MODE DIVISION MULTIPLEXING ZERO FORCING EQUALISATION SCHEME USING LU FACTORIZATION

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Abstrak

Rangkaian optik dianggap sebagai rangkaian tulang belakang utama yang mengendalikan trafik internet di seluruh dunia. Pada masa kini, trafik internet telah mempunyai pertumbuhan tahunan yang besar disebabkan oleh pertambahan peranti-peranti yang terangkai. Teknologi semasa dalam rangkaian optik dipercayai tidak akan mampu untuk mengendalikan pertumbuhan ini dalam masa terdekat sehingga baru-baru ini, teknik pemultipleksan dalam komunikasi optik bergantung kepada teknik modulasi di mana polarisasi, amplitud dan frekuensi isyarat yang digunakan sebagai pembawa data yang utama. Dalam teknik ini, mod cahaya adalah dianggap sebagai kesan tidak diingini menyebabkan penyebaran mod. Sebaliknya, pemultipleksan pembahagian mod (MDM) telah diperkenalkan sebagai pendekatan pemultipleksan yang bergantung kepada penggunaan mod cahaya untuk manfaat miningkatkan produk kapasiti jarak jauh rangkaian optik.

Seperti mana-mana teknologi baru, banyak masalah menghalangnya daripada dipiawaikan dan digunakan secara komersil. Salah satu isu utama MDM adalah gandingan mod, yang merupakan fenomena inventible berlaku apabila tenaga daripada satu mod pemindahan ke satu mod yang lain semasa pembiakan mereka sepanjang gentian optik menyebabkan gangguan antara simbol (ISI) meningkatkan kadar ralat bit (BER) dan mengurangkan prestasi sistem secara keseluruhan. Skim penyamaan yang berbeza telah dicadangkan setakat ini sebagai cubaan untuk mengurangkan kesan mod gandingan kepada isyarat optik MDM. Walau bagaimanapun, mereka mengalami kerumitan perkomputan tinggi dan bergantung kepada latihan isyarat dalam menganggarkan saluran optik yang meningkatkan muatan kekal. Teknik ini bergantung terutamanya kepada algoritma min kuasa dua terkecil (LMS) dan algoritma kuasa dua terkecil rekursif (RLS). Tujuan kajian ini adalah untuk memperkenalkan penyamaan tanpa paksaan berasaskan LU untuk MDM. Kajian sebelum ini dalam domain radio pada pelbagai input pelbagai output (MIMO) dan pemultipleksan pembahagian frekuensi ortogon (OFDM) menunjukkan bahawa skema-skema tanpa paksaan mempunyai kerumitan perkomputan yang rendah berbanding dengan skema-skema semasa kerana mereka menyamakan isyarat tanpa isyarat latihan, sekali gus mengurangkan muatan kekal. Semua idea terdahulu memberi motivasi kepada kajian ini untuk menyesuaikan diri dengan pendekatan ini dalam komunikasi optik.

Kajian ini menggunakan Metodologi Penyelidikan Reka bentuk (DRM) empat-peringkat. Data awal dikumpulkan dari simulator optik, diproses dan digunakan untuk mendapatkan fungsi pemindahan (H) daripada sistem. Kemudian ia digunakan untuk membangunkan skema penyamaan dalam MATLAB. Eksperimentasi pada penyamaan tanpa paksaan berasaskan LU menunjukkan kerumitan O(N) yang lebih rendah daripada RLS yang mempunyai O(N2) dan lebih cepat daripada LMS kerana LMS memerlukan purata 0.0126 saat untuk memproses manakala isyarat tanpa paksaan berasaskan LU memerlukan 0.0029 saat sahaja. Sebaliknya, penyamaan yang dicadangkan mengurangkan kelewatan penyebaran masa saluran, menyebabkan tiga kali kenaikan dalam kapasiti saluran MDM dan kerumitan pengiraan lebih rendah. Sumbangan utama kajian ini adalah pengurangan kerumitan pengiraan skema penyamaan di MDM. Penggunaan skema ini dalam sistem MDM sebenar boleh menghasilkan kos lebih efektif dan pemprosesan isyarat digital (DSP) yang lebih kecil untuk MDM dan boleh mempercepatkan kerja pada

penyeragaman MDM untuk digunakan secara komersial sebagai teknik pemultipleksan untuk rangkaian komunikasi optik.

Abstract

Optical networks is considered as the main backbone networks that handled the Internet traffic worldwide. Currently, the Internet traffic has had huge annual growth due to the increment in connected devices. At this rate, it is believed that the current technology in optical network will not able to handle this growth in the near future. Till recently, multiplexing techniques in the optical communication rely on modulation techniques where polarization, amplitude and frequency of the signal are used as the main data carrier. In these techniques, light modes are considered as an undesired effect causing modal dispersion. In contrast, mode division multiplexing (MDM) was introduced as a multiplexing approach which relies on the utilization of the light modes for the benefit of increasing the capacity-distance product of the optical network.

As per any new technology, it is still facing a lot of problems preventing it from being commercially standardized and used. One of the main MDM issues is the mode coupling, which is an inventible phenomena occurs when the energy of one mode transfers to another mode during their propagation throughout the optical fibre causes inter-symbol interference (ISI), increasing the bit error rate (BER) and reducing the overall system performance. Different equalization schemes have been proposed so far attempting to mitigate the effect of mode coupling on the MDM optical signal. However, they suffer from high computational complexity and rely on training signals in estimating the optical channel which increases the overhead payload. These technique mainly rely on Least Mean Squared (LMS) and Recursive Least Squared (RLS) algorithms. The purpose of this study is to introduce a Zero Forcing LU-based equalization scheme for MDM. Previous research in the radio domain on multiple-input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) demonstrated that zero forcing schemes have low computational complexity compared to current schemes as they equalize the signal without training signals, thus reducing the overhead payload. All of the previous points motivate the work of this study to adapt this approach in optical communications.

The study adopts the four stages of the Design Research Methodology (DRM). The initial data was collected from the optical simulator, processed and used to derive the transfer function (H) of the system. Then it was used to develop the equalization scheme in MATLAB. The experimentation on Zero Forcing LU based equalization scheme shows $O(N)$ complexity which is lower than RLS which has $O(N^2)$ and faster than LMS, in fact, LMS needs an average of 0.0126 seconds to process the signal while ZF LU-based needs 0.0029 seconds only. On the other hand, the proposed equalization reduces the time delay spread of the channel, resulting three times increment in the capacity of the MDM channel and even lower computational complexity. The main contribution of this study is the reduction of the computational complexity of the previous equalization schemes in MDM. Applying this scheme in real MDM systems can produce more cost effective and smaller digital signal processing (DSP) parts for MDM equipment and can accelerate the work on the standardization of MDM for being commercially used as a multiplexing technique for optical communication networks.

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Table of Contents

List of Figures

List of Tables

Chapter One: Introduction

1.1 Research Overview

Two decades since the appearance of the modern commercial Internet in the mid of 1990s, and the worldwide network traffic has increased by an estimated rate exceeding 50% each year [1]. This dramatic growth comes from the appearance of high-definition video streaming, cloud computing, mobile networking, mobile gaming and many other web 2.0 applications. The forecast of the global traffic growth rate for the coming five years is shown in Figure 1 below [2].

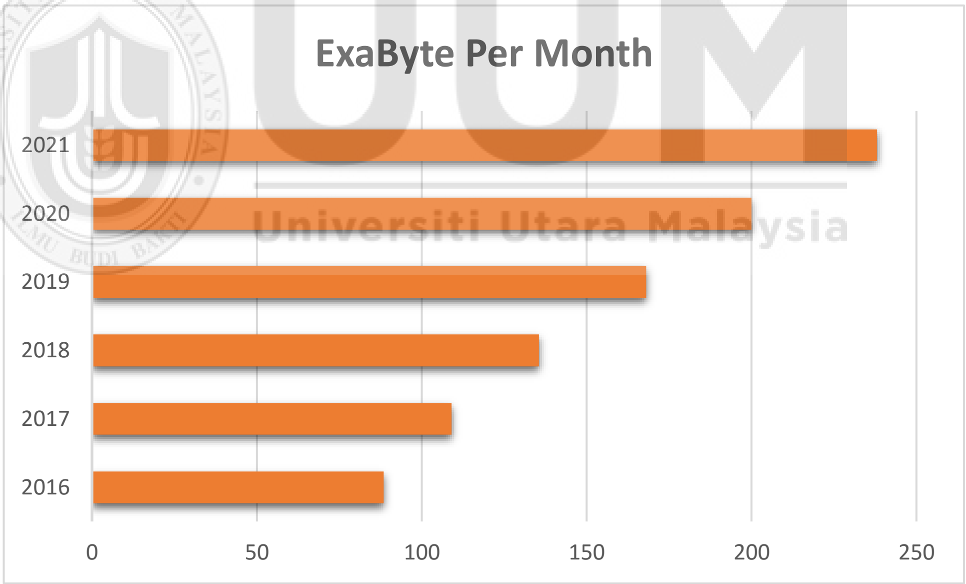


Figure 1 Global Internet Traffic Forecast [2]

To accommodate the bandwidth growth in optical fibre and future proof next generation network infrastructure, researchers are looking for new strategies for multiplexing data through a single optical fibre. Multiplexing schemes decrease the data transmission cost through the network and is a key factor to ensure that technology is able to handle the growth in information technologies [1]. To date, multiplexing in time or time division multiplexing (TDM), polarization or polarization division multiplexing (PDM), wavelength or wave division multiplexing (WDM) and phase dimensions have been explored. In fact, most commercial systems are currently using all of these four dimensions to increase their capacity range [3, 4].

Despite these attempts, the transmission capacity of optical fibre is now approaching its theoretical limits due linear and non-linear fibre effects [4]. In order to maintain the huge demands of high data rate transmission ways, a new multiplexing dimension is now required to overcome these limits [4].

Recently, the introduction of the mode dimension through the mode division multiplexing in multimode fibre has emerged as a promising approach that can double the current capacity records by providing new spatially parallel channels for transmitting independent signals [3]. As shown in Figure 2, using mode division multiplexing can provide dramatic increase in the transmission capacity [5].

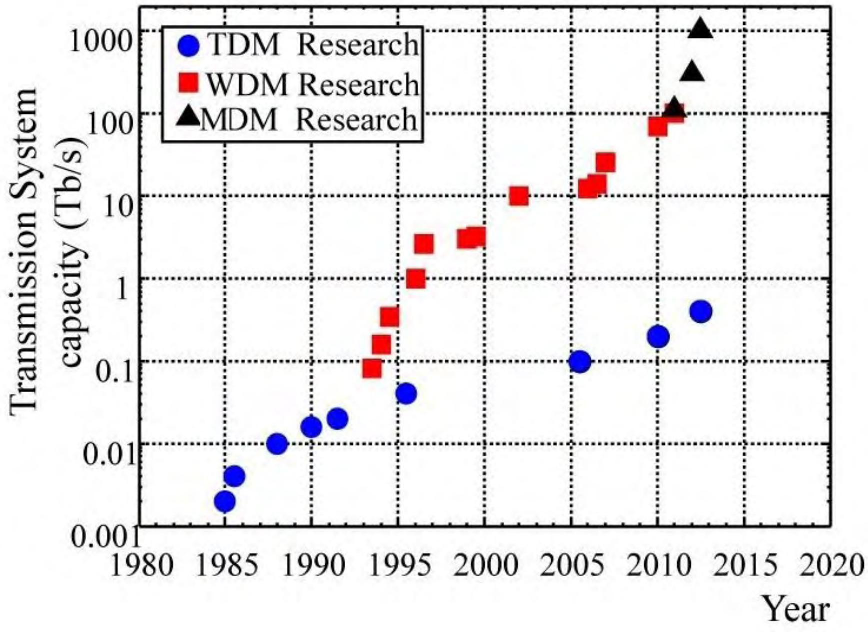


Figure 2 Optical Network Transmission Capacity Per Approach [5]

Mode division multiplexing is realized by utilizing the propagating modes in multimode fibres [6]. An optical mode is defined as a pattern of the optical electric field that propagates without changing, apart from an amplitude change and phase shifts. Different optical modes are mutually orthogonal, so they provide independent spatial dimensions for data transmission [1, 6].

One of the major issues that limit the bandwidth performance of mode division multiplexing is mode coupling which occurs between different modes propagating in long-distance optical fibre links [6]. Many techniques have been explored during the last period trying to compensate for the mode coupling effect and successfully separate the signals.

This study will focus on compensation of mode coupling by filtering the channel impulse response of the system. Generally, performing this action on this signal is known as equalization [7]. Many mode division multiplexing equalization techniques have been developed, mostly based on tapped filters in the time domain [8, 9] and frequency domain [10-12]. This study undertakes a concerted effort to model and develop a new mode division multiplexing equalization technique based on LU factorization to increase the overall system bandwidth-distance product and improve the channel impulse response of the system by reducing the time delay spread of the channel.

1.2 Important Terminologies

The main goal of this study is to mitigate the mode coupling effect in multimode fibres by proposing an equalization technique for mode division multiplexing. This is expected to improve the channel impulse response and enhancing the channel time delay spread of the mode division multiplexing transmission system. The next section will define some essential technologies related to the domain of this study.

1.2.1 Multi-Mode and Single-Mode Fibres

Multimode Fibre is a type of optical fibre manufactured with a larger core diameter compared with the single-mode one [13]. It had been widely used in data centers and local networks [14]. Due to this larger core diameter, an input pulse of light will excite many modes to propagate inside the fibre. Usually these modes arrives as several pulses at the output with different speeds caused by a phenomena known as modal dispersion [15]. Due to this limitation, until recently, multimode fibres are being used for short-haul applications [16]. This is in contrast with single mode fibre which does not suffer from this phenomena, thus, making it the dominant choice for long-haul applications [17].

However, single mode fibre is reaching its theoretical capacity limits. This has motivated researchers to look for new multiplexing approaches for increasing the bandwidth [11].

1.2.2 Mode Division Multiplexing

Mode division multiplexing is an attractive strategy for increasing the capacity of optical fibre links by utilizing light modes propagating inside a multimode fibre [8]. Theoretically, light modes are orthogonal to each other, thus, it is possible to consider each one of these modes as an independent data channel [18]. Long distance propagation of optical modes in multimode fibre causes mode coupling between the different modes, leading to crosstalk and interference [6]. Signal processing techniques can be used to separate the coupled channels [3]. The mode division multiplexing optical transmission system is shown in Figure 4 [6].

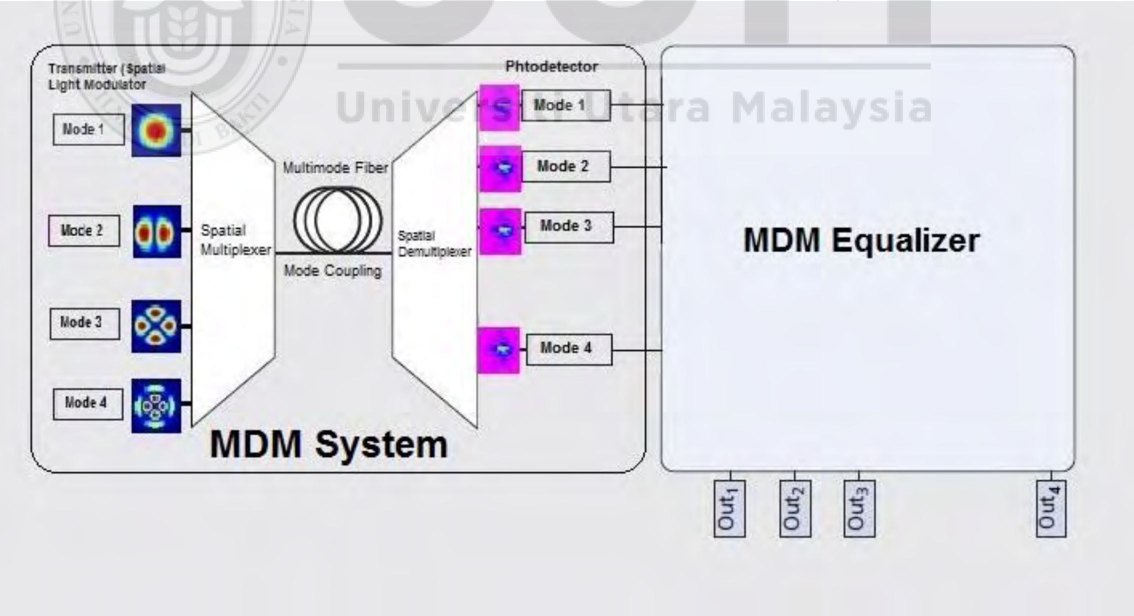


Figure 3 Mode Division Multiplexing Optical Transmission System [6]

1.2.3 Channel Equalization

Noise from mode coupling occurs when modes propagate for long distances through multimode fibre, resulting in crosstalk and interference [19]. Equalization techniques are required for compensating for mode coupling noise at the receiver side, to allow spatial modes to be uncoupled in order to retrieve their initial transmitted channel [8]. Mode division multiplexing equalizers based on adaptive and non-adaptive filters and algorithms are being widely studied for the sake of mode coupling compensation, which is the main purpose of this research and will be explored in more detail in Chapter Two.

