A FAMILY OF GROUP CHAIN ACCEPTANCE SAMPLING PLANS BASED ON TRUNCATED LIFE TEST

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Abstrak

Persampelan penerimaan merupakan prosedur kawalan kualiti berstatistik yang digunakan untuk menentukan sama ada untuk menerima atau menolak sesuatu lot, berdasarkan hasil pemeriksaan sampel. Bagi produk berkualiti tinggi, bilangan penerimaan sifar diambil kira dan ujian hayat ini selalunya diberhentikan pada masa tertentu, yang dipanggil ujian hayat terpangkas. Pelan yang melibatkan bilangan penerimaan sifar dianggap tidak adil terhadap pengeluar kerana kebarangkalian penerimaan lot menurun secara drastik pada kadar kerosakan yang sangat kecil. Untuk mengatasi masalah ini, persampelan berantai yang menggunakan maklumat lot sebelum dan selepas telah diperkenalkan. Bagi pelan persampelan berantai biasa, hanya satu produk yang boleh diperiksa pada satu masa, walaupun secara praktikalnya, penguji mampu memeriksa lebih dari satu produk serentak. Dalam situasi ini, pelan persampelan kumpulan berantai dengan sampel bersaiz kecil menjadi pilihan kerana ia menjimatkan masa dan kos pemeriksaan. Oleh yang demikian, adalah bermanfaat untuk membangunkan beberapa jenis pelan persampelan berantai dalam konteks ujian berkumpulan. Matlamat kajian ini adalah untuk membangunkan pelan persampelan baharu bagi kumpulan berantai (GChSP), kumpulan berantai yang diubahsuai (MGChSP), kumpulan berantai dua sisi (TS-GChSP) dan kumpulan berantai dua sisi yang diubahsuai (TS-MGChSP) menggunakan taburan Pareto jenis ke-2. Empat pelan tersebut juga digeneralisasikan berdasarkan beberapa nilai kadar kerosakan yang telah ditetapkan. Kajian ini melibatkan empat fasa: mengenal pasti beberapa kombinasi reka bentuk parameter; membangunkan prosedur; mendapatkan fungsi cirian pengoperasian; dan mengukur prestasi dengan menggunakan data simulasi dan data hayat yang sebenar. Pelan yang dibangunkan dinilai menggunakan beberapa reka bentuk parameter dan dibandingkan dengan pelan yang telah mantap berdasarkan bilangan kumpulan minimum, g dan kebarangkalian penerimaan lot, $L(p)$. Dapatan menunjukkan kesemua pelan yang dicadangkan mempunyai g yang lebih kecil dan $L(p)$ yang lebih rendah berbanding dengan pelan yang telah mantap. Kesemua pelan tersebut berupaya menjimatkan masa dan kos pemeriksaan, serta memberikan lebih perlindungan kepada pengguna daripada menerima produk yang rosak. Ini seharusnya memberi banyak faedah kepada pengamal industri terutamanya yang melibatkan ujian musnah untuk produk berkualiti tinggi.

Kata kunci: Persampelan berantai, Persampelan penerimaan kumpulan, Lengkung cirian pengoperasian, Ujian hayat terpangkas, Persampelan rantaian dua sisi.

Abstract

Acceptance sampling is a statistical quality control procedure used to accept or reject a lot, based on the inspection result of its sample. For high quality products, zero acceptance number is considered and the life test is often terminated on a specific time, hence called truncated life test. A plan having zero acceptance number is deemed unfair to producers as the probability of lot acceptance drops drastically at a very small proportion defective. To overcome this problem, chain sampling which uses preceding and succeeding lots information was introduced. In ordinary chain sampling plans, only one product is inspected at a time, although in practice, testers can accommodate multiple products simultaneously. In this situation, group chain sampling plan with small sample size is preferred because it saves inspection time and cost. Thus, it is worthwhile to develop the various types of chain sampling plans in the context of group testing. This research aims to develop new group chain (GChSP), modified group chain (MGChSP), two-sided group chain (TS-GChSP) and modified two-sided group chain (TS-MGChSP) sampling plans using the Pareto distribution of the 2nd kind. These four plans are also generalized based on several pre-specified values of proportion defective. This study involves four phases: identifying several combinations of design parameters; developing the procedures; obtaining operating characteristic functions; and measuring performances using both simulated and real lifetime data. The constructed plans are evaluated using various design parameters and compared with the established plan based on the number of minimum groups, g and probability of lot acceptance, $L(p)$. The findings show that all the proposed plans provide smaller g and lower $L(p)$ compared to the established plan. All the plans are able to reduce inspection time and cost, and better at protecting customers from receiving defective products. This would be very beneficial to practitioners especially those involved with destructive testing of high quality products.

Keywords: Chain sampling, Group acceptance sampling, Operating characteristic curve, Truncated life test, Two-sided chain sampling.

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Glossary of Terms

a	Pre-specified testing time
c	Acceptance number
d	Rejection number
g	Total number of groups
$L(p)$	Probability of lot acceptance
r	Group size
t_0	Test termination time
α	Producer's risk
β	Consumer's risk
λ	Shape parameter of Pareto distribution of the 2nd kind
σ	Scale parameter of Pareto distribution of the 2nd kind
μ	Mean lifetime of a product
μ_0	Specified mean lifetime of a product
μ/μ_0	Mean ratio
p	Proportion defective
n	Sample size
AQL	Acceptable quality level
LTPD	Lot tolerance percent defective
AOQL	Average outgoing quality limit

List of Publications

Mughal, A.R., Zain,Z., & Aziz, N. (2015). Time Truncated Group Chain Sampling Strategy for Pareto Distribution of the 2nd kind. *Research Journal of Applied Sciences, Engineering and Technology,* 10(4), 471-474.

Mughal, A. R., Zain, Z., & Aziz, N. (2015). Group Acceptance Sampling Plan for Pareto Distribution of the 2nd kind using Two-Sided Chain Sampling. *International Journal of Applied Engineering Research*, 10(16), 37240-37242.

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Zain, Z., Mughal, A. R., & Aziz, N. (2015, December). Generalized group chain acceptance sampling plan. In *INNOVATION AND ANALYTICS CONFERENCE AND EXHIBITION (IACE 2015): Proceedings of the 2nd Innovation and Analytics Conference & Exhibition* (Vol. 1691, No. 1, p. 050030). AIP Publishing.

Mughal, A. R., Zain, Z., & Aziz, N. (2016). Two-Sided and Modified Two-Sided Group Chain Sampling Plan for Pareto Distribution of the 2nd kind. *Advance and Applications in Statistics.* (Accepted).

Mughal, A. R., Zain, Z., & Aziz, N. (2016). Modified and Generalized Modified Group Chain Sampling Plan based on Truncated Life Test. *Sains Malaysiana*. (In Review).

CHAPTER ONE
INTRODUCTION

In this chapter, the fundamental concepts of quality control and uses of probability distributions in acceptance sampling plans are explained. The objective of the study, methodology and analysis on acceptance sampling plans are also discussed. Several group chain acceptance sampling plans for attributes are developed for experimenters in order to reach the accurate probability of lot acceptance at pre-specified design parameters.

1.1 Background

According to Juran (1951), "Quality means that a product meets customer needs leading to customer satisfaction, and quality also means all the activities in which a business engages in, to ensure that the product meets customer needs. You can think of this second aspect of quality as quality control - ensuring a quality manufacturing process". Quality is a measure of excellence or a state of being free from defects, deficiencies and considerable variations. The quality of a product is brought about by the consistent adherence and verifiable standards to achieve uniformity of production that satisfies consumer or user necessities (Deva and Rebecca, 2012).

The International Organization for Standardization (ISO), founded in 1947, is a worldwide association of national standards which has contributed significantly in recent years (Schilling & Neubauer, 2008). The ISO's standards offer guidance and tools for companies who want to ensure that their products meet customers' requirements and their quality consistently improves. The ISO 2859 and ISO 3951 standards are parts of the series created to address the role of acceptance sampling when dealing with the flow of products with an emphasis on the producer's process. In quality control, acceptance sampling is a common inspection procedure used to either accept or reject a shipped lot, but not to examine the quality of the lot. In acceptance sampling, a random sample is inspected from a lot and, based on the mutually agreed acceptance sampling plan between producer and consumer, the decision is made to either accept or reject the lot. Acceptance sampling was popularized by Dodge and Romig where it was initially practiced by the U.S. military in the testing of bullets in World War II (Schilling & Neubauer, 2008). Suppose that each and every bullet is inspected prior to war, no bullet is at hand for time of action, and that if no bullet is inspected then mishaps may occur.

Acceptance sampling is very useful when the testing is destructive and the cost of inspection is very high, such that it is not feasible to examine the lifetime of each product (100% testing). Therefore, a sample is randomly chosen from the lot for hypothetical acceptance or rejection. The products under examination are destructive (such as electronic products) and it is in the manufacturer's interest to observe the average or mean lifetime of these destructive products. It is not practical to inspect all of the products in a lot and record the defective products or number of failures. The only solution is to randomly select a particular number of products and put them to the test. Based on this information, the producer then either accepts or rejects the whole lot.

According to Mughal and Aslam (2011), acceptance sampling plan is an inspection procedure which consists of lot size, sample size and acceptance or non-acceptance criteria. The minimum sample size, cost of the experiment and probability of lot acceptance are very important elements from an experimenter's point of view for the selection of a suitable acceptance sampling plan. As with other statistical methods, acceptance sampling plans are dependent on the type of data being measured, namely the attribute and variable. Thus, the two major categories of acceptance sampling are:

i. lot-by-lot acceptance sampling of attributes, in which each product in a sample is inspected on a go-no-go basis.

ii. lot-by-lot acceptance sampling of variables, in which each product in a sample is measured on continuous scale such as weight, strength and thickness.

These types of acceptance sampling shall be further described in the following sections, but the emphasis will be on the former as it is pertinent to this study.

1.1.1 Attribute Acceptance Sampling

An attribute acceptance sampling is usually applied to assure the quality level of products submitted by the vendor in order to satisfy pre-specified design parameters such as the acceptance number, testing time, producer's risk and consumer's risk. Each product in a sample is observed on a go-no-go basis for one or more characteristics. The attribute acceptance sampling plan has three design parameters: lot size, N, sample size, n and the acceptance number, c. This plan is carried out as: select a sample of size n from the submitted lot of size N using acceptance number, c.

If the number of defective products is less than c then the lot will be accepted. For example, a manufacturer has a shipment of 1,000 products and wants to inspect the lifetime of 100 products with $c=2$. If there are 0, 1, or 2 defective products, the lot is accepted. However, if more than 2 defectives are found, the entire lot will be rejected. The attribute acceptance sampling plans will be further elaborated in Section 1.3.

1.1.2 Variable Acceptance Sampling

Variable acceptance sampling is considered for use in observing measurable quality characteristics such as weight, strength or thickness. If the variable is, say, a fraction of impurities in raw material where a small number is enviable, the plan is carried out as: select a sample of size n and accept the lot if the average measurement, $\bar{x}$, is less than a specified factor. The decision is based on these criteria: $\bar{x} - k\sigma \geq$LSL or $\bar{x} - k\sigma \leq$USL, where LSL and USL denote the lower and upper regulatory limits. The probability distribution of the variable must be identified, and if it is not based on the normal probability distribution then the conclusion made on this basis would be invalid. The drawback of variable acceptance sampling is that various plans must be developed for every quality characteristic that is under inspection. This may lead to the rejection of a submitted lot even though the recorded sample information is free from defective products.

1.2 Operating Characteristic (OC) Curve

In acceptance sampling, a vital measure of the performance of an acceptance sampling plan is the operating characteristic (OC) curve. This curve draws the probability of accepting the lot, $L(p)$, versus the lot proportion defective, p. Associated with each sampling plan is an OC curve which represents the performance of the acceptance sampling plan against good and poor quality standards. An example of OC curve is shown in Figure 1.1.

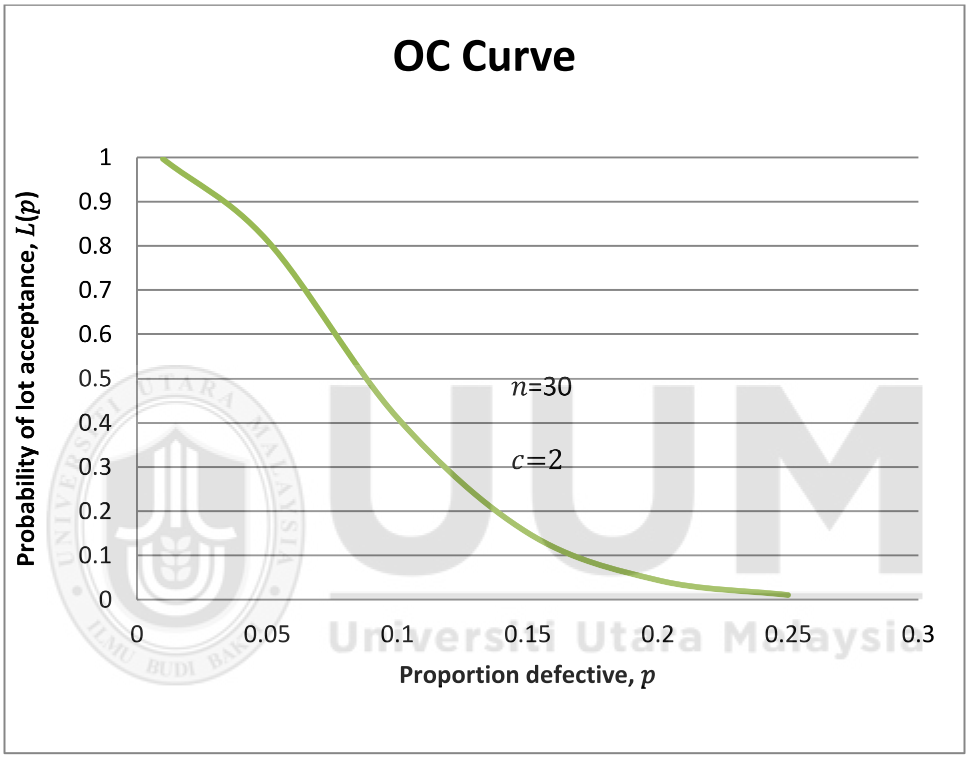

Figure 1.1. OC curve for n=30, c=2

The above figure was developed for the various values of proportion defective and discriminates between good and bad lots for fixed values of acceptance numbers c = 2 and n = 30. If the sample size is 30 and the lot proportion defective is 10%, then the probability of lot acceptance is 0.40. It means that if there are 100 lots each comprising 10% of defective products from the manufacturing process then approximately 40 lots will be accepted. The shape of the OC curve for various values of design parameters plays a very important role in the selection of the most suitable acceptance sampling plan. It also shows the relationship between the required sample size and acceptance number which is either in increasing or decreasing function to each other. The required OC curve based on the acceptance sampling plan can be chosen easily when it passes through the desired or pre-specified design parameters. For instance, one can compare OC curves to choose the appropriate acceptance sampling plan and develop curves for various sample sizes and acceptance numbers. In this scenario where a small sample size is preferred, an acceptance sampling plan with zero acceptance number is desirable. Acceptance sampling plans with acceptance number zero and a smaller sample size is mostly used in situations when the lot inspection is very costly or destructive. The relationship between the proportion defective and probability of lot acceptance are shown in Figure 1.2 for various values of acceptance number.

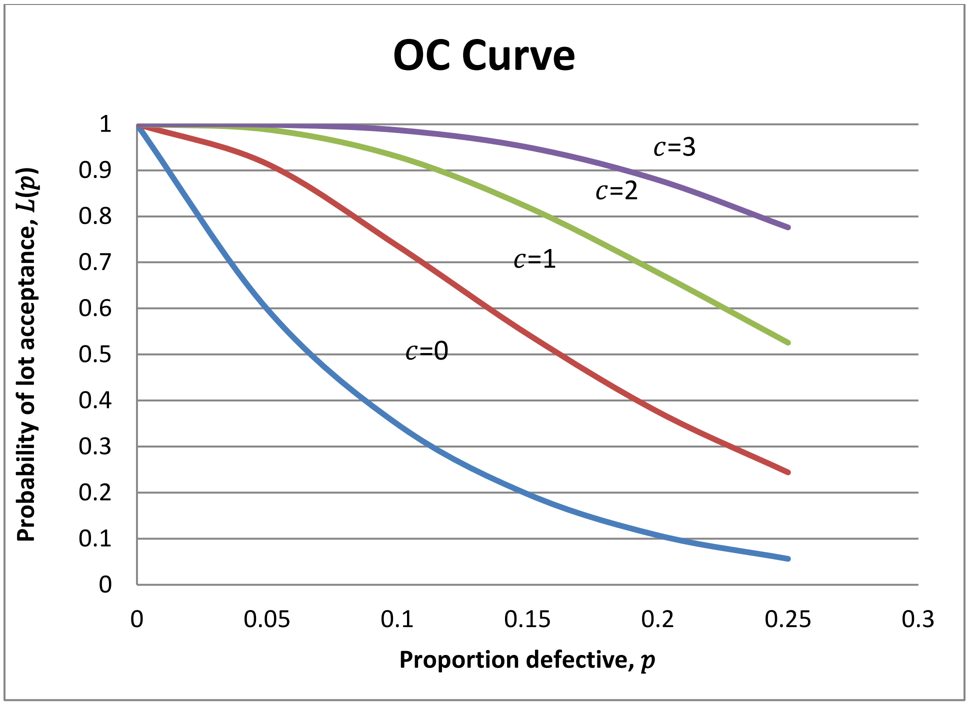

Figure 1.2. OC curve for various values of acceptance number

In Figure 1.2, when $c = 0$, the OC curve is convex throughout and begins to drop very rapidly for small value of proportion defectives. If the proportion defective is 5% and $c = 0$, then the probability of lot acceptance is equal to 0.60, that is, almost 40% of the lots will be rejected (returned to the producer). In this case, the OC curve has no point of inflection, which is often unfair to the producers and may be particularly uneconomical for the consumers.

In an acceptance sampling plan, the required OC curve can usually be obtained by considering the two points: acceptable quality level (AQL) and lot tolerance percent defective (LTPD). AQL represents the consumer's approach to accept the submitted product as having a very small value of proportion defective. Basically, it is the poorest quality level which would be assumed acceptable in the whole process and the probability of rejecting a lot based on such acceptable quality level is called producer's risk, denoted by α. Meanwhile, the worst quality level that could be deemed acceptable for an individual lot refers to LTPD, and the probability of accepting such lot is known as consumer's risk, denoted by β. Refusing the good quality products may cause shortage of supplies which disrupts the consumer's manufacturing process and potentially lead to a poor relationship with the producer. According to Aslam *et al.* (2010a), Mughal and Aslam (2011) and Mughal and Ismail (2013), consumer's risk is generally considered when the main target of the acceptance sampling plans is to obtain the minimum sample size. Meanwhile, producer's risk is assumed in finding the minimum testing time at various quality levels.

1.3 Attribute Acceptance Sampling Plans

As mentioned earlier in Section 1.1.2, the main disadvantage of variable acceptance sampling plan is that the distribution of the under-examined quality characteristic must be known, whereas in attribute acceptance sampling a manufacturer can easily use it to examine the lifetime of products without identifying the lifetime distribution. In the following sections, the major types of attribute acceptance sampling plans are briefly discussed based on the mean lifetime of the product.

1.3.1 Single, Double and Sequential Acceptance Sampling Plan

In single attribute acceptance sampling, only one sample is taken from the submitted lot and this sample information is utilized to either accept or reject the lot. The null and alternative hypotheses are formulated to examine the quality level of a product, $\mu \geq \mu_0$ and $\mu < \mu_0$, where μ and μ_0 are the true and specified average lifetimes, respectively. In this sampling plan, a randomly selected sample is put to test which continues for a pre-assumed testing time, a. The submitted lot is rejected if more failures are observed than the pre-specified acceptance number. If the first sample results cannot lead to a valid decision, then a double acceptance sampling plan is applied to allow another chance in accepting or rejecting the submitted lot. Consider a producer who wants to observe the lifetime of submitted products for a pre-assumed testing time and takes a sample of 50 and 100 products from 1,000 products with c_1=1 and c_2=4, respectively. The lot is accepted if at most one defective product is recorded during the testing time based on the first sample information (out of 50). The lot is rejected if more than one defective product is observed. A second sample is taken if the observed defective products are less than four but greater than one; then accepts the lots if a total of four or fewer defective products are found out of 100; otherwise, reject the lot. The major advantages of double acceptance sampling plan over a single acceptance sampling plan is that it may decrease the overall amount of essential examination and also gives a second chance to a lot for acceptance.

The above mentioned acceptance sampling plan has considered two samples taken from the submitted lot. If more than two samples are taken, it is called multiple sampling plan. In this plan, after completing every stage, the lot is accepted if the number of defective products is less than or equal to the pre-specified acceptance number. This procedure is continued until the last pre-considered sample is chosen and a decision about the lot is made. The main advantage of this plan is that the required sample at each stage is generally smaller than the single and double acceptance sampling plans. In a sequential sampling approach, a sequence of sample size is selected. It is an extension of double and multiple sampling techniques because no upper limit of the number of samples are required. If one product is selected in a sequence then it is named as an item-by-item sequential plan. If more than one product is chosen as a sequence, it is known as group sequential sampling plan. In this plan, the number of defective products and total number of inspected sample size are plotted in a *y*-axis and *x*-axis, respectively. The lot is rejected if the point falls on or above the rejection line, and if the point falls between the acceptance and rejection boundaries then another sample should be taken. In practice, sequential sampling can theoretically continue open-endedly until the lot is 100% examined.

1.3.2 Chain Acceptance Sampling Plan

The use of chain sampling plans is usually suggested when an extremely high quality product is needed. Dodge (1955) introduced a chain sampling plan known as ChSP-1 which makes use of cumulative recorded results of various samples. To overcome the deficiency of a single-acceptance sampling plan when c = 0, the details discussed previously in Section 1.2 with the help of Figure 1.2 is considered. Over the past several decades, the chain sampling plans have been enhanced by many authors. Govindaraju and Lai (1998) developed a modified chain sampling plan (MChSP-1) which provides a more accurate probability of lot acceptance than Dodge (1955) and does not overestimate the probability of lot acceptance for a fixed value of proportion defective. Deva and Rebecca (2012) introduced a two-sided chain sampling plan to give more protection to the producer as well as the consumer based on preceding and succeeding lot information. It converts to the plan developed by Govindaraju and Lai (1998) and also gives the same probability of lot acceptance when the numbers of preceding and succeeding lots are equal.

1.3.3 Group Acceptance Sampling Plan

In common acceptance sampling plans, only a single product is inspected at one time, but in practice it is possible to inspect more than one product at the same time given the availability of testers. In this situation, the submitted products put in a tester are considered as group (multiple testers each accommodating g products) and such plan based on this type of inspection is known as group acceptance sampling plan. According to Mughal and Aslam (2011), this plan is carried out in the following way: a sample $(n=r*g)$ is selected from the lot size, N where the required sample size, n is a multiple of number of testers, r and group size, g. The submitted product is acceptable or sent for consumer's use if the number of defective products, d is less than or equal to the acceptance number. For example, if an experimenter needs to inspect 50 products and he has the facility to examine 5 products at a time, then 5 products are allocated into 10 groups for completing the investigation.

1.4 Determination of Sample Size

The most common question of the experimenter is, "How large is the sample size that I need?" The desired goal of the research can be achieved based on this sample information and the good sample size can also clarify the margin of error. With the help of probability distributions shown in Figure 1.3, the minimum sample size and probability of lot acceptance can be found for the required pre-specified design parameters.

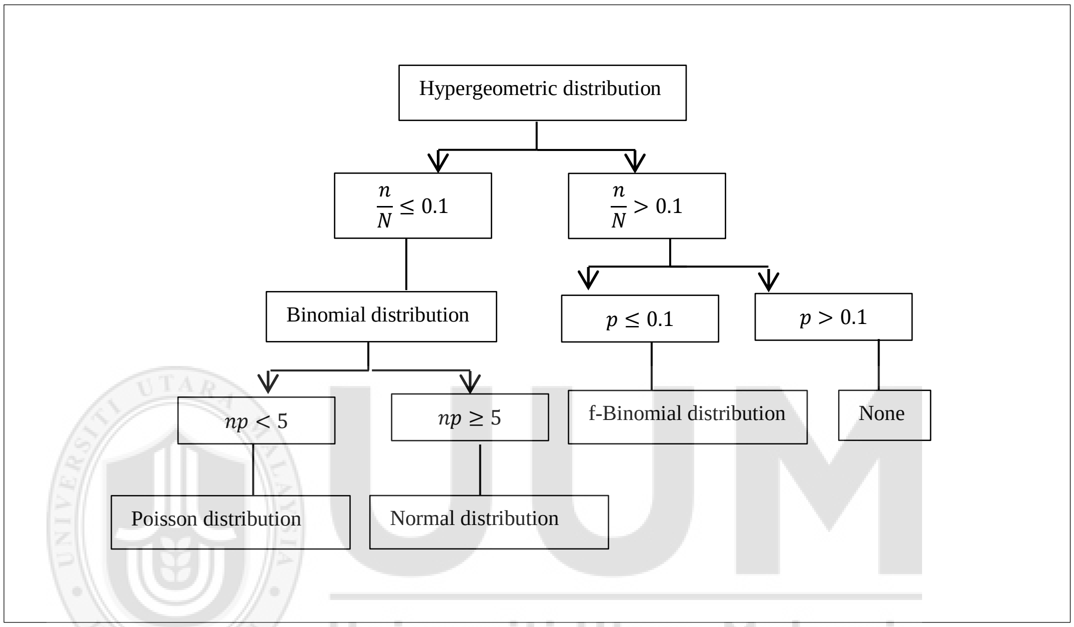

Figure 1.3. Useful approximating distributions in acceptance sampling (Schilling & Neubauer, 2008)

If the decision of a submitted product is classified into two categories, which is accept or reject, then these distributions shown in Figure 1.3 are functional and approximate one to another (Schilling & Neubauer, 2008). The Hypergeometric distribution is fundamental in acceptance sampling plans and applicable when a sample is selected without replacement from a finite lot size. The complement of the Hypergeometric distribution is the Binomial distribution, which is undoubtedly the most applicable distribution in acceptance sampling plans. It is used when a sampling procedure follows an infinite lot size which assumes sampling with a replacement. In situations where the proportion defective approaches zero for a very large sample size, the Poisson distribution is used and it is known as the Poisson approximation to the Binomial distribution. Another approximation to the Binomial distribution is f-binomial distribution, which is applied when the sampling procedure is based on a finite lot size, pre-specified number of defectives and assumes without a replacement process. In this research, the Binomial distribution is considered to find out the required design parameters because the submitted product is classified into two categories and follows an independent selection process.

1.5 Failure Time Distributions

Failure time distributions, or lifetime distributions, are based on mathematical models that illustrate the probability of defectives occurring over time. This function is integrated to find the probability that the failure time takes a value in a known time interval. The failure time rate of electronic components is not systematic and the inspection is terminated when either more defectiveness occurs than the pre-specified acceptance number or the required inspection time is over. Such inspection following this method is called lifetime testing, or, truncated life test. The cumulative distribution function (CDF) can be used to find the value of proportion defective. The selected lifetime distribution and its characteristics (mean, median or specific percentile lifetime) must satisfy the requirements of acceptance sampling plans. In failure time data theory, a well-known probability plot (PP), quantile plot (QQ) and Kolmogorov- Smirnov (K-S) goodness of fit test can be used to investigate the pattern of data that displays the specific behavior with regard to which lifetime distribution is most suitable.

There are many lifetime distributions which have been used in acceptance sampling plans. Baklizi (2003), Mughal *et al.* (2010b), Mughal *et al*. (2011a), Mughal (2011) and Mughal and Aslam (2011) have used the Pareto distribution of the 2nd kind, the Weibull distribution, the Burr type XII distribution, the Exponential distribution and the family of Pareto distributions, respectively. The Pareto distribution of the 2nd kind is discussed here because Aslam *et al.* (2010a) had used this distribution and proved that it provides better results than the established plan developed by Aslam and Jun (2009a) which was based on the Weibull distribution in terms of the required minimum sample size. The PDF and CDF of the Pareto distribution of the 2nd kind are

$$f(t:\sigma,\lambda)=\frac{\lambda}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\lambda+1)} \quad t>0,\sigma>0,\lambda>0; \qquad 1.1$$

$$F(t:\sigma,\lambda)=1-\left(1+\frac{t}{\sigma}\right)^{-\lambda} \quad t>0,\sigma>0,\lambda>0\ , \qquad 1.2$$

where σ and λ are the scale and shape parameters, respectively. The mean of this distribution is

$$\mu=\frac{\sigma}{\lambda-1} \quad , \quad \lambda>1. \qquad 1.3.$$

For the existence of mean, the value of the shape parameter must be greater than 1.

1.6 Problem Statement

Acceptance sampling is a very useful method in monitoring the average life of electronic components, specifically on the average life of the submitted lot, μ, test experimental time, t and the number of defective products, d. For situations in which testing is destructive, sampling plans with small sample sizes are selected. These plans often have acceptance number zero; as a result, the probability of lot acceptance begins to drop very sharply as the lot proportion defective is higher than zero and it is a very intolerable situation from both the producer's and consumer's point of view. In this scenario, appropriate sampling plans are required and the chain sampling plan is the best option as its discriminatory power of OC curves is based on past lot information.

As discussed in Section 1.2, Dodge (1955) developed a chain sampling plan (ChSP-1) which makes use of cumulative recorded results of various samples to overcome the deficiency of a single acceptance sampling plan when c = 0. Moreover, Govindaraju and Lai (1998) as well as Deva and Rebecca (2012) developed various chain sampling plans which were then extended to lifetime distributions by Ramaswamy and Jayasri (2014). In these established chain sampling plans, researchers did not consider the group sampling procedure which would have been very useful and economical. Most of these acceptance sampling plans increase or decrease the probability of lot acceptance at several quality levels but also underestimate and overestimate the probability of lot acceptance at the same value of proportion defective. However, the existing chain sampling plans still need improvement, especially with regards to group acceptance sampling, modified group acceptance sampling, two-sided group acceptance sampling and generalized group acceptance sampling plan.

1.7 Objectives of the Study

In chain acceptance sampling plan, the minimum sample size and probability of lot acceptance are generally obtained for the pre-assumed testing time, consumer's risk and desired acceptance number. The main objectives of this research are to develop

- New group and modified group chain acceptance sampling plans for attributes using the Pareto distribution of the 2^{nd} kind,
- New two-sided group and modified group chain acceptance sampling plans for the Pareto distribution of the 2^{nd} kind, and
- Generalization of the above four plans based on several pre-specified values of proportion defective.

In the first stage, new group and modified group chain acceptance sampling plans for attributes are developed using the Pareto distribution of the 2^{nd} kind based on past lot information. Secondly, two-sided chain factors is integrated to the group chain sampling for attributes based on preceding and succeeding lots. The advantage of this factor is to provide more accurate information regarding the probability of acceptance at different quality levels. New two-sided group and modified group chain acceptance sampling plans for the Pareto distribution of the 2^{nd} kind are constructed and additional information is used from the preceding as well as succeeding lot quality. The minimum group size and probability of lot acceptance are obtained for a pre-specified test termination time, an allowable acceptance number and consumer's risk for various quality levels. In the third method, all four acceptance sampling plans developed in the earlier two methods are generalized based on several pre-specified values of proportion defective. Under these values of proportion defective, p, the design parameters of the proposed plans are found and some comparisons are made with the established plan developed by Mughal and Aslam (2011).

