SWGARCH: AN ENHANCED GARCH MODEL FOR TIME SERIES FORECASTING

MOHAMMED Z. D. SHBIER

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Abstrak

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) adalah salah satu model siri masa yang paling popular untuk ramalan siri masa. Model GARCH menggunakan varians jangka panjang sebagai salah satu berat. Data lampau digunakan untuk mengira varians jangka panjang kerana ia mengandaikan bahawa varians untuk tempoh masa yang panjang adalah sama dengan varians untuk tempoh masa yang singkat. Walau bagaimanapun, ini tidak mencerminkan pengaruh varians harian. Oleh itu, varians jangka panjang perlu diberi penambahbaikan untuk mengambilkira kesan seharian. Kajian ini mencadangkan model *Sliding Window GARCH (SWGARCH)* untuk untuk meningkatkan pengiraan varians dalam model *GARCH*. Model *SWGARCH* mempunyai empat langkah. Langkah pertama adalah untuk menganggarkan parameter model *SWGARCH* dan langkah kedua adalah untuk mengira varians tingkap berdasarkan teknik gelongsor tetingkap. Langkah ketiga adalah untuk mengira pulangan tempoh dan langkah terakhir adalah untuk menanamkan varians baru yang dikira daripada data lampau dalam model yang dicadangkan. Prestasi *SWGARCH* dinilai pada tujuh (7) set data siri masa domain yang berbeza dan dibandingkan dengan empat (4) model siri masa dari segi ralat min kuasa dua dan ralat min peratusan mutlak. Prestasi SWGARCH adalah lebih baik daripada *GARCH, EGARCH, GJR* dan *ARIMA-GARCH* untuk empat (4) set data dari segi ralat min kuasa dua dan untuk lima (5) dari segi ralat min peratusan mutlak. Saiz tetingkap anggaran telah meningkatkan pengiraan varians jangka panjang. Penemuan mengesahkan bahawa *SWGARCH* boleh digunakan untuk ramalan siri masa dalam bidang yang berbeza.

Kata kunci: *GARCH*, Ramalan siri masa, Gelongsor tetingkap, varians jangka panjang

Abstract

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is one of most popular models for time series forecasting. The GARCH model uses the long run variance as one of the weights. Historical data is used to calculate the long run variance because it is assumed that the variance of a long period is similar to the variance of a short period. However, this does not reflect the influence of the daily variance. Thus, the long run variance needs to be enhanced to reflect the influence of each day. This study proposed the Sliding Window GARCH (SWGARCH) model to improve the calculation of the variance in the GARCH model. SWGARCH consists of four (4) main steps. The first step is to estimate the model parameters and the second step is to compute the window variance based on the sliding window technique. The third step is to compute the period return and the final step is to embed the recent variance computed from historical data in the proposed model. The performance of SWGARCH is evaluated on seven (7) time series datasets of different domains and compared with four (4) time series models in terms of mean square error and mean absolute percentage error. Performance of SWGARCH is better than the GARCH, EGARCH, GJR, and ARIMA-GARCH for four (4) datasets in terms of mean squared error and for five (5) datasets in terms of maximum absolute percentage error. The window size estimation has improved the calculation of the long run variance. Findings confirm that SWGARCH can be used for time series forecasting in different domains.

Keywords: GARCH, Time series forecasting, Sliding window, Long run variance.

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List of Abbreviations

AE	Artificial Evolution
ANN	Artificial Neural Network
AR	Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BP	Backward propagation
DID	Drainage and Irrigation Department
DM	Data Mining
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedastic
GANN	Genetic Algorithms with Neural Networks
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GJR	Glosten, Jagannathan, and Runkle
GPS	Global Positioning System
GR-NN	General Regression Neural Network
LRA	Linear Regression Analysis
MA	Moving Average
MAPE	Mean Absolute Percentage Error
MSE	Mean Square Error
PCA	Principal Component Analysis
RMSE	Root Mean Squared Error
SVM	Support Vector Machine
SWGARCH	Sliding Window Generalized Autoregressive Conditional Heteroscedasticity

CHAPTER ONE

INTRODUCTION

The subject of time series analysis has drawn significant attention. Since it is of tremendous interest to practitioners, as well as to academic researchers on this topic, therefore, to make statistical inferences and forecasts of future values of the interested variables are very critical. The main targets of the time series analysis are classified into two steps: (1) identifying the mechanism of the phenomena represented by the numerical data; and (2) attempting to predict the future values of the interested variables by analyzing the past data (Cryer and Chan, 2008).

In order to accomplish both of the targets, explicitly expressed statistical models are required to describe the patterns of the observed dataset. To describe data adequately, statistical models are established based on fundamental principles. Furthermore, goodness-of-fit tests and model selection criteria are developed to verify the adequacy of the selected model in describing the data. Once the identified model is confirmed to be adequate, the prediction of the future values can be obtained by extrapolation.

A time series is a set of observations Y_t, with each observation being recorded at a specified time t (Cryer and Chan, 2008). Time series have always been used in the field of econometrics. Already at the outset, Jan Tinbergen (1939) constructed the first econometric model for the United States, and thus started the scientific research program of empirical econometrics time series models, which have wide applications in science and technology (Kirchgassner, 2007). Examples of time series can be found in almost every field of life, including economics, astronomy, physics, agriculture, disaster, medicine, genetic engineering, and commerce.

To perform forecasting, parametric models are often required to describe the patterns of the observed dataset. In order to describe the data adequately, such statistical models should be established based on fundamental principles.

Mathematical models play an important role in the statistical analysis of data. These models can be deterministic or stochastic. In the time series analysis, the first and most important step is to identify the appropriate class of mathematical models for the data. As in regression problems, model criticism is an important stage in time series model building, where the fitted model is under analysis. To improve the model, there is a need to go through an iterative procedure of identification, estimation, and diagnostic checking. The diagnostic checking not only examines the model for possible errors, but it can also suggest ways to improve the model in the next iterative stage (Box et al., 1994).

The classical linear models used in the prediction could not be used for the variance time series dataset (Cohen et al., 2002). Nonlinear model dependence on a series of prior data observation is of interest to several studies, somewhat because of the possibility of producing a chaotic time series. More significantly, experiential investigations found that nonlinear modeling has the benefit to be used in forecasting (Abarbanel, 1997; Kantz & Schreiber, 2004).

In nonlinear time series modeling, there are models to represent the changes of variance over time long heteroskedasticity. These models represent ARCH and comprise a wide variety of representations (GARCH, EGARCH, GJR). Here, changes in the variability are related to the use of past values of the observed series or long run variance making the prediction (Brooks, 2008).

Other methods used for time series forecasting are ARMA and ARIMA (Percival & Walden, 1993). Here, changes in the variability are related to predicting, which depends on the recent values of the observed series.
The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process is an econometric term developed in 1982 by Robert Engle. It is statistical model and there are several forms of GARCH modeling. The GARCH process is often preferred by financial modeling professionals because it provides a more real-world context than other forms when trying to predict the prices and rates of financial instruments (Bollerslev, 1986).

The general process for a GARCH model involves four steps. The first step is to estimate the model parameters; the second step is to compute a long run variance from the historical data, normally in a period of one year of the data (Hull, 2002); the third step is compute the period return from the daily data; and the last step is to use the recent variance that has been computed from the historical data in the GARCH model for forecasting.

In this study, the sliding window (SW) technique is used to capture the time delay between the cause of the event and the actual event (Keogh et al., 2003). This step is called segmentation. For example, how may recent day's effects the current water level of the river. The water level is the cause and the event is the increase in the current water. Figure 1.1 shows the illustration of a sliding window.

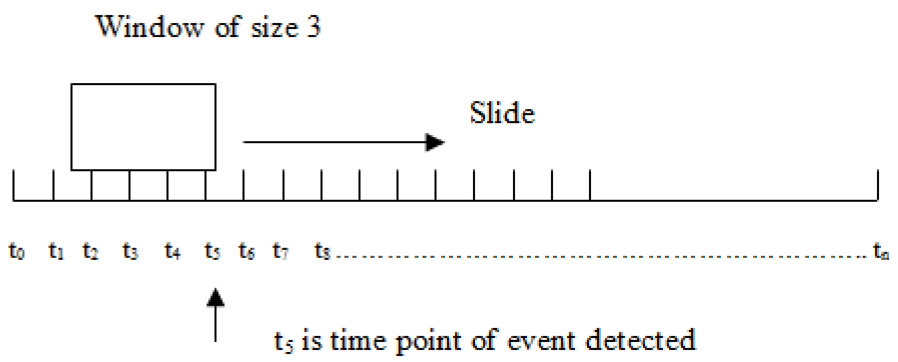

Figure 1.1. An example of sliding window

Therefore, this study has proposed a hybrid model which consists of GARCH model and sliding window technique. The GARCH model is the main algorithm that has been hybridized with sliding window technique for time series forecasting.

1.1 Problem Statement

The first component of the GARCH model is the calculation of the long run variance. The GARCH model has limitation of the long run variance computation based on the historical data. The long run variance is calculated using the whole series. However, using the series does not reflect the influence of daily variance. The variance of one month is similar to the variance of one day back. Therefore, the long run variance needs to be enhanced to calculate the influence of each day differently (Brooks, 2008; Hull, 2015).

Hence, the limitations of the long run variance of the GARCH need to be addressed in order to improve the prediction model. Therefore, in this study, an enhanced GARCH model called SWGARCH model is proposed to overcome the limitation.

The questions of this study are:

- Can a new technique be used to overcome the problem of the long run variance in GARCH model?
- How to develop the enhanced GARCH model by hybridization the new technique and GARCH model?
- Will the performance of the enhanced GARCH model be better than the GARCH and other common hybrid time series forecasting models?

1.2 Research Objective

The objectives of this study are:

- To propose a new technique based on sliding window in calculating the variance in GARCH model.
- To develop an algorithm for the enhanced GARCH model.
- To evaluate the performance of the enhanced GARCH model.

1.3 Scope, Assumption, and Limitation

The scope of the study is to develop a SWGARCH model for time series forecasting. The SWGARCH model is based on the GARCH model. The study focuses on a short-term forecasting of time series data. The data that have been used are: the water level and house price index of Malaysia, house price index of Kuala Lumpur and Florida, daily NASDAQ index, and daily Dow Jones index. The performance of the proposed model is evaluated based on mean square error and mean absolute percentage error and compared with common time series forecasting models.

1.4 Significance of the Research

The outcome of this study is significant because

i. Sliding window variance enables the calculation of variance to improve the forecasting accuracy.
ii. The enhanced model can be used for time series forecasting in several different domains.

1.5 Organization of the Thesis

The thesis is organized as follows. In the second chapter, the relevant literature on time series models are reviewed; while Chapter 3 describes the research methodology used for the research. Also, the suggested models are interpreted and some of their theoretical properties are studied, specifically the sliding window weight. Additionally, the GARCH model is described in this chapter. In the fourth chapter, implementation of SWGARCH model applied for seven case studies is presented. Finally, in Chapter 5, the conclusions of this study are given together with suggestions for future work.

CHAPTER TWO

LITERATURE REVIEW

This chapter presents the reviews of related studies on time series forecasting utilizing stationary and nonstationary, linear, and nonlinear models. The time series analysis is presented in Section 2.1 and time series models are presented in Section 2.2. The ARCH/GARCH models and basic principles from statistics are presented in Section 2.3. Time series modeling approaches discussed in Sections 2.4, while sliding window technique is presented in Section 2.5. The last section presents the summary of this chapter.

2.1 Time Series Analysis

Time series is as a sequence of observations on a variable, regularly taken at equally spread out intervals over time (Falk, 2011). Time series data has a natural temporal ordering and index. Time series analysis includes approaches for examining the time series data in order to extract meaningful statistics, indicators, and other characteristics of the data. Time series predicting is the use of a model to predict future values based on formerly observed data. Time series analysis can be useful to real world values, continuous data, discrete numeric data, or discrete data (Kirchgassner, 2007).

Time series could be found in various domains. The annual crop yield of sugar-beets and their price per ton is an example of time series data recorded in agriculture. Various other examples are exhibited as below: the newspapers' business sections report daily stock prices, weekly interest rates, monthly rates of unemployment, and annual turnovers. Meteorology records hourly wind speeds, daily maximum, and minimum temperature and annual rainfall. Geophysics is continuously observing the shaking or trembling of the earth in order to predict possibly impending earthquakes. An electroencephalogram traces brain waves made by an electroencephalograph in order to detect a cerebral disease, while electrocardiogram traces heart waves. Social sciences survey annual deaths and birth rates, the number of accidents in homes, and various forms of criminal activities. The parameters in a manufacturing process are permanently monitored in order to carry out an online inspection in quality assurance (Falk et al., 2012; Hernandez et al., 2016; Yin and Chen, 2016).

There are clearly many reasons to record and analyze the data of a time series. Among these is the wish to gain a better understanding of the data generating mechanism, the forecasting and prediction of future values, or the best control of a system. The characteristic property of a time series is the fact that the data are not generated independently, their dispersion varies in time, and they are often governed by a trend, i.e. cyclic and seasonal components. Statistical procedures that suppose independent and identically distributed data are, therefore, omitted from the analysis of time series as prepressing data (Falk et al., 2012).

Enormous numbers of various representations are used for time series; however, a common code specifies a time series is Y indexed by natural observations, where the $Y = \ a_1, a_2, a_3, \dots,$ and a_t are the measurements of time series:

$$Y = \ \{a_1, a_2, a_3, \dots, a_t\}$$

A time series analysis and modeling can be done either in the time domain or in the frequency domain. The autocorrelation function and the partial autocorrelation function are time domain concepts, while the spectral density and the power spectral function are frequency domain concepts. In the time domain, the autocorrelation of observations is focused. In the frequency domain, the cyclical movement is concentrated. The same information of a discrete stochastic process can be presented for different insights, and the two forms of time series analysis and modeling are complementary to each other (Cryer and Chan, 2008).

Moreover, time series fitting, analysis, and forecasting methods may be divided into two methods, namely parametric method and nonparametric method. The parametric approaches assume that the underlying stationary stochastic process has a certain structure that can be described using a minor number of parameters. For example, in the Autoregressive (AR) model or Moving Average (MA) model, the task is to estimate the coefficients of the model that describes the stochastic process. By contrast, nonsufficient approaches explicitly estimate the covariance or the spectrum of the process without assuming that the process has any particular structure (Casella et al., 2006).

The approaches of time series analysis and forecasting may also be divided into linear/stationary and nonlinear/nonstationary, and univariate and multivariate. The main purpose of modeling a time series is to predict future values of the time series based on the current and historical values of the time series (Strickland, 2015).

In the linear model, the relationships are modeled using linear predictor utilities, in which unidentified model coefficients are estimated from the recent data. Linear regression is commonly used in modeling the relationship between dependent variables and independent variables (Strickland, 2015).

The nonlinear model normally uses nonlinear regression for modeling. Nonlinear regression is a form of regression analysis, in which observational data are modeled by a function, which is a nonlinear combination of the model coefficient and depends on more than one independent variable. Linear and nonlinear modeling is used for time series analysis. However, nonlinear modeling is the common case for real world case study modeling (Strickland, 2015).

Univariate model analysis is simpler than multivariate model analysis. The main idea is that scalar variables are involved in the analysis. Whereas multivariate analysis is based on multivariate statistics, which involve observation and analysis of more than one variable. Univariate and nonstationary/nonlinear are the common cases of time series analysis and forecasting (Strickland, 2015).

In the context of statistics, econometrics, quantitative finance, seismology, meteorology, and geophysics, the primary goal of time series analysis is forecasting. In the context of signal processing, control engineering, and communication engineering, it is used for signal detection and estimation; while in the context of data mining, pattern recognition, and machine learning, time series analysis can be used for clustering, classification, query by content, anomaly detection, as well as forecasting (Cryer and Chan, 2008).

The characteristic property of a time series is the fact that the data is not generated independently, their dispersion varies in time, they are often governed by a trend, and they have cyclic components. Statistical procedures that suppose independent and identically distributed data are, therefore, excluded from the analysis of time series. This requires proper methods that are summarized under time series analysis (Falk et al., 2012).

2.2 Time Series Models

Time series modeling could have many methods and represent different stochastic processes. When modeling variations in the level of a process, the three broad classes of practical importance are the AR model, integrated (I) model, and MA model. These are the most common classes that depend linearly on preceding data observations (Gershenfeld, 1999). Combinations of these ideas produce the Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models (Mills, 1990; Percival & Walden, 1993). Known as the Box-Jenkins method, the ARMA model involves two parts: an AR model and MA model. ARMA is generally referred to as an ARMA (p, q) model, where p and q represent the order of the models, AR(p), MA(q) (Box & Jenkins, 1976; Box & Jenkins, 1994). The extensions of these classes that deal with vector-valued data are available under the heading of multivariate time series models, and sometimes the preceding acronyms are extended by including an initial *V* for vector, as in *VAR* for Vector Autoregression.

Linear time series models have played an important role in data analysis with a long history. The traditional procedures for time series analysis include the Linear Regression (LR) model, which constructs a bridge formula between a given time series dataset and forecasted value (Cohen et al., 2002). LR is a form of regression analysis; consequently, the function with established regression parameters can be pickled as the minimum of the original time series dataset. The line in the "linear" model may not be a straight line, but rather the way in which the regression coefficients occur in the linear regression formula.

The ARIMA model conceder one of popular non-linear time series. The ARIMA model has been applied numerous times in the research of economic and finance areas, such as electricity market (Jaasa et al., 2011; Sibel & Yayar, 2006), agricultural commodity market (Chen, et al., 2009; Khim-Sen, et al., 2007), and mineral market (Fang & Shen., 2010; Li, 2005).

The nonlinear dependence of the level of a series on prior data observations is of interest, partly because of the possibility of creating a chaotic time series. Nevertheless, more importantly, experiential investigations can show the advantage of using forecasting derived from nonlinear models (Abarbanel, 1997; Kantz & Schreiber, 2004).

Among the other types of nonlinear time series, there are models that represent the changes of variance over time heteroskedasticity. These models represent Autoregressive Conditional Heteroskedasticity (ARCH) and the collection comprises a wide variety of representations (GARCH, EGARCH, GJR, etc.). Here, changes in the variability are related to predict the past values of the observed series. This is in contrast to other possible representations of locally varying variabilities, where the variabilities might be modeled as being driven by a separate time-varying process, as in a doubly stochastic model.

Nonlinear time series model has gained much attention in recent years. Among the successful examples are the ARCH model (Engle, 1982), where the model introduced to capture the serial dependence in conditional variance of a time series, and the threshold autoregressive (TAR) model of Tong (1978) that uses a piecewise linear model to model the conditional mean.

2.3 The ARCH/GARCH Models

The ARCH/GARCH models were proposed in the 1980s by econometricians, such as Robert Engle (2001), who won the Nobel Prize for Economics in 2003 for his work. Since the introduction of the ARCH/GARCH models in econometrics, it has widely been used in many applications, especially for volatility modeling. There are many derivatives of ARCH/GARCH used for different applications and different sets of data, etc. Although, these days, the stochastic volatility models has largely been superseded in academia, the ARCH/GARCH models still have great value and will continue to be used heavily in the industry and finance fields (Bollerslev, 1986).

In order to understand anything about these sorts of models, there is a need to first consider some basic principles from the statistics.

2.3.1 Ordinary Least Squares

Acting as the backbone of a large portion of statistics, sciences, and quantitative analysis in humanities is the humble Ordinary Least Squares (OLS), which is often called linear regression (Wong, 2014).

Economists seek to derive linear relationships between some numbers of variables, because this makes the dynamic changes between them clear in the parameter results. As a result of this simplicity in relationship, there is a strong predictive and explanatory power in the proposed model.

Given any set of data, it would be possible to perfectly fit every single point to some extremely complex function. Nonetheless, this sort of function has no value because it offers little to no explanatory or predictive power. There is simply no way to come up with a coherent relationship between two variables if the equation is some convoluted polynomial with trigonometric functions. Hence, econometricians and other scientists focus on coming up with these simple linear relationships.

The way OLS works is, given some set of observations with n parameters: $\{X_{1,i}\ ,\ X_{2,i}\ , X_{3,i}\ , \dots\ ,\ X_{n-1,i}\ , Y_i\ \}$, where each X_i is to be an independent parameter and Y_i is considered to be the dependent parameter in the true but unknown relationship, $Y_i = \beta_0 + \beta_1 X_{1,i} + \cdots + \beta_{n-1} Xn \quad + u_i$, where β_i is constant and u_i is considered to be an disturbance term. However, since it is impossible to know the true relationship, it is suggested to fit the observations to be close to the real relationship, which is in the form of $\hat{Y}_i = b_0 + b_1 X_1 + \cdots + b_{n-1} Xn -_1$. Then, the equation arises on how to get b_i close to β_i?

The notion of a residual is defined, where $e_i = Y_i - \hat{Y}_i$ and the sum of the squares of the residuals is minimized, i.e. minimize $\sum_{i=1}^{n} e_i^2$ with respect to b_0, b_1, b_n. Minimizing $\sum_{i=1}^{n} e_i$ is less successful since this causes a negative residual to cancel with a positive residual, so $b_0 = \hat{Y}$ and $b_1, \dots, b_n = 0$ can be set and that would be considered a good fit, since that makes the sum of the residuals 0. Minimizing $\sum_{i=1}^{n} |e_i|$ would be feasible, but when it is minimized, the derivatives need to be included, which is very difficult when involving the absolute value (though very possible). Hence, this study chooses to minimize the sum of the squares (Wong, 2014).

2.3.2 The Heteroskedasticity

OLS works great (assuming some preliminary conditions are met), but one assumption that must be made for OLS to work is that the disturbance terms, u_i, homoscedastic [A statistics term indicating that the variance of the errors over the sample are similar], that is, the variance of the errors over the sample are similar $\sigma_{u_i}^2 = \sigma_u^2$ for each i.

However, this is not always a very realistic assumption in real life, since variance is not necessarily always constant. For example, consider the case where a researcher examines the relationship between income and consumption in households. They would likely find that consumption is more closely tied to income in low-income households rather than higher ones, since savings/deficit is likely to be much smaller in absolute value for those households. Then, the variance of those households with higher incomes appears to be much higher, therefore, variance is not constant across the sample (Engle, 2001).

The problem of weighing each data point equally when running statistical tests generally occurs, despite the fact that some of the results may vary from the true model more than others. This makes any statistical analysis inaccurate although the confidence intervals and standard errors will end up being too small. Then, a wrong conclusion might be assumed by thinking there is more precision than there actually is (Wong, 2014).

2.3.3 Autoregressive Models

The generalized AR(*p*) model uses *p* lag variables, which can be written in the form:

$$Y_t = c + \sum_{i=1}^{p} \emptyset_i \; Y_{t-i} + \varepsilon_t \tag{2.1}$$

where *c* is constant, *p* autoregressive terms, $\emptyset_1, \dots, \emptyset_i$ are the model parameters, ε_t is white noise.

The basis behind the AR model comes from the idea that the output/dependent variable is a linear function of its previous values as lag variables. The easiest way to understand this is via an example: the simpler case of an AR model is $AR\ (1)$: $Y_t = c + \emptyset_1 Y_{t-1} + \varepsilon_t$, where $\emptyset_1$ is constant and ε_t is the error term at time *t*, which is considered to be white noise. White noise can be considered as some independent identically distributed random distribution centered around zero. Commonly used is a Gaussian white noise distribution, which is basically a normal distribution with a mean of zero.

Technically, the simplest AR model is AR(0), which is just $Y_t = c + \varepsilon_t$, and is basically just white noise centered around *c*, but this does not give any intuition about how the model should actually look. Since this model can be seen as a case of OLS, therefore it can determine or solve the constants *c* and $\emptyset_i$ by using the OLS procedure detailed above and treat ε as white noise. This sort of model is valuable in financial applications, where the information used to predict the value of some asset is heavily based on the prior values of the asset in earlier time periods. For example, in economics, it can be assumed that the stock price on one day is extremely correlated to the price of the stock from the day before (Cryer and Chan, 2008).

2.3.4 Moving Average Models

The generalized MA(q) model uses q lag error terms, which can be written in the form:

$$Y_t = d + \varepsilon_t + \sum_{i=1}^{p} \theta_i \ \varepsilon_{t-i} \tag{2.2}$$

where *d* is constant, ε_t is white noise, *p* autoregressive terms, $\emptyset_1, \dots, \emptyset_i$ are the model parameters (Wong, 2014).

Unlike the AR models, moving average models utilize past error terms in order to forecast future terms. Of course, this is not a true regression model, because each ε_t is not actually known (again, these $\boldsymbol{\varepsilon}$ terms are considered to be white noise or random shocks). However, a mathematical representation of the MA model can be written, which will be useful later on in the ARMA/ARIMA/ARCH/GARCH models. The simplest (nontrivial) case of MA is MA(1), which can be written in the form $Y_t = d + \varepsilon_t + \theta_1 \varepsilon_{t-1}$, where θ_1 is constant and each ε is a white noise term.

Again, this model is extremely valuable in financial applications, where it is considered that the price of some asset is affected by a sum of stochastic shocks over time, again from the information set. However, unlike in the AR model, OLS cannot be simply applied to solve the θ coefficients, since each ε term is completely unknown. The method of solving MA coefficients involves solving a system of nonlinear equations (Wong, 2014).

2.3.5 ARMA/ARIMA Models

The generalized ARMA(p, q) with p autoregressive and q moving average parameters can be written in the form:

$$Y_t = c + \sum_{i=1}^{p} \emptyset_i \; Y_{t-i} + \sum_{i=1}^{q} \theta_i \;\; \varepsilon_{t-i} \tag{2.3}$$

where c is constant, p autoregressive terms,$\emptyset_1, \dots, \emptyset_i$, q moving-average terms, $\theta_1, \dots, \theta_i$ are the model parameters, ε_t is white noise. This equation used for stationary modelling.

The ARMA model is derived by combining autoregressive terms and moving average terms to create a more complete model (AR + MA = ARMA) (Cryer and Chan, 2008). A very simple case of ARMA is ARMA(1, 1): $Y_t = c + \varepsilon_t + \emptyset_1 Y_{t-1} + \theta_1 \varepsilon_{t-1}$, with one autoregressive term and one moving average term. A possible interpretation of this model could be Y_t being the price of some asset at time t, which is a function of the price of the asset at time $t - 1$ (Y_{t-1}), a random shock at time t, (ε_t), random shock at time $t - 1$, (ε_{t-1}), along with a constant c.

In general, p and q are not large because:

1) The coefficients are likely to get small and are not statistically significant with too many lag terms,
2) The interpretations can get difficult with such large models, and
3) With too many terms, it is possible to lose the predictive power due to overfitting. Overfitting is the case where there are too many parameters and could cause to model the random noise rather than the actual underlying relationships.

ARIMA can be considered to be a further generalization of ARMA. However, to understand this, there is a need to address the topics of stationarity, differencing, etc. (Wong, 2014).

The generalized ARIMA model is hard to write because the difference between the variants is hard to capture explicitly. However, using our previous notation, we can write the general ARIMA (p, d, q) model has p autoregressive terms and q moving average terms, with d degree of differencing in the form:

$$Y_t^{(d)} = c + \sum_{i=1}^{p} \emptyset_i \; Y_{t-i} + \sum_{i=1}^{q} \theta_i \; \varepsilon_{t-i} \tag{2.4}$$

We can usually take d = 1 or at most 2. The ARIMA models is to generalize

ARMA models in the analysis of nonlinear time series data, by combining differencing, moving average terms and autoregressive terms. We can also think of AR, MA, and ARMA all as special cases of the more general ARIMA model. ARIMA models are extremely useful in time series econometrics and statistics and have a variety of applications (Dougherty, 2011).

2.3.6 Stationarity

ARMA are time series models for stationary data, in which, despite the data being stochastic, the probability distribution of the data remains constant. Any time series data with trends or seasonality (regular cycles) cannot be considered to be stationary. In very simple terms, the data should look roughly similar at any point in time. A time series with a cyclic behavior can actually be stationary, as long as it is non-regular (cycles fixed length), so at some point in time, it is impossible to know some sort of peak or trough of the dataset. Most of the time, stationary data stays relatively flat, with a constant variance due to the fact that the probability distribution is constant (Wong, 2014).

2.3.7 Differencing

However, a lot of useful time series data is nonstationary, e.g. stock indices, like the Dow Jones, experience obvious trends over periods of time. Some phenomena experience regular literal seasonal changes, such as the cost of heating oil. Therefore, one way to still work with nonstationary time series data is with differencing. Since this study works with discrete data, there is exactly a notion of derivatives, hence, $Y_t' = Y_t - Y_{t-1}$. (Minor detail, given n observations, the differenced data will have $n - 1$ observations). Differencing allows to potentially stabilize the mean of the time series, so that trends and seasoning can be removed. For example, the Dow Jones data might have trends over some period of time, but on a day-to-day basis, the change in the Dow Jones is very likely to be centered at zero. The logarithms method can be used to normalize variances; therefore, the nonstationary time series data can be turned into stationary time series data that can be worked with.

Of course, differencing will not always work in one process and so there may be a need to reiterate the process. For example, the second differencing is given by $Y_t'' = Y_t' - Y_{t-1}'$, which can be generalized to anything desired. However, in the real world case, the maximum order of ARMA is two, because the explanatory power will be lost when going to the third derivative (Wong, 2014).

The ARCH model was introduced by Engle in 1982, the way that econometricians described variances of models was to use a rolling standard deviation. It is where one could equally weigh all the observations by the standard deviation over some number of previous observations, as below:

$$\sigma_{u_{t+1}} = \frac{1}{n} + \sum_{i=0}^{n} \sigma_{u_{t-i}} \tag{2.5}$$

where *n* is number of observations, σ is variance, u_t recent return.

However, the question is not always whether a model is a good fit for the data, sometimes it is simply to consider the accuracy of the model itself and whether its predictions are valid or not. One method of testing the accuracy is to look at the variance of the error terms.

A good way to think about ARCH is to think of it as a generalization of this formulation, instead of weighing each value equally, ARCH treats the weights as parameters to be estimated. This is more realistic because:

1) Assuming equal weights seems inaccurate since it is presumed that more recent observations are more likely to be more relevant, and
2) Restricting the weights only to some finite number of observations is not ideal.

The general ARCH(n) model of order m is as below:

$$u_t^2 = c + \left(\sum_{i=1}^{n} \alpha_i u_{t-i}^2\right) + \omega_t \tag{2.6}$$

where c is constant, *a* is model parameter, *u* is return, *w* is error.

ARCH is essentially the combination of the AR and MA models that are applied to the disturbance terms.

The generalization of ARCH to GARCH is analogous to the generalization of ARMA to ARIMA-GARCH, which basically says that the best predictor of the variance in the next time period is given by a weighted average of the long-run average variance, the variance predicted in this period by (G)ARCH, and new information given in this period (Wong, 2014).

2.4 Time Series Modeling Approaches

In order to use time series modeling, there are two common approaches. The first approach is use a standalone time series model. Meanwhile, the second approach is to use a hybrid model. The following two sections exhibit the two modeling approaches.

2.4.1 The Time Series Approach

In the study performed by (Babu and Reddy, 2012), three different types of ARIMA models have been used in order to analyze and predict the Average Global Temperature. In their study three models has been used which are ARIMA model, Trend-Based ARIMA model, and Wavelet-based ARIMA model. The ARIMA model used three steps. The first step is making the data stationary by performing the differencing operation. The second step is identifying the suitable values for model order by ACF and PACF. The third step is predicting future values using ARIMA technique. Trend-based ARIMA model consist of two steps. First step smoothening the data has been used as a preprocessing. Second step is predicting future values using ARIMA technique. The Wavelet-based ARIMA model is also consists of two steps. First step is the preprocessing of data using the wavelet technique and the second step is predicting future values using ARIMA technique. The performance of the proposed method was performed based on MAPE, maximum absolute percentage error (MaxAPE), and Mean Absolute Error (MAE). It was concluded that Wavelet-based ARIMA performs the best out of the three models.

Chen et al. (2008) used the ARIMA model for the short-term forecasting of property crime for one city of China. The results of ARIMA are compared with the other two exponential smoothing models, namely, simple exponential smoothing (SES), and Holt two-parameter exponential smoothing (HES). The 50 weeks' property crime recordings are chosen as sample series in order to meet the basic requirements of the ARIMA model. Root Mean Square Error (RMSE) and MAPE are used as performance comparison criteria. The result shows that ARIMA is the best model.

The ARIMA model used in their study is accurate, simple, and fast in computation. It is suitable for this dataset. However, the prediction model used one dataset in this comparison. In this case, it could not be a generic model.

A study conducted by Akpinar and Yumusak (2013) used the ARIMA model for forecasting natural gas consumption in Turkey. Their model is summarized in three steps. The first step is removing the cycling component in time series as data preprocessing. The second step is to split the dataset into six datasets in terms of month period. The third step is applying the ARIMA model using parameters ranging from (0,0,0) to (2,2,2) to each dataset. The last step is merging the results of these models. The compression criteria used in this study are Relative Absolute Error (RAE), MAPE, RMSE, and Standard Percent Error. ARIMA(1,0,1) perform the best model in term of MAPE against others models. The data splitting/merging technique is an efficient way to enhance model performance. Furthermore, the prediction model is not generic and it uses one dataset.

A study done by Xie et al. (2013) developed a seasonal ARIMA model with exogenous variables (SARIMAX) to predict day-ahead electricity prices in the Elspot market, the largest day-ahead market for power trading in the world. Compared with the ARIMA model, the SARIMAX model is a composite technique. The first feature is a seasonal component that is introduced to cope with the weekly effect on price fluctuations. The price dataset used in this study consists of 730 daily observations from 1st January 2010 to 31st December 2012. Four exogenous variables are selected: hydropower production, nuclear power production, thermal power production, and wind power production. Weekly effects have been observed in Elspot prices as seasonal component. Prices tend to be lower on weekends than those of weekdays. The performance of this model is evaluated in terms of MAPE and MaxMAPE. The value of MAPE and MaxMAPE are 1.95% and 8.85%. Furthermore, the errors are also has been compared to the ARIMA models developed by other researchers (Jacasa et al. 2011), the ARIMA model performance was a MAPE of 2.38% and a MaxMAPE of 14.74%. The results show SARIMAX model performs better than ARIMA model.

