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**A FAMILY OF GROUP CHAIN ACCEPTANCE SAMPLING  
PLANS BASED ON TRUNCATED LIFE TEST**

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## Abstrak

Persampelan penerimaan merupakan prosedur kawalan kualiti berstatistik yang digunakan untuk menentukan sama ada untuk menerima atau menolak sesuatu lot, berdasarkan hasil pemeriksaan sampel. Bagi produk berkualiti tinggi, bilangan penerimaan sifar diambil kira dan ujian hayat ini selalunya diberhentikan pada masa tertentu, yang dipanggil ujian hayat terpangkas. Pelan yang melibatkan bilangan penerimaan sifar dianggap tidak adil terhadap pengeluar kerana kebarangkalian penerimaan lot menurun secara drastik pada kadar kerosakan yang sangat kecil. Untuk mengatasi masalah ini, persampelan berantai yang menggunakan maklumat lot sebelum dan selepas telah diperkenalkan. Bagi pelan persampelan berantai biasa, hanya satu produk yang boleh diperiksa pada satu masa, walaupun secara praktikalnya, penguji mampu memeriksa lebih dari satu produk serentak. Dalam situasi ini, pelan persampelan kumpulan berantai dengan sampel bersaiz kecil menjadi pilihan kerana ia menjimatkan masa dan kos pemeriksaan. Oleh yang demikian, adalah bermanfaat untuk membangunkan beberapa jenis pelan persampelan berantai dalam konteks ujian berkumpulan. Matlamat kajian ini adalah untuk membangunkan pelan persampelan baharu bagi kumpulan berantai (GChSP), kumpulan berantai yang diubahsuai (MGChSP), kumpulan berantai dua sisi (TS-GChSP) dan kumpulan berantai dua sisi yang diubahsuai (TS-MGChSP) menggunakan taburan Pareto jenis ke-2. Empat pelan tersebut juga digeneralisasikan berdasarkan beberapa nilai kadar kerosakan yang telah ditetapkan. Kajian ini melibatkan empat fasa: mengenal pasti beberapa kombinasi reka bentuk parameter; membangunkan prosedur; mendapatkan fungsi cirian pengoperasian; dan mengukur prestasi dengan menggunakan data simulasi dan data hayat yang sebenar. Pelan yang dibangunkan dinilai menggunakan beberapa reka bentuk parameter dan dibandingkan dengan pelan yang telah mantap berdasarkan bilangan kumpulan minimum,  $g$  dan kebarangkalian penerimaan lot,  $L(p)$ . Dapatan menunjukkan kesemua pelan yang dicadangkan mempunyai  $g$  yang lebih kecil dan  $L(p)$  yang lebih rendah berbanding dengan pelan yang telah mantap. Kesemua pelan tersebut berupaya menjimatkan masa dan kos pemeriksaan, serta memberikan lebih perlindungan kepada pengguna daripada menerima produk yang rosak. Ini seharusnya memberi banyak faedah kepada pengamal industri terutamanya yang melibatkan ujian musnah untuk produk berkualiti tinggi.

**Kata kunci:** Persampelan berantai, Persampelan penerimaan kumpulan, Lengkung cirian pengoperasian, Ujian hayat terpangkas, Persampelan rantaian dua sisi.

## Abstract

Acceptance sampling is a statistical quality control procedure used to accept or reject a lot, based on the inspection result of its sample. For high quality products, zero acceptance number is considered and the life test is often terminated on a specific time, hence called truncated life test. A plan having zero acceptance number is deemed unfair to producers as the probability of lot acceptance drops drastically at a very small proportion defective. To overcome this problem, chain sampling which uses preceding and succeeding lots information was introduced. In ordinary chain sampling plans, only one product is inspected at a time, although in practice, testers can accommodate multiple products simultaneously. In this situation, group chain sampling plan with small sample size is preferred because it saves inspection time and cost. Thus, it is worthwhile to develop the various types of chain sampling plans in the context of group testing. This research aims to develop new group chain (GChSP), modified group chain (MGChSP), two-sided group chain (TS-GChSP) and modified two-sided group chain (TS-MGChSP) sampling plans using the Pareto distribution of the 2<sup>nd</sup> kind. These four plans are also generalized based on several pre-specified values of proportion defective. This study involves four phases: identifying several combinations of design parameters; developing the procedures; obtaining operating characteristic functions; and measuring performances using both simulated and real lifetime data. The constructed plans are evaluated using various design parameters and compared with the established plan based on the number of minimum groups,  $g$  and probability of lot acceptance,  $L(p)$ . The findings show that all the proposed plans provide smaller  $g$  and lower  $L(p)$  compared to the established plan. All the plans are able to reduce inspection time and cost, and better at protecting customers from receiving defective products. This would be very beneficial to practitioners especially those involved with destructive testing of high quality products.