1.8 Significance of the Study

In this study, the proposed several group chain sampling plans for attribute based on a truncated life test are useful to save cost, time and energy. These plans are able to provide a more accurate probability of lot acceptance with a minimum sample size based on several values of mean ratios and proportion defective. This study offers new methods in acceptance sampling which extends the boundary of knowledge in this field as well as benefit both researchers and practitioners.

1.9 Thesis Outline

In the next chapter, a thorough review on relevant literature is provided. This includes the development of chain and group sampling methods based on the effects of the proportion defective, acceptance number and lifetime distributions on acceptance sampling plans. In Chapter 3, the new plans known as group chain sampling plans with modified and two-sided chain sampling are developed. Procedures and mathematical equations are constructed based on algorithms to investigate the performance of the proposed plan. Chapter 4 focuses on a family of group chain acceptance sampling plans when the submitted product follows the

Pareto distribution of the 2nd kind. The family of generalized chain sampling plans, its application and comparative study of the proposed plans with established plan are then discussed in Chapter 5. In Chapter 6, the conclusion of the results and future research work are presented.

CHAPTER TWO

LITERATURE REVIEW

This chapter provides further descriptions on acceptance sampling plans for attributes that were developed by many researchers based on various lifetime distributions. The methods for evaluating the required design parameters such as minimum sample size and acceptance number of the established acceptance sampling plans are discussed in the following four sections. The first section gives a chronological review of the acceptance sampling plans for attributes based on various lifetime distributions for obtaining the minimum sample size and probability of lot acceptance. The second section presents the economic reliability of acceptance sampling plans in finding the minimum termination time under the restriction of pre-specified design parameters. Next, several chain sampling plans for attributes based on past lot information are discussed with consideration to the different values of proportion defective and lifetime distributions. Lastly, the more recent group acceptance sampling plans for attributes are deliberated for evaluating the minimum number of groups and probability of lot acceptance when the average lifetime of a product follows the lifetime distribution.

2.1 History of Acceptance Sampling Plans

Acceptance sampling has an extensive past, having originated from the Engineering Department of Western Electric’s Bell Telephone Laboratories. In 1924, this foundation greatly contributed to the development of acceptance sampling and some of the members were the fathers of acceptance sampling. The well-known statisticians, H.F. Dodge, W.A. Shewhart, Juran and H.G. Romig were members of this department. During 1925 to 1926, this department presented the concept of single, double and multiple acceptance sampling plans, consumer's risk, producer's risk, probability of acceptance and operating characteristic curves. Dodge and Romig (1941) produced Dodge-Romig tables for single and double sampling Inspection based on consumer's risk and rectification in 1941. Wald (1943) also introduced sequential sampling which is a generalization of multiple acceptance sampling plans.

The single acceptance sampling plan, based on exponential distribution as a lifetime distribution of a submitted lot, was first introduced by Epstein (1954). Two approaches were discussed to find the design parameters. The first approach deals with replacement and the second handles non-replacement situations. In a replacement case, a failed product can be replaced by a new one if it fails during the experimental time. If a failed product cannot be replaced by a new one, it is called a non-replacement case. Formulae were presented for an expected number of observations, testing time, and probability of acceptance based on the mean lifetime.

Later in 1960, Goode and Kao (1960) suggested an extended sampling plan and a reliability sampling plan. The Weibull distribution was used as a lifetime distribution to examine the mean lifetime of a submitted product. These plans were an extension of the established single acceptance sampling plan developed by Epstein (1954) based on exponential distribution as a special case of Weibull. Various tables were provided for an attribute sampling plan based on the Weibull distribution of any desired form of operating characteristics. The methodologies introduced in this study are used to find out the mean lifetime of a submitted product which has presented the lot quality history.

Similarly, Gupta (1962) recommended an acceptance sampling plan based on a truncated life test for pre-specified design parameters. The normal and lognormal distributions were considered in order to find a suitable sample size for the required mean or median life of a product. A wide range of operating characteristic values were obtained for practical implementation to ensure the most appropriate plan for specified circumstances. Various values of producer's risk were assumed and several tables were also presented for the comparison of minimum mean ratios to examine the average lifetime of a product.

During 1962 to 2000, several researchers proposed various acceptance sampling plans using different techniques. Based on the above mentioned plan introduced by Epstein (1954), Kantam *et al.* (2001) developed an acceptance sampling plan when the lifetime of a product follows log-logistic distribution. Various acceptance numbers and test termination were considered and analysis was also presented with the help of different tables. It had been proven through their research that an acceptance sampling plan based on the log-logistic distribution required a lesser amount of sample size compared to the established plans developed by Kantam and Rosaiah (1998).

Later, Baklizi (2003) suggested an acceptance sampling plan based on the Pareto distribution of the 2nd kind as a lifetime of a product. The minimum sample size, probability of lot acceptance and mean ratios were discovered to satisfying the consumer's risk. It was proven that the proposed plan required a smaller sample size than the established plan developed by Kantam *et al.* (2001). By using the same concept, an acceptance sampling plan based on generalized Rayleigh distribution was then developed by Tsai and Wu (2006). The cumulative distribution function suggested by Voda (1976) was used to find the design parameters. Tables were presented for practitioners, but his plan required a greater sample size than Baklizi (2003) and Kantam *et al.* (2001).

As discussed earlier, Baklizi (2003) developed a plan based on the Pareto distribution of the 2nd kind, but Balakrishnan *et al*. (2007) pointed out that Baklizi (2003) had used the scale parameter of the Pareto distribution of the 2nd kind as a mean lifetime and found the design parameters without putting the actual mean value. A generalized Brinbaum-Saunders distribution was proposed and an acceptance sampling plan based on this distribution was developed. Several tables of design parameters were shown for different values of mean ratios. The real application of this distribution was also discussed with the help of probability plot (PP) when the lifetime of a product is based on median lifetime instead of mean.

Meanwhile, Aslam *et al*. (2010a) introduced an acceptance sampling plan when the life of a submitted product is based on a generalized exponential distribution.

Different tables of minimum sample sizes and other design parameters were shown in their research. These tables are helpful for manufacturers when considering a certain median life of a submitted product, its shape parameters and other design parameters. Lio *et al*. (2010) also proposed an acceptance sampling plan for the Burr XII distribution to ensure the lifetime of a product based on a pre-specified percentile lifetime instead of a mean or median. Tables of a minimum sample size and operating characteristic values were constructed which is very helpful for experimenters when dealing with skewed data theory.

The above mentioned acceptance sampling plans are used to determine the required design parameters when the lifetime of a submitted product is based on a specific lifetime distribution. In these plans, different techniques and lifetime distributions are considered for various kinds of data. These procedures are applicable if they fulfil the requirements of pre-specified criteria such as producer's risk, consumer's risk, acceptance number and required testing.

2.2 Economic Reliability Acceptance Sampling Plan for Attributes

Economic reliability acceptance sampling plans deal with another important characteristic of an acceptance sampling plan which is the minimum test termination time based on lifetime distributions. These lifetime distributions can be used to find the best economic reliability acceptance sampling plan which is more economical for researchers in saving testing time. In general, the minimum termination time of the experiment is considered in these plans. An experiment is terminated if either the termination time, t ends or the number of defectives is more than the pre-specified acceptance number, c.

The economic reliability plan suggested by Kantam *et al*. (2006) considers the lifetime of a product which follows a log-logistic distribution. The minimum termination time is found by considering the various pre-specified design parameters such as producer's risk and acceptance number. A comparative study had proven that the proposed plan required a minimum termination time unlike in the established plan developed by Kantam *et al.* (2001). Also, Aslam and Shahbaz (2007) adopted the same plan for the mean lifetime of a submitted product based on a generalized exponential distribution. For the known values of a shape parameter, they proved that their plan was more economical than Kantam *et al.* (2006)'s in terms of the time, cost and labor needed to reach the final decision about the submitted products. In addition, Aslam (2008) developed an economic reliability plan considering a generalized Rayleigh distribution which was more economical in terms of saving the cost of the experiment compared to the established plan developed by Tsai and Wu (2006).

Instead of considering the population mean, Mughal *et al*. (2011) suggested an economic reliability test plan for the Burr type XII distribution where the lifetime of a product is based on a pre-specified percentile lifetime. The minimum termination time is found to ensure that the pre-specified percentile lifetime satisfies producer's risk. The operating characteristics values are discussed for various specified parameters. They proved that the proposed plan is more economical in the sense of the required minimum termination time when compared to that of Lio *et al*. (2010).

2.3 Chain Acceptance Sampling Plans for Attributes

After being introduced in Section 1.3, chain acceptance sampling will be further elaborated in this section. It is to be noted that an independent process and error-free inspection are the basic assumptions of chain sampling plans. It means that all the under-examined products are not correlated with each other and the inspection method is perfect. In the chain sampling technique, the criteria for accepting and rejecting a submitted lot depends on the information of the inspection of immediately preceding samples, in which the submitted product comes from the same manufacturing process and follows an identical independent distribution.

Under certain circumstances when $c = 0$, the chain sampling plan works considerably better than single acceptance sampling plan for very small values of proportion defectives. Its distinguishing feature is that the current lot under assessment can also be accepted if one defective product is found in the sample and the preceding samples are free from defectives. It provides a further chance of a submitted lot on the basis of only one defective product and recovers the poor judgment between good and bad lots. On the basis of sample information taken from the lot, a lot is considered good if it fulfills the pre-specified designed parameters.

As discussed in Chapter 1, Dodge (1955) had introduced a method known as modified chain sampling plan (MChSP-1) as an alternative to the single acceptance sampling plan. The procedure of this plan based on the cumulative information of preceding lots is shown in Figure 2.1.

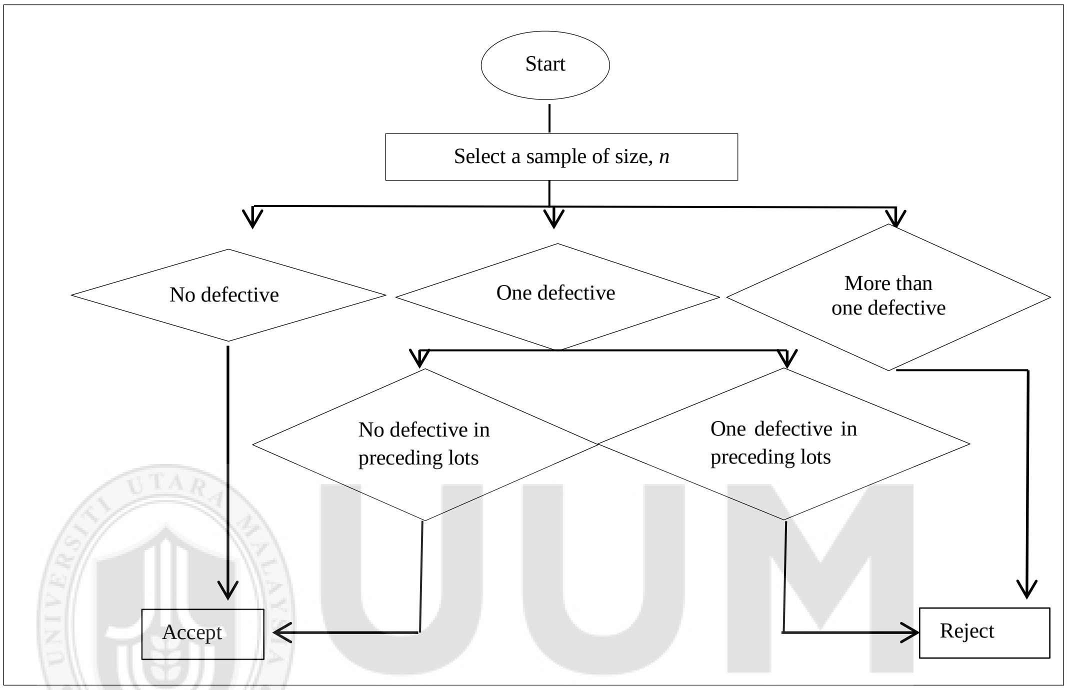

Figure 2.1. Dodge Chain Sampling Plan

In the chain sampling technique, the criteria for accepting and rejecting the submitted lot depends on the information of the inspection of immediately preceding samples. After the rejection of the submitted lot, a new cumulation criterion was introduced by Dodge and Stephens (1964). In this method, a general family of chain sampling plans was proposed based on two-stage chain sampling. This procedure continues until the maximum number of samples (k samples) and the size of samples based on the observations 3, 5, 8, 10, 15, and 20. The k, maximum cumulation of number of samples, varies from 2 to 10. Schilling and Dodge (1969) have introduced several procedures and tables for different acceptance sampling plans with consideration for a normal distribution with a known standard deviation. They also developed a generalized dependent plan using several values of proportion defectives. In this plan, the considered value of acceptance number was equal to two instead of the existing plan which uses $c = 1$. Tables of joint probabilities were shown for n = 4, 5, 8, 10, c = 0, 1, 2 for different proportion defective values. These tables were very helpful for experimenters as they show the effect of various combinations of design parameters.

Soundararajan (1978a) and Soundararajan (1978b) have evaluated procedures and tables for the construction and selection of chain sampling plans. Formerly (1978a), he developed a technique for obtaining the desired operating characteristic values by considering the average outgoing quality limit (AOQL). The AOQL represents the maximum defective for the average outgoing quality in a rectifying inspection, regardless of the incoming quality level. The AOQL of a rectifying inspection is a significant characteristic and is very helpful in constructing a rectifying inspection plan for a specified value of AOQL. Secondly (1978b), Soundararajan proposed two methods based on the required ratio of average quality level to lot tolerance percent defective: AQL/LTPD and also the ratio AQL/AOQL. Based on these required ratios, the design parameters of the proposed plan were discovered which was very helpful to the experimenter for the selection of a desired OC curve. In common chain sampling plans, only two classes of either good or bad products are considered. These plans categorize a submitted lot as accepted or rejected and mostly concern the required value of proportion defective. However, these acceptance sampling plans do not provide any information regarding the proportion of defective products, and fall in the boundary of the required quality limit. Shankar *et al*. (1991) proposed three classes of chain sampling plans to categorize the product as good, bad or marginal (near miss and there is one extremely bad product). For the practical use of this plan, they presented different tables for several combinations of design parameters with regard to the Poisson distribution. They proved that it was an extension of two-class attribute plans and it being very useful for the experimenters when the submitted product is able to be classified as good, marginal or bad.

Meanwhile, Raju (1991) introduced a generalized family of three-stage chain sampling plans, extending the concept of the original plan developed by Dodge (1955). Expressions were derived for the OC curves with cumulative acceptance numbers c_1, c_2, c_3= (0,1,2), (0,1,3), (0,2,3), (1,2,3), (0,1,4), (0,2,4), (0,3,4), (1,2,4), (1,3,4) and (2,3,4). The OC curves were obtained for a cumulative number of samples, k which was the extension of the plan developed by Schilling and Dodge (1969). It was proven that the proposed plan has better discriminating power than a single sampling plan with the same sample size. Much later, Raju and Narasimha (1996) developed a new chain sampling plan that provided the generalization and extension of Dodge (1955) and Dodge and Stephens (1964) idea. This plan was based on the information from one or more preceding samples as well as the current sample to make a decision about the submitted lot. The OC function was derived for a desired combination of design parameters using a two-stage chain plan based on

$(c_{1,}c_{2,})$. Comparisons were constructed with respect to a minimum sample size and discriminating power by considering single and double acceptance sampling plans. The effect of various acceptance numbers and discriminating power of OC function with the established plans were also presented for experimenters. Govindaraju and Lai (1998) then introduced a modified chain sampling plan (MChSP-1) based on a truncated life test as shown in Figure 2.2.

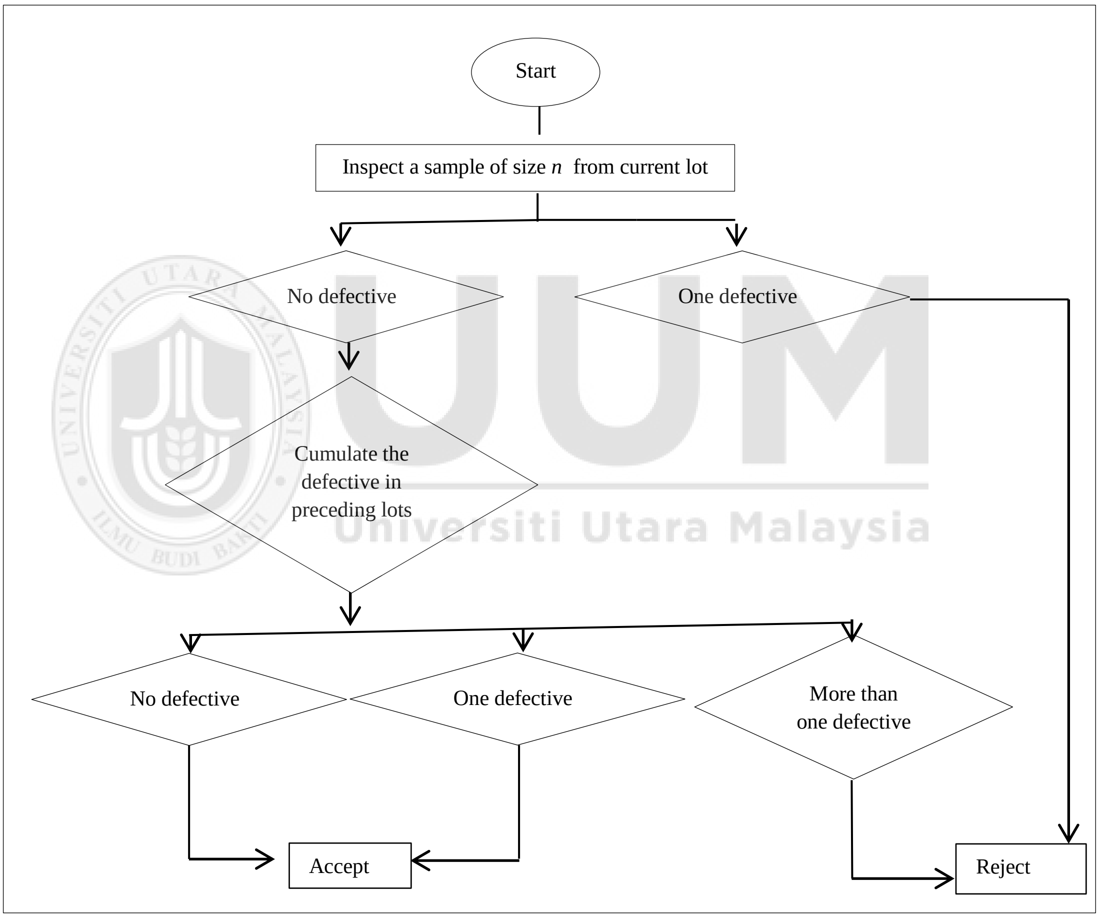

Figure 2.2. Govindaraju and Lai Modified Chain Sampling Plan

They used preceding lot information and derived the operating characteristic curves by considering several values of proportion defective as presented in Figure 2.3.

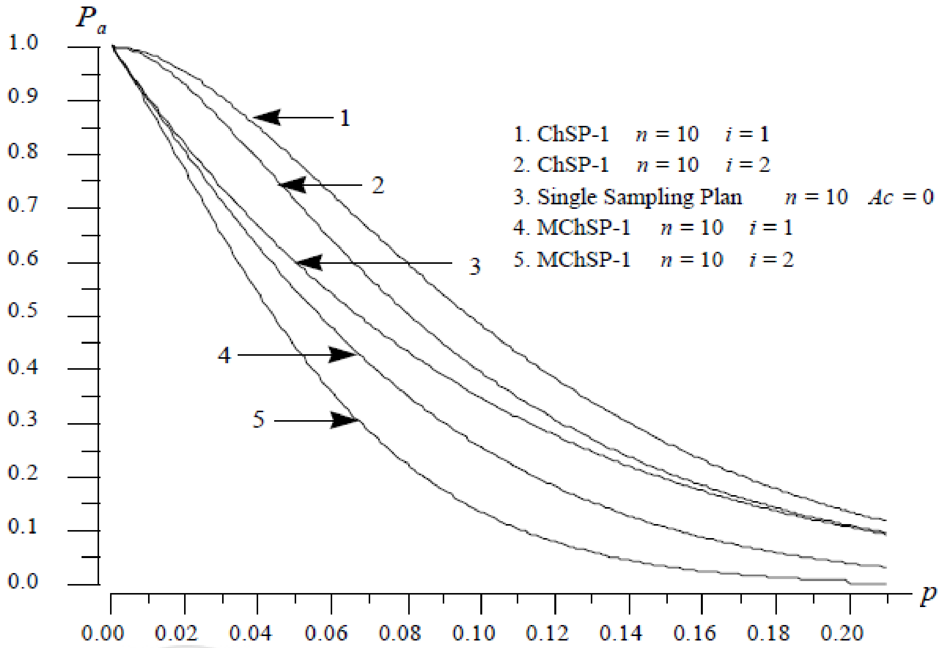

Figure 2.3. Comparison of ChSP-1 and MChSP-1(Source: Govindaraju and Lai, 1998)

In Figure 2.3, OC 1 and OC 2 represent the plan developed by Dodge (1955) while OC 4 and OC 5 denote the plan developed by Govindaraju and Lai (1998). Comparative studies show that the modified chain sampling plan is an improvement of established plans and gives more accurate probability of lot acceptance. If n = 10, i = 1 and p = 0.04, MChSP produces approximately 55% while ChSP gives 85% of probability of lot acceptance, respectively. Meanwhile, in the traditional chain sampling plan, only past lot information is considered, but Deva and Rebecca (2012) suggested a two-sided complete chain sampling plan based on preceding as well as succeeding lot information. The operating characteristic values of a product are derived for various quality levels. The proposed plan provided more safety to the consumer's and producer's risks. It is interesting to note that the proposed plan offers the same probability of lot acceptance of the established plan proposed by Govindaraju and Lai (1998). Several tables and figures were provided by considering various combinations of design parameters which are useful for experimenters.

Recently, Ramaswamy and Jayasri (2014) developed a chain sampling plan based on truncated lifetimes where the lifetime of a product follows a generalized Rayleigh distribution. A minimum sample size and the required acceptance number were obtained when satisfying different values of consumer's risk. The probability of lot acceptance was also found for different values of mean ratios. Later, Ramaswamy and Jayasri (2015) introduced a modified chain sampling plan considering several lifetime distributions. For pre-specified values of test termination time and consumer's risk, minimum sample sizes and operating characteristic values were obtained. Comparisons were made among all considered lifetime distributions based on sample size by considering different combinations of design parameters.

The above discussed plans were proposed by many researchers based on different methodologies by considering several combinations of design parameters. In these plans, the minimum sample size and probability of lot acceptance were found by satisfying producer's risk and consumer's risk. In these plans, only a single product can be inspected at a time, but in a practical situation it is possible to examine more than one product. This technique is briefly discussed in the next section.

2.4 Group Acceptance Sampling Plan for Attributes

As discussed earlier in Chapter 1 (Section 1.3), group acceptance sampling plans have been used to inspect more than one product at the same testing time. In this case, the total number of products that can be inspected is divided into groups according to the number of available testers. The standard method is to adopt a parametric model for the lifetime distribution and then derive the minimum sample size to ensure certain mean life of a submitted product. The Pareto distribution of the 2^{nd} kind, also known as the Lomax distribution, is considered in this research. Aslam *et al*. (2010b) used this distribution and proved that it provides better results than the established plan developed by Aslam and Jun (2009a) based on the Weibull distribution. The minimal group size, operating characteristic values and the optimal ratio of the true mean life to the specified mean life were determined. It was proven to save the cost and time of experimentation and performs well than established plan in terms of the required minimum sample size. It is a heavy-tail probability distribution which is also very useful in business, economics, actuarial science, queuing theory and Internet traffic modeling. Meanwhile, Mughal *et al.* (2010a) used a different method to evaluate the design parameters of the economic reliability group acceptance sampling plan. They considered a truncated life test when the average lifetime of a submitted product is based on a Marshall-Olkin extended Lomax distribution. For a given sample size, acceptance number and producer's risk, the minimum termination time was obtained. It was reported that the proposed plan required a smaller minimum test termination time than the established plan developed by Rao (2009a) when the lifetime of a product follows Marshall-Olkin extended Lomax distribution. Moreover, Mughal *et al.* (2010b) introduced an economic reliability group acceptance sampling plan for the Weibull distribution by considering producer's risk as well as consumer's risk. They claimed that the proposed plan required a minimum termination time than the established plan developed by Aslam & Jun (2009b).

For inspecting the mean lifetime of a submitted product, Mughal (2011) recommended a hybrid group acceptance sampling plan based on an exponential distribution. The minimum sample size and acceptance number were determined by satisfying the consumer's risk. The effect of test termination time on group size and other design parameters was discussed. The proposed plan required a smaller minimum sample size than the established plan developed by Rao (2009b) when the lifetime of a product follows generalized exponential distribution. Furthermore, Aslam *et al.* (2011) used the Poisson and weighted Poisson distributions to examine the lifetime of a product based on the Pareto distribution of the 2^{nd} kind. Comparisons were made among the Poisson and weighted Poisson distributions using different design parameters. Tables were also provided for the selection of a more appropriate OC curve.

Meanwhile, Mughal and Aslam (2011) introduced an efficient group acceptance sampling plan for a family of Pareto distributions and a total number of defective products assumed as groups. The advantage of their proposed plan over the existing plan developed by Aslam *et al.* (2010b) is that it provides lenient inspection for both producer's and consumer's point of view. In their plan, the number of defective products was recorded based on all groups instead of an individual group. Meanwhile, Mughal and Ismail (2013) constructed an economic reliability acceptance sampling plan for a family of Pareto distributions using an efficient group sampling technique (Mughal & Aslam, 2011). The minimum termination time required for a given group and acceptance number was obtained by satisfying the design parameters. The proposed plan required a minimum termination time unlike the existing plan developed by Mughal and Aslam (2011). Mughal *et al.* (2015a) developed an economic reliability group acceptance sampling plan for the Pareto distribution of the 2nd kind using group acceptance sampling. The Poisson and weighted Poisson distributions were used to find the required design parameters for biased data theory. A comparative study of the proposed plan was discussed with the established plan developed by Aslam *et al.* (2011) and proved that the proposed plan required a minimum testing time, which was unlike the established plan.

More recently, based on the above mentioned chain sampling plan developed by Ramaswamy and Jayasri (2014), Mughal *et al.* (2015b) proposed a group chain sampling plan when the lifetime of a product follows the Pareto distribution of the 2nd kind. A minimum sample size and probability of lot acceptance were obtained when satisfying pre-assumed design parameters at various quality levels. It was proven that the proposed plan required a minimum sample size than that of the established plan developed by Ramaswamy and Jayasri (2015). Moreover, Zain *et al.* (2015) developed a generalized group chain sampling plan and extended the established plan introduced by Mughal *et al.* (2015b). The minimum sample size and probability of lot acceptance were found by considering several values of proportion defective when satisfying the pre-specified consumer's risk.

To conclude the overall discussion in this chapter, group acceptance sampling and several methodologies of the chain sampling plan are briefly discussed in the above sections, which are the core topics of our research. The above mentioned plans developed by Dodge (1955), Govindaraju and Lai (1998), Deva and Rebecca (2012) and Mughal *et al.* (2015b) are used to explore the family of group chain sampling plans. The procedures of family of group chain sampling plans are constructed in the next chapter which provides the more appropriate combination of design parameters for experimenters.

CHAPTER THREE

METHDOLOGY

This chapter describes four phases to develop and evaluate the performance of the newly proposed family of group chain sampling plans for attributes. As mentioned in Chapter 1 (Section 1.7), the first phase identifies several combinations of design parameters. In the second phase, the procedures of acceptance sampling plans using the new (i) group chain, (ii) modified group chain, (iii) two-sided group chain and (iv) two-sided modified group chain are developed. The third phase describes the construction of OC functions which follow the Pareto distribution of the 2^{nd} kind. The results are then generalized based on several pre-specified values of proportion defective obtained from the output of Phase II. Finally, in Phase IV, the performances of the proposed plans are measured using real lifetime data.

3.1 Phase I: Identifying Design Parameters

In group acceptance sampling, a lot of size N is considered and a sample of size $n = r * g$ is selected where g and r represents the number of groups and number of testers, respectively. In this testing, the lot is accepted if no more than c defectives are observed; otherwise, the lot is rejected. As discussed earlier in Chapter 1 (Section 1.4), it is desirable to achieve the maximum probability of lot acceptance at the minimum sample size. Hence, this study introduces a family of group chain sampling plans for attributes considering various design parameters: consumers risk, β; pre-specified testing time, a; number of tester, r; allowable preceding lots, i and succeeding lots, j. These design parameters are studied in order to assure that the average life (μ) of a product is higher than the specified life (μ_0). A product is assumed to be good and accepted if $\mu \geq \mu_0$, at the pre-specified design parameters with the minimum values of sample size ($n = r * g$) and more accurate probability of lot acceptance $L(p)$. This can be obtained when satisfying the several combinations of design parameters as presented in Table 3.1.

Table 3.1

Pre-specified values of design parameters

Design Parameters						
Pre-specified testing time, a	0.7	0.8	1.0	1.2	1.5	2.0
Consumer's risk, β	0.25	0.10	0.05	0.01		
Allowable preceding and succeeding lots i, j	1	2	3	4		
Number of testers, r	2	3	4	5		

The procedure of the proposed plans are developed and discussed in subsequent sections based on pre-specified values of design parameters in Table 3.1. It is to be noted that this table has also been used by Mughal *et al.* (2015b), Mughal and Aslam (2011) and Aslam *et al.* (2010a) in their research.

3.2 Phase II: Developing the Acceptance Sampling Procedures

The first objective of this study is to develop new group chain and modified group chain acceptance sampling plans. This can be achieved by initially developing the procedures based on the acceptance number in the lot. Extending the earlier works of

Dodge (1955) and Govindaraju and Lai (1998), the probability of lot acceptance for the new group chain and modified group chain acceptance sampling plans can be derived by using acceptance sampling procedures 3.1 and 3.2 which are illustrated in Figures 3.1 and 3.2, respectively.

Procedure 3.1 GChSP	
Step 1	Find the minimum number of g groups and allocate r products to each group such that the required sample size is $n = r \times g$.
Step 2	Inspect the sample and count the number of defectives, d.
Step 3	If no defective is found in the current sample ($d = 0$), accept the lot.
Step 4	If two or more defectives are found in the current sample ($d > 1$), reject the lot.
Step 5	If one defective is found in the current sample ($d = 1$), but preceding i samples have no defectives, ($d_i = 0$), accept the lot.

(Note: Steps 1 and 2 are common to all of the proposed plans.)