Yin and Chen, (2016) have used EGARCH and ANN for predicting return series of CNY/USD exchange rate. The dataset employed in their study consists of daily exchange rates of CNY/USD from June, 2010 to end of February, 2015, The datasets are collected from http://www.safe.gov.cn. The EGARCH/ANN model can efficiently capture the properties of nonlinearity as well as volatility. Three models have been employed in their study namely Elman Neural Network, Neural Network, and EGARCH. RMSE and MAP have been used as performance criteria. RMSE results for Elman neural network, neural network, and EGARCH are 0.000247, 0.000848, and 0.00082 respectively. By comparing the error indicators, it can be concluded that the Elman Neural Network performs better than the EGARCH-M, neural network

2.4.2 Hybrid Time Series Approach

Hybrid GARCH-Neural Network (GARCH-NN) model has been proposed by Li Si-ming et al. (2012) for the prediction of stocks of the Shenzhen Stock Exchange Index in the Chinese stock market. The hybrid method consists of two steps. First step is the selection of GARCH model according to the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) criteria. Second step is to hybridize with Neural

Networks. The GARCH models are GARCH, EGARH, IGARH, TGARH, and GARH-M. Improving the performance for forecasting is the goal of hybrid model. The advantage of NN models are their ability to model complex nonlinear relationship without a priori assumptions of the nature of the relationship. Mean Squared Error (MSE) is used as performance criteria in the study. From the result, GARCH-M is the best of the GARCH family models. NN model is complex and it requires long computation time.

In the study done by Hajizadeh et al., (2012), hybrid models which incorporate a series of GARCH family model and ANN have been used to examine their ability in enhancing the forecasting the volatility of US Real Estate Investment Trusts market (REITs) market. The GARCH models used in their study are ARCH(1,1), AR(2)-GARCH(1,1), GARCH(1,1), GARCHM(1,1), EGARCH(1,1), TGARCH(1,1), IGARCH(1,1), PGARCH(1,1), and GARCH(1,1). The hybrid models are ANN-ARCH, ANN-ARGARCH, ANN-GARCH, ANN-GARCHM, ANN-EGARCH, ANN-TGARCH, ANN-IGARCH, ANN-PGARCH, and ANN-GARCH models. Results showed that EGARCH model has the highest prediction accuracy for volatility. Furthermore, the hybrid models of ANN-EGARCH model perform outstanding predictive power for the one-step-ahead forecasting in term of RMSE.

Monfared and Enke (2014) proposed a hybrid GJR/Neural Network model for volatility forecasting in the financial market. Three types of Neural Network models have been used in this study. The models are feed-forward with back propagation, generalized regression, and radial basis function. Four datasets between 1977 to 2011 representing real and contemporary periods of market calm and crisis have been employed in the study. Results show that neural networks improved the forecasting ability of the GJR-GARCH during crisis. In low volatility periods, it is recommended that neural networks architectures, as well as the GJR, not be used for forecasting purposes. The hybrid model is not beneficial due to the unnecessary complexity of the model

The study conducted by Lu et al. (2016) amid to compares the forecast performance of volatilities between two types of hybrid ANN and GARCH-type models. They used ANN-EGARCH, ANN-GJR, EGARCH-ANN, and GJR-ANN for forecast the volatilities of log-returns series in Chinese energy market. The Chinese energy index in Shanghai Stock Exchange from 31 December 2013 to 10 March 2016 has been used in their study. In order to evaluate the performance of models in forecasting volatility, RMSE is employed. The results show that ANN-EGARCH is the best model when compared with ANN-GJR EGARCH-ANN, and GJR-ANN.

Chen et al. (2011) used the ARIMA-GARCH hybrid model for traffic flow prediction. The model combines the linear ARIMA model with the nonlinear GARCH model, so that it can capture both the conditional mean and conditional heteroscedasticity of traffic flow series. The performance of the hybrid model is compared with that of the standard ARIMA model in terms of MAE, MSE, and Mean Relative Error (MRE). The results of the ARIMA model and ARIMA-GARCH model prediction performance are relatively similar. The general GARCH model combined with ARIMA of the same order cannot always improve the prediction accuracy. The performance enhancement of ARIMA-GARCH against ARIMA is 2%. In addition, a certain approach has to be developed to give a more efficient prediction performance.

Narendra and Reddy (2014) used the ARIMA-GARCH model for the predictions of the Indian stock. In their model, the MA filter is used to decompose the given time series data into two components: low volatile component, and highly volatile component. The ARIMA model used the low volatile component as input, while GARCH used the highly volatile component as input. The final model used the two outputs of the ARIMA and GARCH models. The proposed model is compared against ARIMA, trend-ARIMA, wavelet-ARIMA, and GARCH models. The performance measures used for comparison are the error measures, mean absolute percentage error (MAPE), MaxAPE, MAE, and RMSE. For example, the MSE of ARIMA-GARCH, GARCH, ARIMA, trend-ARIMA, wavelet-ARIMA, and GARCH models are 0.1976, 0.3630, 2.4, 0.2108, and 0.2011, respectively. The results obtained confirmed that the prediction accuracy is better compared to the other models. Unfortunately, the prediction model comprises two ARIMA-GARCH models, which are complicated in nature and consist a long processing of computation. Furthermore, the prediction model is not generic and can only predict on one type of data. Therefore, a certain approach has to be developed in finding a generic and efficient prediction model.

In the study done by Areekul et al. (2009), a combination of the ARIMA and ANN models is used for predicting short-term electricity prices. This model is examined by using the data of Australian National Electricity Market, New South Wales regional in 2006. A comparison of the forecasting performance with the proposed ARIMA and ARIMA-ANN models is presented. The performance based on MAPE, MAE and RMSE of the ARIMA-ANN model is better in accuracy than the ARIMA model. The results of the ARIMA-ANN model showed that there is a small percentage of improvement over the ARIMA model. Thus, this combination model gives better predictions than the ARIMA model forecasts, and its overall forecasting capability is improved. Experimental results indicate that the combined model can be an effective way to improve forecasting accuracy that can be achieved by either of the models used separately. Hong-qiong and Tian-hao (2007) used the hybrid ARIMA-ANN model for short-term traffic flow forecasting. ARIMA is used for linear prediction and ANN is used for nonlinear prediction. The final forecasting computed by sums the output of the two models. The performance of the hybrid model is compared against the individual models based on MSE and MAPE. The values of these models, ARIMA-ANN, ARIMA, and ANN, in terms of MSE are 0.063490, 0.113427, and 0.109432, respectively. The values in terms of MAPE are 0.072845, 0.132167, and 0.095831, respectively. Therefore, ARIMA-ANN gave the best performance when compared to ARIMA and ANN. However, ARIMA-ANN model is complicated and requires lengthy processing (computation) and the model was only tested on one dataset.

Puspitasari et al. (2012) presented a forecasting model for half-hourly electricity load in Java-Bali Indonesia by using the hybrid ARIMA-ANFIS model. Their algorithm applied half-hourly electricity load data in Java-Bali from 1st January 2009 to 31st December 2010, which is measured in mega watt. The hybrid ARIMA-ANFIS model involves three steps. First, the ARIMA model is used based on the Box-Jenkins methodology. Second, the residuals of ARIMA are applied as input for the ANFIS model. As for the last step, the final forecast is calculated by combining the forecast of ARIMA in the first step and the forecast of ANFIS at the second step.

Rahman et al. (2013) compared the ARIMA and ANFIS models for forecasting the weather conditions in Dhaka, Bangladesh. Maximum temperature, minimum temperature, humidity, and air pressure are used in their research between 2000 and 2009. The comparison is done in terms of sum of square error (SSE), R^2 error, RMSE, and MAE. The ARIMA model has better performance than the ANFIS model.

Nguyen et al. (2013) used ANN and interval type-2 fuzzy system (IT2-FLS) for stock price forecasting. The hybrid approach consisted of two components. The first component of the hybrid ANN model is used to select inputs that are highly relevant to the dependent variables. The IT2-FLS is employed as the second component of the hybrid forecasting method. The IT2-FLS's parameters are initialized through the deployment of the k-means clustering method and they are adjusted by the genetic algorithm. The result show that the hybrid performs better than ANN.

In a study done by Khandelwal et al. (2015), they used a hybrid model consisting of Discrete Wavelet Transform (DWT), ARIMA and ANN models. The model is used for forecasting the annual number of lynx trapped in the Mackenzie river district; the weekly exchange rates from British pound to US Dollar; the monthly mining data of India; and the Average Monthly Temperature of Las Vegas, US. At first, DWT is used to decompose the in-sample training dataset of the time series into linear (detailed) and nonlinear (approximate) parts. Then, the ARIMA and ANN models are used to separately recognize and predict the reconstructed detailed and approximate components, respectively. In this manner, the proposed approach tactically utilizes the unique strengths of DWT, ARIMA, and ANN to improve the forecasting accuracy. The results shows clearly demonstrate that the method has yielded notably better forecasts than ARIMA, ANN, and Zhang's hybrid model (Zhang, 2003). The model performance comparison done in term of MSE are 0.00434, 0.01819, 0.01774, and 0.01550, for ARIMA, ANN, and Zhang's model, respectively.

The prediction model comprises three DWT-ARIMA-ANN models, which are complicated and consist a long processing of computation. Moreover, the ANN model is complicated in nature. On the other hand, the model has been used for four datasets and the prediction performance is good. Consequently, the model could be a generic model for prediction.

2.5 Sliding Window Technique

Changbao et al. (2016) used Fourier analysis algorithm based on sliding window to overcome shortcoming of traditional time domain sequence component detection algorithm. Symmetrical component method was used in calculating the sequence component of the fundamental phasor in this algorithm after using the calculation of continuous average sliding window of Fourier analysis to separate the real component and the imaginary component of the fundamental phasor from three input signals. The Matlab simulation indicated that using sequence component detection algorithm based on sliding window had high accuracy and satisfying real time dynamic performance stability.

Shatkay and Zdonik (1996) have used sliding window for approximate queries and representations for large time series and obtaining break points in places where the behavior of the sequence changes significantly in approximate queries. In their study, they have developed a breaking algorithm which is used for searching patterns of behavior in dataset. Sliding window has been employed to obtaining break points in queries. In this case, they emphasized on the need for queries that are based on patterns of behavior rather than on specific values, and the need to reduce the amount of stored and scanned data while increasing the speed of access

Extensive review and empirical comparison of time series segmentation algorithms from a data mining perspective has been studied by Keogh et al. (2003) Three major approaches to time series segmentation have been reviewed. The approaches are sliding window, top-down, and bottom-up. The main problem with the sliding window algorithm is its inability to look ahead and lacking the global view. The bottom-up and the top-down approaches produce better results, but are offline and require the scanning of the entire dataset. This is impractical or may even be unfeasible in a data mining context, where data are in the order of terabytes or arrive in continuous streams. The authors proposed a novel approach in which they capture the online nature of sliding window and yet retain the superiority of bottom-up. Empirically results show the new approach to be superior to all others.

In a study by Tang et al. (2009), they propose a new approach for genetic association analysis that is based on a variable-sized sliding-window framework and employs principal component analysis to find the optimum window size. The performance of the proposed method has been evaluated against single marker method and a variable-length Markov chain method in term of type I error rate (false positive). The variable-sized sliding-window approach proved to be the best model.

2.6 Summary

The ARCH/GARCH models were proposed in the 1980s by econometricians, such as Robert Engle (2001), who won the Nobel Prize for Economics in 2003 for his work. Since the introduction of ARCH/GARCH model, it has been widely used for volatility modeling in different applications. GARCH model is one of the most used model for time series forecasting. Hybridization of GARCH model is the most popular approach for volatility modeling.

CHAPTER THREE

METHODOLOGY

This chapter presents the framework and methodology of this study. Furthermore, the dataset description, evaluation measure, and numeric example are also presented. The rest of this chapter is organized as follows. Section 3.1 presents the research framework while Section 3.2 describes the enhance SWGARCH model development. The algorithm on of SWGARCH is presented in Section 3.3. Section 3.4 provides the evaluation procedure used in this study while the data use in the evaluation are described in Section 3.5. Numeric examples are presented in Section 3.6. Finally, Section 3.7 summaries this chapter.

3.1 The Research Framework

The research framework is the roadmap of the research that aims to provide guidance to researchers for conducting research (Forrester, 2006). In accordance with the purpose of this study which is to develop an algorithm for the enhance GARCH model, thus to achieve this goal, there are three phases of the research work. For every phase there is an objective that will be achieved. The research framework is depicted in Figure 3.1.

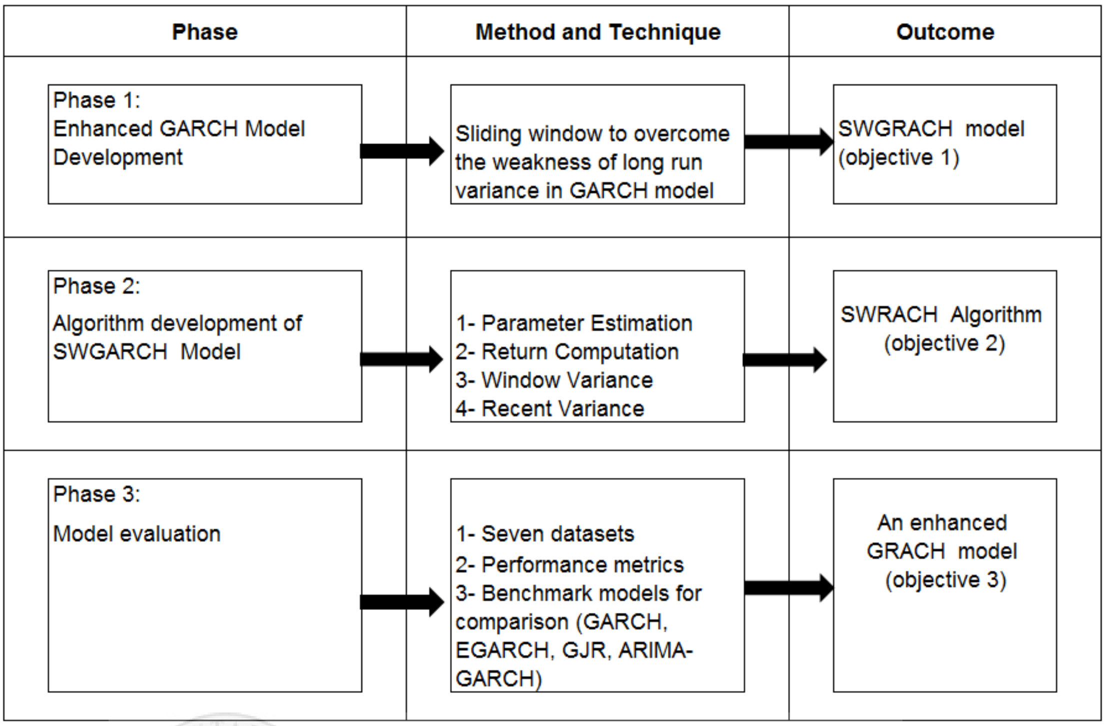

Figure 3.1. Research Framework

This study proposes an enhanced GARCH model, which is the hybridization between GARCH model and sliding window technique. The three main phases of the research work are: (1) enhanced GARCH model development; (2) algorithm development of SWGARCH model; and (3) evaluating of SWGARCH model. Each phase of the framework has its own research method. The following sections describe the phases in the research framework.

3.2 Enhanced GARCH Model Development

The weakness of the GARCH model (the calculation of variance) is fixed by changing a component of the variance (i.e. the long run variance) with a component called window variance. The window variance is calculated using the sliding window technique. This would incorporate more recent return which will provide greater weight. This window variance (V_w) can be calculated as shown in Equation (3.1).

$$V_w = u_t^2 \cdot W_1 + u_{t-1}^2 \cdot W_2 + \cdots + u_{t-n}^2 . W_n \tag{3.1}$$

where u_{t-i} is return and W_{t-i} (i = 1,2,3,… , n) is window weight subject to

$$W_t + W_{t-1} + W_{t-2} + \cdots + W_{t-n} = 1 \tag{3.2}$$

Each time series well have their own values for the window variance. For example, one year is the window size for economic time series, while three days is the window size for flooding (Ku Mahamud et al., 2009).

The enhanced model known as SWGARCH calculates the variance $\mathrm{\sigma_n^2}$ using window variance (V_W) as well as from the period return ($\mathrm{u_{n-1}^2}$) and recent variance ($\mathrm{\sigma_{n-1}^2}$) as shown in Equation (3.3).

$$\sigma_n^2 = \gamma V_w + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{3.3}$$

where V_w is window variance, u_{n-1} is return and σ_{n-1} is recent variance. The parameters γ, α and β are assigned to window variance, return, and recent variance, respectively. In SWGARCH model, σ_n^2 is based on the most recent observation of return.

The return u_n is defined as the continuously compounded return during day *i* (between the end of day *i*-1 and end of day *i*). In other words, the return is the gain or loss in a particular period.

3.3 Algorithm Development of SWGARCH Model

Algorithm development of SWGARCH model involves four (4) steps as shown in Figure 3.2.

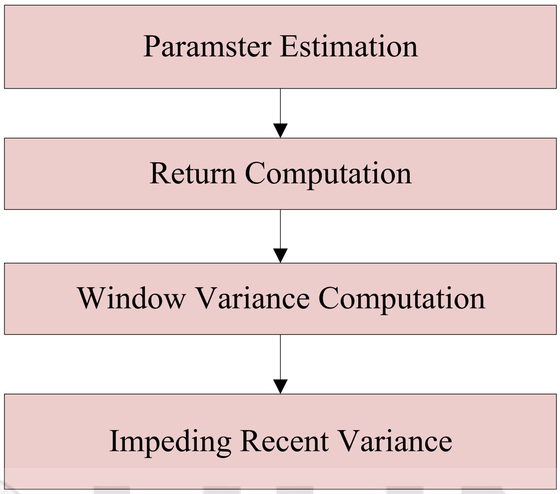

Figure 3.2. SWGARCH algorithm

The first step is to estimate the model parameters followed by the computation of the period return in the second step. The third step is to compute the window variance and the final step is to impeding the recent variance from historical data. Equation (3.3) shows SWGARCH model to calculate the variance. Estimating the SWGARCH parameters, return computation, computation of sliding window variance, and recent variance are explained in the following sections.

3.3.1 Estimating SWGARCH Parameters

The parameters γ, α and β are estimated from the sample of historical data. The approach used is known as the Maximize Likelihood Method (MLM). It involves choosing values for the parameters that have maximum chance or likelihood of the data (Hull, 2015). Maximize likelihood is an iterative searching tool to find the parameters in the model that maximize the expression as shown in Equation (3.4).

$$\sum_{i=1}^{m}\left[-ln(\delta_i) - \frac{u_i^2}{\delta_i}\right] \quad (3.4)$$

where u_i is the return and δ_i is the change in values at time t of observation i. δ_i is calculated as in Equation (3.5).

$$\delta_i = (S_i - S_{i-1}) / S_{i-1} \quad (3.5)$$

where S_i is the actual value at time i. The likelihood measure (L) is given by Equation (3.6).

$$L = -ln(v_i) - v_i^2 / v_i \quad (3.6)$$

where v_i is variance of day i. Using MLM, The sum of γ, α, and β is equal to 1.

3.3.2 The Return Computation

Data other than the ones that have been used in estimating the parameters will have to be used in the return calculation. Accounting to Hull, (2015), the return value at time t for each observation is computed by squaring the period return which is calculated as shown in Equation (3.7).

$$period\ return\ at\ time\ t = ln\left(\frac{s_t}{s_{t-1}}\right) \quad (3.7)$$

where s_t is the actual value at time t.

3.3.3 Computation of Sliding Window Variance

SWGARCH model introduces a new method by using sliding window variance in obtaining the weights for forecasting the future value. This method is composed of three steps:

a) Estimate the window size.

b) Calculate weight of each observation within the window.

c) Multiply each weight by the return, and then compute the sum of the multiplied values.

Principal component analysis has been used to estimate the window size (Tang et al., 2009). The window size is identified where there is a big drop in the scree plot of the variance. The older an observation, the less weight it is given in arithmetic. Weight of each observation within the window composed of two steps. First step is to calculate the total of window size weights (TL) using Equation (3.8).

$$TL = w_n + w_{n-1} + w_{n-2} + \cdots + w_1 = 1 + 2 + 3 + \cdots + n \tag{3.8}$$

where w_n is weight at day n. Second step is to normalize the weight as shown in Equation (3.9).

$$W_i = \frac{w_i}{TL} \tag{3.9}$$

where w_i is the weight for each observation. Multiplying each weight by the return and compute the sum of the multiplied values is shown in Equation (3.1) is the result of window variance.

3.3.4 Recent Variance

The recent variance is used as the third component of SWGARCH model. SWGARCH variance calculation is a recursive procedure approach.

3.3.5 SWGARCH Algorithm

This study has implemented low level hybridization where GARCH is the main algorithm which during its implementation will integrate the sliding window technique to enhance the variance. The hybrid algorithm is called SWGARCH and this hybrid algorithm will refine the long run variance in GARCH by changing the weights. Figure 3.3 displays the pseudocode for SWGARCH algorithm.

Step 1	Load data; //Estimate model parameters γ , α , β	
Step 2	Call estimate parameters *gamma*, *alpha*, *beta* (sample data)	
Step 3	While not EOF	
	//Compute period return	
Step 4	pr= *ln*(s_t/s_{t-1}); //Compute return	Equation (3.7)
Step 5	r=(pr)^2;	
	//Compute Window Variance *//Estimate window size*	
Step 6	ws=n; //example n=3	
Step 7	$r_1=s_{t-1};r_2=s_{t-2};r_3=s_{t-3}$;	
Step 8	$w_1=3/6;w_2=2/6;w_3=1/6$;	Equation (3.9)
Step 9	$w_v= (r_1 * w_1) + (r_2 * w_2) + (r_3 * w_3)$;	Equation (3.1)
	//Compute model variance	
Step 10	swgarch=(*gamma* * w_v) + (*alpha* * r) + (*beta* * $swgarch_{n-1}$);	Equation (3.3)
Step 11	End While	

Figure 3.3. SWGARCH pseudocode

The first step is load data for SQL server database. Second step is to call the MLM model parameters estimation. Third step is start loop for all data in the dataset. Fourth step is to compute period return using Equation (3.7). Fifth step is calculating the return. Sixth step is estimation of the window size. The window size estimation details were presented in Section 3.1.5. Seventh step is to retrieve recent three return values. Eighth step is normalizing weight for window variance. Ninth step is computing window variance using Equation (3.1). Eleventh step is the end of the dataset loop. The SWGARCH has been implementing using C# and the programming code is presented in Appendix A.

3.4 Evaluation of SWGARCH Model

The descriptions of the datasets, evaluation metrics and the benchmark models are presented in this section. A dataset is use to display the proposed hybrid algorithm.

3.4.1 Datasets

In this study, seven datasets have been collected, namely Senara Station, Kuala Nerang Station, NASDAQ Index, Dow Jones Index, Malaysia House Price Index (HPI), Kuala Lumpur HPI, and Florida HPI. All datasets are checked for missing values and outlier's values and in these cases; there are no missing or outlier values.

The first and the second time series datasets used in this study are environment datasets. These datasets consist of the water level data recorded at Senara Station and Kuala Nerang Station, respectively. Each consists of 366 daily water level values. The Senara station water level ranges between 0.52 and 4.19 in Meter. While the Kuala

Nerang station water level ranges between 12.26 and 19.32 in Meter. The two datasets were taken between the period of 1st January 2007 and 31th December 2007. The two datasets are collected from Hydrology and Water Resources Division of the Department of Irrigation and Drainage, Malaysia (National Flood Monitoring Centre, 2007).

The third and the fourth time series datasets used are economic datasets. The third dataset is the NASDAQ Index and the fourth is the Dow Jones Index. The two datasets consist of 252 close price values. The NASDAQ Index values range between 44.10 and 59.48. While the Dow Jones Index values range between 38.31 and 55.96. The two datasets were taken between the period of 1st January 2015 and 31th December 2015. The two datasets are available on http://finance.yahoo.com.

The fifth, sixth, and seventh time series datasets are economic datasets. The fifth dataset is Kuala Lumpur HPI in Malaysia, the sixth is Florida HPI in the USA, and the seventh dataset is Malaysia HPI. The three datasets consist of 52 weekly values. The Kuala Lumpur HPI values range between 54 and 100. While the Florida HPI values range between 63 and 100. The Malaysia HPI values range between 93.7 and 220.2. The Kuala Lumpur HPI and Florida HPI datasets were taken between the period of 1st January 2015 and 31th December 2015. The Malaysia HPI dataset was taken between the period of the 1st quarter of 1999 and the 2nd quarter of 2015.

Kuala Lumpur HPI and Florida HPI datasets are available on https://www.google.com/trends. Malaysia HPI dataset have been gathered from

National Property Information Center, Ministry of Finance Malaysia, and available on http://napic.jpph.gov.my (National Property Information Centre, 2015)

3.4.2 Evaluation Metrics and Benchmark Models

The MSE and MAPE are used for comparing and assessing the performance and reliability of the model between the real data and the predicted data. MSE and MAPE are common performance evaluation criteria for time series (Doorley et al., 2014; Feng and Jones, 2015; Kapila Tharanga Rathnayaka et al., 2015). MSE is expressed in Equation (3.10).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2 \qquad (3.10)$$

where, $\hat{Y}$ is a vector of n predictions and Y is the vector of observed values corresponding to the inputs to the function that generates the predictions. MAPE is as expressed in Equation (3.11).

$$MAPE = \frac{1}{n}\sum_{t-1}^{n}\frac{|e_t|}{V_t} \cdot 100 \qquad (3.11)$$

If the MAPE calculated value is less than 10%, it is interpreted as highly accurate forecasting, between 10–20% is good forecasting, between 20–50% is reasonable forecasting, and over 50% is inaccurate forecasting (Kapila Tharanga Rathnayaka et al., 2015; Yorucu, 2003).

Benchmark models that have been used for performance comparison are GARCH, EGARCH, GJR, and ARIMA-GARCH. These are common models for time series forecasting.

The hold-out method or split sample method is the simplest of all the resampling methods considered in this study. It involves a single random split or partition of the data into a training set with proportion P and a test set with proportion $1 - P$ (Pang and Jung, 2013). We have chosen to use 20% for the training set (parameter estimation) and the rest as the evaluation set.

3.4.3 Numeric Example

This section presents a numeric example for SWGARCH model. The Standard & Poor's 500 Index (S&P 500) dataset which is an index of 500 stocks seen as a leading indicator of United State equities and a reflection of the performance of the large cap universe has been employed for this purpose. The S&P 500 is a market value weighted index and one of the common benchmarks for the United State stock market as well as for research. The dataset has been taken between the period of 1st January 2015 and 31th December 2015, and consists of 252 index values. The dataset is available on http://finance.yahoo.com. Sample of the dataset is shown in Table 3.1. In Table 3.1, first column of the table record the date. The second column shows the Close Price Index.

Table 3.1

Sample of S&P 500 Index Dataset

Date	Close Price Index
Fri, Jan 02, 2015	2,058.19995
Mon, Jan 05, 2015	2,020.57996
Tue, Jan 06, 2015	2,002.60999
Wed, Jan 07, 2015	2,025.90002
Thu, Jan 08, 2015	2,062.13989
Fri, Jan 09, 2015	2,044.81006

Mon, Jan 12, 2015	2,028.26001
Tue, Jan 13, 2015	2,023.03003
Wed, Jan 14, 2015	2,011.27002
Thu, Jan 15, 2015	1,992.67004

The following sections display the steps in obtaining the forecasted value of the index.

3.4.3.1 Estimating SWGARCH Parameters

Table 3.2 shows how the calculation could be organized in estimating the parameters. The first and second columns in the table show the day and the price index (S_i) for the day respectively. The third column records the change in rate (S_i) at the end of day_i where $\delta_i = (S_i - S_{i-1})/ S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for day_i based on the change rate. On day 3, we start things off by setting the variance equal to u_2^2. On subsequence days, Equation (3.3) is used. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 3.2

Estimation of parameters in SWGARCH model

Date	$\mathbf{Day}_i$	S_i	δ_i	$v_i = \sigma_i^2$	$-ln(v_i) - v_i^2 \div v_i$
Fri, Jan 02, 2015	1	2,058.19995			
Mon, Jan 05, 2015	2	2,020.57996	-0.01828		
Tue, Jan 06, 2015	3	2,002.60999	-0.00889		
Wed, Jan 07, 2015	4	2,025.90002	0.01163	0.00012	7.89695
Thu, Jan 08, 2015	5	2,062.13989	0.01789	0.00020	6.91226
Fri, Jan 09, 2015	6	2,044.81006	-0.00840	0.00020	8.14870
Mon, Jan 12, 2015	7	2,028.26001	-0.00809	0.00010	8.56801

Tue, Jan 13, 2015	8	2,023.03003	-0.00258	0.00005	9.76219
Wed, Jan 14, 2015	9	2,011.27002	-0.00581	0.00003	9.25106
Thu, Jan 15, 2015	10	1,992.67004	-0.00925	0.00005	8.20436
				Sum	58.74353

The values shown in the fifth column of Table 3.2 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.54055 , \qquad \alpha = 0.25473, \qquad \beta = 0.20472$$

3.4.3.2 The Return Calculation

Table 3.3 displays another 5 days of S&P 500 Index data where the first column displays day index, column two displays Close Price. Period return is displayed in the third column. The Return of each day of index price is computed by squaring the period return of the third column of Table 3.3. This table shows the sequence of index value from day 11 to day 15. In this case, the returns from day 12 to day 15 are 0.0, 0.00002, 0.00023, and 0.00003 respectively.

Table 3.3
Computation of Return

Date	**Day**	**Close Price**	**Period Return** $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$
Fri, Jan 16, 2015	11	2,019.42004	
Tue, Jan 20, 2015	12	2,022.55005	0.00155
Wed, Jan 21, 2015	13	2,032.12000	0.00472
Thu, Jan 22, 2015	14	2,063.14990	0.01515
Fri, Jan 23, 2015	15	2,051.82007	-0.00551

3.4.3.3 Computation of Sliding Window Variance

Table 3.4 displays the five component numbers and variance explained of the S&P 500 Index.

Table 3.4
S&P 500 Index Variance

Component Number	Variance Explained (%)
1	84.4715
2	5.9630
3	3.5458
4	3.0847
5	2.9350

For the S&P 500 Index dataset, the big drop in variance explained after component 2 as shown in Figure 3.4. Components 2 through 5 appear at the base of the cliff composed of Component 2. The value of three components 1-2 is 90.4345% of the total variance. This supports the second component as a window size.

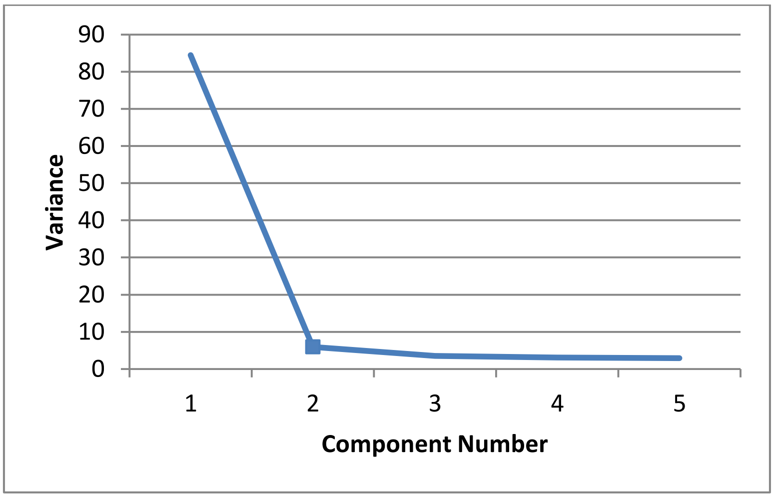

Figure 3.4. Variance plot for S&P 500 Index dataset

The sample of returns for four days are shown in Table 3.5. The two windows are $day_{15} - day_{14}$, and $day_{14} - day_{13}$ as $window_1\ ,(W_1)$, and $window_2\ ,(W_2)$ respectively.

Table 3.5

Sample Data from Sliding Window for S&P 500 Index

Date	**Day**	**Return**
Fri, Jan 16, 2015	11	
Tue, Jan 20, 2015	12	0.00000
Wed, Jan 21, 2015	13	0.00002
Thu, Jan 22, 2015	14	0.00023
Fri, Jan 23, 2015	15	0.00003

Next, derived from Equation (3.8), the total of window size weight given is determined:

$$\mathrm{TL} = \mathrm{w}_2 + \mathrm{w}_1 = 1 + 2 = 3$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$W_1 = \frac{w_1}{TL} = \frac{2}{3} = 0.67$$

$$W_2 = \frac{w_2}{TL} = \frac{1}{3} = 0.33$$

Further, derived from Equation (3.1), the sliding window variance for day_n $(n = 15)$ is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2)$$

where $u_{15}^2 = 0.00003$, $u_{14}^2 = 0.00023$

$$V_W = (0.67 \times 0.00003) + (0.33 \times 0.00023) = 0.0001$$

The value of $V_w = 0.00010$, or 0.01%. In other words, the window variance for day 15 is 0.01%.

3.4.3.4 Recent Variance

The recent variance i.e. variance for day 14 can be calculated using data from Table 3.3. Thus $\sigma_{14}^2 = 0.00023$

3.4.3.5 SWGARCH Variance

Derived from Equation (3.3), the variance for day 15 is calculated as follows:

$$\sigma_{15}^2 = (0.54055 \times 0.0001) + (0.25473 \times 0.00023) + (0.20472 \times 0.00023) = 0.00016$$

3.4.3.6 The Forecasting

In forecasting the closing price on day 16, the variance on day 15 will have to be calculated. This can be done using Equation (3.12) as shown.

$$E[\sigma_{n+t}^2] = V_w + (\alpha + \beta)^t \text{ x } (\sigma_n^2 - V_w) \tag{3.12}$$

where *n* represents the day of the calculated SWGARCH variance and *t* represents the additional time that will reflects the forecasted day. In this case, *n* = 15 and *t* = 1 because the forecasted values that of interest is for day 16. Thus the expected variance for day 15 is given by

$$\text{E}[\sigma_{16}^2] = 0.00010 + (0.45945)^1 x \,(0.00016 - 0.00010) = 0.00012$$

The forecasted closing price index of day 16 is computed as follows:

$$2{,}051.82007 + (2{,}051.82007 \times 0.00012) = 2052.21538$$

Thus, the forecasted index value of day 16 is 2052.0663.

The MSE and MAPE of SWGARCH model are 0.12477and 0.00949 respectively.

Table 3.6 shows the results of using SWGARCH model for the first ten datasets of the S&P 500 Index. The MSE value of this sample result shown in the error column is 0.27429. This supports that the model's forecasted value is almost as accurate as the actual dataset.

