A MULTIPLE CHANNEL QUEUEING MODEL UNDER AN UNCERTAIN ENVIRONMENT WITH MULTICLASS ARRIVALS FOR SUPPLYING DEMANDS IN A CEMENT INDUSTRY

ZEINA MUEEN MOHAMMED

DOCTOR OF PHILOSOPHY
UNIVERSITI UTARA MALAYSIA
2018

Awang Had Salleh
Graduate School
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Abstrak

Dalam tahun-tahun kebelakangan ini, penggunaan simen telah meningkat di kebanyakan negara Asia, termasuk Malaysia. Terdapat banyak faktor yang mempengaruhi bekalan tuntutan permintaan yang meningkat dalam industri simen, seperti kesesakan lalu lintas, logistik, cuaca dan kerosakan mesin. Faktor-faktor ini menghalang kelancaran dan kecekapan bekalan, terutamanya semasa kesesakan puncak di pintu masuk utama industri di mana wujud masa giliran dan menunggu akibat ketidakupayaan untuk memenuhi tarikh akhir. Unsur-unsur asas, seperti kadar ketibaan dan kadar perkhidmatan yang tidak dapat ditentukan lebih awal harus dipertimbangkan di bawah persekitaran yang tidak pasti. Kaedah-kaedah penyelesaian termasuk teknik giliran konvensional, model penjadualan dan simulasi tidak dapat merumus ukuran prestasi sistem giliran industri simen. Oleh itu, satu prosedur baru bagi selang subset kabur direkabentuk dan digabung ke dalam model giliran dengan mengambil kira kadar ketibaan dan kadar perkhidmatan. Hasilnya, satu model giliran berbilang saluran dengan berbilang kelas ketibaan, (M_1, M_2)/G/C/2Pr, di bawah persekitaran yang tidak pasti dibangunkan. Model ini dapat menganggar ukuran prestasi kadar ketibaan produk secara pukal iaitu Kelas Satu dan produk secara kampit iaitu Kelas Dua dalam sistem giliran perusahaan pengilang simen. Bagi model giliran kabur (M_1, M_2)/G/C/2Pr, dua teknik penyahkaburan, iaitu Pengaturcaraan Parameter Tak Linear dan Pemangkatan Teguh digunakan untuk menukar sistem giliran kabur kepada sistem giliran krisp. Ini menghasilkan tiga sub-model yang dicadangkan, iaitu sub-model 1, *MCFQ-2Pr*, sub-model 2, *MCCQ-ESR-2Pr* dan sub-model 3, *MCCQ-GSR-2Pr*. Model-model ini memberikan nilai-nilai krisp yang optimum untuk ukuran prestasi. Dalam menganggar prestasi keseluruhan sistem, satu langkah tambahan diperkenalkan melalui model *TrMF-UF* yang menggunakan satu faktor utiliti yang berdasarkan selang subset kabur dan pendekatan Potong- α. Justeru, model-model ini membantu para pembuat keputusan untuk menghadapi permintaan pesanan di bawah persekitaran yang tidak pasti dalam industri pembuatan simen dan menangani peningkatan jumlah yang diperlukan pada masa akan datang.

Kata kunci: Model giliran berbilang saluran, Selang subset kabur, Teknik nyahkabur, Faktor utiliti, Keutamaan giliran

Abstract

In recent years, cement consumption has increased in most Asian countries, including Malaysia. There are many factors which affect the supply of the increasing order demands in the cement industry, such as traffic congestion, logistics, weather and machine breakdowns. These factors hinder smooth and efficient supply, especially during periods of peak congestion at the main gate of the industry where queues occur as a result of inability to keep to the order deadlines. Basic elements, such as arrival and service rates, that cannot be predetermined must be considered under an uncertain environment. Solution approaches including conventional queueing techniques, scheduling models and simulations were unable to formulate the performance measures of the cement queueing system. Hence, a new procedure of fuzzy subset intervals is designed and embedded in a queuing model with the consideration of arrival and service rates. As a result, a multiple channel queueing model with multiclass arrivals, $(M_1, M_2)/G/C/2Pr$, under an uncertain environment is developed. The model is able to estimate the performance measures of arrival rates of bulk products for Class One and bag products for Class Two in the cement manufacturing queueing system. For the $(M_1, M_2)/G/C/2Pr$ fuzzy queueing model, two defuzzification techniques, namely the Parametric Nonlinear Programming and Robust Ranking are used to convert fuzzy queues into crisp queues. This led to three proposed sub-models, which are sub-model 1, *MCFQ-2Pr*, sub-model 2, *MCCQ-ESR-2Pr* and sub-model 3, *MCCQ-GSR-2Pr*. These models provide optimal crisp values for the performance measures. To estimate the performance of the whole system, an additional step is introduced through the *TrMF-UF* model utilizing a utility factor based on fuzzy subset intervals and the α-cut approach. Consequently, these models help decision-makers deal with order demands under an uncertain environment for the cement manufacturing industry and address the increasing quantities needed in future.

Keywords: Multiple channel queueing model, Fuzzy subset intervals, Defuzzification techniques, Utility factor, Priority queue

Acknowledgement

In the name of ALLAH, the Beneficent, the Merciful.

All praise to Almighty Allah, the Most Gracious and Merciful, who is omnipresent, for giving me the strength and determination to complete this study. This work simply could not have been possible without the assistance and encouragement from many others. Many people and institutions contributed their time and their expertise to the completion of this thesis. No words can express adequately my sense of indebtedness; yet I feel I shall be failing in my obligation if I do not put on record my gratitude to the following persons: First and foremost, I would like to thank the most important people that have made this thesis possible. The person is my great supervisor, Associate Professor Dr. Razamin Ramli. I sincerely thank her for her support and guidance throughout my PhD journey. I would also like to thank my other great supervisor, Dr. Nerda Zura Zaibidi, for her motivation and inspiration in supervising me during the difficult early stages of my journey.

I would like to thank my mother; may God prolong her life and my father, God's mercy on him, for their unconditional love, support and encouragement throughout my life, especially during my academic career. Thanks also to the Dean and management staff of the School of Quantitative Sciences.

Thank you all from the bottom of my heart.

May Allah bless you all.

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List of Abbreviations

Chi-Square	Chi-Sq
CIMA	Cement Industries of Malaysia
DSW	Dong, Shah and Wong
ESR	Exponential Service Rates
FCFS	First-Come -First-Service
FQ	Fuzzy Queueing
GSR	Gamma Service Rates
LB	Lower Bound
LR	Left and Right
LS	Left Side
Max	Maximum
Min	Minimum
MCCQ	Multiple Channel Crisp Queueing
MCFQ	Multiple Channel Fuzzy Queueing
MF	Membership Function
MSE	Mean Square Error
PM	Performance Measures
PNLP	Parametric Non-Linear Programming
PQC	Priority Queueing Concept
Pr	Priority
RPT	Residual Processing Time
RR	Robust Ranking
RS	Right Side
Tp	Trapezoidal
Tr	Triangular
UB	Upper Bound
UF	Utility Factor

List of Publications

- **Conference Proceedings**

1. Mueen, Z., Ramli, R., & Zaibidi, N. Z. (2015, December). Performance measurements of single server fuzzy queues with unreliable server using left and right method. I*n AIP Conference Proceedings* (Vol. 1691, No. 1, p. 030019). AIP Publishing.

- **Journals Articles**

1. Mueen, Z., Ramli, R., & Zaibidi, N. Z. (2016). Minimizing waiting times using multiple fuzzy queueing model with supply priorities. *Far East Journal of Mathematical Science*, *99*(8), 1177-1193. [Scopus (Q4) indexed].

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CHAPTER ONE
INTRODUCTION

A Cement production has undergone tremendous development from its beginnings some 2,000 years ago and is viewed as a very important material in various countries. Today's annual global cement production has reached 2.8 billion tonnes, and is expected to increase to some four billion tonnes annually (Schneider, Romer, Tschudin, & Bolio, 2011). While, the use of cement has a very long history, the industrial production of cement in the manufacturing sector only started in the middle of the 19th century.

This manufacturing sector is the cradle for a technological change, innovation and economic growth of any developing country. This is because innovative ideas that can transform the economic status of any developing country are usually conceived and commercialized in this sector, making it the true engine of developmental growth, technological advancement and economic prosperity. Out of the many industries that make up this sector, the cement industry has been identified as the foundation for any rapid structural and infrastructural developmental growth in both developed and developing countries (Crafts & Venables, 2003). This is due to the fact that economic expansion leads to industrialization, which usually creates the need for increased cement consumption, hence making this material a buzzword in the construction world.

The contents of the thesis is for internal user only

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APPENDICES

Appendix A: Secondary data from the cement factory (2013-2014)

Date	Daily arrivals of trucks (bulk)	Daily arrivals of trucks (bag)	Daily Average service time (hour/truck)
1/1/2013	21	13	1.1833
2/1/2013	20	17	1.5000
3/1/2013	17	12	1.1833
4/1/2013	25	9	1.4667
5/1/2013	12	21	1.2833
6/1/2013	23	27	1.5500
7/1/2013	25	8	1.6333
8/1/2013	20	21	0.8833
9/1/2013	49	4	0.9333
10/1/2013	38	9	1.0333
11/1/2013	18	14	0.9000
12/1/2013	10	24	1.2667
13/1/2013	21	11	0.8833
14/1/2013	5	9	0.9667
15/1/2013	20	31	0.7333
16/1/2013	36	27	1.3500
17/1/2013	31	21	0.9667
18/1/2013	32	29	0.8833
19/1/2013	24	23	1.0000
20/1/2013	30	29	1.3833
21/1/2013	23	6	0.7333
22/1/2013	43	23	0.7333
23/1/2013	40	12	0.8167
24/1/2013	29	22	1.1667
25/1/2013	15	20	0.7333
26/1/2013	13	14	0.8000
27/1/2013	32	14	0.7333
28/1/2013	10	13	1.3500
29/1/2013	21	23	0.7333
30/1/2013	17	23	1.6167
31/1/2013	28	27	1.5167
1/2/2013	17	24	0.7333
2/2/2013	20	12	0.8167
3/2/2013	27	12	1.5500
4/2/2013	36	6	0.8333
5/2/2013	21	31	1.0000