1.2.4 Zero Forcing Equalization

Zero Forcing Equalizer is an equalization algorithm used in communication systems, which inverts the frequency response of the channel. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel [20]. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case [20, 21].

1.2.5 LU Factorization

LU Factorization is a mathematical method mainly used to solve simultaneous system equations. Computers usually solve square systems of linear equations by depending on LU factorization. Thus, it is also a key method of finding the inverse of a given matrices or computing its determinant [22]. A technique based on this mathematical method will be used in developing the mode division multiplexing equalizer of this research.

1.3 Research Motivation

Due to the dramatic increment of network traffic passes through the optical fibre backbones [23], it is essential to explore mode division multiplexing as a new multiplexing paradigm that will increase the aggregate bandwidth of optical systems in order to allow future traffic growth [23]. Intensive research has been undertaken worldwide to enhance and standardize mode division multiplexing so that it may be commercially deployed in future infrastructure for ultra-high capacities. It is expected that the work on mode division multiplexing will satisfy future communication industry needs that single mode fibres are incapable of due to nonlinear effects [24].

Many mode division multiplexing hardware enablers like mode multiplexer [25], spatial photodetectors [26], photonic crystal lanterns [27], and spatial light modulators [28, 29] have been investigated but thus far, most of them are not in their final shape and suffer from modal crosstalk due to mode coupling [30]. To mitigate these limitations, a new equalization technique will be proposed.

1.4 Problem Statement

Multimode fibres have not been deployed in long-distance applications due to inter-modal dispersion and mode coupling [1, 31]. To mitigate these limitations, mode division multiplexing has been proposed as a new multiplexing approach to enhance the distance-bandwidth capacity product in Multimode fibres [1, 32]. Mode division multiplexing is realized for increasing system capacity by utilizing each optical mode as an individual channel capable of transmitting digital data, thus providing parallel transmission of data channels [1, 33].

As previously mentioned, many mode division multiplexing hardware enablers such as mode couplers [25, 34], spatial photodetectors [26], photonic crystal lanterns [27] and spatial light modulators [28, 29] have been studied. However, thus far, most of them are still not in their final shape and suffer from modal crosstalk due to mode coupling [18, 30, 35, 36]. Mode coupling is an inevitable physical effect in multimode fibres due to fibre imperfections occurring during the manufacturing process [12, 30, 37]. As a result of mode coupling, particular light modes exchange their powers with each other in a random way through their propagating inside the fibre, causing inter-symbol-interference (ISI) between the data symbols and increases the time delay spread in the channel impulse response pulse and reducing the bandwidth of the system. [18, 30, 38, 39].

Equalization is one of the methods used in mitigating ISI [7]. Many mode division multiplexing equalization techniques have been developed, mostly based on tapped filters in the time domain [8, 9] and frequency domain [10-12]. However, the versatility of these equalization techniques are still limited by the algorithms used. Current techniques are based on algorithms such as Least Mean Squares (LMS) [8, 10-12] and Recursive Least Squares (RLS) [10, 11]. Most of these algorithms suffer from high calculation complexity which lead to increase processing time and affect the system performance [8-12, 40].

LMS has only one user-adjustable parameter which is the step size [41]. Putting small step size ensures the algorithm convergence and stability but slow down the convergence time, on the other hand, putting large step size will increase the convergence time but the algorithm will be unstable and might not converge properly [11]. RLS converges faster than LMS as it works “recursively’ by depending on the data coming from the current and the previous samples, however, this increases the computational complexity of the algorithm [11].

1.5 Research Questions

This research strives to answer the following research questions:

- How to derive the channel transfer function of the MDM system to provide the receiver with the needed equalization base?
- How to develop low complexity equalization scheme based on Zero Forcing (ZF) in mode division multiplexing by using LU factorization?
- Will the performance of the MDM system be enhanced after using the Zero Forcing (ZF) equalization scheme based on LU factorization?

1.6 Research Objectives

This research embarks on the following research objectives:

- To derive the channel transfer function of the MDM system to provide the receiver with the needed equalization base.
- To develop a Zero Forcing (ZF) based equalization scheme in mode division multiplexing by using LU factorization to reduce the calculations complexity in restoring the signal.
- To evaluate the performance of the ZF equalization scheme based on LU factorization.

1.7 Research Scope

This research is intended to introduce a new LU equalization technique for mode division multiplexing at the physical layer of the OSI layers based on Zero Forcing equalization model in order to enhance the system’s channel impulse response and reducing the time delay spread of the channel.

1.8 Significance of Research

The new LU equalization technique is expected to compensate for the detrimental effects of mode coupling effect which is the bottleneck for capacity gains in other multimode fibre multiplexing systems. This is anticipated to drive mode division multiplexing as the next multiplexing dimension for long-distance applications in future optical networks. In addition, devices with low complexity needs less energy for processing the data, thus, the manufacturing of smaller and more cost effect equipment can be achieved.

Chapter Two: Literature Review

Utilizing the mode dimension, termed mode division multiplexing, is a new promising approach to achieve ultra-high transmission capacities critical for satisfying optical communication industry demands [42].

This chapter discusses the physical limitations of mode division multiplexing in Section 2.1 which leads to the need for equalization schemes in mode division multiplexing to compensate for these physical limitations. The role of equalization is discussed in Section 2.2, followed by discussions on equalization filters in Section 2.3 and equalization algorithms in Section 2.4. Section 2.5 provides a critical review of previous equalization schemes in mode division multiplexing systems and previous LU factorization equalization schemes in radio OFDM systems.

2.1 Mode Division Multiplexing Limitations

Current mode division multiplexing research attempts to control of the propagation of modes inside optical fibre to enhance the transmission performance [43]. Nonetheless, both multimode fibres and few-mode fibres suffer from mode coupling and modal dispersion, which are major bandwidth limitation factors in multimode fibres and FMFs in mode division multiplexing [19]. To further understand the cause of these bandwidth limitations, modal coupling and modal dispersion will be briefly explained.

2.1.1 Mode Coupling

Mode coupling is an inevitable physical phenomenon arising from multimode fibre manufacturing defects such as microbends and optical fibre cross section ellipticity [18, 35, 36]. Mode coupling causes particular modes to exchange the power randomly through their propagating within the fibre, leading to inter-symbol interference (ISI) between symbols carried by these particular modes causing higher impulse response width and longer time delay spread, and reducing the capacity-distance product of the system [18, 30, 38, 39]. Because of mode coupling, even when a light pulse is launched at a multimode fibre input as a single spatial mode, it propagates through a multimode fibre as a superposition of many spatial modes. Mode coupling must either be avoided by careful design of all these components, or mitigated by adaptive optical signal processing [44]. The mode coupling effect, its impact on the channel impulse response, time delay spread and how can equalization mitigate these problems are all shown in Figure 5 [45].

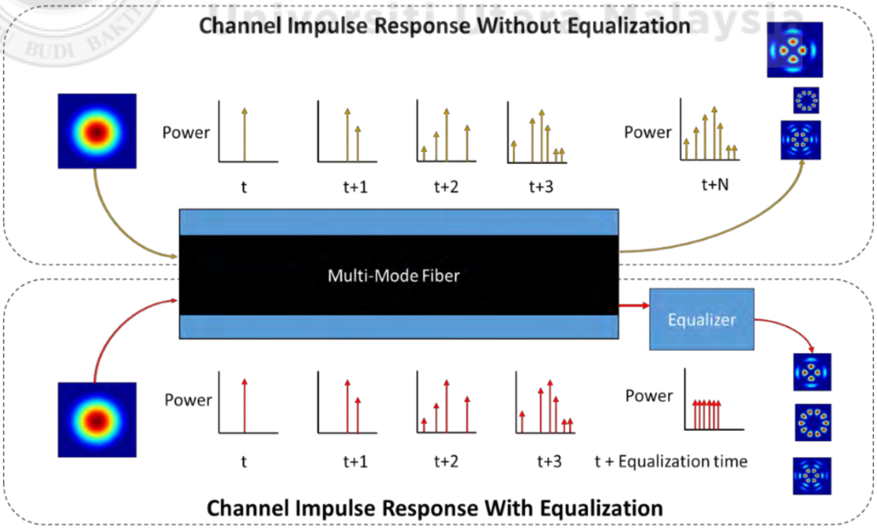


Figure 4 Mode Coupling Effect and its Effect on Channel Impulse Response with and without Equalization [45]

2.1.2 Modal Dispersion

On the other hand, *modal dispersion* is caused by the spread in the pulse caused by the different speeds at which the modes propagate, causing them to reach the fibre with different time delay [14].

As a result, at the multimode fibre output, the light pulse will spread in time and prohibit high data rates as the pulses will overlap due to inter-symbol interference [30, 46].

To mitigate modal dispersion and mode coupling in multimode fibres and few-mode fibres, equalization techniques may be used. This will be discussed in more details in the upcoming section [19].

2.2 Role of Equalization in Compensating Physical Limitations in Mode Division Multiplexing

To compensate the effects of modal dispersion and mode coupling, equalization is crucial. Channel equalizers can be considered as a promising approach that can be used to mitigate mode coupling effects [6].

Generally, an equalizer consists of a filter which is the part responsible for mitigating the unwanted effects in an effort to restore the signal to its original transmitted shape [47], and an adaptive algorithm which is needed to update the filter parameters in order to reflect the changes which occur in the time-varying channel of the optical fibre cable [47].

However, in mode division multiplexing cases where many modes (data streams) are propagating at the same time within the fibre, the equalizer complexity will increase with the increment of these propagating modes [48]. Current researchers are seeking to find an equalization technique with less complexity [49, 50].

The upcoming sections will provide details of current equalization filter structures and will highlight the possible techniques to decrease the equalizer complexity.

2.3 Equalization Filters

The first part of a mode division multiplexing equalizer is the adaptive equalization filter. The main feature of this filter is its capability of self-adjusting its transfer function, giving it the ability to self-learn and change filter parameters to adapt to different signal characteristics.

Adaptive filters are used to obtain the desired signal without any undesired noise by reducing the noise that is corrupting the signal. The process is shown in Figure 6 below [51] . There are two inputs for the adaptive filter: $x(n)$ and $d(n)$. The input $x(n)$ is the reference signal or the input signal before equalization, which corresponds to the undesired noise that needs to be filtered out. This reference signal is not exactly the same as the noise corrupting the signal, so it cannot just be removed from the corrupted signal to get the desired signal. The reference signal can have a different phase, amplitude, or time delay than the actual noise signal. The input $d(n)$ is the primary input, which corresponds to the desired signal or the desired impulse response plus the undesired noise.

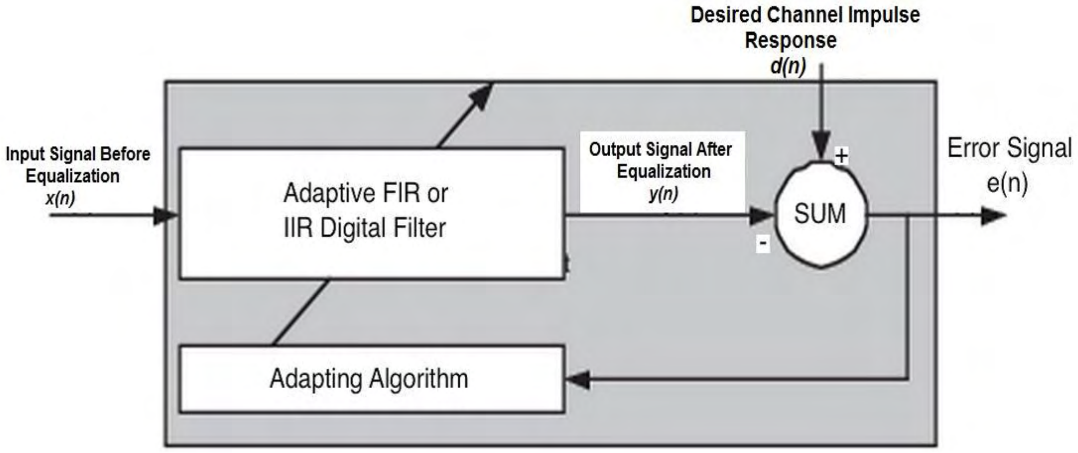


Figure 5 Adaptive Equalization Filter [51]

Instead of just subtracting the reference signal from the corrupted signal, the adaptive equalization filter predicts the noise that is present in the corrupted signal and then subtracts it. The reference signal $x(n)$ is used to help make the prediction because it is chosen to be similar to the noise that is corrupting the signal. Also, the reference signal $x(n)$ is filtered in order to account for the phase, amplitude, and time delay. Filtering the reference signal results in $y(\mathrm{n})$, which is the prediction of the noise that is corrupting the signal. Once the reference signal is filtered, it is subtracted from the corrupted signal in order to obtain $e(n)$, which is the error signal. The error signal $e(n)$, which is the difference between $d(n)$ and $y(\mathrm{n})$, is the output of the system. If done correctly, this can be used to recover the desired part of the signal which is the desired impulse response from the undesired noise by filtering it out [52].

An adaptive equalization filters may be broadly classified into two types, Finite Impulse Response (FIR) equalization filter and Infinite Impulse Response (IIR) equalization filter

FIR digital filters find wide uses in digital communication systems for uses such as channel equalization, matched filtering, pulse shaping and channelization because of their high stability and linear phase characteristics. It is preferred that the filters used in digital communication systems to consume less power and operate at high speed [53].

On the other hand, the IIR filters achieve the needed filter specification with lower filter order, this will reduce the computational needs and/or the hardware complexity, also, they have smaller group delay in compared with FIR [54].

2.4 Equalization Algorithms

Many equalization algorithms has been developed for the sake of ISI compensation in wireless and optical domains [20]. Based on the way of dealing with the received signal, these algorithms can be categorized into three main categories, Minimum Mean Square Error (MMSE) or the adaptive equalization algorithms, Blind equalization algorithms and Zero Forcing (ZF) equalization algorithms [20, 55]

2.4.1 Adaptive Equalization Algorithms

When an adaptive filter is used for channel equalization, the relationship between the input and output signals of a filter is iteratively modeled. In this process, the filter coefficients of the adaptive filter are self-adjusted using an adaptive algorithm. The coefficients are adjusted in order to minimize the power of $e(n)$. Multiple adaptive algorithms were researched and tested, and the two most used adaptive algorithms are the Least Mean Squares (LMS) algorithm [8, 10-12]and the Recursive Least Squares (RLS) algorithm [10, 11]. The LMS algorithm involves less computationally demanding calculations and is less complex than the RLS algorithm, thus causing the LMS algorithm to needless memory and computational resources than the RLS algorithm [56].

However, the correlation matrix of the input signal for the LMS has a large eigenvalue spread, which can degrade the convergence of the adaptive filter. The RLS algorithm does not have the large eigenvalue spread problem, potentially allowing for a faster convergence speed. The RLS algorithm is more complex than the LMS algorithm but has the potential to perform better in time varying environments [57].