**Keywords:** Chain sampling, Group acceptance sampling, Operating characteristic curve, Truncated life test, Two-sided chain sampling.

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## Glossary of Terms

$a$	Pre-specified testing time
$c$	Acceptance number
$d$	Rejection number
$g$	Total number of groups
$L(p)$	Probability of lot acceptance
$r$	Group size
$t_0$	Test termination time
$\alpha$	Producer's risk
$\beta$	Consumer's risk
$\lambda$	Shape parameter of Pareto distribution of the 2 <sup>nd</sup> kind
$\sigma$	Scale parameter of Pareto distribution of the 2 <sup>nd</sup> kind
$\mu$	Mean lifetime of a product
$\mu_0$	Specified mean lifetime of a product
$\mu/\mu_0$	Mean ratio
$p$	Proportion defective
$n$	Sample size
AQL	Acceptable quality level
LTPD	Lot tolerance percent defective
AOQL	Average outgoing quality limit

## List of Publications

Mughal, A.R., Zain,Z., & Aziz, N. (2015). Time Truncated Group Chain Sampling Strategy for Pareto Distribution of the 2<sup>nd</sup> kind. *Research Journal of Applied Sciences, Engineering and Technology*, 10(4), 471-474.

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Mughal, A. R., Zain, Z., & Aziz, N. (2016). Two-Sided and Modified Two-Sided Group Chain Sampling Plan for Pareto Distribution of the 2<sup>nd</sup> kind. *Advance and Applications in Statistics*. (Accepted).

Mughal, A. R., Zain, Z., & Aziz, N. (2016). Modified and Generalized Modified Group Chain Sampling Plan based on Truncated Life Test. *Sains Malaysiana*. (In Review).





# **CHAPTER ONE**

## **INTRODUCTION**

In this chapter, the fundamental concepts of quality control and uses of probability distributions in acceptance sampling plans are explained. The objective of the study, methodology and analysis on acceptance sampling plans are also discussed. Several group chain acceptance sampling plans for attributes are developed for experimenters in order to reach the accurate probability of lot acceptance at pre-specified design parameters.

### **1.1 Background**

According to Juran (1951), “Quality means that a product meets customer needs leading to customer satisfaction, and quality also means all the activities in which a business engages in, to ensure that the product meets customer needs. You can think of this second aspect of quality as quality control - ensuring a quality manufacturing process”. Quality is a measure of excellence or a state of being free from defects, deficiencies and considerable variations. The quality of a product is brought about by the consistent adherence and verifiable standards to achieve uniformity of production that satisfies consumer or user necessities (Deva and Rebecca, 2012).

The International Organization for Standardization (ISO), founded in 1947, is a worldwide association of national standards which has contributed significantly in recent years (Schilling & Neubauer, 2008). The ISO’s standards offer guidance and tools for companies who want to ensure that their products meet customers’