6/2/2013	29	12	1.3000
7/2/2013	23	29	0.9167
8/2/2013	5	4	0.8500
9/2/2013	0	0	0.0000
10/2/2013	0	0	0.0000
11/2/2013	0	0	0.0000
12/2/2013	8	4	1.1000
13/2/2013	15	14	0.7667
14/2/2013	30	6	1.3167
15/2/2013	10	4	0.7333
16/2/2013	25	5	0.8000
17/2/2013	0	0	0.0000
18/2/2013	34	17	0.9833
19/2/2013	29	29	1.4833
20/2/2013	44	24	1.6000
21/2/2013	46	28	1.0833
22/2/2013	36	24	0.9333
23/2/2013	37	12	1.0667
24/2/2013	30	8	0.8833
25/2/2013	32	20	0.9667
26/2/2013	31	6	0.9167
27/2/2013	39	20	0.9000
28/2/2013	26	13	1.0667
1/3/2013	36	20	0.9167
2/3/2013	25	9	1.1000
3/3/2013	21	19	0.7500
4/3/2013	25	15	0.8000
5/3/2013	40	5	1.3167
6/3/2013	38	30	0.9667
7/3/2013	49	14	1.0667
8/3/2013	43	24	1.3167
9/3/2013	30	21	1.0333
10/3/2013	19	25	1.4333
11/3/2013	24	21	1.6167
12/3/2013	39	9	0.9000
13/3/2013	32	23	1.5167
14/3/2013	30	26	0.7333
15/3/2013	20	30	0.9333
16/3/2013	36	21	0.7500
17/3/2013	34	23	0.8167
18/3/2013	8	23	1.1000
19/3/2013	30	4	0.8833
20/3/2013	45	28	0.7500

21/3/2013	24	33	0.8333
22/3/2013	25	25	1.1167
23/3/2013	25	6	1.1833
24/3/2013	14	15	1.0667
25/3/2013	28	13	1.6000
26/3/2013	12	10	1.1500
27/3/2013	15	20	1.2333
28/3/2013	8	17	0.7500
29/3/2013	20	14	0.8833
30/3/2013	9	28	1.5833
31/3/2013	15	6	1.2833
1/4/2013	18	7	0.7333
2/4/2013	25	32	0.8333
3/4/2013	50	24	1.3333
4/4/2013	24	20	1.8333
5/4/2013	9	17	0.8167
6/4/2013	23	15	0.7333
7/4/2013	28	24	0.9833
8/4/2013	25	15	1.3000
9/4/2013	31	10	0.7500
10/4/2013	41	23	1.0333
11/4/2013	21	19	0.9500
12/4/2013	30	23	0.7333
13/4/2013	34	13	1.0833
14/4/2013	32	17	1.2333
15/4/2013	30	25	1.1167
16/4/2013	31	26	1.0000
17/4/2013	22	26	1.5000
18/4/2013	20	30	1.3667
19/4/2013	28	17	1.3667
20/4/2013	5	17	1.3333
21/4/2013	18	29	1.3000
22/4/2013	4	20	1.3333
23/4/2013	16	13	1.1500
24/4/2013	6	26	0.9667
25/4/2013	11	15	0.7333
26/4/2013	5	26	0.7333
27/4/2013	5	22	0.7833
28/4/2013	6	14	1.0667
29/4/2013	13	8	1.4167
30/4/2013	25	7	1.0833
1/5/2013	5	28	1.3000
2/5/2013	20	6	0.9667

3/5/2013	20	15	0.9167
4/5/2013	6	15	0.7333
5/5/2013	5	28	0.9333
6/5/2013	8	2	1.0333
7/5/2013	37	21	1.4833
8/5/2013	29	24	1.1833
9/5/2013	23	12	0.7833
10/5/2013	14	19	0.8500
11/5/2013	9	26	1.0000
12/5/2013	9	18	1.0000
13/5/2013	29	10	1.5000
14/5/2013	14	22	0.8167
15/5/2013	20	31	0.8667
16/5/2013	24	32	0.7833
17/5/2013	23	29	1.5000
18/5/2013	21	28	1.5000
19/5/2013	15	18	1.3333
20/5/2013	22	10	1.6167
21/5/2013	39	20	0.8333
22/5/2013	43	29	1.3500
23/5/2013	39	21	1.2500
24/5/2013	37	24	1.1500
25/5/2013	6	13	0.7333
26/5/2013	15	29	1.1833
27/5/2013	21	14	1.0833
28/5/2013	26	27	1.2500
29/5/2013	10	32	1.9167
30/5/2013	13	25	1.2500
31/5/2013	31	19	0.9667
1/6/2013	15	28	1.0167
2/6/2013	11	23	0.9000
3/6/2013	26	13	0.9333
4/6/2013	7	24	1.1167
5/6/2013	15	20	1.6667
6/6/2013	30	28	1.5000
7/6/2013	16	28	1.3667
8/6/2013	46	28	1.2500
9/6/2013	17	28	1.6167
10/6/2013	15	28	0.7667
11/6/2013	18	28	1.0833
12/6/2013	41	24	0.9667
13/6/2013	16	30	0.7667
14/6/2013	23	33	1.4833

15/6/2013	18	30	1.4667
16/6/2013	36	26	0.9000
17/6/2013	11	30	0.9833
18/6/2013	39	24	0.8167
19/6/2013	17	32	1.5000
20/6/2013	16	31	1.0000
21/6/2013	7	24	0.9333
22/6/2013	6	28	1.0000
23/6/2013	22	6	1.1667
24/6/2013	6	19	1.6667
25/6/2013	9	30	0.9333
26/6/2013	12	27	0.8500
27/6/2013	12	25	0.9333
28/6/2013	17	23	0.8333
29/6/2013	13	26	0.8333
30/6/2013	6	23	0.9833
1/7/2013	20	12	1.2667
2/7/2013	32	24	1.6667
3/7/2013	18	23	0.9333
4/7/2013	20	21	0.7333
5/7/2013	36	5	1.6333
6/7/2013	18	30	1.2500
7/7/2013	18	13	1.5500
8/7/2013	31	4	1.5833
9/7/2013	37	26	1.2500
10/7/2013	36	25	0.9500
11/7/2013	16	17	1.3667
12/7/2013	13	27	1.5833
13/7/2013	11	23	1.4667
14/7/2013	11	30	1.5000
15/7/2013	14	5	1.0000
16/7/2013	23	15	0.8333
17/7/2013	20	31	1.0000
18/7/2013	23	32	0.9333
19/7/2013	34	28	1.5667
20/7/2013	37	15	0.9000
21/7/2013	18	24	0.7667
22/7/2013	20	3	0.8000
23/7/2013	20	32	0.9667
24/7/2013	15	11	0.9333
25/7/2013	25	16	1.5167
26/7/2013	10	6	1.5500
27/7/2013	14	11	1.4333

28/7/2013	7	8	1.1167
29/7/2013	12	6	0.8667
30/7/2013	17	7	1.4333
31/7/2013	31	19	0.9000
1/8/2013	7	26	0.9000
2/8/2013	13	33	1.1500
3/8/2013	16	19	1.3500
4/8/2013	26	21	1.6000
5/8/2013	4	15	1.4333
6/8/2013	18	3	1.4833
7/8/2013	0	0	0.0000
8/8/2013	0	0	0.0000
9/8/2013	0	0	0.0000
10/8/2013	4	1	0.8500
11/8/2013	13	11	1.5833
12/8/2013	26	15	0.7333
13/8/2013	45	22	1.5000
14/8/2013	26	12	1.5167
15/8/2013	32	19	0.8667
16/8/2013	49	4	1.5167
18/8/2013	0	0	0.0000
17/8/2013	8	4	0.9000
19/8/2013	38	15	0.9667
20/8/2013	43	29	1.0000
21/8/2013	32	30	1.1667
22/8/2013	14	33	1.4167
23/8/2013	19	32	0.7333
24/8/2013	12	27	1.1333
25/8/2013	23	27	1.4667
26/8/2013	31	7	1.6333
27/8/2013	8	27	1.5333
28/8/2013	19	27	1.5000
29/8/2013	19	24	0.7833
30/8/2013	32	26	1.3500
31/8/2013	14	10	1.6000
1/9/2013	32	3	1.5000
2/9/2013	22	16	1.2000
3/9/2013	32	30	1.4000
4/9/2013	35	22	0.7333
5/9/2013	29	25	0.8333
6/9/2013	37	7	1.1500
7/9/2013	19	15	1.2333
8/9/2013	17	12	1.3667

9/9/2013	18	12	0.8500
10/9/2013	46	22	1.5000
11/9/2013	42	16	0.8667
12/9/2013	38	7	1.4167
13/9/2013	30	12	1.1667
14/9/2013	12	31	1.4667
15/9/2013	6	26	1.1333
16/9/2013	12	19	1.0167
17/9/2013	13	6	1.0167
18/9/2013	39	18	1.5167
19/9/2013	19	31	1.0167
20/9/2013	47	29	0.7333
21/9/2013	27	18	0.9333
22/9/2013	17	23	1.1833
23/9/2013	14	5	1.2500
24/9/2013	27	18	1.0333
25/9/2013	43	30	1.2333
26/9/2013	28	22	0.8000
27/9/2013	20	21	1.5000
28/9/2013	16	30	0.7333
29/9/2013	23	15	0.9333
30/9/2013	27	15	1.0000
1/10/2013	47	18	1.0333
2/10/2013	40	10	1.0333
3/10/2013	28	18	1.1000
4/10/2013	35	21	1.5167
5/10/2013	15	25	0.9000
6/10/2013	35	13	1.1500
7/10/2013	15	7	1.3500
8/10/2013	26	25	0.8500
9/10/2013	36	18	1.4667
10/10/2013	13	18	1.0333
11/10/2013	37	22	1.5167
12/10/2013	35	30	1.4667
13/10/2013	6	30	1.4500
14/10/2013	6	10	1.4000
15/10/2013	20	8	1.1333
16/10/2013	12	3	1.2333
17/10/2013	12	33	1.5000
18/10/2013	37	19	1.4667
19/10/2013	42	8	0.9667
20/10/2013	18	8	0.9667
21/10/2013	23	13	1.0833

22/10/2013	23	30	1.5000
23/10/2013	12	21	1.1500
24/10/2013	30	16	1.2000
25/10/2013	9	21	1.4167
26/10/2013	28	26	1.0667
27/10/2013	7	33	0.7333
28/10/2013	18	9	0.8667
29/10/2013	19	30	1.0833
30/10/2013	35	18	1.3000
31/10/2013	30	27	1.0833
1/11/2013	27	19	1.3500
2/11/2013	17	16	1.3500
3/11/2013	26	16	0.9500
4/11/2013	33	16	1.1667
5/11/2013	28	32	0.9000
6/11/2013	7	23	1.2167
7/11/2013	33	32	1.4833
8/11/2013	48	27	1.0167
9/11/2013	12	35	1.2667
10/11/2013	22	11	1.0333
11/11/2013	10	6	1.4500
12/11/2013	12	15	1.0167
13/11/2013	16	9	1.4167
14/11/2013	10	16	1.2167
15/11/2013	13	16	1.2833
16/11/2013	20	31	1.3667
17/11/2013	26	15	1.4333
18/11/2013	4	8	1.2333
19/11/2013	20	19	1.3167
20/11/2013	23	15	1.0833
21/11/2013	6	19	1.4500
22/11/2013	19	9	1.4667
23/11/2013	23	4	0.9667
24/11/2013	12	5	1.5000
25/11/2013	7	1	1.1667
26/11/2013	28	11	0.7333
27/11/2013	29	12	1.2833
28/11/2013	19	21	1.0167
29/11/2013	21	11	1.4500
30/11/2013	31	8	0.9500
1/12/2013	17	16	0.8500
2/12/2013	12	6	1.1000
3/12/2013	26	17	1.2333