Table 3.6

Sample Model Performance for S&P 500 Dataset

Date	**Day**	**Index**	**Forecast Index**	**| Error |**
Fri, Jan 23, 2015	15	2051.82007		
Mon, Jan 26, 2015	16	2057.09009	2,052.0663	5.0238
Tue, Jan 27, 2015	17	2029.55005	2,029.6104	0.0603
Wed, Jan 28, 2015	18	2002.16003	2,002.3662	0.2062
Thu, Jan 29, 2015	19	2021.25000	2,021.6007	0.3507
Fri, Jan 30, 2015	20	1994.98999	1,995.2543	0.2643
Mon, Feb 02, 2015	21	2020.84998	2,021.1290	0.2790
Tue, Feb 03, 2015	22	2050.03003	2,050.3673	0.3372
Wed, Feb 04, 2015	23	2041.51001	2,041.8905	0.3805
Thu, Feb 05, 2015	24	2062.52002	2,062.7337	0.2136
			Average Error	**0.7906**

3.4.3.7 SWGARCH Model Comparison

In this case study, the performance of SWGARCH against ARIMA/GARCH in terms of MSE and MAPE is compared.

Table 3.7

Experimental Results

Model	MSE	MAPE
SWGARCH	0.12477	0.00949
ARIMA-GARCH	1.00596	0.03908

From Table 3.7, the results show that SWGARCH model performs better than ARIMA/GARCH model to forecast the index value in S&P 500 dataset according to MSE and MAPE.

3.5 Summary

The weakness of GARCH model has been solved by hybridization of GARCH model and sliding window technique. The algorithm of SWGARCH has been developed and tested using S&P 500 index dataset. The results have shown that SWGARCH model is able to forecast with good accuracy in term of MSE and MAPE.

CHAPTER FOUR

EXPERIMENT AND RESULTS

In this chapter, the performance evaluation of SWGARCH model is discussed. The results of the evaluation are compared with well-known time series models. Seven different datasets are used from two domains to estimate the sliding window variance and are used in the SWGARCH model to forecasting future values. The experimental design is provided in Section 4.2 and the tests of the method are described in Sections 4.2 – 4.8. The discussion and summary of the chapter are presented in Section 4.10 and Section 4.11, respectively.

4.1 Experimental Design

Seven datasets were used in the experiment. This section provides the results of all the experiments performed in this study. The purpose of first and second datasets of the experiments was to apply the SWGARCH model for environment datasets. The third to seventh datasets of the experiments was to apply the SWGARCH model for economic datasets.

4.2 Case Study of Senara Dataset in North Malaysia

Sample data of water level for Senara Dataset from Day 1 to Day 100 is shown in Figure 4.1.

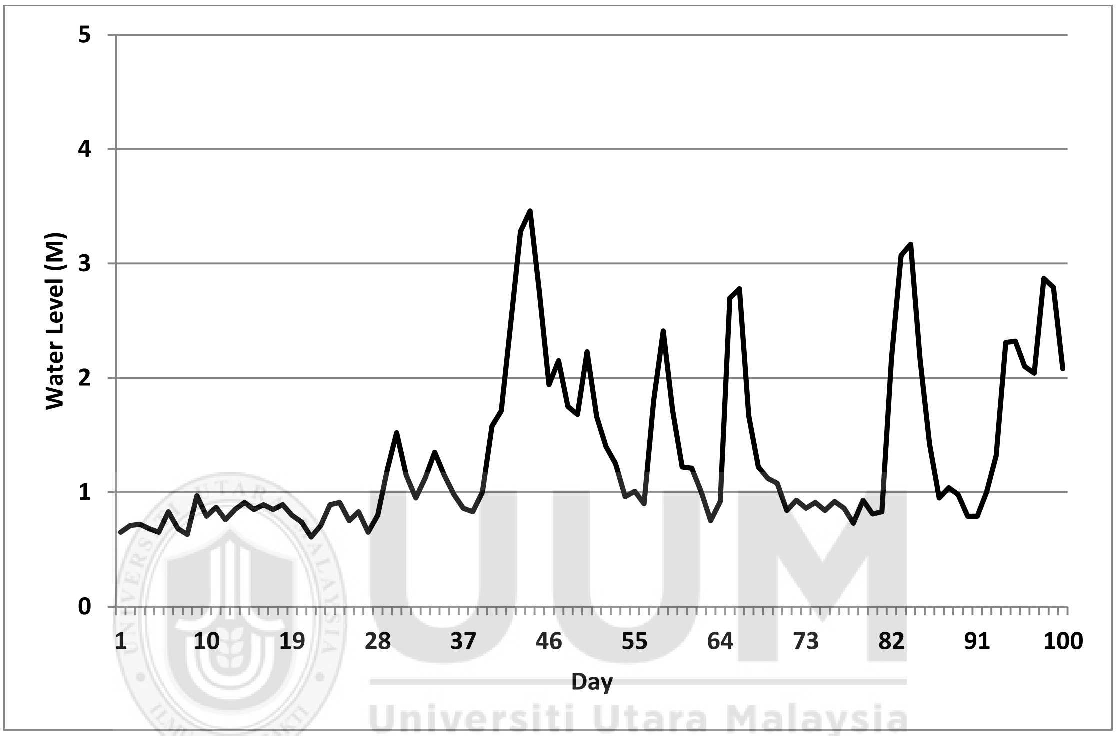

Figure 4.1. Senara sample data

4.2.1 Estimating SWGARCH Parameters

Table 4.1 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the day index and the water level (S_i) for the day respectively. The third column records the change in rate (S_i) at the end of day_i where $\delta_i = (S_i - S_{i-1})/ S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for day_i based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.1

Parameters Calculation for Senara Dataset

Date	Day_i	S_i	u_i	δ_i	$-ln(v_i) - v_i^2 \div v_i$
Mon, Jan 01, 2007	1	0.84			
Tue, Jan 02, 2007	2	0.91	0.08333		
Wed, Jan 03, 2007	3	0.70	-0.23077		
Thu, Jan 04, 2007	4	0.76	0.08571	0.02241	3.47049
Fri, Jan 05, 2007	5	0.71	-0.06579	0.00960	4.19517
Sat, Jan 06, 2007	6	0.73	0.02817	0.00317	5.50288
⋮	⋮	⋮	⋮	⋮	⋮
Mon, Mar 12, 2007	71	0.67	0.08065	0.02638	3.38867
Tue, Mar 13, 2007	72	0.62	-0.07463	0.00816	4.12585
Wed, Mar 14, 2007	73	0.73	0.17742	0.01138	1.70959
				Sum	237.5676

The values shown in the fifth column of Table 4.14.1 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.49850, \qquad \alpha = 0.35480, \qquad \beta = 0.01213$$

4.2.2 The Return Calculation

Table 4.2 displays another 5 days of Senara data where the first column display day index, column two displays water level. Period return is displayed in the third column. The Return of each day of water level is computed by squaring the period return of the third column of Table 4.2. This table shows the sequence of index value during 5 days. In this case, the return from day 74 to day 78 are: 0.00780, 0.0088, 0.00020, 0.00327, and 0.00204 respectively.

Table 4.2

Computation of Return

Date	Day i	Water Level	Period Return $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	Variance
Thu, Mar 15, 2007	74	0.65		
Fri, Mar 16, 2007	75	0.71	0.08829	
Sat, Mar 17, 2007	76	0.72	0.01399	
Sun, Mar 18, 2007	77	0.68	-0.05716	0.00327
Mon, Mar 19, 2007	78	0.65	-0.04512	0.00226

4.2.3 Computation of Sliding Window Variance

Table 4.3 displays the component numbers and variance explained for Senara Dataset water level.

Table 4.3

Senara Dataset Water Level Variance

Component Number	Variance Explained (%)
1	50.6316
2	30.47057
3	11.75715
4	4.998144
5	2.14254

For the Senara dataset, the big drop in variance explained after component 3 is shown in Figure 4.2 components 3 through 5 appear at the base of the cliff composed of Component 3. The value of three components 1-3 is 92.86% of the total variance. This supports the third component as a window size.

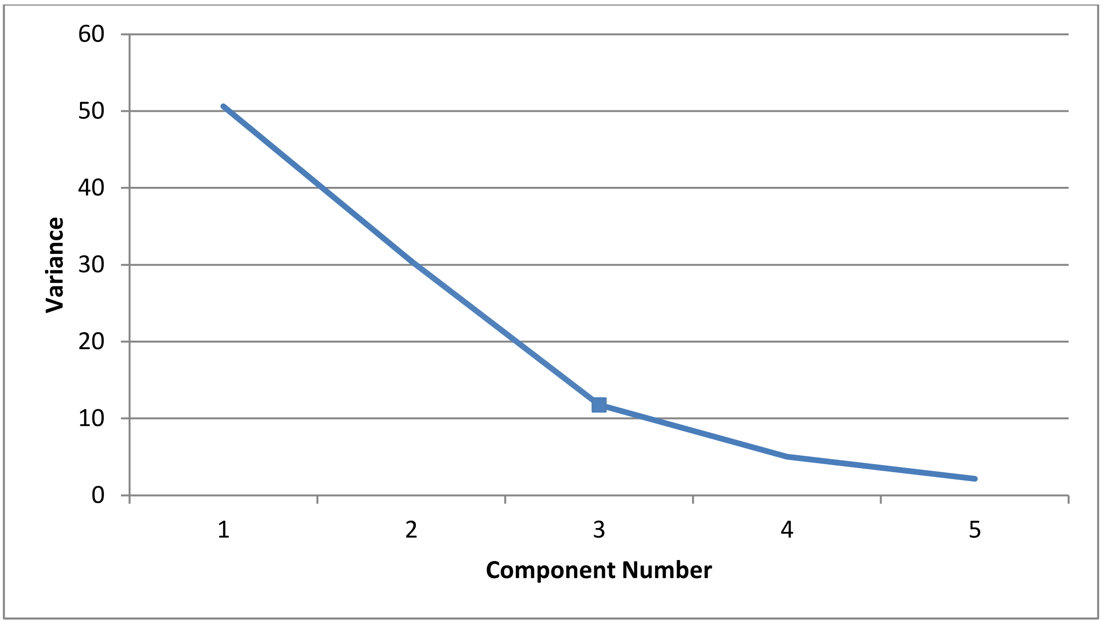

Figure 4.2. Variance plot for Senara dataset

The sample of returns for five days are shown in Table 4.4. The three windows are: $day_3 - day_5$, $day_2 - day_4$, and $day_1 - day_3$ as $window_1$ (W_1), $window_2$ (W_2), and $window_3$ (W_3) respectively.

Table 4.4

Sample Data from Sliding Window for Senara Dataset

Day	Return
74	
75	0.00780
76	0.00020
77	0.00327
78	0.00204

Next, derived from equation (3.8) , the total of window size weight given is determined as follows:

$$\text{TL} = \text{w}_1 + \text{w}_2 + \text{w}_3 = 1 + 2 + 3 = 6$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_1 = \frac{w_1}{TL} = \frac{1}{6} = 0.1667$$

$$Norm_2 = \frac{w_2}{TL} = \frac{2}{6} = 0.33$$

$$Norm_3 = \frac{w_3}{TL} = \frac{3}{6} = 0.50$$

Further, derived from Equation (3.1) , the sliding window variance for day_n $(n = 78)$ is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2) + (W_3 \times u_{n-2}^2)$$

where $u_n^2 = 0.00204$, $u_{n-1}^2 = 0.00327, u_{n-2}^2 = 0.00020$

$$V_W = (0.5 \times 0.00204) + (0.33 \times 0.00327) + (0.1667 \times 0.00020) = 0.00213$$

The value of $V_w = 0.00213$, or 0.213% which shows that the window variance for day 78 is 0.213%.

4.2.4 Recent Variance

The recent variance i.e. variance for day 77 can be calculated using data from Table 4.2. Thus $\sigma_{76}^2 = 0.00327$.

4.2.5 SWGARCH Variance

The variance for day 78 is calculated using Equation (3.3) as follows:

$$\sigma_{78}^2 = (0.49850 \times 0.00213) + (0.35480 \times 0.00327) + (0.01213 \times 0.00327) = 0.00226$$

4.2.6 The Forecasting

In the forecasting the water level on day 79, the variance on day 78 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{78}^2] = 0.00213 + 0.36693^1(0.00226 - 0.00213) = 0.00310$$

The forecasted water level of day 79 is computed as follows:

$$0.68 + (0.68 \times 0.00310) = 0.65201$$

4.3 Case Study of Kuala Nerang Dataset in North Malaysia

Sample data of water level for Kuala Nerang Dataset from day 1 to day 100 is shown in Figure 4.3.

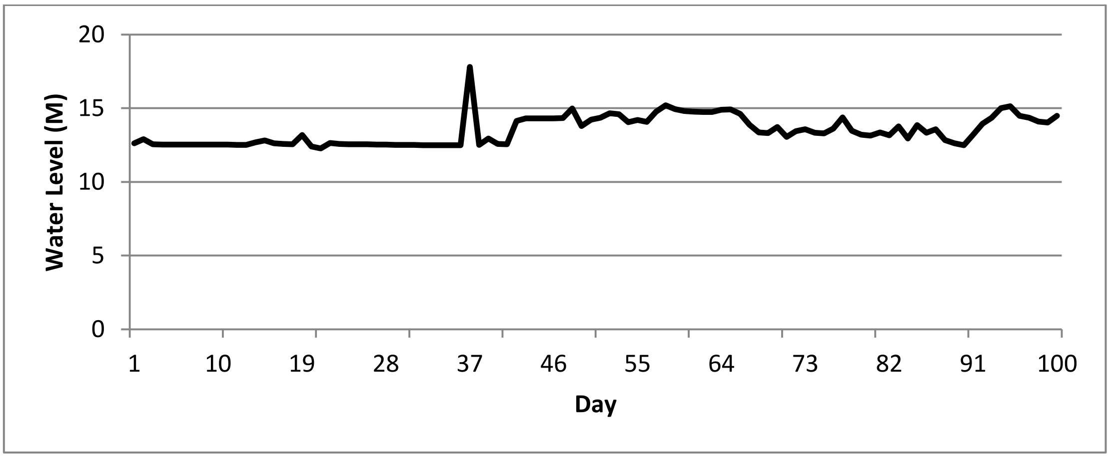

Figure 4.3. Kuala Nerang sample data

4.3.1 Estimating SWGARCH Parameters

Table 4.5 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the day and the water level (S_i) for the day respectively. The third column records the change in rate (S_i) at the end of day_i where $\delta_i = (S_i - S_{i-1})/ S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for day_i based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.5

Parameters Calculation for Kuala Nerang Dataset

Date	**Day**$_i$	S_i	δ_i	$v_i = \sigma_i^2$	$-ln(v_i) - v_i^2 \div v_i$
Mon, Jan 01, 2007	1	13.13			
Tue, Jan 02, 2007	2	13.11	-0.00152		
Wed, Jan 03, 2007	3	13.11	0.00000		
⋮	⋮	⋮	⋮	⋮	⋮
Mon, Mar 12, 2007	71	12.48	0.00000	0.00000	13.83752
Tue, Mar 13, 2007	72	12.48	0.00000	0.00000	14.46607
Wed, Mar 14, 2007	73	17.80	0.42628	0.02817	-2.88192
				Sum	681.3982

The values shown in the fifth column of Table 4.5 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.38750, \qquad \alpha = 0.26980, \qquad \beta = 0.33658$$

4.3.2 The Return Calculation

Table 4.6 displays another 6 days of Kuala Nerang data where the first column display day index, column two displays water level. Period return is displayed in the third column. The Return of each day of water level is computed by squaring the period return of the third column of Table 4.6. This table shows the sequence of index value during 6 days. In this case, the return from day 2 to day 6 are 0.00120, 0.00084, 0.00001, 0.01425, and 0.000014, respectively.

Table 4.6

Computation of Return

Date	**Day$_i$**	**Water Level**	**Period Return** $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	**Variance**
Thu, Mar 15, 2007	74	12.50		
Fri, Mar 16, 2007	75	12.94	0.03459	
Sat, Mar 17, 2007	76	12.57	-0.02901	
Sun, Mar 18, 2007	77	12.54	-0.00239	
Mon, Mar 19, 2007	78	14.13	0.11938	0.01425
Tue, Mar 20, 2007	79	14.30	0.01196	0.01035

4.3.3 Computation of Sliding Window Variance

Table 4.7 displays the component numbers and variance explained for Kuala Nerang water level.

Table 4.7

Kuala Nerang Water Level Variance

Component Number	Variance Explained (%)
1	33.1437
2	23.1902
3	18.5763
4	18.1928
5	6.870

For the Kuala Nerang dataset, the big drop in variance explained after component 4 is as shown in Figure 4.4. Components 4 through 5 appear at the base of the cliff composed of Component 4. The value of three components 1-4 is 93.10% of the total variance. This supports the fourth component as a window size.

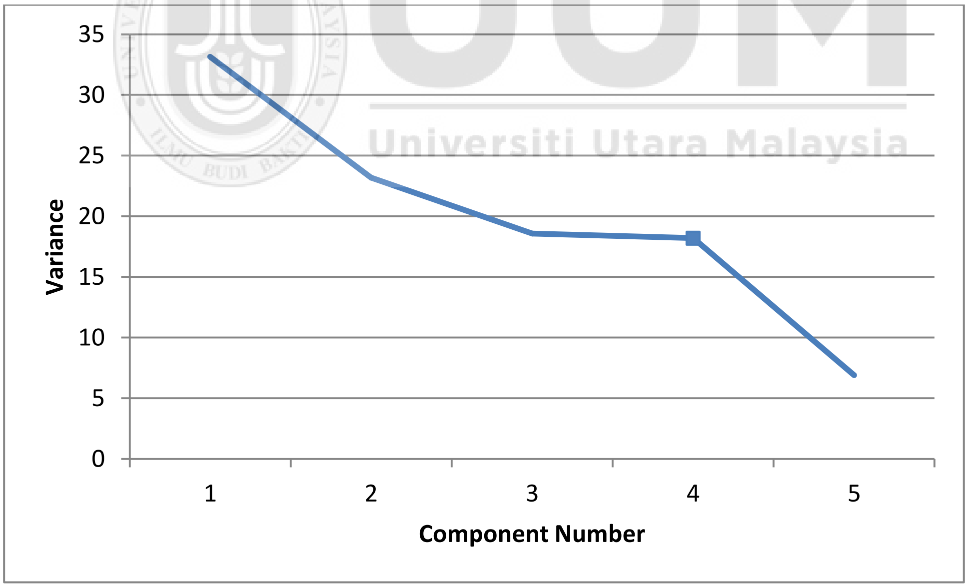

Figure 4.4. Variance plot for Kuala Nerang dataset

The sample of return for five days are shown in Table 4.8. The four windows are: $day_4 - day_6$, $day_3 - day_5$, $day_2 - day_4$, and $day_1 - day_3$ as $window_1$ (W_1), $window_2$ (W_2) , $window_3$ (W_3) , and $window_4$ (W_4) respectively.

Table 4.8

Sample Data from Sliding Window for Kuala Nerang Dataset

Date	Day	Return
Thu, Mar 15, 2007	74	
Fri, Mar 16, 2007	75	0.00120
Sat, Mar 17, 2007	76	0.00084
Sun, Mar 18, 2007	77	0.00001
Mon, Mar 19, 2007	78	0.01425
Tue, Mar 20, 2007	79	0.00014

Next, derived from Equation 3.8, the total of window size weight given is determined as follows:

$$\mathrm{TL} = \mathrm{w}_1 + \mathrm{w}_2 + \mathrm{w}_3 + \mathrm{w}_4 = 1 + 2 + 3 + 4 = 10$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_1 = \frac{w_1}{TL} = \frac{1}{10} = 0.1$$

$$Norm_2 = \frac{w_2}{TL} = \frac{2}{10} = 0.2$$

$$Norm_3 = \frac{w_3}{TL} = \frac{3}{10} = 0.3$$

$$Norm_4 = \frac{w_3}{TL} = \frac{4}{10} = 0.4$$

Further, derived from Equation (3.1), the sliding window variance for day_n $(n = 79)$ is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2) + (W_3 \times u_{n-2}^2)\,(W_4 \times u_{n-3}^2)$$

where $u_n^2 = 0.00014, u_{n-1}^2 = 0.01425, u_{n-2}^2 = 0.00001, u_{n-3}^2 = 0.00084.$

$$V_W = (0.4 \times 0.00014) + (0.3 \times 0.01425) + (0.2 \times 0.00001) + (0.1 \times 0.00084) = 0.00442$$

The value of $V_w = 0.0442$, or 4.42%. In other words, the window variance for day 79 is 4.42%.

4.3.4 Recent Variance

The recent variance i.e. variance for day_{79} can be calculated using data from Table 4.8. Thus $\sigma_{n-1}^2 = 0.01425$.

4.3.5 SWGARCH Variance

The variance for day 79 is calculated using Equation (3.3) as follows:

$$\sigma_{79}^2 = (0.38750 \times 0.00442) + (0.26980 \times 0.01425) + (0.33658 \times 0.01425) = 0.01035$$

4.3.6 The Forecasting

In forecasting the water level on day 80, the variance on day 79 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{80}^2] = 0.00442 + 0.60638^1(0.01035 - 0.00442) = 0.01100$$

The forecasted water level of day 80 is computed as follows:

$$14.13 + (14.13 \times 0.01100) = 14.45729$$

4.4 Case Study of House Price Index for Kuala Lumpur in Malaysia

Sample data of price index for KL HPI Dataset from week 1 to week 33 is shown in Figure 4.5.

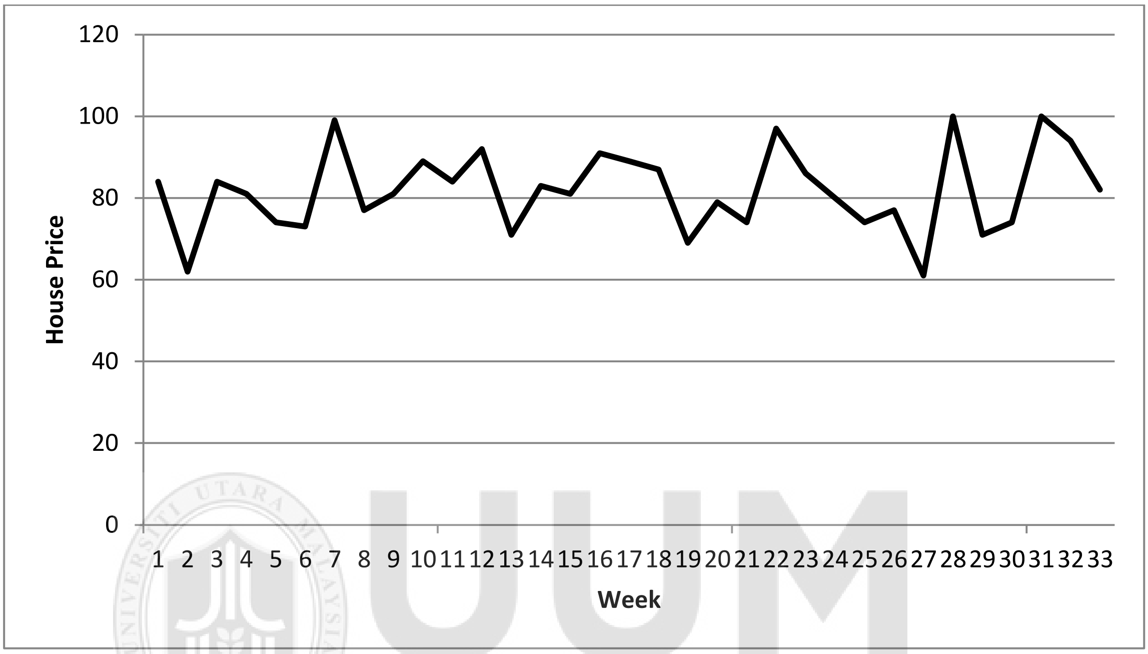

Figure 4.5. KL HPI sample data

4.4.1 Estimating SWGARCH Parameters

Table 4.9 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the week and the price index (S_i) for the week respectively. The third column records the change in rate (S_i) at the end of $week_i$ where $\delta_i = (S_i - S_{i-1})/S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for week$_i$ based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.9

Parameters Calculation for KL HPI

Date	$Week_i$	S_i	u_i	δ_i	$-ln(v_i) - v_i^2 \div v_i$
Sun, Jan 15, 2012	1	41.00			
Sun, Jan 22, 2012	2	63.00	0.53659		
Sun, Jan 29, 2012	3	67.00	0.06349		
Sun, Feb 05, 2012	4	60.00	-0.10448	0.05487	2.70383
Sun, Feb 12, 2012	5	84.00	0.40000	0.02922	-1.94330
⋮	⋮	⋮	⋮	⋮	⋮
Sun, Dec 23, 2012	50	72.00	0.14286	0.01539	2.84803
Sun, Dec 30, 2012	51	29.00	-0.59722	0.01228	-24.64408
Sun, Jan 06, 2013	52	47.00	0.62069	0.08980	-1.88007
				Sum	-243.0502

The values shown in the fifth column of Table 4.9 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.00005, \qquad \alpha = 0.23500, \qquad \beta = 0.48564$$

4.4.2 The Return Calculation

Table 4.10 displays another 5 weeks of KL HPI data where the first column display week index, column two displays house price. Period return is displayed in the third column. The Return of each week of price index is computed by squaring the period return of the third column in Table 4.10. This table shows the sequence of index value during 6 weeks. In this case, the return from week 53 to week 57 are 0.00104, 0.00430, 0.00272, and 0.07768 respectively.

Table 4.10

Computation of Return

Date	Week $_i$	House Price	Period Return $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	Variance
Sun, Jan 13, 2013	53	61.00		
Sun, Jan 20, 2013	54	63.00	0.03226	
Sun, Jan 27, 2013	55	59.00	-0.06560	
Sun, Feb 03, 2013	56	56.00	-0.05219	0.00272
Sun, Feb 10, 2013	57	74.00	0.27871	0.00196

4.4.3 Computation of Sliding Window Variance

Table 4.11 displays the component numbers and variance explained for KL index.

Table 4.11

KL Index Variance

Component Number	Variance Explained (%)
1	27.2930
2	23.7399
3	23.4490
4	16.7951
5	8.7231

For the KL HPI dataset, the big drop in variance explained after component 4 is shown in Figure 4.6. Components 4 through 5 appear at the base of the cliff composed of component 4. The value of four components 1-4 is 91.28% of the total variance. This supports the fourth component as a window size.

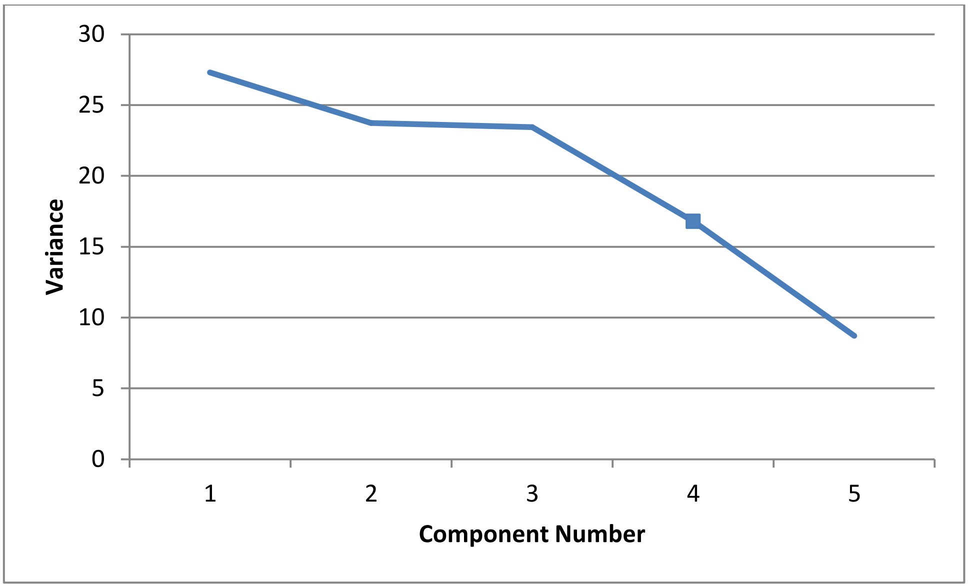

Figure 4.6. Variance plot for KL HPI dataset

The sample of returns for six weeks are shown in Table 4.12. The four windows are: $day_4 - day_6$, $day_3 - day_5$, $day_2 - day_4$, and $day_1 - day_3$ as $window_1$ (W_1) , $window_2$ (W_2) , $window_3$ $(W_{3)}$, and $window_4$ (W_4) respectively.

Table 4.12

Sample Data from Sliding Window for KL HPI

Date	Week $_i$	Return
Sun, Jan 13, 2013	53	
Sun, Jan 20, 2013	54	0.00104
Sun, Jan 27, 2013	55	0.00430
Sun, Feb 03, 2013	56	0.00272
Sun, Feb 10, 2013	57	0.07768
Sun, Feb 17, 2013	58	0.09928

Next, derived from Equation (3.8), the total of window size weight given is determined as follows:

$$\mathrm{TL} = \mathrm{w}_1 + \mathrm{w}_2 + \mathrm{w}_3 = 1 + 2 + 3 + 4 = 10$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_1 = \frac{w_1}{TL} = \frac{1}{10} = 0.1$$

$$Norm_2 = \frac{w_2}{TL} = \frac{2}{10} = 0.2$$

$$Norm_3 = \frac{w_3}{TL} = \frac{3}{10} = 0.3$$

$$Norm_4 = \frac{w_3}{TL} = \frac{4}{10} = 0.4$$

Further, derived from Equation (3.1), sliding window variance in KL HPI for week 58 is calculated as in the following equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2) + (W_3 \times u_{n-2}^2)\ (W_4 \times u_{n-3}^2)$$

where $u_n^2 = 0.09928$, $u_{n-1}^2 = 0.07768$, $u_{n-2}^2 = 0.00272$, $u_{n-3}^2 = 0.00430$.

$$V_W = (0.4 \times 0.09928) + (0.3 \times 0.07768) + (0.2 \times 0.00272) + (0.1 \times 0.00430) = 0.06786$$

The value of $V_w = 0.06786$, or 6.786%. In other words, the window variance for week 58 is 6.786%.

4.4.4 Recent Variance

The recent variance i.e. variance for week 57 can be calculated using data from Table 4.10. Thus $\sigma_{57}^2 = 0.07768$.

4.4.5 SWGARCH Variance

The variance for week 57 is calculated using Equation (3.3) as follows:

$$\sigma_n^2 = (0.00005 \times 0.06786) + (0.23500 \times 0.07768) + (0.48564 \times 0.07768) = 0.05598$$

4.4.6 The Forecasting

In the forecasting the price index on week 58, the variance on week 57 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{58}^2] = 0.06786 + 0.72064^1(0.05598 - 0.06786) = 0.05930$$

The forecasted price index of day 58 is computed as follows:

$$54 + (54 \times 0.05930) = 57.53267$$

4.5 Case Study of House Price Index for Florida in the USA

Figure 4.7 shows the weekly HPI for Florida in the USA between January 2015 and December 2015.

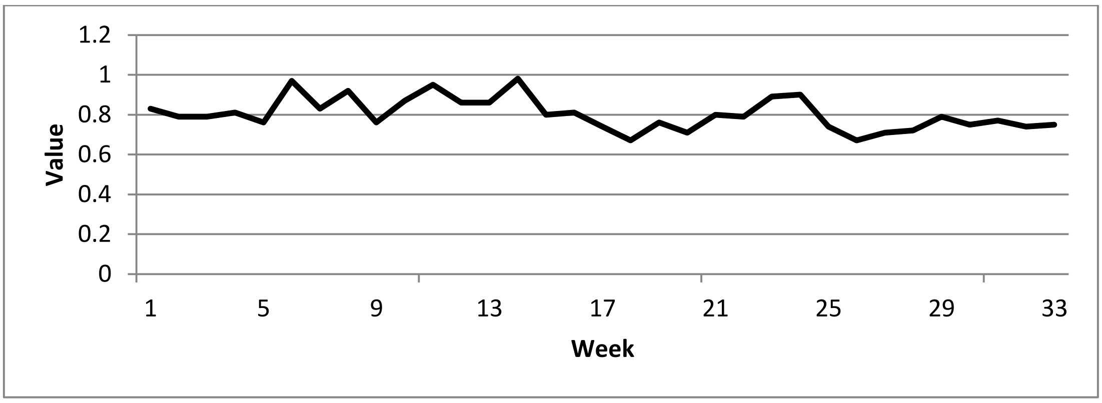

Figure 4.7. Sample Florida HPI data

4.5.1 Estimating SWGARCH Parameters

Table 4.13 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the week and the price index (S_i) for the week respectively. The third column records the change in rate (S_i) at the end of $week_{\,i}$ where $\delta_i = (S_i - S_{i-1})/\,S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for $week_{\,i}$ based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.13

Parameters Calculation for Florida HPI

Date	$Week_i$	S_i	u_i	δ_i	$-ln(v_i) - v_i^2 \div v_i$
Sun, Jan 15, 2012	1	39.00			
Sun, Jan 22, 2012	2	51.00	0.30769		
Sun, Jan 29, 2012	3	40.00	-0.21569		
Sun, Feb 05, 2012	4	51.00	0.27500	0.06897	1.57760
Sun, Feb 12, 2012	5	43.00	-0.15686	0.05396	2.46354
Sun, Feb 19, 2012	6	39.00	-0.09302	0.02387	3.37249
⋮	⋮	⋮	⋮	⋮	⋮
Sun, Dec 23, 2012	50	55.00	0.05769	0.09875	2.28147
Sun, Dec 30, 2012	51	36.00	-0.34545	0.05423	0.71399
Sun, Jan 06, 2013	52	52.00	0.44444	0.12551	0.50154
				Sum	99.3610

The values shown in the fifth column of Table 4.13 were calculated in the final iteration of search for the optimal γ, α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.59865, \qquad \alpha = 0.35480, \qquad \beta = 0.00246$$

4.5.2 The Return Calculation

Table 4.14 displays another 5 weeks of Florida HPI data where the first column display week index, column two displays price index. Period return is displayed in the third column. The Return of each week of price index is computed by squaring the period return of the third column of Table 4.14. This table shows the sequence of index value during 5 weeks. In this case, the returns from week 54 to week 57 are 0.01976, 0.00529, 0.07944, and 0.03612, respectively.