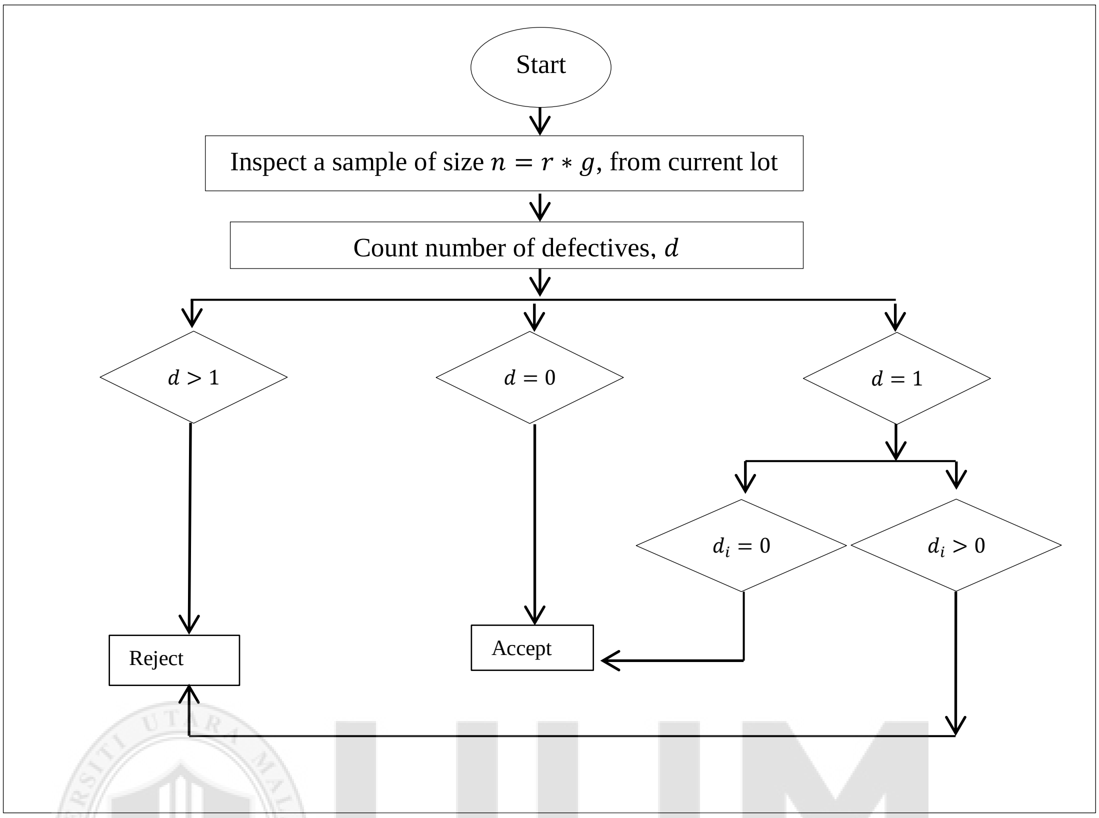

Figure 3.1. Acceptance sampling procedure for GChSP

Procedure 3.2 MGChSP	
Step 1	Find the minimum number of g groups and allocate r products to each group such that the required sample size is $n = r \times g$.
Step 2	Inspect the sample and count the number of defectives, d.
Step 3	If no defective is found in the current sample $(d = 0)$ and the immediately preceding i samples have no defectives, $(d_i = 0)$, accept the lot.
Step 4	If no defective is found in the current sample $(d = 0)$, while the preceding i samples have only one defective $(d_i = 1)$, accept the lot.
Step 5	If one or more defectives are found in the current sample $(d > 0)$, reject the lot.

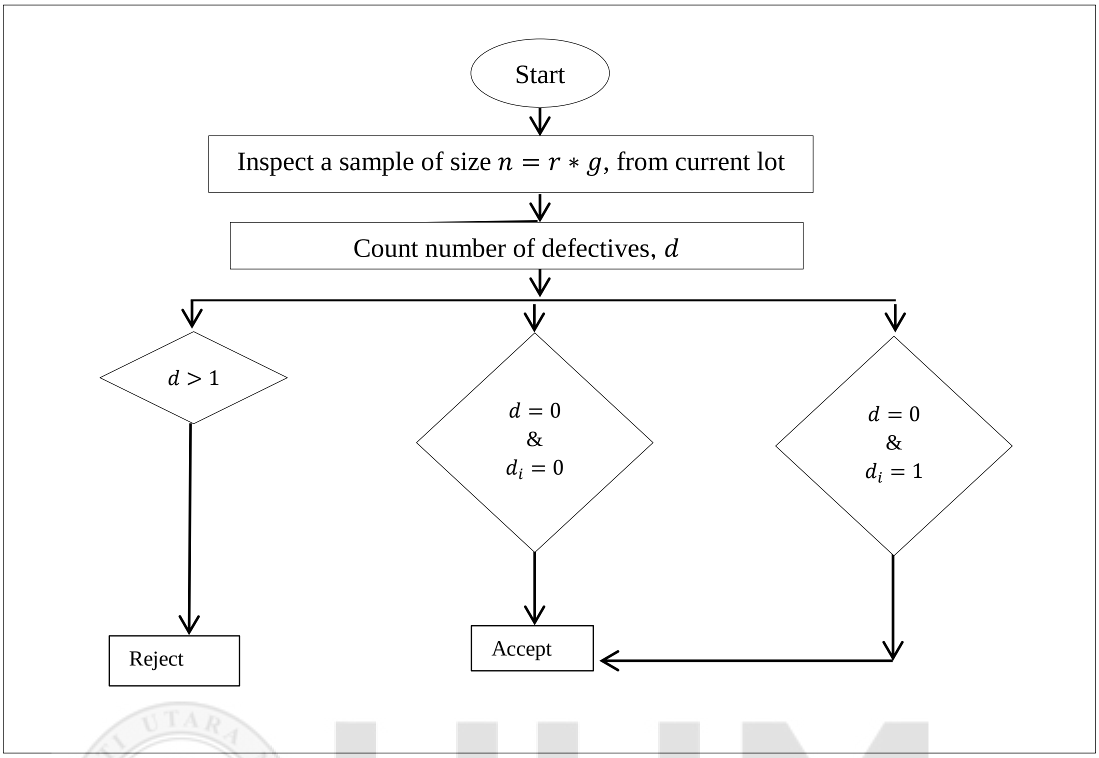

Figure 3.2. Acceptance sampling procedure for MGChSP

The advantage of MChSP is that it gives a more accurate probability of lot acceptance than ChSP as it does not overestimate the probability of lot acceptance for a required value of proportion defective. To fulfill the second objective, the procedures based on the new two-sided group chain and two-sided modified group chain sampling plans are developed. These procedures consider preceding, current and succeeding lots information as stated in procedures 3.3 and 3.4 and also shown in Figures 3.3 and 3.4, respectively.

Procedure 3.3 TS-GChSP	
Step 1	Find the minimum number of g groups and allocate r products to each group such that the required sample size is $n = r \times g$.
Step 2	Inspect the sample and count the number of defectives, D, which is the sum of current, preceding i and succeeding j defectives.
Step 3	Accept the lot if the current sample as well as preceding i and succeeding j samples have zero defectives: $D = 0$.
Step 4	If two or more defectives are found ($D > 1$), reject the lot.
Step 5	Also accept the lot if one defective is observed to be in either preceding sample i or succeeding sample j but the current sample is free from defectives.

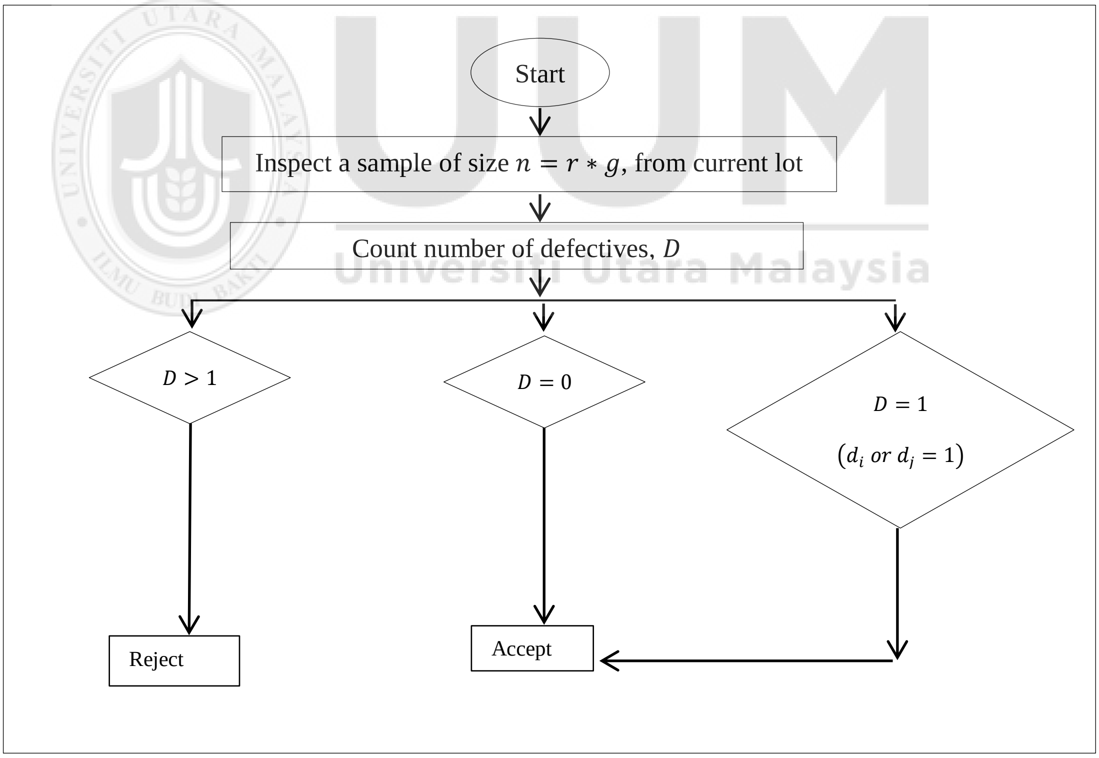

Figure 3.3. Acceptance sampling procedure for TSGChSP

Procedure 3.4 TS-MGChSP

Step 1	Find the minimum number of g groups and allocate r products to each group such that the required sample size is $n = r \times g$.
Step 2	Inspect the sample and count the number of defectives, D, which is the sum of current, preceding i and succeeding j defectives.
Step 3	Accept the lot if the current sample as well as preceding i and succeeding j samples have $D = 0$.
Step 4	If two or more defectives are found ($D > 1$), reject the lot.
Step 5	Also accept the lot if only one defective product occurs in the current sample while the rest of the samples have no defective products.

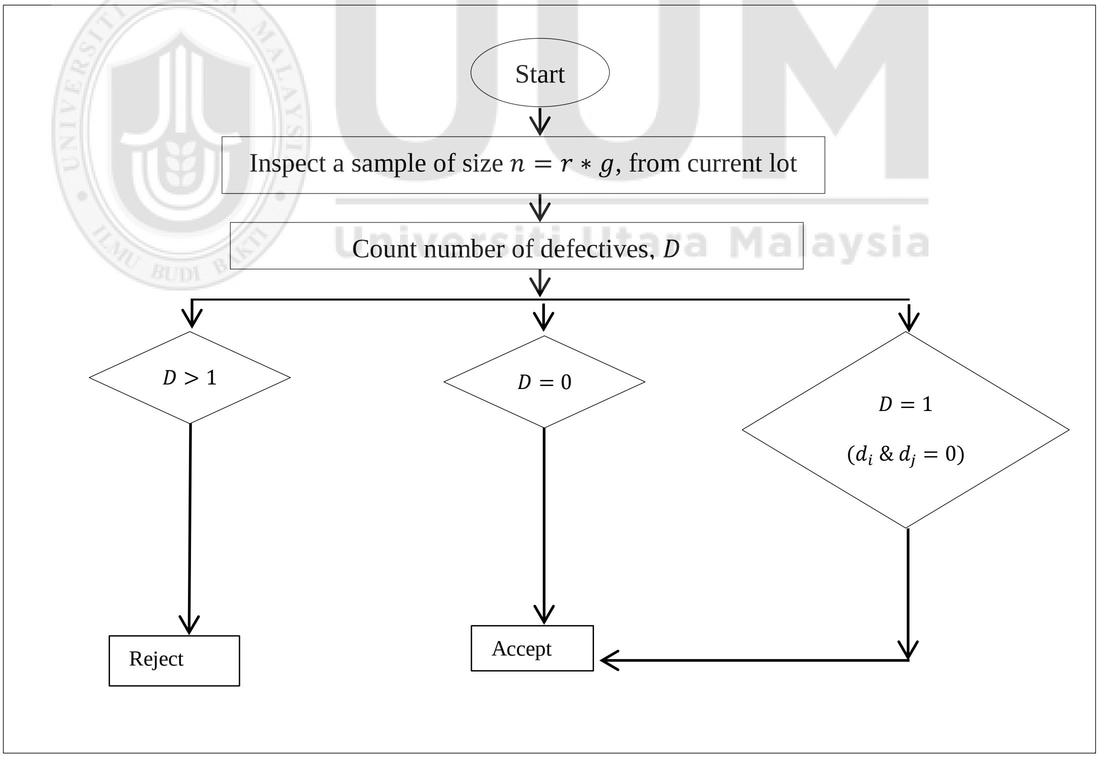

Figure 3.4. Acceptance sampling procedure for TSMGChSP

The minimum number of groups and probability of lot acceptance for the two-sided group chain and modified two-sided group chain sampling plans are obtained by using acceptance sampling procedures 3.3 and 3.4.

3.3 Phase III: Obtaining Operating Characteristic Function Using Lifetime Distribution

In order to achieve the probability of lot acceptance for zero and one defective products, Binomial distribution is applied. This is applicable when the submitted lot size is large, the process is based on independent inspection and the inspection outcomes are categorized into two mutually exclusive outcomes. Then, the probability of lot acceptance can be written in the following form

$$L(p) = \sum_{c=0}^{1} \binom{rg}{c} p^c (1-p)^{rg-c} \quad 3.1$$

where p is the proportion defective. After solving Equation 3.1 for zero and one defective product, the probability of lot acceptance for each is

$$P_0 = (1-p)^{r*g} \text{ and} \quad 3.2$$

$$P_1 = (r*g*p)(1-p)^{(r*g)-1} \quad 3.3$$

In order to find the proportion defective, p, the CDF of the lifetime distribution is required. As mentioned in Chapter 1 (Section 1.5), there are many lifetime distributions but the Pareto distribution of the 2nd kind is discussed here because it provides a smaller minimum sample size than other distributions (Aslam *et al*. 2010a). By using Equation 1.2 (from Chapter 1), the proportion defective, p of the Pareto distribution of the 2nd kind can be written as

$$p = F(t{:}\,\sigma, \lambda) = 1 - \left(1 + \frac{t}{\sigma}\right)^{-\lambda}. \qquad 3.4$$

It is appropriate to determine the termination time, t_0 as a multiple of the specified life, μ_0 such that $t_0 = a\mu_0$. As discussed earlier in Chapter 1 (Section 1.2), when the main objective is to obtain a minimum sample size and more accurate probability of lot acceptance, consumer's risk is taken into account. The consumer's risk (probability of accepting the bad lot) also defines the poorest quality level that the consumer can tolerate. The minimum values of sample size $n = r * g$ and the probability of lot acceptance are found by solving the following inequality based on the pre-specified value of consumer's risk.

$$L(p) = \sum_{c=0}^{1} \binom{rg}{c} p^c (1-p)^{rg-c} \leq \beta. \qquad 3.5$$

After the required minimum sample size is obtained, the experimenter may need to find the accurate probability of lot acceptance for the desired quality level of a submitted product. For fixed values of design parameters, the operating characteristic values such a function of mean ratio,(μ/μ_0) can also be found. A summary of the existing plans is shown in Figure 3.5.

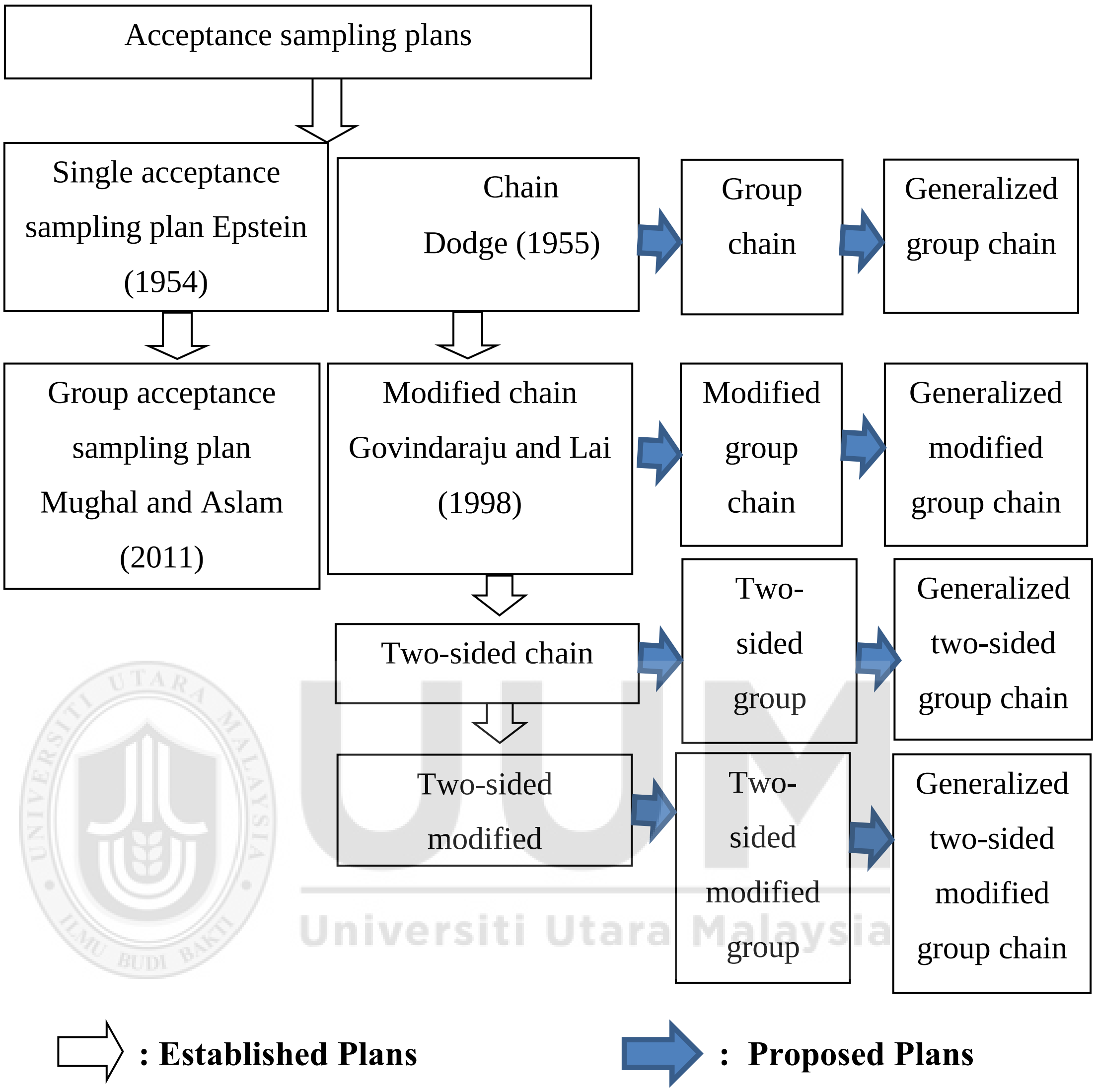

Figure 3.5. Established and proposed acceptance sampling plans

From Figure 3.5, the white arrows represent the established plans and the blue arrows denote the proposed plans, respectively. In this study, the group, modified group, two-sided group and modified two-sided group chain sampling plans are developed. Initially, the Pareto distribution of the 2nd kind was considered in examining the average lifetime of a submitted product and would then be generalized for several pre-specified values of proportion defective.

3.4 Phase IV: Measuring Performance

This study proposes a family of group chain acceptance sampling plans for attributes, which can be utilized when a multi-product tester is used for a truncated life test. Several approaches are considered in obtaining the design parameters such as minimum sample size and more accurate probability of lot acceptance. Computer-based programs are used to evaluate the design parameters of the proposed plans under the conditions of a binomial distribution. Furthermore, numerical analysis on the performance of the proposed plans using sample size, probability of lot acceptance and operating characteristic curves based on real lifetime data are also discussed in this research.

CHAPTER FOUR
GROUP CHAIN SAMPLING PLANS BASED ON PARETO DISTRIBUTION OF THE 2ND KIND

The main objective of this chapter is to investigate the minimum sample size and accurate probability of lot acceptance for a family of group chain acceptance sampling plans. The procedures discussed in the previous chapter, which form the core structure of this research, are now further developed and evaluated for the lifetime of a submitted product which follows the Pareto distribution of the 2nd kind. The numerical analysis for the proposed acceptance sampling plans: (i) group chain, (ii) modified group chain, (iii) two-sided group chain and (iv) two-sided modified group chain are described in the subsequent sections based on Binomial distribution. The following Sections 4.2 to 4.5 describe the four proposed plans to examine the lifetime of submitted product, in order to obtain the minimum number of groups and probability of zero and one defective product. As already discussed in Chapter 2 (Section 2.3), it is assumed that the lot comes from a repetitive manufacturing process under the same conditions and that the producer has a good reputation in the market.

4.1 Group Chain Sampling Plan (GChSP)

In this section, group chain sampling plan (GChSP) is developed to ensure that the mean lifetime, μ, of a submitted product is higher than the specified mean lifetime, μ_0, that is $\mu \geq \mu_0$. For convenience, the abbreviation of the proposed plan, GChSP is used throughout the thesis. Based on Procedure 3.1 (Section 3.2 on page 51), the probability of zero and one defective product for GChSP can be written in the following form by using the probability law of addition,

$$L(p) = P(d = 0) + \{P(d = 1)|P(d = 0)_i\}. \quad 4.1$$

In a sample of size $n = r * g$, the submitted lot will be accepted if the current sample contains no defective products. The lot is also accepted if the current lot has only one defective but the preceding lot, i, contains no defective products. This procedure for i = 2, can be illustrated in Figure 4.1, where D and $\overline{D}$ denote the defective and non-defective products respectively.

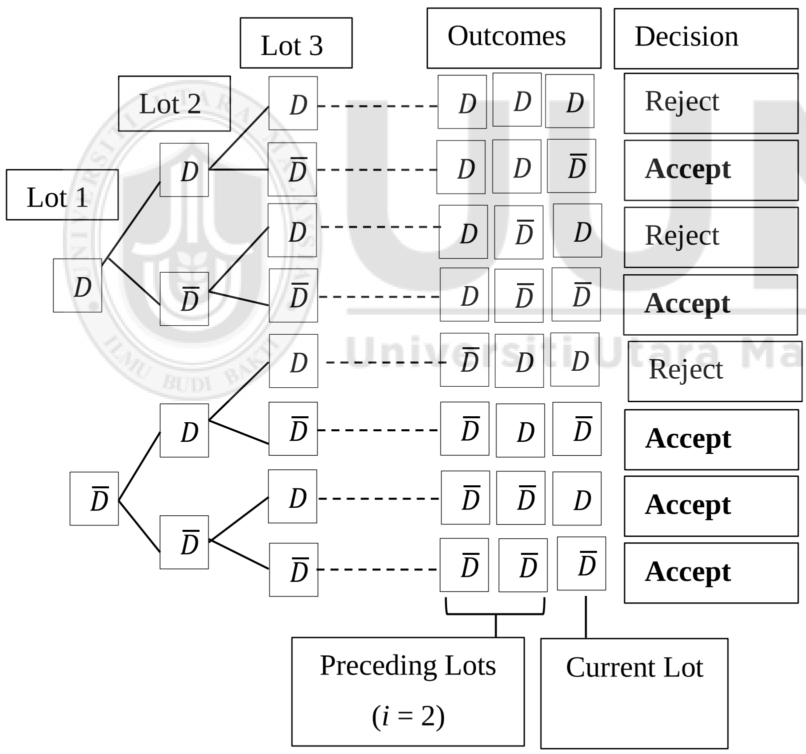

Figure 4.1. A tree diagram of chain sampling

With reference to Figure 4.1, when i = 2, the outcomes $\{\overline{D}\overline{D}\overline{D}, \overline{D}D\overline{D}, D\overline{D}\overline{D}, DD\overline{D}, \overline{D}\overline{D}D\}$ meet the acceptance criteria for chain sampling.

Thus, the probability of lot acceptance using GChSP can be written in the following form,

$$L(p)_{GChSP} = \{P_{0,(r*g)} + P_{0,(r*g)}P_{0,(r*g)}P_{1,(r*g)}\}. \quad 4.2$$

Upon simplification, it is expressed as,

$$L(p)_{GChSP} = \left\{P_{0,(r*g)} + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2\right\}. \quad 4.3$$

Based on the above Equation 4.3, the general expression of probability of lot acceptance of GChSP is,

$$L(p)_{GChSP} = \left\{P_{0,(r*g)} + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^i\right\} \quad 4.4$$

Considering Binomial distribution, Equation 4.4 can be rewritten in the following form,

$$L(p)_{GChSP} =$$

$$\left[\binom{r*g}{0} p^0(1-p)^{r*g} + \binom{r*g}{1} p^1(1-p)^{(r*g)-1}\left[\binom{r*g}{0} p^0(1-p)^{r*g}\right]^i\right] \quad 4.5$$

Upon simplification of the above Equation 4.5, then,

$$L(p)_{GChSP} = (1-p)^{(r*g)} + (r*g)(p)(1-p)^{(r*g)-1}(1-p)^{(r*g)*i} \quad 4.6.$$

In order to find the proportion defective, p, the CDF and mean of the lifetime distribution are required. The CDF and mean of Pareto distribution of the 2nd kind in respective order,

$$F(t{:}\,\sigma,\lambda) = 1 - \left(1 + \frac{t}{\sigma}\right)^{-\lambda},\ t > 0, \sigma > 0, \lambda > 0\ , \quad 4.7$$

$$\mu = \frac{\sigma}{\lambda - 1} \quad 4.8$$

where σ, and λ, are the scale and shape parameters respectively. For simplicity, the test termination time, t_0, can be represented as a multiple of the specified life, μ_0 and pre-specified testing time, a. It can be written in the following form,

$$t_0 = a\mu_0 \text{ .} \qquad 4.9$$

Using Equations 4.7, 4.8 and 4.9, the proportion defective, p, can be written as,

$$p = 1 - \left(1 + \frac{a\mu_0}{(\lambda-1)\mu}\right)^{-\lambda} \quad ; \qquad 4.10$$

$$p = 1 - \left[1 + \frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda}. \qquad 4.11$$

It is to be noted that for existence of the mean, the value of the shape parameter of the Pareto distribution of the 2nd kind must be greater than one ($\lambda > 1$). By using Equation 4.11, the proportion defective, p, can be obtained for a pre-specified testing time, a, and mean ratio of one, $(\mu/\mu_0) = 1$. The calculated values of p are presented in Table 4.1 below.

Table 4.1

Lot proportion defective, p

	a					
λ	0.7	0.8	1.0	1.2	1.5	2.0
2	0.6540	0.6914	0.7500	0.7934	0.8400	0.8889
3	0.5936	0.63336	0.7037	0.7559	0.8134	0.8750
4	0.5678	0.6115	0.6836	0.7397	0.8025	0.8704

As shown in Table 4.1, reading vertically downward, the proportion defective decreases when the value of shape parameter of Pareto distribution of the 2nd kind increases (λ = 2 to 4). Reading across horizontally, the proportion defective increases with pre-specified testing time, at all values of λ. Based on these values, the minimum number of groups, g, are obtained using Equation 4.6, when satisfying the following inequality,

$$L(p)_{GChSP} = (1-p)^{(r*g)} + (r*g)(p)(1-p)^{(r*g)-1}(1-p)^{(r*g)*i} \leq \beta. \qquad 4.12$$

For various values of consumer's risk, β; allowable number of preceding lots, i; number of testers, r; and shape parameters of Pareto distribution of the 2nd kind, λ = 2, the minimum number of groups, g, is obtained and displayed in Table 4.2 based on the values in Table 4.1.

Table 4.2

Number of minimum groups,g, required for GChSP when $\lambda = 2$

			a					
β	r	i	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	2	2	2	2	2
	3	2	**2**	2	2	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1

From Table 4.2, the number of groups required for the GChSP is quite similar for different values of design parameters, but it decreases when the number of pre-specified testing time, consumer's risk, preceding lots and number of testers increases. Suppose the average lifetime of a product is the same as its specified lifetime, $\mu = \mu_0$= 10,000 hours, β = 0.01, a = 0.7, r = 3, i = 2, $\lambda = 2$, and $g = 2$ (in bold). Then a sample of six products is drawn from the lot where 3 testers are located into 2 groups. Based on this information, after 7,000 hours ($t_0 = a\mu_0$) of testing, the submitted lot will be accepted if no defectives are observed or if one defective occurs in the current sample, but no defectives are recorded in the two preceding samples. For the same design parameters, the minimum number of groups, g, is obtained and displayed in Tables 4.3 to 4.4 for the various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3$, 4).

Table 4.3

Number of minimum groups,g, required for GChSP when $\lambda = 3$

β	r	i	a = 0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	2	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	3	2	2	2	2
	3	2	2	2	2	2	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1

Table 4.4

Number of minimum groups,g, required for GChSP when $\lambda = 4$

			a					
β	*r*	*i*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	2	1
	3	2	2	2	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	3	2	2	2	1
	3	2	2	2	2	2	1	1
	4	3	2	2	2	1	1	1
	5	4	2	1	1	1	1	1

In Table 4.2, the number of groups required for the GChSP is also very similar for different values of shape parameter as shown in Tables 4.3 and 4.4. Using the numbers of groups from Tables 4.2 to 4.4, the probability of lot acceptance is obtained for the desired value of mean ratio. The choices of design parameter values are considered only for comparison purposes. For various values of mean ratio (μ/μ_0 = 1, 2, 4, 6, 8, 10, 12), the probability of lot acceptance known as operating characteristic is presented in Tables 4.5 to 4.7.

Table 4.5

Operating characteristic values for r = 3, i = 2, when $\lambda = 2$

β	g	a	μ/μ_0 1	2	4	6	8	10	12
0.25	1	0.7	0.0418	0.1763	0.4426	0.6174	0.7256	0.7950	0.8415
	1	0.8	0.0296	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0157	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0088	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0041	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0014	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.10	1	0.7	0.0418	0.1763	0.4426	0.6174	0.7256	0.7950	0.8415
	1	0.8	0.0296	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0157	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0088	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0041	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0014	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.05	2	0.7	0.0017	0.0274	0.1513	0.2939	0.4190	0.5201	0.6000
	1	0.8	0.0296	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0157	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0088	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0041	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0014	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.01	2	0.7	**0.0017**	0.0274	0.1513	0.2939	0.4190	0.5201	**0.6000**
	2	0.8	0.0009	0.0177	0.1159	0.2415	0.3594	0.4596	0.5419
	2	1.0	0.0002	0.0077	0.0698	0.1657	0.2663	0.3594	0.4411
	1	1.2	0.0088	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0041	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0014	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641

From Table 4.5, it can be shown that when the mean ratio increases, the probability of lot acceptance increases. Referring to β = 0.01, g = 2, a = 0.7, i = 2, r = 3, and $\lambda = 2$, the probability of lot acceptance is 0.0017 (in bold) when the mean ratio of average lifetime and the specified average lifetime of a product are equal to 1 or $\mu/\mu_0 = 1$. The probability of lot acceptance increases from 0.0017 to 0.6000 (in bold), when the mean ratio increases from 1 to 12. It indicates that the chances of lot acceptance increases to sixty percent when the lifetime of product is twelve times of the average lifetime. For other values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3$, 4), the probability of lot acceptance is obtained and presented in Tables 4.6 and 4.7.