4/12/2013	19	25	1.2500
5/12/2013	17	30	1.1667
6/12/2013	23	19	1.1667
7/12/2013	13	21	1.1167
8/12/2013	23	22	0.8500
9/12/2013	26	3	1.3500
10/12/2013	28	17	1.2333
11/12/2013	14	13	1.3667
12/12/2013	28	15	1.4167
13/12/2013	14	21	1.2333
14/12/2013	22	6	1.1667
15/12/2013	26	19	1.1333
16/12/2013	14	16	1.0500
17/12/2013	17	25	0.9667
18/12/2013	28	21	1.0167
19/12/2013	14	8	0.9500
20/12/2013	9	3	0.9500
21/12/2013	12	12	1.0167
22/12/2013	14	12	0.8500
23/12/2013	14	21	1.1333
24/12/2013	14	33	1.2333
25/12/2013	14	14	1.3167
26/12/2013	22	14	1.1500
27/12/2013	14	22	0.9833
28/12/2013	22	25	1.1500
29/12/2013	22	13	0.9500
30/12/2013	17	19	1.0500
31/12/2013	43	22	1.0167
1/1/2014	12	18	0.5833
2/1/2014	8	15	0.5500
3/1/2014	10	9	0.6000
4/1/2014	9	10	1.0167
5/1/2014	13	30	1.0167
6/1/2014	9	34	0.7000
7/1/2014	19	13	0.5333
8/1/2014	12	32	1.0833
9/1/2014	20	10	0.6667
10/1/2014	4	20	0.9167
11/1/2014	15	26	0.5333
12/1/2014	16	29	0.8167
13/1/2014	11	18	1.3667
14/1/2014	16	19	1.0667
15/1/2014	7	22	0.6167

16/1/2014	10	15	0.5167
17/1/2014	3	7	1.2333
18/1/2014	5	32	0.9667
19/1/2014	7	30	0.7167
20/1/2014	7	22	0.7333
21/1/2014	7	13	0.7500
22/1/2014	4	15	1.3500
23/1/2014	8	18	0.6833
24/1/2014	11	5	0.5500
25/1/2014	11	7	0.5500
26/1/2014	7	14	0.7833
27/1/2014	9	14	0.6333
28/1/2014	7	9	0.8167
29/1/2014	0	0	0.0000
30/1/2014	0	0	0.0000
31/1/2014	0	0	0.0000
1/2/2014	0	0	0.0000
2/2/2014	0	0	0.0000
3/2/2014	11	18	0.5667
4/2/2014	7	31	0.8333
5/2/2014	9	25	0.6333
6/2/2014	9	8	0.6333
7/2/2014	8	25	0.8667
8/2/2014	0	0	0.0000
9/2/2014	11	24	0.5667
10/2/2014	10	15	0.6000
11/2/2014	8	43	0.7667
12/2/2014	13	4	0.5333
13/2/2014	6	23	1.0667
14/2/2014	12	19	0.6000
15/2/2014	12	24	0.5833
16/2/2014	7	17	0.9500
17/2/2014	11	24	0.6667
18/2/2014	12	9	0.6167
19/2/2014	5	16	0.5667
20/2/2014	5	31	1.1667
21/2/2014	8	25	1.2000
22/2/2014	6	28	0.8333
23/2/2014	3	33	0.7500
24/2/2014	14	10	0.5667
25/2/2014	13	20	0.5833
26/2/2014	15	16	0.5500
27/2/2014	9	19	0.9000

28/2/2014	17	19	0.5333
1/3/2014	9	29	0.9000
2/3/2014	14	12	0.6000
3/3/2014	3	40	0.8667
4/3/2014	5	35	1.0833
5/3/2014	5	20	0.8667
6/3/2014	34	37	0.9667
7/3/2014	38	19	0.6667
8/3/2014	27	23	0.9667
9/3/2014	27	18	0.5333
10/3/2014	29	28	0.7667
11/3/2014	34	14	0.5333
12/3/2014	7	16	0.5167
13/3/2014	6	28	0.6167
14/3/2014	9	22	0.5833
15/3/2014	11	37	0.9333
16/3/2014	4	23	0.6000
17/3/2014	6	21	1.2333
18/3/2014	13	23	0.7167
19/3/2014	9	17	1.0667
20/3/2014	10	26	0.9667
21/3/2014	10	23	0.9667
22/3/2014	17	27	0.6000
23/3/2014	9	16	1.1000
24/3/2014	21	13	0.5500
25/3/2014	14	22	0.8167
26/3/2014	16	27	0.6667
27/3/2014	16	18	0.6667
28/3/2014	14	17	0.7667
29/3/2014	13	42	0.8333
30/3/2014	13	23	0.8500
31/3/2014	14	19	0.7667
1/4/2014	16	16	0.6667
2/4/2014	13	29	0.9333
3/4/2014	22	13	0.5500
4/4/2014	10	10	1.1167
5/4/2014	16	16	0.6833
6/4/2014	14	22	0.7833
7/4/2014	12	12	1.0167
8/4/2014	19	17	0.6167
9/4/2014	15	24	0.7167
10/4/2014	14	31	0.9000
11/4/2014	14	24	0.8500

12/4/2014	14	18	0.9333
13/4/2014	14	8	0.8833
14/4/2014	3	2	0.5167
15/4/2014	14	15	0.9167
16/4/2014	21	18	0.6333
17/4/2014	16	19	0.8333
18/4/2014	19	41	0.6667
19/4/2014	17	22	0.8000
20/4/2014	17	24	0.8833
21/4/2014	18	22	0.7667
22/4/2014	10	24	1.3333
23/4/2014	21	20	0.6333
24/4/2014	17	31	0.8333
25/4/2014	15	39	0.9500
26/4/2014	13	33	1.0167
27/4/2014	12	29	1.1333
28/4/2014	13	20	1.0333
29/4/2014	21	31	0.6500
30/4/2014	15	18	0.9833
1/5/2014	16	12	0.8333
2/5/2014	20	48	0.6833
3/5/2014	10	19	1.4500
4/5/2014	18	12	0.9167
5/5/2014	20	42	0.7167
6/5/2014	14	18	1.0333
7/5/2014	14	27	1.0333
8/5/2014	22	16	0.6667
9/5/2014	18	24	0.9000
10/5/2014	25	19	0.6000
11/5/2014	24	30	0.6500
12/5/2014	18	9	0.9167
13/5/2014	23	18	0.6500
14/5/2014	22	19	0.6667
15/5/2014	21	13	0.7167
16/5/2014	20	16	0.8333
17/5/2014	12	8	1.4000
18/5/2014	18	36	0.8000
19/5/2014	10	18	1.2333
20/5/2014	18	13	0.8667
21/5/2014	13	19	1.1667
22/5/2014	12	20	1.3167
23/5/2014	22	11	0.6833
24/5/2014	25	10	0.6333

25/5/2014	12	24	1.3500
26/5/2014	19	35	0.9500
27/5/2014	20	22	0.8500
28/5/2014	13	30	1.2833
29/5/2014	13	7	1.3167
30/5/2014	27	14	0.6333
31/5/2014	27	10	0.6667
1/6/2014	25	29	0.7000
2/6/2014	23	27	0.8000
3/6/2014	19	21	0.9667
4/6/2014	33	10	0.5667
5/6/2014	25	30	0.7333
6/6/2014	12	14	1.5500
7/6/2014	16	15	1.1333
8/6/2014	16	28	1.2000
9/6/2014	20	35	0.9333
10/6/2014	27	24	0.6667
11/6/2014	19	33	0.9500
12/6/2014	19	30	0.9500
13/6/2014	25	37	0.7333
14/6/2014	23	27	0.8333
15/6/2014	16	20	1.1500
16/6/2014	13	16	1.4500
17/6/2014	23	28	0.8500
18/6/2014	23	26	0.8833
19/6/2014	27	20	0.6833
20/6/2014	24	27	0.8500
21/6/2014	16	20	1.2167
22/6/2014	21	18	1.0000
23/6/2014	30	6	0.6333
24/6/2014	12	21	1.2333
25/6/2014	21	43	0.9333
26/6/2014	17	6	1.1500
27/6/2014	20	5	0.9500
28/6/2014	21	17	1.0000
29/6/2014	22	33	0.9333
30/6/2014	28	16	0.7167
1/7/2014	19	29	1.1833
2/7/2014	21	31	1.0000
3/7/2014	21	33	1.0000
4/7/2014	6	9	1.0000
5/7/2014	26	22	0.8500
6/7/2014	25	37	0.9333

7/7/2014	21	29	1.0833
8/7/2014	24	24	0.9667
9/7/2014	26	27	0.8833
10/7/2014	25	30	1.0000
11/7/2014	38	13	0.6167
12/7/2014	26	29	0.9000
13/7/2014	12	6	1.4167
14/7/2014	21	23	1.0667
15/7/2014	34	3	0.7000
16/7/2014	16	15	1.5000
17/7/2014	23	15	0.9667
18/7/2014	21	29	1.0833
19/7/2014	27	27	0.9333
20/7/2014	20	10	1.1500
21/7/2014	20	10	1.1833
22/7/2014	26	12	0.9667
23/7/2014	24	18	1.0000
24/7/2014	28	36	0.9333
25/7/2014	29	34	0.8833
26/7/2014	0	0	0.0000
27/7/2014	0	0	0.0000
28/7/2014	0	0	0.0000
29/7/2014	0	0	0.0000
30/7/2014	19	7	1.3500
31/7/2014	29	29	0.9000
1/8/2014	29	25	0.8833
2/8/2014	0	0	0.0000
3/8/2014	17	14	1.5000
4/8/2014	18	26	1.5167
5/8/2014	6	7	1.3333
6/8/2014	33	16	0.7833
7/8/2014	20	17	1.2167
8/8/2014	28	36	0.9667
9/8/2014	17	16	1.7167
10/8/2014	21	17	1.2833
11/8/2014	33	31	0.8333
12/8/2014	25	33	1.0333
13/8/2014	22	5	1.2500
14/8/2014	32	23	0.9000
15/8/2014	28	22	0.9667
16/8/2014	28	27	1.0667
17/8/2014	19	17	1.5167
18/8/2014	19	23	1.7500