Table 4.14

Computation of Return

Date	Week	House Price	Period Return $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	variance
Sun, Jan 13, 2013	53	61.00		
Sun, Jan 20, 2013	54	53.00	-0.14058	
Sun, Jan 27, 2013	55	57.00	0.07276	
Sun, Feb 03, 2013	56	43.00	-0.28185	0.07944
Sun, Feb 10, 2013	57	52.00	0.19004	0.05541

4.5.3 Computation of Sliding Window Variance

Table 4.15 displays the component numbers and variance explained for Florida HPI.

Table 4.15

Florida Price Variance

Component Number	Variance Explained Percent
1	30.1560 %
2	23.3692 %
3	17.9412 %
4	16.1765 %
5	12.3569 %

For the Florida dataset, the big drop in variance explained after component 4 is shown in Figure 4.18. Components 4 through 5 appear at the base of the cliff composed of component 3. The value of three components 1-4 is 87.6431% of the total variance. This supports the fourth component as a window size.

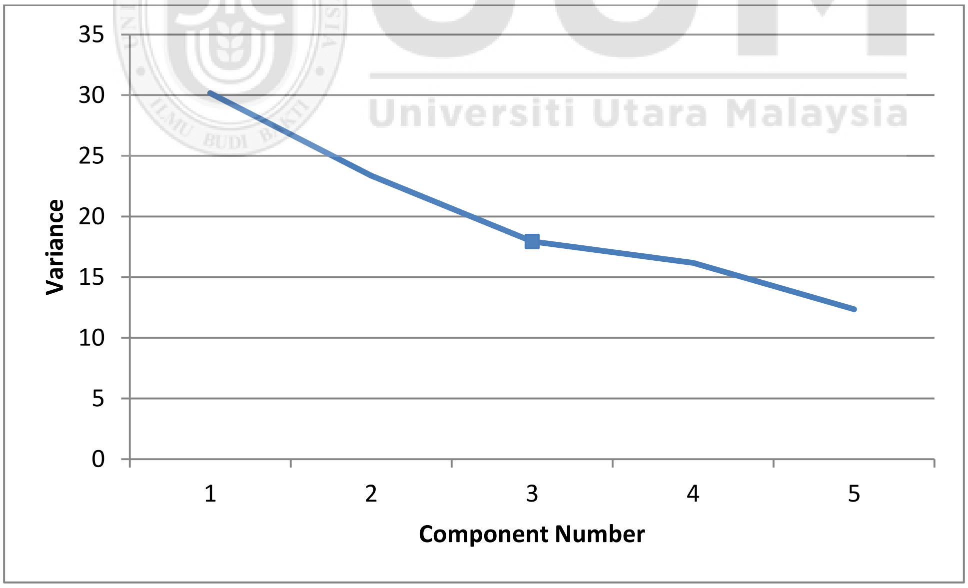

Figure 4.8. Variance plot for Florida dataset

The sample of returns for five days are shown in Table 4.16. The four windows are:

$day_4 - day_6$, $day_3 - day_5$, $day_2 - day_4$, and $day_1 - day_3$ as $window_1$ W_1 , $window_2$ W_2 , $window_3$ W_3 , and $window_4$ W_4 respectively.

Table 4.16

Sample Data from Sliding Window for Florida HPI

Date	$Week_i$	Return
Sun, Jan 13, 2013	53	
Sun, Jan 20, 2013	54	0.01976
Sun, Jan 27, 2013	55	0.00529
Sun, Feb 03, 2013	56	0.07944
Sun, Feb 10, 2013	57	0.03612

Next, derived from Equation (3.8), the total of window size weight given is determined as follows:

$$\mathrm{TL} = \mathrm{w}_1 + \mathrm{w}_2 + \mathrm{w}_3 + \mathrm{w}_4 = 1 + 2 + 3 + 4 = 10$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_1 = \frac{w_1}{TL} = \frac{1}{10} = 0.1$$

$$Norm_2 = \frac{w_2}{TL} = \frac{2}{10} = 0.2$$

$$Norm_3 = \frac{w_3}{TL} = \frac{3}{10} = 0.3$$

$$Norm_4 = \frac{w_3}{TL} = \frac{4}{10} = 0.4$$

Further, derived from Equation (3.1), sliding windows variance in Florida for week 57 is calculated as in the following equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2) + (W_3 \times u_{n-2}^2)\,(W_4 \times u_{n-3}^2)$$

where $u_n^2 = 0.03612$, $u_{n-1}^2 = 0.07944$, $u_{n-2}^2 = 0.00529$, $u_{n-3}^2 = 0.01976$.

$$V_W = (0.4 \times 0.03612) + (0.3 \times 0.0347)0.07944 + (0.2 \times 0.00529)$$
$$+ (0.1 \times 0.01976) = 0.04516$$

The value of $V_w = 0.04516$, or 4.516%. In other words, the window variance for Week 57 is 4.516%.

4.5.4 Recent Variance

The recent variance i.e. variance for week 56 can be calculated using data from Table 4.14. Thus $\sigma_{56}^2 = 0.07944$.

4.5.5 SWGARCH Variance

The variance for day 6 is calculated using Equation (3.3) as follows:

$$\sigma_{57}^2 = (0.4567 \times 0.04516) + (0.0270 \times 0.07944) + (0.5167 \times 0.07944) = 0.05541$$

4.5.6 The Forecasting

In forecasting the price index on week 58, the variance for week 57 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{58}^2] = 0.04516 + 0.35726^1(0.05541 - 0.04516) = 0.05715$$

The forecasted price index of week 58 is computed as follows:

$$52 + (052 \times 0.05715) = 54.97204$$

4.6 Case Study of Malaysia House Price Index

Figure 4.9 shows sample of Malaysia HPI between 2003 Q1 and 2015 Q2.

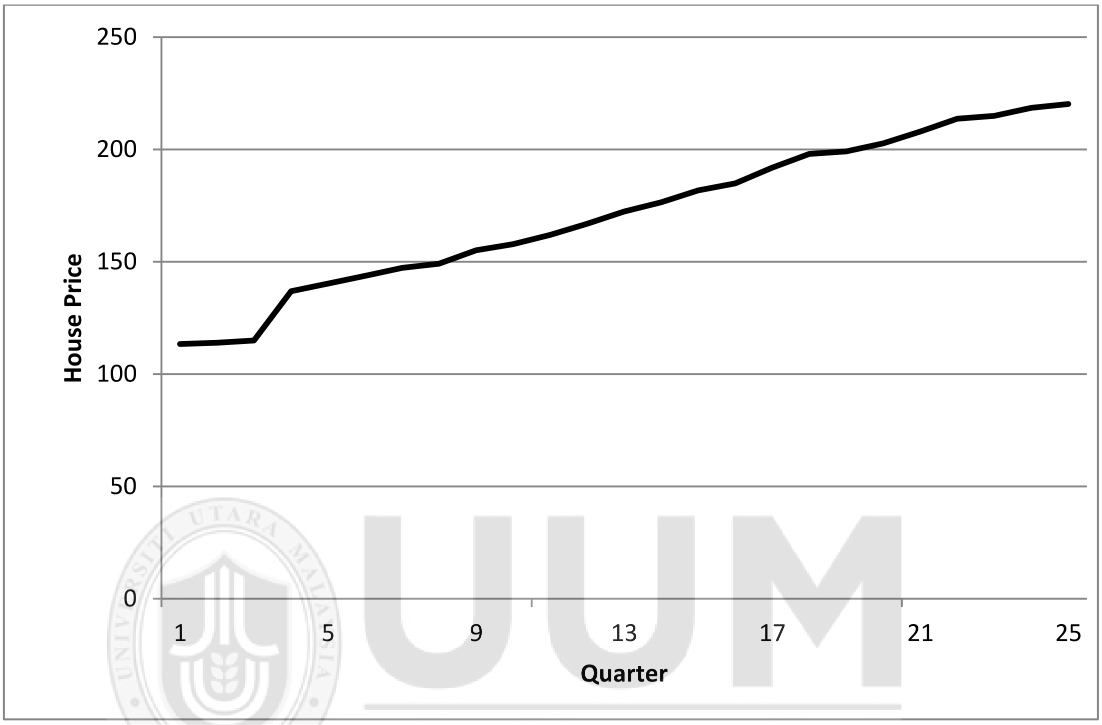

Figure 4.9. Sample Malaysia HPI

4.6.1 Estimating SWGARCH Parameters

Table 4.17 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the quarter and the price index (S_i) for the week respectively. The third column records the change in rate (S_i) at the end of quarter $_i$ where $\delta_i = (S_i - S_{i-1})/ S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for quarter $_i$ based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.17

Parameters Calculation for Malaysia HPI

$Quarter$	$Quarter_i$	S_i	u_i	δ_i	$-\ln(v_i) - v_i^2 \div v_i$
1999 Q1	1	93.40			
1999 Q2	2	93.70	0.00321		
1999 Q3	3	95.20	0.01601	0.00018	7.18659
1999 Q4	4	96.90	0.01786	0.00017	6.78205
2000 Q1	5	97.90	0.01032	0.00018	8.04202
2000 Q2	6	100.70	0.02860	0.00013	2.68382
2000 Q3	7	101.40	0.00695	0.00027	8.03475
2000 Q4	8	101.60	0.00197	0.00017	8.62874
2001 Q1	9	100.60	-0.00984	0.00011	8.23774
2001 Q2	10	101.40	0.00795	0.00009	8.62262
2001 Q3	11	102.40	0.00986	0.00007	8.16177
2001 Q4	12	101.90	-0.00488	0.00006	9.28881
2002 Q1	13	102.40	0.00491	0.00004	9.48480
				Sum	77.9671

The values shown in the fifth column of Table 4.17 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.00486, \qquad \alpha = 0.23500, \qquad \beta = 0.60344$$

4.6.2 The Return Calculation

Table 4.18 displays another 6 days of Malaysia HPI data where the first column display quarter index, column two displays price index. Period return is displayed in the third column. The Return of each quarter of water level is computed by squaring the period return of the third column of Table 4.18. This table shows the sequence of index value during 6 quarters. In this case, the return on 2010 Q2 to 2011 Q4 are 0.0006, 0.0006, 0.0006, 0.0002, 0.0016, and 0.0003, respectively.

Table 4.18

Computation of Return

Quarter	Quarter_i	House Price	Period Return $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	Variance
2002 - Q2	14	103.80		
2002 - Q3	15	105.60	0.01719	
2002 - Q4	16	107.30	0.01597	0.00026
2003 - Q1	17	107.20	-0.00093	0.00021

4.6.3 Computation of Sliding Window Variance

Table 4.19 shows the component numbers and variance explained of Malaysia HPI.

Table 4.19

Malaysia HPI PCA Variance Explained

Component Number	Variance Explained Percent
1	89.4426%
2	6.9555%
3	2.2876%
4	0.8336%
5	0.4804%

For the Malaysia HPI dataset, the big drop in variance explained after component 2 is shown in Figure 5.11. Components 2 – 5 appear at the base of the cliff composed of

Component 2, which accounts for 96.39% of the total variance. This supports the first component as a window size. This study used the window size of second quarter.

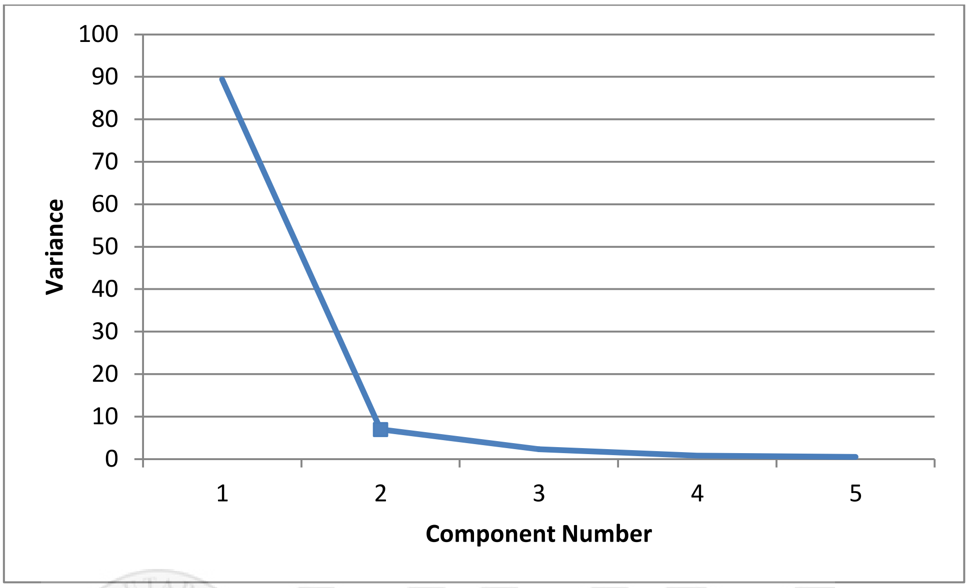

Figure 4.10. Variance plot for Malaysia HPI dataset

The sample of return for four days are shown in Table 4.20. The two windows are $Quater_{15} - Quater_{14}$, and $Quater_{14} - Quater_{13}$ as $window_1\ (W_1)$, and $window_2\ (W_2)$ respectively.

Table 4.20

Sample Data from Sliding Window for Malaysia HPI

Quarter	Quarter Number	Return
2002 Q2	14	
2002 Q3	15	0.00030
2002 Q4	16	0.00026
2003 Q1	17	0.00000

Next, derived from Equation (3.8), the total of window size weight given is determined:

$$\mathrm{TL} = \mathrm{w}_2 + \mathrm{w}_1 = 1 + 2 = 3$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_2 = \frac{w_2}{TL} = \frac{1}{3} = 0.33$$

$$Norm_1 = \frac{w_1}{TL} = \frac{2}{3} = 0.67$$

Further, derived from Equation (3.1), the sliding window variance for day_n $(n = 17)$ is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2)$$

where $u_{17}^2 = 0.0, u_{16}^2 = 0.00026$

$$V_W = (0.67 \times 0.0) + (0.33 \times 0.00026) = 0.00008$$

The value of $V_w = 0.00008$, or 0.08%. In other words, the window variance for Quarter 17 is 0.08%.

4.6.4 Recent Variance

The recent variance i.e. variance for Quarter 17 can be calculated using data from Table 4.18. Thus $\sigma_{16}^2 = 0.00026$.

4.6.5 SWGARCH Variance

The variance for day 6 is calculated using Equation (3.3) as follows:

$\sigma_{17}^{2} = (0.00486 \times 0.00008) + (0.23500 \times 0.00026) + (0.60344 \times 0.00026) = 0.00021$

4.6.6 The Forecasting

In the forecasting the price index on Quarter 18, the variance on Quarter 17 will have to be calculated. This can be done using Equation (3.12) as shown.

$$E[\sigma_{18}^{2}] = 0.0008 + 0.83844^{1}(0.00021 - 0.0008) = 0.0006$$

The forecasted price index of Quarter 18 is computed as follows:

$$107.30 + (107.30 \times 0.00021) = 107.22757$$

4.7 Case Study of NASDAQ Index

Figure 4.11 shows a sample of the NASDAQ index data from day 1 to day 100.

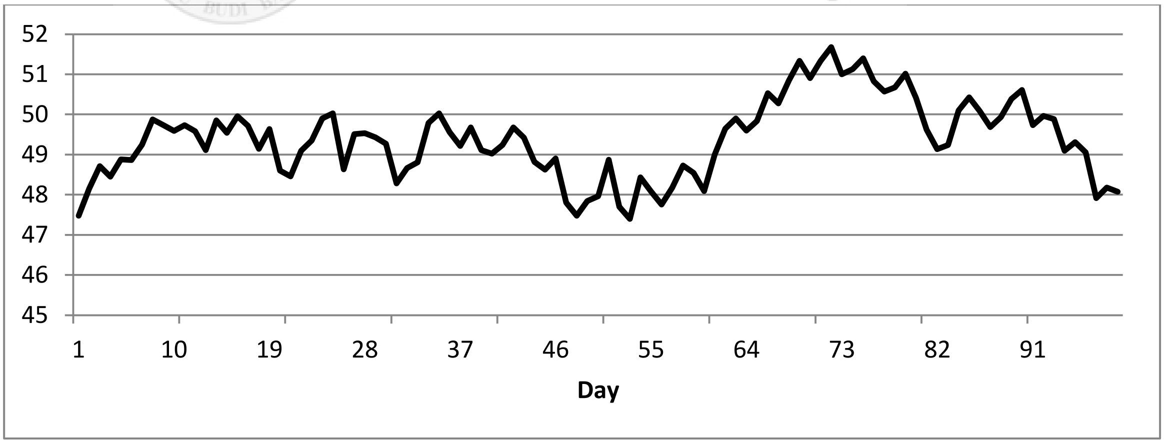

Figure 4.11. Sample NASDAQ Index data

4.7.1 Estimating SWGARCH Parameters

Table 4.21 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the day and the price index (S_i) for the day respectively. The third column records the change in rate (S_i) at the end of day $_i$ where $\delta_i = (S_i - S_{i-1})/\, S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for quarter $_i$ based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.21

Parameters Calculation for NASDAQ Index

Date	Day_i	S_i	u_i	δ_i	$-ln(v_i) - v_i^2 \div v_i$
Fri, Jan 02, 2015	1	4,726.81006			
Mon, Jan 05, 2015	2	4,652.56982	-0.01571		
Tue, Jan 06, 2015	3	4,592.74023	-0.01286	0.00019	7.69659
Wed, Jan 07, 2015	4	4,650.47022	0.01257	0.00017	7.75132
Thu, Jan 08, 2015	5	4,736.18994	0.01843	0.00023	6.90855
⋮	⋮	⋮	⋮	⋮	⋮
Thu, Mar 12, 2015	48	4,893.29004	-0.00440	0.00007	8.41414
Fri, Mar 13, 2015	49	4,871.75977	0.01185	0.00005	9.51066
Mon, Mar 16, 2015	50	4,929.50977	-0.01571	0.00008	7.64796
				Sum	419.3754

The values shown in the fifth column of the table were calculated in the final iteration of search for the optimal values of γ, α, and β and are given by:

$$\gamma = 0.61965, \qquad \alpha = 0.12470, \qquad \beta = 0.24570$$

4.7.2 The Return Calculation

Table 4.22 displays another 6 days of NASDAQ index data where the first three columns display day index, close price index and period return respectively. The return of each day of price index is computed by squaring the period return. This table shows the sequence of index value during 6 days. In this case, the return from day 2 to day 6 are 0.00003, and 0.00003, respectively.

Table 4.22

Computation of Return

Date	Day_i	Close Price	Period Return $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	Variance
Tue, Mar 17, 2015	51	4,937.43018		
Wed, Mar 18, 2015	52	4,982.83008	0.00915	
Thu, Mar 19, 2015	53	4,992.37988	0.00191	0.00000
Fri, Mar 20, 2015	54	5,026.41992	0.00680	0.00002

4.7.3 Computation of Sliding Window Variance

Table 4.23 displays the component numbers and variance explained for NASDAQ index.

Table 4.23

Senara Dataset Water Level Variance

Component Number	**Variance Explained (%)**
1	92.6259
2	4.61957
3	1.3643
4	0.8232
5	0.5668

For the NASDAQ dataset, the big drop in variance explained after component 2 is shown in Figure 4.12. Components 2 through 5 appear at the base of the cliff composed of component 3. The value of two components 1-2 is 97.25% of the total variance. This supports the second component as a window size.

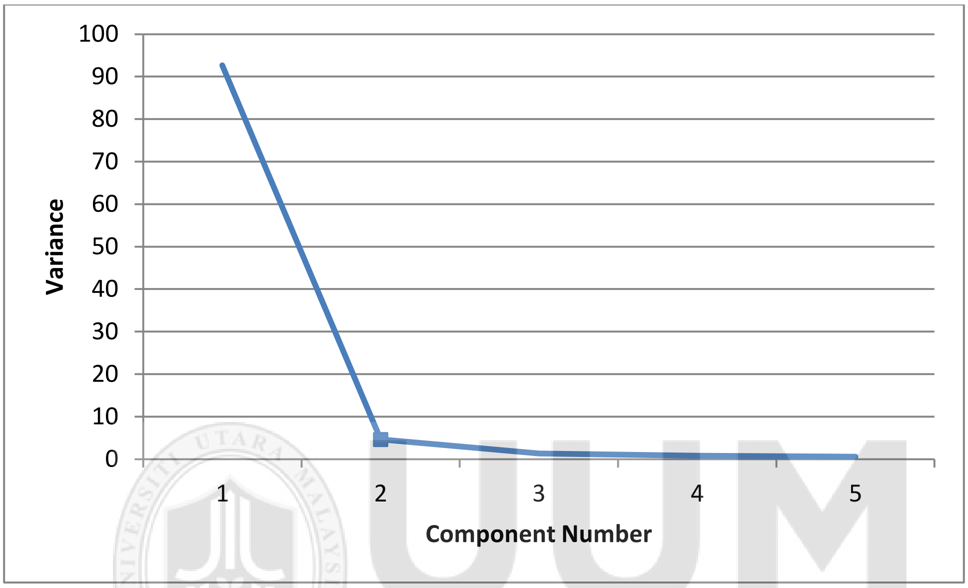

Figure 4.12. Variance plot for NASDAQ dataset

The sample of returns for five days are shown in Table 4.24. The two windows are $day_3 - day_5$, and $day_2 - day_4$ as $window_1(W_1)$, and $window_2\,(W_2)$ respectively.

Table 4.24

Sample Data from Sliding Window for NASDAQ Index

Date	Day	Return
Tue, Mar 17, 2015	51	
Wed, Mar 18, 2015	52	0.00008
Thu, Mar 19, 2015	53	0.00000
Fri, Mar 20, 2015	54	0.00005

Next, derived from Equation (3.8), the total of window size weight given is determined:

$$\mathrm{TL} = \mathrm{w}_2 + \mathrm{w}_1 = 1 + 2 = 3$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_2 = \frac{w_2}{TL} = \frac{1}{3} = 0.33$$

$$Norm_1 = \frac{w_1}{TL} = \frac{2}{3} = 0.67$$

Further, derived from Equation (3.1), the sliding window variance for day_n ($n = 54$) is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2)$$

where $u_{54}^2 = 0.00005$, $u_{53}^2 = 0.0$

$$V_W = (0.67 \times 0.00005) + (0.33 \times 0.0) = 0.00003$$

The value of $V_w = 0.00003$, or 0.003%. In other words, the window variance for day 54 is 0.003%.

4.7.4 Recent Variance

The recent variance i.e. variance for day 54 can be calculated using data from Table 4.24. Thus $\sigma_{n-1}^2 = 0.000004$.

4.7.5 SWGARCH Variance

Derived from Equation (3.3).The variance for day 54 is calculated using Equation (3.3) as follows:

$$\sigma_{54}^2 = (0.8230 \times 0.000004) + (0.1542 \times 0.000004) + (0.0228 \times 0.000004)$$
$$= 0.00002$$

4.7.6 The Forecasting

In the forecasting the price index on day 55, the variance on day 54 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{54}^2] = 0.000004 + 0.37040^1(0.00002 - 0.000004) = 0.00003$$

The forecasted price index of day 55 is computed as follows:

$$5026.41992 + (5026.41992 \times 0.0003) = 5011.11113$$

4.8 Case Study of Dow Jones Index

Sample of the Dow Jones Index is as displayed in Figure 4.13.

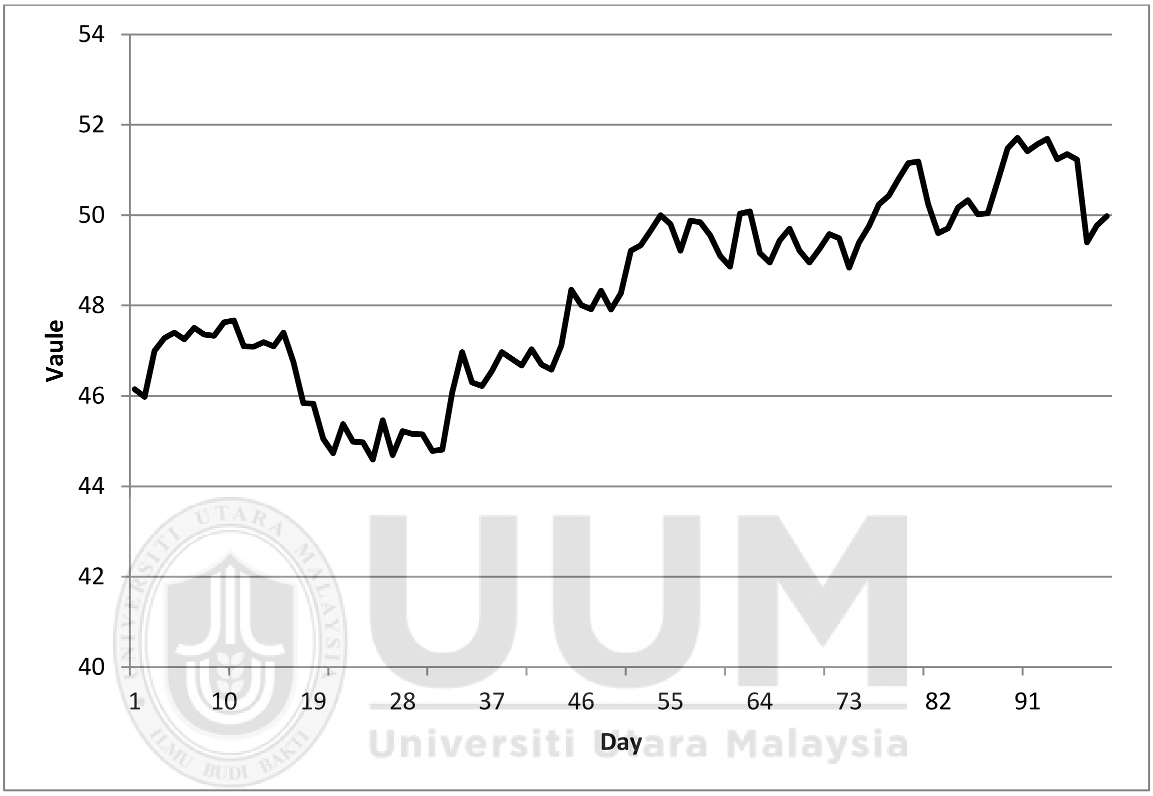

Figure 4.13. Sample Dow Jones Index data

4.8.1 Estimating SWGARCH Parameters

Table 4.25 shows sample data used for the calculation of parameters estimation. The first and second columns in the table show the day and the price index (S_i) for the day respectively. The third column records the change in rate (S_i) at the end of day $_i$ where $\delta_i = (S_i - S_{i-1})/\, S_{i-1}$. The forth column records the estimate of variance rate, $v_i = \sigma_i^2$, for quarter $_i$ based on the change rate. The fifth column tabulates the likelihood measure (L) and its can be obtained using Equation (3.6). We are interested in choosing $\gamma, \alpha,$ and β to maximize the sum of the numbers in the fifth column. This involves an iterative search procedure.

Table 4.25

Parameters Calculation for Dow Jones Index

Date	$\boldsymbol{Day_i}$	$\boldsymbol{S_i}$	$\boldsymbol{u_i}$	$\boldsymbol{\delta_i}$	$\boldsymbol{-ln(v_i) - v_i^2 \div v_i}$
Fri, Jan 02, 2015	1	17,832.99023			
Mon, Jan 05, 2015	2	17,501.65039	-0.01858		
Tue, Jan 06, 2015	3	17,371.64063	-0.00743	0.00015	8.43323
Wed, Jan 07, 2015	4	17,584.51953	0.01225	0.00012	7.77123
Thu, Jan 08, 2015	5	17,907.86914	0.01839	0.00022	6.88173
⋮	⋮	⋮	⋮	⋮	⋮
Thu, Mar 12, 2015	48	17,895.22070	0.01473	0.00013	7.28079
Fri, Mar 13, 2015	49	17,749.31055	-0.00815	0.00013	8.43236
Mon, Mar 16, 2015	50	17,977.41992	0.01285	0.00012	7.65959
				Sum	418.4264

The values shown in the fifth column of Table 4.25 were calculated in the final iteration of search for the optimal γ , α, and β. In this dataset, the optimal values of the parameters are

$$\gamma = 0.61965, \qquad \alpha = 0.12470, \qquad \beta = 0.24570$$

4.8.2 The Return Calculation

Table 4.26 displays another 6 days of Dow Jones Index data where the first column display day index, column two displays index price. Period return is displayed in the third column. The Return of each day of price index is computed by squaring the period return of the third column of Table 4.26. This table shows the sequence of index value during 4 days. In this case, the return from day 51 to day 54 are 0.00016, 0.00004, and 0.00009, respectively.

Table 4.26

Computation of Return

Date	$\boldsymbol{Day}_{i}$	**Close Price**	**Period Return** $u_i = ln\left(\frac{s_i}{s_{i-1}}\right)$	**Variance**
Tue, Mar 17, 2015	51	17,849.08008		
Wed, Mar 18, 2015	52	18,076.18945	0.01264	
Thu, Mar 19, 2015	53	17,959.02930	-0.00650	0.00004
Fri, Mar 20, 2015	54	18,127.65039	0.00935	0.00006

4.8.3 Computation of Sliding Window Variance

Table 4.27 displays the component numbers and variance explained for Dow Jones Index.

Table 4.27

Dow Jones Index Variance

Component Number	**Variance Explained (%)**
1	79.5044
2	11.5894
3	3.5786
4	2.7361
5	2.5912

For the Dow Jones Index dataset, the big drop in variance explained after component 2 is shown in Figure 4.14. Components 2 through 5 appear at the base of the cliff composed of component 3. The value of three components 1-2 is 91.1 % of the total variance. This supports the second component as a window size.

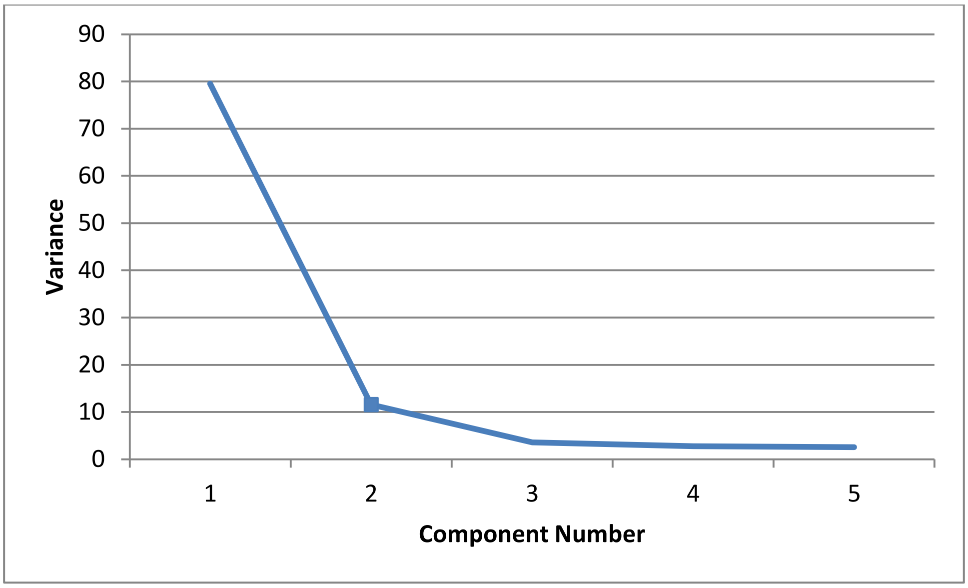

Figure 4.14. Variance plot for Dow Jones Index

The sample of returns for five days are shown in Table 4.28. The two windows are $day_3 - day_5$, and $day_2 - day_4$ as $window_1\,(W_1)$, and $window_2\,(W_2)$ respectively.

Table 4.28

Sample Data from Sliding Window for Dow Jones Index

Date	Day_i	**Return**
Tue, Mar 17, 2015	51	
Wed, Mar 18, 2015	52	0.00016
Thu, Mar 19, 2015	53	0.00004
Fri, Mar 20, 2015	54	0.00009

Next, derived from Equation (3.8), the total of window size weight given is determined:

$$\mathrm{TL} = \mathrm{w}_2 + \mathrm{w}_1 = 1 + 2 = 3$$

Derived from Equation (3.9), each weight is normalized as shown in the equation below:

$$Norm_2 = \frac{w_2}{TL} = \frac{1}{3} = 0.33$$

$$Norm_1 = \frac{w_1}{TL} = \frac{2}{3} = 0.67$$

Further, derived from Equation (3.1), the sliding window variance for day_n $(n = 54)$ is calculated as in the equation below:

$$V_w = (W_1 \times u_n^2) + (W_2 \times u_{n-1}^2)$$

where $u_{54}^2 = 0.00009$, $u_{53}^2 = 0.00004$

$$V_W = (0.67 \times 0.00009) + (0.33 \times 0.00004) = 0.00007$$

The value of $V_w =$ 0.00007, or 0.007%. In other words, the window variance for day 54 is 0.007%.

4.8.4 Recent Variance

The recent variance i.e. variance for day 54 can be calculated using data from Table 4.26. Thus $\sigma_{53}^2 = 0.00004$.

4.8.5 SWGARCH Variance

The variance for day 54 is calculated using Equation (3.3) as follows:

$\sigma_{54}^2 = (0.61965 \times 0.00007) + (0.12470 \times 0.00001) + (0.24570 \times 0.00004) = 0.0011$

4.8.6 The Forecasting

In forecasting the price index on day 55, the variance on day 54 will have to be calculated. This can be done using Equation (3.12) as shown.

$$\mathrm{E}[\sigma_{54}^2] = 0.00007 + 0.37040^1(0.0011 - 0.00007) = 0.00007$$

The forecasted price index of day 55 is computed as follows:

$$18127.6504 + (18127.6504 \times 0.00007) = 18117.2721$$

4.9 SWGARCH Model Performance

The following sections discuss the performance of the SWGARCH model for Senara station river water level, Kuala Nerang station river water level, KL HPI, Florida HPI, Malaysia HPI, NASDAQ index, and Dow Jones index case studies.

4.9.1 The Performance of Senara Station Case Study

Figure 4.15 shows the sample result of the real data of the model for 50 days. The MSE is 0.053292 and the MAPE is 6.0134 for the river water level at Senara station. The output of the model for Senara water level is as shown in the figure. This confirms that the model's propagated predicted value is almost as accurate as the actual dataset.