One of the popular equalization adaptive update algorithm is the *least mean square* (LMS) algorithm which is currently being used in many equalization application for both optical and radio domains. The premise of LMS algorithms is that it estimates the gradient vector in the steepest descent algorithm and uses them to update the weights. The goal of LMS algorithm is to minimize the mean square error [51].

The largest advantage of LMS is its low computation and cost. However, it has only one adjustable variable, step size, which is employed to control the convergence rate, and the convergence rate is slower compared to RLS. RLS is more complicated in computation in LMS. But it can achieve better convergence speed [51].

The LMS algorithm converges on the optimal solution by updating coefficients, also known as filter weights. This optimal solution occurs when the error signal between the desired signal and the output signal is minimized. These coefficients are updated in each iteration in order to get to the optimal channel impulse response. To update the coefficients, the LMS algorithm performs three main steps, which include calculating $y(n)$

from the adaptive filter, calculating *e(n)*, and then finally updating the filter coefficients [56, 58].

The largest advantage of LMS is its low computation and cost which can reduce the complexity of the mode division multiplexing equalizer. However, the convergence rate is slower compared to RLS. RLS is more complicated in computation in LMS. But it can achieve better convergence speed [51].

As with the LMS algorithm, the RLS algorithm uses some of the fundamental adaptive filter concepts. The RLS algorithm converges on the optimal solution by updating coefficients for each iteration of the algorithm until the optimal solution is reached. Similarly, in order to update the coefficients, the RLS algorithm performs three main steps, which include calculating *y(n)* from the adaptive filter, calculating *e(n)*, and then finally updating the filter coefficients. Although the main overall process is similar for the LMS and RLS algorithms, the method used to update the coefficients are significantly different [57, 59].

Due to the need for storing current and previous values, more memory is required when using the RLS algorithm. However, the RLS algorithm has a significantly better convergence time compared to the convergence time of the LMS which make it better in equalizing the expected high number of modes in an mode division multiplexing system, however, the algorithm is considered to be complicated and many recent and previous studies are working to reduce its complexity for better utilization in the mode division multiplexing equalization field. [11].

2.4.2 Blind Equalization Algorithm

When a training sequence is not available, probabilistic and statistic properties of the transmitted data can be exploited to adapt the equalizer coefficients. This is known as blind equalization[20]

The exclusion of a training sequence does not come without a price. Apart from being generally more complex, the two main disadvantages with blind equalization compared to trained equalization are slow convergence and possible convergence to local minima [20].

2.4.3 Zero Forcing Equalization Algorithm

As mentioned before, Zero Forcing Equalizer (ZFE) is an equalization algorithm used in digital communication system. This equalizer works by applying the effect of channel inverse on the received signal in order mitigate the channel's impact on the transmitted signal[20]. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case [20, 21]. This will be useful when ISI is significant compared to noise as in the case of an optical channel. For a channel with transfer function H, the zero forcing equalizer W is constructed such that $W = 1/H$. Thus the combination of channel and equalizer gives a flat frequency response and linear phase $H \times W = 1$. With this, the input signal is multiplied by the reciprocal of H. This is intended to remove the effect of channel from the received signal, in particular the ISI [20].

2.5 Discussion on Equalization Schemes

In addition to the details mentioned previously in this chapter regarding mode division multiplexing equalization and its mechanism, the next section will highlight the current approaches in contact with mode division multiplexing equalization in the optical domain. It will also stipulate previous similar work done in the radio domain to provide the needed comparison between the two domains when it comes to channel equalization.

2.5.1 Equalization Schemes in Mode Division Multiplexing

Having a comprehensive literature review of the developments in mode division multiplexing equalization field is insightful for understanding the different configurations and functionalities. It will also highlight the advantages and disadvantages of previous equalization techniques in order to have a complete array of techniques to choose for adoption and to understand which aspects to avoid.

To achieve the goal of this study, many recent articles working on mode division multiplexing equalization had been studied.

In [10], a 6X6 mode division multiplexing equalizer has been introduced. It shows that with precise choosing of the data training, it is possible to reduce the algorithm complexity and enhance the performance of the system. It also has a comparison between the TDE and FDE and mentioning the main advantages and disadvantages of each one. Mode division multiplexing FDE performs better due to its capability of reducing the complexity and handling large number of data streams (modes). It is also shows that using

LMS with FDE can improve performance, however, LMS is still considered to have poor performance but less complexity comparing with RLS algorithm.

In [12], the author was able to successfully compensate the mode coupling effect by using 3X3 mode division multiplexing equalizer. It shows that the system performance is directly affected by the light mode type and order being propagated inside the fibre. It also shows that the used equalizer is performing well under weak mode coupling regime but it has poor performance under strong regime.

In [11], the author compares the performance of both LMS and RLS in mode division multiplexing equalization. Both of the two algorithms have advantage and disadvantages that already mentioned before. LMS have less complexity but slow and unstable. RLS is more stable and faster algorithms but with significantly higher complexity. The work in this article was based on having different scenarios, each scenario has different number of propagating modes (6, 12, 20 and 30 modes in each scenario) and different type of mode division multiplexing equalizer to match the difference in the number of the propagating modes in each scenario.

In [8], a performance test to LMS algorithm was taken place. It still shows poor performance and instability. However, the authors were able to improve the performance of LMS by carefully determining the training data sequence.

In [9], the article shows that choosing the number of taps (iterations) for the equalization filter plays a role in reducing the complexity even if the filter is used in the complicated

TDE, however, having high number of filter taps is not recommended as it may increase the complexity, so choosing them should be in a very careful way.

In [40], the author shows the combination of mode division multiplexing hardware enablers and software algorithms can reduce the system complexity. In the case of this article, the author used a combination of SLM and mode division multiplexing equalization filter and proved that the combination reduces the overall system complexity. However, the used technique performs well under low number of excited modes only.

Table 1 below summarizes the above literature in terms of the equalization filter, equalization algorithm and the main findings.

Table 1 Summary of Comparison of Recent Research Conducted with Mode Division Multiplexing Equalization.

No.	Equalization Filter	Equalization Algorithm	Criticism	Author and year
1	FIR equalization filter	LMS Equalization algorithm	LMS algorithm slow and unstable.	N. Bai, E. Ip, M.-j. Li, T. Wang, and G. Li, 2013

2	FIR Equalization filter	LMS, RLS, CMA and a combination between RLS and CMA (RLSCMA)	The used technique of RLSCMA is good under weak coupling regime but not under the strong one.	H. Zhao, L. Zhang, B. Liu, Q. Zhang, Y. Wang, Q. Tian, *et al.*, 2014
3	FIR Equalization Filter	Compare the performance of RLS and LMS	Both of RLS and LMS have limitations in speed and complexity	J. M. Kahn and S. Ö. Arı, 2015
4	FIR Equalization Filter	LMS Algorithm with in the time domain.	Proper data initialization can improve the performance even when poor algorithm like LMS is used.	S. Randel, R. Ryf, A. Sierra, P. J. Winzer, A. H. Gnauck, C. a. Bolle, *et al.*, 2011

5	FIR Equalization filter	LMS Algorithm with in the time domain.	-Filter taps number is an essential parameter that can enhance the performance -High taps number increase complexity.	R. Ryf, S. Randel, a. H. Gnauck, C. Bolle, R.-J. Essiambre, P. J. Winzer, *et al.*, 2011
6	FIR equalization Filter in combination with SLM	CMA Equalization Algorithm	The combination between the two techniques mentioned in the objectives reduced the equalizer complexity.	K. Shi, G. Gordon, M. Paskov, J. Carpenter, T. D. Wilkinson, and B. C. Thomsen, 2013

The studies mentioned in the above table mainly depends on using LMS and RLS as the adaptive algorithms to update the coefficients of the FIR or IIR filters so these filter are able to equalize the optical channel.

Mostly, all of these ways have disadvantages in terms of system complexity or convergence speed. On the other hand, none of them tries to use the direct utilization of LU factorization in the channel equalization. In fact, matrix factorization and inversion methods like LU or QR factorization had been used in the radio domain and proved to be a good solution for system complexity and speed when they are used in combination with the other algorithms or standalone [60-63].

The next section will highlight the use of such techniques in the radio domain.

2.5.2 Equalization Schemes Based on LU Factorization in Orthogonal Frequency Division Multiplexing

Exploring the development of equalization schemes based on LU factorization in OFDM system in radio communications will provide researchers with a critical overview of possible configurations used for implementation and open issues. Understanding these components will bridge the researchers' knowledge on the possible combinations and parameters, allowing for the most appropriate configuration for adaption and particular aspects to avoid.

In [62], a comparison of the performance of IIR and FIR equalization filters with LMS and RLS algorithms in the radio domain is taken place. Results shows that integrating matrix factorization methods like LU or QR factorization can help to significantly reduce the system complexity with keeping the needed performance. It also shows that using IIR equalization filter with QR-RLS algorithm gives good results and better than the other scenarios.

In [60], a test of directly implementing matrix factorization method as an independent equalizer is taken place. QR factorization was used without an FIR or IIR to equalize the radio channel. In fact, this way performs well and better than the other ordinary ways of having an adaptive equalization filter with adaptive equalization algorithm like RLS or LMS.

In [63], matrix factorization methods were used to enhance the performance of the adaptive equalization algorithms. It shows that performing the matrix inversion progressively on the training stream by using QR factorization (symbol by symbol) instead of waiting for the whole stream can enhance the system complexity up to 75% and without losing the overall performance.

In [61], a comparison of the equalization techniques in the time domain has taken place. It shows that using matrix factorization like LU or QR in combination with RLS can reduce the system complexity.

[64] mentions that using Zero Forcing algorithm shows low computational complexity in the equalization part. It was used to equalize different wireless channels with different capacities and modulations.

On the other hand, [21] shows that Zero Forcing algorithm is very efficient in mitigating the ISI in a wireless channel. These findings were based on the comparison of the channel behavior before and after adding the ZFE to the system.

Table 2 summarizes the above equalization schemes in terms of the equalization filter, equalization algorithm and main findings.

Table 2 Summary of Comparison of Previous Research Conducted on Equalization Schemes Based on LU Factorization in OFDM.

No.	Equalization Filter	Equalization Algorithm	Findings	Author and year
1	IIR Equalization filter	QR-RLS in TDE	QR-RLS -IIR gives better BER performance than LMS based FIR filter.	D. Rawal, C. Vijaykumar, and K. K. Arya, 2007

2	Direct Implementation of QR factorization as a filter	QR factorization	BER performance is significantly reduced compared with other equalization methods like FIR based LMS filter	M. a. L. Davies, Sangarapillai and Foster, Joanne and Chambers, Jonathon and McWhirter, John, 2009
3	FIR Equalization Filter	QR factorization as a matrix inversion method	Using this method was significantly reduces the computational complexity on the last training symbol by up to 75% without any BER performance degradation.	Xue, K. Bae, K. Kim, and H. Yang, 2013
4	FIR Filter	LMS, RLS and QR-RLS	The QR-RLS method gives better BER performance than LMS based FIR filter within almost the same computational complexity.	D. Rawal and C. Vijaykumar, 2008

5	Zero Forcing Equalizer	ZFE with Automatic Repeat Request (ARQ) and Cyclic Redundancy Check (CRC)	The ZFE shows low complexity in equalizing different wireless channels with different capacities and modulations.	Giwan Choi, Wei Zhang and Xiaoli Ma, 2012
6	Zero Forcing Equalizer	ZF Algorithm	The ZFE was able to mitigate the ISI of the wireless channel under observation, the results shows decrement in the BER after adding the equalizer to the system.	Pragya Sharma, 2012

The comprehensive study on both multi-channel systems in optical and radio channels reflects the similarity between the two domains in terms of multipath fading and the distortion of the received signal due to ISI. Thus, the mechanics of equalization in radio domain is expected to work in the optical domain if properly configured.

Despite the advantages of LU, QR and ZF equalization for OFDM systems in radio communications, their prowess in ISI compensation has not been harnessed for optical mode division multiplexing transmission. This provides a fertile ground for further investigation of LU equalization and their adaptation for optical mode division multiplexing systems. Equalizers based on Zero Forcing [21, 64], LU or QR factorization have been proven to perform rapidly [65] and at a reduced computational complexity [65]. On the other hand, ZF equalizers does not need any training signals which will reduce the overhead payload [21]. The merits of LU, QR factorization and ZF equalizers and the similar multi-path nature of both OFDM and mode division multiplexing systems are strong motivating factors for further investigation on these algorithms for adaption in optical mode division multiplexing systems.

2.6 Summary

Recent work on mode division multiplexing equalization techniques and previous radio equalization techniques have been presented. Current mode division multiplexing equalization techniques based on RLS and LMS algorithms shows high complexity or slow performance which may not be sufficiently accurate for mitigating mode coupling.

On the other hand, previous research combining LU and QR factorization with RLS and the other adaptive algorithm and the use of ZFE in the radio domain is shown to reduce the complexity and increase the performance speed. Based on this, this research aims to adopt these LU factorization techniques to be utilized in the mode division multiplexing equalization domain.

Chapter Three: Research Methodology

The goal of this study is to propose a Zero Forcing LU-based equalization model capable of compensate the modal dispersion and mode coupling effect in mode division multiplexing. LU-based equalizers are considered to be a promising technique that can ensure better reduction of mode coupling, impulse response width and time delay spread in mode division multiplexing system.

Achieving these objectives requires a rigid methodology and this is the focus of this chapter. This research harnesses the Design Research Methodology (DRM) to achieve its objectives. The chapter begins with an overall research approach as shown in Section 3.1. Section 3.2 describes all stages of the DRM. A summary is provided at the end of this chapter.

3.1 Research Approach

The target of this study is to find a low complexity mode division multiplexing equalizer for mode division multiplexing. To date, intensive research has been done in investigating mode division multiplexing in few-mode fibres (FMF). Six modes of light only are propagating inside the FMF at the same time, compared with hundreds of modes in the case of mode division multiplexing in multimode fibre [4]. The computational complexity of the mode division multiplexing equalizer is proportionally increased with the increment of the propagating modes [66].

In this research, an improved mode division multiplexing equalization technique is proposed. It is based on LU factorization method which is expected to reduce the equalizer complexity and improve the capacity of current mode division multiplexing systems. This research follows the definition of the research design by Blessing and Adib in [67] and [68] respectively.

In order to achieve the goal of this research, a solid understanding of the problem and how channel equalization works with its main open issues is highly imperative. This is helpful in exploring the proper equalizer that can reduce the issues previously mentioned.

DRM framework is shown in Figure 6 below. The figure also shows the link between each stage, the main methods and procedures and its outcomes. The light arrows shows the process flow while the bold arrows shows the main methods and outcomes.