19/8/2014	26	13	1.0500
20/8/2014	24	15	1.2167
21/8/2014	28	27	1.0667
22/8/2014	16	12	1.2667
23/8/2014	22	29	1.3333
24/8/2014	22	23	1.3333
25/8/2014	32	31	0.9333
26/8/2014	36	23	0.8500
27/8/2014	34	30	0.9000
28/8/2014	38	16	0.7833
29/8/2014	20	14	1.8667
30/8/2014	36	17	0.9000
31/8/2014	26	6	1.1500
1/9/2014	17	17	1.2333
2/9/2014	29	14	1.0833
3/9/2014	26	13	1.2500
4/9/2014	32	16	0.9667
5/9/2014	22	27	1.4167
6/9/2014	25	15	1.3000
7/9/2014	27	8	1.0833
8/9/2014	22	23	1.5000
9/9/2014	27	14	1.1500
10/9/2014	28	13	1.1833
11/9/2014	36	16	0.9333
12/9/2014	21	7	1.2500
13/9/2014	32	22	1.0667
14/9/2014	23	23	1.4667
15/9/2014	35	21	0.9833
16/9/2014	34	18	0.9667
17/9/2014	30	7	1.0333
18/9/2014	17	12	1.3333
19/9/2014	29	27	1.2167
20/9/2014	27	28	1.2833
21/9/2014	43	13	0.8000
22/9/2014	43	12	0.8000
23/9/2014	23	9	1.4167
24/9/2014	26	14	1.3500
25/9/2014	29	21	1.3000
26/9/2014	36	20	1.0167
27/9/2014	28	24	1.3333
28/9/2014	37	5	1.0167
29/9/2014	38	30	1.0167
30/9/2014	37	34	1.0167

1/10/2014	26	10	1.4667
2/10/2014	23	23	1.2500
3/10/2014	36	10	1.0333
4/10/2014	32	18	1.1500
5/10/2014	0	0	0.0000
6/10/2014	47	10	0.8500
7/10/2014	38	23	1.0167
8/10/2014	30	24	1.3667
9/10/2014	4	23	1.3667
10/10/2014	70	14	2.5833
11/10/2014	40	4	0.9833
12/10/2014	48	12	0.8500
13/10/2014	29	22	1.3500
14/10/2014	32	17	1.1667
15/10/2014	58	13	0.7333
16/10/2014	49	24	0.8333
17/10/2014	29	6	1.3667
18/10/2014	30	25	1.3833
19/10/2014	3	8	1.0333
20/10/2014	4	20	1.0667
21/10/2014	4	26	1.4000
22/10/2014	7	9	1.3000
23/10/2014	3	25	1.5000
24/10/2014	13	36	1.2167
25/10/2014	40	20	1.0667
26/10/2014	41	21	1.0333
27/10/2014	47	14	0.9500
28/10/2014	37	29	1.0833
29/10/2014	45	24	0.9833
30/10/2014	33	27	1.2667
31/10/2014	48	16	0.9000
1/11/2014	31	16	1.4000
2/11/2014	31	16	1.4667
3/11/2014	28	23	1.2667
4/11/2014	22	13	1.6167
5/11/2014	24	20	1.3000
6/11/2014	52	10	0.8500
7/11/2014	28	14	1.3667
8/11/2014	46	29	0.9833
9/11/2014	31	22	1.5167
10/11/2014	42	17	1.0167
11/11/2014	33	15	1.3500
12/11/2014	35	29	1.2667

13/11/2014	40	10	1.1167
14/11/2014	34	26	1.3500
15/11/2014	47	14	0.9667
16/11/2014	41	21	1.1167
17/11/2014	43	5	1.1000
18/11/2014	26	16	1.3667
19/11/2014	38	30	1.1833
20/11/2014	43	17	1.0667
21/11/2014	27	21	1.2167
22/11/2014	38	29	1.1833
23/11/2014	40	8	1.1333
24/11/2014	37	18	1.2667
25/11/2014	43	29	1.1333
26/11/2014	33	18	1.5167
27/11/2014	48	18	0.9833
28/11/2014	44	31	1.0167
29/11/2014	53	19	0.9000
30/11/2014	30	19	1.3000
1/12/2014	49	10	1.0000
2/12/2014	49	32	1.1333
3/12/2014	35	14	1.4000
4/12/2014	43	27	1.1333
5/12/2014	39	26	1.1833
6/12/2014	45	10	1.0833
7/12/2014	39	22	1.3167
8/12/2014	55	24	0.9333
9/12/2014	39	21	1.3000
10/12/2014	45	25	1.1333
11/12/2014	52	10	1.0000
12/12/2014	45	26	1.0667
13/12/2014	17	19	0.8667
14/12/2014	36	26	1.4333
15/12/2014	12	9	1.4500
16/12/2014	36	21	1.3333
17/12/2014	42	14	1.2833
18/12/2014	43	18	1.2000
19/12/2014	45	32	1.1500
20/12/2014	31	16	1.2333
21/12/2014	21	10	0.5000
22/12/2014	53	19	0.9333
23/12/2014	38	26	1.3333
24/12/2014	38	25	1.2000
25/12/2014	52	15	1.2000

26/12/2014	43	18	1.1500
27/12/2014	31	19	1.4000
28/12/2014	60	14	0.6000
29/12/2014	54	24	1.0000
30/12/2014	42	18	1.3333
31/12/2014	34	15	1.5167
Total	16223	13879	

Appendix B: Codes for fuzzification phase using Matlab

```
format compact
B = xlsread('C:\Users\zeina\Desktop\my computer\Fuzzification step matlab\ALL
DATA.xlsx','DATA2013-DATA2014');
A = B(any(B,2),:);    % Exclude the zero rows
Asc = sort(A,1);      % Sort each of columns matrix in ascending order
Lamda1 = (Asc(:,1));  % Lamda1
LBm1=min(Lamda1)      % min of Lamda1
UBmx1=max(Lamda1)     % Max of Lamda1
MeanL1=mean2(Lamda1)  % Mean of Lamda1
AvL11 = Lamda1 < MeanL1;   % specifying the values under MeanL1 with 1 and 0 for else
AvL1=Lamda1(AvL11);        % Assign these values to AvL1
AvLmL1=mean2(AvL1)         % mean Value of all Lamda1< Mean of Lamda1
LM11 = Lamda1 < AvLmL1;
LM111=Lamda1(LM11);
LML1=mode(LM111)
LM22 = (AvLmL1< Lamda1 & Lamda1<  MeanL1);  %A(A<9 & A>2)
LM222=Lamda1(LM22);
LML12=mode(LM222)
AvHL11=Lamda1 > MeanL1;    % specifying the values upper MeanL1 with 1 and 0 for else
AvHL1=Lamda1(AvHL11);      % Assign these values to AvHL1
AvHmL1=mean2(AvHL1)        % mean Value of all Lamda1> Mean of Lamda1
UM11 = (AvHmL1< Lamda1 & Lamda1<  UBmx1);
UM111=Lamda1(UM11);
UML1=mode(UM111)
UM22 = (Lamda1 < AvHmL1  & Lamda1>  MeanL1);  %A(A<9 & A>2)
UM222=Lamda1(UM22);
UM2L1=mode(UM222)
%%
Lamda2 = (Asc(:,2));  % Lamda2
LBm2=min(Lamda2)      % Min of Lamda2
UBmx2=max(Lamda2)     % Max of Lamda2
MeanL2=mean2(Lamda2)  % Mean of Lamda2
AvL21 = Lamda2 < MeanL2;   % specifying the values under MeanL1 with 1 and 0 for else
AvL2=Lamda2(AvL21);        % Assign these values to AvL1
AvLmL2=mean2(AvL2)         % mean Value of all Lamda1< Mean of Lamda1
LM21 = Lamda2 < AvLmL2;
LM211=Lamda2(LM21);
LM2=mode(LM211)
LM2L2 = (AvLmL2< Lamda2 & Lamda2<  MeanL2);  %A(A<9 & A>2)
LM2L22=Lamda1(LM2L2);
LML2=mode(LM2L22)
AvHL21=Lamda2 > MeanL2;    % specifying the values upper MeanL1 with 1 and 0 for else
AvHL2=Lamda2(AvHL21);      % Assign these values to AvHL1
AvHmL2=mean2(AvHL2)        % mean Value of all Lamda1> Mean of Lamda1
UM21 = (AvHmL2< Lamda2 & Lamda2<  UBmx2);
UM211=Lamda2(UM21);
UML2=mode(UM211)
UML22 = (Lamda2 < AvHmL2  & Lamda2>  MeanL2);  %A(A<9 & A>2)
UML222=Lamda2(UML22);
UML22=mode(UML222)
    %%
Mu = (Asc(:,4));      % Mu
LBmM=min(Mu)          % Min of Mu
UBmxM=max(Mu)         % Max of Mu
```

```
MeanMu=mean2(Mu)      % Mean of Mu
AvMu11 = Mu < MeanMu;   % specifying the values under MeanL1 with 1 and 0 for else
AvMu111=Mu(AvMu11);        % Assign these values to AvL1
AvMu1=mean2(AvMu111)        % mean Value of all Lamda1< Mean of Lamda1
LMMu11 = Mu < AvMu1;
LMMu111=Mu(LMMu11);
LMMu1=mode(LMMu111)

 LMMu22 = (AvMu1< Mu & Mu< MeanMu);  %A(A<9 & A>2)
 LMMu222=Mu(LMMu22);
LMMu2=mode(LMMu222)
AvHMu11=Mu > MeanMu;    % specifying the values upper MeanL1 with 1 and 0 for else
AvHMu111=Mu(AvHMu11);      % Assign these values to AvHL1
AvHMu1=mean2(AvHMu111)       % mean Value of all Lamda1> Mean of Lamda1
UMMu11 = (AvHMu1< Mu & Mu< UBmxM);
UMMu111=Mu(UMMu11);
UMMu1=mode(UMMu111)
UMMu22 = (Mu < AvHMu1  & Mu>  MeanMu);  %A(A<9 & A>2)
UMMu222=Mu(UMMu22);
UMMu2=mode(UMMu222)
LB = [LBm1;LBm2;LBmM];
LM1 = [LML1;LM2;LMMu1];
Avg.L= [AvLmL1;AvLmL2;AvMu1];
LM2= [LML12;LML2;LMMu2];
Avg.M= [MeanL1;MeanL2;MeanMu];
UM2= [UM2L1;UML22;UMMu2];
Avg.H= [AvHmL1;AvHmL2;AvHMu1];
UM1= [UML1;UML2;UMMu1];
UB = [UBmx1;UBmx2;UBmxM];
data=[LB,LM1,Avg.L,LM2,Avg.M,UM2,Avg.H,UM1,UB];

% Create the column and row names in cell arrays
f = figure;
colnames = {'LB';'LM1';'Avg.L';'LM2';'Avg.M';'UM2';'Avg.H';'UM1';'UB'};
rnames = {'Lamda1';'Lamda2';'Mu'};
% Create the uitable
t = uitable(f, 'Data', data, 'ColumnName', colnames,'RowName', rnames,...
                 'Position', [20 20 760 100]);
x0=300;
y0=80;
width=600;
height=100;
set(gcf,'units','points','position',[x0,y0,width,height])
```