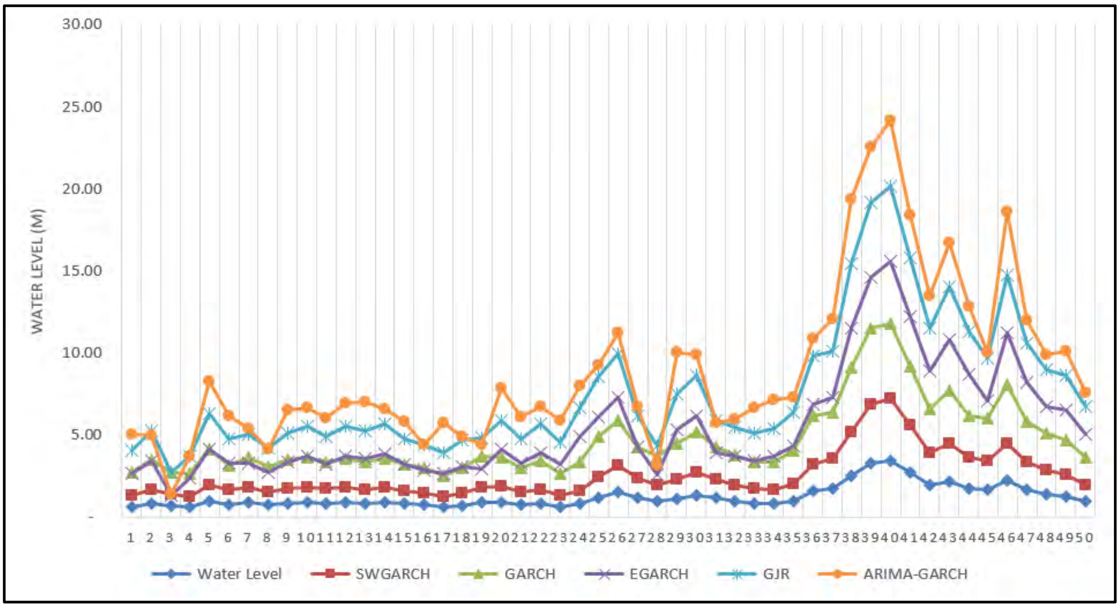

Figure 4.15. Actual and forecast water level for Senara station

Table 5.50 depicts the results of forecasting using the SWGARCH model for first ten datasets of Senara Station. The first column in the table records the day number. The second column records the water level value. The third column records the forecasted value. The fourth column records the error values, which are computed as follows:

$$\text{Error} = |\text{Actual value} - \text{forecasted value}|$$

The MSE value of this sample result shown in the error column is 3.87380%. This supports that the model's propagated forecasted value is almost as accurate as the actual dataset. The full results are provided in Appendix B.

Table 4.29

Sample Model Performance for Senara Station

Date	Day $_i$	Water Level	Forecast Value	| Error |
Mon, Mar 19, 2007	78	0.65		
Tue, Mar 20, 2007	79	0.83	0.83181	0.21769
Wed, Mar 21, 2007	80	0.68	0.69744	2.56491
Thu, Mar 22, 2007	81	0.63	0.65547	4.04316
Fri, Mar 23, 2007	82	0.97	0.99578	2.65787
Sat, Mar 24, 2007	83	0.79	0.85625	8.38590
Sun, Mar 25, 2007	84	0.87	0.95058	9.26240
Mon, Mar 26, 2007	85	0.76	0.79533	4.64902
Tue, Mar 27, 2007	86	0.85	0.86453	1.70957
Wed, Mar 28, 2007	87	0.91	0.92250	1.37369
			Average Error	3.87380

4.9.2 The Performance of Kuala Nerang Case Study

Figure 4.16 shows the sample result of the real data of the model for 50 days. The MSE is 0.05329 and the MAPE is 6.01342 for the river water level at Kuala Nerang station. The output of the model for Kuala Nerang water level is clearly explained as shown in the figure. This confirms that the model's propagated predicted value is almost as accurate as the actual dataset.

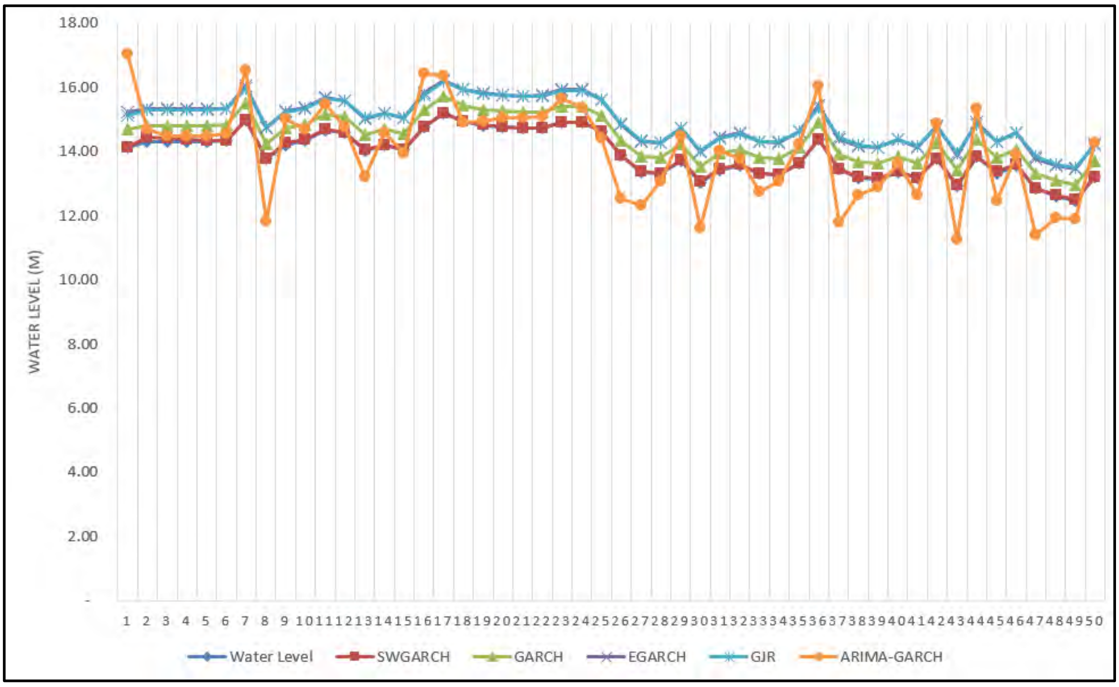

Figure 4.16. Actual and forecast water level for Kuala Nerang station

Table 4.30 shows the results of using the SWGARCH model for the first ten datasets of Kuala Nerang station. The MSE value of this sample result shown in the error column is 0.25535%. This supports that the model's propagated predicted value is almost as accurate as the actual dataset. The full results are provided in Appendix C.

Table 4.30

Sample Model Performance for Kuala Nerang Station

Date	Day i	Water Level	Forecast Value	| Error |
Tue, Mar 20, 2007	79	14.3		
Wed, Mar 21, 2007	80	14.3	14.42472	0.80169
Thu, Mar 22, 2007	81	14.3	14.36660	0.39555
Fri, Mar 23, 2007	82	14.3	14.33664	0.18619
Sat, Mar 24, 2007	83	14.3	14.33635	0.04431
Sun, Mar 25, 2007	84	15	14.98221	0.01478

Mon, Mar 26, 2007	85	13.8	13.78751	0.05451
Tue, Mar 27, 2007	86	14.2	14.25591	0.25254
Wed, Mar 28, 2007	87	14.3	14.38809	0.33535
Thu, Mar 29, 2007	88	14.7	14.69098	0.21130
			Average Error	0.25535

4.9.3 The Performance of KL HPI Case Study

Figure 4.17 shows the sample result of the real data of the model from week 1 to week 33. The MSE is 0.00168 and the MAPE is 0.16439 for the output of the KL HPI model. This confirms that the model's propagated forecasted value is almost as accurate as the actual dataset.

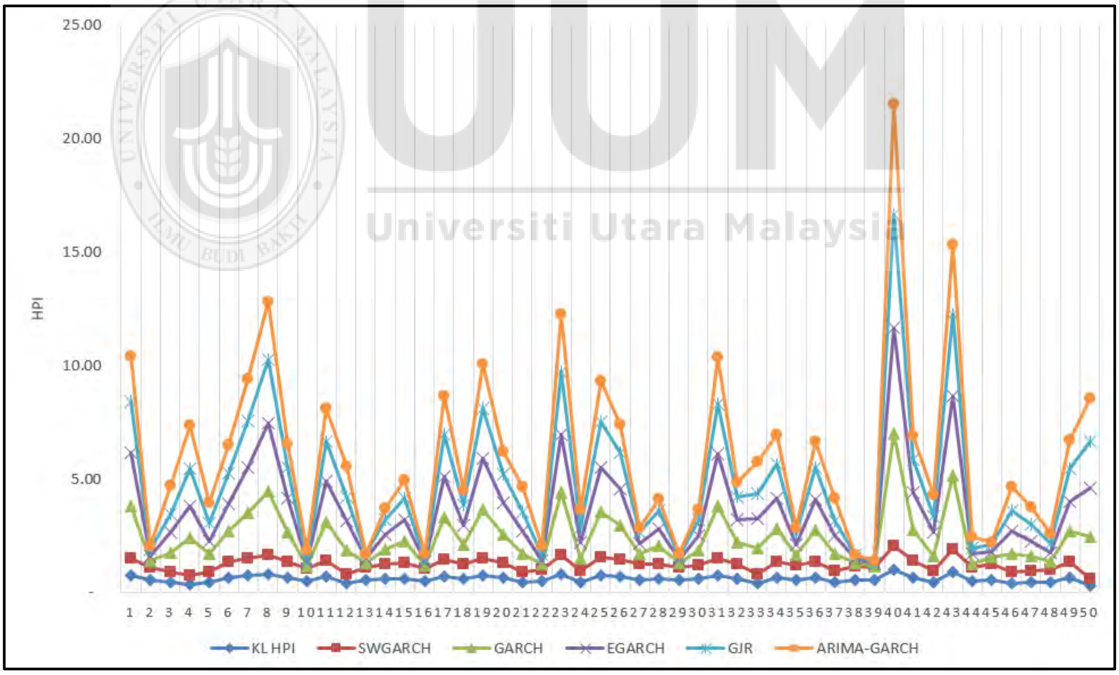

Figure 4.17. Actual and forecast values for KL House Price

Table 4.31 shows the results of using the SWGARCH model for the first ten datasets of the KL House Price Index. Again, it can be seen that the performance of

SWGARCH is good. The MSE value of this sample result shown in the error column is 5.5164%. The full results are provided in Appendix D.

Table 4.31

Sample Model Performance for KL House Price Index

Date	$Week_i$	House Price	Forecast Value	| Error |
Sun, Feb 17, 2013	58	54		
Sun, Feb 24, 2013	59	43	0.45550	5.93018
Sun, Mar 03, 2013	60	35	0.36947	5.56263
Sun, Mar 10, 2013	61	43	0.45701	6.28038
Sun, Mar 17, 2013	62	65	0.68832	5.89566
Sun, Mar 24, 2013	63	72	0.76061	5.64033
Sun, Mar 31, 2013	64	80	0.85345	6.68115
Sun, Apr 07, 2013	65	65	0.67814	4.32996
Sun, Apr 14, 2013	66	50	0.53277	6.55317
Sun, Apr 21, 2013	67	69	0.70914	2.77384
			Average Error	5.5164

4.9.4 The Performance of Florida HPI Case Study

Figure 4.18 shows the sample result of the real data of the model for 37 weeks. The MSE is 21.80 and the MAPE is 5.1993 for the output of the Florida HPI model. This confirms that the model's propagated forecast value is almost as accurate as the actual dataset.

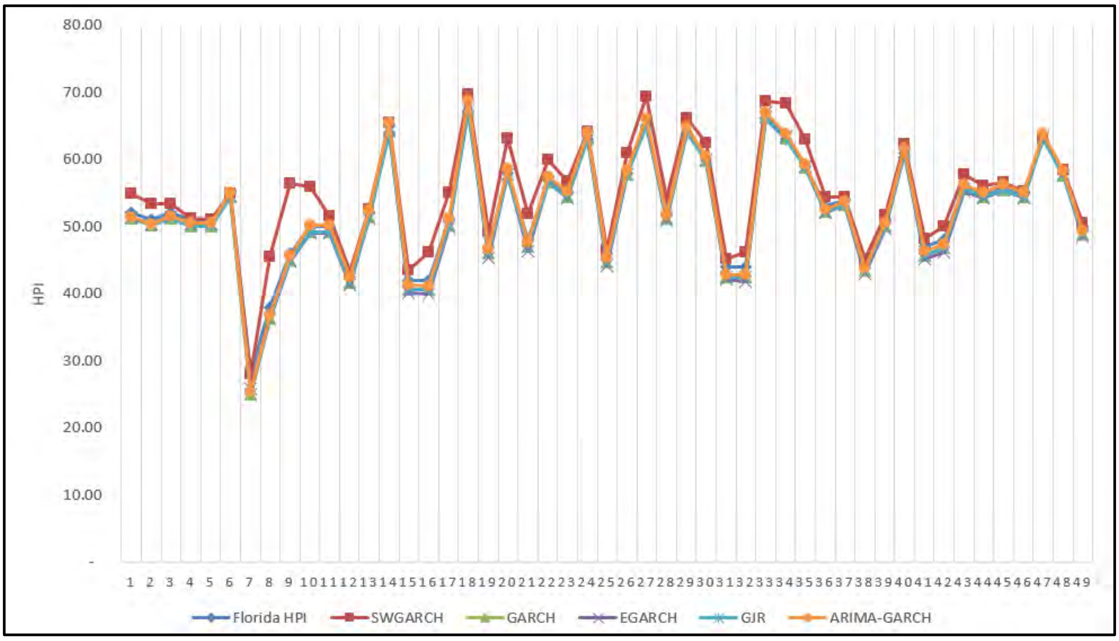

Figure 4.18. Actual and forecast value for Florida HPI

Table 4.32 describes the results of production using the SWGARCH model for the first ten datasets of Florida HPI. The MSE value of this sample result shown in the error column is 6.9788%. The full results are provided in Appendix D.

Table 4.32

Sample Model Performance for Florida HPI

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Feb 10, 2013	57	52		
Sun, Feb 17, 2013	58	51	53.48991	4.88219
Sun, Feb 24, 2013	59	52	53.37082	2.63619
Sun, Mar 03, 2013	60	51	51.28087	0.55072
Sun, Mar 10, 2013	61	51	51.01904	0.03733
Sun, Mar 17, 2013	62	55	55.01148	0.02086
Sun, Mar 24, 2013	63	28	28.06989	0.24961
Sun, Mar 31, 2013	64	38	45.50719	19.75576

Sun, Apr 07, 2013	65	46	56.46512	22.75025
Sun, Apr 14, 2013	66	50	55.96340	11.92680
			Average Error	6.97886

4.9.5 The Performance of Malaysia HPI Case Study

Figure 4.19 shows the sample result of the real data of the model from 2003 - Q1 to 2015 – Q2. The MSE is 21.7955 and the MAPE is 5.1993 for the Malaysia house price model.

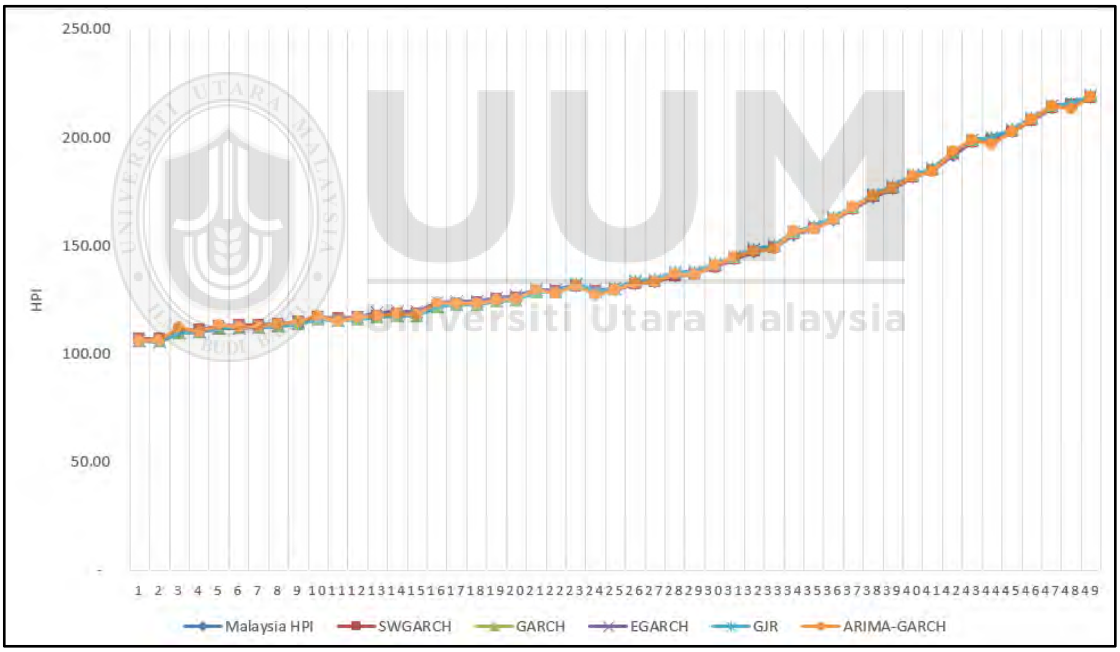

Figure 4.19. Actual and forecast value for Malaysia HPI

Table 4.33 displays the results of production using the SWGARCH model for the first ten datasets of Malaysia HPI. The MSE value of this sample result shown in the error column is 1.44%. This supports that the model's propagated forecasted value is almost as accurate as the actual dataset. The full results are given in Appendix E.

Table 4.33

Sample Model Performance for Malaysia House Price Index

Date	Quarter i	House Price	Forecast Value	| Error |
2003 Q1	17	107.2		
2003 Q2	18	107.1	107.12071	0.01933
2003 Q3	19	110.6	110.61202	0.01087
2003 Q4	20	111.2	111.22007	0.01805
2004 Q1	21	112.8	112.83439	0.03049
2004 Q2	22	113.1	113.12022	0.01788
2004 Q3	23	113.4	113.41653	0.01458
2004 Q4	24	114	114.00952	0.00835
2005 Q1	25	115	115.00627	0.00546
2005 Q2	26	116.9	116.90542	0.00463
			Average Error	0.01440

4.9.6 The Performance of NASDAQ Index Case Study

*Figure 4.20*Figure 4.20 shows the sample result of the real data of the model from day 1 to day 100. The MSE is 1.1110 and the MAPE is 0.01209 for the NASDAQ index model.

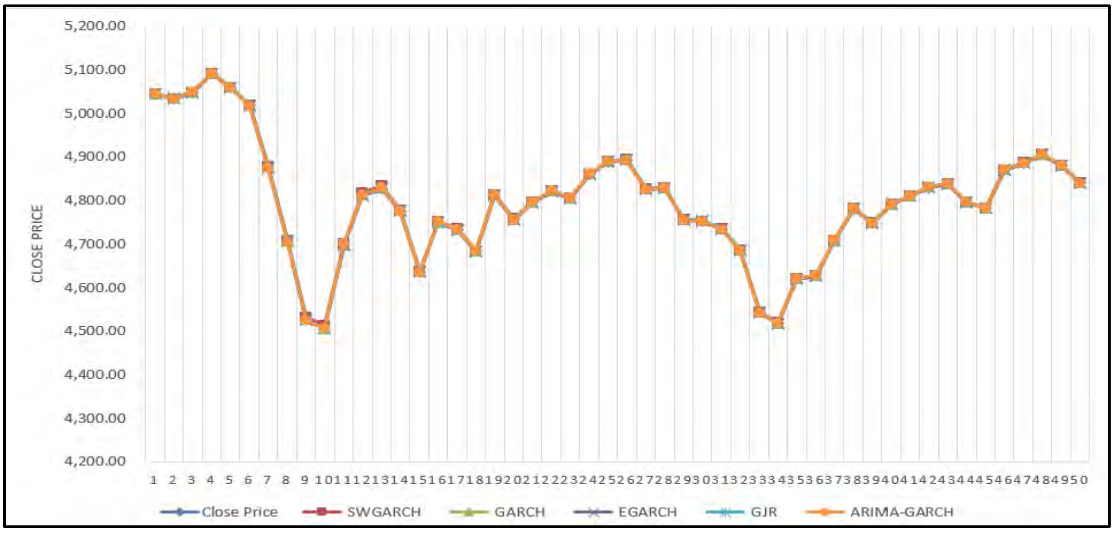

Figure 4.20. Actual and forecast value for NASDAQ Index

Table 4.34 shows the results of using the SWGARCH model for the first ten datasets of the NASDAQ Index. The MSE value of this sample result shown in the error column is 0.984%. This supports that the model's forecasted value is almost as accurate as the actual dataset. The full results are provided in Appendix F.

Table 4.34

Sample Model Performance for NASDAQ Index

Date	**Day *i***	**Close Price**	**Forecast Value**	**| Error |**
Mon, Mar 23, 2015	55	5,010.9702		
Tue, Mar 24, 2015	56	4,994.7300	4994.84294	0.00226
Wed, Mar 25, 2015	57	4,876.5200	4876.57565	0.00114
Thu, Mar 26, 2015	58	4,863.3599	4864.98866	0.03349
Fri, Mar 27, 2015	59	4,891.2202	4892.27493	0.02156
Mon, Mar 30, 2015	60	4,947.4399	4947.65827	0.00441
Tue, Mar 31, 2015	61	4,900.8799	4901.33579	0.00930
Wed, Apr 01, 2015	62	4,880.2300	4880.72864	0.01022
Thu, Apr 02, 2015	63	4,886.9399	4887.17897	0.00489
Mon, Apr 06, 2015	64	4,917.3198	4917.38164	0.00126
			Average Error	0.00984

4.9.7 The Performance of Dow Jones Index Case Study

Figure 4.21 shows the sample result of the real data of the model from day 1 to day 100. The MSE is 8.93591 and the MAPE is 0.00955 for the Dow Jones index model.

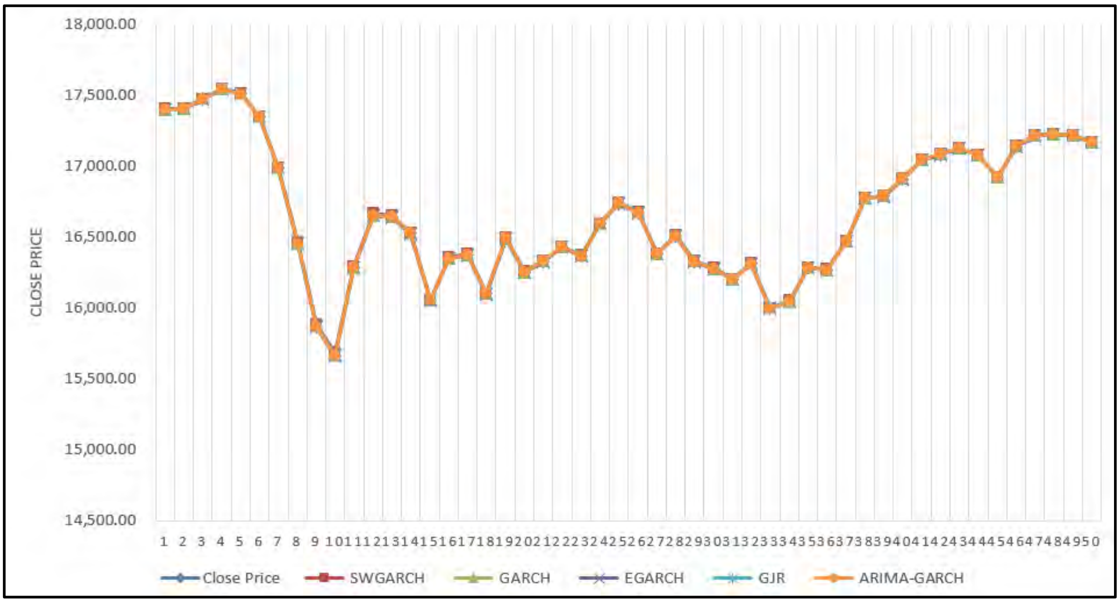

Figure 4.21. Actual and forecast value for Dow Jones Index

Table 4.35 displays the results of using the SWGARCH model for the first ten datasets of the Dow Jones Index. The MSE value of this sample result shown in the error column is 0.792%. This supports that the model's forecasted value is almost as accurate as the actual dataset. The full results are provided in Appendix G.

Table 4.35

Sample Model Performance for Dow Jones Index

Date	**Day *i***	**Close Price**	**Forecast Value**	**| Error |**
Mon, Mar 23, 2015	55	18,116.0391		
Tue, Mar 24, 2015	56	18,011.1406	17718.95603	0.00235
Wed, Mar 25, 2015	57	17,718.5391	17681.19694	0.01678
Thu, Mar 26, 2015	58	17,678.2305	17714.48699	0.01031
Fri, Mar 27, 2015	59	17,712.6602	17976.58015	0.00150
Mon, Mar 30, 2015	60	17,976.3105	17778.42887	0.01299
Tue, Mar 31, 2015	61	17,776.1191	17700.89229	0.01533
Wed, Apr 01, 2015	62	17,698.1797	17764.41223	0.00660

Thu, Apr 02, 2015	63	17,763.2402	17881.24162	0.00219
Mon, Apr 06, 2015	64	17,880.8496	17876.00108	0.00325
			Average Error	0.00792

4.10 Model Comparison

The results from this experiment are as tabulated in Table 4.36 and Table 3.37 in term of MSE and MAPE. In term of MSE, SWGARCH provides the best results for four (4) datasets out of seven (7) datasets while GARCH, EGARCH and GJR each produced one (1) best result as shown in Table 4.36.

Table 4.36

MSE Model Performance

Dataset	SWGARCH	GARCH	EGARCH	GJR	ARIMA-GARCH
Senara	**0.00168**	1.0041	1.0728	1.0039	1.0173
Kuala Nerang	**0.0017**	1.0013	1.0005	0.9999	1.0145
KL HPI	**0.0032**	1.02619	1.0163	1.04265	1.02764
Florida	21.7955	1.0088	**0.9882**	1.0088	1.0027
Malaysia HPI	**0.00270**	1.0739	1.0345	1.0739	0.9222
NASDAQ	1.1110	1.0002	1.0728	**0.9999**	1.0173
Dow Jones	8.9359	**0.9997**	1.0728	1.0000	1.0173

In term of MAPE, SWGARCH provides the best results for five (5) datasets out of seven (7) datasets while GARCH and ARIMA-GARCH each produced one (1) best result as shown in Table 4.37.

Table 4.37

MAPE Model Performance

Dataset	**SWGARCH**	**GARCH**	**EGARCH**	**GJR**	**ARIMA-GARCH**
Senara	**6.0134**	79.7457	79.2571	79.7665	71.0448
Kuala Nerang	**0.16439**	7.3516	7.3487	7.3615	5.2755
KL HPI	**0.070**2	1.49498	1.49714	1.50538	1.50404
Florida	5.1993	**1.2938**	1.3341	1.2938	1.3293
Malaysia HPI	**0.02405**	0.7012	0.7024	0.7012	0.5289
NASDAQ	**0.01209**	0.0201	0.0180	0.0202	0.0158
Dow Jones	0.00955	0.0057	0.0051	0.0058	**0.0045**

In order to compare and represent the performance of the proposed algorithms visually, a geometric mean is calculated to normalize the MSE, and MAPE values of the seven datasets (Izakian, Abraham, & Snsel, 2009). In addition, the difference between the SWGARCH algorithm and the proposed hybrid algorithms are calculated to provide the enhancement of each algorithm in terms of percentage.

Figure 4.22 displays the results of the five algorithms in terms of the best MSE value which shows that the worse performance produced by EGARCH algorithm. The algorithms GARCH, GJR, and ARIMA-GARCH show similar performance to each other which is better than EGARCH algorithm. The SWGARCH algorithm achieved the best performance in terms of best MSE value.

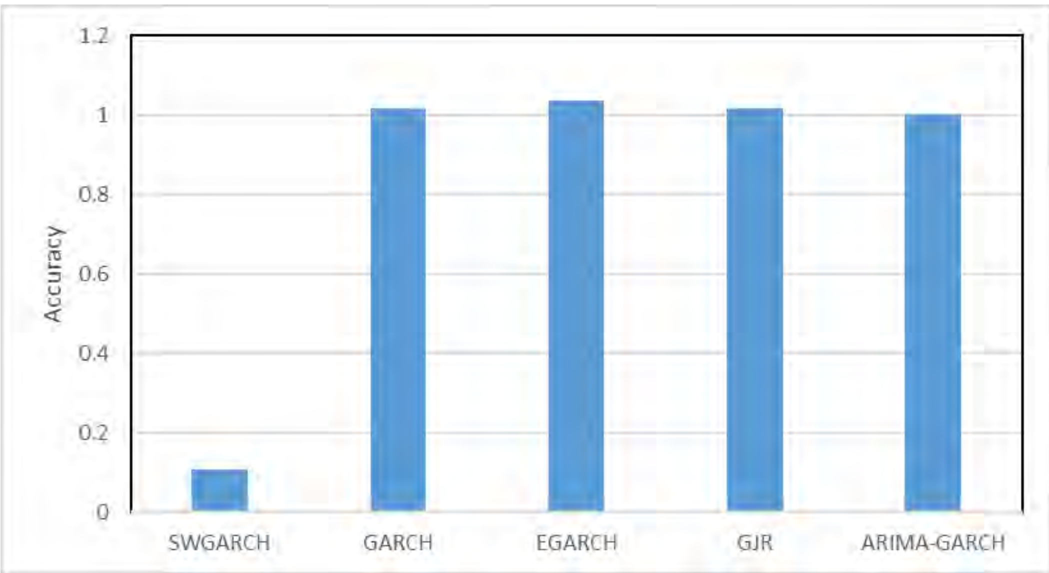

Figure 4.22. Geometric mean for the best MSE values

Figure 4.23 represents the enhancement of each algorithm which is expressed in terms of percentage. Each algorithm is compared with the GARCH algorithm in terms of best MSE value enhancement. Figure 4.23 shows that SWGARCH algorithm enhanced 89.39% followed by EGARCH 2%, GJR 0.2%, and ARIMA-GARCH 1.37%. This enhancement indicates that SWGARCH algorithm increased the performance.

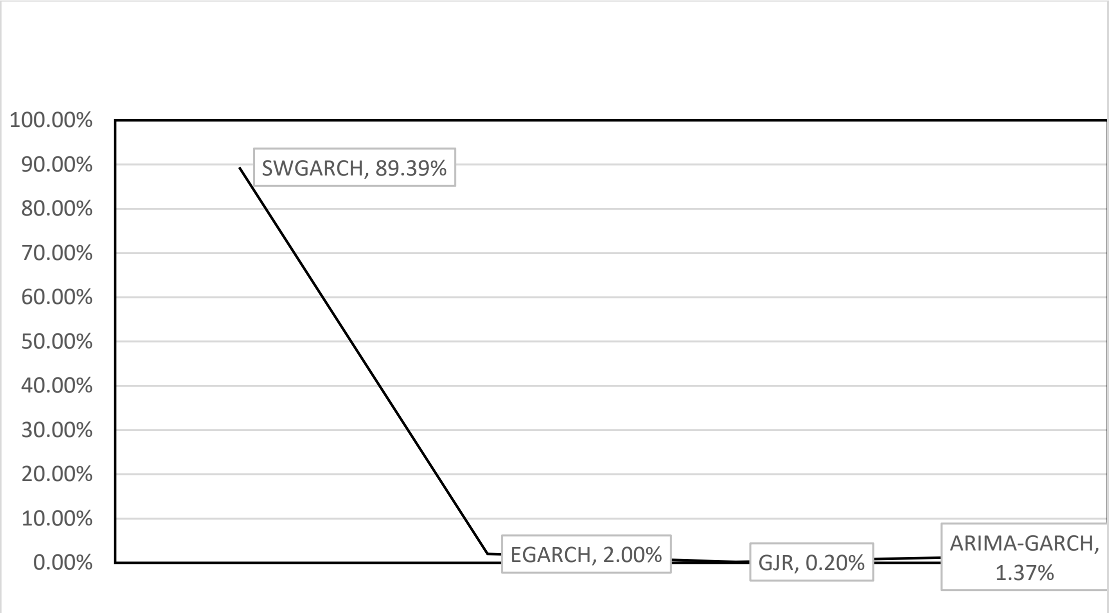

Figure 4.23. The percentage enhancement of each algorithm in terms of the best MSE

Figure 4.23 displays the results of the five algorithms in terms of the best MAPE value. The worse performance are produced by GARCH and GJR algorithms. The EGARCH and ARIMA-GARCH algorithms show better performances than GARCH and GJR algorithms. The SWGARCH algorithm achieved the best performance in terms of best MAPE value as shown in Figure 4.23.

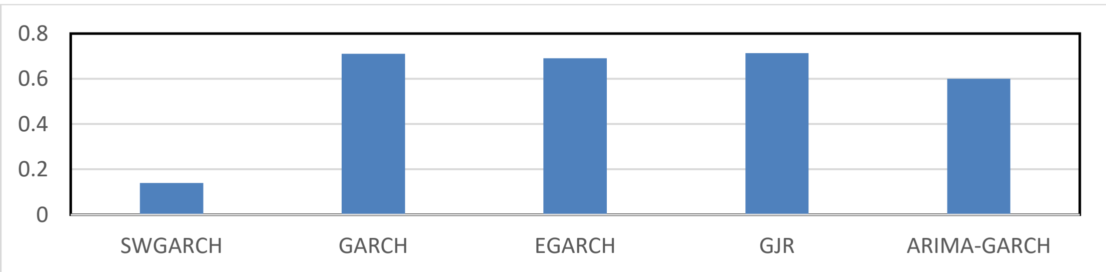

Figure 4.24. Geometric mean for the best MAPE values

Figure 4.23 represents the enhancement of each algorithm which is expressed in terms of percentage. Each algorithm is compared with the GARCH algorithm in terms of best MAPE value enhancement. The Figure 4.25 shows that SWGARCH algorithm enhanced 80.41% followed by EGARCH 2.74%, GJR 0.44%, and ARIMA-GARCH 15.43%. This enhancement indicates that SWGARCH algorithm increased the performance.