Table 4.6

Operating characteristic values for r = 3, i = 2, when $\lambda = 3$

			μ/μ_0						
β	g	a	1	2	4	6	8	10	12
0.25	1	0.7	0.0685	0.2582	0.5592	0.7207	0.8096	0.8626	0.8964
	1	0.8	0.0490	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.0261	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0146	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0065	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0020	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.10	1	0.7	0.0685	0.2582	0.5592	0.7207	0.8096	0.8626	0.8964
	1	0.8	0.0490	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.0261	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0146	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0065	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0020	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.05	2	0.7	0.0045	0.0555	0.2395	0.4125	0.5441	0.6401	0.7104
	1	0.8	0.0490	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.0261	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0146	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0065	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0020	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.01	2	0.7	**0.0045**	0.0555	0.2395	0.4125	0.5441	0.6401	0.7104
	2	0.8	0.0023	0.0378	0.1914	0.3522	0.4834	0.5837	0.6598
	2	1.0	0.0007	0.0180	0.1244	0.2584	0.3811	0.4834	0.5658
	2	1.2	0.0002	0.0089	0.0825	0.1914	0.3013	0.3996	0.4834
	1	1.5	0.0065	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0020	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804

Table 4.7

Operating characteristic values for r = 3, i = 2, when $\lambda = 4$

			μ/μ_0						
β	g	a	1	2	4	6	8	10	12
0.25	1	0.7	0.0828	0.2974	0.6056	0.7579	0.8381	0.8846	0.9138
	1	0.8	0.0596	0.2442	0.5475	0.7125	0.8038	0.8583	0.8931
	1	1.0	0.0319	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.0177	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0077	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0022	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.10	1	0.7	0.0828	0.2974	0.6056	0.7579	0.8381	0.8846	0.9138
	1	0.8	0.0596	0.2442	0.5475	0.7125	0.8038	0.8583	0.8931
	1	1.0	0.0319	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.0177	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0077	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0022	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.05	2	0.7	0.0065	0.0719	0.2823	0.4637	0.5939	0.6851	0.7500
	2	0.8	0.0034	0.0501	0.2294	0.4020	0.5346	0.6319	0.7034
	1	1.0	0.0319	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.0177	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0077	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0022	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.01	2	0.7	**0.0065**	0.0719	0.2823	0.4637	0.5939	0.6851	0.7500
	2	0.8	0.0034	0.0501	0.2294	0.4020	0.5346	0.6319	0.7034
	2	1.0	0.0010	0.0248	0.1536	0.3027	0.4317	0.5346	0.6148
	2	1.2	0.0003	0.0126	0.1044	0.2294	0.3486	0.4506	0.5346
	1	1.5	0.0077	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0022	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261

Tables 4.6 and 4.7 further clarify the influence of mean ratio on the probability of lot acceptance. The probability of lot acceptance increases when the mean ratio of the products increases. It is evident that the probability of lot acceptance also increases when the value of shape parameter increases. Considering β = 0.01, g = 2, a = 0.7, i = 2, r = 3, and $\lambda = 3$, the probability of lot acceptance is 0.0045, when μ/μ_0 = 1, as mentioned in Table 4.5. For the same design parameters, when the value of shape parameter increases from 2 to 4 the probability of lot acceptance increases from 0.0045 to 0.0065 as shown in Tables 4.6 and 4.7. It shows very low increments in probability of lot acceptance with regard to higher values of proportion defective.

The effect of different values of mean ratio and shape parameter on probability of lot acceptance is illustrated in Figure 4.2.

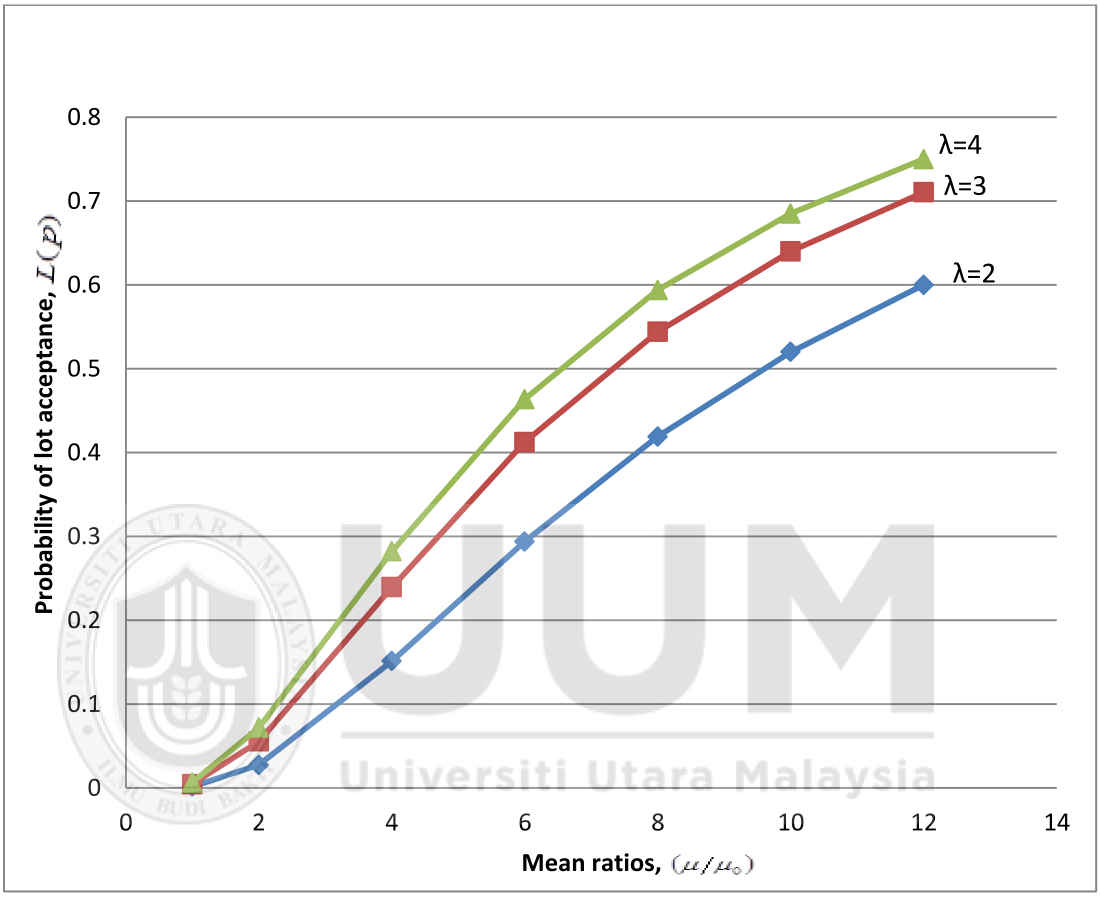

Figure 4.2. Probability of lot acceptance versus various values of mean ratios for GChSP

Examining the above Figure 4.2, the probability of lot acceptance of a submitted product increases when the mean ratio and shape parameter increases. In contrast, when the mean ratio and shape parameter decrease, more lots are expected to be rejected. For example, when the true average life increases from 1 to 12 times of specified average life, the probability of lot acceptance increases from 0.0017 to

0.6000 when λ =2. Meanwhile, when the shape parameter increases from 2 to 4, the probability of lot acceptance increases from 0.6000 to 0.7500 (from Tables 4.5-4.7) respectively. For the fixed value of mean ratio and with the same design parameters as mentioned in Tables 4.5 and 4.7, the probability of lot acceptance is found for various values of preceding lots (i = 1, 2, 3), and is presented in Table 4.8.

Table 4.8

Operating characteristic values for μ/μ_0=1, r=3 when λ =2

β	g	a	i = 1	2	3
0.25	1	0.7	0.0512	0.0418	0.0414
	1	0.8	0.0352	0.0296	0.0294
	1	1.0	0.0178	0.0157	0.0156
	1	1.2	0.0097	0.0088	0.0088
	1	1.5	0.0044	0.0041	0.0041
	1	2.0	0.0014	0.0013	0.0013
0.10	1	0.7	**0.0512**	0.0418	**0.0414**
	1	0.8	0.0352	0.0296	0.0294
	1	1.0	0.0178	0.0157	0.0156
	1	1.2	0.0097	0.0088	0.0088
	1	1.5	0.0044	0.0041	0.0041
	1	2.0	0.0014	0.0013	0.0013
0.05	2	0.7	0.0018	0.0017	0.0017
	1	0.8	0.0352	0.0296	0.0294
	1	1.0	0.0178	0.0157	0.0156
	1	1.2	0.0097	0.0088	0.0088
	1	1.5	0.0044	0.0041	0.0041
	1	2.0	0.0014	0.0013	0.0013
0.01	2	0.7	0.0018	0.0017	0.0017
	2	0.8	0.0009	0.0008	0.0008
	2	1.0	0.0003	0.0002	0.0002
	1	1.2	0.0097	0.0088	0.0088
	1	1.5	0.0044	0.0041	0.0041
	1	2.0	0.0014	0.0013	0.0013

In Table 4.8, the probability of lot acceptance decreases when the number of preceding lots, pre-specified testing time and consumer's risk increase. Consider β = 0.10, g = 1, a = 0.7, r = 3, i = 1, $\mu/\mu_0 = 1$ and $\lambda = 2$ where the probability of lot acceptance is 0.0512. The chance of lot acceptance decreases from 5% to 4%, when the preceding lot increases from 1 to 3. There is a strong indication that if the lot has greater value of proportion defective (poorer quality), the chances of lot acceptance is very low and tends to be at zero for higher values of preceding lots. Based on the same design parameters, the probability of lot acceptance is also found and presented in Tables 4.9 to 4.10 for larger values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3, 4$).

Table 4.9

Operating characteristic values for μ/μ_0 = 1, r = 3 when $\lambda = 3$

β	g	a	i = 1	2	3
0.25	1	0.7	0.0869	0.0685	0.0672
	1	0.8	0.0607	0.0490	0.0484
	1	1.0	0.0308	0.0261	0.0260
	1	1.2	0.0165	0.0146	0.0146
	1	1.5	0.0071	0.0065	0.0065
	1	2.0	0.0020	0.0019	0.0019
0.10	1	0.7	**0.0869**	0.0685	0.0672
	1	0.8	0.0607	0.0490	0.0484
	1	1.0	0.0308	0.0261	0.0260
	1	1.2	0.0165	0.0146	0.0146
	1	1.5	0.0071	0.0065	0.0065
	1	2.0	0.0020	0.0019	0.0019
0.05	2	0.7	0.0047	0.0045	0.0045
	1	0.8	0.0607	0.0490	0.0484
	1	1.0	0.0308	0.0261	0.0260
	1	1.2	0.0165	0.0146	0.0146
	1	1.5	0.0071	0.0065	0.0065
	1	2.0	0.0020	0.0019	0.0019
0.01	2	0.7	0.0047	0.0045	0.0045
	2	0.8	0.0024	0.0023	0.0023
	2	1.0	0.0007	0.0007	0.0007
	2	1.2	0.0002	0.0002	0.0002
	1	1.5	0.0071	0.0065	0.0065
	1	2.0	0.0020	0.0019	0.0019

Table 4.10

Operating characteristic values for μ/μ_0 = 1, r = 3 when $\lambda = 4$

β	g	a	i = 1	2	3
0.25	1	0.7	0.1064	0.0828	0.0809
	1	0.8	0.0749	0.0596	0.0587
	1	1.0	0.0382	0.0319	0.0317
	1	1.2	0.0203	0.0177	0.0176
	1	1.5	0.0084	0.0077	0.0077
	1	2.0	0.0022	0.0022	0.0022
0.10	1	0.7	**0.1064**	0.0828	0.0809
	1	0.8	0.0749	0.0596	0.0587
	1	1.0	0.0382	0.0319	0.0317
	1	1.2	0.0203	0.0177	0.0176
	1	1.5	0.0084	0.0077	0.0077
	1	2.0	0.0022	0.0022	0.0022
0.05	2	0.7	0.0069	0.0065	0.0065
	2	0.8	0.0036	0.0034	0.0034
	1	1.0	0.0382	0.0319	0.0317
	1	1.2	0.0203	0.0177	0.0176
	1	1.5	0.0084	0.0077	0.0077
	1	2.0	0.0022	0.0022	0.0022
0.01	2	0.7	0.0069	0.0065	0.0065
	2	0.8	0.0036	0.0034	0.0034
	2	1.0	0.0010	0.0010	0.0010
	2	1.2	0.0003	0.0003	0.0003
	1	1.5	0.0084	0.0077	0.0077
	1	2.0	0.0022	0.0022	0.0022

Similarly, as shown in Table 4.8, the probability of lot acceptance decreases when the number of preceding lots increases, but it increases when the shape parameter increases, as observed from Tables 4.9 and 4.10. Assuming, β = 0.10, g = 1, a = 0.7, r = 3, i = 1, $\mu/\mu_0 = 1$, and $\lambda = 3$, the probability of lot acceptance is 0.0869 as observed in Table 4.9. For the same design parameters, the probability of lot acceptance increases from 0.0869 to 0.1064 when the shape parameter increases from 2 to 4, as shown in Table 4.10. Meanwhile, the probability of lot acceptance decreases when the number of preceding lots increases from 1 to 3, as clearly portrayed in Figure 4.3 below.

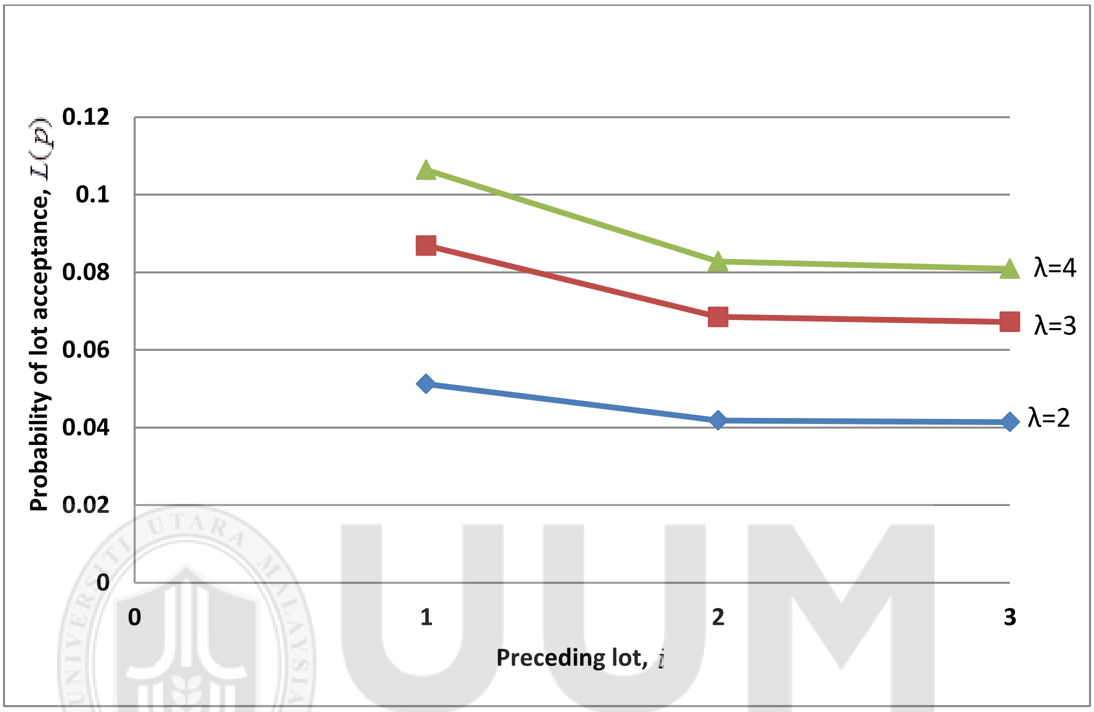

Figure 4.3. Probability of lot acceptance versus preceding lot for GChSP

As observed in Figure 4.3, the probability of lot acceptance of a submitted product decreases when the number of preceding lot increases. At β = 0.10, g = 1, a = 0.7, i = 1, r = 3, and $\lambda = 2$, the probability of lot acceptance is 0.0512 from Table 4.8. As shown in Figure 4.3 above, the probability of lot acceptance decreases from 5% to 4% when the number of preceding lot increases from 1 to 2. This means that when the number of preceding lots increases the chances of lot acceptance decreases and contributes very small change in probability of lot acceptance. It does not make much difference to the chances of accepting the current lot when more preceding lots are considered. This tendency is the opposite for the larger values of shape parameters. For the same above mentioned design parameter, the probability of lot acceptance increases from 0.0512 to 0.1064 when the shape parameter increases from 2 to 4 respectively.

4.2 Modified Group Chain Sampling Plan (MGChSP)

According to Procedure 3.2 (as stated in Chapter 3), the final outcomes for a modified group chain sampling plan (MGChSP) can be written in the forms $\{\bar{D}\bar{D}\bar{D}, \bar{D}D\bar{D}, D\bar{D}\bar{D}\}$, illustrated in Figure 4.2,

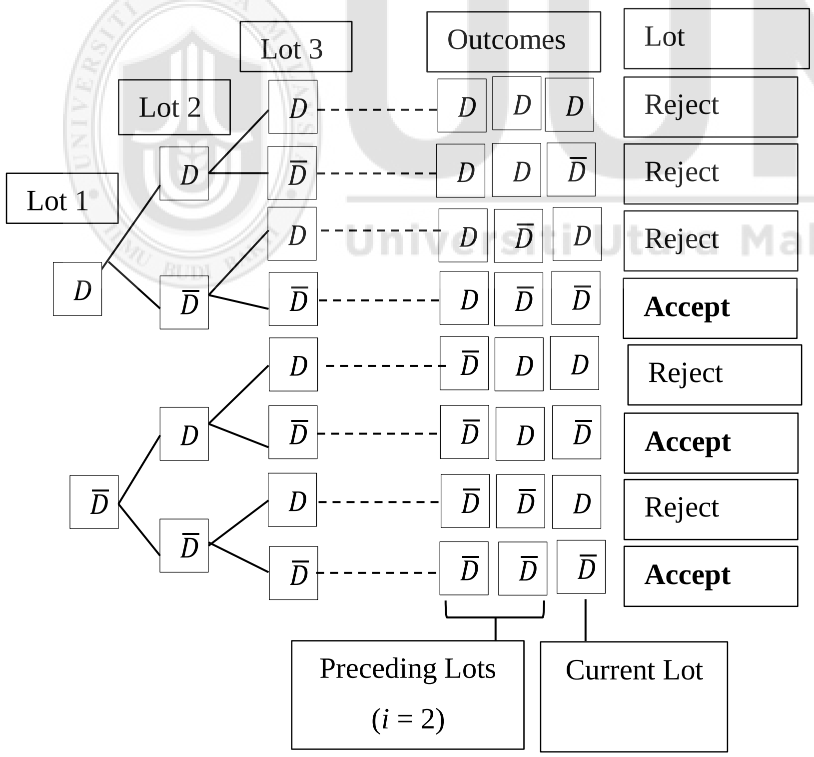

Figure 4.4. A tree diagram of modified chain sampling

Based on Figure 4.4, the probabilities of lot acceptance for MGChSP are,

$$L(p)_{MGChSP} = \{P(d=0)|P(d=0)_i\} + \{P(d=0)|P(d=1)_i\} \;; \quad 4.13$$

$$L(p)_{MGChSP} = \{P_{0,(r*g)}P_{0,(r*g)}P_{0,(r*g)} + P_{0,(r*g)}P_{1,(r*g)}P_{0,(r*g)} + P_{1,(r*g)}P_{0,(r*g)}P_{0,(r*g)}\} \;; \quad 4.14$$

$$L(p)_{MGChSP} = \left\{\left(P_{0,(r*g)}\right)^3 + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2 + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2\right\} \;; \quad 4.15$$

$$L(p)_{MGChSP} = \left\{\left(P_{0,(r*g)}\right)^3 + 2P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2\right\}. \quad 4.16$$

Based on the above Equation 4.16, the general expression of probability of lot acceptance for MGChSP is,

$$L(p)_{MGChSP} = \left\{\left(P_{0,(r*g)}\right)^{(i+1)} + i\,P_{1,(r*g)}\left(P_{0,(r*g)}\right)^i\right\}. \quad 4.17$$

Considering Binomial distribution, the above Equation 4.17 converts to the following form,

$$L(p)_{MGChSP} = \begin{bmatrix} \left\{\binom{r*g}{0} p^0(1-p)^{r*g}\right\}^{(i+1)} + \\ i\left\{\binom{r*g}{1} p^1(1-p)^{(r*g)-1}\right\}\left\{\binom{r*g}{0} p^0(1-p)^{r*g}\right\}^i \end{bmatrix}; \quad 4.18$$

$$L(p)_{MGChSP} = \left[\left\{(1-p)^{(r*g)}\right\}^{(i+1)} + i\left\{(r*g)(p)(1-p)^{(r*g)-1}\right\}\left\{(1-p)^{(r*g)}\right\}^i\right]. \quad 4.19$$

After simplification of the above Equation 4.19, the probability of lot acceptance for MGChSP is,

$$L(p)_{MGChSP} = (1-p)^{(r*g)(i+1)}[1 + i(r*g)(p)/(1-p)]. \quad 4.20$$

For pre-specified values of testing time and shape parameter of the Pareto distribution of the 2nd kind already discussed earlier in Table 4.1, the minimum number of groups, g, is found based on the following Equation 4.21,

$L(p)_{MGChSP} =$

$$\left[\left\{(1-p)^{(r*g)}\right\}^{(i+1)} + i\left\{(r*g)(p)(1-p)^{(r*g)-1}\right\}\left\{(1-p)^{(r*g)}\right\}^{i}\right] \leq \beta. \quad 4.21$$

For various values of β, i, and r, the minimum number of groups, g, is presented in Tables 4.11 to 4.13.

Table 4.11

Number of minimum groups, g, required for MGChSP when $\lambda = 2$

β	r	i	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	2	2	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 4.11 showed the relationship between different values of the design parameters. The number of groups required for the MGChSP is similar but it decreases when the pre-specified testing time, consumers' risk, number of testers and number of preceding lots increase. Assuming the average life of a product, $\mu = \mu_0$ = 10,000 hours and the other pre-specified design parameters are β = 0.01, a = 0.7, r = 3, i = 2, $\lambda = 2$, and $g = 1$, then a sample of size 3 products drawn from the lot where 3 testers are located into 1 group, as shown in Table 4.11. Based on this information,

the submitted lot will be accepted if no defective is observed in the preceding sample as well as current sample. The lot is also acceptable if one defective is observed in the preceding lot but no defective in the current sample during 7,000 hours of testing. The number of groups required for the MGChSP for various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3, 4$), are provided in Tables 4.12 and 4.13.

Table 4.12

Number of minimum groups, g, required for MGChSP when λ =3

β	r	i	*a*					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	2	2	2	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 4.13

Number of minimum groups, g, required for MGChSP when λ =4

β	r	i	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	2	2	2	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

From Tables 4.12 and 4.13, the required number of groups for the MGChSP is similar compared to Table 4.11 for different values of shape parameter. Using these values of g in Tables 4.11 and 4.13, the probability of lot acceptance is obtained for the desired quality level. For various values of mean ratios (μ/μ_0 = 1, 2, 4, 6, 8, 10, 12), the probability of lot acceptance is presented in Tables 4.14 to 4.16.

Table 4.14

Operating characteristic values for r=3, i=2, g=1 when λ =2

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	**0.0009**	0.0267	0.1801	0.3405	0.4630	0.5530	**0.6200**
0.25	0.8	0.0004	0.0158	0.1367	0.2844	0.4064	0.5000	0.5716
0.10	1.0	0.0001	0.0057	0.0788	0.1974	0.3113	0.4064	0.4833
0.05	1.2	0.0000	0.0021	0.0457	0.1367	0.2371	0.3285	0.4064
0.01	1.5	0.0000	0.0005	0.0205	0.0788	0.1569	0.2371	0.3113
	2.0	0.0000	0.0000	0.0057	0.0319	0.0788	0.1367	0.1974

Table 4.14 shows that the probability of lot acceptance increases when the mean ratio of the products increases. Considering that β = 0.01, g = 1, a = 0.7, r = 3, i = 2, and $\lambda = 2$, the probability of lot acceptance is 0.0009 when $\mu/\mu_0 = 1$ from Table 4.14. The probability of lot acceptance increases from 0.0009 to 0.6200 when the mean ratio increases from 1 to 12. This is evident when the mean lifetime of a product is twelve times more than of the average lifetime, then the probability of lot acceptance will be increased by about 62%. This percentage suggests that the chances of lot acceptance increases for higher values of mean ratios. Based on the values of g in Tables 4.12 and 4.13, the probability of lot acceptance is also obtained for $\lambda = 3$, and 4, as shown in Tables 4.14 and 4.16 respectively.

Table 4.15

Operating characteristic values for r = 3, i = 2, g = 1 when $\lambda = 3$

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	**0.0030**	0.0608	0.2821	0.4570	0.5735	0.6526	**0.7088**
0.25	0.8	0.0013	0.0390	0.2277	0.3994	0.5211	0.6066	0.6684
0.05	1.0	0.0003	0.0162	0.1473	0.3027	0.4274	0.5211	0.5917
0.05	1.2	0.0001	0.0068	0.0947	0.2277	0.3481	0.4450	0.5211
0.01	1.5	0.0000	0.0019	0.0487	0.1473	0.2536	0.3481	0.4274
	2.0	0.0000	0.0026	0.0162	0.0705	0.1473	0.2277	0.3027

Table 4.16

Operating characteristic values for r = 3, i = 2, g = 1 when $\lambda = 4$

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	0.0047	0.0815	0.3282	0.5037	0.6149	0.6886	0.7401
0.25	0.8	0.0021	0.0540	0.2710	0.4472	0.5654	0.6459	0.7032
0.05	1.0	0.0004	0.0237	0.1829	0.3496	0.4748	0.5654	0.6321
0.05	1.2	0.0001	0.0104	0.1224	0.2710	0.3959	0.4920	0.5654
0.01	1.5	0.0000	0.0031	0.0664	0.1829	0.2984	0.3959	0.4748
	2.0	0.0000	0.0004	0.0237	0.0933	0.1829	0.2710	0.3496

Similar to earlier observations (Table 4.1), the probability of lot acceptance increases when the mean ratios and shape parameter increases as shown in Tables 4.15 and 4.16. Considering β = 0.01, g = 1, a = 0.7, i = 2, r = 3, and $\lambda = 3$, the probability of lot acceptance is 0.0030. The chances of lot acceptance increases from 0.3% to 0.71% when the mean ratio increase from 1 to 12 from Table 4.15. It means that when the average lifetime of product increases, it offers higher chance of lot acceptance. For the same design parameters when λ =4, the probability of lot acceptance increases from 0.0030 to 0.0047. This increasing trend is also illustrated in Figure 4.5.

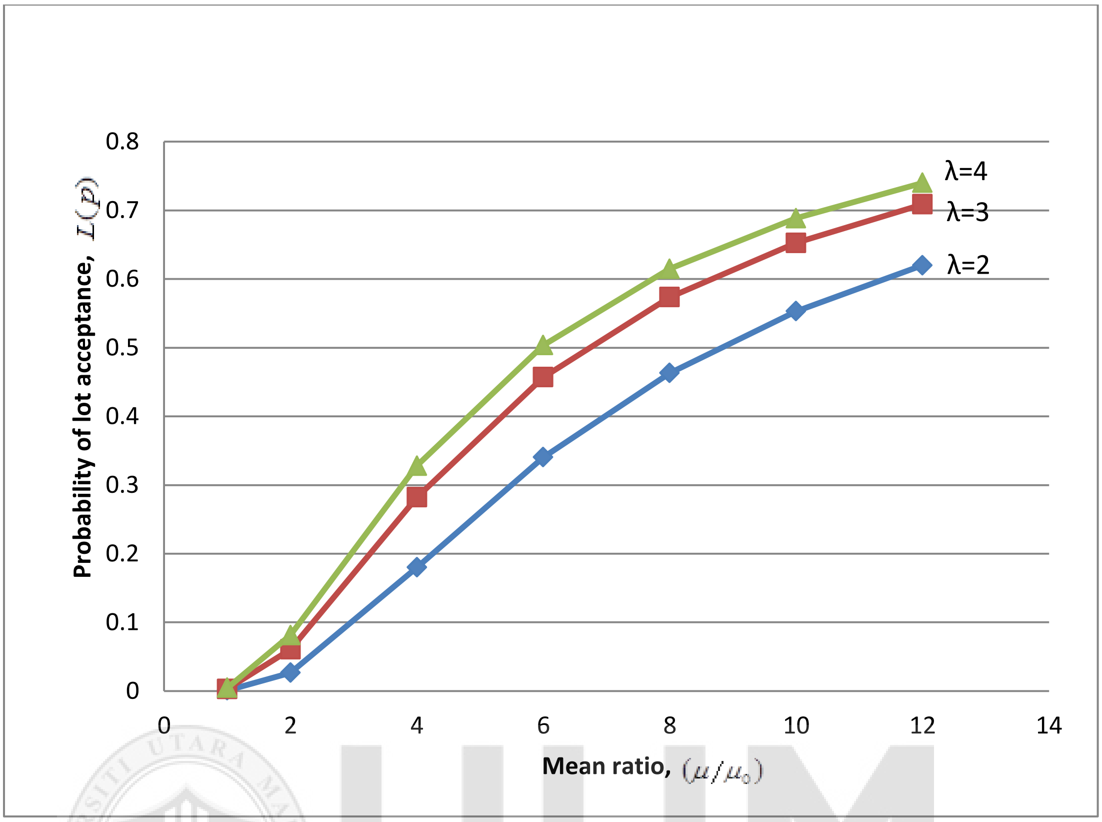

Figure 4.5. Probability of lot acceptance versus mean ratios for MGChSP

The effect of mean ratio and the value of shape parameter on the probability of lot acceptance are illustrated in above Figure 4.5. The probability of lot acceptance of a submitted product increases when the value of mean ratio and shape parameter increases. If the true average lifetime increases from 1 to 12 times of specified average life then the probability of lot acceptance increases from 0.0009 to 0.6200 when, $\lambda = 2$. Meanwhile, when the shape parameter increases from 2 to 4, the probability of lot acceptance also increases from 0.6200 to 0.7401. For the same design parameters as mentioned in Tables 4.14 to 4.16, the probability of lot acceptance is established for various values of preceding lots, (i = 1, 2, 3), and presented in Tables 4.17 to 4.19.

Table 4.17

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 2$

β	a	i		
		1	2	3
	0.7	**0.0114**	0.0009	**0.0001**
0.25	0.8	0.0067	0.0004	0.0000
0.10	1.0	0.0024	0.0001	0.0000
0.05	1.2	0.0010	0.0000	0.0000
0.01	1.5	0.0003	0.0000	0.0000
	2.0	0.0001	0.0000	0.0000

In Table 4.17, the probability of lot acceptance decreases until zero when the number of preceding lot and pre-specified testing time increases. The MGChSP provides a strict inspection such that the probability of lot acceptance decreases very rapidly when the lot contains greater proportion defective. Consider β = 0.10, g = 1, a = 0.7, i = 1, r = 3, $\mu/\mu_0 = 1$, and $\lambda = 2$, the probability of lot acceptance is 0.0114 from Table 4.17. The probability of lot acceptance decreases from 0.0114 to 0.0001 when the number of preceding lot increases from 1 to 3. For the same design parameters, the probability of lot acceptance also decreases from 0.0114 to 0.0001when pre-specified testing time increases from 0.7 to 2.0. It shows that the greater value of preceding lots and pre-specified testing time reduces the probability of lot acceptance of a product. By considering various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3,\ 4$), the probability of lot acceptance is obtained and shown in Tables 4.18 to 4.19.