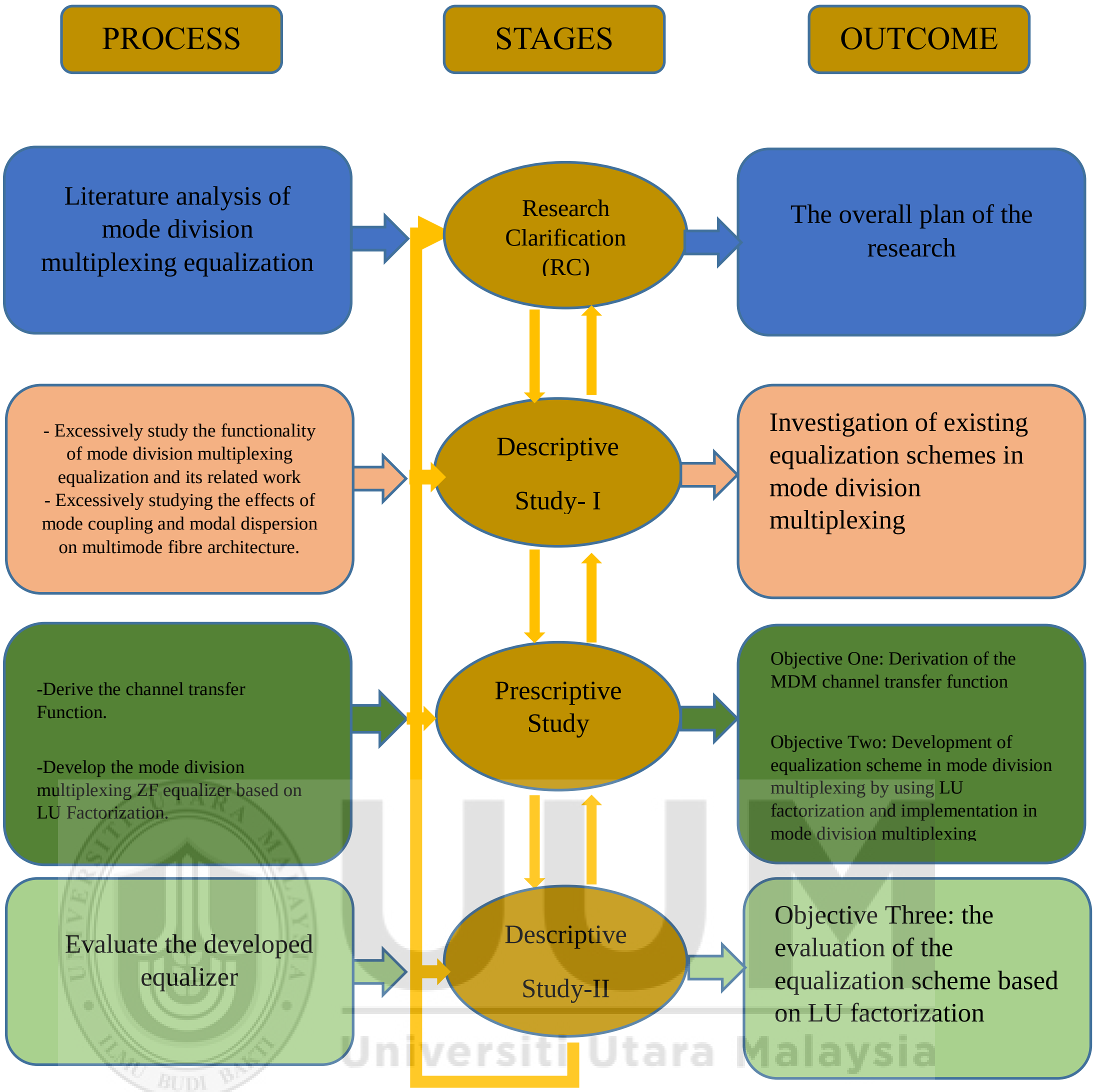


Figure 6 Research Approach

3.2 Stages of Design Research Methodology

The following subsections discuss each stage in the DRM, which may be divided into four stages, starting with the Research Clarification (RC) in Section 3.2.1, then Descriptive Study I (DS-I) in Section 3.2.2. Section 3.2.3 discusses the Prescriptive Study (PS). Then, finally the Descriptive Study II (DS-II) will be explained in Section 3.2.4 of this chapter

3.2.1 Research Clarification

The first stage of DRM is the RC stage. In this stage, defining the problem and determining the objectives can be deduced by literature research reports, and previously evaluated relevance. This part provides a rigid and essential base for the next research process. Six steps are involved in RC as shown in Figure 7.

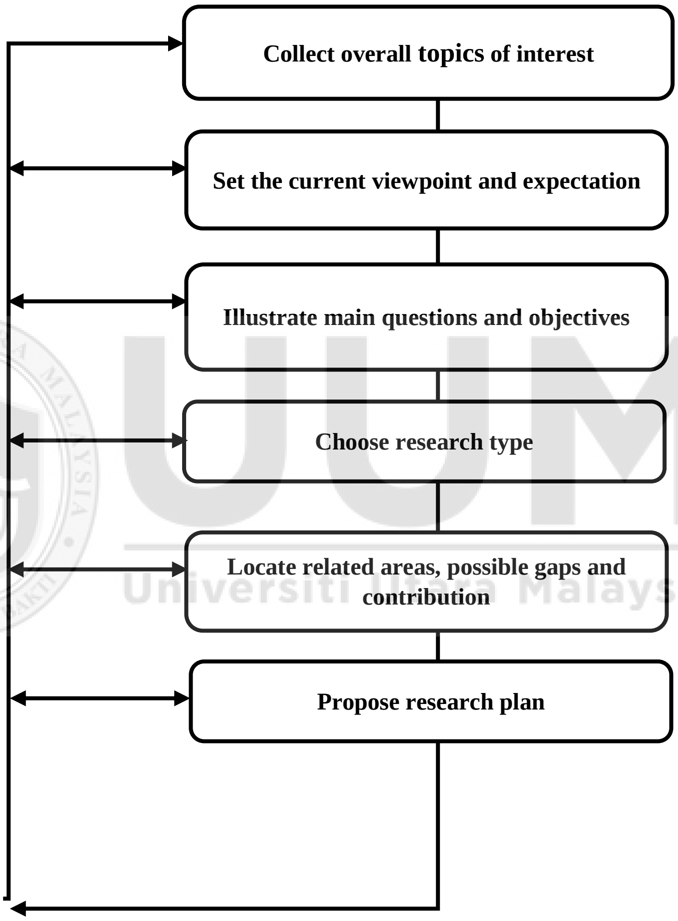


Figure 7 the Research Clarification Stage Steps

The outcomes from the RC stage are:

1) Mode division multiplexing research’s motivation and the research’s focus.
2) The research problem along with research questions in relation with research objectives for mode division multiplexing equalization in multimode fibre.
3) The research scope, type of research and research methods for mode division multiplexing equalization in multimode fibre.
4) The expected research contribution and deliverables for mode division multiplexing equalization in multimode fibre.

3.2.2 Descriptive Study-I

When the RC stage is completed, the overall research plan will be obtained, which will provide a solid ground to start the second stage of the research which is the DS-I. By the completion of DS-I stage, critical understanding of the current situation would have been gained. Critical review of the literature review in the research domain as well as experimental studies also involves in this stage. Within the progress of this stage, many details on the main work of mode division multiplexing equalization and LU factorization will be highlighted, considering its crucial issues and the possible solutions based on previous work in mode division multiplexing approach. The steps of this stage is shown in Figure 8 below.

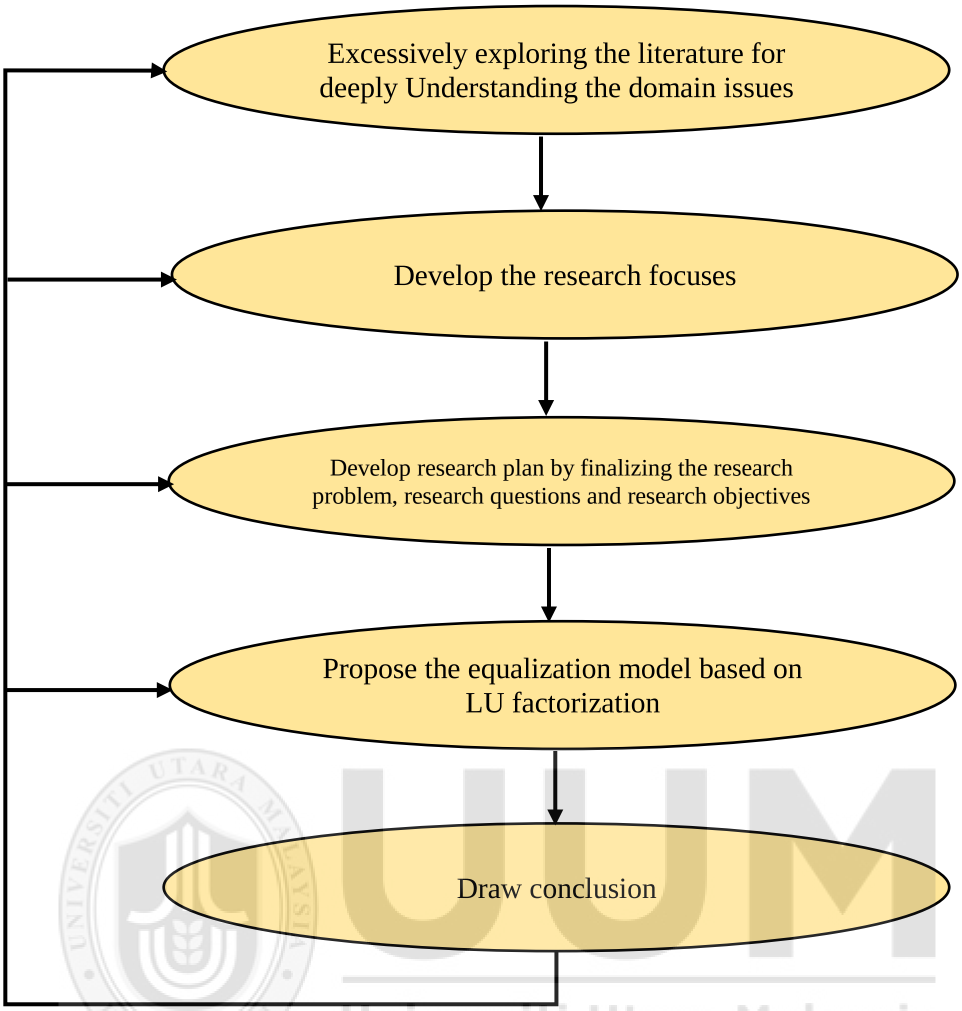


Figure 8 Main Steps in Descriptive Study I (DS-I)

3.2.3 Prescriptive Study

The main stage of DRM is PS, it shows the implementation steps needed in designing and developing the Zero Forcing LU-based equalizer and the adopted methods in performing this process. A block diagram that defines the main procedures of PS according to this research methodology is shown in Figure 9 below.

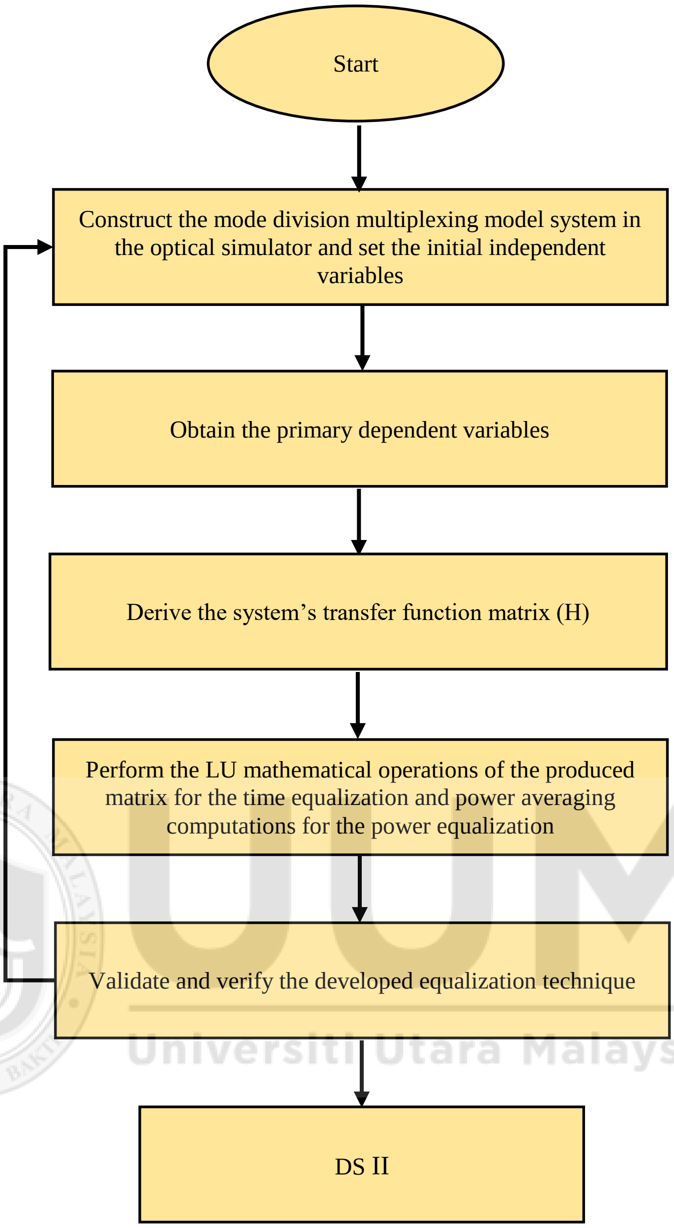


Figure 9 Prescriptive Study Steps

3.2.3.1 Experimental Design

The first step of PS includes the constructing of the needed mode division multiplexing model by using software simulation. Within this step, the setting of the independent variables and deriving the primary dependent variables takes place.

The simulation model is constructed using Optisystem software. Sets the transmitter, channel and receiver blocks within the simulator environment and adding the needed perturbations to simulate the defects in the fiber channel is also taken place at this stage. The main output from the optical simulator is the Power Coupling Coefficients values which is the main primary dependent variable which is used to create the main secondary dependent variable, the transfer function (H). Thus part will be discussed in more details in Chapter Four.

The second step is the creation of the transfer function (H) which is one of the main input parameters for the proposed equalizer. This is to be done by applying mathematical calculations on the primary dependent variables gained from the first step of this stage. This stage also includes the algorithms coding in MATLAB. This code is used to perform the proposed equalization scheme in the extracted data from the optical simulator.

Using MATLAB to perform the LU factorization method on the secondary dependent variables is the third process of this stage to achieve the time equalization stage, moreover, further averaging computations on the obtained powers will be taken place for the power equalization stage.

3.2.3.2 Verification and Validation

The Verification and Validation (V&V) is the fourth and the final step of this stage. Model verification is the confirmation that the developed model can be altered from one shape into another, as required, with the needed accuracy. In this research, the MDM model's validation study will take place by implementing the mode division multiplexing model into two different optical simulators. The two scenarios will have exactly the same independent variables, then the work will be valid if the same dependent variables are obtained. On the other hand, the derived transfer function will be validated by comparing it with previous valid ones from the literature. The algorithms will be validated by comparing it with other valid algorithms under the same operational environment.

For the Verification stage, different scenarios based on different in independent variables will take place. The work will be also verified by repeating the experimental steps several times before having the final observation. On the other hand, the algorithms will be verified by making sure that it is capable of recovering the MDM signal at each time when the different transfer function is introduced.

3.2.4 Descriptive Study-II

DS-II shows the evaluation of the developed equalizer. By completing this stage, the third objective of this research will be achieved. As per [67], the evaluation of performance is a vital stage in the evaluation of any research. The main steps of this stage is shown in Figure 10 below.

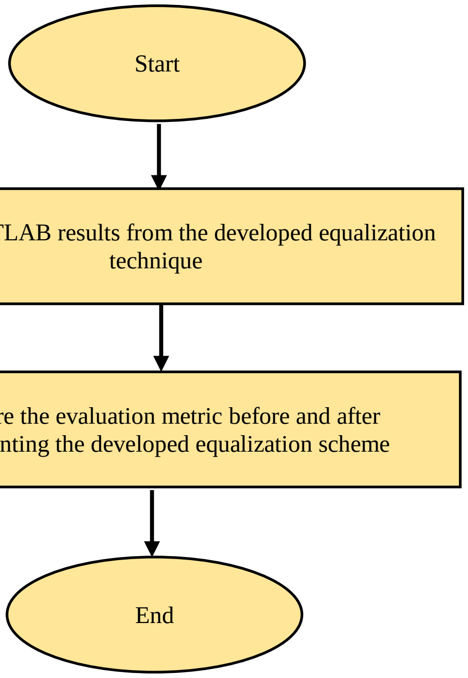


Figure 10 Main Evaluation Steps

3.3 Implementation and Evaluation Techniques

In communication systems, the implementation and performance evaluation is considered essential. It is possible to carry out with this phase by using three main approaches individually or in combination. The three approaches are analytical modelling, measurement and simulation [69].