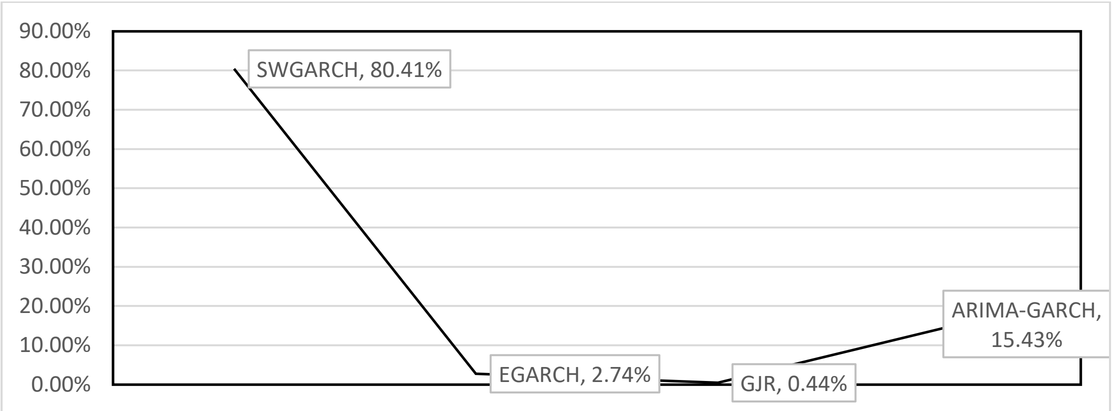

Figure 4.25. The percentage enhancement of each algorithm in terms of the best MAPE

4.11 Summary

Overall results show that SWGRACH provides the best performance when compared with other common hybrid algorithms for the datasets that have been applied in terms MSE and MAPE.

CHAPTER FIVE

CONCLUSION AND FUTURE WORK

Time series forecasting has experienced major changes for the past decades. Related studies covered issues on time series forecasting utilizing various models, covering from stationary to nonstationary models. Due to the linear structure of the statistical model, it is inefficient in dealing with the forecasting of the time series data of interest. This led to models' nonstationary modeling, especially for volatility modeling. The GARCH model is the most popular model used for volatility modeling. Nonetheless, several weaknesses of GARCH make it suffer long run variance. By using SWGARCH, the disadvantage of GARCH is addressed. However, an efficient optimization algorithm is needed in order to increase the capability of SWGARCH in the prediction task. For that matter, particular concerns are given on parameter estimation and sliding window of the SWGARCH parameters using the likelihood estimation algorithm, and PCA, which has been proven to overcome the limitation of the manual approach. Based on the literature review that has been done in Chapter 2, the proposed model is designed, which is presented in Chapter 3 and Chapter 4.

5.1 Research Contribution

The prediction accuracy of time series data is vital, by using real datasets of Senara and Kuala Nerang, NASDAQ Index and Dow Jones Index, HPI of Kuala Lumpur and HPI of Florida. This is to achieve Objective 1. As to address the highlighted problem, enhancements that are introduced to the GARCH model are proven to be beneficial to the problem under study.

The sliding window technique has been hybridized with GARCH model to overcome the log run variance, which fulfilled objective 2. Experimental results showed that the performance of SWGARCH is better than GARCH, EGARCH, GJR, and ARIMA-GARCH in most of the case studies datasets, which confirmed that SWGARCH can be used as an enhancement model for time series forecasting. Thus, objective 3 is achieved.

5.2 Future Work

Based on the experiment and results presented in Chapter 4, the proposed SWGARCH is proven to be more superiorly relative to the metrics utilized as compared to the other identified prediction techniques. This indicates that SWGARCH possess significant implications to the problem of interest. However, there is always possible potential applications that can be explored in the future and improve its limitation.

First, SWGARCH is used for short-term forecasting effectively. More enhancements are needed to enable SWGARCH to be utilized for long-term forecasting.

Second, in this study, the experiments that have been conducted involved environment and economic datasets. Thus, it would be interesting to test the efficiency and applicability of SWGARCH on renewable commodities datasets such as finance datasets.

Lastly, the hybridization of SWGARCH and clustering technique could be used to enhance the performance of this model.

REFERENCES

Abarbanel, H. (1997). *Analysis of Observed Chaotic Data*. Cambridge University Press.

Akpinar, M., & Yumusak, N. (2013). Forecasting household natural gas consumption with ARIMA model: A case study of removing cycle. In *2013 7th International Conference on Application of Information and Communication Technologies* (pp. 1–6). IEEE. doi:10.1109/ICAICT.2013.6722753

Areekul, P., Senjyu, T., Toyama, H., & Yona, A. (2009). Combination of artificial neural network and ARIMA time series models for short term price forecasting in deregulated market. In *2009 Transmission & Distribution Conference & Exposition: Asia and Pacific* (pp. 1–4). IEEE. doi:10.1109/TD-ASIA.2009.5356936

Awartani, B. M., & Corradi, V. (2005). Predicting the volatility of the S&P-500 stock index via GARCH models: The role of asymmetries. *International Journal of Forecasting*, *21*(1), 167–183. doi:10.1016/j.ijforecast.2004.08.003

Babu, C., & Reddy, B. (2012). Predictive data mining on Average Global Temperature using variants of ARIMA models. In *IEEE-International Conference On Advances In Engineering, Science And Management (ICAESM - 2012)* (pp. 256–260).

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*(3), 307–327. doi:10.1016/0304-4076(86)90063-1

Box, P., & Jenkins, M. (1976). *Time-Series Analysis: Forecasting and Control* (2nd ed.). San Francisco: Holden-Day.

Box, P., Jenkins, M., & Reinsel, C. (1994). *Time Series Analysis: Forecasting and Control* (3rd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Brooks, C. (2008). *Introductory Econometrics for Finance*.

Casella, G., Fienberg, S., & Olkin, I. (2006). *Springer Texts in Statistics*. *Design* (Vol. 102). doi:10.1016/j.peva.2007.06.006

Centre National Property Information. (2015). The Malaysian House Price Index By House Type (1988 - 2015). Retrieved December 2, 2016, from http://napic.jpph.gov.my/

Changbao, Z., Jun, Z., & Guoli, L. (2016). Research on sequence component detection algorithm based on sliding-window mean-value. In *2016 IEEE 11th Conference on Industrial Electronics and Applications (ICIEA)* 531–533. IEEE. doi:10.1109/ICIEA.2016.7603641

Chen, J., Yong, X., Yang, S., & Liu, L. (2009). Analysis and forecast about mineral product price based on time series method. *Journal of Kunming University of Science and Technology*, *34*(6), 9–14.

Chen, P., Yuan, H., & Shu, X. (2008). Forecasting Crime Using the ARIMA Model.

Cohen, J., Cohen, P., West, G., & Aiken, S. (2002). *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*. Routledge.

Cryer, J. D., & Chan, K.-S. (2008a). *Time Series Analysis: With Applications in R* (2nd ed.).

Cryer, J. D., & Chan, K.-S. (2008b). *Time Series Analysis. With Applications to R. Design*. doi:10.1007/978-0-387-75959-3

Doorley, R., Pakrashi, V., Caulfield, B., & Ghosh, B. (2014). Short-term forecasting of bicycle traffic using structural time series models. In *17th International IEEE Conference on Intelligent Transportation Systems (ITSC)* (pp. 1764–1769). IEEE. doi:10.1109/ITSC.2014.6957948

Dougherty, C. (2011). *Introduction to Econometrics* (4th Editio.). Oxford University Press.

Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, *50*, 987–1007.

Engle, R. (2001). GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics. *Journal of Economic Perspectives*, *15*(4), 157–168. doi:10.1257/jep.15.4.157

Evans, T., & McMillan, D. G. (2007). Volatility forecasts: the role of asymmetric and long-memory dynamics and regional evidence. *Applied Financial Economics*, *17*(March 2015), 1421–1430. doi:10.1080/09603100601007149

Falk, M. (University of W. (2011). A First Course on Time Series Analysis, 364.

Falk, M., Marohn, F., Michel, R., Hofmann, D., & Macke, M. (2012). *A First Course on Time Series Analysis Examples with SAS by Chair of Statistics*. University of Wurzburg.

Fang, L., & Shen., L. (2010). Analysis of the recent trends of mineral resources asset prices in China based on the ARIMA method. *Journal of China Mining Magazine*, *8*(26–29).

Feng, Y., & Jones, K. (2015). Comparing multilevel modelling and artificial neural networks in house price prediction. In *2015 2nd IEEE International Conference on Spatial Data Mining and Geographical Knowledge Services (ICSDM)* (pp. 108–114). IEEE. doi:10.1109/ICSDM.2015.7298035

Forrester, E. C. (2006). A process research framework. *oftware Engineering Institute, Carnegie Mellon University*.

Gershenfeld, N. (1999). *The Nature of Mathematical Modeling*. Cambridge University Press.

Hajizadeh, E., Seifi, A., Fazel Zarandi, M. H., & Turksen, I. B. (2012). A hybrid modeling approach for forecasting the volatility of S&P 500 index return. *Expert Systems with Applications*, *39*(1), 431–436. doi:10.1016/j.eswa.2011.07.033

Hao, C., Fangxing, L., Qiulan, W., & Wang, Y. (2011). Short term load forecasting using regime-switching GARCH models. In *2011 IEEE Power and Energy Society General Meeting* (pp. 1–6). IEEE. doi:10.1109/PES.2011.6039457

Hernandez, J., Ovando, M., Acosta, F., & Pancardo, P. (2016). Water Level Meter for Alerting Population about Floods. In *2016 IEEE 30th International Conference on Advanced Information Networking and Applications (AINA)* (pp. 879–884). IEEE. doi:10.1109/AINA.2016.76

Hong-qiong, H., & Tian-hao, T. (2007). Short-term Traffic Flow Forecasting Based on ARIMA-ANN. *2007 IEEE International Conference on Control and Automation*, 2370–2373.

Hull, J. C. (2002). *Options, Futures, and Other Derivatives* (5th ed.). Prentice Hall.

Hull, J. C. (2015). *Options, Futures and Other Derivatives*. *Pearson* (9th ed.). Pearson.

Jaasa, T., Androcec, I., & Sprci, P. (2011). Electricity price forecasting - ARIMA method approach. In *Proceeding of the 8th International Conference on the European Energy Market* (pp. 222–225).

Jacasa, T., Androcec, I., & Sprcic, P. (2011). Electricity price forecasting-ARIMA model approach. *8th International Conference on the European Energy Market*, 222–225.

Kantz, H., & Schreiber, T. (2004). *Nonlinear Time Series Analysis* (2nd ed.). Cambridge University Press.

Kapila Tharanga Rathnayaka, M., Seneviratna, D., Jianguo, W., & Arumawadu, H. I. (2015). A hybrid statistical approach for stock market forecasting based on Artificial Neural Network and ARIMA time series models. In *2015 International Conference on Behavioral, Economic and Socio-cultural Computing (BESC)* (pp. 54–60). IEEE. doi:10.1109/BESC.2015.7365958

Keogh, E., Chu, S., Hart, D., & Pazzani, M. (2003). Segmenting Time Series: A Survey and Novel Approach. *Data Mining in Time Series Databases*, 1–21. doi:10.1.1.12.9924

Khandelwal, I., Adhikari, R., & Verma, G. (2015). Time Series Forecasting Using Hybrid ARIMA and ANN Models Based on DWT Decomposition. *Procedia Computer Science*, *48*, 173–179. doi:10.1016/j.procs.2015.04.167

Khim-Sen, L., Shitan, M., & Huzaimi, H. (2007). Time series methodling and forecasting of Sarawak black pepper price. *Journal of Munich Personal RePEc Archive*, *791*, 1–16.

Kirchgassner, G. (2007). *Introduction to Modern Time Series Analysis*. Springer Berlin Heidelberg New York.

Ku Mahamud, K. R., Zakaria, N., Katuk, N., & Shbier, M. (2009). Flood Pattern Detection Using Sliding Window Technique. In *2009 Third Asia International Conference on Modelling & Simulation* (pp. 45–50). doi:10.1109/AMS.2009.15

Li, B. (2005). Economic forecast. *Economic Management Press*, 45–48.

Li, S., Lin, Z., Xiao, Z., & Ma, J. (2012). The use of GARCH-neural network model for forecasting the volatility of bid-ask spread of the Chinese stock market. In *2012 International Conference on Management Science & Engineering 19th Annual Conference Proceedings* (pp. 1899–1903). doi:10.1109/ICMSE.2012.6414430

Lu, X., Que, D., & Cao, G. (2016). Volatility Forecast Based on the Hybrid Artificial Neural Network and GARCH-type Models. *Procedia - Procedia Computer Science*, 1044–1049. doi:10.1016/j.procs.2016.07.145

McMillan, D., Speight, A., & Apgwilym, O. (2000). Forecasting UK stock market volatility. *Applied Financial Economics*, *10*(4), 435–448. doi:10.1080/09603100050031561

Mills T. (1990). *Time Series Techniques for Economists*. Cambridge University Press.

Monfared, S. A., & Enke, D. (2014). Volatility Forecasting Using a Hybrid GJR-GARCH Neural Network Model. *Procedia Computer Science*, *36*, 246–253. doi:10.1016/j.procs.2014.09.087

Narendra, B., & Reddy, E. (2014). Selected Indian stock predictions using a hybrid ARIMA-GARCH model. In *2014 International Conference on Advances in Electronics Computers and Communications* (pp. 1–6). IEEE. doi:10.1109/ICAECC.2014.7002382

National Flood Monitoring Centre. (2007). Hydrology and Water Resources Division of the Department of Irrigation and Drainage. Retrieved December 2, 2016, from http://infobanjir.water.gov.my/

Nguyen, T., Khosravi, A., Nahavandi, S., & Creighton, D. (2013). Neural network and interval type-2 fuzzy system for stock price forecasting. In *2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (pp. 1–8). doi:10.1109/FUZZ-IEEE.2013.6622370

Pang, H., & Jung, S.-H. (2013). Sample Size Considerations of Prediction-Validation Methods in High-Dimensional Data for Survival Outcomes. *Genetic Epidemiology*, *37*(3), 276–282. doi:10.1002/gepi.21721

Percival, B., & Walden, T. (1993). *Spectral Analysis for Physical Applications*. Cambridge University Press.

Puspitasari, I., Akbar, M. S., & Lee, M. H. (2012). Two-level seasonal model based on hybrid ARIMA-ANFIS for forecasting short-term electricity load in Indonesia. In *2012 International Conference on Statistics in Science, Business and Engineering (ICSSBE)* (pp. 1–5). IEEE. doi:10.1109/ICSSBE.2012.6396642

Rahman, M., Islam, S., Nadvi, S. Y., & Rahman, R. M. (2013). Comparative Study of ANFIS and ARIMA Model for Weather Forecasting in Dhaka.

Shatkay, H., & Zdonik, S. B. (1996). Approximate queries and representations for large data sequences. *Proceedings of the Twelfth International Conference on Data Engineering*, 536–545. doi:10.1109/ICDE.1996.492204

Sibel, H., & Yayar, R. (2006). Forecasting of Sunflower Oil Price in Turkey. *Application and Science Research*, *2*(9), 572–578.

Strickland, J. (2015). *Predictive Analytics using R*. Lulu.com.

Tang, R., Feng, T., Sha, Q., & Zhang, S. (2009). A Variable-Sized Sliding-Window Approach for Genetic Association Studies via Principal Component Analysis. *Annals of Human Genetics*, *73*(6), 631–637. doi:10.1111/j.1469-1809.2009.00543.x

Taylor, J. W., & Taylor, J. W. (2004). Smooth Transition Exponential Smoothing Smooth Transition Exponential Smoothing, *23*, 385–394.

Tinbergen, J. (1939). A Method and Its Application to Business Cycle Theory. *Statistical Analysis of Business Cycle Theories*.

Wong, B. (2014). Introduction to (Generalized) Autoregressive Conditional Heteroskedasticity Models in Time Series Econometrics.

Xie, M., Sandels, C., Zhu, K., & Nordstrom, L. (2013). A seasonal ARIMA model with exogenous variables for elspot electricity prices in Sweden. In *2013 10th International Conference on the European Energy Market (EEM)* (pp. 1–4). IEEE. doi:10.1109/EEM.2013.6607293

Yin, D., & Chen, W. (2016). The forecast of USD/CNY exchange rate based on the Elman neural network with volatility updating. In *2016 35th Chinese Control Conference (CCC)* (pp. 9562–9566). IEEE. doi:10.1109/ChiCC.2016.7554875

Yorucu, V. (2003). The Analysis of Forecasting Performance by Using Time Series Data for Two Mediterranean Islands. *Review of Social, Economic & Business Studies*, *2*, 175–196.

Zhang, G. (2003). Time series forecasting using a hybrid ARIMA and neural network model. *Neurocomputing*, *50*, 159–175.

Zou, W., & Guo, M. (2014). Using CARR model and GARCH model to forecast volatility of the stock index: Evidence from China's Shanghai stock market. In *2014 International Conference on Management Science & Engineering 21th Annual Conference Proceedings* (pp. 1106–1112). IEEE. doi:10.1109/ICMSE.2014.6930352

APPENDIX A: SWGARCH Algorithm

Window variance procedure

```
while (@@FETCH_STATUS = 0)
begin
       select @Return1=[Return] from [dbo].[CaseData]
       Where CaseID=@CaseID and    DataID=@DataID
       select @Return=[Return] from [dbo].[CaseData]
       Where CaseID=@CaseID and DataID=@DataID-1
       select @Return3=[Return] from [dbo].[CaseData]
       Where CaseID=@CaseID and DataID=@DataID-2

       set @WindowVariance =(@Return1 *.5) +(@Return *.33)+(@Return3 *.1667)

       fetch next from cur into @DataID,@DataVal
end
```

SWGARCH Procedure

```
fetch next from cur into @DataID,@DataVal
while (@@FETCH_STATUS = 0)
begin
    select @WindowVariance=WindowVariance from [dbo].[CaseData]
    Where CaseID=@CaseID and DataID=@DataID
    select @WindowVariance1=WindowVariance from [dbo].[CaseData]
    Where CaseID=@CaseID and DataID=@DataID-1
    select @Variance1=Variance from [dbo].[CaseData]
    Where CaseID=@CaseID and DataID=@DataID-1
    select @Return=[Return] from [dbo].[CaseData]
    Where CaseID=@CaseID and DataID=@DataID
    select @Return1=[Return] from [dbo].[CaseData]
    Where CaseID=@CaseID and DataID=@DataID-1
set @i=@i+1
if @i=1
       set @Variance =@WindowVariance1
else
       set @Variance =(@WindowVariance * @gamma)+(@Variance1 * @beta)+(@Return1
*@alpha )

fetch next from cur into @DataID,@DataVal
end
```

Forecast Procedure

```
fetch next from cur into @DataID
while (@@FETCH_STATUS = 0)
begin
       ForecastVariance=round(windowvariance+(@alpha+@beta)*(swgarch-
       windowvariance),4),
       Forecast=round([DataVal] + ([DataVal] *
       round(windowvariance+(@alpha+@beta)*
       (swgarch-windowvariance),4)) ,4)
fetch next from cur into @DataID
end
```

APPENDIX B: Performance for Senara Station

Model Performance for Senara Station

Date	Day $_i$	Water Level	Forecast Value	| Error |
Mon, Mar 19, 2007	78	0.65	0.65	0.0020
Tue, Mar 20, 2007	79	0.83	0.83	0.0018
Wed, Mar 21, 2007	80	0.68	0.68	0.0174
Thu, Mar 22, 2007	81	0.63	0.63	0.0255
Fri, Mar 23, 2007	82	0.97	0.97	0.0258
Sat, Mar 24, 2007	83	0.79	0.79	0.0662
Sun, Mar 25, 2007	84	0.87	0.87	0.0806
Mon, Mar 26, 2007	85	0.76	0.76	0.0353
Tue, Mar 27, 2007	86	0.85	0.85	0.0145
Wed, Mar 28, 2007	87	0.91	0.91	0.0125
Thu, Mar 29, 2007	88	0.85	0.85	0.0080
Fri, Mar 30, 2007	89	0.89	0.89	0.0049
Sat, Mar 31, 2007	90	0.85	0.85	0.0029
Sun, Apr 01, 2007	91	0.89	0.89	0.0021
Mon, Apr 02, 2007	92	0.8	0.8	0.0016
Tue, Apr 03, 2007	93	0.74	0.74	0.0043
Wed, Apr 04, 2007	94	0.61	0.61	0.0045
Thu, Apr 05, 2007	95	0.71	0.71	0.0137
Fri, Apr 06, 2007	96	0.89	0.89	0.0224
Sat, Apr 07, 2007	97	0.91	0.91	0.0321
Sun, Apr 08, 2007	98	0.75	0.75	0.0179
Mon, Apr 09, 2007	99	0.83	0.83	0.0187
Tue, Apr 10, 2007	100	0.65	0.65	0.0125
Wed, Apr 11, 2007	101	0.8	0.8	0.0269
Thu, Apr 12, 2007	102	1.2	1.2	0.0516
Fri, Apr 13, 2007	103	1.52	1.52	0.1408

Date	Day $_i$	Water Level	Forecast Value	| Error |
Sat, Apr 14, 2007	104	1.15	1.15	0.1089
Sun, Apr 15, 2007	105	0.95	0.95	0.0731
Mon, Apr 16, 2007	106	1.13	1.13	0.0609
Tue, Apr 17, 2007	107	1.35	1.35	0.0509
Wed, Apr 18, 2007	108	1.15	1.15	0.0346
Thu, Apr 19, 2007	109	0.98	0.98	0.0268
Fri, Apr 20, 2007	110	0.86	0.86	0.0216
Sat, Apr 21, 2007	111	0.83	0.83	0.0173
Sun, Apr 22, 2007	112	1	1	0.0109
Mon, Apr 23, 2007	113	1.58	1.58	0.0269
Tue, Apr 24, 2007	114	1.71	1.71	0.1701
Wed, Apr 25, 2007	115	2.49	2.49	0.2270
Thu, Apr 26, 2007	116	3.28	3.28	0.2924
Fri, Apr 27, 2007	117	3.46	3.46	0.3062
Sat, Apr 28, 2007	118	2.75	2.75	0.1407
Sun, Apr 29, 2007	119	1.94	1.94	0.0645
Mon, Apr 30, 2007	120	2.15	2.15	0.1531
Tue, May 01, 2007	121	1.75	1.75	0.1057
Wed, May 02, 2007	122	1.68	1.68	0.0645
Thu, May 03, 2007	123	2.23	2.23	0.0427
Fri, May 04, 2007	124	1.66	1.66	0.0652
Sat, May 05, 2007	125	1.4	1.4	0.0951
Sun, May 06, 2007	126	1.25	1.25	0.0723
Mon, May 07, 2007	127	0.96	0.96	0.0278
Tue, May 08, 2007	128	1.01	1.01	0.0380
Wed, May 09, 2007	129	0.9	0.9	0.0277
Thu, May 10, 2007	130	1.8	1.8	0.0290
Fri, May 11, 2007	131	2.41	2.41	0.4862
Sat, May 12, 2007	132	1.72	1.72	0.3940
Sun, May 13, 2007	133	1.22	1.22	0.1795
Mon, May 14, 2007	134	1.21	1.21	0.1279

Date	Day_i	Water Level	Forecast Value	| Error |
Tue, May 15, 2007	135	1	1	0.0631
Wed, May 16, 2007	136	0.75	0.75	0.0234
Thu, May 17, 2007	137	0.92	0.92	0.0445
Fri, May 18, 2007	138	2.7	2.7	0.1491
Sat, May 19, 2007	139	2.78	2.78	1.3931
Sun, May 20, 2007	140	1.67	1.67	0.7857
Mon, May 21, 2007	141	1.22	1.22	0.3257
Tue, May 22, 2007	142	1.12	1.12	0.1622
Wed, May 23, 2007	143	1.08	1.08	0.0848
Thu, May 24, 2007	144	0.84	0.84	0.0145
Fri, May 25, 2007	145	0.93	0.93	0.0254
Sat, May 26, 2007	146	0.86	0.86	0.0256
Sun, May 27, 2007	147	0.91	0.91	0.0140
Mon, May 28, 2007	148	0.84	0.84	0.0044
Tue, May 29, 2007	149	0.92	0.92	0.0044
Wed, May 30, 2007	150	0.86	0.86	0.0055
Thu, May 31, 2007	151	0.73	0.73	0.0044
Fri, Jun 01, 2007	152	0.93	0.93	0.0130
Sat, Jun 02, 2007	153	0.81	0.81	0.0286
Sun, Jun 03, 2007	154	0.83	0.83	0.0290
Mon, Jun 04, 2007	155	2.17	2.17	0.0348
Tue, Jun 05, 2007	156	3.07	3.07	1.1658
Wed, Jun 06, 2007	157	3.17	3.17	1.3288
Thu, Jun 07, 2007	158	2.17	2.17	0.3833
Fri, Jun 08, 2007	159	1.42	1.42	0.1080
Sat, Jun 09, 2007	160	0.95	0.95	0.1246
Sun, Jun 10, 2007	161	1.04	1.04	0.1642
Mon, Jun 11, 2007	162	0.98	0.98	0.0912
Tue, Jun 12, 2007	163	0.79	0.79	0.0215
Wed, Jun 13, 2007	164	0.79	0.79	0.0170
Thu, Jun 14, 2007	165	1	1	0.0191

Date	Day $_i$	Water Level	Forecast Value	| Error |
Fri, Jun 15, 2007	166	1.32	1.32	0.0384
Sat, Jun 16, 2007	167	2.31	2.31	0.1241
Sun, Jun 17, 2007	168	2.32	2.32	0.3859
Mon, Jun 18, 2007	169	2.1	2.1	0.2859
Tue, Jun 19, 2007	170	2.04	2.04	0.0969
Wed, Jun 20, 2007	171	2.87	2.87	0.0128
Thu, Jun 21, 2007	172	2.79	2.79	0.1374
Fri, Jun 22, 2007	173	2.08	2.08	0.0980
Sat, Jun 23, 2007	174	1.5	1.5	0.0775
Sun, Jun 24, 2007	175	1.13	1.13	0.0885
Mon, Jun 25, 2007	176	1.11	1.11	0.0971
Tue, Jun 26, 2007	177	1.13	1.13	0.0532
Wed, Jun 27, 2007	178	1.26	1.26	0.0144
Thu, Jun 28, 2007	179	1.04	1.04	0.0052
Fri, Jun 29, 2007	180	0.85	0.85	0.0168
Sat, Jun 30, 2007	181	0.87	0.87	0.0287
Sun, Jul 01, 2007	182	0.84	0.84	0.0182
Mon, Jul 02, 2007	183	0.7	0.7	0.0045
Tue, Jul 03, 2007	184	0.7	0.7	0.0099
Wed, Jul 04, 2007	185	0.84	0.84	0.0113
Thu, Jul 05, 2007	186	0.83	0.83	0.0151
Fri, Jul 06, 2007	187	0.87	0.87	0.0116
Sat, Jul 07, 2007	188	0.89	0.89	0.0050
Sun, Jul 08, 2007	189	0.9	0.9	0.0010
Mon, Jul 09, 2007	190	1.07	1.07	0.0006
Tue, Jul 10, 2007	191	1.09	1.09	0.0134
Wed, Jul 11, 2007	192	1.39	1.39	0.0169
Thu, Jul 12, 2007	193	1.06	1.06	0.0301
Fri, Jul 13, 2007	194	0.94	0.94	0.0505
Sat, Jul 14, 2007	195	0.95	0.95	0.0413
Sun, Jul 15, 2007	196	0.89	0.89	0.0143

Date	Day_i	Water Level	Forecast Value	| Error |
Mon, Jul 16, 2007	197	1.99	1.99	0.0076
Tue, Jul 17, 2007	198	1.73	1.73	0.4600
Wed, Jul 18, 2007	199	1.21	1.21	0.3242
Thu, Jul 19, 2007	200	0.94	0.94	0.1408
Fri, Jul 20, 2007	201	0.75	0.75	0.0601
Sat, Jul 21, 2007	202	1.63	1.63	0.1044
Sun, Jul 22, 2007	203	1.99	1.99	0.5476
Mon, Jul 23, 2007	204	2.22	2.22	0.5877
Tue, Jul 24, 2007	205	2.23	2.23	0.2327
Wed, Jul 25, 2007	206	1.66	1.66	0.0175
Thu, Jul 26, 2007	207	1.35	1.35	0.0503
Fri, Jul 27, 2007	208	1.89	1.89	0.0989
Sat, Jul 28, 2007	209	1.78	1.78	0.1342
Sun, Jul 29, 2007	210	2.41	2.41	0.1272
Mon, Jul 30, 2007	211	2.15	2.15	0.1174
Tue, Jul 31, 2007	212	1.89	1.89	0.0806
Wed, Aug 01, 2007	213	1.62	1.62	0.0401
Thu, Aug 02, 2007	214	1.3	1.3	0.0237
Fri, Aug 03, 2007	215	1.17	1.17	0.0370
Sat, Aug 04, 2007	216	1.13	1.13	0.0308
Sun, Aug 05, 2007	217	1.05	1.05	0.0122
Mon, Aug 06, 2007	218	0.89	0.89	0.0038
Tue, Aug 07, 2007	219	0.84	0.84	0.0113
Wed, Aug 08, 2007	220	0.82	0.82	0.0107
Thu, Aug 09, 2007	221	0.78	0.78	0.0042
Fri, Aug 10, 2007	222	0.71	0.71	0.0012
Sat, Aug 11, 2007	223	0.73	0.73	0.0034
Sun, Aug 12, 2007	224	0.7	0.7	0.0029
Mon, Aug 13, 2007	225	0.66	0.66	0.0015
Tue, Aug 14, 2007	226	0.72	0.72	0.0016
Wed, Aug 15, 2007	227	0.76	0.76	0.0036

Date	Day $_i$	Water Level	Forecast Value	| Error |
Thu, Aug 16, 2007	228	0.91	0.91	0.0043
Fri, Aug 17, 2007	229	1.26	1.26	0.0195
Sat, Aug 18, 2007	230	1.11	1.11	0.0628
Sun, Aug 19, 2007	231	1.17	1.17	0.0626
Mon, Aug 20, 2007	232	1.6	1.6	0.0356
Tue, Aug 21, 2007	233	1.33	1.33	0.0577
Wed, Aug 22, 2007	234	1.39	1.39	0.0745
Thu, Aug 23, 2007	235	1.19	1.19	0.0334
Fri, Aug 24, 2007	236	0.99	0.99	0.0152
Sat, Aug 25, 2007	237	0.89	0.89	0.0212
Sun, Aug 26, 2007	238	0.89	0.89	0.0192
Mon, Aug 27, 2007	239	0.97	0.97	0.0090
Tue, Aug 28, 2007	240	0.78	0.78	0.0036
Wed, Aug 29, 2007	241	1.15	1.15	0.0257
Thu, Aug 30, 2007	242	1.43	1.43	0.1166
Fri, Aug 31, 2007	243	1.31	1.31	0.1131
Sat, Sep 01, 2007	244	1.07	1.07	0.0461
Sun, Sep 02, 2007	245	1.71	1.71	0.0452
Mon, Sep 03, 2007	246	2.21	2.21	0.2368
Tue, Sep 04, 2007	247	1.6	1.6	0.1929
Wed, Sep 05, 2007	248	1.2	1.2	0.1193
Thu, Sep 06, 2007	249	1.04	1.04	0.0882
Fri, Sep 07, 2007	250	0.91	0.91	0.0510
Sat, Sep 08, 2007	251	1.31	1.31	0.0354
Sun, Sep 09, 2007	252	1.22	1.22	0.0783
Mon, Sep 10, 2007	253	0.98	0.98	0.0566
Tue, Sep 11, 2007	254	0.81	0.81	0.0324
Wed, Sep 12, 2007	255	0.7	0.7	0.0243
Thu, Sep 13, 2007	256	0.82	0.82	0.0245
Fri, Sep 14, 2007	257	1.62	1.62	0.0386
Sat, Sep 15, 2007	258	2.71	2.71	0.5478

Date	Day $_i$	Water Level	Forecast Value	| Error |
Sun, Sep 16, 2007	259	3.03	3.03	0.9005
Mon, Sep 17, 2007	260	4.19	4.19	0.7349
Tue, Sep 18, 2007	261	4.16	4.16	0.3525
Wed, Sep 19, 2007	262	3.9	3.9	0.1714
Thu, Sep 20, 2007	263	3.37	3.37	0.0549
Fri, Sep 21, 2007	264	2.51	2.51	0.0262
Sat, Sep 22, 2007	265	1.92	1.92	0.0855
Sun, Sep 23, 2007	266	1.68	1.68	0.1126
Mon, Sep 24, 2007	267	2.02	2.02	0.0971
Tue, Sep 25, 2007	268	1.48	1.48	0.0459
Wed, Sep 26, 2007	269	1.49	1.49	0.0828
Thu, Sep 27, 2007	270	1.2	1.2	0.0522
Fri, Sep 28, 2007	271	1.2	1.2	0.0391
Sat, Sep 29, 2007	272	0.92	0.92	0.0173
Sun, Sep 30, 2007	273	1.29	1.29	0.0455
Mon, Oct 01, 2007	274	1.76	1.76	0.1319
Tue, Oct 02, 2007	275	3.86	3.86	0.3664
		Water Level	Average Error	0.1088

APPENDIX C: Performance for Kuala Nerang

Model Performance for Kuala Nerang

Date	Day *i*	Water Level	Forecast Value	| Error |
Tue, Mar 20, 2007	79	14.3	14.3	0.1573
Wed, Mar 21, 2007	80	14.3	14.31	0.1147
Thu, Mar 22, 2007	81	14.3	14.31	0.0566
Fri, Mar 23, 2007	82	14.3	14.31	0.0266
Sat, Mar 24, 2007	83	14.3	14.33	0.0063
Sun, Mar 25, 2007	84	15	14.98	0.0022
Mon, Mar 26, 2007	85	13.8	13.78	0.0075
Tue, Mar 27, 2007	86	14.2	14.22	0.0359
Wed, Mar 28, 2007	87	14.3	14.34	0.0481
Thu, Mar 29, 2007	88	14.7	14.66	0.0310
Fri, Mar 30, 2007	89	14.6	14.59	0.0170
Sat, Mar 31, 2007	90	14	14.04	0.0069
Sun, Apr 01, 2007	91	14.2	14.19	0.0082
Mon, Apr 02, 2007	92	14.1	14.06	0.0096
Tue, Apr 03, 2007	93	14.8	14.77	0.0060
Wed, Apr 04, 2007	94	15.2	15.19	0.0127
Thu, Apr 05, 2007	95	14.9	14.94	0.0179
Fri, Apr 06, 2007	96	14.8	14.81	0.0136
Sat, Apr 07, 2007	97	14.8	14.76	0.0083
Sun, Apr 08, 2007	98	14.7	14.73	0.0035
Mon, Apr 09, 2007	99	14.7	14.73	0.0013
Tue, Apr 10, 2007	100	14.9	14.9	0.0005
Wed, Apr 11, 2007	101	14.9	14.91	0.0006
Thu, Apr 12, 2007	102	14.6	14.62	0.0008
Fri, Apr 13, 2007	103	13.9	13.88	0.0018
Sat, Apr 14, 2007	104	13.4	13.36	0.0112
Sun, Apr 15, 2007	105	13.3	13.3	0.0198
Mon, Apr 16, 2007	106	13.7	13.72	0.0163