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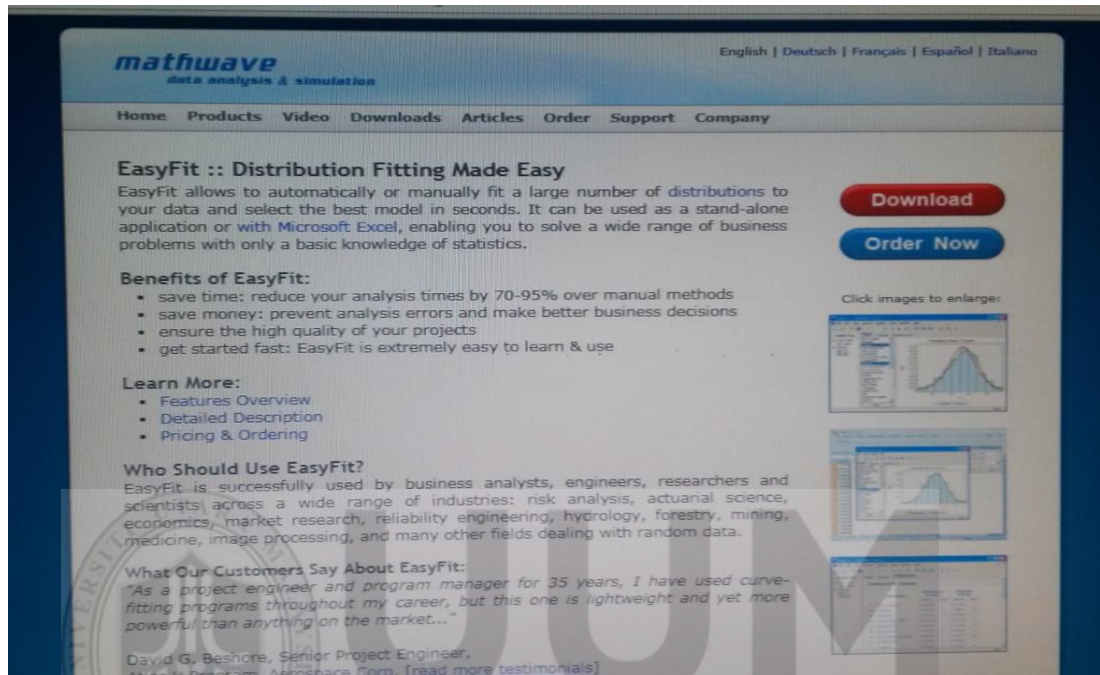
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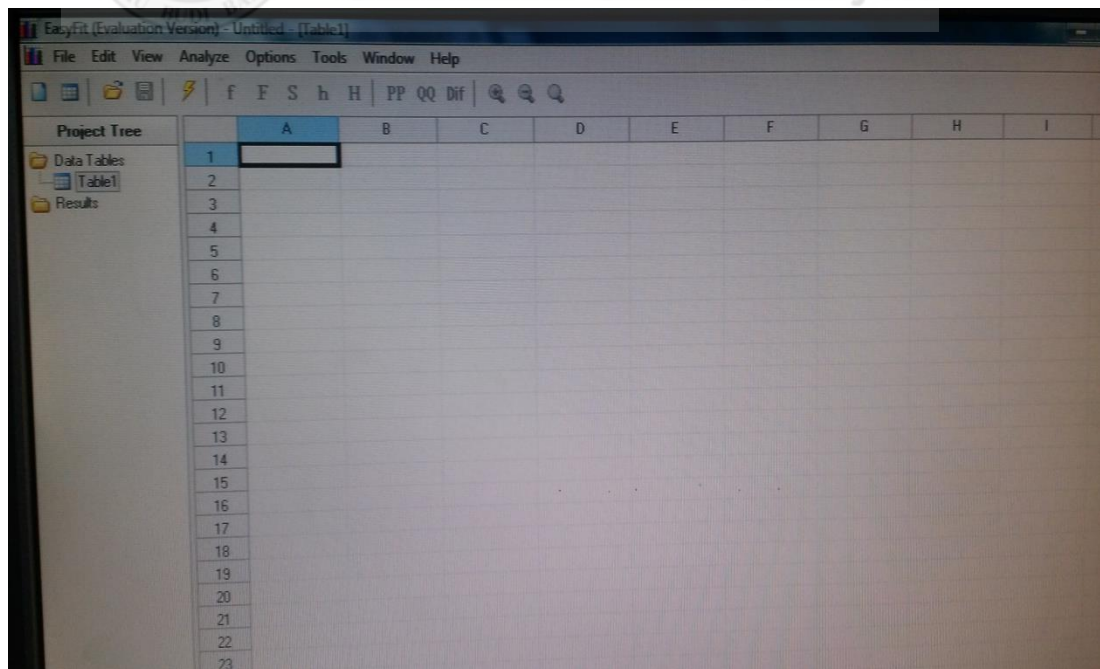
## APPENDIX A