Appendix C: The histogram for intervals (i.e. low, medium and high) of service rates (gamma and exponential distributions) for 2013-2014 data

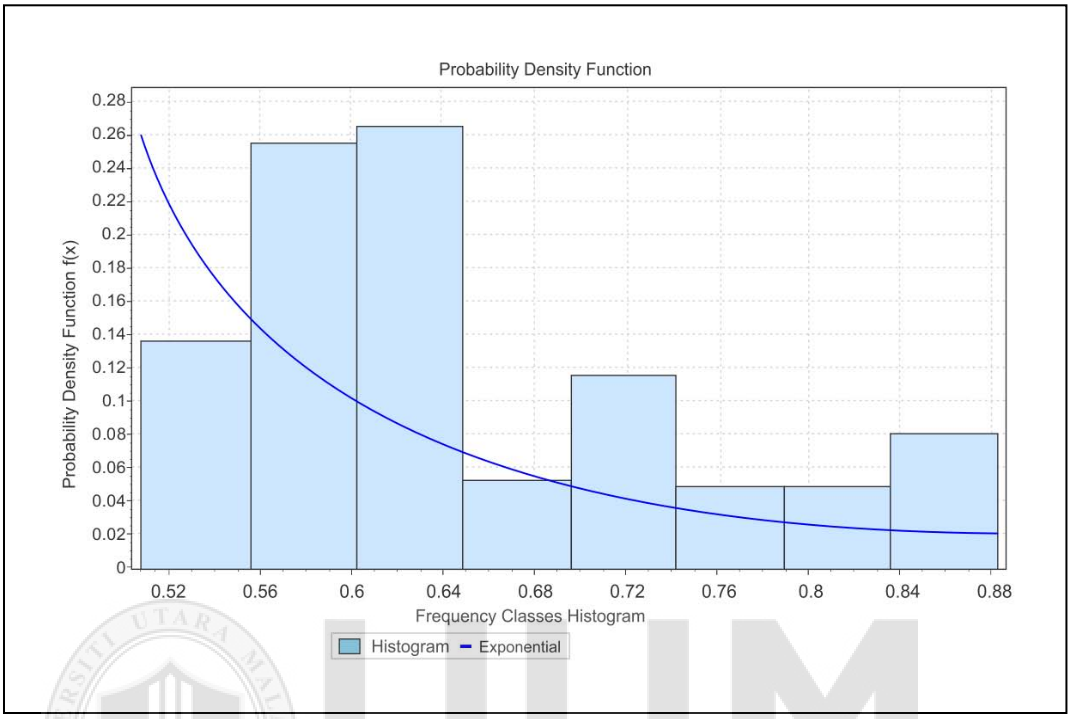

Figure C1. Exponential Service Rates of the Low Interval for 2013 Data

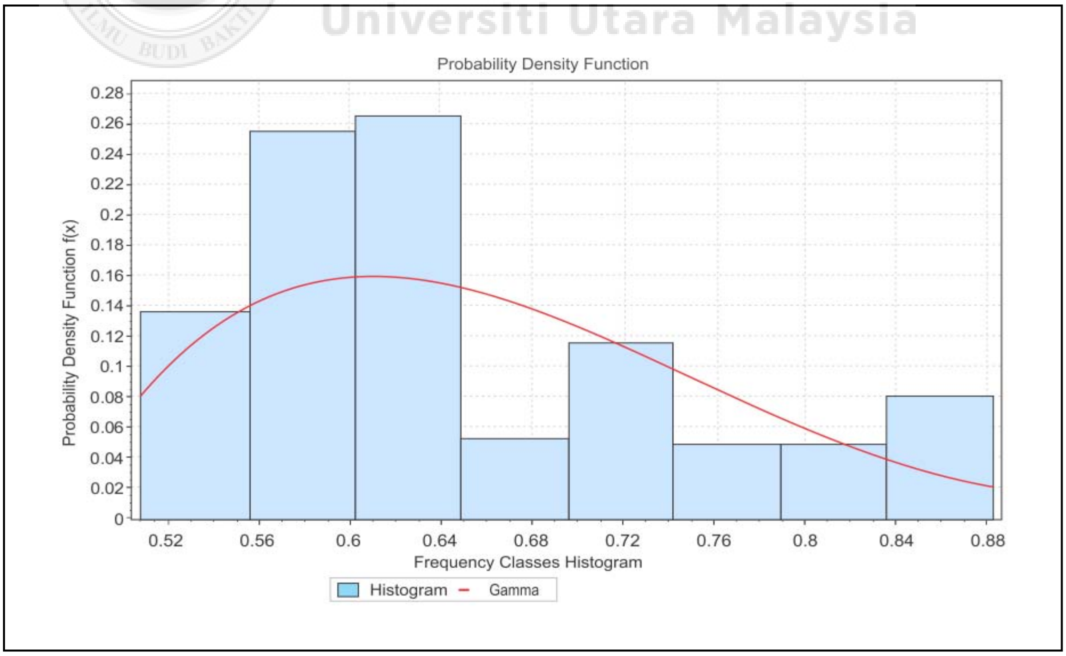

Figure C.2. Gamma Service Rates of the Low Interval for 2013 data

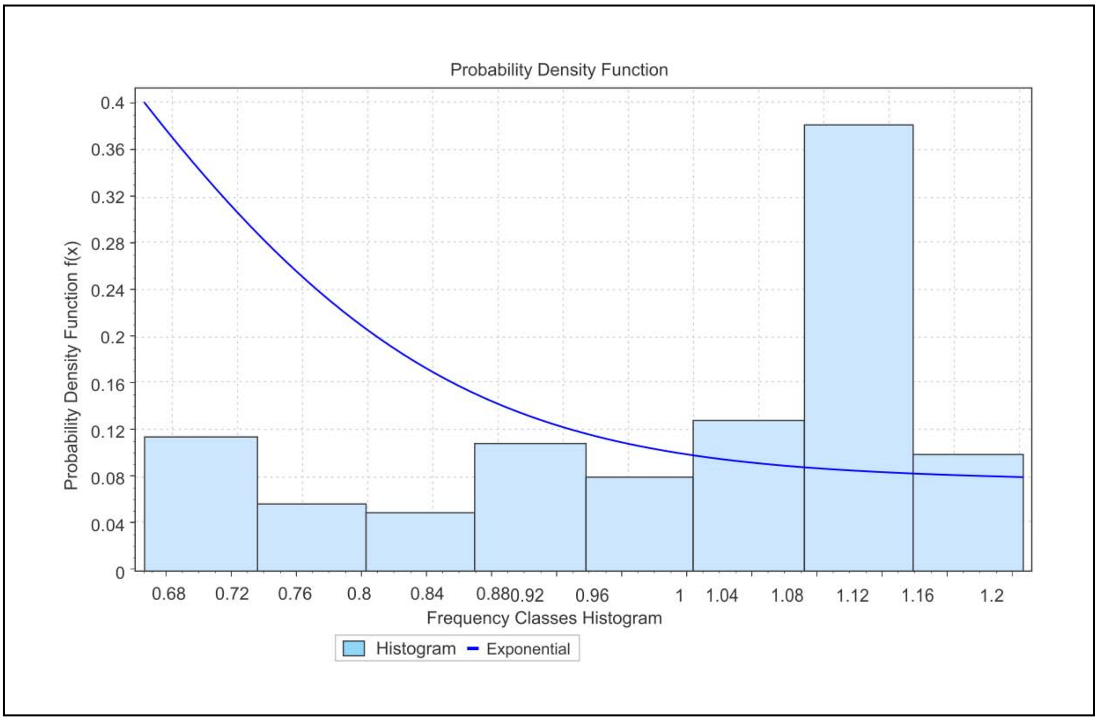

Figure C.3. Exponential Service Rates of the Medium Interval for 2013 Data

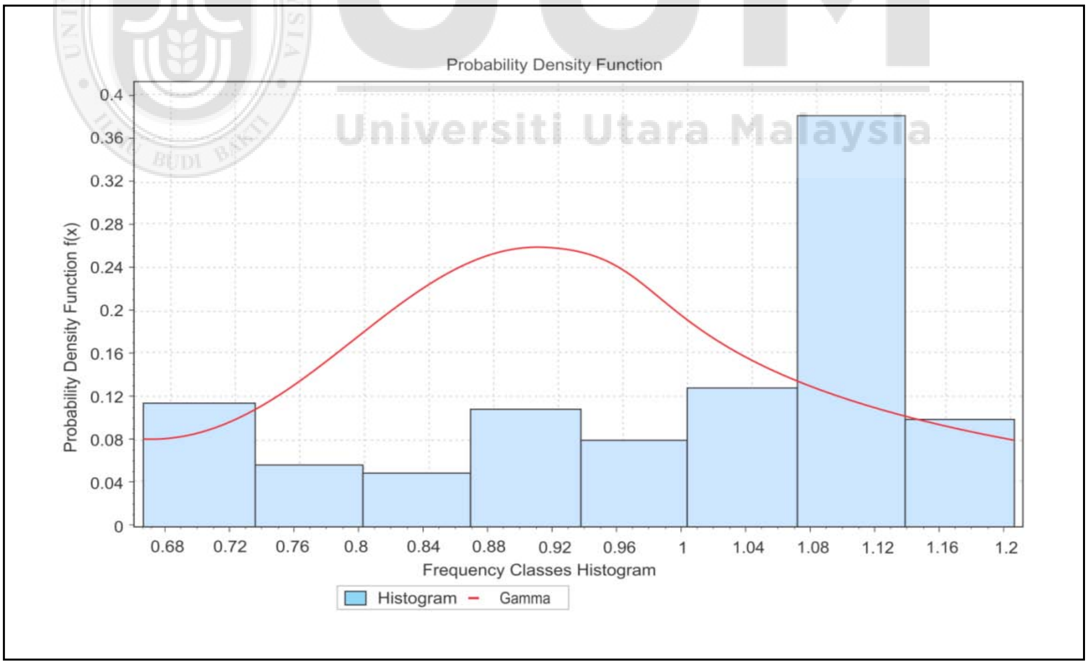

Figure C.4. Gamma Service Rates of the Medium Interval for 2013 Data

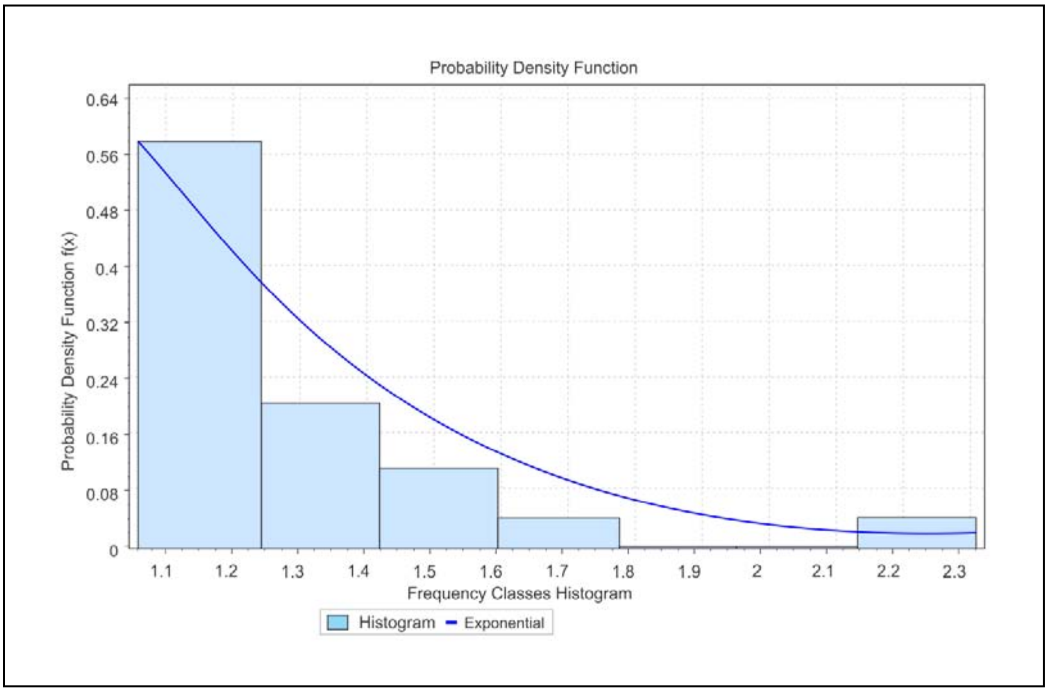

Figure C.5. Exponential Service Rates of the High Interval for 2013 Data

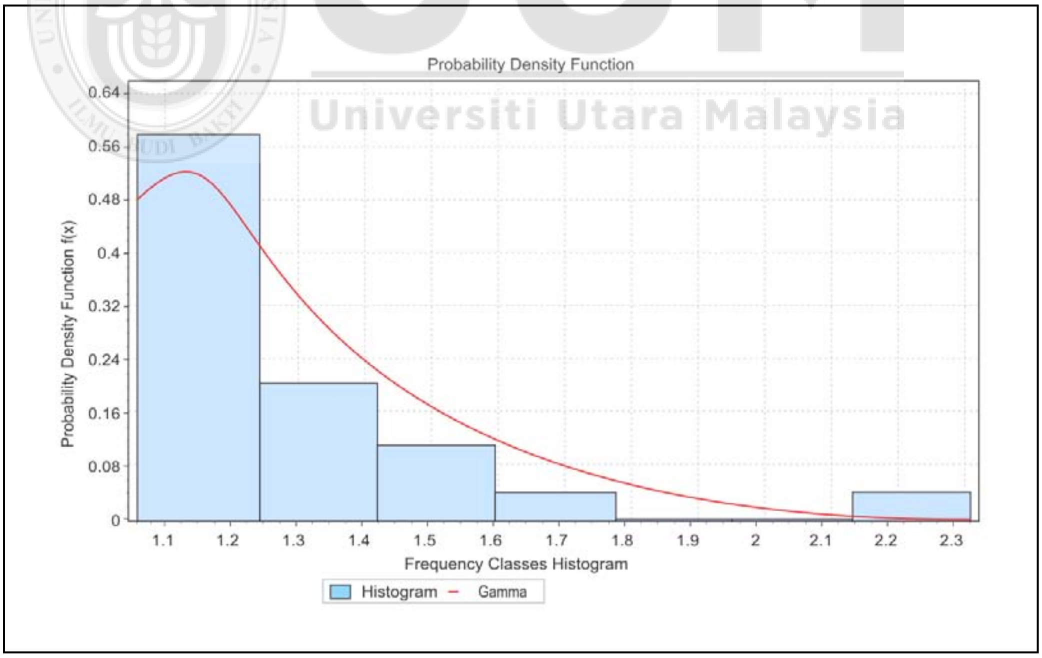

Figure C.6. Gamma Service Rates of the High Interval for 2013 Data

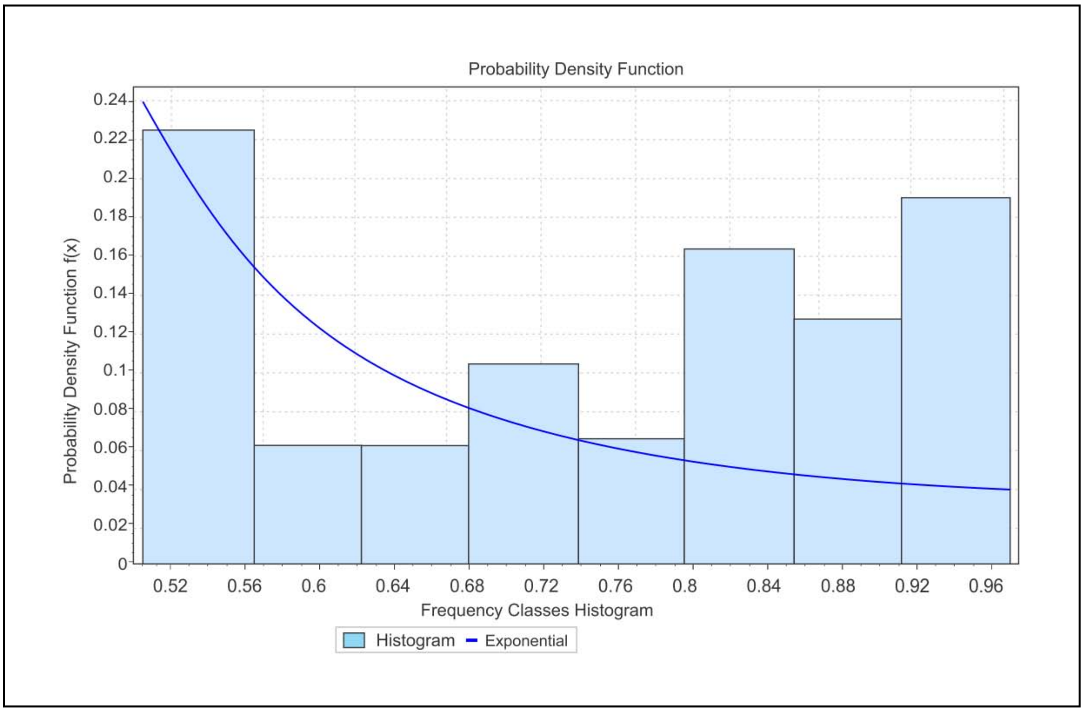

Figure C.7. Exponential Service Rates of the Low Interval for 2014 Data

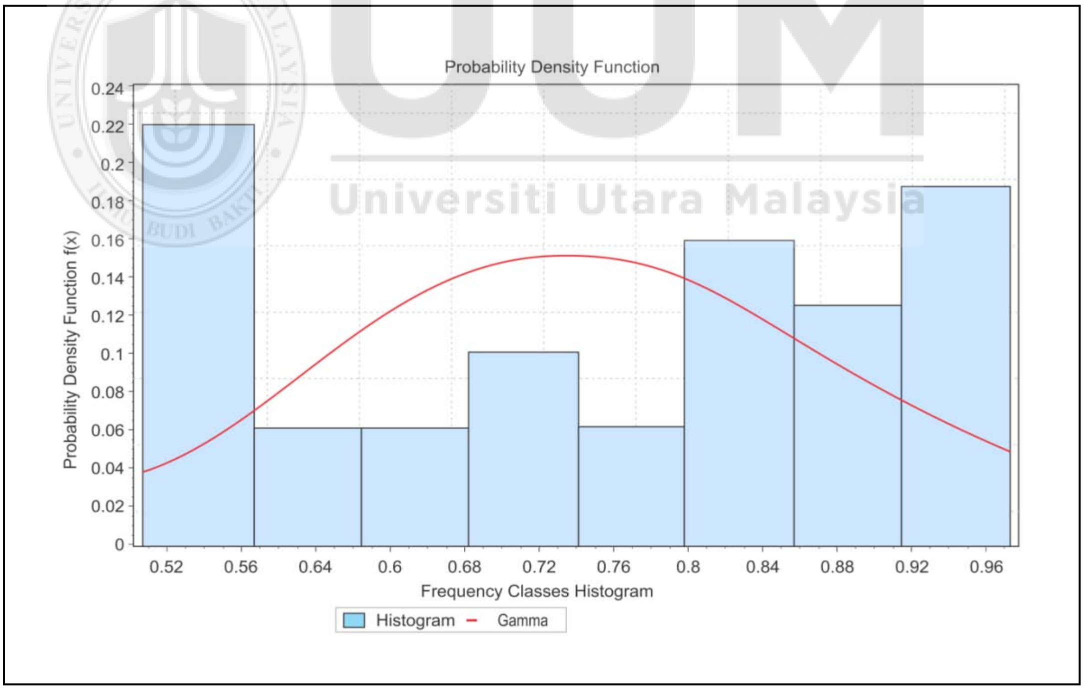

Figure C.8. Gamma Service Rates of the Low Interval for 2014 Data

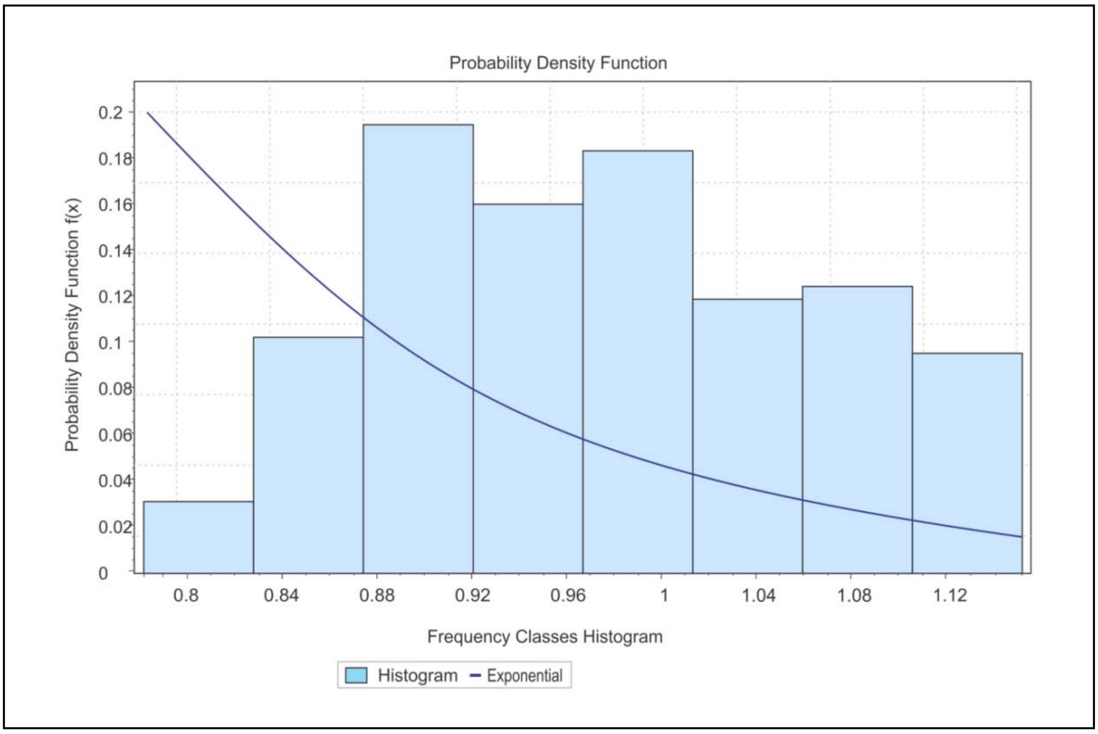

Figure C.9. Exponential Service Rates of the Medium Interval for 2014 Data

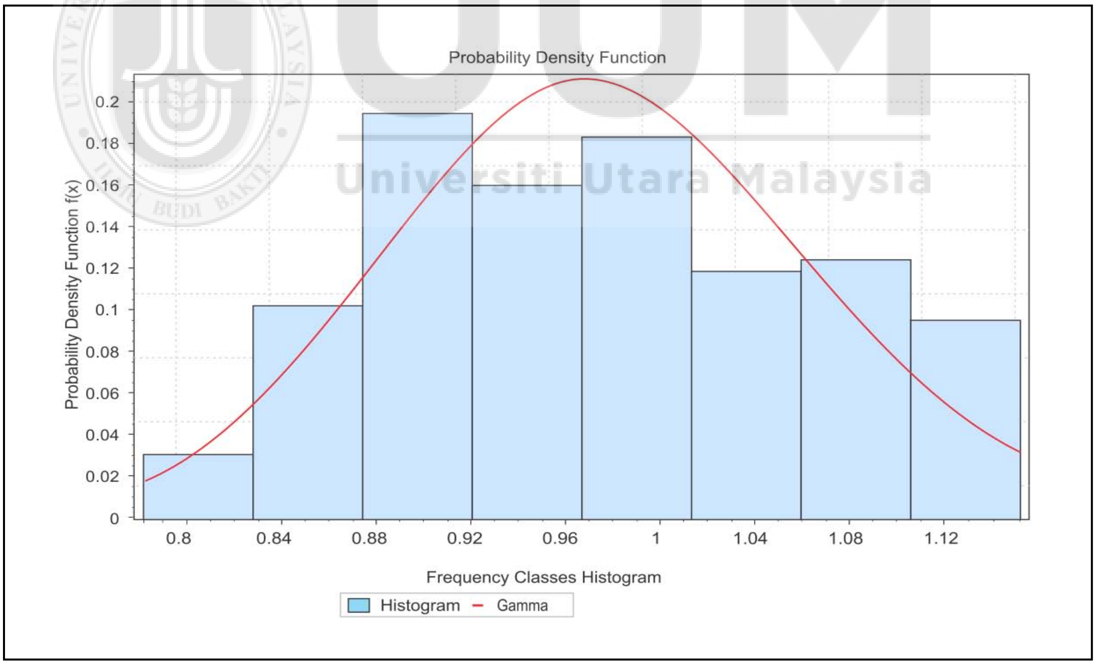

Figure C.10. Gamma Service Rates of the Medium Interval for 2014 Data

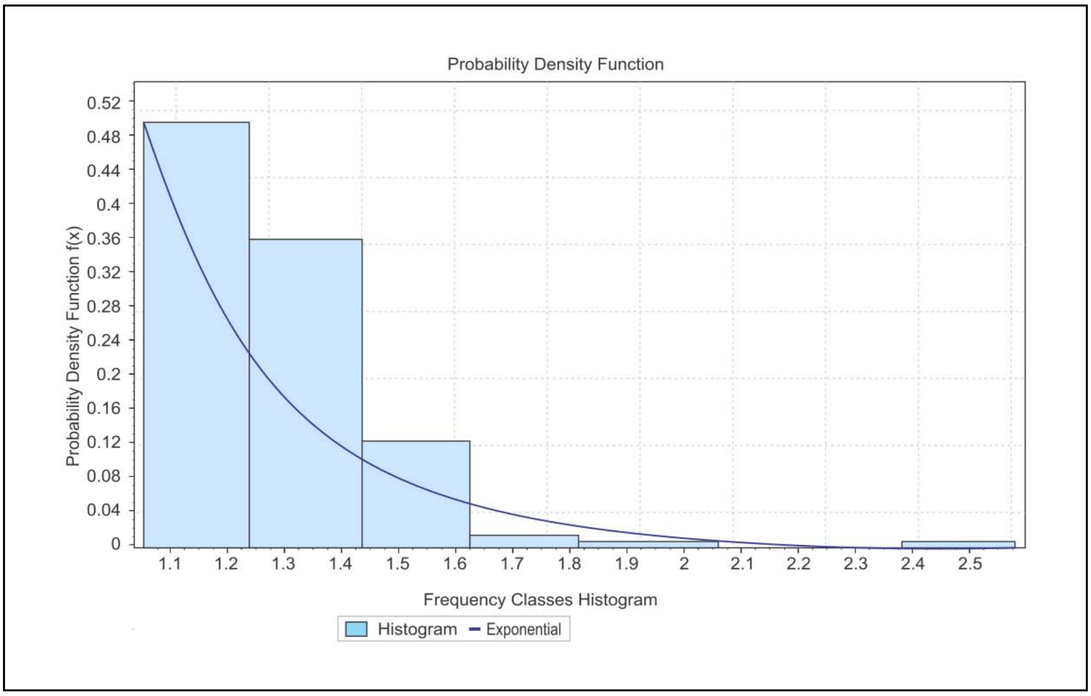

Figure C.11. Exponential Service Rates of the High Interval for 2014 Data

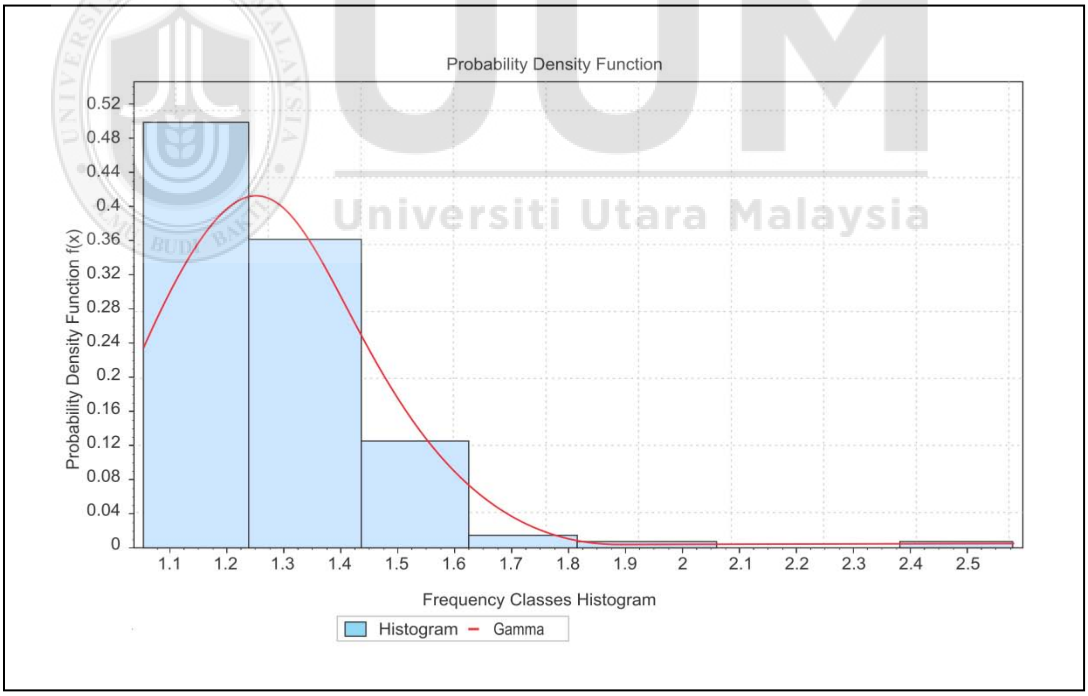

Figure C.12. Gamma Service Rates of the High Interval for 2014 Data

Appendix D: The values of performance measures for sub-model 1

$$
\begin{aligned}
D = {} & 1462797729792\mu^{9} + 40000000\lambda_1^{9} + 1537734375\lambda_2^{9} + 3110400000\mu^{2}\lambda_1^{7} + 17418240000\mu^{3}\lambda_1^{6} \\
& + 78382080000\mu^{4}\lambda_1^{5} + 282175488000\mu^{5}\lambda_1^{4} + 790091366400\mu^{6}\lambda_1^{3} + 1625330810880\mu^{7}\lambda_1^{2} \\
& + 53144100000\mu^{2}\lambda_2^{7} + 198404640000\mu^{3}\lambda_2^{6} + 595213920000\mu^{4}\lambda_2^{5} + 1428513408000\mu^{5}\lambda_2^{4} \\
& + 2666558361600\mu^{6}\lambda_2^{3} + 3656994324480\mu^{7}\lambda_2^{2} + 24603750000\lambda_1^{2}\lambda_2^{7} + 38272500000\lambda_1^{3}\lambda_2^{6} \\
& + 38272500000\lambda_1^{4}\lambda_2^{5} + 25515000000\lambda_1^{5}\lambda_2^{4} + 11340000000\lambda_1^{6}\lambda_2^{3} + 3240000000\lambda_1^{7}\lambda_2^{2} + 432000000\mu\lambda_1^{8} \\