Date	Day *i*	Water Level	Forecast Value	| Error |
Tue, Apr 17, 2007	107	13	13.04	0.0113
Wed, Apr 18, 2007	108	13.4	13.43	0.0168
Thu, Apr 19, 2007	109	13.6	13.57	0.0200
Fri, Apr 20, 2007	110	13.3	13.32	0.0139
Sat, Apr 21, 2007	111	13.3	13.28	0.0083
Sun, Apr 22, 2007	112	13.6	13.62	0.0044
Mon, Apr 23, 2007	113	14.4	14.38	0.0042
Tue, Apr 24, 2007	114	13.5	13.45	0.0140
Wed, Apr 25, 2007	115	13.2	13.19	0.0318
Thu, Apr 26, 2007	116	13.1	13.14	0.0331
Fri, Apr 27, 2007	117	13.4	13.36	0.0194
Sat, Apr 28, 2007	118	13.2	13.16	0.0094
Sun, Apr 29, 2007	119	13.8	13.76	0.0046
Mon, Apr 30, 2007	120	12.9	12.94	0.0091
Tue, May 01, 2007	121	13.8	13.84	0.0250
Wed, May 02, 2007	122	13.3	13.33	0.0407
Thu, May 03, 2007	123	13.6	13.57	0.0424
Fri, May 04, 2007	124	12.8	12.84	0.0264
Sat, May 05, 2007	125	12.6	12.61	0.0226
Sun, May 06, 2007	126	12.5	12.48	0.0202
Mon, May 07, 2007	127	13.2	13.2	0.0122
Tue, May 08, 2007	128	13.9	13.94	0.0177
Wed, May 09, 2007	129	14.4	14.35	0.0301
Thu, May 10, 2007	130	15	15	0.0312
Fri, May 11, 2007	131	15.1	15.12	0.0274
Sat, May 12, 2007	132	14.5	14.47	0.0196
Sun, May 13, 2007	133	14.3	14.34	0.0161
Mon, May 14, 2007	134	14.1	14.09	0.0147
Tue, May 15, 2007	135	14	14.03	0.0083
Wed, May 16, 2007	136	14.5	14.48	0.0054
Thu, May 17, 2007	137	14.4	14.44	0.0056

Date	Day i	Water Level	Forecast Value	| Error |
Fri, May 18, 2007	138	14.7	14.65	0.0065
Sat, May 19, 2007	139	14.5	14.49	0.0041
Sun, May 20, 2007	140	14.8	14.78	0.0033
Mon, May 21, 2007	141	13.8	13.8	0.0030
Tue, May 22, 2007	142	13.7	13.73	0.0191
Wed, May 23, 2007	143	14.6	14.6	0.0280
Thu, May 24, 2007	144	14.6	14.61	0.0295
Fri, May 25, 2007	145	14.7	14.7	0.0291
Sat, May 26, 2007	146	14.8	14.77	0.0139
Sun, May 27, 2007	147	14.8	14.75	0.0069
Mon, May 28, 2007	148	14.7	14.71	0.0017
Tue, May 29, 2007	149	15	14.95	0.0007
Wed, May 30, 2007	150	12.6	12.58	0.0011
Thu, May 31, 2007	151	13.6	13.61	0.1034
Fri, Jun 01, 2007	152	13.7	13.69	0.1787
Sat, Jun 02, 2007	153	13.6	13.64	0.1204
Sun, Jun 03, 2007	154	13.6	13.6	0.0625
Mon, Jun 04, 2007	155	13.9	13.94	0.0193
Tue, Jun 05, 2007	156	14.1	14.05	0.0076
Wed, Jun 06, 2007	157	13.4	13.38	0.0051
Thu, Jun 07, 2007	158	13.2	13.21	0.0106
Fri, Jun 08, 2007	159	14	14	0.0147
Sat, Jun 09, 2007	160	13.5	13.52	0.0196
Sun, Jun 10, 2007	161	13.7	13.65	0.0260
Mon, Jun 11, 2007	162	15.8	15.76	0.0204
Tue, Jun 12, 2007	163	13.7	13.7	0.0806
Wed, Jun 13, 2007	164	13.3	13.33	0.1747
Thu, Jun 14, 2007	165	15.2	15.15	0.1853
Fri, Jun 15, 2007	166	15.4	15.42	0.1680
Sat, Jun 16, 2007	167	16.4	16.4	0.1501
Sun, Jun 17, 2007	168	16.4	16.36	0.0868

Date	Day *i*	Water Level	Forecast Value	| Error |
Mon, Jun 18, 2007	169	14	14.02	0.0496
Tue, Jun 19, 2007	170	13.6	13.59	0.0992
Wed, Jun 20, 2007	171	13.7	13.71	0.1366
Thu, Jun 21, 2007	172	13.5	13.53	0.0766
Fri, Jun 22, 2007	173	13.5	13.45	0.0393
Sat, Jun 23, 2007	174	13	13.01	0.0100
Sun, Jun 24, 2007	175	12.9	12.87	0.0071
Mon, Jun 25, 2007	176	12.8	12.78	0.0071
Tue, Jun 26, 2007	177	12.9	12.85	0.0042
Wed, Jun 27, 2007	178	12.8	12.76	0.0023
Thu, Jun 28, 2007	179	12.7	12.66	0.0010
Fri, Jun 29, 2007	180	12.6	12.58	0.0007
Sat, Jun 30, 2007	181	12.5	12.53	0.0007
Sun, Jul 01, 2007	182	12.5	12.49	0.0005
Mon, Jul 02, 2007	183	12.9	12.85	0.0003
Tue, Jul 03, 2007	184	13.5	13.45	0.0029
Wed, Jul 04, 2007	185	14.1	14.11	0.0118
Thu, Jul 05, 2007	186	14.4	14.4	0.0224
Fri, Jul 06, 2007	187	14.4	14.42	0.0219
Sat, Jul 07, 2007	188	14.4	14.42	0.0130
Sun, Jul 08, 2007	189	14.4	14.42	0.0057
Mon, Jul 09, 2007	190	14.6	14.57	0.0017
Tue, Jul 10, 2007	191	13.2	13.18	0.0008
Wed, Jul 11, 2007	192	13.1	13.09	0.0338
Thu, Jul 12, 2007	193	12.7	12.73	0.0496
Fri, Jul 13, 2007	194	12.6	12.56	0.0300
Sat, Jul 14, 2007	195	13	12.95	0.0188
Sun, Jul 15, 2007	196	12.8	12.76	0.0091
Mon, Jul 16, 2007	197	12.7	12.71	0.0079
Tue, Jul 17, 2007	198	12.7	12.65	0.0045
Wed, Jul 18, 2007	199	13.1	13.07	0.0023

Date	Day i	Water Level	Forecast Value	| Error |
Thu, Jul 19, 2007	200	13.8	13.76	0.0046
Fri, Jul 20, 2007	201	14.6	14.62	0.0160
Sat, Jul 21, 2007	202	13.9	13.94	0.0304
Sun, Jul 22, 2007	203	13.2	13.2	0.0352
Mon, Jul 23, 2007	204	13.6	13.61	0.0371
Tue, Jul 24, 2007	205	13.6	13.6	0.0318
Wed, Jul 25, 2007	206	13.3	13.26	0.0180
Thu, Jul 26, 2007	207	12.9	12.92	0.0100
Fri, Jul 27, 2007	208	13.9	13.91	0.0086
Sat, Jul 28, 2007	209	13.5	13.51	0.0246
Sun, Jul 29, 2007	210	13.8	13.82	0.0351
Mon, Jul 30, 2007	211	13.5	13.5	0.0233
Tue, Jul 31, 2007	212	13.8	13.75	0.0156
Wed, Aug 01, 2007	213	13.3	13.31	0.0084
Thu, Aug 02, 2007	214	13	12.99	0.0082
Fri, Aug 03, 2007	215	12.8	12.84	0.0093
Sat, Aug 04, 2007	216	12.7	12.72	0.0070
Sun, Aug 05, 2007	217	12.6	12.63	0.0042
Mon, Aug 06, 2007	218	12.6	12.56	0.0021
Tue, Aug 07, 2007	219	12.5	12.52	0.0011
Wed, Aug 08, 2007	220	12.9	12.86	0.0006
Thu, Aug 09, 2007	221	12.9	12.86	0.0026
Fri, Aug 10, 2007	222	12.8	12.82	0.0036
Sat, Aug 11, 2007	223	12.8	12.81	0.0020
Sun, Aug 12, 2007	224	12.8	12.79	0.0011
Mon, Aug 13, 2007	225	12.8	12.79	0.0003
Tue, Aug 14, 2007	226	12.8	12.79	0.0001
Wed, Aug 15, 2007	227	12.8	12.8	0.0000
Thu, Aug 16, 2007	228	12.8	12.78	0.0000
Fri, Aug 17, 2007	229	12.8	12.8	0.0000
Sat, Aug 18, 2007	230	12.6	12.56	0.0000

Date	Day *i*	Water Level	Forecast Value	| Error |
Sun, Aug 19, 2007	231	12.5	12.48	0.0011
Mon, Aug 20, 2007	232	12.9	12.9	0.0019
Tue, Aug 21, 2007	233	13.2	13.24	0.0049
Wed, Aug 22, 2007	234	12.7	12.66	0.0081
Thu, Aug 23, 2007	235	12.6	12.59	0.0128
Fri, Aug 24, 2007	236	12.5	12.46	0.0130
Sat, Aug 25, 2007	237	12.5	12.45	0.0071
Sun, Aug 26, 2007	238	12.7	12.73	0.0037
Mon, Aug 27, 2007	239	12.7	12.73	0.0026
Tue, Aug 28, 2007	240	12.8	12.8	0.0028
Wed, Aug 29, 2007	241	12.7	12.74	0.0016
Thu, Aug 30, 2007	242	13	12.97	0.0010
Fri, Aug 31, 2007	243	12.8	12.83	0.0014
Sat, Sep 01, 2007	244	12.8	12.81	0.0021
Sun, Sep 02, 2007	245	14.1	14.08	0.0017
Mon, Sep 03, 2007	246	13.8	13.82	0.0319
Tue, Sep 04, 2007	247	13.2	13.17	0.0466
Wed, Sep 05, 2007	248	12.8	12.78	0.0338
Thu, Sep 06, 2007	249	12.6	12.59	0.0273
Fri, Sep 07, 2007	250	13.6	13.59	0.0156
Sat, Sep 08, 2007	251	13.6	13.62	0.0283
Sun, Sep 09, 2007	252	12.7	12.7	0.0314
Mon, Sep 10, 2007	253	12.6	12.58	0.0322
Tue, Sep 11, 2007	254	12.5	12.5	0.0320
Wed, Sep 12, 2007	255	13.2	13.22	0.0164
Thu, Sep 13, 2007	256	13.3	13.33	0.0187
Fri, Sep 14, 2007	257	13.3	13.29	0.0182
Sat, Sep 15, 2007	258	13.8	13.76	0.0104
Sun, Sep 16, 2007	259	13.7	13.66	0.0093
Mon, Sep 17, 2007	260	14.9	14.92	0.0084
Tue, Sep 18, 2007	261	15	15.01	0.0340

Date	Day *i*	Water Level	Forecast Value	| Error |
Wed, Sep 19, 2007	262	14.8	14.75	0.0465
Thu, Sep 20, 2007	263	13.6	13.6	0.0245
Fri, Sep 21, 2007	264	12.8	12.84	0.0339
Sat, Sep 22, 2007	265	12.7	12.67	0.0458
Sun, Sep 23, 2007	266	12.7	12.7	0.0359
Mon, Sep 24, 2007	267	13	12.95	0.0198
Tue, Sep 25, 2007	268	13.5	13.52	0.0089
Wed, Sep 26, 2007	269	13.5	13.45	0.0103
Thu, Sep 27, 2007	270	13.4	13.42	0.0114
Fri, Sep 28, 2007	271	13.3	13.33	0.0062
Sat, Sep 29, 2007	272	13.3	13.28	0.0031
Sun, Sep 30, 2007	273	13.7	13.7	0.0010
Mon, Oct 01, 2007	274	14.1	14.07	0.0039
Tue, Oct 02, 2007	275	14.4	14.39	0.0081
		Water Level	Average Error	0.0229

APPENDIX D: Performance for KL House Price Index

Model Performance for KL House Price Index

Date	Week *i*	Forecast Value	Forecast Value	| Error |
Sun, Feb 17, 2013	58	57.5327	54	3.5327
Sun, Feb 24, 2013	59	45.5500	43	2.5500
Sun, Mar 03, 2013	60	36.9469	35	1.9469
Sun, Mar 10, 2013	61	45.7006	43	2.7006
Sun, Mar 17, 2013	62	68.8322	65	3.8322
Sun, Mar 24, 2013	63	76.0610	72	4.0610
Sun, Mar 31, 2013	64	85.3449	80	5.3449
Sun, Apr 07, 2013	65	67.8145	65	2.8145
Sun, Apr 14, 2013	66	53.2766	50	3.2766
Sun, Apr 21, 2013	67	70.9140	69	1.9140
Sun, Apr 28, 2013	68	41.6317	40	1.6317
Sun, May 05, 2013	69	57.4563	53	4.4563
Sun, May 12, 2013	70	65.0703	58	7.0703
Sun, May 19, 2013	71	66.9294	61	5.9294
Sun, May 26, 2013	72	55.5613	50	5.5613
Sun, Jun 02, 2013	73	72.7097	70	2.7097
Sun, Jun 09, 2013	74	61.8397	60	1.8397
Sun, Jun 16, 2013	75	75.9349	73	2.9349
Sun, Jun 23, 2013	76	66.5534	64	2.5534
Sun, Jun 30, 2013	77	45.2582	43	2.2582
Sun, Jul 07, 2013	78	50.8542	49	1.8542
Sun, Jul 14, 2013	79	83.6201	79	4.6201
Sun, Jul 21, 2013	80	47.6478	45	2.6478
Sun, Jul 28, 2013	81	82.5951	72	10.5951
Sun, Aug 04, 2013	82	76.7458	67	9.7458
Sun, Aug 11, 2013	83	65.8111	56	9.8111
Sun, Aug 18, 2013	84	67.3786	59	8.3786
Sun, Aug 25, 2013	85	57.6906	53	4.6906

Sun, Sep 01, 2013	86	58.9646	58	0.9646
Sun, Sep 08, 2013	87	75.3857	74	1.3857
Sun, Sep 15, 2013	88	61.8443	61	0.8443
Sun, Sep 22, 2013	89	41.0141	40	1.0141
Sun, Sep 29, 2013	90	68.7079	66	2.7079
Sun, Oct 06, 2013	91	61.4773	56	5.4773
Sun, Oct 13, 2013	92	71.7900	65	6.7900
Sun, Oct 20, 2013	93	48.4989	44	4.4989
Sun, Oct 27, 2013	94	58.8619	53	5.8619
Sun, Nov 03, 2013	95	55.1715	52	3.1715
Sun, Nov 10, 2013	96	104.0800	100	4.0800
Sun, Nov 17, 2013	97	72.8375	66	6.8375
Sun, Nov 24, 2013	98	49.1381	43	6.1381
Sun, Dec 01, 2013	99	99.0936	88	11.0936
Sun, Dec 08, 2013	100	60.7438	48	12.7438
Sun, Dec 15, 2013	101	68.0577	54	14.0577
Sun, Dec 22, 2013	102	50.5006	41	9.5006
Sun, Dec 29, 2013	103	53.0540	43	10.0540
Sun, Jan 05, 2014	104	53.0014	46	7.0014
Sun, Jan 12, 2014	105	67.9590	66	1.9590
Sun, Jan 19, 2014	106	30.3680	29	1.3680
Sun, Jan 26, 2014	107	50.1643	45	5.1643
Sun, Feb 02, 2014	108	59.5172	49	10.5172
Sun, Feb 09, 2014	109	68.7862	58	10.7862
Sun, Feb 16, 2014	110	52.5328	42	10.5328
Sun, Feb 23, 2014	111	31.7882	29	2.7882
Sun, Mar 02, 2014	112	32.8233	31	1.8233
Sun, Mar 09, 2014	113	58.5263	55	3.5263
Sun, Mar 16, 2014	114	42.6374	39	3.6374
Sun, Mar 23, 2014	115	49.2102	43	6.2102
Sun, Mar 30, 2014	116	54.1105	50	4.1105
Sun, Apr 06, 2014	117	69.9657	62	7.9657

Sun, Apr 13, 2014	118	50.7576	48	2.7576
Sun, Apr 20, 2014	119	59.8061	58	1.8061
Sun, Apr 27, 2014	120	31.1190	30	1.1190
Sun, May 04, 2014	121	74.7539	69	5.7539
Sun, May 11, 2014	122	58.3326	48	10.3326
Sun, May 18, 2014	123	78.3673	62	16.3673
Sun, May 25, 2014	124	79.1756	62	17.1756
Sun, Jun 01, 2014	125	70.8739	56	14.8739
Sun, Jun 08, 2014	126	33.1776	31	2.1776
Sun, Jun 15, 2014	127	85.9131	80	5.9131
Sun, Jun 22, 2014	128	44.2953	37	7.2953
Sun, Jun 29, 2014	129	90.3761	67	23.3761
Sun, Jul 06, 2014	130	75.6610	53	22.6610
Sun, Jul 13, 2014	131	83.9165	57	26.9165
Sun, Jul 20, 2014	132	91.7468	72	19.7468
Sun, Jul 27, 2014	133	51.7252	45	6.7252
Sun, Aug 03, 2014	134	51.5630	48	3.5630
Sun, Aug 10, 2014	135	65.5848	61	4.5848
Sun, Aug 17, 2014	136	77.3911	73	4.3911
Sun, Aug 24, 2014	137	64.4650	59	5.4650
Sun, Aug 31, 2014	138	49.3640	48	1.3640
Sun, Sep 07, 2014	139	53.1416	51	2.1416
Sun, Sep 14, 2014	140	66.9696	65	1.9696
Sun, Sep 21, 2014	141	56.6351	55	1.6351
Sun, Sep 28, 2014	142	71.3394	69	2.3394
Sun, Oct 05, 2014	143	58.3570	57	1.3570
Sun, Oct 12, 2014	144	59.3163	57	2.3163
Sun, Oct 19, 2014	145	37.9928	37	0.9928
Sun, Oct 26, 2014	146	58.4440	56	2.4440
Sun, Nov 02, 2014	147	68.0251	63	5.0251
Sun, Nov 09, 2014	148	75.0632	70	5.0632
Sun, Nov 16, 2014	149	59.8863	55	4.8863

Sun, Nov 23, 2014	150	68.4836	64	4.4836
Sun, Nov 30, 2014	151	67.8825	66	1.8825
Sun, Dec 07, 2014	152	51.0163	50	1.0163
Sun, Dec 14, 2014	153	40.2553	39	1.2553
Sun, Dec 21, 2014	154	63.1826	61	2.1826
Sun, Dec 28, 2014	155	45.1633	43	2.1633
Sun, Jan 04, 2015	156	59.3000	54	5.3000
Sun, Jan 11, 2015	157	57.5546	53	4.5546
Sun, Jan 18, 2015	158	65.7629	60	5.7629
Sun, Jan 25, 2015	159	62.1223	59	3.1223
Sun, Feb 01, 2015	160	50.2396	49	1.2396
Sun, Feb 08, 2015	161	54.5119	54	0.5119
Sun, Feb 15, 2015	162	43.7034	43	0.7034
Sun, Feb 22, 2015	163	58.8194	58	0.8194
Sun, Mar 01, 2015	164	48.6917	47	1.6917
Sun, Mar 08, 2015	165	61.2847	59	2.2847
Sun, Mar 15, 2015	166	60.6998	58	2.6998
Sun, Mar 22, 2015	167	53.5063	51	2.5063
Sun, Mar 29, 2015	168	72.8151	71	1.8151
Sun, Apr 05, 2015	169	53.8194	52	1.8194
Sun, Apr 12, 2015	170	75.0898	72	3.0898
Sun, Apr 19, 2015	171	52.8881	50	2.8881
Sun, Apr 26, 2015	172	42.5419	39	3.5419
Sun, May 03, 2015	173	43.5470	40	3.5470
Sun, May 10, 2015	174	46.0525	43	3.0525
Sun, May 17, 2015	175	54.8755	52	2.8755
Sun, May 24, 2015	176	52.5032	51	1.5032
Sun, May 31, 2015	177	68.9116	68	0.9116
Sun, Jun 07, 2015	178	55.9688	55	0.9688
Sun, Jun 14, 2015	179	57.1150	55	2.1150
Sun, Jun 21, 2015	180	67.5893	66	1.5893
Sun, Jun 28, 2015	181	58.0675	56	2.0675

Sun, Jul 05, 2015	182	56.5228	55	1.5228
Sun, Jul 12, 2015	183	65.8611	65	0.8611
Sun, Jul 19, 2015	184	61.1048	60	1.1048
Sun, Jul 26, 2015	185	42.7356	42	0.7356
Sun, Aug 02, 2015	186	68.3963	67	1.3963
Sun, Aug 09, 2015	187	64.0136	60	4.0136
Sun, Aug 16, 2015	188	45.2336	42	3.2336
Sun, Aug 23, 2015	189	59.8710	55	4.8710
Sun, Aug 30, 2015	190	64.6345	58	6.6345
Sun, Sep 06, 2015	191	52.2871	50	2.2871
Sun, Sep 13, 2015	192	49.6687	47	2.6687
Sun, Sep 20, 2015	193	54.7761	53	1.7761
Sun, Sep 27, 2015	194	47.4620	47	0.4620
Sun, Oct 04, 2015	195	58.8404	58	0.8404
Sun, Oct 11, 2015	196	57.7437	57	0.7437
Sun, Oct 18, 2015	197	58.0549	57	1.0549
Sun, Oct 25, 2015	198	59.6727	59	0.6727
Sun, Nov 01, 2015	199	52.7734	52	0.7734
Sun, Nov 08, 2015	200	59.1958	59	0.1958
Sun, Nov 15, 2015	201	49.3206	49	0.3206
Sun, Nov 22, 2015	202	65.6986	65	0.6986
Sun, Nov 29, 2015	203	61.5064	60	1.5064
Sun, Dec 06, 2015	204	76.3068	74	2.3068
Sun, Dec 13, 2015	205	63.8028	62	1.8028
Sun, Dec 20, 2015	206	69.8092	67	2.8092
Sun, Dec 27, 2015	207	58.0666	57	1.0666
Sun, Jan 03, 2016	208	69.6255	68	1.6255
Sun, Jan 10, 2016	209	73.6061	72	1.6061
Sun, Jan 17, 2016	210	58.8734	58	0.8734
Sun, Jan 24, 2016	211	82.5743	81	1.5743
Sun, Jan 31, 2016	212	68.3473	66	2.3473
Sun, Feb 07, 2016	213	62.4875	60	2.4875

Sun, Feb 14, 2016	214	61.5096	59	2.5096
Sun, Feb 21, 2016	215	60.6060	58	2.6060
Sun, Feb 28, 2016	216	54.9446	54	0.9446
Sun, Mar 06, 2016	217	49.2807	49	0.2807
Sun, Mar 13, 2016	218	33.1220	33	0.1220
Sun, Mar 20, 2016	219	58.2113	57	1.2113
Sun, Mar 27, 2016	220	51.6566	48	3.6566
Sun, Apr 03, 2016	221	69.5375	63	6.5375
Sun, Apr 10, 2016	222	65.3253	59	6.3253
Sun, Apr 17, 2016	223	59.4583	53	6.4583
Sun, Apr 24, 2016	224	56.6151	55	1.6151
Sun, May 01, 2016	225	56.7278	55	1.7278
Sun, May 08, 2016	226	64.4324	64	0.4324
Sun, May 15, 2016	227	63.5158	63	0.5158
Sun, May 22, 2016	228	64.4709	64	0.4709
Sun, May 29, 2016	229	62.2364	62	0.2364
Sun, Jun 05, 2016	230	49.3827	49	0.3827
Sun, Jun 12, 2016	231	54.3852	54	0.3852
Sun, Jun 19, 2016	232	74.1371	73	1.1371
Sun, Jun 26, 2016	233	68.3843	67	1.3843
Sun, Jul 03, 2016	234	61.5316	59	2.5316
Sun, Jul 10, 2016	235	72.4453	71	1.4453
Sun, Jul 17, 2016	236	61.2828	59	2.2828
Sun, Jul 24, 2016	237	76.4407	75	1.4407
Sun, Jul 31, 2016	238	51.3091	50	1.3091
Sun, Aug 07, 2016	239	44.0796	42	2.0796
Sun, Aug 14, 2016	240	57.5107	54	3.5107
Sun, Aug 21, 2016	241	57.0806	54	3.0806
Sun, Aug 28, 2016	242	68.7609	64	4.7609
Sun, Sep 04, 2016	243	49.2324	48	1.2324
Sun, Sep 11, 2016	244	69.6310	67	2.6310
Sun, Sep 18, 2016	245	49.9119	48	1.9119

Sun, Sep 25, 2016	246	91.3691	86	5.3691
Sun, Oct 02, 2016	247	65.4366	59	6.4366
Sun, Oct 09, 2016	248	75.0752	65	10.0752
Sun, Oct 16, 2016	249	56.1347	50	6.1347
Sun, Oct 23, 2016	250	54.8158	48	6.8158
Sun, Oct 30, 2016	251	51.4877	48	3.4877
Sun, Nov 06, 2016	252	60.1662	59	1.1662
Sun, Nov 13, 2016	253	62.8589	61	1.8589
Sun, Nov 20, 2016	254	53.7671	53	0.7671
Sun, Nov 27, 2016	255	63.6224	63	0.6224
Sun, Dec 04, 2016	256	67.5064	66	1.5064
Sun, Dec 11, 2016	257	86.0262	85	1.0262
Sun, Dec 18, 2016	258	57.0492	56	1.0492
Sun, Dec 25, 2016	259	63.8169	61	2.8169
Sun, Jan 01, 2017	260	58.0440	55	3.0440
		Average Error	Average Error	3.9043

APPENDIX E: Performance for Florida House Price Index

Model Performance for Florida House Price Index

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Feb 10, 2013	57	52	54.9720	2.9720
Sun, Feb 17, 2013	58	51	53.4899	2.4899
Sun, Feb 24, 2013	59	52	53.3708	1.3708
Sun, Mar 03, 2013	60	51	51.2809	0.2809
Sun, Mar 10, 2013	61	51	51.0190	0.0190
Sun, Mar 17, 2013	62	55	55.0115	0.0115
Sun, Mar 24, 2013	63	28	28.0699	0.0699
Sun, Mar 31, 2013	64	38	45.5072	7.5072
Sun, Apr 07, 2013	65	46	56.4651	10.4651
Sun, Apr 14, 2013	66	50	55.9634	5.9634
Sun, Apr 21, 2013	67	50	51.5680	1.5680
Sun, Apr 28, 2013	68	43	43.3481	0.3481
Sun, May 05, 2013	69	52	52.5587	0.5587
Sun, May 12, 2013	70	64	65.5865	1.5865
Sun, May 19, 2013	71	42	43.5342	1.5342
Sun, May 26, 2013	72	42	46.1513	4.1513
Sun, Jun 02, 2013	73	51	55.0229	4.0229
Sun, Jun 09, 2013	74	67	69.7883	2.7883
Sun, Jun 16, 2013	75	47	49.2255	2.2255
Sun, Jun 23, 2013	76	58	63.2058	5.2058
Sun, Jun 30, 2013	77	48	51.8944	3.8944
Sun, Jul 07, 2013	78	57	59.9359	2.9359
Sun, Jul 14, 2013	79	55	56.8524	1.8524
Sun, Jul 21, 2013	80	63	64.1208	1.1208
Sun, Jul 28, 2013	81	46	46.5825	0.5825
Sun, Aug 04, 2013	82	58	60.9060	2.9060
Sun, Aug 11, 2013	83	65	69.3019	4.3019
Sun, Aug 18, 2013	84	52	54.1722	2.1722

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Aug 25, 2013	85	64	66.1995	2.1995
Sun, Sep 01, 2013	86	60	62.4441	2.4441
Sun, Sep 08, 2013	87	44	45.1702	1.1702
Sun, Sep 15, 2013	88	44	46.1604	2.1604
Sun, Sep 22, 2013	89	66	68.6409	2.6409
Sun, Sep 29, 2013	90	63	68.3060	5.3060
Sun, Oct 06, 2013	91	59	63.0292	4.0292
Sun, Oct 13, 2013	92	53	54.3954	1.3954
Sun, Oct 20, 2013	93	54	54.3788	0.3788
Sun, Oct 27, 2013	94	45	45.2466	0.2466
Sun, Nov 03, 2013	95	51	51.8176	0.8176
Sun, Nov 10, 2013	96	61	62.2432	1.2432
Sun, Nov 17, 2013	97	47	48.1713	1.1713
Sun, Nov 24, 2013	98	48	50.1361	2.1361
Sun, Dec 01, 2013	99	56	57.8280	1.8280
Sun, Dec 08, 2013	100	55	56.1066	1.1066
Sun, Dec 15, 2013	101	56	56.5569	0.5569
Sun, Dec 22, 2013	102	55	55.2026	0.2026
Sun, Dec 29, 2013	103	63	63.0202	0.0202
Sun, Jan 05, 2014	104	58	58.4686	0.4686
Sun, Jan 12, 2014	105	50	50.5266	0.5266
Sun, Jan 19, 2014	106	64	64.9526	0.9526
Sun, Jan 26, 2014	107	62	64.2388	2.2388
Sun, Feb 02, 2014	108	46	47.3138	1.3138
Sun, Feb 09, 2014	109	52	54.4608	2.4608
Sun, Feb 16, 2014	110	69	71.9733	2.9733
Sun, Feb 23, 2014	111	64	67.4059	3.4059
Sun, Mar 02, 2014	112	57	59.1301	2.1301
Sun, Mar 09, 2014	113	49	49.9580	0.9580
Sun, Mar 16, 2014	114	61	61.9828	0.9828
Sun, Mar 23, 2014	115	58	59.8472	1.8472

Date	Week i	House Price	Forecast Value	| Error |
Sun, Mar 30, 2014	116	52	53.2497	1.2497
Sun, Apr 06, 2014	117	63	63.8215	0.8215
Sun, Apr 13, 2014	118	53	54.1144	1.1144
Sun, Apr 20, 2014	119	55	56.6276	1.6276
Sun, Apr 27, 2014	120	59	60.0687	1.0687
Sun, May 04, 2014	121	71	71.4945	0.4945
Sun, May 11, 2014	122	74	75.2506	1.2506
Sun, May 18, 2014	123	66	67.0220	1.0220
Sun, May 25, 2014	124	62	62.6962	0.6962
Sun, Jun 01, 2014	125	67	67.4882	0.4882
Sun, Jun 08, 2014	126	54	54.3271	0.3271
Sun, Jun 15, 2014	127	71	72.6299	1.6299
Sun, Jun 22, 2014	128	61	64.1727	3.1727
Sun, Jun 29, 2014	129	68	71.2123	3.2123
Sun, Jul 06, 2014	130	57	58.4394	1.4394
Sun, Jul 13, 2014	131	63	64.3536	1.3536
Sun, Jul 20, 2014	132	56	57.0496	1.0496
Sun, Jul 27, 2014	133	73	74.0597	1.0597
Sun, Aug 03, 2014	134	56	58.0847	2.0847
Sun, Aug 10, 2014	135	65	68.9578	3.9578
Sun, Aug 17, 2014	136	46	48.2263	2.2263
Sun, Aug 24, 2014	137	66	70.6460	4.6460
Sun, Aug 31, 2014	138	76	84.2074	8.2074
Sun, Sep 07, 2014	139	47	50.7169	3.7169
Sun, Sep 14, 2014	140	49	55.1640	6.1640
Sun, Sep 21, 2014	141	62	68.0908	6.0908
Sun, Sep 28, 2014	142	64	67.6853	3.6853
Sun, Oct 05, 2014	143	74	75.7307	1.7307
Sun, Oct 12, 2014	144	47	47.8173	0.8173
Sun, Oct 19, 2014	145	63	69.1131	6.1131
Sun, Oct 26, 2014	146	59	66.3282	7.3282

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Nov 02, 2014	147	55	58.6624	3.6624
Sun, Nov 09, 2014	148	66	67.0701	1.0701
Sun, Nov 16, 2014	149	64	65.0804	1.0804
Sun, Nov 23, 2014	150	70	71.0313	1.0313
Sun, Nov 30, 2014	151	61	61.5246	0.5246
Sun, Dec 07, 2014	152	57	57.6578	0.6578
Sun, Dec 14, 2014	153	83	83.9032	0.9032
Sun, Dec 21, 2014	154	44	46.8639	2.8639
Sun, Dec 28, 2014	155	52	64.0137	12.0137
Sun, Jan 04, 2015	156	71	85.0043	14.0043
Sun, Jan 11, 2015	157	51	56.6510	5.6510
Sun, Jan 18, 2015	158	56	61.0772	5.0772
Sun, Jan 25, 2015	159	76	80.7514	4.7514
Sun, Feb 01, 2015	160	57	60.3771	3.3771
Sun, Feb 08, 2015	161	37	39.7721	2.7721
Sun, Feb 15, 2015	162	60	67.6366	7.6366
Sun, Feb 22, 2015	163	67	79.6289	12.6289
Sun, Mar 01, 2015	164	62	69.9221	7.9221
Sun, Mar 08, 2015	165	74	77.0434	3.0434
Sun, Mar 15, 2015	166	49	49.8644	0.8644
Sun, Mar 22, 2015	167	57	61.9293	4.9293
Sun, Mar 29, 2015	168	89	96.4686	7.4686
Sun, Apr 05, 2015	169	75	83.9104	8.9104
Sun, Apr 12, 2015	170	66	72.4144	6.4144
Sun, Apr 19, 2015	171	54	56.5651	2.5651
Sun, Apr 26, 2015	172	93	95.6193	2.6193
Sun, May 03, 2015	173	82	93.9243	11.9243
Sun, May 10, 2015	174	74	83.8895	9.8895
Sun, May 17, 2015	175	53	55.8317	2.8317
Sun, May 24, 2015	176	74	78.0207	4.0207
Sun, May 31, 2015	177	64	70.0718	6.0718