### Procedure of Using EasyFit - Distribution Fitting Software

#### Step 1: Download the software

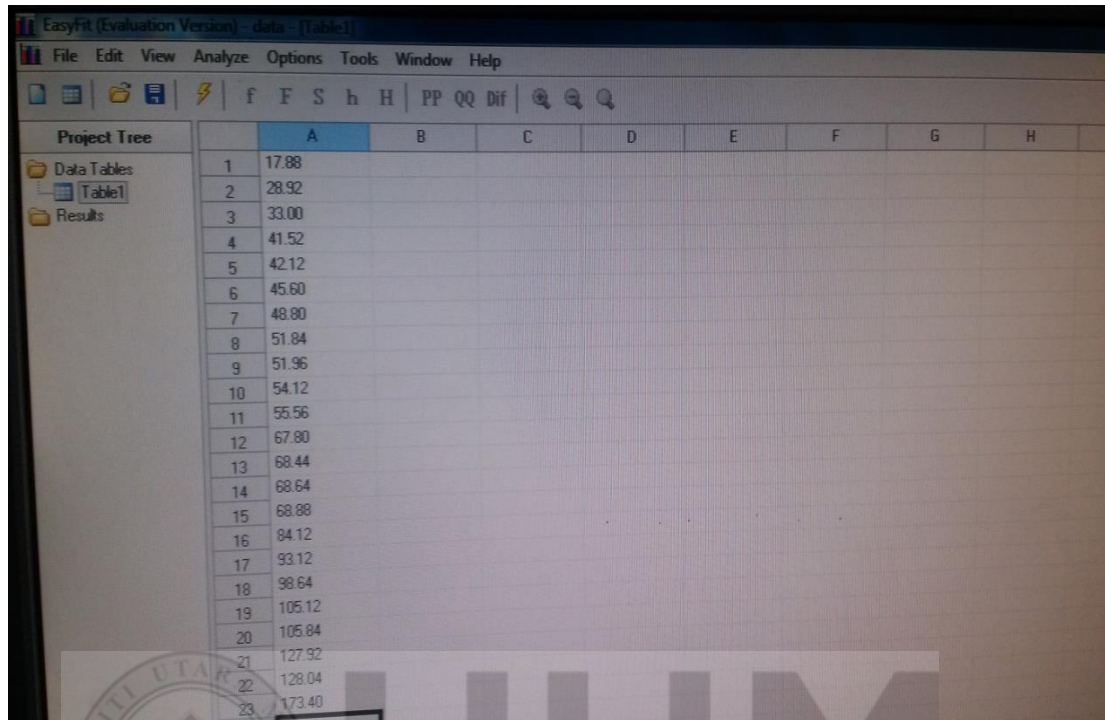


#### Step 2: Open the spreadsheet

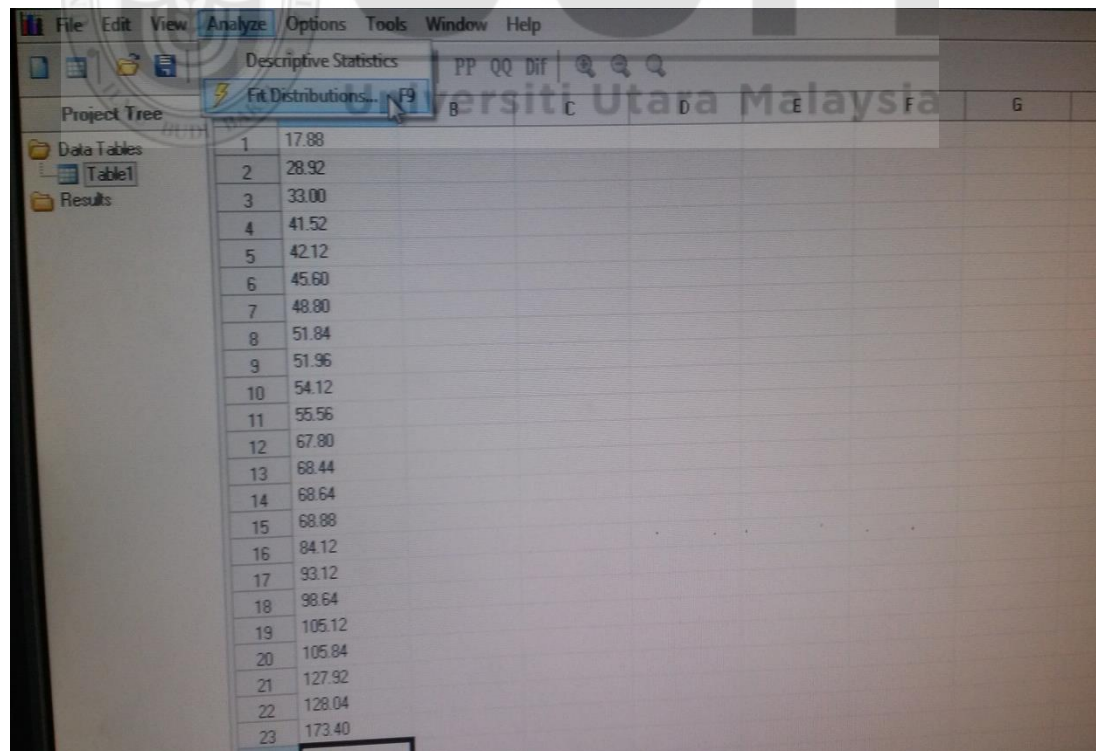




Step 3: Enter the data



Step 4: Select fit distribution options



## Step 5: Get the required results

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Data				Goodness of Fit - Summary				Fitting Results					
2	17.88						#	Distribution	Parameters					
3	28.92	5#	Distribution	Kolmogorov-Smirnov Statistic				1 Pareto2	a=18293 b=133.37					
4	33							2 Burr	k=0.28338 a=4.4062 b=38.723					
5	41.52							3 Burr(4P)	k=0.36332 a=0.85281					
6	42.12		1 Pareto2		0.23537				b=2.1618 g=17.88					
7	45.6		2 Inv. Gaussian (3P)		0.24914			4 Cauchy	s=22.408 m=60.038					
8	48.8		3 Inv. Gaussian		0.26832			5 Chi-Squared	n=252					
9	51.84		4 Gen. Gamma (4P)		0.27032			6 Chi-Squared (2P)	n=92 g=-31.863					
10	51.96		5 Weibull (3P)		0.27129			7 Dagum	k=183.33 a=19436 b=2.3054					
11	54.12		6 Pareto		0.2888			8 Dagum (4P)	k=3.6174 a=1.3733					
12	55.56		7 Levy (2P)		0.28947				b=19.603 g=9.4152					
13	67.8		8 Gamma (3P)		0.33437			9 Error	k=10 s=648.93 m=252.85					
14	68.44		9 Chi-Squared (2P)		0.35376			10 Error Function	k=0.00109					
15	68.64		10 Kumaraswamy		0.36506			11 Exponential	k=0.00395					
16	68.88		11 Faigue Life (3P)		0.3663			12 Exponential (2P)	k=0.00426 g=17.88					