3.3.1 Options for Implementation and Evaluation

Three main methods are widely used for the implementation and evaluation of research. Analytical modelling is a way of representing the system by mathematical equations. Analytical modelling is feasible for simple systems but not recommended for complicated systems as additional work is needed to simplify and make assumptions [70], thus affecting the representation of the actual system [71]. On the other hand, measurement is another approach which may be used for the implementation and evaluation. Measurements may be classified into two types: the first type is conducted by working on real live systems and the second type is by using specially designed laboratory environment for performing the needed experiments. In fact, the second way is more common but not suitable for large and complicated systems [72]. The third implementation and evaluation is by means of simulation. Simulations depend on special kind of software that have the ability to represent and explain events as if it occurs in a real environment. The flexibly offered by software for simulations and the recent credibility simulations have rendered this the main approach for implementation and evaluation by many researchers [73, 74].

3.3.2 Proposed Technique for Design and Evaluation of LU-Based Mode Division Multiplexing Equalizer

Network simulators are being utilized all over the work by many researchers and designer for the sake of education, commerce and industry purposes. They are needed to simulate any part of the network. Many simulator are being used every day to approximate the outcome of real scenarios paving the way for designers and researchers for real time enquiry or performance [75]. Some of network simulator are free like OMNET++, Global Mobile simulator (GloMoSim), and Network Simulator 3(NS3), while others are not free like MATLAB, Optisystem, and Optsim. Optsim and Optisystem are the simulation tool that will be use during the implementation of this research.

Optsim [76] and Optisystem [77] are a wide-ranging software design suite that permit users to design, test, and simulate optical links of advanced optical networks. Optsim and Optisystem are considered to be a standalone products that do not rely on other simulation frameworks, however, they can be integrated with other simulation software like MATLAB for more functionality. They are physical layer simulators based on the actual modelling of fibre optic communication network. These software have powerful new simulation circumstances and like real components and systems. A copy of these software are installed in the optical network laboratory's computer and they are used to design and simulate the proposed work.

As previously mentioned, Optsim will be the main optical simulator to be used during the design and implementation, while Optisystem will work as a supporting environment during the Verification and Validation (V&V) stage.

On the other hand, MATLAB will be used in the derivation of the secondary dependent variables, data analysis and development of the LU-based mode division multiplexing equalizer.

MATLAB is a simulation platform for execution mathematical calculations. Its name came from "Matrix Laboratory" which highlights that MATLAB was initially programmed to carry on with the mathematical calculations related with matrices in a more comfortable way than the other method of doing such calculations at time of MATLAB's first version like C and FORTRAN. However of its initial purposes, MATLAB developers added many new functions to the software that make it a very powerful tool and one of the best mathematical simulation environments which is being used by many researcher, students and designers [78].

In addition to MATLAB capability of performing graphical and numerical computations, MATLAB can also be considered as a procedural programming language that combining an efficient programming structure with pre-defined mathematical commands [78].

3.3.3 Evaluation Metrics

The evaluation of the proposed equalizer will be taken place by calculating the time delay spread and the computational complexity for the time equalization part and the standard deviation for the power equalization part.

Time delay spread is the measurement of the duration between the arrival of the first mode and the last mode in MDM signal. Reducing time delay spread increases the channel capacity and reduces the complexity due to the fact that the number of the propagating modes will be also decreased [79, 80] and as shown in more details in Chapter four.

On the other hand, the computational complexity is the measurement of the needed mathematical operations or the needed time to get an algorithm converged. Having low computational complexity will increase the system performance as less time and processing is needed to achieve the target from an algorithm and it is the main goal behind this research.

3.4 Summary

This chapter shows detailed research steps based on DRM to ensure the research objectives are feasible and can be implemented as planned. This research focuses on the development of the proposed LU-Based equalizer for mode division multiplexing according to the four stages in DRM.

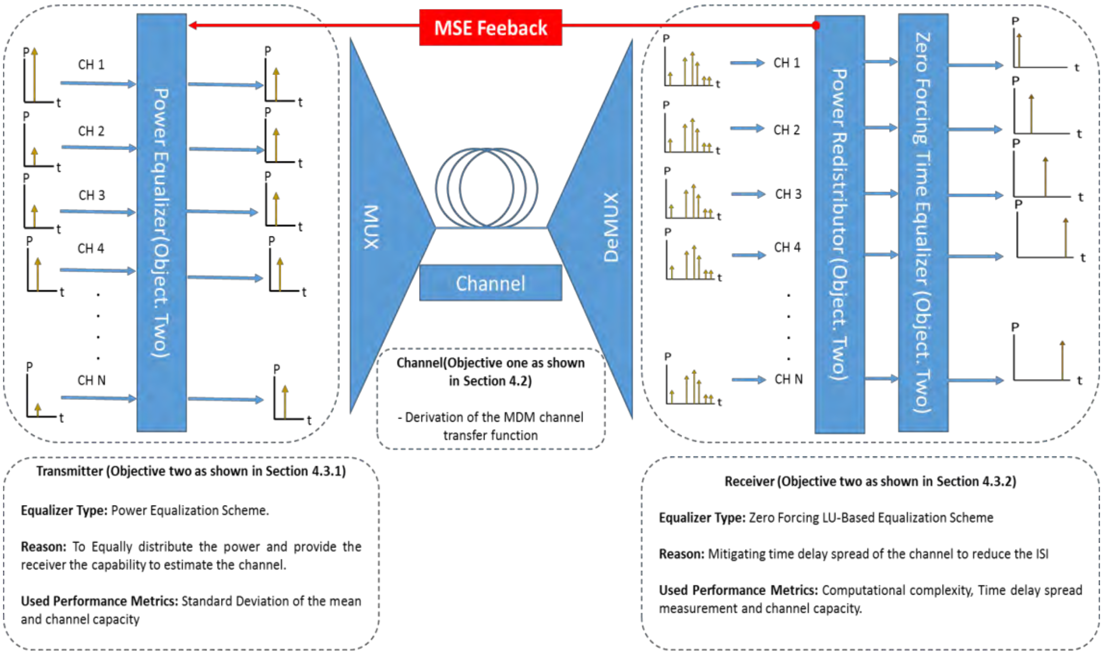
Chapter Four: Implementation, Results and Evaluation

After putting the needed guidelines to achieve the objectives of this research in chapter three, this chapter proposes a two-stage mode division multiplexing equalization scheme based on LU factorization and identical power distribution. The proposed scheme focuses on enhancing the performance of the MDM systems by mitigating the unwanted effect of mode coupling following an algorithm with low complexity.

As mentioned in chapter 2, many techniques has been used so far trying to compensate the mode coupling effect in the fiber channel, however, they are suffering from high complexity or instability. On the other hand, many techniques based on LU and QR factorization had been used on the radio domain and shows significant results in reducing the complexity and enhancing the stability of the used equalization algorithm.

Section 4.1 highlights the background on the LU factorization algorithm in terms of its applications, and how it works in finding the matrix inversion and achieving the mode division multiplexing ZF time equalization. Later in this chapter and specifically in Section 4.2, the implementation of the first objective is taken place, while Section 4.3 shows the implementation of the second objective

Sections 4.4 and 4.5 shows the evaluation of the developed algorithms is illustrated. Finally, the chapter ends with the summary in Section 4.5. Figure 11 below shows the schematic diagram of the MDM system after implementing the proposed equalizers.



Channel(Objective one as shown in Section 4.2)

- Derivation of the MDM channel transfer function

Transmitter (Objective two as shown in Section 4.3.1)

Equalizer Type: Power Equalization Scheme.

Reason: To Equally distribute the power and provide the receiver the capability to estimate the channel.

Used Performance Metrics: Standard Deviation of the mean and channel capacity

Receiver (Objective two as shown in Section 4.3.2)

Equalizer Type: Zero Forcing LU-Based Equalization Scheme

Reason: Mitigating time delay spread of the channel to reduce the ISI

Used Performance Metrics: Computational complexity, Time delay spread measurement and channel capacity.

Figure 11 Proposed MDM System Schematic Diagram

4.1 LU Factorization

As previously mentioned, this research focuses on the development of ZF equalization scheme. This approach works under the concept of having the deconvolution of the channel effect by applying its inverse on the signal. Fast and low complexity technique is needed in finding this inversion in MDM system to provide high performance operation.

LU Factorization or decomposition as described in some contexts is a mathematical algorithm mainly used in solving linear equations and finding the matrix inversion. Finding matrix inversion is a very complicated task if done by using the ordinary methods, it can take long and complicated steps to achieve it especially when dealing with large matrices, however, this task can be achieved easily by using LU factorization.

LU factorization decomposes any square matrix into Lower (L) and Upper (U) triangular matrices. The result of the decomposition can be used in solving linear equations and/or finding the matrix inversion by:

For matrix A,

$$AA^{-1} = I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

where I is the identity matrix

Let’s assume that $A^{-1} = x$ and $I = b$, it is possible to find the inverse of *A* by:

$$Ax = b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

where A is an N X N square matrix. Performing the LU factorization on A decomposes it into an N X N lower triangular L and upper triangular matrix U, then:

$$LUx = b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

let,

$$y = Ux \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Then,

$$Ly = b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

to solve y by using forward substituting method , and

$$Ux = y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...6$$

To solve x (A^{-1}) by using back substituting method

4.2 Derivation of the Channel Transfer Function

The implementation of this part goes through the steps mentioned in the below flowchart:

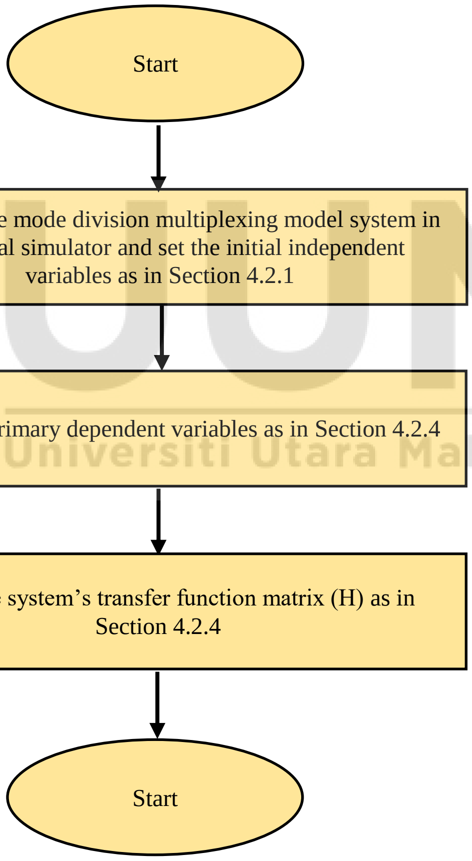


Figure 12 Transfer Function Derivation Steps

4.2.1 Mode Division Multiplexing Simulation Model

In order to get the initial data needed to test, implement, validate and verify the developed algorithms, it is mandatory to have data extracted from an MDM system. Simulation was chosen as the experimenting environment of this research, thus, the MDM model was constructed by using Optisystem®.

The model was constructed to match as much of the real operating environment as possible. The model was mainly consisting three parts, the transmitter, the optical media and the receiver. Spatial vertical-cavity surface-emitting laser (Spatial VCSEL) was chosen because of its ability to generate the light modes that will be injected inside the optical media. As MDM is relying mainly on utilizing the propagating light modes inside the multimode fibers, the last was chosen as the model's fiber medium. Finally, the receiver side is consisting of the spatial receiver component. The main parts of the MDM model in shown in Figure 13 below.

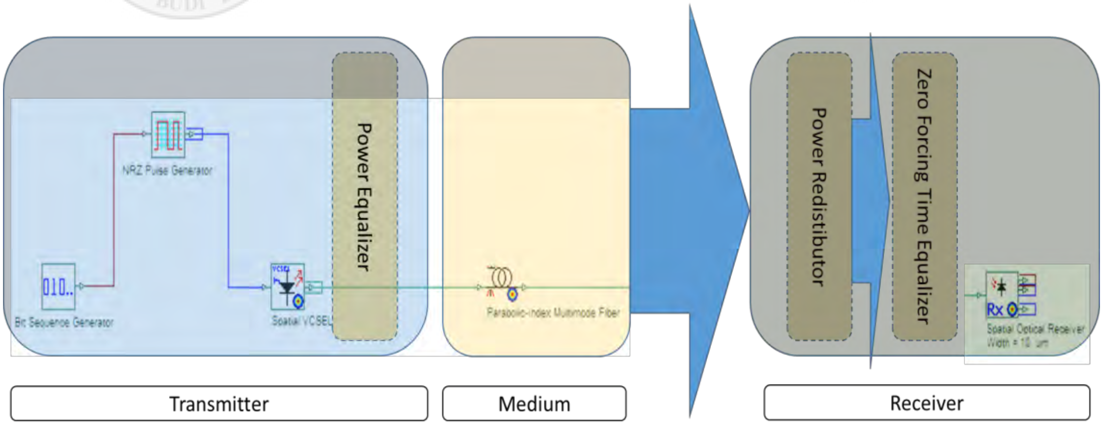


Figure 13 Main Simulator's Components of MDM System

On the other hand the main components used during the modelling process is shown in Table 3 below:

Table 3 main component used in modelling

Component Name	Function
Bit generator	Generates the transmission bitrate
Spatial VCSEL	Generates optical modes and a signal light source
Multimode fiber	Optical transmitting medium
Optical power meter	Measures optical signal power
Eye diagram analyzer	Measures Bit Error Rate (BER),shows signal's eye diagram and other performance metrics

On the other hand, before running the simulation, the parameters below had been set:

- Transmission bitrate: 10Gbps which is a high enough bitrate to simulate what is intended from real MDM system.
- Laser wavelength: 1552 nm in order to match the wavelength used for long-haul systems [81].
- Optical fiber length: 8.5 km
- Number of generated modes in the transmitter: 10 modes had been generated by the spatial VCSEL.

- Modes power ratios: This parameter determines how much power from the total will go to each of the generated modes. It was set to the average value in a way to simulate the effect of the power equalization algorithm.
- Fiber core size: 31.25 µm to match the standard multimode optical fiber cables [81].
- Fiber cladding size: 31.25 µm to match the standard multimode optical fiber cables[81].
- Core refractive index: 1.4142, it was chosen based on the standard values of the multimode optical fiber cables[82].
- Delta value: this value is representing the difference between the refractive indexes of the core and the cladding, this value has been set to 0.1%. This will affect the number of the guided modes that will be excited inside the multimode fiber, setting it to 0.1% will excite 10 modes, while setting it to 0.16% will increase the number of excited modes to 15 modes and so on.

4.2.2 Perturbations Induction

In order to have data as if they are got from a real MDM model, mode coupling and other intended system's perturbations had been induced and as below:

- Mode coupling: As mentioned in chapter two, mode coupling is an inevitable physical phenomenon arising from multimode fiber manufacturing defects such as microbends and optical fiber cross section ellipticity [18, 35, 36]. In order to simulate mode coupling effect, the multimode fiber was segmented into a several parts of random lengths, then, these parts were connected to each other by using special simulator’s component called “spatial connector”. This component has the ability in offsetting the laser beam by a very small amount within the cross-section of the fiber core. By doing this, the simulation of the bending in the real fiber optics is achieved [83], thus, will induce mode coupling into the system.

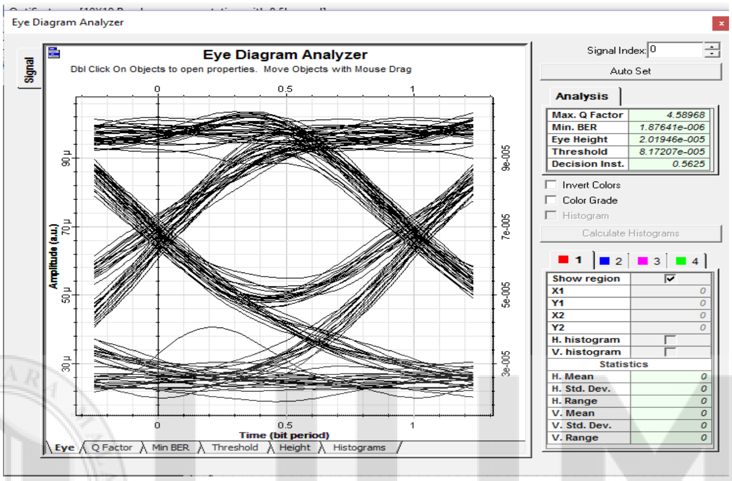
- Unequal power distribution: in order to simulate the light source’s inability of allocate equal power to all of the generated modes. In this model, it was assumed that the VCSEL incapability of generating modes with equal powers was has been resolved by the power equalization algorithm.