& + 2194196594688\mu^{8}\lambda_1 + 11071687500\mu\lambda_2^{8} + 3291294892032\mu^{8}\lambda_2 + 9226406250\lambda_1\lambda_2^{8} + 540000000\lambda_1^{8}\lambda_2 \\
& + 59049000000\mu\lambda_1\lambda_2^{7} + 5184000000\mu\lambda_1^{7}\lambda_2 + 4875992432640\mu^{7}\lambda_1\lambda_2 + 137781000000\mu\lambda_1^{2}\lambda_2^{6} \\
& + 183708000000\mu\lambda_1^{3}\lambda_2^{5} + 153090000000\mu\lambda_1^{4}\lambda_2^{4} + 81648000000\mu\lambda_1^{5}\lambda_2^{3} + 27216000000\mu\lambda_1^{6}\lambda_2^{2} \\
& + 248005800000\mu^{2}\lambda_1\lambda_2^{6} + 32659200000\mu^{2}\lambda_1^{6}\lambda_2 + 793618560000\mu^{3}\lambda_1\lambda_2^{5} + 156764160000\mu^{3}\lambda_1^{5}\lambda_2 \\
& + 1984046400000\mu^{4}\lambda_1\lambda_2^{4} + 587865600000\mu^{4}\lambda_1^{4}\lambda_2 + 3809369088000\mu^{5}\lambda_1\lambda_2^{3} + 1693052928000\mu^{5}\lambda_1^{3}\lambda_2 \\
& + 5333116723200\mu^{6}\lambda_1\lambda_2^{2} + 3555411148800\mu^{6}\lambda_1^{2}\lambda_2 + 496011600000\mu^{2}\lambda_1^{2}\lambda_2^{5} + 551124000000\mu^{2}\lambda_1^{3}\lambda_2^{4} \\
& + 367416000000\mu^{2}\lambda_1^{4}\lambda_2^{3} + 146966400000\mu^{2}\lambda_1^{5}\lambda_2^{2} + 1322697600000\mu^{3}\lambda_1^{2}\lambda_2^{4} + 1175731200000\mu^{3}\lambda_1^{3}\lambda_2^{3} \\
& + 587865600000\mu^{3}\lambda_1^{4}\lambda_2^{2} + 2645395200000\mu^{4}\lambda_1^{2}\lambda_2^{3} + 1763596800000\mu^{4}\lambda_1^{3}\lambda_2^{2} + 3809369088000\mu^{5}\lambda_1^{2}\lambda_2^{2}
\end{aligned}
$$

$$
\begin{aligned}
N_1 = {} & 5851190919168\mu^{10} + 1537734375\lambda_2^{10} - 345600000\mu^{2}\lambda_1^{8} + 829440000\mu^{3}\lambda_1^{7} + 17418240000\mu^{4}\lambda_1^{6} \\
& + 125411328000\mu^{5}\lambda_1^{5} + 601974374400\mu^{6}\lambda_1^{4} + 2076811591680\mu^{7}\lambda_1^{3} + 5038525513728\mu^{8}\lambda_1^{2} \\
& + 44286750000\mu^{2}\lambda_2^{8} + 212576400000\mu^{3}\lambda_2^{7} + 793618560000\mu^{4}\lambda_2^{6} + 2380855680000\mu^{5}\lambda_2^{5} \\
& + 5714053632000\mu^{6}\lambda_2^{4} + 10666233446400\mu^{7}\lambda_2^{3} + 14627977297920\mu^{8}\lambda_2^{2} + 24603750000\lambda_1^{2}\lambda_2^{8} \\
& + 38272500000\lambda_1^{3}\lambda_2^{7} + 38272500000\lambda_1^{4}\lambda_2^{6} + 25515000000\lambda_1^{5}\lambda_2^{5} + 11340000000\lambda_1^{6}\lambda_2^{4} + 3240000000\lambda_1^{7}\lambda_2^{3} \\