Table 4.18

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 3$

		i		
β	a	1	2	3
	0.7	**0.0243**	0.0030	**0.0003**
0.25	0.8	0.0146	0.0013	0.0001
0.10	1.0	0.0055	0.0003	0.0000
0.05	1.2	0.0022	0.0001	0.0000
0.01	1.5	0.0006	0.0000	0.0000
	2.0	0.0001	0.0000	0.0000

Table 4.19

Operating characteristic values for μ/μ_0=1, r=3, g=1 when λ =4

		i		
β	a	1	2	3
	0.7	**0.0322**	0.0047	0.0005
0.25	0.8	0.0197	0.0021	0.0002
0.10	1.0	0.0075	0.0004	0.0000
0.05	1.2	0.0030	0.0001	0.0000
0.01	1.5	0.0008	0.0000	0.0000
	2.0	0.0001	0.0000	0.0000

Similar to Table 4.17, the probability of lot acceptance decreases when the number of preceding lots and pre-specified testing time increases. It shows increasing behaviour when the value of shape parameter increases as shown in Tables 4.18 and 4.19. For β = 0.10, g = 1, a = 0.7, i = 1, r = 3, $\mu/\mu_0 = 1$, and $\lambda = 3$, the probability of lot acceptance is 0.0243 from Table 4.18. The probability of lot acceptance decreases from 0.0243 to 0.0003 when the number of preceding lot increases from 1 to 3. For the same design parameters, the probability of lot acceptance increases from 0.0114 to 0.0322 when the shape parameter increases from 2 to 4. This tendency is shown in Figure 4.6.

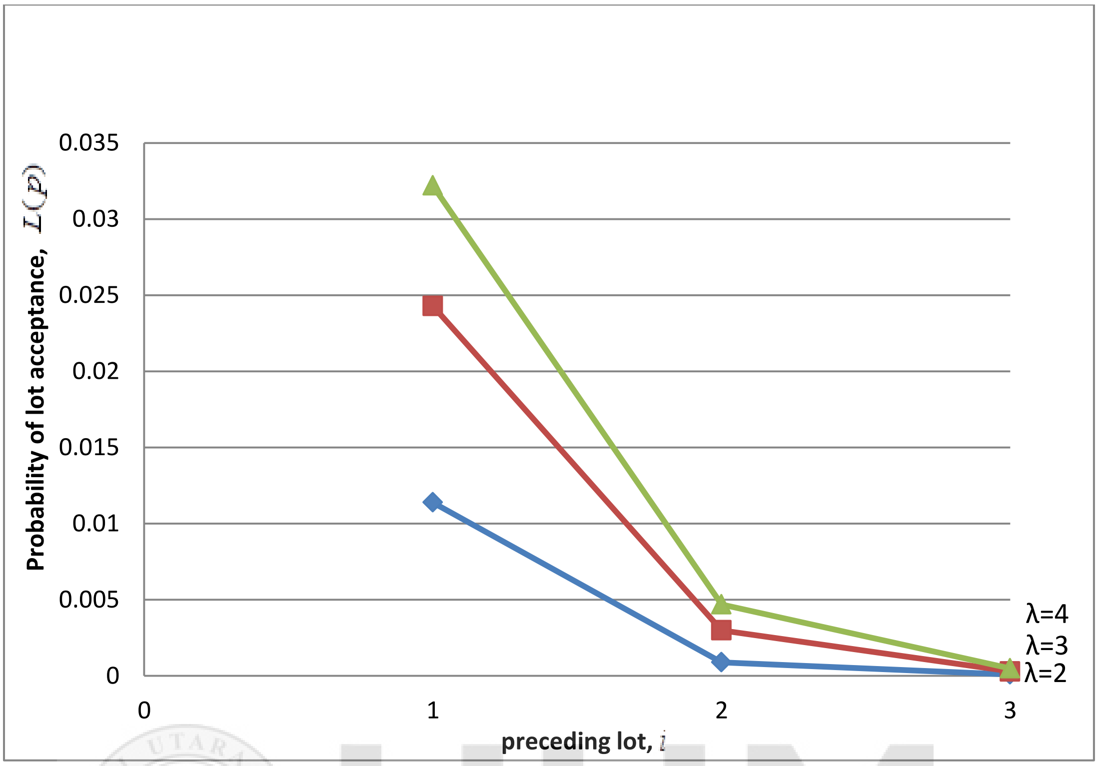

Figure 4.6. Probability of lot acceptance versus preceding lot for MGChSP

From reviewing Figure 4.6, the probability of lot acceptance of a submitted product decreases when the number of preceding lot increases. This curve shows that, if the number preceding lot increases from 1 to 3, then the probability of lot acceptance decreases from 0.0114 to 0.0001 respectively when β = 0.10, g = 1, a = 0.7, i = 1, r = 3, $\mu/\mu_0 = 1$, and $\lambda = 2$. It means that most of the lots are rejected and provided with the similar values of probability of lot acceptance when the higher numbers of preceding lots are considered. On the other hand, the chance of lot acceptance increases from 1% to 3% when the shape parameter increases from 2 to 4 for the same pre-specified design parameters.

4.3 Two-Sided Group Chain Sampling Plan (TS-GChSP)

In Sections 4.2 and 4.3, only preceding lot information was considered, however this section will includes succeeding lot information as well as the consideration of consumer's risk. Based on the Procedure 3.3 (from Chapter 3), two-sided group chain sampling plan (TS-GChSP) are proposed using the cumulative information of preceding as well as succeeding lots. The probability of zero and one defective product for TS-GChSP can be written in the following form by using probability law of addition,

$$L(p)_{TS-GChSP} = \left\{P(d=0)\middle|P(d=0)_{i,j}\right\} + \left\{P(d=0)\middle|P(d=1)_{i,j}\right\} \quad 4.22$$

In the sample of size, $n = r * g$, the submitted lot will be accepted if the current, preceding, i, and succeeding, j, lots have no defective product. The lot is also accepted if the current lot has zero defective but either preceding, i, or succeeding, j, lots have only one defective product. The above mentioned procedure is illustrated in Figure 4.7, where, D, and $\overline{D}$, denote the defective and non-defective products respectively.

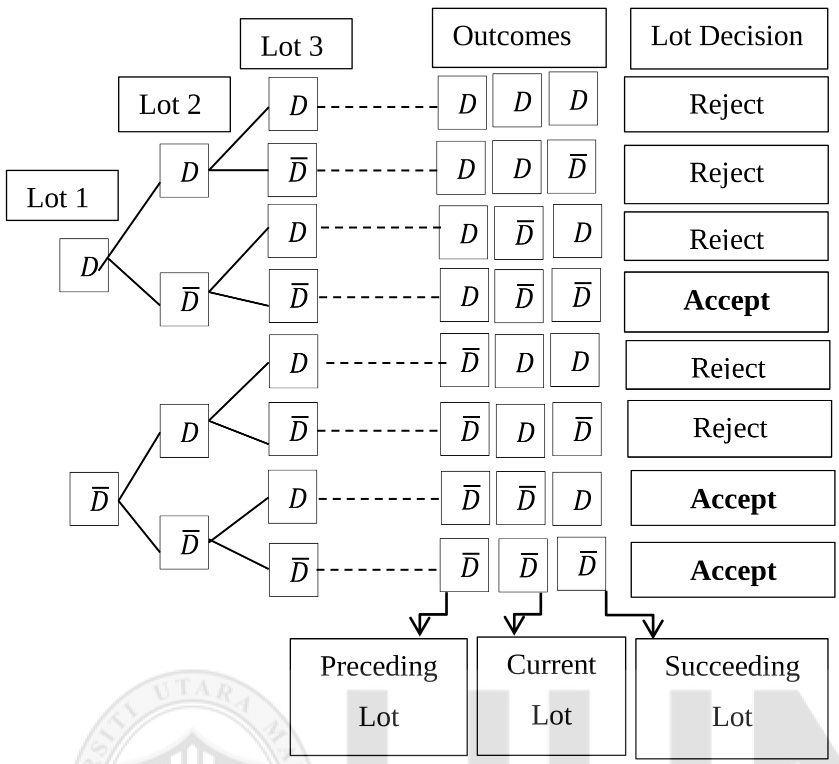

Figure 4.7. A schematic structure of two-sided chain sampling

As depicted in Figure 4.7 when $i = j = 1$, the lot can be accepted based on the these outcomes, $\{\bar{D}\bar{D}\bar{D}, \bar{D}\bar{D}D, D\bar{D}\bar{D}\}$; hence, the probability of lot acceptance for TS-GChSP can be written in the following form,

$$L(p)_{TS-GChSP} = \{(P_{0,(r*g)})(P_{0,(r*g)})(P_{0,(r*g)}) + (P_{1,(r*g)})(P_{0,(r*g)})(P_{0,(r*g)}) + (P_{0,(r*g)})(P_{0,(r*g)})(P_{1,(r*g)})\}; \quad 4.23$$

After simplification of the above equation 4.23, then it becomes,

$$L(p)_{TS-GChSP} = \left\{(P_{0,(r*g)})^3 + P_{1,(r*g)}(P_{0,(r*g)})^2 + P_{1,(r*g)}(P_{0,(r*g)})^2\right\}; \quad 4.24$$

$$L(p)_{TS-GChSP} = \left\{(P_{0,(r*g)})^3 + 2P_{1,(r*g)}(P_{0,(r*g)})^2\right\}. \quad 4.25.$$

Based on the above Equation 4.25, the general expression of probability of lot acceptance of TS-GChSP is,

$$L(p)_{TS-GChSP} = \left\{\left(P_{0,(r*g)}\right)^{(i+j+1)} + (i+j)\,(P_{1,(r*g)})\left(P_{0,(r*g)}\right)^{(i+j)}\right\} \qquad 4.26.$$

Considering that the Binomial distribution under the condition, $i = j$, the above equation transforms into the following,

$$L(p)_{TS-GChSP} =$$

$$\left\{\binom{r*g}{0}\, p^0(1-p)^{(r*g)}\right\}^{(2i+1)} + 2i\left\{\binom{r*g}{1}\, p^1(1-p)^{(r*g)-1}\right\}$$

$$\left\{\binom{r*g}{0}\, p^0(1-p)^{(r*g)}\right\}^{2i} \qquad 4.27,$$

$$L(p)_{TS-GChSP} =$$

$$(1-p)^{(r*g)(2i+1)} + 2i\left\{(r*g)(p)(1-p)^{(r*g)-1}\right\}\left\{(1-p)^{(r*g)(2i)}\right\} \qquad 4.28.$$

After simplifying of the above equation 4.28, the probability of lot acceptance of TS-GChSP is,

$$L(p)_{\mathrm{TS-GChSP}} = (1-p)^{(r*g)(2i+1)}\{1 + 2i(r*g)(p)/(1-p)\} \qquad 4.29.$$

Using the pre-specified proportion defective, p, from Table 4.1, the minimum number of groups, g, is found based on Equation 4.30 below,

$$L(p)_{TS-GChSP} = (1-p)^{(r*g)(2i+1)}\{1 + 2i(r*g)(p)/(1-p)\} \le \beta \qquad 4.30.$$

For various values of β, i, and r, the minimum number of groups, g, is presented in Table 4.20.

Table 4.20

Number of minimum groups required for TS-GChSP when $\lambda = 2$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

In Table 4.20, the required number of groups for TS-GChSP is almost similar but it decreases when the value of pre-specified testing, consumer's risk, preceding, and succeeding lots and number of testers increases. Consider β = 0.01, a = 0.7, r = 2, i = j = 1, and $\lambda = 2$ where the required number of groups is 2, while a = 0.7, r = 3, i = j =2 and the required number of groups is 1 as depicted in Table 4.20. This means that when the number of tester, preceding and succeeding lots increases, a smaller number of groups are needed to reach a valid conclusion. Assuming that average life of a product, $\mu = \mu_0$=10,000 hours, β = 0.01, a = 0.7, r = 2, i = j =1, $\lambda =$2, and g = 2, then a sample of size 4 products drawn from the lot where 2 testers are located into 2 groups from Table 4.20 . Based on this information, the submitted lot will be accepted if no defective is observed in preceding, current as well as succeeding samples. The lot is also acceptable if one defective is recorded either in preceding or succeeding samples, but no defective occurs in current sample throughout 7,000 hours. For various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3,\ 4$), the required number of groups for the TS-GChSP are obtained in Tables 4.21 and 4.22.

Table 4.21

Number of minimum groups required for TS-GChSP when $\lambda = 3$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 4.22

Number of minimum groups required for TS-GChSP when $\lambda = 4$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

From Tables 4.21 and 4.22, the number of groups required for the TS-GChSP is quite similar for different values of shape parameter and also shows the same results found in Table 4.20. Using these numbers of group, the probability of lot acceptance is obtained for the desired quality level. For various values of mean ratios (μ/μ_0 = 1, 2, 4, 6, 8, 10, 12), the probability of lot acceptance is presented in Tables 4.23 to 4.25.

Table 4.23

Operating characteristic values for r = 3, $i = j$= 1,g=1 when $\lambda = 2$

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	**0.0009**	0.0267	0.1801	0.3405	0.4630	0.5530	**0.6200**
0.25	0.8	0.0004	0.0158	0.1367	0.2844	0.4064	0.5000	0.5716
0.10	1.0	0.0001	0.0057	0.0788	0.1974	0.3113	0.4064	0.4833
0.05	1.2	0.0000	0.0021	0.0457	0.1367	0.2371	0.3285	0.4064
0.01	1.5	0.0000	0.0005	0.0205	0.0788	0.1569	0.2371	0.3113
	2.0	0.0000	0.0000	0.0057	0.0319	0.0788	0.1367	0.1974

The effect of various values of mean ratio and pre-specified testing time is shown in Table 4.23. The probability of lot acceptance increases as the mean ratio increases, but decreases when the pre-specified testing time increases. Assuming β = 0.01, g = 1, a = 0.7, $i = j$=1, r = 3, $\mu/\mu_0 = 1$, and $\lambda = 2$, the probability of lot acceptance is 0.0009 from Table 4.23. The probability of lot acceptance increases from 0.0009 to 0.6200 when the mean ratio increase from 1 to 12. Meanwhile, the probability of lot acceptance decreases from 0.0009 to 0.0000 when pre-specified testing time increases from 0.7 to 2.0. It is noted that the proposed TS-GChSP converted to the above MGChSP yields similar results when the number of preceding lot is equal to the number of succeeding lot or $i = j$. For the same design parameters, the probability of lot acceptance is obtained and placed in Tables 4.24 to 4.25 for various values of shape parameter of the Pareto distribution of the 2nd kind.

Table 4.24

Operating characteristic values for r = 3, $i = j$= 1,g = 1 when $\lambda = 3$

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	**0.0030**	0.0608	0.2821	0.4570	0.5735	0.6526	**0.7088**
0.25	0.8	0.0013	0.0390	0.2277	0.3994	0.5211	0.6066	0.6684
0.05	1.0	0.0003	0.0162	0.1473	0.3027	0.4274	0.5211	0.5917
0.05	1.2	0.0001	0.0068	0.0947	0.2277	0.3481	0.4450	0.5211
0.01	1.5	0.0000	0.0019	0.0487	0.1473	0.2536	0.3481	0.4274
	2.0	0.0000	0.0026	0.0162	0.0705	0.1473	0.2277	0.3027

Table 4.25 *Operating characteristic values for r = 3, $i = j$= 1,g = 1 when $\lambda = 4$*

β	a	μ/μ_0						
		1	2	4	6	8	10	12
	0.7	**0.0047**	0.0815	0.3282	0.5037	0.6149	0.6886	0.7401
0.25	0.8	0.0021	0.0540	0.2710	0.4472	0.5654	0.6459	0.7032
0.05	1.0	0.0004	0.0237	0.1829	0.3496	0.4748	0.5654	0.6321
0.05	1.2	0.0001	0.0104	0.1224	0.2710	0.3959	0.4920	0.5654
0.01	1.5	0.0000	0.0031	0.0664	0.1829	0.2984	0.3959	0.4748
	2.0	0.0000	0.0004	0.0237	0.0933	0.1829	0.2710	0.3496

The results of Tables 4.24 to 4.25 show the probability of lot acceptance for different value of shape parameters. Consider, β = 0.01, g = 1, a = 0.7, $i = j$= 1, r = 3, $\mu/\mu_0 = 1$, and λ =3 where the probability of lot acceptance is 0.0030 from Table 4.24. The probability of lot acceptance increases from 0.0030 to 0.7088 when the mean ratio increase from 1 to 12. For the same above mentioned design parameters the probability of lot acceptance also increases from 0.0009 to 0.0047 when the value of shape parameter increases from 2 to 4. This increasing trend is illustrated in Figure 4.8.

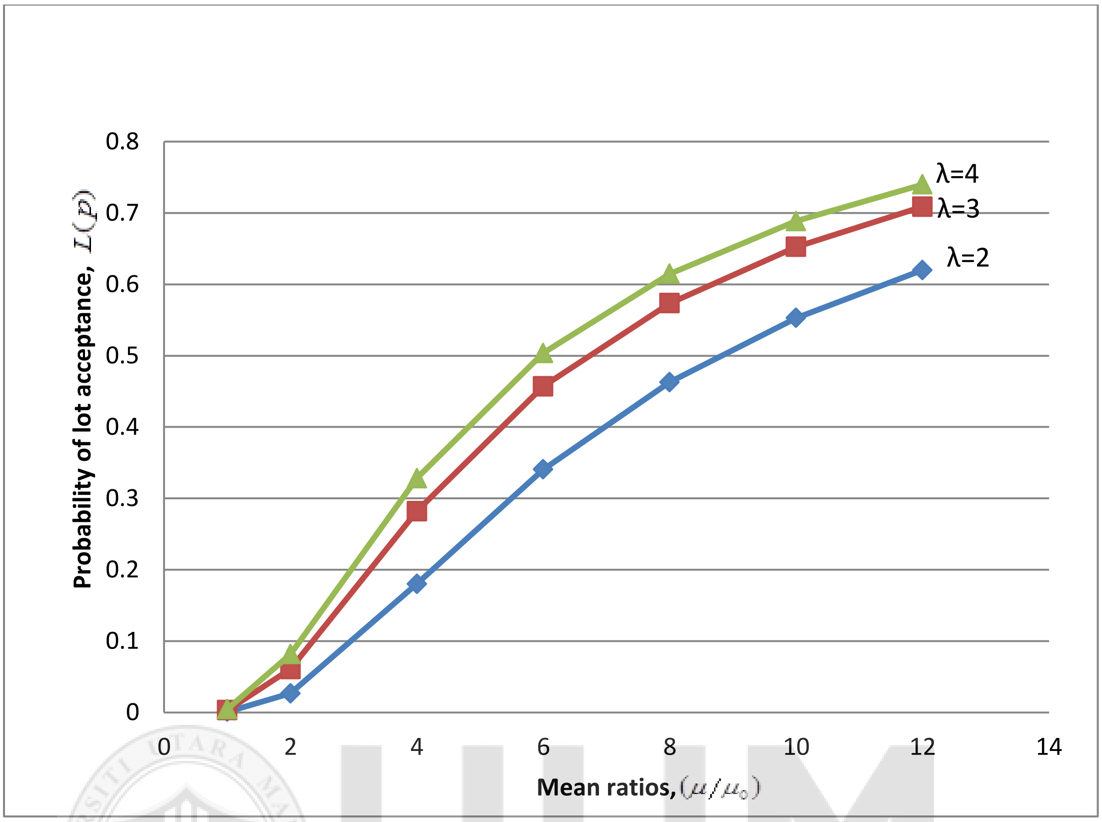

Figure 4.8. Probability of lot acceptance versus mean ratios for TS-GChSP

After observing Figure 4.7, the probability of lot acceptance of a submitted product increases when the mean ratio and the value of shape parameter increases. The true average life increases from 1 to 12 times of specified average life then the probability of lot acceptance increases from 0.0009 to 0.6200 when $\lambda = 2$. Meanwhile when the shape parameter increases from 2 to 4, the probability of lot acceptance also increases from 0.6200 to 0.7401 for the same design parameters when $\mu/\mu_0 = 12$. It can be seen that the probability of lot acceptance increases when the mean ratio increases and the greater value of shape parameter produce the higher probability of lot acceptance than the smaller one. For the same design parameters as mentioned in Tables 4.23 and 4.25, the probability of lot acceptance is found for various values of preceding and succeeding lots, ($i = j = 1,\ 2$), and presented in Tables 4.26 to 4.28.

Table 4.26

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 2$

β	a	i = 1	i = 2
	0.7	**0.0009**	**0.0000**
0.25	0.8	0.0004	0.0000
0.10	1.0	0.0000	0.0000
0.05	1.2	0.0000	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

According to the observations of Table 4.26, the probability of lot acceptance decrease and monotonically approaches to zero when the number preceding, succeeding lots and pre-specified testing time increase. For β = 0.10, g = 1, a = 0.7, $i = j$= 1, r = 3 and $\lambda = 2$, the probability of lot acceptance is 0.0009 when μ/μ_0 =1, from Table 4.26. The probability of lot acceptance decreases 0.0009 to 0.0000 when the number of preceding and succeeding lot increases from 1 to 2. The probability of lot acceptance also decreases from 0.0009 to 0.0000 when pre-specified testing time increases from 0.7 `to 2.0. It means that either number of preceding and succeeding lots or pre-specified testing time increases, the probability of lot acceptance decreases until it reaches zero. Based on these results, the probability of lot acceptance is found and shown in Tables 4.27 to 4.28 for various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3$, 4).

Table 4.27

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 3$

		i	
β	*a*	1	2
	0.7	**0.0030**	**0.0000**
0.25	0.8	0.0013	0.0000
0.10	1.0	0.0003	0.0000
0.05	1.2	0.0001	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

Table 4.28

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 4$

		i	
β	*a*	1	2
	0.7	**0.0047**	0.0001
0.25	0.8	0.0021	0.0000
0.10	1.0	0.0004	0.0000
0.05	1.2	0.0001	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

Similar to Table 4.26, the probability of lot acceptance decreases as the number preceding, succeeding lots increases from Tables 4.27 to 4.28. On the other hand, this shows the increasing trend when the value of shape parameter increases. Consider β = 0.10, g = 1, a = 0.7, $i = j$= 1, r = 3 and $\lambda = 3$, the probability of lot acceptance is 0.0030 when $\mu/\mu_0 = 1$, from Table 4.27. The probability of lot acceptance decreases from 0.0030 to 0.0000 when the number of preceding and succeeding lot increases from 1 `to 2. Meanwhile when the shape parameter increases from 2 to 4, the probability of lot acceptance also increases from 0.0009 to

0.0047 for the same above mentioned design parameters. This trend is portrayed in the following Figure 4.9.

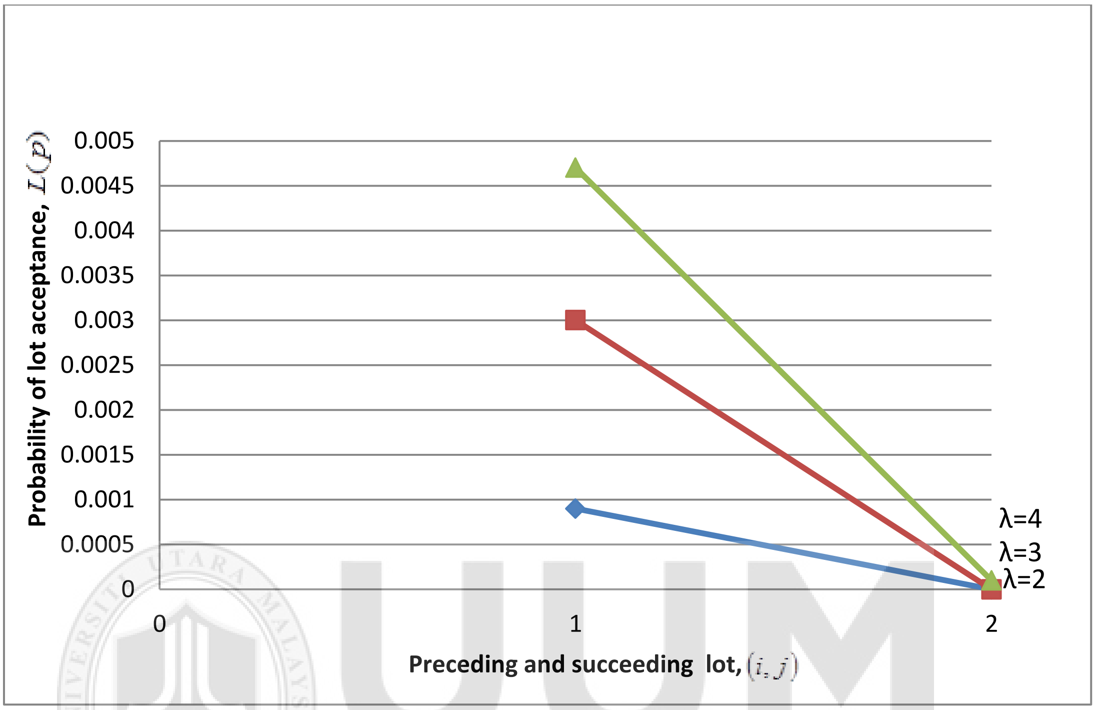

Figure 4.9. Probability of lot acceptance versus preceding and succeeding lot for TS-GChSP

From examining the above Figure 4.9, the probability of lot acceptance of a submitted product decreases when the number preceding and succeeding lot increases. It means most of the lots are rejected when the number preceding and succeeding lot increases. The probability of lot acceptance decreases 0.0009 to 0.0000 when the number of preceding lot increases from 1 to 2 but it increases 0.0009 to 0.0047 when the shape parameter increases from 2 to 4 respectively.

4.4 Two-Sided Modified Group Chain Sampling Plan (TS-MGChSP)

In this section, a two-sided modified group chain sampling plan (TS-MGChSP) is proposed using the cumulative information of preceding as well as succeeding lots as described in Procedure 3.4 (Chapter 3). The decision about the submitted lot, which is either accept or reject, is made based on the current, preceding, i, and the succeeding, j, samples of information. The probability of zero and one defective product for TS-MGChSP can be written in the following form by using probability law of addition,

$$L(p)_{TS-MGChSP} = \{P(d=0)|P(d=0)_{i,j}\} + \{P(d=1)|P(d=0)_{\ i,j}\} \quad 4.31$$

In a sample of size, $n = r * g$, the submitted lot is accepted if the current sample as well as the preceding, i, and the succeeding, j, samples contain no defective product. The lot is also accepted if the current lot has one defective but preceding, i, and succeeding, j, lots have no defective products as shown in Figure 4.10 .

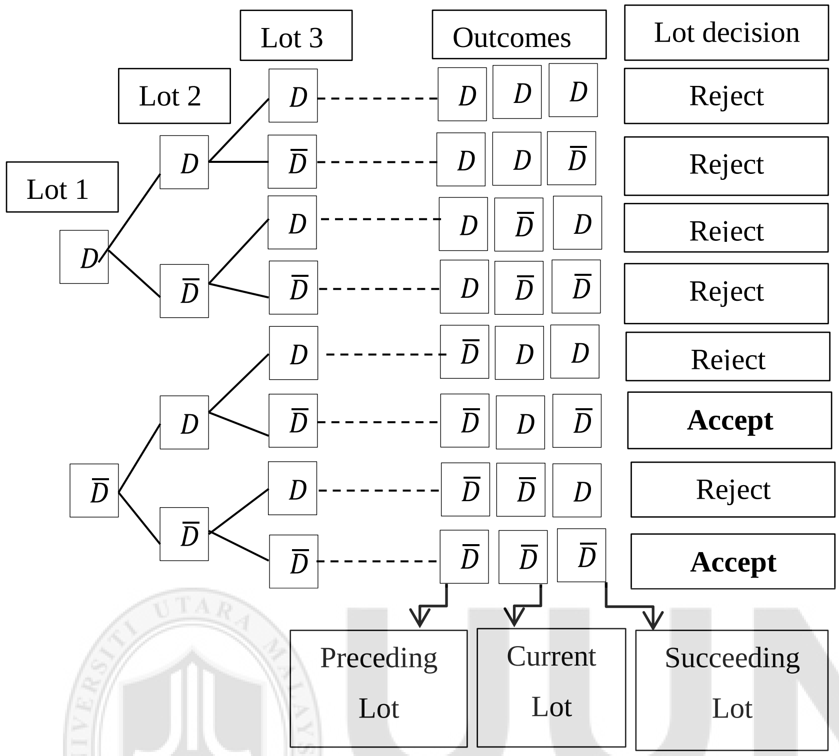

Figure 4.10. A schematic structure of two-sided chain sampling

According to Figure 4.10 when $i = j = 1$, based on these outcomes, $\{\bar{D}\bar{D}\bar{D}, \bar{D}D\bar{D}\}$, the probability of lot acceptance of TS-MGChSP can be written in the following form,

$$L(p)_{TS-MGChSP} =$$

$$\{(P_{0,(r*g)})(P_{0,(r*g)})(P_{0,(r*g)}) + (P_{0,(r*g)})(P_{1,(r*g)})(P_{0,(r*g)})\}; \qquad 4.32$$

after simplification of the above equation, then it becomes,

$$L(p)_{\text{TS-MGChSP}} = \left\{\left(P_{0,(r*g)}\right)^3 + P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2\right\}. \qquad 4.33$$

Based on the above equation, the general expression of probability of lot acceptance of TS-GChSP

$$L(p)_{TS-MGChSP} = \left\{\left(P_{0,(r*g)}\right)^{(i+j+1)} + \left(P_{1,(r*g)}\right)\left(P_{0,(r*g)}\right)^{(i+j)}\right\} \quad 4.34$$

Considering that the Binomial distribution under the condition, $i = j$, then the above equation converts to the following forms,

$$L(p)_{TS-MGChSP} =$$

$$\left\{\binom{r*g}{0} p^0(1-p)^{(r*g)}\right\}^{(2i+1)} +$$

$$\left\{\binom{r*g}{1} p^1(1-p)^{(r*g)-1}\right\}\left\{\binom{r*g}{0} p^0(1-p)^{(r*g)}\right\}^{2i} \quad 4.35$$

$$L(p)_{TS-MGChSP} =$$

$$(1-p)^{(r*g)(2i+1)} + \left\{(r*g)(p)(1-p)^{(r*g)-1}\right\}\left\{(1-p)^{(r*g)(2i)}\right\} \quad 4.36$$

After simplification of the above equation 4.36, the probability of lot acceptance for TS-MGChSP becomes,

$$L(p)_{TS-MGChSP} = (1-p)^{(r*g)(2i+1)}\{1 + (r*g)(p)/(1-p)\} \quad 4.37$$

Using the pre-specified proportion defective, p, from Table 4.1, the minimum number of groups, g, are found based on Equation 4.38,

$$L(p)_{\mathrm{TS}-MGChSP} = (1-p)^{(r*g)(2i+1)}\{1 + (r*g)(p)/(1-p)\} \leq \beta \quad 4.38$$

For various values of β, i, and r, the minimum number of groups, g, presented in Tables 4.29 to 4.31.

Table 4.29

Number of minimum groups required for TS-MGChSP when $\lambda = 2$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

In Table 4.29, the number of groups required for the TS-MGChSP is similar for different value of design parameters. Assuming the average life of a product, $\mu = \mu_0$= 10,000 hours and other above mentioned pre-specified design parameters are β = 0.01, a = 0.7, r = 2, $i = j = 1$, $\lambda = 2$ and $g = 1$, from Table 4.29, then a sample of size 2 products drawn from the lot where 2 testers are located into 1 group. Using this information, the submitted lot will be accepted if no defective is observed in preceding, current as well as succeeding sample. The lot is also acceptable if one defective occurs in current sample but no defective is recorded in preceding and succeeding sample during 7,000 hours. Based on these design parameters the number of groups required for the TS-MGChSP is obtained and placed in Tables 4.30 and 4.31for various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3, 4$).

Table 4.30

Number of minimum groups required for TS-MGChSP when $\lambda = 3$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 4.31

Number of minimum groups required for TS-MGChSP when $\lambda = 4$

β	r	$i = j$	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Similar to Table 4.29, the required number of groups for the TS-MGChSP for different value of design parameters but it decreases when the pre-specified testing time, consumer's risk, number of preceding, succeeding lots and number of testers increases shown in Tables 4.30 and 4.31. Considering, β = 0.01, a = 0.7, r = 2, $i = j$ =1, and $\lambda = 3$, the required number of groups are 2. For the same design parameters, when r = 3, the required number of groups is 1, from Table 4.30. It is clear indication that when the number of preceding lots, succeeding lots and number of tester increases, a small number of groups are required to reach the valid conclusion about the submitted lot. Using these numbers of group, the probability of lot acceptance is obtained for the desired quality level. For various values of mean ratios (μ/μ_0 = 1, 2, 4, 6, 8, 10, 12) and Pareto distribution of the 2nd kind ($\lambda = 2$, 3, 4) the probability of lot acceptance is presented in Table 4.32.