4.2.3 Initial Observation and Extracted Data

After running the simulation of the constructed model, an initial observation is obtained and as below:

- The system is running but with high bit error rate of 1.8×10^{-6}, the high Bit Error Rate (BER) is the result of inducing mode coupling into the system, practically, normal values of BER as per that ITU should be not more than 1×10^{-9} [84]
- The measured received power is 4.868 dB, this measurement will be used in determining the input signal (P_m) that will be used in testing the developed equalizers
- Each one of the generated modes (channel), excited 10 guides during its propagation inside the multimode fiber.

The random mode coupling can be recognized from the random energy transfer between the modes in each channel, thus, instead of having the one mode that had been generated by the transmitter, 10 modes with different powers were detected at the receiver side and due to this, higher BER, unorganized and narrower eye diagram is obtained and as shown in Figure 14 below.



Signal Index: 0

Analysis	
Max. Q Factor	4.58968
Min. BER	1.87641e-006
Eye Height	2.01946e-005
Threshold	8.17207e-005
Decision Inst.	0.5625

☐ Invert Colors
☐ Color Grade
☐ Histogram

Show region	☑
X1	0
Y1	0
X2	0
Y2	0
H. histogram	☐
V. histogram	☐
Statistics	
H. Mean	0
H. Std. Dev.	0
H. Range	0
V. Mean	0
V. Std. Dev.	0
V. Range	0

Figure 14 Observed System Eye Diagram

On the other hand, the below are the initial raw data that had been extracted from the model:

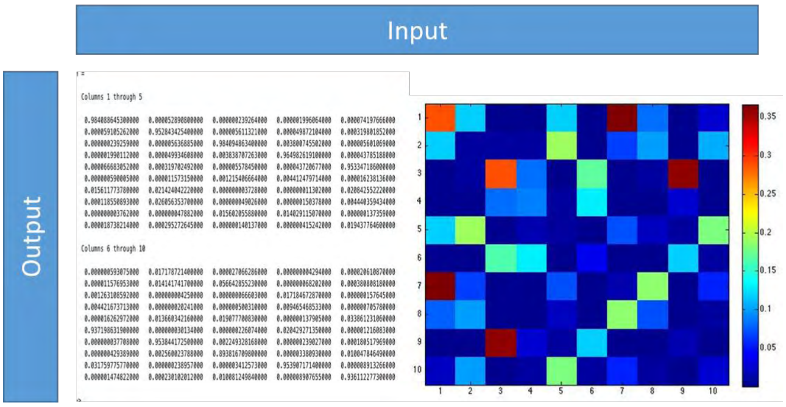
- Power Coupling Coefficient (PCC): this is the main data extracted from the model that will be used to create the input signal matrix
- Propagation delay: this data will be used in calculating the time delay spread performance metric.

4.2.4 Mathematical Representation of the Received Signal

As previously mentioned in Section 4.2.1, the difference between the refractive indexes of the cladding and the core were set in a way that the guided modes inside the optical fiber is equal to the number of the generated modes inside the VCSEL, thus, the received signal is considered as a 10×10 MDM signal and can be mathematically represented by a 10×10 square matrix of PCCs which can be processed by the LU-based equalizer that deals with square matrices only.

$$\begin{pmatrix} PCC_{11} & \cdots & PCC_{110} \\ \vdots & \ddots & \vdots \\ PCC_{101} & \cdots & PCC_{1010} \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..7$$

Each column vector of this matrix is showing the ten guided modes of a channel, for example, the first column vector is showing the PCCs of the first channel, and the second column vector is for the second channel and so on. The received signal is shown in Figure 15 below, the mode coupling can be observed form the color dissipation across the channel matrix.



```
Columns 1 through 5

 0.984088645300000   0.000052890800000   0.000000239264000   0.000001996064000   0.000074197666000
 0.000059105262000   0.952843425400000   0.000005611321000   0.000049872104000   0.000319801852000
 0.000000239259000   0.000005636885000   0.984094863400000   0.003800745502000   0.000005601069000
 0.000001990112000   0.000049934608000   0.003838707263000   0.964982619100000   0.000043785188000
 0.000066683052000   0.000319702492000   0.000005578450000   0.000043720677000   0.953347186000000
 0.000000590005000   0.000011573150000   0.001215406664000   0.004412479714000   0.000016238136000
 0.015611773780000   0.021424042220000   0.000000003728000   0.000000011302000   0.020842552220000
 0.000118550893000   0.026056353700000   0.000000049026000   0.000000150378000   0.004440359434000
 0.000000003762000   0.000000047882000   0.015602055880000   0.014029115070000   0.000000137359000
 0.000018738214000   0.000295272645000   0.000000140137000   0.000000415242000   0.019437764600000

Columns 6 through 10

 0.000000593075000   0.017178721400000   0.000027066286000   0.000000004294000   0.000020610870000
 0.000011576953000   0.014141741700000   0.056642855230000   0.000000068202000   0.000380808180000
 0.001263108592000   0.000000004250000   0.000000066603000   0.017184672870000   0.000000157645000
 0.004421673713000   0.000000020241000   0.000000500310000   0.009465468533000   0.000000705780000
 0.000016262972000   0.013660342160000   0.019077700830000   0.000000137905000   0.033861231060000
 0.937198631900000   0.000000030134000   0.000000226074000   0.020429271350000   0.000001216083000
 0.000000037708000   0.953844172500000   0.002249328168000   0.000000239027000   0.000180517969000
 0.000000429389000   0.002560023788000   0.893816709800000   0.000003380930000   0.010047846490000
 0.031759775770000   0.000000238957000   0.000003412573000   0.953907171400000   0.000008913266000
 0.000001474822000   0.000230102012000   0.010081249840000   0.000008907655000   0.936112277300000
```

Figure 15 10×10 *MDM Signal as it comes out of the optical fiber*

In this research, the channel transfer function is constructed by dividing the received signal on the transmitted signal and as below:

For any communication system,

$$y = Hx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

where y is the output signal, H is the channel transfer function and x is the input signal

let, $y = R_{redist.}$

where $R_{redist.}$ is the power redistributed received signal matrix and,

$R_{redist.} = H \times P_m$, where P_m is transmitted mode power

then,

$$H = \frac{R_{redist.}}{P_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

First step of the implementation is the coding of Algorithm 4.1 and Algorithm 4.2 in MATLAB®. Second step is the extraction of the initial data from the simulated optical model. After the above, the extracted data has been exported to MATLAB® in order use it to construct the received signal matrix and the channel transfer function.

In this research, A 10X10 transfer function is mathematically derived based on the extracted data of the system's PCCs, this compared with the 8X8 transfer function in [85]which was constructed based on experimental observation and the derivation of its mathematical representation is based on the light electric field. The derived equation is used to physically optimize the parameters of the Spatial Light Modulator (SLM) to successfully mitigate mode coupling and reducing ISI.

On the other hand, [86] derived the system transfer function base on the light intensity by capturing the light intensity of the different modes using special camera, which lead to the fact that the derivation is also based on experimental observation.

To validate the calculated transfer function, a comparison with previous work based on the normalized power measurement is taken place. The normalized powers in [86] are varied from 0-0.1 within 0.5ps compared with the range of this research which is from 0-0.12 within the same period, however, the derived transfer function is shorter pulse with only 0.5ps compared with 50ps in [86], having shorter pulses decreases the possible overlapping, thus, the chances of inducing ISI to the channel is also reduced. On the other hand, the derived transfer function in this research is a short period pulse of about 5.5×10^{-13} seconds with hill's shape which is similar to the pulse obtained in [87]. In [8], a 6X6 signal was obtained, the transfer function of the obtained signal is consisting of 36 guided modes with different magnitudes reaching the other end of the fibre at different times which is very similar to the case of this research with the difference that a 10X10 signal is obtained. The comparisons shows that the obtained transfer function from this research is within the normal range of the other valid transfer function of the optical communications systems.

The channel transfer function is considered to be a mandatory parameter that need to be known by the equalizer for successful mode coupling mitigation. As mentioned in Section 4.2 above, the LU-based equalizer finds the inverse of the channel transfer function in order to mitigate the channel effect (mode coupling) on the signal.

4.3 Zero Forcing Equalization Scheme Development

The next section shows the steps of the ZFE development, it starts with Section 4.3.1 where the implementation of the MDM Power Equalization Algorithm (MDM-PE) is taken place, the section ends with the discussion of the obtained results out of this algorithm.

The second step in Section 4.3.2 where the implementation of the MDM Time Equalization Algorithm (MDM-TE) is taken place. In the same way of the first step, this section ends with the discussion of the obtained results.

4.3.1 Power Equalization Algorithm

The currently used spatial light sources are usually suffering from its inability to generate modes with equal powers [88] resulting modes with different powers at the receiver side that can complicate the detection process afterwards.

On the other hand, according to [79, 80], long haul communication systems has no Channel State Information (CSI) known, which means that the transmitter have no information about the surviving mode and the receiver has information about the channel [80].

In order to let the receiver estimates the channel and make Equation 9 to be valid, a unified power distribution should be obtained in both the transmitter and the receiver. To achieve this, two conditions should be taken into consideration. First, the transmitter need to keep an equal power distribution between all of the transmitted modes. [79, 80]. Second, the received powers need to be redistributed to keep this unified power distribution [89], as it is observed form the experiments related to this study that under strong mode coupling regime, there is power degradation in some of the received modes at the receiver.

To achieve the above conditions, a two-stage power equalization algorithm is proposed. Initially, this algorithm is equally allocate the powers between the modes upon the beginning of the transmission. This is can be done by taking the mathematical average of the powers at the transmitter and let an amplifier to do the amplifying and the attenuation of the signals accordingly and as shown in the first part of Algorithm 4.1, thus, the first condition is achieved. Once the signal is received, the other part of the algorithm works on the redistribution of the received powers, practically, this is can be done amplifying the effected channels and mathematically by adding the difference in the powers values and taking the channel with the highest powers as a reference and as shown in the second part of Algorithm 4.1 below, this achieves the second condition mentioned above.

Furthermore, the redistribution part calculates the Mean Square Error (MSE) for the signal before and after the redistribution, this information is fed back to the transmitter to enhance the power allocation for the next transmitting period.

Algorithm 4.1 Mode Division Multiplexing Power Equalization (MDM-PE)

Part 1. Transmitter side

Step 1. Compute the mathematical average of the powers of each channel

$$P_m = \frac{\sum P_t}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 10$$

where P_m is the average mode power, P_t is the total transmitted power and N is the number of the system's channels.

Step 2. Amplify or attenuate the power of each mode based on the calculated average power in order to have the same power in all modes.

Part 2. Receiver Side

Step 3. Redistribute the received powers to be equally distributed on the channel matrix

$$P_{norm} = P_m + (\mathrm{P}_m \times k) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 11$$

where P_{norm} is the redistributed power, P_m is the power of each channel and k is a factor equals to $\frac{P_{\max}}{P_m} - 1$

Step 4. Amplify the power based on the redistributed value for the effected channels to derive the redistributed received signal matrix R_{norm}

Step 5. Calculate the MSE value of each channel and feed it back to the transmitter part based on the below:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(R_{norm} - R)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 12$$

where R_{norm} is the redistributed received signal and R is the received signal before redistribution.

Step 6. Amplify or attenuate the power of the next transmission period for each channel based on the MSE value.

After completing the coding of the above algorithm in MATLAB®, the transmitted is inserted to the power equalizer which its main task is to redistribute the powers equally between the channels, this is the first part of the algorithm. The second part is taken place at the receiver side where the distorted signal is received. The main task of this part is to redistribute the power between the channels based on the channel with the maximum power. By this point, the received signal is redistributed and ready to be equalized by the time equalizer.

The practical implementation of the first part is done by assigning the same powers for all channels in the transmitter component at the simulator. In the case of this study, each mode is assigned with $\frac{1}{10}\times P_t$ which is equal to 0.4868 dB

On the other hand, the results of the redistributions parts is shown in Figure 16 and 17 below, the last raw in these binary images shows the effect of the redistribution process on the received signal.

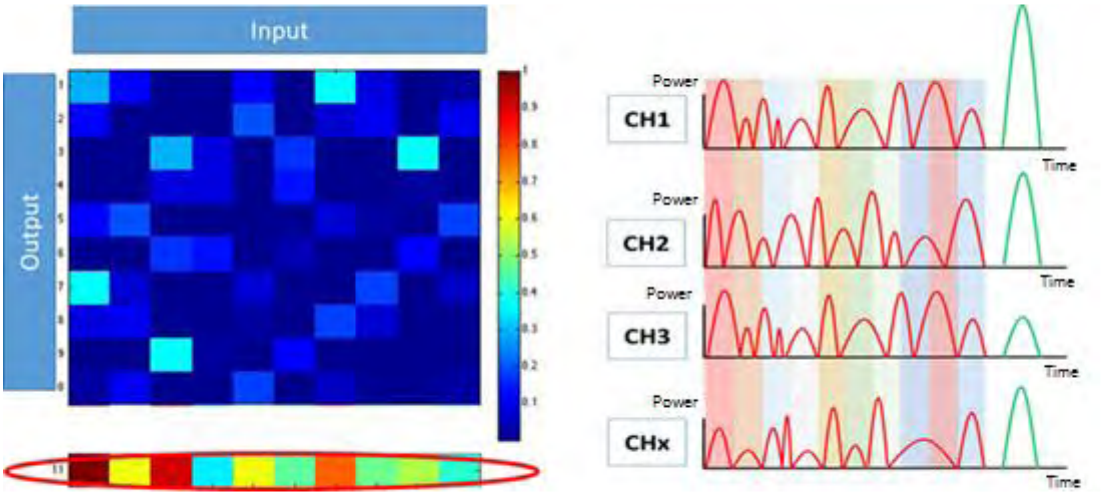


Figure 16 Received Signal before Power Redistribution

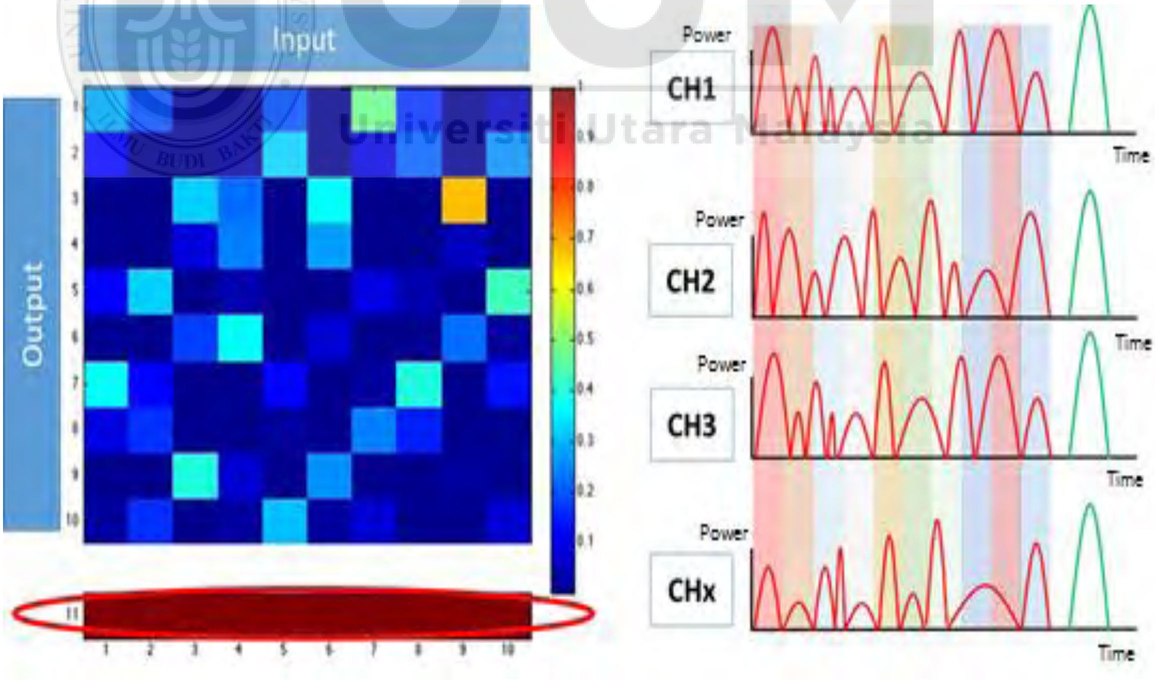


Figure 17 Received Signal after Power Redistribution

The second part task does not stop at this point only, it has another task which the calculation of the MSE value of the received channel before and after the redistribution. The MSE information is used to optimize the transmitter for the next transmission periods by attenuating the powers of the channels less effected by mode coupling while amplifying the most effected channels to keep the equal distribution valid at the receiver side. MSE calculations is based on Equation 12 above and as shown in Table 4 below.