17	84.12		12 Dagum		0.38245			13 Faigue Life	a=14671 b=140.61					
18	99.12		13 Levy		0.38539			14 Faigue Life (3P)	a=1.7333 b=108.2 g=12.943					
19	98.64		14 Faigue Life		0.40547			15 Frechet	a=1.192 b=46.725					
20	105.12		15 Gumbel Max		0.4093			16 Frechet (3P)	a=1.3469 b=51.601 g=2.5763					
21	105.84		16 Burr (4P)		0.42473			17 Gamma	a=0.15182 b=1665.4					
22	127.92		17 Gen. Gamma		0.4326			18 Gamma (3P)	a=0.25339 b=1678.9 g=17.88					
23	128.04		18 Power Function		0.4521			19 Gen. Extreme Value	k=0.04681 s=33.58 m=51.257					
24	173.4		19 Uniform		0.45534			20 Gen. Gamma	k=1.107 a=0.31737 b=1665.4					
25			20 Normal		0.46872			21 Gen. Gamma (4P)	k=0.01331 a=0.41479					
26			21 Logistic		0.47529				b=571.92 g=17.88					
27			22 Hypersecant		0.48084			22 Gen. Pareto	k=0.03243 s=37.69 m=27.33					
28			23 Exponential		0.48267			23 Gumbel Max	s=505.97 m=-39.201					
29			24 Reciprocal		0.49386			24 Gumbel Min	s=505.97 m=544.9					
30			25 Error		0.49949			25 Hypersecant	s=648.93 m=252.85					
31			26 Laplace		0.49949			26 Inv. Gaussian	k=38.389 m=252.85					
32			27 Exponential (2P)		0.50574			27 Inv. Gaussian (3P)	k=45.407 m=240.8 g=12.056					
33			28 Error Function		0.51039			28 Johnson SB	g=1.8883 d=0.24833					
34			29 Johnson SB		0.52231				k=3718.0 v=23.482					
35			30 Rayleigh (2P)		0.53063			29 Kumaraswamy	a1=0.30397 a2=1.6371					