Table 4.32

Operating characteristic values for r = 3, $i = j$ = 1and g = 1

λ	β	a	μ/μ_0 1	2	4	6	8	10	12
2	0.25	0.7	**0.0005**	0.0156	0.1175	0.2389	0.3420	0.4245	**0.4902**
	0.10	0.8	0.0002	0.0091	0.0871	0.1948	0.2932	0.3752	0.4423
	0.05	1.0	0.0000	0.0032	0.0484	0.1299	0.2157	0.2932	0.3601
	0.01	1.2	0.0000	0.0012	0.0273	0.0871	0.1590	0.2293	0.2932
		1.5	0.0000	0.0003	0.0119	0.0484	0.1012	0.159	0.2157
		2.0	0.0000	0.0001	0.0032	0.0188	0.0484	0.0871	0.1299
3	0.25	0.7	**0.0016**	0.0368	0.1930	0.3367	0.4441	0.5238	**0.5845**
	0.10	0.8	0.0007	0.0232	0.1520	0.2873	0.3945	0.4767	0.5405
	0.05	1.0	0.0001	0.0093	0.0944	0.2090	0.3110	0.3945	0.4620
	0.01	1.2	0.0000	0.0038	0.0589	0.1520	0.2450	0.3262	0.3945
		1.5	0.0000	0.0011	0.0292	0.0944	0.1713	0.2450	0.3110
		2.0	0.0000	0.0001	0.0093	0.0430	0.0944	0.1520	0.2090
4	0.25	0.7	**0.0026**	0.0502	0.2291	0.3785	0.4851	0.5623	0.6200
	0.10	0.8	0.0012	0.0326	0.1845	0.3281	0.4363	0.5169	0.5782
	0.05	1.0	0.0002	0.0138	0.1195	0.2462	0.3524	0.4363	0.5025
	0.01	1.2	0.0001	0.0060	0.0774	0.1845	0.2843	0.3678	0.4363
		1.5	0.0000	0.0017	0.0404	0.1195	0.2056	0.2843	0.3524
		2.0	0.0000	0.0002	0.0138	0.0580	0.1195	0.1845	0.2462

The observations of Table 4.32 present the pattern of probability of lot acceptance when the value of mean ratio increases. Assuming that β = 0.01, g = 1, a = 0.7, $i = j$=1, r = 3, μ/μ_0 = 1, and $\lambda = 2$, the probability of lot acceptance is 0.0005 as shown in Table 4.32. The probability of lot acceptance increases from 0.0005 to 0.4902 when the mean ratio increase from 1 to 12 and signifying that the greater mean ratio would lead to the higher probability of lot acceptance. Similarly, the probability of lot acceptance increases when the mean ratio and the value of shape parameters increase are shown in Table 4.32. Consider β = 0.01, g = 1, a = 0.7, $i = j$= 1, r = 3, μ/μ_0 = 1, and $\lambda = 3$ where the probability of lot acceptance is 0.0016. The probability of lot acceptance increases 0.0016 to 0.5845 when the mean ratio increase from 1 to 12. For the same design parameters the probability of lot acceptance also increases from 0.0005 to 0.0026 when the value of shape parameter increases from 2 to 4. This increasing trend is illustrated in Figure 4.11.

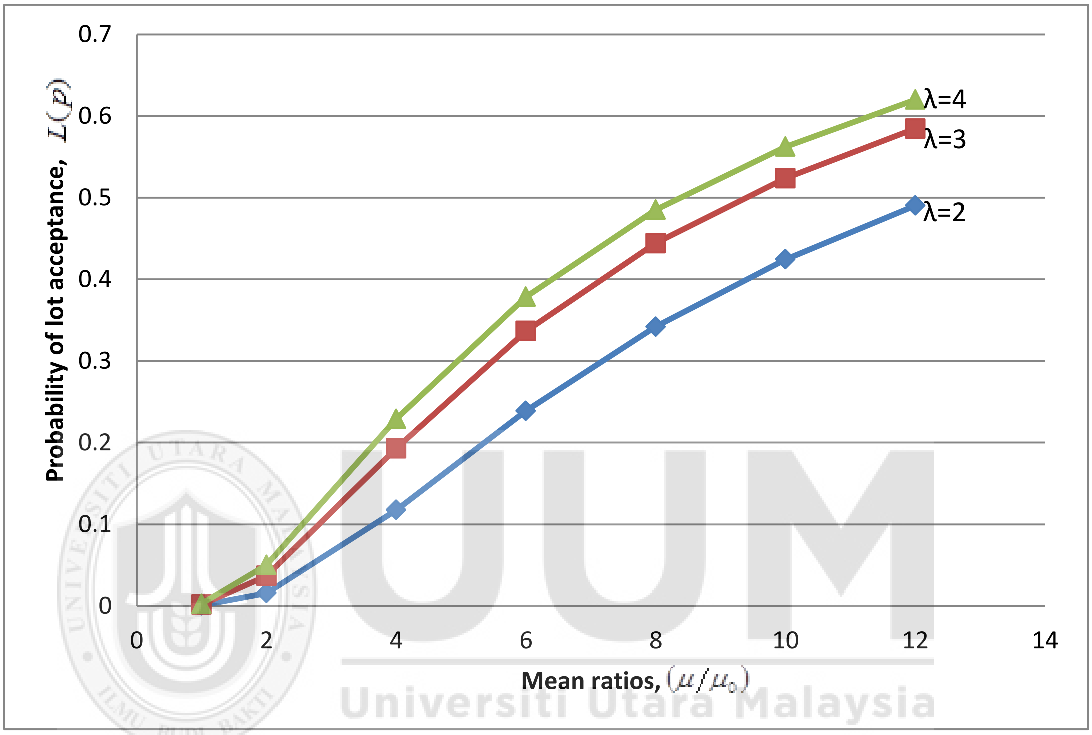

Figure 4.11. Probability of lot acceptance versus mean ratios for TS-MGChSP

In Figure 4.11, the probability of lot acceptance of a submitted product increases when the mean ratio and the value of shape parameter increases. The probability of lot acceptance increases from 0.0005 to 0.4902 when the mean ratio increased from 1 to 12, for λ =2. Meanwhile, when the shape parameter increases from 2 to 4, the probability of lot acceptance also increases from 0.0005 to 0.0026 respectively. For the same design parameters as mentioned in Table 4.32, the probability of lot acceptance is found for various values of preceding and succeeding lots, ($i = j$ =1, 2)which is presented in Table 4.33.

Table 4.33

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 2$

		$i = j$	
β	a	1	2
	0.7	**0.0005**	**0.0000**
0.25	0.8	0.0002	0.0000
0.10	1.0	0.0000	0.0000
0.05	1.2	0.0000	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

From Table 4.33, the probability of lot acceptance decreases as the number of preceding, succeeding lot and pre-specified testing time increases. For β = 0.10, g = 1, a = 0.7, $i = j$= 1, r = 3, μ/μ_0 = 1, and λ = 2, the probability of lot acceptance is 0.0005 from Table 4.35. The probability of lot acceptance decreases from 0.0005 to 0.0000 when the number preceding and succeeding lot increases from 1 to 2. The probability of lot acceptance also decreases from 0.0005 to 0.0000 when pre-specified testing time increases from 0.7 to 2.0. By considering various values of shape parameter of the Pareto distribution of the 2nd kind ($\lambda = 3$, 4), the probability of lot acceptance is obtained and shown in Tables 4.34 and 4.35.

Table 4.34

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 3$

		$i = j$	
β	a	1	2
	0.7	**0.0016**	**0.0000**
0.25	0.8	0.0007	0.0000
0.10	1.0	0.0001	0.0000
0.05	1.2	0.0000	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

Table 4.35

Operating characteristic values for μ/μ_0 = 1, r = 3, g = 1 when $\lambda = 4$

		i	
β	a	1	2
	0.7	**0.0026**	0.0000
0.25	0.8	0.0012	0.0000
0.10	1.0	0.0002	0.0000
0.05	1.2	0.0001	0.0000
0.01	1.5	0.0000	0.0000
	2.0	0.0000	0.0000

Similar to Table 4.33, the probability of lot acceptance decreases as the number of preceding, succeeding lots and pre-specified testing time increases as shown in Tables 4.34 and 4.35. It increases when the value of shape parameter of Pareto distribution of the 2nd kind increases. If β = 0.10, g = 1, a = 0.7, $i = j$= 1, r = 3, μ/μ_0 = 1, and $\lambda = 3$, the probability of lot acceptance is 0.0016 from Table 4.33. The probability of lot acceptance decreases from 0.0016 to 0.0000 when the number of preceding and succeeding lot increases from 1 to 2. For the same design parameters the chances of lot acceptance increases from 0.05% to 0.26% when value of shape parameter increases from 2 to 4 and this trend is shown in Figure 4.12.

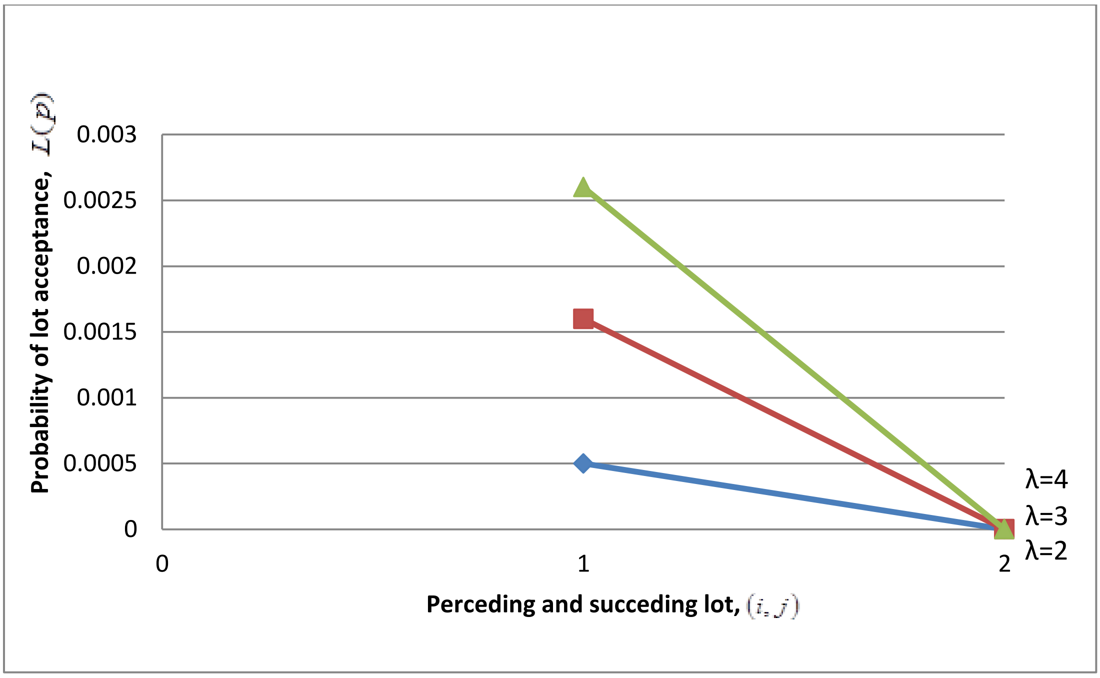

Figure 4.12. Probability of lot acceptance versus preceding and succeeding lot for TS-MGChSP

From inspecting Figure 4.10, the probability of lot acceptance of a submitted product decreases when the number of preceding and succeeding lot increases. Most of the lots are rejected when the inspection contains a greater number of preceding and succeeding lots. These curves show that, if the number preceding lot increases from 1 to 2, the probability of lot acceptance slightly decreases from 0.0005 to 0.0000 when, $\lambda = 2$. The probability of lot acceptance increases from 0.0005 to 0.0026 when the shape parameter increases from 2 to 4.

In the next chapter, four generalized sampling plans are proposed (based on the four plans discussed previously) to inspect the lifetime of a submitted product by considering several pre-specified values of proportion defective. The minimum number of groups, probabilities of lot acceptance and their comparisons are shown in tables and figures.

CHAPTER FIVE
GENERALIZED GROUP CHAIN SAMPLING PLANS

In Chapter 4, the group chain sampling plan (GChSP), modified group chain sampling plan (MGChSP), two-sided group chain sampling plan (TS-GChSP) and two-sided modified group chain sampling plan (TS-MGChSP) were presented. It is to be noted that each of the plans considers only the specified value of proportion defective based on Pareto distribution of the 2^{nd} kind for pre-specified values of testing time, mean ratio and shape parameters. However, in practice, the value of proportion defective varies from lot to lot. Therefore, in this chapter, several values of proportion defective are considered. Based on procedures 3.1 to 3.4 (Chapter 3), generalized group chain sampling plans are proposed. Sections 5.1 to 5.4 describe the (i) generalized GChSP, (ii) generalized MGChSP, (iii) generalized TS-GChSP and (iv) generalized TS-MGChSP respectively. Finally, in Section 5.5, a real lifetime data set is used to illustrate all the proposed plans and graphical results are provided for comparison purposes.

5.1 Generalized Group Chain Sampling Plan (GGChSP)

Using the pre-specified values of proportion defective, p, the minimum number of groups, g, are found for GGChSP based on Equation 4.12 (as mentioned in page 64) when satisfying the other design parameters. The values of different combination of design parameters based on previous studies and various values of β, i, and r, the minimum number of groups, g, are presented in Table 5.1.

Table 5.1

Number of minimum groups required for GGChSP

			β			
p	r	i	0.25	0.10	0.05	0.01
0.001	2	1	830	1245	1562	2324
	3	2	488	775	1000	1534
	4	3	352	576	749	1151
	5	4	279	461	599	921
0.005	2	1	166	249	312	464
	3	2	98	155	200	307
	4	3	71	115	150	230
	5	4	56	92	119	184
0.010	2	1	83	**124**	156	232
	3	2	49	**78**	100	153
	4	3	35	58	75	115
	5	4	28	46	60	92
0.015	2	1	55	83	104	154
	3	2	33	52	67	102
	4	3	24	39	50	77
	5	4	19	31	40	61
0.020	2	1	42	62	78	116
	3	2	25	39	50	76
	4	3	18	29	38	57
	5	4	14	23	30	46
0.025	2	1	33	50	62	92
	3	2	20	31	40	61
	4	3	14	23	30	46
	5	4	11	19	24	37
0.030	2	1	28	41	52	77
	3	2	17	26	33	51
	4	3	12	19	25	38
	5	4	10	16	20	31
0.035	2	1	24	35	44	66
	3	2	14	22	29	44
	4	3	10	17	22	33
	5	4	8	13	17	26
0.040	2	1	21	31	39	57
	3	2	12	19	25	38
	4	3	9	15	19	29
	5	4	7	12	15	23
0.045	2	1	19	28	34	51
	3	2	11	17	22	34
	4	3	8	13	17	26
	5	4	5	11	14	21
0.050	2	1	17	25	31	46
	3	2	9	16	20	30
	4	3	7	12	15	23
	5	4	6	9	12	18

			β			
p	r	i	0.25	0.10	0.05	0.01
	2	1	15	23	28	42
0.055	3	2	9	14	18	28
	4	3	7	11	14	21
	5	4	5	9	11	17
	2	1	14	21	26	38
0.060	3	2	8	13	17	25
	4	3	6	10	13	19
	5	4	5	8	10	15
	2	1	13	19	24	35
0.065	3	2	8	12	15	23
	4	3	6	9	12	18
	5	4	5	7	9	14
	2	1	12	18	22	33
0.070	3	2	7	11	14	22
	4	3	5	8	11	16
	5	4	4	7	9	13
	2	1	11	17	20	30
0.075	3	2	7	10	13	20
	4	3	5	8	10	15
	5	4	4	6	8	12
	2	1	11	15	19	28
0.080	3	2	6	10	12	19
	4	3	5	7	9	14
	5	4	4	6	8	12
	2	1	10	15	18	27
0.085	3	2	6	9	12	18
	4	3	4	7	9	13
	5	4	4	6	7	11
	2	1	9	14	17	25
0.090	3	2	6	9	11	17
	4	3	4	7	9	13
	5	4	3	5	7	10
	2	1	9	13	16	24
0.095	3	2	5	8	11	16
	4	3	4	6	8	12
	5	4	3	5	6	10
	2	1	8	12	15	23
0.100	3	2	5	8	10	15
	4	3	4	6	8	11
	5	4	3	5	6	9
	2	1	6	8	10	15
0.150	3	2	4	5	7	10
	4	3	3	4	5	8
	5	4	2	3	4	6
	2	1	4	6	8	11
0.200	3	2	3	4	5	7
	4	3	2	3	4	6
	5	4	2	3	3	5
	2	1	3	5	6	9
0.250	3	2	2	3	4	6
	4	3	2	2	3	4
	5	4	1	2	3	4

			β			
p	r	i	0.25	0.10	0.05	0.01
	2	1	3	4	5	7
0.300	3	2	2	3	3	5
	4	3	1	2	3	4
	5	4	1	2	2	3
	2	1	2	2	3	4
0.350	3	2	2	2	3	4
	4	3	1	1	2	2
	5	4	1	1	1	2

As shown in Table 5.1, horizontally, a larger number of groups is required to achieve a smaller value of consumer's risk. Meanwhile, the number of groups decreases when the number of preceding lots, number of testers and pre-specified proportion defective increases. For example, when β = 0.10, p = 0.010, r = 2, i = 1, a total of 124 groups is required (where sample size = 248), whereas for the same consumer's risk and proportion defective, but at r = 3, i = 2, only 78 groups are required (sample size = 234). This means that when the number of preceding lots and the number of testers increase, a small number of groups (hence sample size) is required to reach a valid conclusion about the submitted lot. Assuming the pre-specified design parameters are p=0.010, r=3, and i=2, then a sample of 234 products is drawn from the lot and tested in 78 groups, each allocated into 3 testers. Based on this information, the submitted lot will be accepted, if no defective is observed or if one defective occurs in the current sample but no defectives are recorded in the preceding two samples. The minimum number of groups for various values of proportion defective and consumer's risk are presented in Table 5.2. The choices of design parameter values are considered only for comparison purpose.

Table 5.2

Number of minimum groups for r = 3 and i = 2

p	β	g	β	g	β	g	β	g
0.001	0.25	488	0.10	775	0.05	1000	0.01	1534
0.005		98		155		200		307
0.010		49		78		100		153
0.015		33		52		67		102
0.020		25		39		50		76
0.025		20		31		40		61

In Table 5.2, the number of groups decreases when the proportion defective and consumer’s risk increase. Considering that the consumer’s risk is 0.10 (10%) and proportion defective is 0.001 (0.1%), the required number of groups is 775. The number of groups decreases from 775 to 155 when proportion defective increases from p = 0.001 to p = 0.005. At a proportion defective of 0.001 (0.1%), when consumer’s risk decreases from 0.25 to 0.01, the number of groups increases from 488 to 1534. This indicates that larger sample size is required for increased customer protection (reduced consumer’s risk). This trend is also displayed in Figure 5.1.

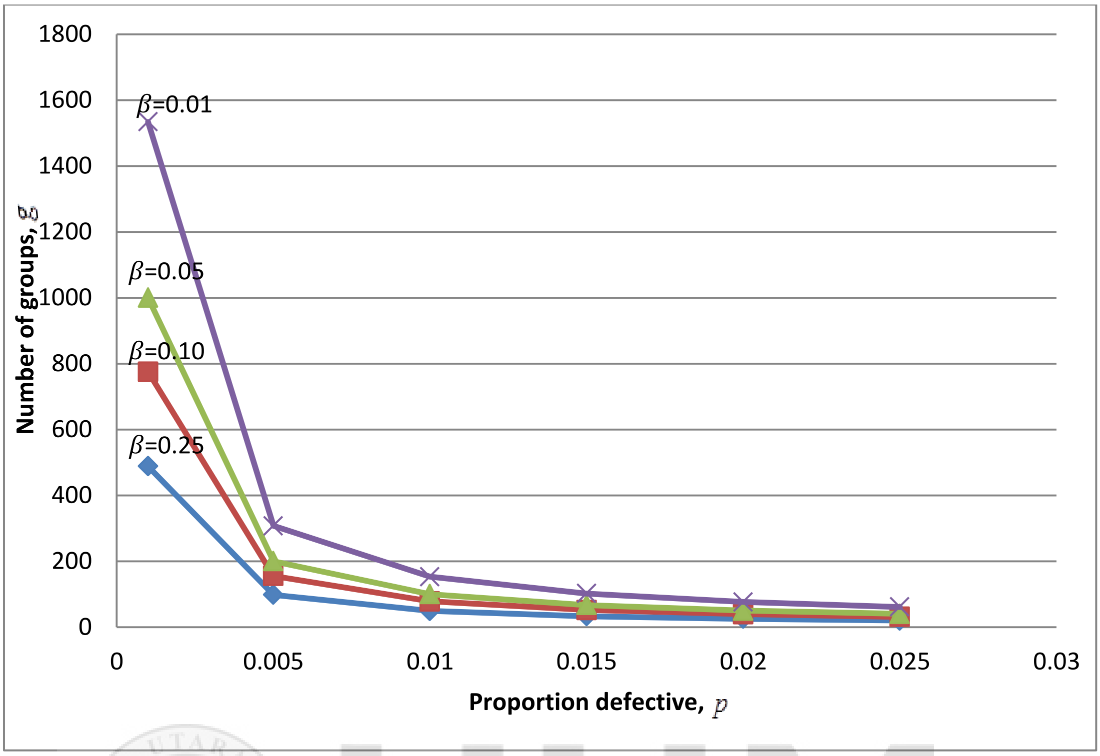

Figure 5.1. Number of groups versus proportion defective for GGChSP

Figure 5.1 shows that the number of groups decreases when the proportion defective increases for a pre-specified value of consumer's risk. For a fixed proportion defective the number of groups increases when the consumer's risk decreases. These curves show that, if the proportion defective increases from 0.1% to 2.5%, the number of groups decreases from 488 to 20 when consumer's risk is 25%. Next, for a fixed value of proportion defective 0.1%, the number of groups increases from 488 to 1534 when consumer's risk decreases from 25% to 1% respectively. On the other hand, the required number of groups monotonically decreases and provided the group size remains the same when the value of proportion defective increases at various values of consumer's risk. Based on the values of proportion defective considered in Table 5.2 when i = 2, the effect of probability of lot acceptance for fixed values of sample size suggested as by Montgomery (2009) is shown in Table 5.3.

Table 5.3

Operating characteristic values for GGChSP

	$n = r * g$							
p	10	20	30	40	50	100	150	200
0.001	0.9998	0.9990	0.9979	0.9963	0.9943	0.9789	0.9564	0.9285
0.005	0.9943	0.9790	0.9564	0.9285	0.8968	0.7175	0.5505	0.4166
0.010	0.9791	0.9284	0.8623	0.7899	0.7169	0.4165	0.2379	0.1388
0.015	0.9565	0.8621	0.7527	0.6456	0.5486	0.2370	0.1062	0.0490
0.020	0.9284	0.7891	0.6449	0.5180	0.4135	0.1374	0.0486	0.0176
0.025	0.8963	0.7150	0.5467	0.4124	0.3107	0.0808	0.0225	0.0063

The probability of lot acceptance decreases when the proportion defective and sample size increases as shown in Table 5.3. At small values of proportion defective and sample size very small changes are observed in probability of lot acceptance. For example, if p=0.001 and n=10, the probability of lot acceptance is 0.9998 and it decreases only 0.9990 when n=20. For subsequent tables, discussion will focus on n=50 to onwards. Considering that the proportion defective of a lot is 0.1%, the chance of lot acceptance will be approximately 99% when $n = 50$. This means that if there are 100 lots each consisting of 0.1% of defective product from the manufacturing process, then approximately 1 lot will be rejected. For the same value of sample size the probability of lot acceptance decreases from 0.9943 to 0.3107 when the proportion defective increases from 0.001 to 0.025. Meanwhile the probability of lot acceptance also decreases from 0.9943 to 0.9285 when sample size increases from 50 to 200 and proportion defective is equal to 0.001. This trend is presented in Figure 5.2.

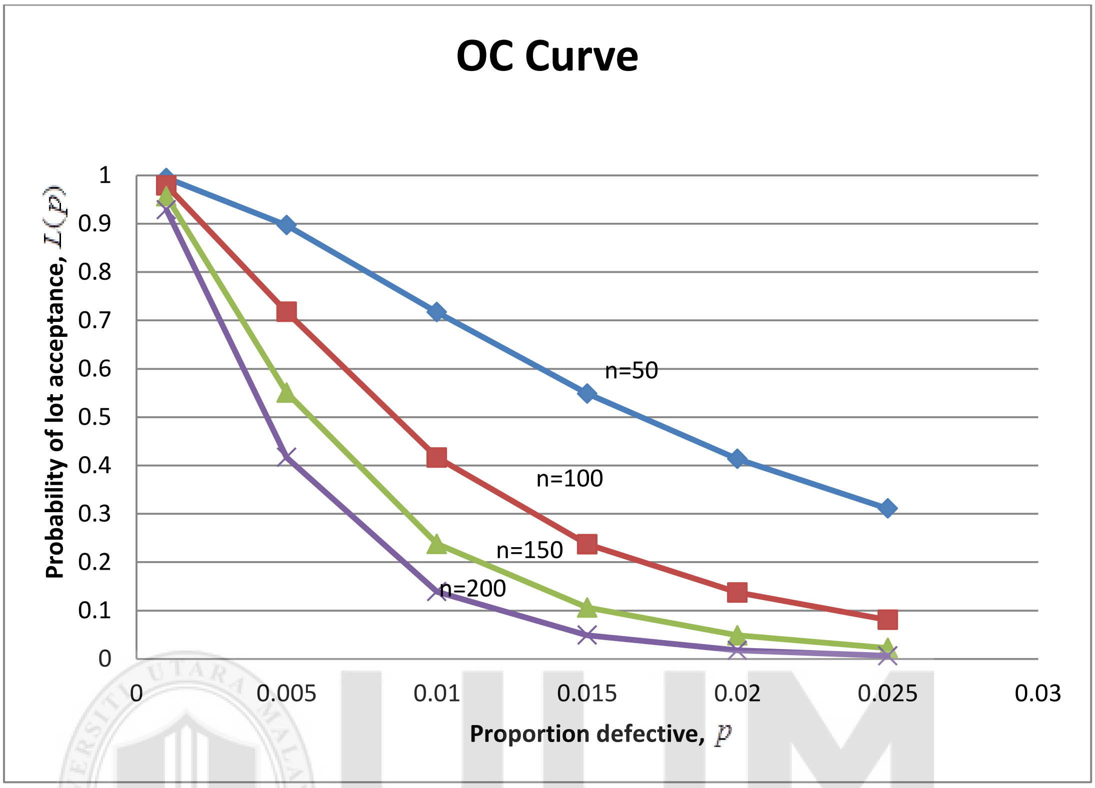

Figure 5.2. Probability of lot acceptance versus proportion defective for GGChSP

Figure 5.2 shows several OC curves for GGChSP with different values of sample size. The probability of lot acceptance decreases when the proportion defective increases. It is easy to see that for a fixed value of sample size, the probability of lot acceptance also decreases for higher proportion defectives. On the other hand, for a fixed value of proportion defective, the chance of lot acceptance decreases when sample size increases. It means that the probability of lot acceptance of a submitted product is monotonically smaller for a greater sample size.

5.2 Generalized Modified Group Chain Sampling Plan (GMGChSP)

Similar to the earlier section, a generalized modified group chain sampling plan (GMGChSP) is proposed. Using the pre-specified values of proportion defective, p, the minimum number of groups, g, are found based on Equation 4.21 (mentioned previously in page 77). For various values of β, i, and r, the minimum number of groups, g, is presented in Table 5.4.

Table 5.4

Number of minimum groups required for GMGChSP

			β			
p	r	i	0.25	0.10	0.05	0.01
0.001	2	1	527	818	1028	1497
	3	2	258	390	484	694
	4	3	152	226	279	398
	5	4	100	147	181	257
0.005	2	1	106	164	206	299
	3	2	52	78	97	139
	4	3	31	46	56	80
	5	4	20	30	37	52
0.010	2	1	53	**82**	103	149
	3	2	26	**39**	49	69
	4	3	16	23	28	40
	5	4	10	15	19	26
0.015	2	1	35	55	69	100
	3	2	18	26	33	46
	4	3	11	15	19	27
	5	4	7	10	12	17
0.020	2	1	27	41	51	75
	3	2	13	20	25	35
	4	3	8	12	14	20
	5	4	5	8	9	13
0.025	2	1	21	33	41	60
	3	2	11	16	20	28
	4	3	7	9	12	16
	5	4	4	6	8	11
0.030	2	1	18	27	34	50
	3	2	9	13	16	23
	4	3	5	8	10	14
	5	4	4	5	6	9
0.035	2	1	15	24	29	43
	3	2	8	11	14	20
	4	3	5	7	8	12
	5	4	3	5	6	8

			β			
p	r	i	0.25	0.10	0.05	0.01
	2	1	13	21	26	37
0.040	3	2	7	10	12	18
	4	3	4	6	7	10
	5	4	3	4	5	7
	2	1	12	16	18	23
0.045	3	2	6	9	11	16
	4	3	4	5	7	9
	5	4	3	4	4	6
	2	1	11	17	21	30
0.050	3	2	6	8	10	14
	4	3	3	5	6	8
	5	4	2	3	4	6
	2	1	10	15	19	27
0.055	3	2	5	7	9	13
	4	3	3	5	5	8
	5	4	2	3	4	5
	2	1	9	14	17	25
0.060	3	2	5	7	8	12
	4	3	3	4	5	7
	5	4	2	3	3	5
	2	1	8	13	16	23
0.065	3	2	4	6	8	11
	4	3	3	4	5	6
	5	4	2	3	3	4
	2	1	8	12	15	21
0.070	3	2	4	6	7	10
	4	3	3	4	4	6
	5	4	2	3	3	4
	2	1	7	11	14	20
0.075	3	2	4	6	7	9
	4	3	2	3	4	6
	5	4	2	2	3	4
	2	1	7	10	13	19
0.080	3	2	4	5	6	9
	4	3	2	3	4	5
	5	4	2	2	3	4
	2	1	7	10	12	17
0.085	3	2	3	5	6	8
	4	3	2	3	4	5
	5	4	2	2	3	3
	2	1	6	9	11	16
0.090	3	2	3	5	6	8
	4	3	2	3	3	5
	5	4	2	2	2	3
	2	1	6	9	11	16
0.095	3	2	3	4	5	7
	4	3	2	3	3	5
	5	4	2	2	2	3
	2	1	6	8	10	15
0.100	3	2	3	4	5	7
	4	3	2	3	3	4
	5	4	1	2	2	3

					β	
p	r	i	0.25	0.10	0.05	0.01
0.150	2	1	4	6	7	10
	3	2	2	3	4	5
	4	3	1	2	2	3
	5	4	1	1	2	2
0.200	2	1	3	4	5	7
	3	2	2	2	3	4
	4	3	1	2	2	2
	5	4	1	1	1	2
0.250	2	1	2	3	4	6
	3	2	1	2	2	3
	4	3	1	1	2	2
	5	4	1	1	1	1
0.300	2	1	2	3	3	5
	3	2	1	2	2	2
	4	3	1	1	1	2
	5	4	1	1	1	1
0.350	2	1	1	2	2	3
	3	2	1	1	2	2
	4	3	1	1	1	1
	5	4	1	1	1	1

From Table 5.4, when the value of pre-specified proportion defective, number of testers and number of preceding lots increase, the required number of groups for GMGChSP decreases but it increases when the consumers risk decreases. Considering that the consumer's risk, β = 0.10, p = 0.010, r = 2, i = 1, the required number of groups is 82; on the other hand if p = 0.010, r = 3, i = 2, the required number of groups is 39, as shown in Table 5.4. It means that when the number of preceding lots and number of tester increases, a small number of groups are required to reach the valid conclusion about the submitted lot. If p = 0.010, r = 3, and i = 2, then a sample size of 117 products drawn from the lot where 3 testers are located into 39 groups. Based on this information, the submitted lot will be accepted if no defective is observed in preceding as well as current samples. The lot is also accepted if one defective occurs in the preceding sample but with no defective is recorded in current sample during 7,000 hours of testing. The effect of proportion defective and consumer's risk on the number of groups are presented in Table 5.5.