Table 4 Mean Squared Error (MSE) per Channel

Channel	MSE
Channel 1	3.88607×10^{-6}
Channel 2	0.15376
Channel 3	0.00709
Channel 4	0.41655
Channel 5	0.15591
Channel 6	0.28858
Channel 7	0.05406
Channel 8	0.27498
Channel 9	0.21846
Channel 10	0.34802

4.3.2 Time Equalization algorithm

After performs the power redistribution by the power equalization algorithm, the powers of the received signal is equally distributed again and ready to be equalized by the time equalization algorithm. The main task of the algorithm is to find the inverse of the channel matrix H and as shown in Algorithm 4.2.

Algorithm 4.2 Mode Division Multiplexing Time Equalization (MDM-TE)

Step 1. Find the channel matrix H from the received signal based on Equation 9 above

Step 2. Compute the L and U triangular matrices of the obtained H, then finds the inverse channel matrix H^{-1}

For matrix H,

$$HH^{-1} = I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 13$$

where I is the identity matrix

Let’s assume that $H^{-1} = z$ and $I = c$, it is possible to find the inverse of *A* by:

$$Hz = c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 14$$

where H is an N X N square matrix. Performing the LU factorization on A decomposes it into an N X N lower triangular L and upper triangular matrix U, then:

$$LUz = c \tag{15}$$

let,

$$y = Uz \tag{16}$$

Then,

$$Ly = c \tag{17}$$

to solve y by using forward substituting method , and

$$Uz = y \tag{18}$$

To solve x (H^{-1}) by using back substituting method

Step 3. Mitigate the mode coupling effect from the received channel by multiplying it by H^{-1}

$$\bar{R} = R_{\mathrm{Redist.}} \times H^{-1} \tag{19}$$

where $\bar{R}$ is the time equalized signal

In contrast with Figure 18 and 19 below where the mode coupling is observed, the output from the time equalizer (Algorithm 4.2) is shown in Figure 20 and 21 below. The gained signal is consist of 10X10 matrix having values only at its diagonal and almost zeros elsewhere, with the note that the values at the diagonal are equal to input powers for each channel at the transmitter side sbtracted by the value of the line attenuation. The signal before the time equalization can be represented as below:

$$\begin{pmatrix} PCC_{11} & \cdots & PCC_{1n} \\ \vdots & \ddots & \vdots \\ PCC_{m1} & \cdots & PCC_{mn} \end{pmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 20$$

On the other hand, the equalized signal can be given by:

$$\begin{pmatrix} P_{m1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & P_{m10} \end{pmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 21$$

where P_{mX} is the power of each mode

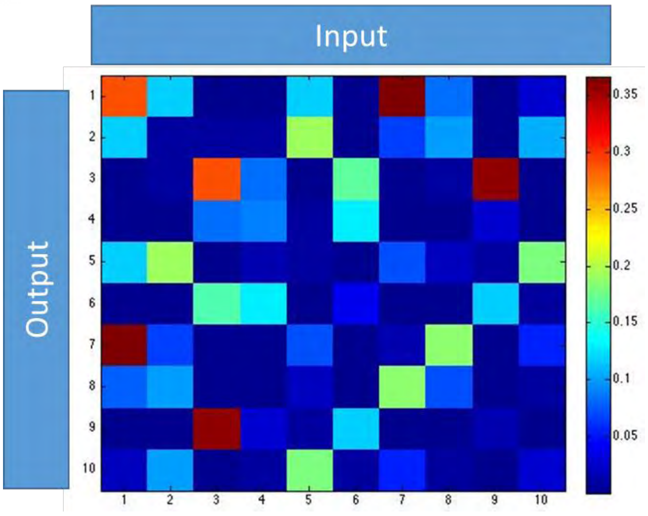
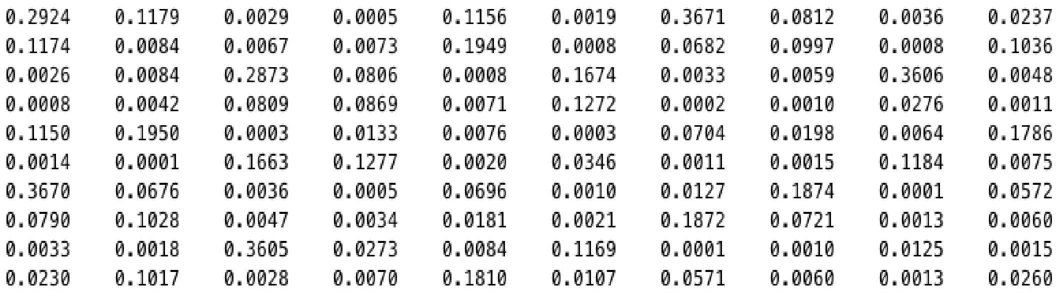


Figure 18 Crosstalk Matrix before Time Equalization shows the effect of the Mode Coupling on the Signal



```
0.2924    0.1179    0.0029    0.0005    0.1156    0.0019    0.3671    0.0812    0.0036    0.0237
0.1174    0.0084    0.0067    0.0073    0.1949    0.0008    0.0682    0.0997    0.0008    0.1036
0.0026    0.0084    0.2873    0.0806    0.0008    0.1674    0.0033    0.0059    0.3606    0.0048
0.0008    0.0042    0.0809    0.0869    0.0071    0.1272    0.0002    0.0010    0.0276    0.0011
0.1150    0.1950    0.0003    0.0133    0.0076    0.0003    0.0704    0.0198    0.0064    0.1786
0.0014    0.0001    0.1663    0.1277    0.0020    0.0346    0.0011    0.0015    0.1184    0.0075
0.3670    0.0676    0.0036    0.0005    0.0696    0.0010    0.0127    0.1874    0.0001    0.0572
0.0790    0.1028    0.0047    0.0034    0.0181    0.0021    0.1872    0.0721    0.0013    0.0060
0.0033    0.0018    0.3605    0.0273    0.0084    0.1169    0.0001    0.0010    0.0125    0.0015
0.0230    0.1017    0.0028    0.0070    0.1810    0.0107    0.0571    0.0060    0.0013    0.0260
```

Figure 19 Signal PCCs before Time-Equalization

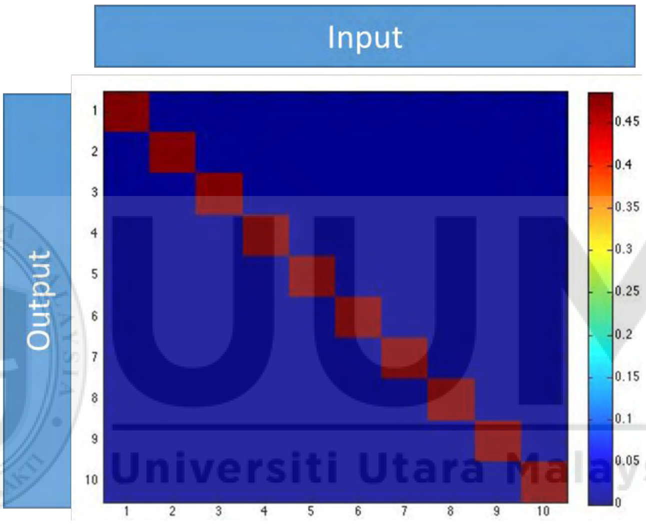
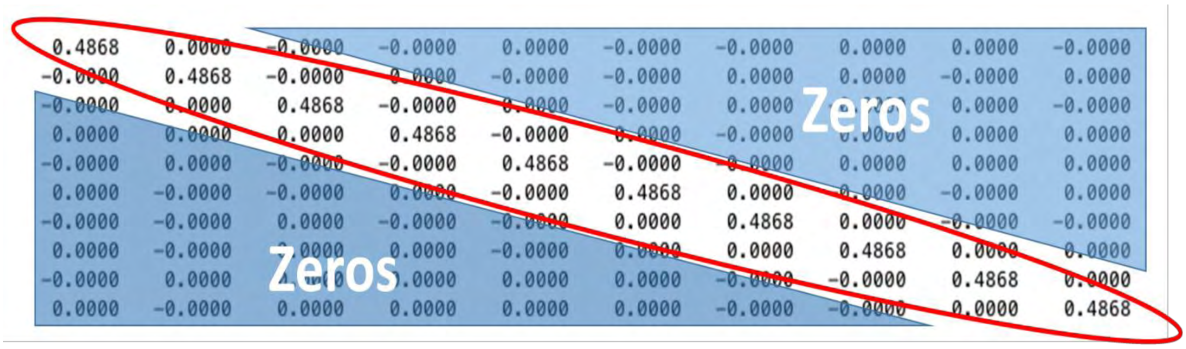


Figure 20 *Crosstalk Matrix after Time Equalization*



```
 0.4868    0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000
-0.0000    0.4868   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
-0.0000    0.0000    0.4868   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000
 0.0000    0.0000    0.0000    0.4868   -0.0000    0.0000   -0.0000    0.0000    0.0000    0.0000
-0.0000    0.0000   -0.0000   -0.0000    0.4868   -0.0000   -0.0000    0.0000    0.0000    0.0000
 0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.4868    0.0000   -0.0000   -0.0000    0.0000
-0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.4868    0.0000   -0.0000   -0.0000
 0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000    0.4868    0.0000    0.0000
-0.0000    0.0000   [illegible]    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.4868    0.0000
 0.0000   -0.0000    0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0000    0.4868
```

Figure 21 Signal PCCs after Time-Equalization

As mentioned before, the equalized channels have powers only in its diagonal and zeros elsewhere, in fact these powers are matching what is intended to receive in an ideal case where no mode coupling exists.

4.4 Evaluation

The evaluation of this research is taken place by the checking of each one of the developed algorithms individually. As previously mentioned in chapter three, the LU-based time equalization algorithm is evaluated by checking three performance metrics: computational complexity, time delay spread and the channel capacity. The evaluation steps are illustrated in Section 4.4.1 below.

On the other hand, Section 4.4.2 illustrates the evaluation of the power equalization algorithm which is taken place by measuring the maximum channel capacity and standard deviation of the gained powers of the received signal before and after the power equalization.

4.4.1 LU-Based Time Equalization Algorithm Evaluation

The evaluation of this algorithm has been taken place by calculating its computational complexity, comparing it with other similar algorithms used within the field of MDM equalization, measuring the spread time delay and the channel capacity of the equalized signal versus the not equalized one.

Firstly, in order to calculate the computational complexity, three different optical scenarios with different signal sizes (10X10, 15X15 and 21X21) had been constructed and simulated in the optical simulator, then implement their output data into MATLAB. Having different signal matrix size is needed to check the algorithm's speed for each channel.

In the first scenario, ten channels were generated in the transmitter side, then injected into an optical fiber capable of handling ten guided modes for each channel, thus, a 10X10 MDM channel matrix were gained. After running the simulation, it was found that the algorithm took between 0.001111 - 0.001472 seconds before stop running.

The other two scenarios were taken place by checking the algorithm performance with larger MDM channel matrices, 15X15 and 21X21 for the second and third scenarios respectively.

Ten attempts had been taken place for each scenario showing that there's slight difference in the CPU time, this difference is due to the performance fluctuation of the used computer, so the average speed is taken into consideration. The CPU time for each scenario is shown in Table 5 below.

Table 5 Time equalization algorithm CPU time per scenario

Scenario	CPU time range for ten attempts in seconds	Average CPU time
10X10 MDM Signal	0.001111 - 0.001472	0.0012915
15X15 MDM Signal	0.002729 - 0.003074	0.0029015
21X21 MDM Signal	0.007007 - 0.009141	0.0080740

Visualizing the above readings is shown in Figure 22 below. From observing the figure and the readings, the algorithm CPU time trend to increase linearly with the increment in its input data which means that the algorithm is O(N). Previous studies shows that RLS have O(N^2) and LMS is also O(N) [90, 91] but showing slower performance as shown in the coming paragraph.

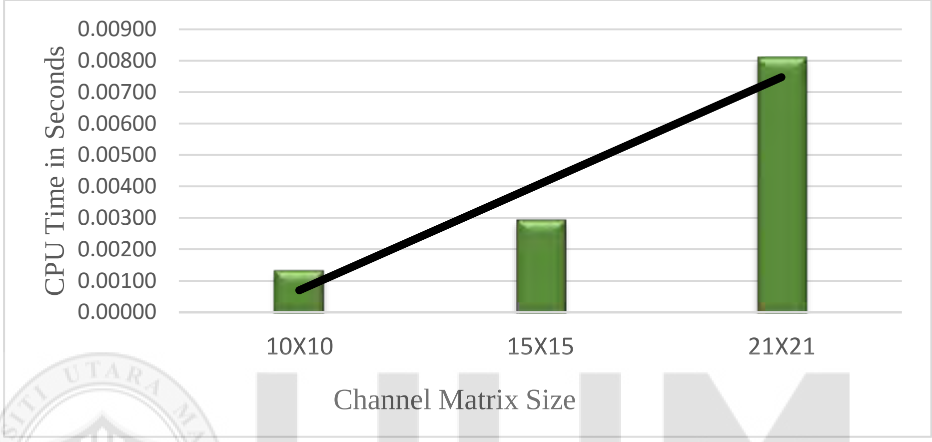


Figure 22 Time equalization algorithm CPU time

On the other hand, the comparison between our algorithm and the currently used algorithms in MDM equalization shows significant improvement in the equalization's algorithm CPU time and as shown in Table 6 and Figure 23 below.

Table 6 CPU Time comparison between RLS, LMS and LU Based ZF Algorithms

Algorithm	CPU time in seconds for 10X10 MDM Signal	CPU time in seconds for 15X15 MDM Signal	CPU time in seconds for 21X21 MDM Signal
RLS equalization algorithm	0.0124000	0.017501	0.025632
LMS equalization algorithm	0.0105000	0.012653	0.018432
LU-Based ZF equalization algorithm	0.0012915	0.0029015	0.008074

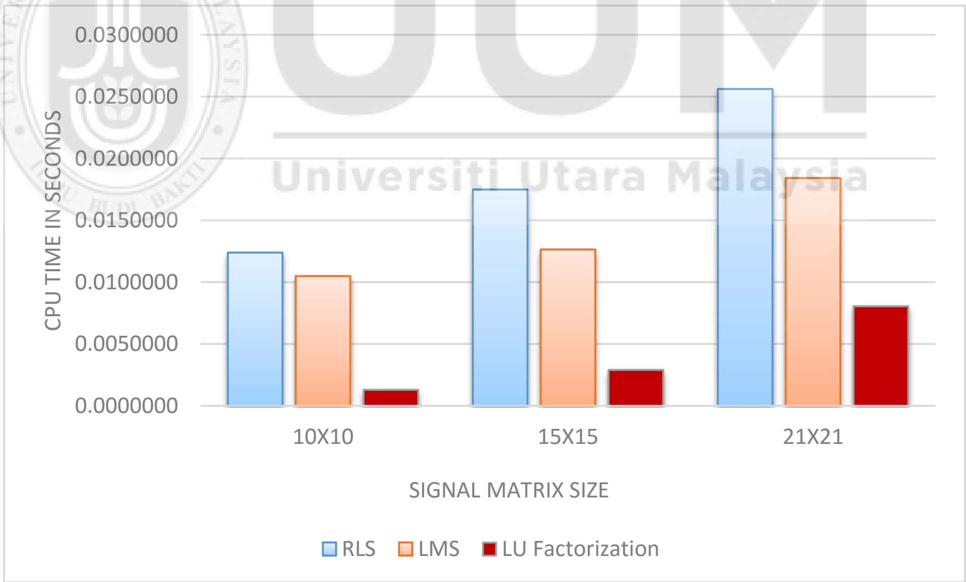


Figure 23 CPU Time Comparison

The above result shows that LU-based Equalization algorithm is ten times and nine times faster than RLS and LMS equalization algorithms respectively, with the note that RLS and LMS works under best scenario condition by setting their taps number to only single tap. During experimentation, tap number is found to be the main parameter which is effecting the CPU time for RLS and LMS, in fact setting the tap's number to more than one in RLS algorithm have a parabolic effect on its computational complexity which makes it have $O(N^2)$ as mentioned before.

