47, pp. 120-127, 2009.

[99] N. M. C. a. J. Major, "A Comprehensive Overview on Different Network Simulators," *International Journal of Engineering Science & Technology,* vol. 5, 2013.

[100] R. D. Group, "OptSim User Guide," ed: RSoft Design Group, Inc., 2010.

[101] Optiwave, "OptiSystem User's Reference," vol. 13, ed, 2014.

[102] D. Redfern and C. Campbell, "The MATLAB® 5 Handbook," 2012.

[103] S. Vazan, "Parallel beam to beam uniformity correction," ed: Google Patents, 2004.

[104] C. Kao and P. S. J. Russell, "FIBER OPTICS," *EDN Magazine,* vol. 14, 1981.

[105] B. E. A. Saleh, *Fibre-Optics* 1991.

[106] O. Porcar Pujol, "Mode Division Multiplexing using MIMO Processing," 2012.

[107] E. E. Morales-Delgado, S. Farahi, I. N. Papadopoulos, D. Psaltis, and C. Moser, "Delivery of focused short pulses through a multimode fiber," *Optics express,* vol. 23, pp. 9109-9120, 2015.

[108] A. C. Carusone, "An equalizer adaptation algorithm to reduce jitter in binary receivers," *Circuits and Systems II: Express Briefs, IEEE Transactions on,* vol. 53, pp. 807-811, 2006.

[109] J.-y. Wang, R.-h. Lai, C.-m. Chen, P.-a. Ting, Y.-h. Huang, a. S. Qr, *et al.*, "A 2 × 2 - 8 × 8 Sorted QR Decomposition Processor for MIMO Detection," *Asscc,* pp. 5-8, 2010.

[110] C. NA, "Matrix inversion using qr decomposition by parabolic synthesis," Department of Electrical and Information Technology Faculty of Engineering, LTH, Lund University 2012.

[111] N. Hema, J. U. Kidav, and B. Lakshmi, "VLSI Architecture for Broadband MVDR Beamformer," *Indian Journal of Science and Technology,* vol. 8, 2015.

[112] A. Alaimo, V. Artale, C. Milazzo, A. Ricciardello, and L. Trefiletti, "Mathematical modeling and control of a hexacopter," in *Unmanned Aircraft Systems (ICUAS), 2013 International Conference on*, 2013, pp. 1043-1050.

[113] R. A. Panicker, J. M. Kahn, and S. P. Boyd, "Compensation of multimode fiber dispersion using adaptive optics via convex optimization," *Journal of Lightwave technology,* vol. 26, pp. 1295-1303, 2008.

[114] A. Tarighat, R. C. Hsu, A. Shah, A. H. Sayed, and B. Jalali, "Fundamentals and challenges of optical multiple-input multiple-output multimode fiber links," *IEEE communications Magazine,* vol. 45, 2007.

[115] H. Zhao, L. Zhang, B. Liu, Q. Zhang, Y. Wang, Q. Tian*, et al.*, "MIMO signal processing for mode division multiplexing with RLSCMA algorithm " *IEEE,* pp. 1-3, 2014.

[116] A. A. Juarez, C. A. Bunge, S. Warm, and K. Petermann, "Perspectives of principal mode transmission in mode-division-multiplex operation," *Optics express,* vol. 20, pp. 13810-13824, 2012.

[117] J. M. Kahn and K.-P. Ho, "Mode coupling in coherent mode-division-multiplexed systems: Impact on capacity and signal processing complexity," in *Signals, Systems and Computers (ASILOMAR), 2012 Conference Record of the Forty Sixth Asilomar Conference on*, 2012, pp. 650-653.

[118] O. Balci, "Validation, verification, and testing techniques throughout the life cycle of a simulation study," *Annals of operations research,* vol. 53, pp. 121-173, 1994.

[119] D. Zarbouti, G. Tsoulos, and D. Kaklamani, "Theory and Practice of MIMO Wireless Communication Systems," *MIMO System Technology for Wireless Communications,* p. 29, 2006.

[120] B. Miller and D. Ranum, *Problem Solving with Algorithms and Data Structures*, 2013.

[121] R. C. Pandey, "Study and Comparison of various sorting algorithms," THAPAR UNIVERSITY PATIALA, 2008.

[122] D. Harel and Y. A. Feldman, *Algorithmics: the spirit of computing*: Pearson Education, 2004.

[123] E. Horowitz, & Sahni, S., "Fundamentals of Data Structures," 1983.

[124] M. H. B. OMAR, "An Innovative Signal Detection Algorithm in Facitating The Cognitive Radio Functionality for Wireless Regional Area Networe Using Singular Value Decomposition," UUM College of Arts and Sciences, Universiti Utara Malaysia, 2011.