& + 540000000\lambda_1^{8}\lambda_2^{2} - 128000000\mu\lambda_1^{9} + 7801587892224\mu^{9}\lambda_1 + 6150937500\mu\lambda_2^{9} + 13165179568128\mu^{9}\lambda_2 \\
& + 9226406250\lambda_1\lambda_2^{9} + 40000000\lambda_1^{9}\lambda_2 + 29524500000\mu\lambda_1\lambda_2^{8} - 1296000000\mu\lambda_1^{8}\lambda_2 + 17309773135872\mu^{8}\lambda_1\lambda_2 \\
& + 59049000000\mu\lambda_1^{2}\lambda_2^{7} + 61236000000\mu\lambda_1^{3}\lambda_2^{6} + 30618000000\mu\lambda_1^{4}\lambda_2^{5} - 9072000000\mu\lambda_1^{6}\lambda_2^{3} \\
& - 5184000000\mu\lambda_1^{7}\lambda_2^{2} + 200766600000\mu^{2}\lambda_1\lambda_2^{7} - 1036800000\mu^{2}\lambda_1^{7}\lambda_2 + 859753440000\mu^{3}\lambda_1\lambda_2^{6} \\
& + 26127360000\mu^{3}\lambda_1^{6}\lambda_2 + 2777664960000\mu^{4}\lambda_1\lambda_2^{5} + 235146240000\mu^{4}\lambda_1^{5}\lambda_2 + 6983843328000\mu^{5}\lambda_1\lambda_2^{4} \\
& + 1222760448000\mu^{5}\lambda_1^{4}\lambda_2 + 13459770777600\mu^{6}\lambda_1\lambda_2^{3} + 4401937612800\mu^{6}\lambda_1^{3}\lambda_2 + 18894470676480\mu^{7}\lambda_1\lambda_2^{2} \\
& + 10970982973440\mu^{7}\lambda_1^{2}\lambda_2 + 385786800000\mu^{2}\lambda_1^{2}\lambda_2^{6} + 404157600000\mu^{2}\lambda_1^{3}\lambda_2^{5} + 244944000000\mu^{2}\lambda_1^{4}\lambda_2^{4} \\
& + 81648000000\mu^{2}\lambda_1^{5}\lambda_2^{3} + 10886400000\mu^{2}\lambda_1^{6}\lambda_2^{2} + 1454967360000\mu^{3}\lambda_1^{2}\lambda_2^{5} + 1322697600000\mu^{3}\lambda_1^{3}\lambda_2^{4} \\
& + 685843200000\mu^{3}\lambda_1^{4}\lambda_2^{3} + 195\,955200000\mu^{3}\lambda_1^{5}\lambda_2^{2} + 3968092800000\mu^{4}\lambda_1^{2}\lambda_2^{4} + 2939328000000\mu^{4}\lambda_1^{3}\lambda_2^{3} \\
& + 1175731200000\mu^{4}\lambda_1^{4}\lambda_2^{2} + 8042001408000\mu^{5}\lambda_1^{2}\lambda_2^{3} + 4514807808000\mu^{5}\lambda_1^{3}\lambda_2^{2} + 11682065203200\mu^{6}\lambda_1^{2}\lambda_2^{2}
\end{aligned}
$$

$$
\begin{aligned}
N_2 =\ & 105321436545024\mu^{11} + 80000000\lambda_1^{11} - 1267200000\mu^2\lambda_1 - 8709120000\mu^3\lambda_1^8 - 37324800000\mu^4\lambda_1^7 \\
& - 62705664000\mu^5\lambda_1^6 + 451480780800\mu^6\lambda_1^5 + 4605103964160\mu^7\lambda_1^4 + 22267032109056\mu^8\lambda_1^3 \\