Table 5.5

Number of minimum groups for r = 3 and i = 2

p	β	g	β	g	β	g	β	g
0.001		258		390		484		694
0.005		52		78		97		139
0.010	0.25	26	0.10	39	0.05	49	0.01	69
0.015		18		26		33		46
0.020		13		20		25		35
0.025		11		16		20		28

From Table 5.5, the number of groups decreases when the proportion defective increases Meanwhile, the number of groups increases when the consumers risk decreases for a specified value of proportion defective. Consider, β = 0.10, r = 3, and $i = 2$, where the required number of groups is 390 and p = 0.001. The number of groups decreases from 390 to 16 when the proportion defective increases from p = 0.001, to p = 0.025. For a fixed value of proportion defective p = 0.001, the number of groups increases from 258 to 694 when the consumer's risk decreases from 0.25 to 0.01. This trend is also shown in Figure 5.3.

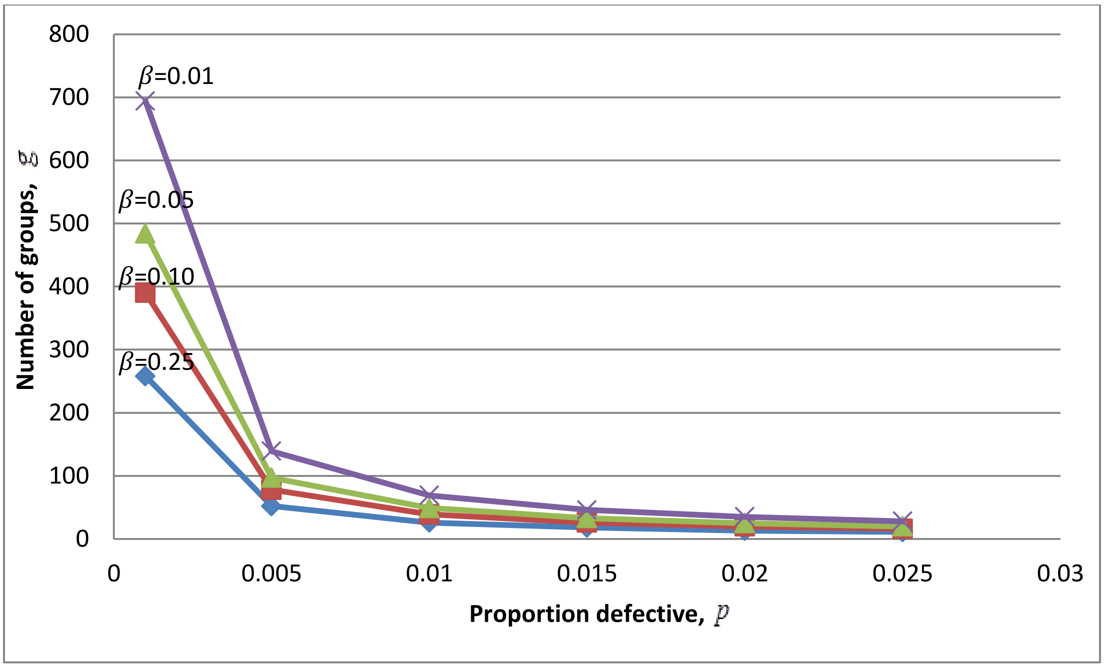

Figure 5.3. Number of groups versus proportion defective for GMGChSP

After observing the above Figure 5.3, it is concluded that the number of groups decreases when the proportion defective increases for a pre-specified consumer's risk. For a fixed value of proportion defective the number of groups increases when the consumer's risk decreases. These curves show that when the proportion defective increases, the number of groups decreases and eventually the values become closer regardless of the consumer's risk. Its main reason is that for higher values of proportion defective the number of groups becomes similar and produces the same result because the probability of lot acceptance decreases very quickly. On the other hand, if consumer's risk decreases from 0.25 to 0.01 then the number of groups increases from 258 to 694 when, p = 0.001. Based on the values of proportion defective presented in Table 5.5 when i=2, the effect of probability of lot acceptance for fixed values of sample size is shown in Table 5.6.

Table 5.6

Operating characteristic values for GMGChSP

	$n = r * g$							
p	10	20	30	40	50	100	150	200
0.001	0.9899	0.9794	0.9688	0.9579	0.9468	0.8890	0.8289	0.7683
0.005	0.9469	0.8891	0.8289	0.7683	0.7084	0.4457	0.2628	0.1487
0.010	0.8891	0.7682	0.6500	0.5413	0.4451	0.1481	0.0438	0.0121
0.015	0.8290	0.6498	0.4911	0.3617	0.2614	0.0434	0.0062	0.0008
0.020	0.7681	0.5405	0.3611	0.2331	0.1469	0.0119	0.0008	0.0000
0.025	0.7078	0.4434	0.2600	0.1462	0.0799	0.0031	0.0001	0.0000

It can be observed from Table 5.6, the probability of lot acceptance decreases when the proportion defective and sample size increases. Considering that p = 0.001, and $n = 50$, the probability of lot acceptance is 0.9468. For the same value of sample size, the probability of lot acceptance decreases from 0.9468 to 0.0799 when the proportion defective increases from 0.001 to 0.025. Meanwhile, the probability of lot acceptance also decreases from 0.9468 to 0.7683 when the sample size increases from 50 to 200 and the proportion defective is equal to 0.001. This trend is displayed in Figure 5.4.

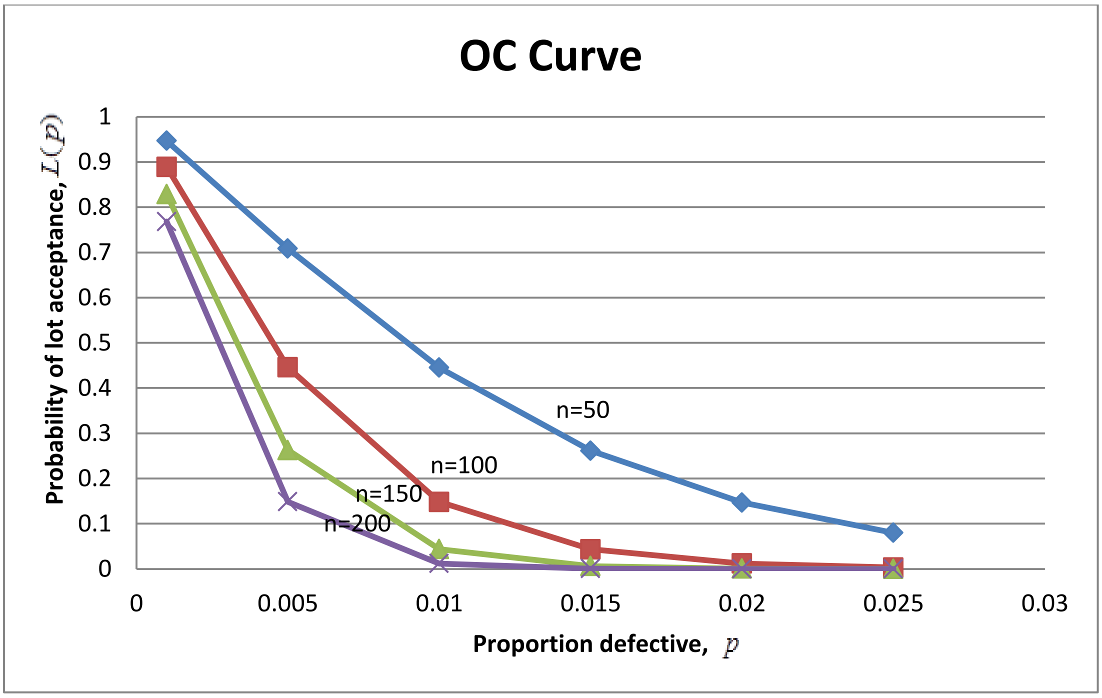

Figure 5.4. Probability of lot acceptance versus proportion defective for GMGChSP

Figure 5.4 shows the several OC curves for GMGChSP with various values of sample size. The probability of lot acceptance decreases when the proportion defective increases for a fixed value of sample size. Furthermore the probability of lot acceptance also decreases when the sample size increases. It is easy to see that plans with a small proportion defective and sample size have a greater probability of lot acceptance than the plans for a large proportion defective and sample size.

5.3 Generalized Two-Sided Group Chain Sampling Plan (GTS-GChSP)

Generalized two-sided group chain sampling plan (GTS-GChSP) is proposed in this section based on pre-specified values of proportion defective, p. The minimum number of groups, g, are found using Equation 4.30. For various values of β, i, j and r, the minimum number of groups, g, is presented in Table 5.7.

Table 5.7

Number of minimum groups required for GTS-GChSP

			β			
p	r	$i = j$	0.25	0.10	0.05	0.01
0.001	2	1	387	584	726	1040
	3	2	166	245	302	428
	4	3	91	134	164	232
	5	4	58	84	103	145
0.005	2	1	78	117	145	208
	3	2	33	49	61	86
	4	3	19	27	33	47
	5	4	12	17	21	29
0.010	2	1	39	**59**	73	104
	3	2	17	**25**	31	43
	4	3	10	14	17	24
	5	4	6	9	11	15
0.015	2	1	26	39	49	69
	3	2	11	17	20	29
	4	3	7	9	11	16
	5	4	4	6	7	10
0.020	2	1	20	29	37	52
	3	2	9	13	15	22
	4	3	5	7	9	12
	5	4	3	5	6	8
0.025	2	1	16	23	29	42
	3	2	7	10	12	16
	4	3	4	6	7	10
	5	4	3	4	5	6
0.030	2	1	13	20	24	35
	3	2	6	9	10	15
	4	3	3	5	6	8
	5	4	2	3	4	5
0.035	2	1	11	17	21	30
	3	2	5	7	9	13
	4	3	3	4	5	7
	5	4	2	3	3	5

			β			
p	r	$i = j$	0.25	0.10	0.05	0.01
	2	1	10	15	18	26
0.040	3	2	5	7	8	11
	4	3	3	4	5	6
	5	4	2	3	3	4
	2	1	9	13	16	23
0.045	3	2	4	6	7	10
	4	3	2	3	4	6
	5	4	2	2	3	4
	2	1	8	12	15	21
0.050	3	2	4	5	6	9
	4	3	2	3	4	5
	5	4	2	2	3	3
	2	1	7	11	13	19
0.055	3	2	3	5	6	8
	4	3	2	3	3	5
	5	4	2	2	2	3
	2	1	7	10	12	17
0.060	3	2	3	4	5	7
	4	3	2	3	3	4
	5	4	1	2	2	3
	2	1	6	9	11	16
0.065	3	2	3	4	5	7
	4	3	2	3	3	4
	5	4	1	2	2	3
	2	1	6	9	11	15
0.070	3	2	3	4	5	6
	4	3	2	2	3	4
	5	4	1	2	2	3
	2	1	6	8	10	14
0.075	3	2	3	4	4	6
	4	3	2	2	3	3
	5	4	1	2	2	2
	2	1	5	8	9	13
0.080	3	2	3	3	4	6
	4	3	2	2	2	3
	5	4	1	2	2	2
	2	1	5	7	9	12
0.085	3	2	2	3	4	5
	4	3	2	2	2	3
	5	4	1	1	2	2
	2	1	5	7	8	12
0.090	3	2	2	3	4	5
	4	3	1	2	2	3
	5	4	1	1	2	2
	2	1	4	6	8	11
0.095	3	2	2	3	4	6
	4	3	1	2	2	3
	5	4	1	1	2	2
	2	1	4	6	7	10
0.100	3	2	2	3	3	4
	4	3	1	1	2	2
	5	4	1	1	1	2

					β	
p	r	$i = j$	0.25	0.10	0.05	0.01
	2	1	3	4	5	7
0.150	3	2	1	2	2	3
	4	3	1	1	1	2
	5	4	1	1	1	1
	2	1	2	3	4	5
0.200	3	2	1	1	2	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	2	3	3	4
0.250	3	2	1	1	1	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	2	2	3	4
0.300	3	2	1	1	1	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	1	1	2	2
0.350	3	2	1	1	1	1
	4	3	1	1	1	1
	5	4	1	1	1	1

From Table 5.7, the number of groups required for the GTS-GChSP varies for various values of consumers risk but decreases when the number of preceding and succeeding lots, number of testers and pre-specified proportion defective increase. Considering that β = 0.10, p = 0.010, r = 2, $i = j$ = 1, the required number of groups is 59, on the other hand if p = 0.010, r = 3, $i = j$ = 2, the required number of groups is 25, as shown in Table 5.7. This means that when the number of preceding lots and number of tester increases, a small number of groups are required to reach the valid conclusion about the submitted lot. Assuming the average life of a product, $\mu = \mu_0$ = 10,000 hours and other above mentioned pre-specified design parameters are p = 0.010, a =0.7, r = 2, and $i = j$ = 1, then a sample size of 118 products are drawn from the lot where 2 testers are located into each of the 59 groups. Based on this information, the submitted lot will be accepted if no defectives are observed in preceding, current as well as succeeding samples. The lot is also accepted if one defective occurs either preceding or succeeding lot but no defective is recorded in current sample over 7,000 hours. The influence of proportion defective and consumer's risk to the number of groups is presented in Table 5.8.

Table 5.8

Minimum number of groups for r = 3, and i = j = 1

p	β	g	β	g	β	g	β	g
0.001	0.25	258	0.10	390	0.05	484	0.01	694
0.005		52		78		97		139
0.010		26		39		49		69
0.015		18		26		33		46
0.020		13		20		25		35
0.025		11		16		20		28

The number of groups decreases when the proportion defective increases and increases when the consumers risk decreases for a specified value of proportion defective as shown in Table 5.8. Consider, β = 0.10, r = 3, and $i = j = 1$ where the required number of groups is 390 when, p = 0.001. The number of groups decreases from 390 to 16 when proportion defective increases from, p = 0.001, to p = 0.025. For a fixed proportion defective p = 0.001, the number of groups increases from 258 to 694 when consumer's risk decreases from 0.25 to 0.01. This trend is also illustrated in Figure 5.5.

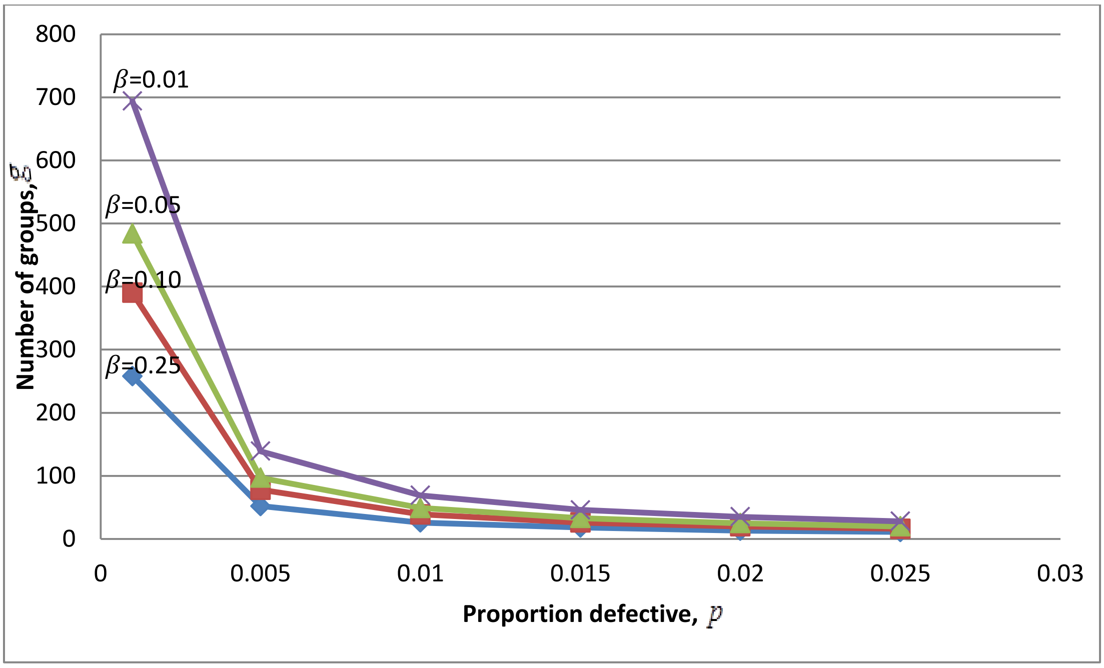

Figure 5.5. Number of groups versus proportion defective for GTS-GChSP

From inspecting the above Figure 5.5, it is evident that the number of groups decreases when the proportion defective increases for a pre-specified consumer's risk. For a fixed proportion defective the number of groups increases when the consumer's risk decreases. These curve shows that, if the proportion defective increases from 0.001 to 0.025, then the number of groups decreases from 258 to 11 when, β = 0.25. If consumer's risk decreases from 0.25 to 0.01 then number of groups increases from 258 to 694 when, p=0.001. The required number of groups decreases and similar number of groups is given when the value of proportion defective increases for different values of consumer's risk. Based on the values of proportion defective presented in Table 5.8 when, $i = j$= 1, the effect of probability of lot acceptance for fixed values of sample size is shown in Table 5.9.

Table 5.9

Operating characteristic values for GTS-GChSP when, i =j = 1

	$n = r * g$							
p	10	20	30	40	50	100	150	200
0.001	0.9899	0.9794	0.9688	0.9579	0.9468	0.8890	0.8289	0.7683
0.005	0.9469	0.8891	0.8289	0.7683	0.7084	0.4457	0.2628	0.1487
0.010	0.8891	0.7682	0.6500	0.5413	0.4451	0.1481	0.0438	0.0121
0.015	0.8290	0.6498	0.4911	0.3617	0.2614	0.0434	0.0062	0.0008
0.020	0.7681	0.5405	0.3611	0.2331	0.1469	0.0119	0.0008	0.0000
0.025	0.7078	0.4434	0.2600	0.1462	0.0799	0.0031	0.0001	0.0000

The probability of lot acceptance decreases when the proportion defective and sample size increases as presented in Table 5.9. Assuming that p=0.001 and n =50, the probability of lot acceptance is 0.9468. For the same value of sample size, the probability of lot acceptance decreases from 0.9468 to 0.0799 when the proportion defective increases from 0.001 to 0.025. Meanwhile, the probability of lot acceptance also decreases from 0.9468 to 0.7683 when sample size increases from 50 to 200 and proportion defective is equal to 0.001. This trend is shown in Figure 5.6.

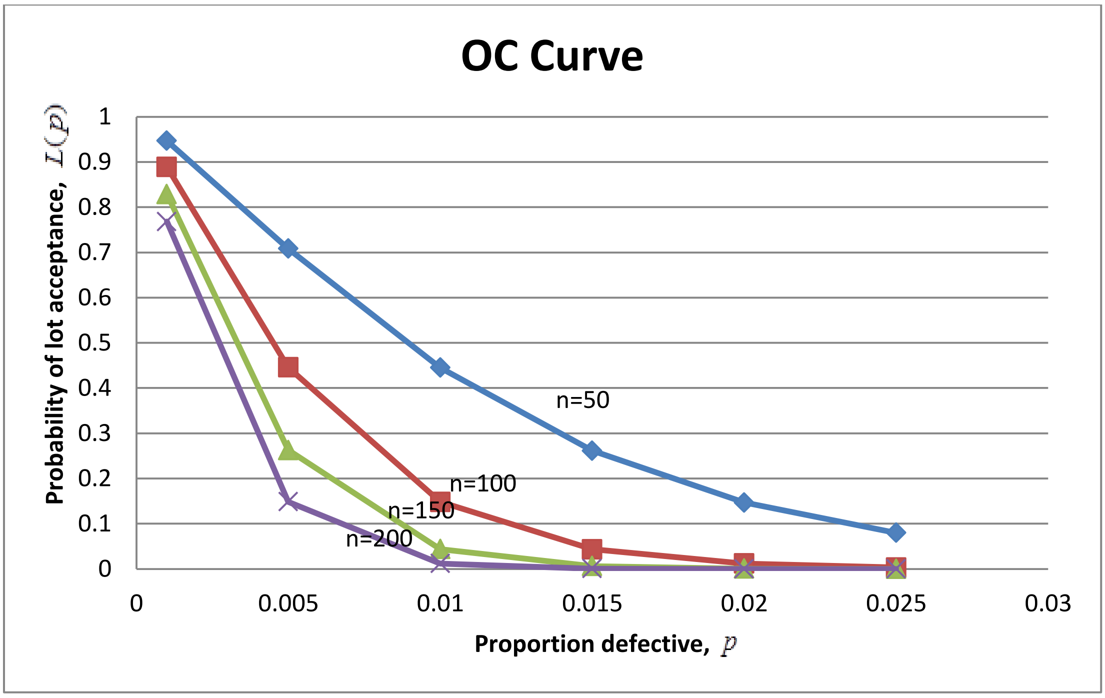

Figure 5.6. Probability of lot acceptance versus proportion defective for GTS-GChSP

Figure 5.6 shows several OC curves for GTS-GChSP with various values of sample size. The probability of lot acceptance decreases when the proportion defective increases for a fixed value of sample size. The probability of lot acceptance also decreases when the sample size increases. It is easy to see that plans with small proportion defective and sample size have a greater probability of lot acceptance than the plans for large proportion defective and sample size.

5.4 Generalized Two-Sided Modified Group Chain Sampling Plan (GTS-MGChSP)

Generalized two-sided modified group chain sampling plan (GTS-MGChSP) is proposed in this section for when the lifetime of submitted product is based on any lifetime distribution. Using pre-specified proportion defective, p, the minimum number of groups, g, is found based on Equation 4.38 (Chapter 4). For various values of β, i, j and r, the minimum number of groups, g, is presented in Table 5.10.

Table 5.10

Number of minimum groups required for GTS-MGChSP

			β			
p	r	$i = j$	0.25	0.10	0.05	0.01
0.001	2	1	312	499	636	943
	3	2	112	183	236	355
	4	3	57	94	121	184
	5	4	35	57	74	113
0.005	2	1	63	100	127	189
	3	2	23	37	47	71
	4	3	12	19	25	37
	5	4	7	12	15	23
0.010	2	1	32	50	64	94
	3	2	12	19	24	36
	4	3	6	10	13	19
	5	4	4	6	8	12
0.015	2	1	21	34	43	63
	3	2	8	13	16	24
	4	3	4	7	9	13
	5	4	3	4	5	8
0.020	2	1	16	25	32	47
	3	2	6	10	12	18
	4	3	3	5	6	10
	5	4	2	3	4	6
0.025	2	1	13	20	26	38
	3	2	5	8	10	15
	4	3	3	4	5	8
	5	4	2	3	3	5
0.030	2	1	11	17	21	32
	3	2	4	7	8	12
	4	3	2	4	4	7
	5	4	2	2	3	4

			β			
p	r	$i = j$	0.25	0.10	0.05	0.01
	2	1	9	15	18	27
0.035	3	2	4	6	7	10
	4	3	2	3	4	6
	5	4	1	2	3	4
	2	1	8	13	16	24
0.040	3	2	3	5	6	9
	4	3	2	3	3	5
	5	4	1	2	2	3
	2	1	7	11	14	21
0.045	3	2	3	4	6	8
	4	3	2	3	3	5
	5	4	1	2	2	3
	2	1	7	10	13	19
0.050	3	2	3	4	5	7
	4	3	2	2	3	4
	5	4	1	2	2	3
	2	1	6	9	12	17
0.055	3	2	2	4	5	7
	4	3	2	2	3	4
	5	4	1	2	2	2
	2	1	6	9	11	16
0.060	3	2	2	3	4	6
	4	3	1	2	2	3
	5	4	1	1	2	2
	2	1	5	8	10	15
0.065	3	2	2	3	4	6
	4	3	1	2	2	3
	5	4	1	1	2	2
	2	1	5	7	9	14
0.070	3	2	2	3	4	5
	4	3	1	2	2	3
	5	4	1	1	2	2
	2	1	5	7	9	13
0.075	3	2	2	3	4	5
	4	3	1	2	2	3
	5	4	1	1	1	2
	2	1	4	7	8	12
0.080	3	2	2	3	3	5
	4	3	1	2	2	3
	5	4	1	1	1	2
	2	1	4	6	8	11
0.085	3	2	2	3	3	5
	4	3	1	2	2	3
	5	4	1	1	1	2
	2	1	4	6	7	11
0.090	3	2	2	2	3	4
	4	3	1	1	2	2
	5	4	1	1	1	2
	2	1	4	6	7	10
0.095	3	2	2	2	3	4
	4	3	1	1	2	2
	5	4	1	1	1	2

			β			
p	r	$i = j$	0.25	0.10	0.05	0.01
	2	1	3	5	7	10
0.100	3	2	2	2	3	4
	4	3	1	1	2	2
	5	4	1	1	1	2
	2	1	2	4	4	6
0.150	3	2	1	2	2	3
	4	3	1	1	1	2
	5	4	1	1	1	1
	2	1	2	3	3	5
0.200	3	2	1	1	2	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	2	2	3	4
0.250	3	2	1	1	1	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	1	2	2	3
0.300	3	2	1	1	1	2
	4	3	1	1	1	1
	5	4	1	1	1	1
	2	1	1	1	1	2
0.350	3	2	1	1	1	1
	4	3	1	1	1	1
	5	4	1	1	1	1

In Table 5.10, the number of groups required for the GTS-MGChSP varies for various values of consumers risk but decreases when the number of preceding and succeeding lots, number of testers and pre-specified proportion defective increases. Considering that the consumer's risk, β = 0.10, p = 0.010, r = 2, $i = j$ = 1, the required number of groups is 50. On the other hand if p = 0.010, r = 3, $i = j$ = 2, the required number of groups is 19, as shown in Table 5.10. This means that when the number of preceding lots and number of tester increases, a small number of groups is required to reach a valid conclusion about the submitted lot. Assuming that the average life of a product, $\mu = \mu_0$= 10,000 hours and that other above mentioned pre-specified design parameters are p = 0.010, r = 2, and $i = j$ = 1, a sample size of 100 products is drawn from the lot where 2 testers are allocated into 50 groups. Based on this information, the submitted lot will be accepted if no defective is observed in preceding, current as well as succeeding sample. The lot is also acceptable if one defective occurs in current sample but no defective is recorded in preceding and succeeding sample. Based on these results, the effect of probability of lot acceptance for fixed values of sample size and proportion defective when, $i = j$= 1 is shown in Table 5.11.

Table 5.11

Minimum number of groups for r = 3 and i = j = 1

p	β	g	β	g	β	g	β	g
0.001		208		332		424		629
0.005		42		67		84		126
0.010	0.25	21	0.10	34	0.05	43	0.01	63
0.015		14		23		29		42
0.020		11		17		22		32
0.025		9		14		17		25

The number of groups decreases when the proportion defective increases and increases when the consumers risk decreases for a specified value of proportion defective as presented in Table 5.11. Consider, β = 0.10, r=3, and $i = j =$1 in which the required number of groups is 332 when, p = 0.001. The number of groups decreases from 332 to 14 when proportion defective increases from, p = 0.001, to p = 0.025. Next, for a fixed proportion defective, p = 0.001, the number of groups increases from 208 to 629 when consumer's risk decreases from 0.25 to 0.01. This trend is also illustrated in Figure 5.7.

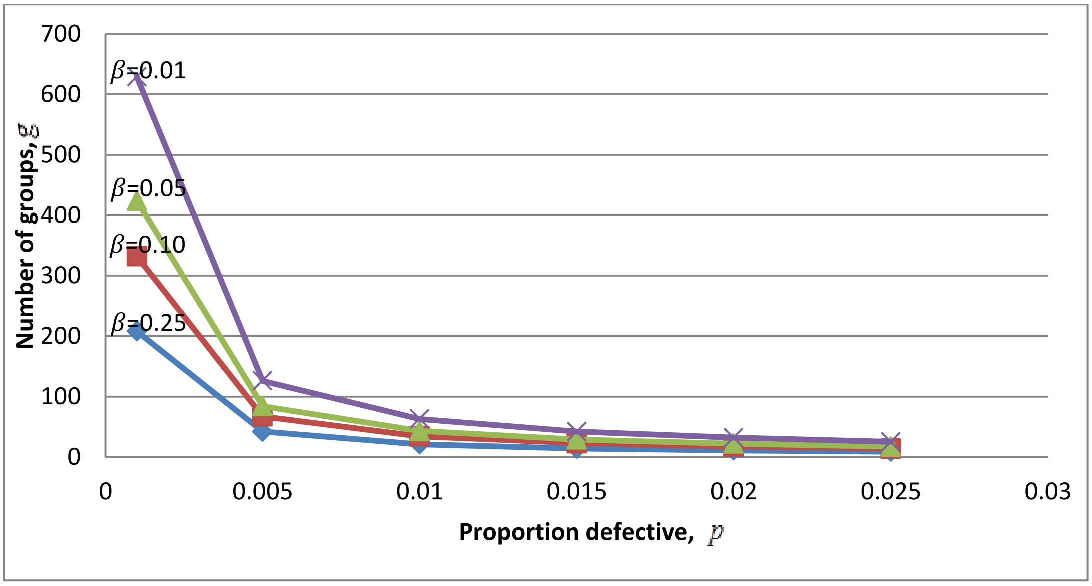

Figure 5.7. Number of groups versus proportion defective for GTS-MGChSP

From Figure 5.7, the number of group's decreases when the proportion defective increases for a pre-specified consumer's risk. On the other hand, for a fixed proportion defective the number of groups increases when the consumer's risk decreases. These curve shows that, as the proportion defective increases from 0.001 to 0.025, and the number of groups decreases from 208 to 9 when, $\beta = 0.25$. If consumer's risk decreases from 0.25 to 0.01 then the number of groups increases from 208 to 629 when, $p = 0.001$. Based on the values of proportion defective presented in Table 5.11 when, $i = j = 1$, the probability of lot acceptance is obtained and shown in Table 5.12 for various values of sample size.

Table 5.12

Operating characteristic values for GTS-MGChSP when, i = j = 1

	$n = r * g$							
p	10	20	30	40	50	100	150	200
0.001	0.9801	0.9606	0.9413	0.9224	0.9037	0.8149	0.7332	0.6585
0.005	0.9036	0.8147	0.7329	0.6581	0.5899	0.3340	0.1838	0.0991
0.010	0.8144	0.6577	0.5274	0.4203	0.3333	0.0986	0.0273	0.0073
0.015	0.7322	0.5268	0.3738	0.2624	0.1825	0.0271	0.0037	0.0005
0.020	0.6568	0.4190	0.2617	0.1608	0.0976	0.0071	0.0005	0.0000
0.025	0.5879	0.3312	0.1812	0.0971	0.0512	0.0018	0.0001	0.0000

The probability of lot acceptance decreases when the proportion defective and sample size increases based on Table 5.12. Assuming that p = 0.001 and $n = 50$, the required probability of lot acceptance is 0.9037. For the same value of sample size, the chances of lot acceptance decreases from about 90% to 5% when the proportion defective increases from 0.001 to 0.025. Meanwhile, the probability of lot acceptance also decreases from 0.9037 to 0.6525 when sample size increases from 50 to 200 and proportion defective is equal to 0.001. This trend is displayed in Figure 5.8.