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Jun 07, 2015	178	65	69.5930	4.5930
Sun, Jun 14, 2015	179	61	62.5091	1.5091
Sun, Jun 21, 2015	180	64	64.3111	0.3111
Sun, Jun 28, 2015	181	54	54.1445	0.1445
Sun, Jul 05, 2015	182	70	70.9721	0.9721
Sun, Jul 12, 2015	183	69	71.8292	2.8292
Sun, Jul 19, 2015	184	71	73.2595	2.2595
Sun, Jul 26, 2015	185	81	81.8184	0.8184
Sun, Aug 02, 2015	186	51	51.3982	0.3982
Sun, Aug 09, 2015	187	95	104.3952	9.3952
Sun, Aug 16, 2015	188	82	102.9845	20.9845
Sun, Aug 23, 2015	189	85	101.8751	16.8751
Sun, Aug 30, 2015	190	58	61.7692	3.7692
Sun, Sep 06, 2015	191	69	73.5706	4.5706
Sun, Sep 13, 2015	192	72	77.2528	5.2528
Sun, Sep 20, 2015	193	75	77.5576	2.5576
Sun, Sep 27, 2015	194	59	59.3421	0.3421
Sun, Oct 04, 2015	195	86	88.2021	2.2021
Sun, Oct 11, 2015	196	52	56.4014	4.4014
Sun, Oct 18, 2015	197	67	78.7147	11.7147
Sun, Oct 25, 2015	198	77	88.6732	11.6732
Sun, Nov 01, 2015	199	83	88.8889	5.8889
Sun, Nov 08, 2015	200	74	75.4492	1.4492
Sun, Nov 15, 2015	201	73	73.7835	0.7835
Sun, Nov 22, 2015	202	87	87.5469	0.5469
Sun, Nov 29, 2015	203	75	76.1364	1.1364
Sun, Dec 06, 2015	204	49	50.0816	1.0816
Sun, Dec 13, 2015	205	64	69.8272	5.8272
Sun, Dec 20, 2015	206	62	68.6928	6.6928
Sun, Dec 27, 2015	207	80	84.4530	4.4530
Sun, Jan 03, 2016	208	82	85.1565	3.1565

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Jan 10, 2016	209	57	58.5407	1.5407
Sun, Jan 17, 2016	210	59	62.9070	3.9070
Sun, Jan 24, 2016	211	73	76.9995	3.9995
Sun, Jan 31, 2016	212	77	79.9944	2.9944
Sun, Feb 07, 2016	213	61	62.2185	1.2185
Sun, Feb 14, 2016	214	67	69.0711	2.0711
Sun, Feb 21, 2016	215	68	69.7960	1.7960
Sun, Feb 28, 2016	216	79	79.9076	0.9076
Sun, Mar 06, 2016	217	92	93.0108	1.0108
Sun, Mar 13, 2016	218	57	58.0934	1.0934
Sun, Mar 20, 2016	219	64	71.0975	7.0975
Sun, Mar 27, 2016	220	63	69.4877	6.4877
Sun, Apr 03, 2016	221	76	78.9262	2.9262
Sun, Apr 10, 2016	222	63	64.0786	1.0786
Sun, Apr 17, 2016	223	100	102.9529	2.9529
Sun, Apr 24, 2016	224	79	87.7614	8.7614
Sun, May 01, 2016	225	90	100.4684	10.4684
Sun, May 08, 2016	226	66	70.0055	4.0055
Sun, May 15, 2016	227	84	88.7173	4.7173
Sun, May 22, 2016	228	71	75.7404	4.7404
Sun, May 29, 2016	229	86	90.2781	4.2781
Sun, Jun 05, 2016	230	69	71.4615	2.4615
Sun, Jun 12, 2016	231	84	87.3507	3.3507
Sun, Jun 19, 2016	232	63	65.6285	2.6285
Sun, Jun 26, 2016	233	81	85.7196	4.7196
Sun, Jul 03, 2016	234	73	77.8558	4.8558
Sun, Jul 10, 2016	235	83	86.5184	3.5184
Sun, Jul 17, 2016	236	83	84.7067	1.7067
Sun, Jul 24, 2016	237	71	71.5900	0.5900
Sun, Jul 31, 2016	238	82	83.0505	1.0505
Sun, Aug 07, 2016	239	61	62.1516	1.1516

Date	Week *i*	House Price	Forecast Value	| Error |
Sun, Aug 14, 2016	240	85	89.2065	4.2065
Sun, Aug 21, 2016	241	69	74.9330	5.9330
Sun, Aug 28, 2016	242	65	69.9589	4.9589
Sun, Sep 04, 2016	243	86	89.0232	3.0232
Sun, Sep 11, 2016	244	66	68.7247	2.7247
Sun, Sep 18, 2016	245	95	100.9504	5.9504
Sun, Sep 25, 2016	246	59	64.7096	5.7096
Sun, Oct 02, 2016	247	66	76.6654	10.6654
Sun, Oct 09, 2016	248	62	69.2784	7.2784
Sun, Oct 16, 2016	249	63	65.4825	2.4825
Sun, Oct 23, 2016	250	63	63.2227	0.2227
Sun, Oct 30, 2016	251	64	64.0427	0.0427
Sun, Nov 06, 2016	252	60	60.0086	0.0086
Sun, Nov 13, 2016	253	59	59.1113	0.1113
Sun, Nov 20, 2016	254	62	62.1155	0.1155
Sun, Nov 27, 2016	255	42	42.0742	0.0742
Sun, Dec 04, 2016	256	62	66.0930	4.0930
Sun, Dec 11, 2016	257	59	66.5204	7.5204
Sun, Dec 18, 2016	258	69	74.8630	5.8630
Sun, Dec 25, 2016	259	61	63.0309	2.0309
Sun, Jan 01, 2017	260	78	79.3193	1.3193
			Average Error	3.3444

APPENDIX F: Performance for Malaysia House Price Index

Model Performance for Malaysia House Price Index

Date	Quarter	House Price	Forecast Value	| Error |
2003 Q1	17	107.2	107.2276	0.0276
Q2	18	107.1	107.1207	0.0207
Q3	19	110.6	110.6120	0.0120
Q4	20	111.2	111.2201	0.0201
2004 Q1	21	112.8	112.8344	0.0344
Q2	22	113.1	113.1202	0.0202
Q3	23	113.4	113.4165	0.0165
Q4	24	114	114.0095	0.0095
2005 Q1	25	115	115.0063	0.0063
Q2	26	116.9	116.9054	0.0054
Q3	27	116.4	116.4083	0.0083
Q4	28	116.9	116.9108	0.0108
2006 Q1	29	117.7	117.7062	0.0062
Q2	30	118.5	118.5047	0.0047
Q3	31	118.8	118.8044	0.0044
Q4	32	122.4	122.4037	0.0037
2007 Q1	33	123.4	123.4144	0.0144
Q2	34	123.7	123.7302	0.0302
Q3	35	125.2	125.2165	0.0165
Q4	36	125.9	125.9119	0.0119
2008 Q1	37	128.7	128.7112	0.0112
Q2	38	128.9	128.9139	0.0139
Q3	39	131.4	131.4203	0.0203
Q4	40	129	129.0153	0.0153
2009 Q1	41	129.6	129.6231	0.0231
Q2	42	132.2	132.2213	0.0213
Q3	43	133.3	133.3179	0.0179
Q4	44	136.1	136.1220	0.0220

2010 Q1	45	136.9	136.9199	0.0199
Q2	46	140.3	140.3238	0.0238
Q3	47	143.7	143.7233	0.0233
Q4	48	147.2	147.2401	0.0401
2011 Q1	49	149.1	149.1471	0.0471
Q2	50	155.1	155.1463	0.0463
Q3	51	157.8	157.8576	0.0576
Q4	52	161.9	161.9871	0.0871
2012 Q1	53	167	167.0673	0.0673
Q2	54	172.4	172.4798	0.0798
Q3	55	176.5	176.5971	0.0971
Q4	56	181.7	181.8001	0.1001
2013 Q1	57	184.9	184.9918	0.0918
Q2	58	191.8	191.8907	0.0907
Q3	59	198	198.0919	0.0919
Q4	60	199.1	199.2260	0.1260
2014 Q1	61	202.7	202.8076	0.1076
Q2	62	208	208.0684	0.0684
Q3	63	213.6	213.6708	0.0708
Q4	64	215	215.0843	0.0843
2015 Q1	65	218.5	218.5768	0.0768
			Average Error	0.0393

APPENDIX G: Performance for NASDAQ Index

Model Performance for NASDAQ Index

Date	**Day *i***	**Close Price**	**Forecast Value**	**| Error |**
Mon, Mar 23, 2015	55	5,010.9702	5,011.1111	0.1409
Tue, Mar 24, 2015	56	4,994.7300	4,994.8429	0.1130
Wed, Mar 25, 2015	57	4,876.5200	4,876.5756	0.0556
Thu, Mar 26, 2015	58	4,863.3599	4,864.9887	1.6288
Fri, Mar 27, 2015	59	4,891.2202	4,892.2749	1.0547
Mon, Mar 30, 2015	60	4,947.4399	4,947.6583	0.2183
Tue, Mar 31, 2015	61	4,900.8799	4,901.3358	0.4559
Wed, Apr 01, 2015	62	4,880.2300	4,880.7286	0.4987
Thu, Apr 02, 2015	63	4,886.9399	4,887.1790	0.2390
Mon, Apr 06, 2015	64	4,917.3198	4,917.3816	0.0618
Tue, Apr 07, 2015	65	4,910.2300	4,910.3513	0.1213
Wed, Apr 08, 2015	66	4,950.8198	4,950.8983	0.0785
Thu, Apr 09, 2015	67	4,974.5601	4,974.7661	0.2060
Fri, Apr 10, 2015	68	4,995.9800	4,996.1726	0.1927
Mon, Apr 13, 2015	69	4,988.2500	4,988.3588	0.1088
Tue, Apr 14, 2015	70	4,977.2900	4,977.3385	0.0485
Wed, Apr 15, 2015	71	5,011.0200	5,011.0440	0.0239
Thu, Apr 16, 2015	72	5,007.7900	5,007.9323	0.1422
Fri, Apr 17, 2015	73	4,931.8101	4,931.8953	0.0852
Mon, Apr 20, 2015	74	4,994.6001	4,995.2821	0.6820
Tue, Apr 21, 2015	75	5,014.1001	5,014.9951	0.8950
Wed, Apr 22, 2015	76	5,035.1699	5,035.5569	0.3869
Thu, Apr 23, 2015	77	5,056.0601	5,056.1827	0.1227
Fri, Apr 24, 2015	78	5,092.0801	5,092.1770	0.0969
Mon, Apr 27, 2015	79	5,060.2500	5,060.4355	0.1855
Tue, Apr 28, 2015	80	5,055.4199	5,055.6330	0.2131
Wed, Apr 29, 2015	81	5,023.6401	5,023.7265	0.0864
Thu, Apr 30, 2015	82	4,941.4199	4,941.5450	0.1251

Date	Day *i*	Close Price	Forecast Value	| Error |
Fri, May 01, 2015	83	5,005.3901	5,006.2506	0.8605
Mon, May 04, 2015	84	5,016.9302	5,017.9185	0.9883
Tue, May 05, 2015	85	4,939.3301	4,939.7003	0.3702
Wed, May 06, 2015	86	4,919.6401	4,920.3832	0.7431
Thu, May 07, 2015	87	4,945.5400	4,946.0393	0.4993
Fri, May 08, 2015	88	5,003.5498	5,003.7093	0.1595
Mon, May 11, 2015	89	4,993.5698	4,994.0269	0.4570
Tue, May 12, 2015	90	4,976.1899	4,976.4592	0.2693
Wed, May 13, 2015	91	4,981.6899	4,981.7609	0.0710
Thu, May 14, 2015	92	5,050.7998	5,050.8339	0.0341
Fri, May 15, 2015	93	5,048.2900	5,048.8482	0.5582
Mon, May 18, 2015	94	5,078.4399	5,078.7962	0.3563
Tue, May 19, 2015	95	5,070.0298	5,070.1718	0.1420
Wed, May 20, 2015	96	5,071.7402	5,071.8237	0.0835
Thu, May 21, 2015	97	5,090.7900	5,090.8049	0.0149
Fri, May 22, 2015	98	5,089.3599	5,089.4041	0.0442
Tue, May 26, 2015	99	5,032.7500	5,032.7769	0.0269
Wed, May 27, 2015	100	5,106.5898	5,106.9608	0.3709
Thu, May 28, 2015	101	5,097.9800	5,098.8377	0.8577
Fri, May 29, 2015	102	5,070.0298	5,070.4584	0.4286
Mon, Jun 01, 2015	103	5,082.9302	5,083.0724	0.1422
Tue, Jun 02, 2015	104	5,076.5200	5,076.6077	0.0877
Wed, Jun 03, 2015	105	5,099.2300	5,099.2559	0.0259
Thu, Jun 04, 2015	106	5,059.1201	5,059.1846	0.0645
Fri, Jun 05, 2015	107	5,068.4600	5,068.6802	0.2202
Mon, Jun 08, 2015	108	5,021.6299	5,021.7590	0.1291
Tue, Jun 09, 2015	109	5,013.8701	5,014.1384	0.2682
Wed, Jun 10, 2015	110	5,076.6899	5,076.8617	0.1717
Thu, Jun 11, 2015	111	5,082.5098	5,082.9860	0.4762
Fri, Jun 12, 2015	112	5,051.1001	5,051.3966	0.2965
Mon, Jun 15, 2015	113	5,029.9702	5,030.1157	0.1455

Date	Day *i*	Close Price	Forecast Value	| Error |
Tue, Jun 16, 2015	114	5,055.5498	5,055.6805	0.1306
Wed, Jun 17, 2015	115	5,064.8799	5,064.9972	0.1174
Thu, Jun 18, 2015	116	5,132.9502	5,133.0147	0.0645
Fri, Jun 19, 2015	117	5,117.0000	5,117.5380	0.5380
Mon, Jun 22, 2015	118	5,153.9702	5,154.3388	0.3686
Tue, Jun 23, 2015	119	5,160.0898	5,160.2988	0.2090
Wed, Jun 24, 2015	120	5,122.4102	5,122.5228	0.1126
Thu, Jun 25, 2015	121	5,112.1899	5,112.3637	0.1738
Fri, Jun 26, 2015	122	5,080.5098	5,080.6252	0.1154
Mon, Jun 29, 2015	123	4,958.4702	4,958.5991	0.1288
Tue, Jun 30, 2015	124	4,986.8701	4,988.6415	1.7714
Wed, Jul 01, 2015	125	5,013.1201	5,014.3118	1.1917
Thu, Jul 02, 2015	126	5,009.2100	5,009.4678	0.2578
Mon, Jul 06, 2015	127	4,991.9399	4,992.0276	0.0877
Tue, Jul 07, 2015	128	4,997.4600	4,997.5095	0.0495
Wed, Jul 08, 2015	129	4,909.7598	4,909.7883	0.0285
Thu, Jul 09, 2015	130	4,922.3999	4,923.2934	0.8935
Fri, Jul 10, 2015	131	4,997.7002	4,998.2959	0.5957
Mon, Jul 13, 2015	132	5,071.5098	5,072.2576	0.7478
Tue, Jul 14, 2015	133	5,104.8901	5,105.9709	1.0808
Wed, Jul 15, 2015	134	5,098.9399	5,099.5192	0.5793
Thu, Jul 16, 2015	135	5,163.1802	5,163.3219	0.1418
Fri, Jul 17, 2015	136	5,210.1401	5,210.6355	0.4954
Mon, Jul 20, 2015	137	5,218.8599	5,219.4126	0.5527
Tue, Jul 21, 2015	138	5,208.1201	5,208.3189	0.1987
Wed, Jul 22, 2015	139	5,171.7700	5,171.8128	0.0428
Thu, Jul 23, 2015	140	5,146.4102	5,146.5701	0.1600
Fri, Jul 24, 2015	141	5,088.6299	5,088.7948	0.1650
Mon, Jul 27, 2015	142	5,039.7798	5,040.2047	0.4249
Tue, Jul 28, 2015	143	5,089.2100	5,089.7283	0.5184
Wed, Jul 29, 2015	144	5,111.7300	5,112.2125	0.4825

Date	Day *i*	Close Price	Forecast Value	| Error |
Thu, Jul 30, 2015	145	5,128.7798	5,129.0424	0.2626
Fri, Jul 31, 2015	146	5,128.2798	5,128.3747	0.0949
Mon, Aug 03, 2015	147	5,115.3799	5,115.4109	0.0310
Tue, Aug 04, 2015	148	5,105.5498	5,105.5732	0.0234
Wed, Aug 05, 2015	149	5,139.9399	5,139.9640	0.0241
Thu, Aug 06, 2015	150	5,056.4399	5,056.5795	0.1396
Fri, Aug 07, 2015	151	5,043.5400	5,044.4035	0.8634
Mon, Aug 10, 2015	152	5,101.7998	5,102.3312	0.5314
Tue, Aug 11, 2015	153	5,036.7900	5,037.2398	0.4498
Wed, Aug 12, 2015	154	5,044.3901	5,045.1271	0.7370
Thu, Aug 13, 2015	155	5,033.5601	5,033.9006	0.3405
Fri, Aug 14, 2015	156	5,048.2402	5,048.2978	0.0575
Mon, Aug 17, 2015	157	5,091.7002	5,091.7440	0.0438
Tue, Aug 18, 2015	158	5,059.3501	5,059.5833	0.2332
Wed, Aug 19, 2015	159	5,019.0498	5,019.3052	0.2554
Thu, Aug 20, 2015	160	4,877.4902	4,877.7573	0.2670
Fri, Aug 21, 2015	161	4,706.0400	4,708.3789	2.3388
Mon, Aug 24, 2015	162	4,526.2500	4,530.9615	4.7115
Tue, Aug 25, 2015	163	4,506.4902	4,512.6949	6.2047
Wed, Aug 26, 2015	164	4,697.5400	4,700.4851	2.9451
Thu, Aug 27, 2015	165	4,812.7100	4,817.8751	5.1651
Fri, Aug 28, 2015	166	4,828.3198	4,833.1004	4.7806
Mon, Aug 31, 2015	167	4,776.5098	4,777.9137	1.4039
Tue, Sep 01, 2015	168	4,636.1001	4,636.6176	0.5175
Wed, Sep 02, 2015	169	4,749.9800	4,752.6667	2.6867
Thu, Sep 03, 2015	170	4,733.5000	4,736.6868	3.1868
Fri, Sep 04, 2015	171	4,683.9199	4,685.1379	1.2180
Tue, Sep 08, 2015	172	4,811.9302	4,812.4132	0.4830
Wed, Sep 09, 2015	173	4,756.5298	4,758.7538	2.2240
Thu, Sep 10, 2015	174	4,796.2500	4,797.9323	1.6823
Fri, Sep 11, 2015	175	4,822.3398	4,822.9146	0.5748

Date	Day *i*	Close Price	Forecast Value	| Error |
Mon, Sep 14, 2015	176	4,805.7598	4,806.0241	0.2644
Tue, Sep 15, 2015	177	4,860.5200	4,860.6341	0.1141
Wed, Sep 16, 2015	178	4,889.2402	4,889.6355	0.3952
Thu, Sep 17, 2015	179	4,893.9502	4,894.2842	0.3340
Fri, Sep 18, 2015	180	4,827.2300	4,827.3198	0.0898
Mon, Sep 21, 2015	181	4,828.9502	4,829.4885	0.5383
Tue, Sep 22, 2015	182	4,756.7202	4,757.0532	0.3330
Wed, Sep 23, 2015	183	4,752.7402	4,753.3978	0.6575
Thu, Sep 24, 2015	184	4,734.4800	4,734.8859	0.4059
Fri, Sep 25, 2015	185	4,686.5000	4,686.5851	0.0851
Mon, Sep 28, 2015	186	4,543.9702	4,544.2769	0.3067
Tue, Sep 29, 2015	187	4,517.3198	4,519.9775	2.6577
Wed, Sep 30, 2015	188	4,620.1602	4,621.8918	1.7316
Thu, Oct 01, 2015	189	4,627.0801	4,628.6667	1.5866
Fri, Oct 02, 2015	190	4,707.7798	4,708.7134	0.9336
Mon, Oct 05, 2015	191	4,781.2598	4,782.1943	0.9345
Tue, Oct 06, 2015	192	4,748.3599	4,749.5638	1.2040
Wed, Oct 07, 2015	193	4,791.1499	4,791.7665	0.6166
Thu, Oct 08, 2015	194	4,810.7900	4,811.1580	0.3679
Fri, Oct 09, 2015	195	4,830.4702	4,830.6836	0.2134
Mon, Oct 12, 2015	196	4,838.6401	4,838.7375	0.0974
Tue, Oct 13, 2015	197	4,796.6099	4,796.6554	0.0456
Wed, Oct 14, 2015	198	4,782.8501	4,783.0698	0.2197
Thu, Oct 15, 2015	199	4,870.1001	4,870.2613	0.1612
Fri, Oct 16, 2015	200	4,886.6899	4,887.6391	0.9491
Mon, Oct 19, 2015	201	4,905.4702	4,906.0973	0.6271
Tue, Oct 20, 2015	202	4,880.9702	4,881.0962	0.1260
Wed, Oct 21, 2015	203	4,840.1201	4,840.2339	0.1138
Thu, Oct 22, 2015	204	4,920.0498	4,920.3024	0.2526
Fri, Oct 23, 2015	205	5,031.8599	5,032.7743	0.9144
Mon, Oct 26, 2015	206	5,034.7002	5,036.6755	1.9753

Date	Day *i*	Close Price	Forecast Value	| Error |
Tue, Oct 27, 2015	207	5,030.1499	5,031.1410	0.9911
Wed, Oct 28, 2015	208	5,095.6899	5,095.8080	0.1181
Thu, Oct 29, 2015	209	5,074.2700	5,074.7892	0.5192
Fri, Oct 30, 2015	210	5,053.7500	5,054.1199	0.3699
Mon, Nov 02, 2015	211	5,127.1499	5,127.2673	0.1174
Tue, Nov 03, 2015	212	5,145.1299	5,145.7890	0.6591
Wed, Nov 04, 2015	213	5,142.4800	5,142.9155	0.4355
Thu, Nov 05, 2015	214	5,127.7402	5,127.8075	0.0673
Fri, Nov 06, 2015	215	5,147.1201	5,147.1582	0.0381
Mon, Nov 09, 2015	216	5,095.2998	5,095.3602	0.0604
Tue, Nov 10, 2015	217	5,083.2402	5,083.5688	0.3286
Wed, Nov 11, 2015	218	5,067.0200	5,067.2305	0.2105
Thu, Nov 12, 2015	219	5,005.0801	5,005.1408	0.0607
Fri, Nov 13, 2015	220	4,927.8799	4,928.3336	0.4538
Mon, Nov 16, 2015	221	4,984.6201	4,985.5940	0.9739
Tue, Nov 17, 2015	222	4,986.0200	4,986.8694	0.8494
Wed, Nov 18, 2015	223	5,075.2002	5,075.5006	0.3004
Thu, Nov 19, 2015	224	5,073.6401	5,074.5979	0.9578
Fri, Nov 20, 2015	225	5,104.9199	5,105.5196	0.5997
Mon, Nov 23, 2015	226	5,102.4800	5,102.6565	0.1765
Tue, Nov 24, 2015	227	5,102.8101	5,102.8976	0.0876
Wed, Nov 25, 2015	228	5,116.1401	5,116.1521	0.0120
Fri, Nov 27, 2015	229	5,127.5200	5,127.5430	0.0230
Mon, Nov 30, 2015	230	5,108.6699	5,108.6979	0.0280
Tue, Dec 01, 2015	231	5,156.3101	5,156.3612	0.0512
Wed, Dec 02, 2015	232	5,123.2202	5,123.5012	0.2810
Thu, Dec 03, 2015	233	5,037.5298	5,037.8125	0.2827
Fri, Dec 04, 2015	234	5,142.2700	5,143.2086	0.9386
Mon, Dec 07, 2015	235	5,101.8101	5,103.5998	1.7897
Tue, Dec 08, 2015	236	5,098.2402	5,099.2768	1.0366
Wed, Dec 09, 2015	237	5,022.8701	5,023.0847	0.2146

Date	**Day *i***	**Close Price**	**Forecast Value**	**| Error |**
Thu, Dec 10, 2015	238	5,045.1699	5,045.8516	0.6816
Fri, Dec 11, 2015	239	4,933.4702	4,933.9370	0.4668
Mon, Dec 14, 2015	240	4,952.2300	4,953.7398	1.5099
Tue, Dec 15, 2015	241	4,995.3599	4,996.3365	0.9766
Wed, Dec 16, 2015	242	5,071.1299	5,071.4797	0.3498
Thu, Dec 17, 2015	243	5,002.5498	5,003.3685	0.8187
Fri, Dec 18, 2015	244	4,923.0801	4,924.0367	0.9566
Mon, Dec 21, 2015	245	4,968.9199	4,970.0413	1.1214
Tue, Dec 22, 2015	246	5,001.1099	5,001.8770	0.7672
Wed, Dec 23, 2015	247	5,045.9302	5,046.2734	0.3432
Thu, Dec 24, 2015	248	5,048.4902	5,048.8314	0.3411
Mon, Dec 28, 2015	249	5,040.9902	5,041.1547	0.1644
Tue, Dec 29, 2015	250	5,107.9399	5,107.9669	0.0270
Wed, Dec 30, 2015	251	5,065.8501	5,066.3667	0.5166
Thu, Dec 31, 2015	252	5,007.4102	5,007.9294	0.5193
			Average Error	0.5878

APPENDIX H: Performance for Dow Jones Index

Model Performance for Dow Jones Index

Day	Adj Close	Forecast Value	| Error |
1	46.15	46.1885	0.0385
2	45.98	45.9899	0.0099
3	47	47.0105	0.0105
4	47.28	47.2900	0.0100
5	47.4	47.4085	0.0085
6	47.25	47.2719	0.0219
7	47.51	47.5129	0.0029
8	47.36	47.3611	0.0011
9	47.33	47.3309	0.0009
10	47.63	47.6322	0.0022
11	47.67	47.6711	0.0011
12	47.1	47.1029	0.0029
13	47.09	47.0940	0.0040
14	47.19	47.1915	0.0015
15	47.1	47.1065	0.0065
16	47.4	47.4010	0.0010
17	46.75	46.7542	0.0042
18	45.84	45.8500	0.0100
19	45.83	45.8393	0.0093
20	45.05	45.0664	0.0164
21	44.73	44.7515	0.0215
22	45.38	45.3874	0.0074
23	44.99	45.0072	0.0172
24	44.97	44.9754	0.0054
25	44.59	44.6006	0.0106
26	45.46	45.4708	0.0108
27	44.69	44.7011	0.0111
28	45.22	45.2331	0.0131

Day	Adj Close	Forecast Value	| Error |
29	45.16	45.1805	0.0205
30	45.15	45.1640	0.0140
31	44.78	44.7873	0.0073
32	44.81	44.8113	0.0013
33	46.08	46.0940	0.0140
34	46.97	46.9914	0.0214
35	46.3	46.3164	0.0164
36	46.22	46.2602	0.0402
37	46.55	46.5696	0.0196
38	46.97	46.9818	0.0118
39	46.82	46.8223	0.0023
40	46.67	46.6733	0.0033
41	47.03	47.0349	0.0049
42	46.69	46.6925	0.0025
43	46.58	46.5819	0.0019
44	47.12	47.1255	0.0055
45	48.35	48.3665	0.0165
46	48.01	48.0230	0.0130
47	47.92	47.9331	0.0131
48	48.33	48.3619	0.0319
49	47.91	47.9156	0.0056
50	48.27	48.2732	0.0032
51	49.21	49.2218	0.0118
52	49.33	49.3403	0.0103
53	49.66	49.6673	0.0073
54	50	50.0190	0.0190
55	49.8	49.8022	0.0022
56	49.21	49.2154	0.0054
57	49.88	49.8881	0.0081
58	49.84	49.8452	0.0052
59	49.55	49.5590	0.0090

Day	Adj Close	Forecast Value	| Error |
60	49.1	49.1106	0.0106
61	48.86	48.8624	0.0024
62	50.03	50.0432	0.0132
63	50.08	50.0934	0.0134
64	49.16	49.1729	0.0129
65	48.95	48.9814	0.0314
66	49.44	49.4462	0.0062
67	49.7	49.7183	0.0183
68	49.21	49.2145	0.0045
69	48.95	48.9570	0.0070
70	49.25	49.2535	0.0035
71	49.58	49.5863	0.0063
72	49.49	49.4926	0.0026
73	48.84	48.8454	0.0054
74	49.4	49.4074	0.0074
75	49.76	49.7650	0.0050
76	50.24	50.2521	0.0121
77	50.43	50.4386	0.0086
78	50.8	50.8049	0.0049
79	51.15	51.1564	0.0064
80	51.19	51.1922	0.0022
81	50.24	50.2495	0.0095
82	49.6	49.6110	0.0110
83	49.71	49.7163	0.0063
84	50.17	50.1897	0.0197
85	50.33	50.3398	0.0098
86	50.02	50.0222	0.0022
87	50.04	50.0447	0.0047
88	50.75	50.7547	0.0047
89	51.48	51.4891	0.0091
90	51.71	51.7159	0.0059

Day	Adj Close	Forecast Value	| Error |
91	51.41	51.4226	0.0126
92	51.57	51.5810	0.0110
93	51.69	51.6919	0.0019
94	51.24	51.2433	0.0033
95	51.35	51.3519	0.0019
96	51.23	51.2312	0.0012
97	49.4	49.4278	0.0278
98	49.77	49.7931	0.0231
99	49.98	49.9946	0.0146
100	49.97	50.0324	0.0624
101	49.18	49.1890	0.0090
102	49.45	49.4558	0.0058
103	48.16	48.1754	0.0154
104	48.88	48.9071	0.0271
105	50	50.0218	0.0218
106	50.52	50.5657	0.0457
107	51.59	51.6169	0.0269
108	50.67	50.7091	0.0391
109	50.8	50.8157	0.0157
110	49.77	49.8016	0.0316
111	49.14	49.1652	0.0252
112	48.73	48.7387	0.0087
113	48.63	48.6518	0.0218
114	46.45	46.4943	0.0443
115	44.71	44.7683	0.0583
116	44.38	44.4210	0.0410
117	45.37	45.4810	0.1110
118	46.13	46.2076	0.0776
119	46.12	46.1330	0.0130
120	45.77	45.7945	0.0245
121	45.19	45.2058	0.0158

Day	Adj Close	Forecast Value	| Error |
122	45.41	45.4137	0.0037
123	45.84	45.8459	0.0059
124	45.32	45.3307	0.0107
125	44.25	44.2632	0.0132
126	45.36	45.3838	0.0238
127	44.05	44.0833	0.0333
128	44.45	44.4931	0.0431
129	43.74	43.7785	0.0385
130	43.67	43.7101	0.0401
131	44.22	44.2293	0.0093
132	43.52	43.5371	0.0171
133	42.92	42.9284	0.0084
134	42.47	42.4828	0.0128
135	41.45	41.4721	0.0221
136	39.24	39.2994	0.0594
137	38.31	38.3647	0.0547
138	39.93	40.0088	0.0788
139	42.41	42.6149	0.2049
140	42.79	42.8832	0.0932
141	42.56	42.6612	0.1012
142	40.52	40.6960	0.1760
143	41.23	41.2746	0.0446
144	41.66	41.6874	0.0274
145	40.93	41.0307	0.1007
146	42.88	42.9343	0.0543
147	42.59	42.6286	0.0386
148	42.56	42.5915	0.0315
149	42.58	42.6666	0.0866
150	41.9	41.9081	0.0081
151	42.42	42.4264	0.0064
152	43.75	43.7699	0.0199

Day	Adj Close	Forecast Value	| Error |
153	43.01	43.0398	0.0298
154	42.12	42.1450	0.0250
155	42.81	42.8610	0.0510
156	42.41	42.4313	0.0213
157	41.41	41.4394	0.0294
158	41.33	41.3492	0.0192
159	41.16	41.1687	0.0087
160	39.31	39.3619	0.0519
161	39.84	39.8712	0.0312
162	41.65	41.7011	0.0511
163	42.22	42.3350	0.1150
164	43.67	43.7178	0.0478
165	45.49	45.6209	0.1309
166	46.25	46.3008	0.0508
167	46.65	46.7209	0.0709
168	46.95	47.0287	0.0787
169	46.25	46.2691	0.0191
170	45.78	45.7893	0.0093
171	45.69	45.6956	0.0056
172	46.28	46.2936	0.0136
173	46.74	46.7491	0.0091
174	46.78	46.7834	0.0034
175	46.55	46.5585	0.0085
176	46.85	46.8556	0.0056
177	46.64	46.6414	0.0014
178	49.04	49.0875	0.0475
179	49.43	49.4743	0.0443
180	49.56	49.5869	0.0269
181	49.8	49.9179	0.1179
182	50.46	50.4698	0.0098
183	50	50.0054	0.0054

Day	Adj Close	Forecast Value	| Error |
184	50.76	50.7686	0.0086
185	50.68	50.6929	0.0129
186	51.06	51.0676	0.0076
187	50.47	50.4843	0.0143
188	50.7	50.7036	0.0036
189	50.93	50.9348	0.0048
190	50.79	50.7972	0.0072
191	50.8	50.8015	0.0015
192	50.79	50.7911	0.0011
193	49.54	49.5518	0.0118
194	50.24	50.2539	0.0139
195	50.74	50.7514	0.0114
196	50.74	50.7730	0.0330
197	52.22	52.2474	0.0274
198	52.38	52.3996	0.0196
199	52.38	52.3889	0.0089
200	52.06	52.1009	0.0409
201	52.44	52.4432	0.0032
202	51	51.0160	0.0160
203	51.09	51.1054	0.0154
204	51.21	51.2206	0.0106
205	52.48	52.5297	0.0497
206	51.37	51.3900	0.0200
207	50.94	50.9554	0.0154
208	52.37	52.4201	0.0501
209	51.11	51.1573	0.0473
210	50	50.0302	0.0302
211	55.96	56.2780	0.3180
212	53.94	54.2282	0.2882
213	52.43	52.6253	0.1953
214	50.38	51.0320	0.6520

Day	Adj Close	Forecast Value	| Error |
215	49.68	49.7958	0.1158
216	49.82	49.8800	0.0600
217	49.36	49.4381	0.0781
218	48.75	48.7652	0.0152
219	50.07	50.0875	0.0175
220	50.59	50.6096	0.0196
221	51.6	51.6241	0.0241
222	51.39	51.4324	0.0424
223	51.31	51.3203	0.0103
224	52.32	52.3469	0.0269
225	51.2	51.2166	0.0166
226	51.02	51.0321	0.0121