On the other hand, LMS step size value is found to be the main parameter that causes the algorithm to have unstable behavior when having big step size number [11]. This is because the algorithm will jump over large number of symbols before start processing again. LU-based equalization algorithm has no step size parameter as it works symbol by symbol which shows no stability issues during operation.

Secondly, the comparison between the time spread delay metric of the equalized signal with the non-equalized signal is taken place. Time delay spread is the delay in time between the receiving of the first guided mode and the last one in an MDM channel. As mentioned before, each MDM channel excites 10 guided modes during its propagation inside the multimode optical fiber. These modes reaches the other end of the fiber at different times resulting ISI.

Observing one of the MDM channels before the time equalization shows that there is a time delay spread of 5.5×10^{-13} seconds between the first arrived guided mode and the last arrived one for each channel. The mitigation of the mode coupling in the time equalized signal resulting only one dominant guided mode for each channel, thus, time delay spread value of each channel was reduced to zero which avoiding any possible ISI due to the modes overlapping. Figures 24-A shows the time delay spread for the first channel of our model before the time equalization, while Figure 24-B shows the same channel after the time equalization.

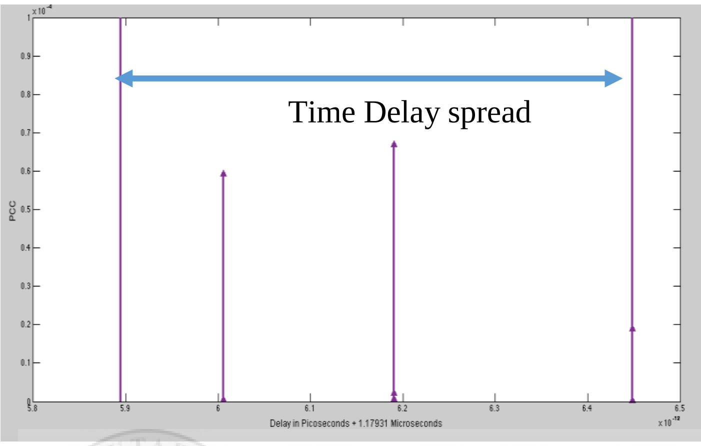


Figure 24-A before Equalization

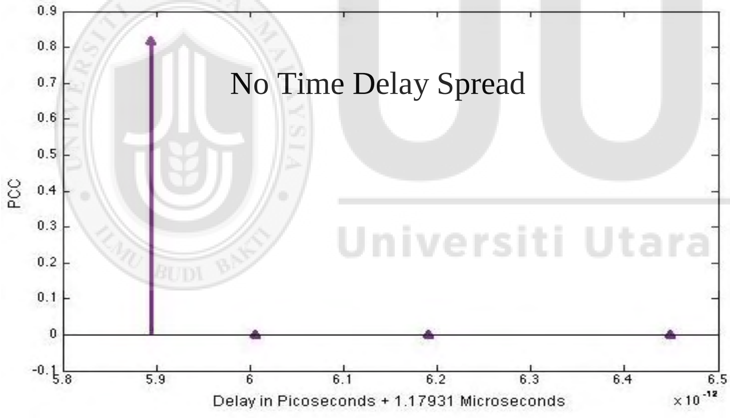


Figure 24-B after Equalization

Figure 24 Time Delay Spread Comparison

On the other hand, the dominant modes are still reaching the other end of the fibre at different times which avoids the symbol overlapping which causing ISI. Figure 25 below shows the comparison in the guided modes overlapping between the channels before and after the ZF time equalization.

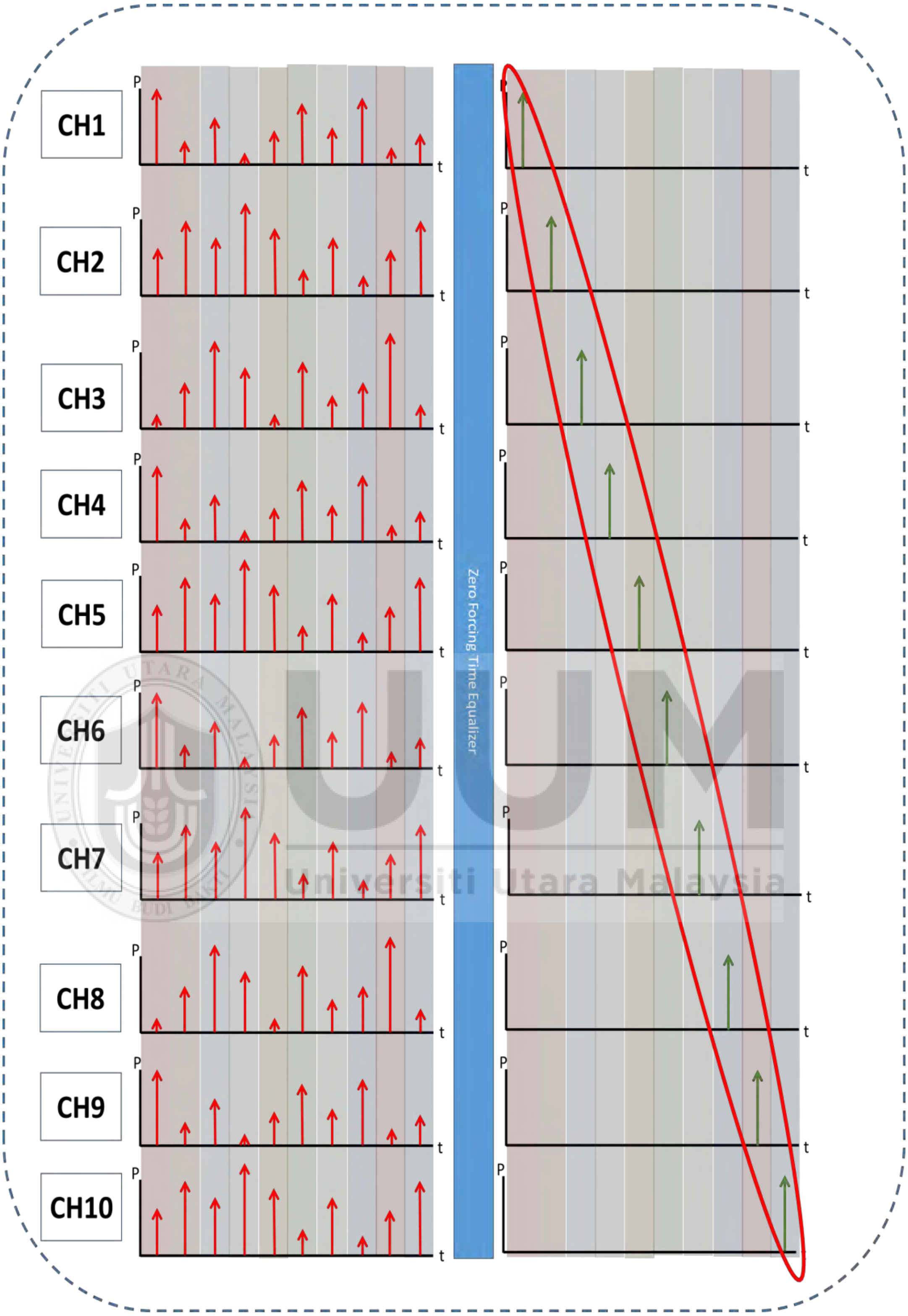


Figure 25 ISI is found in the Left Side (Before Equalization) and no ISI in the right Side (After Equalization)

According to [79, 80], as the symbol time is reduced when reducing the time delay spread in the channel, further reduction in the computational complexity of the equalization algorithm will be taken place.

As previously mentioned in this section, the symbols of the equalized signal are within their time window and with no overlapping, this is has significant improvement to what has been achieved in [12, 85, 92] where the equalized signals still have some minor overlapping. In [12, 92], the equalized signal constellation still has some scattering while in [85], the equalized signal is considered as a long tale signal which is an indication that the symbols still interfering with each other. On the other hand, as per [93], the ideal propagation with no mode coupling is described by a diagonal matrix with values on the diagonal only and zeros elsewhere, this is similar to what is achieved by this research.

On the other hand, as previously mentioned in this section, mitigating the mode coupling resulting only one dominant mode in each channel instead of ten. Based on Equation 22 below [79, 80], the overall channel capacity of the time equalized signal was improved by $\log_2(D)$ comparing with the same signal before the time equalization. Moreover, according to [94, 95], the channel capacity can further increased by the reduction in the time delay spread of the channel.

$$C = \log_2(1 + \frac{p_t}{D}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 22$$

where C is the channel capacity in bits/hertz, p_t is the signal to noise ratio of the channel and D is the number of guided modes in the channel.

4.4.2 Power Equalization Algorithm Evaluation

The standard deviation measurement can be used as a performance metric to show how much the parameter under observation is far from a specific reference. In our case, our reference is the power average value, the standard deviation is equal to:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 23$$

where N is the number channels, x_i is the power of the ith channel and μ is the average power.

Taking the standard deviation of the average of each channel shows significant improvement in the system performance as the standard deviation is reduced from 0.1935 to zero, thus, the detection of the signal will be enhanced [80].

Figure 26 below shows the comparison between the standard deviation measurement before and after performing the power equalization.

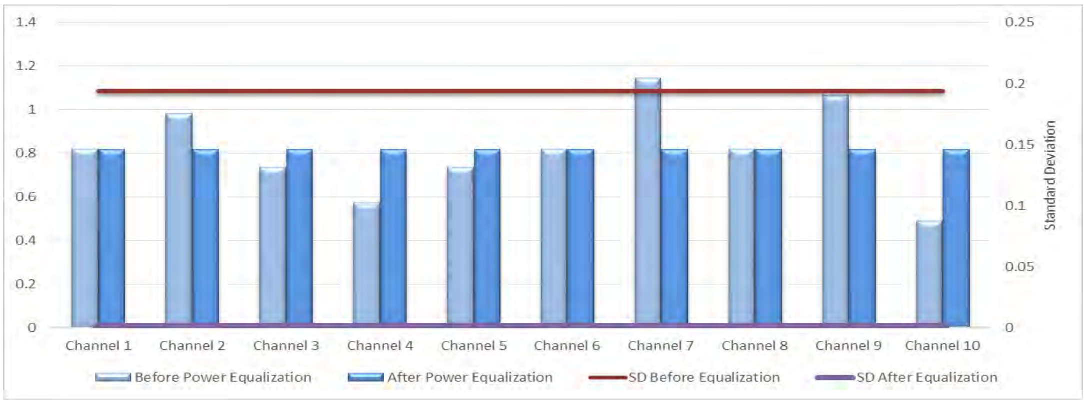


Figure 26 Standard Deviation versus Powers for Each Channel

On the other hand, it was thought that having equal power of the modes in each channel will further enhance the channel capacity by depending this time on the signal bandwidth, thus, further evaluation of this algorithm has been taken place by trying to compute the maximum capacity that the system can handled based on Shanon law:

$$C = B\log(1+SNR) \text{ ....24}$$

where C is the channel, B is the system bandwidth and SNR is the signal to noise ratio.

The system's bandwidth can be measured by taking the Fourier transform of the signal to convert it into the frequency domain, then taking the 3dB point from the obtained graph. However, it was found that the channel bandwidth remains the same before and after the power equalization, and as no changes occurred in the other parameters of Equation 24 above and no further improvement in the channel capacity was detected.

4.5 Summary

This chapter highlights the development stages of mode division multiplexing equalization scheme, the used algorithms. This is fulfill the requirement of the second objective of this research. It also highlight the steps of constructing the optical system model with the main aspects that had been followed to achieve proper simulation environment and deriving the system's transfer function H.

The last part of the chapter highlights the evaluation steps of the developed scheme by checking its performance against several performance metrics which fulfill the requirement for the third and last objective of this research. The gained results show significant improvement in the mode division multiplexing system where the time delay spread, channel capacity and the channel's power profile were enhanced by using algorithms with low complexity compared with the other algorithms.

Chapter Five: Conclusion and Future Works

The aim of this thesis is to develop an equalization scheme for MDM depending on simulation environment and evaluate the developed scheme by checking its performance with several performance metrics. This chapter shows the conclusion came out from the work on this research and as shown in the coming sections. Section 5.1 shows the research findings and the importance of having two-stage equalization scheme. In Section 5.2 the contributions made of this research are highlighted. Finally, Section 5.3 provides some suggestions for further studies.

5.1 Summary of the Research

MDM equalization field is still new and emerging field. It provides researches the rigid ground to explore. Till recently, the developed equalization schemes for MDM are suffering from high computational complexities and/or low stability. The developed algorithms in this research show that in order to equalize any MDM signal, two-stage equalization scheme is needed. The first stage is named MDM Time Equalization (MDM-TE). The goal form this stage is to mitigate the mode coupling effect that leads the receiver to detect many unguided modes, thus, only one mode is detected after this stage resulting a significant reduction in the time delay spread of each channel. According to [94, 95] and as shown previously in chapter 4, reducing time delay spread increases the channel capacity and reduces the complexity [79, 80].

However, the gained signal after the MDM-TE still have different powers which still need to be equalized. MDM Power Equalization (MDM-PE) algorithm is developed based on the idea of the water-filling algorithms currently used in the wireless domain.

The goal form this algorithm is to re-allocate the received powers equally between the channels. As the algorithm is mainly dealing with having the average of power, it is evaluated by depending on the standard deviation of the powers which is becoming zero after running the algorithm on the received signal.

After performing the two stages on the MDM signal, the mode coupling has been successfully mitigated, thus, reducing the possible Inter-Symbol Interference (ISI). The reduction of ISI and mode coupling enhances the Mode Division Multiplexing (MDM) as a new and emerging approach that can increase the distance-bandwidth product of the system and make the use of multimode fibers for long distance application a possible thing to go through.

5.2 Research Contributions

The overall contribution of this research is the development of low complexity equalization scheme for MDM. On the other hand, the research also highlights the similarity between the MDM in optical domain with OFDMA in wireless domain. It shows MDM signal can be treated as a MIMO signal. Previous MIMO signals processing that based on matrix factorization had been re-developed to match the needs on the optical domain. On the other hand, the water-filling algorithm idea in the wireless domain have been utilized for power equalization in MDM. The adaptation of these ideas contributes in mitigating the mode coupling effect in optical fibers and leads to better detection of the data signal by the receiver side.

5.3 Future Works

The proposed scheme in this research improves the optical system capability in mitigating mode coupling effect in multimode fibers. However, there are some limitations as well as pending works that can be pursued for future work. This section outlines some possible extensions.

The LU-based factorization equalizer is developed with this research as a MATLAB code only. Individual component that can be integrated in commercial optical simulators can be done to enhance the working environment of these simulators.

Also, converting the simulation model into real component can be done by completing its logic circuit design then designing and building its equivalent electronic circuit.

On the other hand, designing a multimode optical fiber cable capable of having the capability of propagating specific number of guided modes in order to generate an NXN MDM signals can be achieved. Having real component is very useful and can provides the researchers the ability to study the performance of the developed algorithm by observing real MDM signals.

Achieving the above goals can improve the MDM system performance by enhancing the research environment even more that can lead to accelerate the work on the standardization of MDM to be used in real commercial optical systems.

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