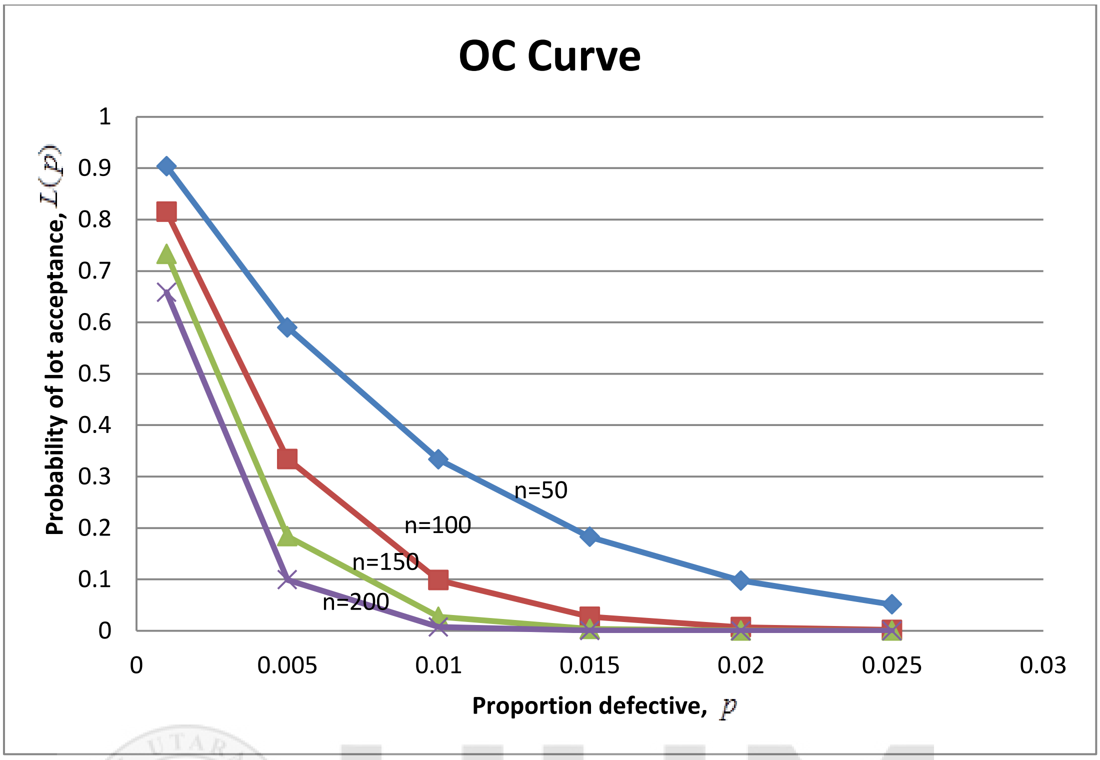

Figure 5.8. Probability of lot acceptance versus proportion defective for GTS-MGChSP

Several OC curves for GTS-MGChSP with various values of sample size are shown in Figure 5.8. The probability of lot acceptance decreases when the proportion defective increases for a fixed value of sample size. The probability of lot acceptance also decreases when the sample size increases. It is easy to see that plans with a small proportion defective and sample size have a greater probability of lot acceptance than the plans for large proportion defective and sample size.

5.5 Comparison of Proposed Plans

In this section, graphical representations are considered to compare the performance and behavior of the probability of lot acceptance and proportion defective for the proposed plans. The results from the proposed plans are based on different combination of design parameters and real lifetime data. A comparison is made between the GChSP, MGChSP, TS-GChSP, TS-MGChSP and established plan developed by Mughal and Aslam (2011) using a real lifetime data set. The observations of this data set are based on the number of million revolutions before failure for 23 ball bearings in the truncated life tests discussed by Rao and Ramesh (2016), as shown in Table 5.13.

Table 5.13

Number of million revolutions before failure for each of the 23 ball bearings

Ball bearings	Million revolutions before failure	Ball bearings	Million revolutions before failure	Ball bearings	Million revolutions before failure
1	17.88	9	51.96	17	93.12
2	28.92	10	54.12	18	98.64
3	33.00	11	55.56	19	105.12
4	41.52	12	67.80	20	105.84
5	42.12	13	68.44	21	127.92
6	45.60	14	68.64	22	128.04
7	48.80	15	68.88	23	173.40
8	51.84	16	84.12		

The Kolmogorov-Smirnov (K-S) goodness of fit test is used to confirm which lifetime distribution is most appropriate for the data in Table 5.13. Based on the results of (EasyFit - Distribution Fitting Software, shown in Appendix A.), the K-S statistic for the Pareto distribution of the 2^{nd} kind is 0.2358 with tabulated value of 0.3295 at 1% level of significance. The K-S statistic is less than the tabulated value so that the Pareto distribution of the 2^{nd} kind provides best fit for submitted products instead of the other several lifetime distributions shown in Table 5.14.

Table 5.14

Goodness of fit-summary

Lifetime Distributions	Kolmogorov- Smirnov Statistic	Lifetime Distributions	Kolmogorov-Smirnov Statistic
Pareto 2^{nd} kind	0.23587	Normal	0.46872
Inv. Gaussian (3Parameter)	0.24914	Logistic	0.47529
Inv. Gaussian	0.26892	Hypersecant	0.48084
Gen. Gamma (4 Parameter)	0.27032	Exponential	0.48267
Weibull (3 Parameter)	0.27129	Reciprocal	0.49366
Pareto	0.29880	Error	0.49949
Levy (2 Parameter)	0.29947	Laplace	0.49949
Gamma (3 Parameter)	0.33437	Exponential (2 Parameter)	0.50574
Chi-Squared (2 Parameter)	0.35376	Error Function	0.51099
Kumaraswamy	0.36506	Johnson SB	0.52291
Fatigue Life (3 Parameter)	0.36630	Rayleigh (2 Parameter)	0.53063
Dagum	0.38245	Gamma	0.53827
Levy	0.38539	Gumbel Min	0.53887
Fatigue Life	0.40547	Beta	0.67274
Gumbel Max	0.40930	Rayleigh	0.69759
Burr (4 Parameter)	0.42473	Pert	0.70408
Gen. Gamma	0.43260	Triangular	0.81569
Power Function	0.45210	Rice	0.85780
Uniform	0.45534	Chi-Squared	0.91996

The shape, λ and scale, σ, parameter of the Pareto distribution of the 2^{nd} kind are evaluated using maximum likelihood estimation (MLE) and can be written in the following forms,

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} log\left(1 + \frac{t}{\sigma}\right) = 0 \qquad 5.1$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} - \left(\frac{n}{\sum_{i=1}^{n} log\left(1+\frac{t}{\sigma}\right)} + 1\right) \sum_{i=1}^{n} \frac{-t/\sigma^2}{\left(1+\frac{t}{\sigma}\right)} = 0 \qquad 5.2$$

Using iteration method for real lifetime data in Table 5.14, the required parameter of the Pareto distribution of the 2nd kind are, λ = 1.6293 ≅ 2, and σ = 133.97. Using the information of β = 0.10, r = 3, i = 2, and $i = j$= 1, the required probability of lot acceptance of the proposed plans and established plan developed by Mughal and Aslam (2011) are shown in Table 5.15.

Table 5.15

Comparison of probability of lot acceptance

a	p	GChSP $L(p)$	MGChSP $L(p)$	TS-GChSP $L(p)$	TS-MGChSP $L(p)$	Mughal and Aslam (2011) $L(p)$
0.7	0.6540	0.0418	0.0009	0.0009	0.0005	0.0212
0.8	0.6914	0.0296	0.0004	0.0004	0.0002	0.0125
1.0	0.7500	0.0157	0.0001	0.0001	0.0000	0.0046
1.2	0.7934	0.0088	0.0000	0.0000	0.0000	0.0019
1.5	0.8400	0.0041	0.0000	0.0000	0.0000	0.0686
2.0	0.8889	0.0014	0.0000	0.0000	0.0000	0.0343

From Table 5.15, the probability of lot acceptance decreases when the pre-specified testing time and proportion defective increases. The values for the specified design parameters are borrowed from Mughal and Aslam (2011) for comparison purposes. For higher values of proportion defective, the MGChSP provides the minimum probability of lot acceptance, unlike the GChSP and the established plan developed by Mughal and Aslam (2011). The GChSP and TS-GChSP also give the minimum probability of lot acceptance and the TS-GChSP gives the same value as the MGChSP when $i = j$. It is to be noted that the TS-MGChSP provides the minimum probability of lot acceptance compared to other proposed and established plans and offers more strict inspection according to the consumer's point of view. It is mentioned in Chapter 2 (Section 2.1) that Baklizi (2003) also developed an ordinary acceptance sampling plan for Pareto distribution of the 2^{nd} kind. He directly used the value of scale parameter, σ, of Pareto distribution of the 2^{nd} kind for examining the mean lifetime of a product instead of solving Equation 4.8 which is $\sigma = \mu(\lambda - 1)$. This conflicts with the basic concept of acceptance sampling plan and may misguide experimenters according to Balakrishnan *et al.*(2007) and cannot be compared with the proposed plans. For the same above mentioned design parameters, the pattern of the probability of lot acceptance is displayed in Figure 5.9 for various values of mean ratios when a = 1.0.

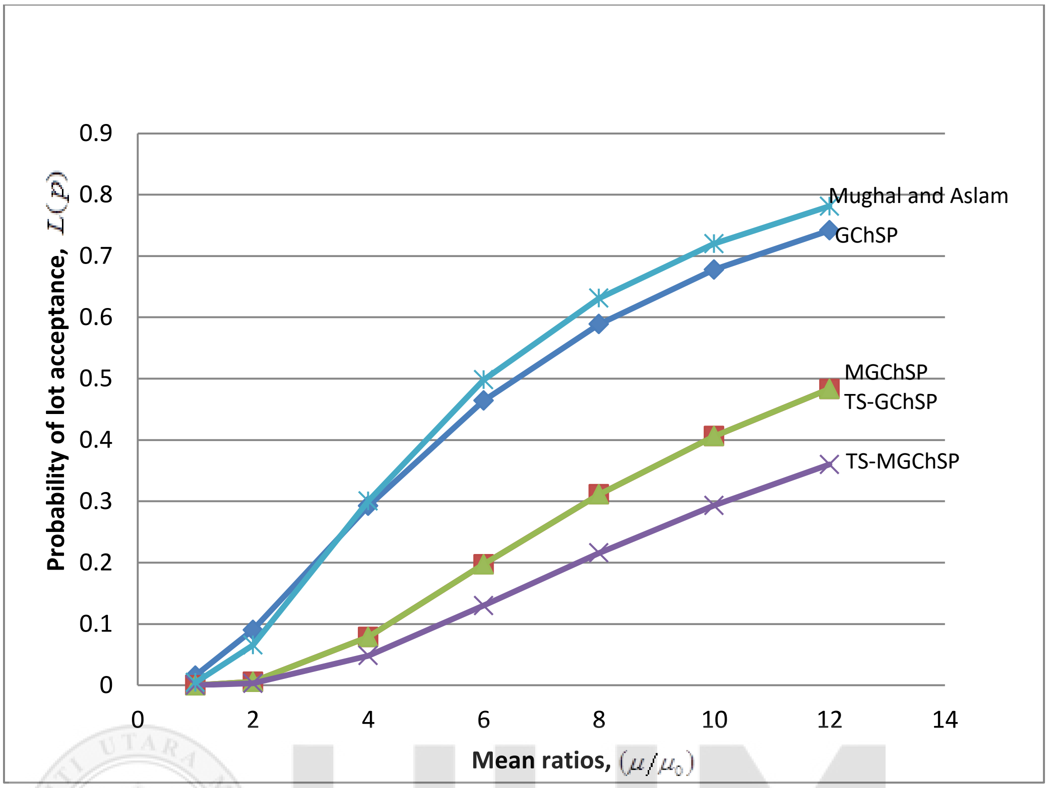

Figure 5.9. Probability of lot acceptance versus mean ratios of the proposed plans

Figure 5.9 shows that when the mean ratio increases, the probability of lot acceptance also increases for a fixed proportion defective based on Pareto distribution of the 2nd kind. It is to be noted that, TS-GChSP converts to MGChSP and gives the same probability of lot acceptance with index $2i$, when $i = j$, that is the same number of lots are considered in preceding and succeeding. Based on the above mentioned design parameters (β = 0.10, r = 3, i = 2, and $i = j$ = 1) the minimum number of groups of the proposed plans (GGChSP, GMGChSP, GTS-GChSP, and GTS-MGChSP) and the established plan developed by Mughal and Aslam (2011) is shown in Table 5.16 for comparison purposes.

Table 5.16

Comparisons of number of groups

p	GGChSP	GMGChSP	GTS-GChSP	GTS-MGChSP	Mughal and Aslam (2011)
0.001	775	390	390	332	1296
0.005	155	78	78	67	259
0.010	78	39	39	34	130
0.015	52	26	26	23	86
0.020	39	20	20	17	65
0.025	31	16	16	14	52

From Table 5.16, it can be observed that the minimum number of groups decreases when the proportion defective increases (moving downward). As anticipated, this observation is true for all types of acceptance sampling plans. Reading across the table horizontally, it is evident that the four proposed plans provide substantially smaller number of groups compared to the established plan (Mughal & Aslam, 2011). This trend is observed for all values of proportion defective. It is clearly evident that at all values of proportion defective, the GTS-MGChSP requires the smallest number of groups among all the proposed as well as the established plans. Hence, GTS-MGChSP offers the smallest sample size and more accurate probability of lot acceptance which is most beneficial for consumers.

5.6 Discussion

The proposed plans suggest a practically straightforward methodology on the investigation of submitted lots based on a truncated life test. The advantages of proposed plans are that they (i) are simple computations (ii) are practically interpretable and economical (iii) use the maximum information about the submitted lot and (iv) give the exact probability of lot acceptance based on lifetime distribution and various values of proportion defectives. By design, these methods are proficient at analyzing the sample size and probability of lot acceptance. Therefore, an attempt has been made to propose several acceptance sampling plans based on various lot accepting criteria. The design aspects of these proposed plans are given in detail which is firstly based on the Pareto distribution of the 2^{nd} kind and then generalized for several pre-specified values of proportion defective. A binomial distribution is used to find out the minimum sample size and probability of lot acceptance. Comparative analyses among the proposed and established plans are also provided which are helpful for experimenters to achieve a more discriminatory OC curve to lead to a minimum sample size and more accurate probability of lot acceptance.

CHAPTER SIX
CONCLUSIONS AND FURTHER WORK

A family of group chain sampling plans are proposed firstly based on lifetime distribution and then generalized for various values of proportion defectives. The proposed plans can be employed when multiple products are examined simultaneously; hence, they are more economical due to saving of testing time and cost. The implementation of the proposed plans in the perspective of group acceptance sampling plan (GASP) has been supported by Mughal and Aslam (2011), Mughal and Ismail (2013). The GASP is very helpful to examine the high quality product from infinite lot and can be applied in chain sampling plan. The chain sampling plan has been classified into only two numbers, c = 0, 1, as discussed by Dodge (1955), Govindaraju and Lai (1998), Deva and Rebecca (2012), Ramaswamy and Jayasri (2014) and Ramaswamy and Jayasri (2015).

For the selection of the desired plan, various combination of design parameters based on several sampling procedures are discussed. Three relationships are recognized from these results. First, higher values of pre-specified proportion defective, testing time and consumer's risk produced the minimum sample size for the fixed value of other design parameters. Secondly, the findings show that all the proposed plans provide smaller g and lower $L(p)$ compared to the established plan.. On the other hand, the probability of lot acceptance increases when the values of mean ratios increase. Third, when the numbers of preceding and succeeding lots are equal, two-sided modified group chain sampling plan converts to modified group chain sampling plan with index $2i$ and deliver the same information regarding the submitted product, which is consistent with earlier finding by Deva and Rebecca (2012) for ordinary sampling plan.

The practical implementation and validation of the proposed plans are described in Chapter 5, Section 5.5 for industrial uses. This real data example indicates that the proposed plans are able to deal with truncated life test based on lifetime distributions. The proposed plans also provide a comparable performance to established plans and among each other, such as minimum number of sample size and probability of lot acceptance.

The proposed GChSP and MGChSP are the first effort in applying group acceptance sampling in the chain sampling to examine multiple products at the same time. These proposed plans produced minimum sample size which can save inspection time, energy, labour and cost. The proposed plans: TS-GChSP and TS-MGChSP can replace other established plans when the average lifetime of a submitted product is based on truncated life test. The proposed generalized plans GGChSP, GMGChSP, GTS-GChSP and GTS-MGChSP are also systematic procedures based on several pre-specified values of proportion defective and useful for practitioners to inspect the products with the help of additional information such as the preceding as well as succeeding lot quality history.

This research has focused on group acceptance sampling development for improved performance but future research might explore the possible extension of the proposed plans. Some other acceptance sampling plans are needed to enhance group chain sampling plan in terms of minimum test termination time instead of sample size. Using the same design parameters, the minimum test termination time can be found to satisfy the producer's risk. Also, this research measured the lifetime of a submitted product by considering Pareto distribution of the 2nd kind. Binomial distribution is considered to find the required design parameters. It would be valuable to reproduce this research for several other lifetime distributions.

Furthermore, the proposed plans can be extended using three classes of attribute chain sampling so-called good, marginal and bad. In practice, submitted products follow the pattern of randomization, replication and random categories. To handle such inspection, weighted distribution and skewed data theories based on proposed plans may be developed for the field of acceptance sampling.

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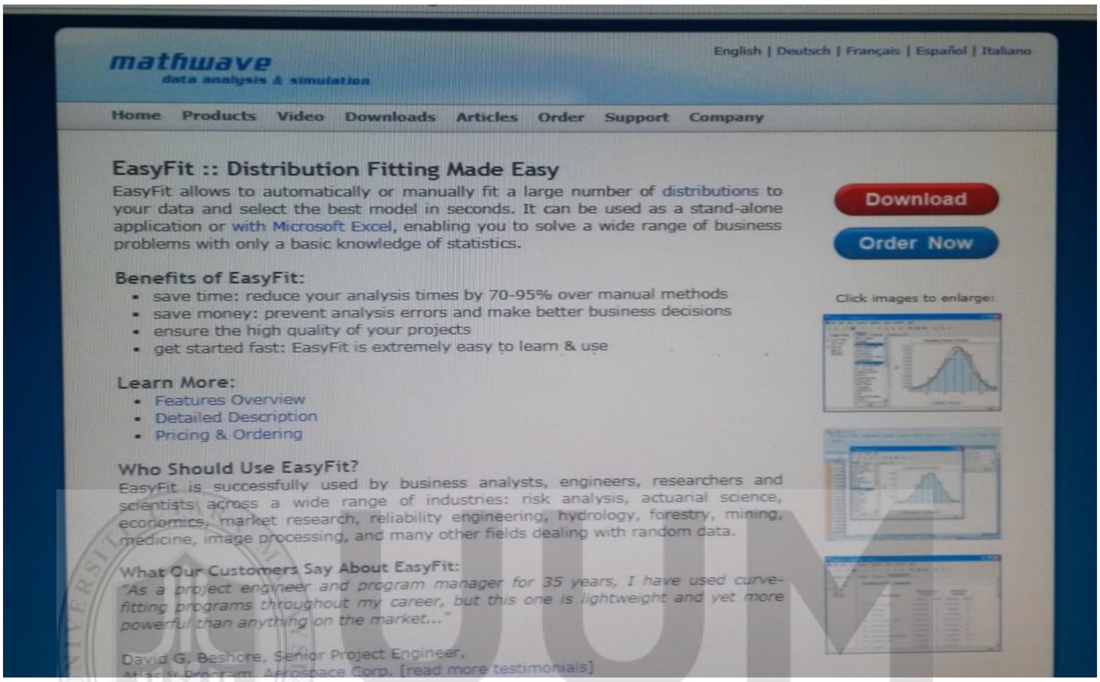
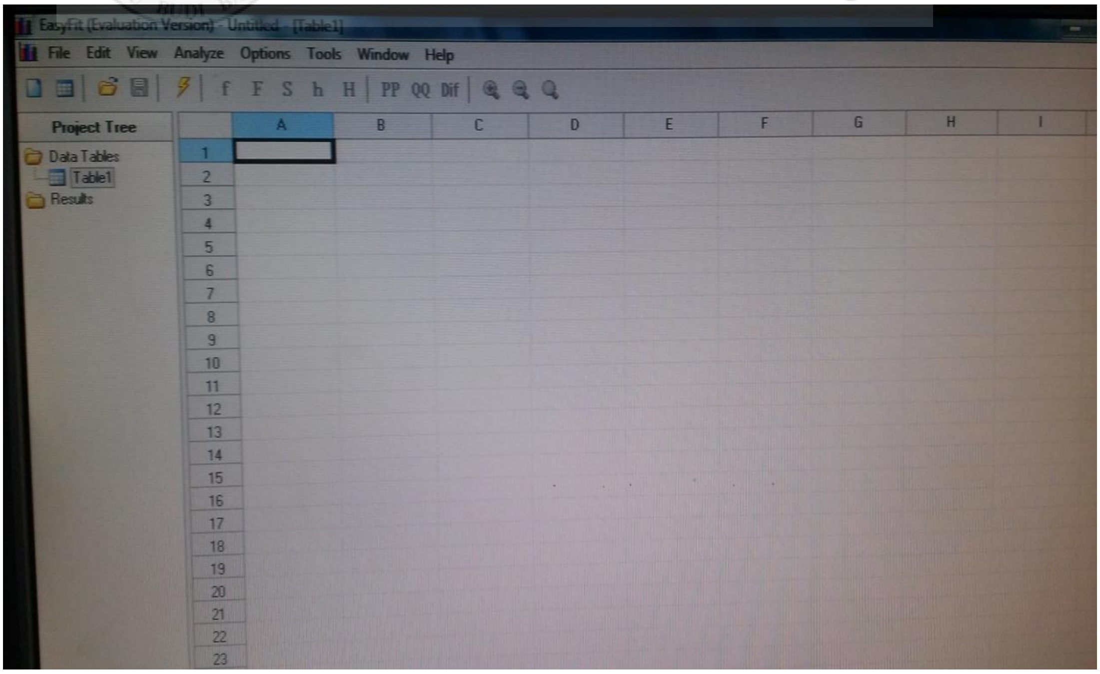
APPENDIX A

Procedure of Using EasyFit - Distribution Fitting Software

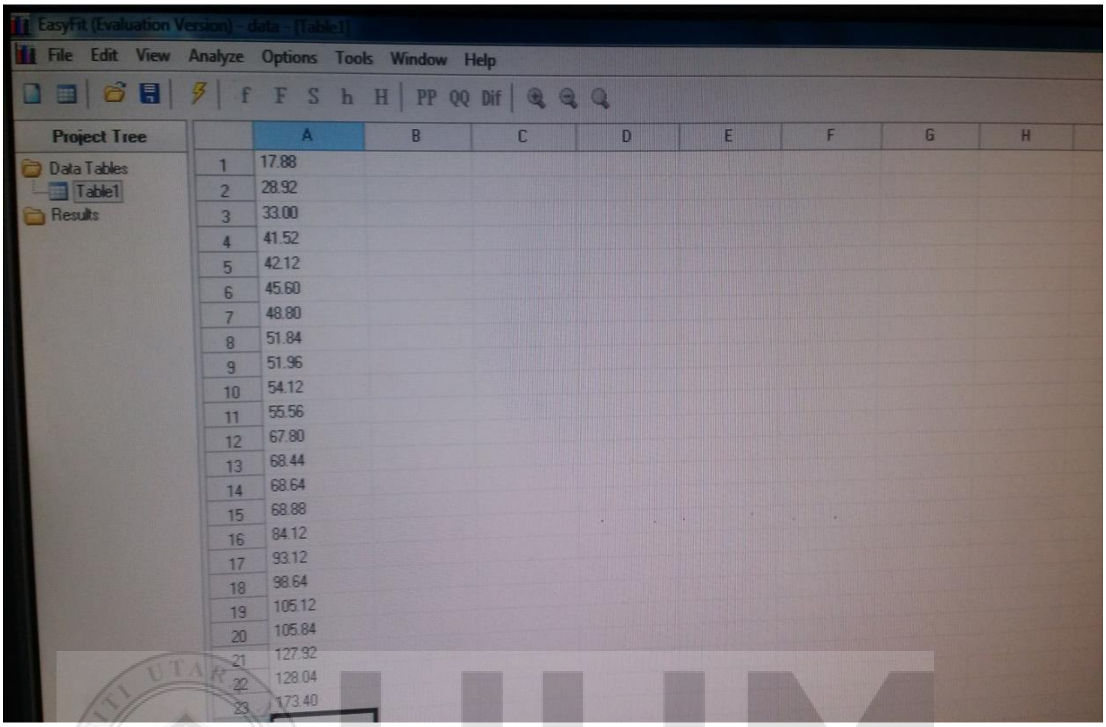
Step 1: Download the software



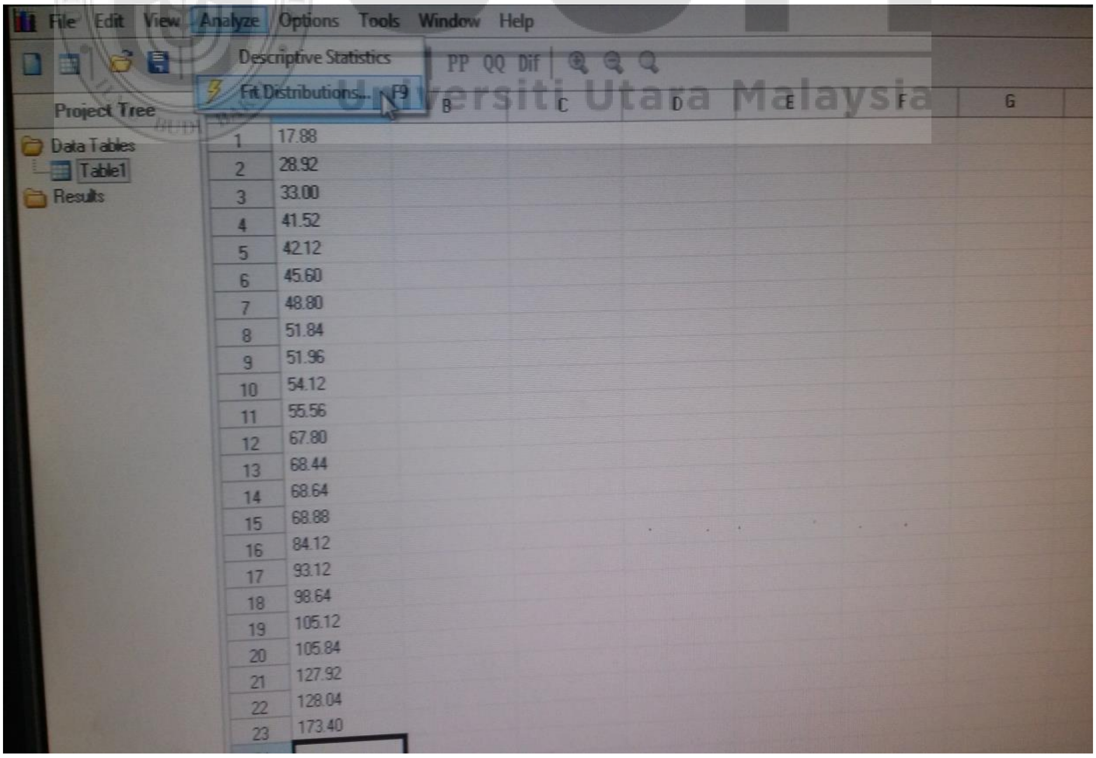
Step 2: Open the spreadsheet



Step 3: Enter the data



Step 4: Select fit distribution options



Step 5: Get the required results

	A	B	C	D	E	F	G	H	I	J
1	Data				Goodness of Fit - Summary				Fitting Results	
2	17.88							#	Distribution	Parameters
3	28.92		Sr#	Distribution	Kolmogorov Smirnov Statistic			1	Pareto 2	a=1.6293 b=133.97
4	33							2	Burr	k=0.26938 a=4.4062 b=39.729
5	41.52							3	Burr (4P)	k=0.36392 a=0.85281
6	42.12		1	Pareto 2	0.23587					b=2.1618 g=17.88
7	45.6		2	Inv. Gaussian (3P)	0.24914			4	Cauchy	s=22.408 m=60.098
8	48.8		3	Inv. Gaussian	0.26892			5	Chi-Squared	n=252
9	51.84		4	Gen. Gamma (4P)	0.27032			6	Chi-Squared (2P)	n=92 g=-31.863
10	51.96		5	Weibull (3P)	0.27129			7	Dagum	k=193.33 a=1.9496 b=2.3054
11	54.12		6	Pareto	0.2988			8	Dagum (4P)	k=3.6174 a=1.3793
12	55.56		7	Levy (2P)	0.29947					b=19.603 g=9.4152
13	67.8		8	Gamma (3P)	0.33437			9	Error	k=1.0 s=648.93 m=252.85
14	68.44		9	Chi-Squared (2P)	0.35376			10	Error Function	h=0.00109
15	68.64		10	Kumaraswamy	0.36506			11	Exponential	l=0.00395
16	68.88		11	Fatigue Life (3P)	0.3663			12	Exponential (2P)	l=0.00426 g=17.88
17	84.12		12	Dagum	0.38245			13	Fatigue Life	a=1.4671 b=140.61
18	93.12		13	Levy	0.38539			14	Fatigue Life (3P)	a=1.7999 b=108.2 g=12.943
19	98.64		14	Fatigue Life	0.40547			15	Frechet	a=1.192 b=46.725
20	105.12		15	Gumbel Max	0.4093			16	Frechet (3P)	a=1.3469 b=51.601 g=2.5763
21	105.84		16	Burr (4P)	0.42473			17	Gamma	a=0.15182 b=1665.4
22	127.92		17	Gen. Gamma	0.4326			18	Gamma (3P)	a=0.25399 b=1678.9 g=17.88
23	128.04		18	Power Function	0.4521			19	Gen. Extreme Value	k=0.84681 s=33.58 m=51.257
24	173.4		19	Uniform	0.45534			20	Gen. Gamma	k=1.107 a=0.31737 b=1665.4
25			20	Normal	0.46872			21	Gen. Gamma (4P)	k=0.81331 a=0.41479
26			21	Logistic	0.47529					b=511.92 g=17.88
27			22	Hypersecant	0.48084			22	Gen. Pareto	k=0.83243 s=37.69 m=27.93
28			23	Exponential	0.48267			23	Gumbel Max	s=505.97 m=-39.201
29			24	Reciprocal	0.49366			24	Gumbel Min	s=505.97 m=544.9
30			25	Error	0.49949			25	Hypersecant	s=648.93 m=252.85
31			26	Laplace	0.49949			26	Inv. Gaussian	l=38.389 m=252.85
32			27	Exponential (2P)	0.50574			27	Inv. Gaussian (3P)	l=45.407 m=240.8 g=12.056
33			28	Error Function	0.51099			28	Johnson SB	g=1.6883 d=0.24833
34			29	Johnson SB	0.52291					l=3719.0 x=23.482
35			30	Rayleigh (2P)	0.53063			29	Kumaraswamy	a1=0.30997 a2=1.6971