& + 67288695570432\mu^9\lambda_1^2 - 88573500000\mu^2\lambda_2^9 - 159432300000\mu^3\lambda_2^8 + 255091680000\mu^4\lambda_2^7 \\
& + 3571283520000\mu^5\lambda_2^6 + 17142160896000\mu^6\lambda_2^5 + 54854914867200\mu^7\lambda_2^4 + 126166304194560\mu^8\lambda_2^3 \\
& + 204060283305984\mu^9\lambda_2^2 + 30754687500\lambda_1^2\lambda_2^9 + 92264062500\lambda_1^3\lambda_2^8 + 164025000000\lambda_1^4\lambda_2^7 \\
& + 191362500000\lambda_1^5\lambda_2^6 + 153090000000\lambda_1^6\lambda_2^5 + 85050000000\lambda_1^7\lambda_2^4 + 32400000000\lambda_1^8\lambda_2^3 \\
& + 8100000000\lambda_1^9\lambda_2^2 + 384000000\mu\lambda_1^{10} + 122875009302528\mu^{10}\lambda_1 + 210642873090048\mu^{10}\lambda_2 \\
& + 4613203125\lambda_1\lambda_2^{10} + 1200000000\lambda_1^{10}\lambda_2 + 14762250000\mu\lambda_1\lambda_2 + 5184000000\mu\lambda_1^9\lambda_2 \\
& + 236973232226304\mu^9\lambda_1\lambda_2 + 88573500000\mu^9\lambda_1^2\lambda_2^8 + 236196000000\mu\lambda_1^3\lambda_2^7 + 367416000000\mu\lambda_1^4\lambda_2^6 \\
& + 367416000000\mu\lambda_1^5\lambda_2^5 + 244944000000\mu\lambda_1^6\lambda_2^4 + 108864000000\mu\lambda_1^7\lambda_2^3 + 31104000000\mu\lambda_1^8\lambda_2^2 \\
& - 504868950000\mu^2\lambda_1\lambda_2^8 - 18662400000\mu^2\lambda_1^8\lambda_2 - 892820880000\mu^3\lambda_1\lambda_2^7 - 100776960000\mu^3\lambda_1^7\lambda_2 \\
& + 595213920000\mu^4\lambda_1\lambda_2^6 - 313528320000\mu^4\lambda_1^6\lambda_2 + 11428107264000\mu^5\lambda_1\lambda_2^5 + 47998050508800\mu^6\lambda_1\lambda_2^4 \\
& + 6094990540800\mu^6\lambda_1^4\lambda_2 + 125252055613440\mu^7\lambda_1\lambda_2^3 + 36976275947520\mu^7\lambda_1^3\lambda_2 \\
& + 218322561171456\mu^8\lambda_1\lambda_2^2 + 122875009302528\mu^8\lambda_1^2\lambda_2 - 1275458400000\mu^2\lambda_1^2\lambda_2^7 \\
& - 1873821600000\mu^2\lambda_1^3\lambda_2^6 - 1763596800000\mu^2\lambda_1^4\lambda_2^5 - 1102248000000\mu^2\lambda_1^5\lambda_2^4 - 457228800000\mu^2\lambda_1^6\lambda_2^3 \\
& - 121305600000\mu^2\lambda_1^7\lambda_2^2 - 2182451040000\mu^3\lambda_1^2\lambda_2^6 - 3042204480000\mu^3\lambda_1^3\lambda_2^5 - 2645395200000\mu^3\lambda_1^4\lambda_2^4 \\
& - 1469664000000\mu^3\lambda_1^5\lambda_2^3 - 509483520000\mu^3\lambda_1^6\lambda_2^2 - 1322697600000\mu^4\lambda_1^3\lambda_2^4 - 1763596800000\mu^4\lambda_1^4\lambda_2^3 \\
& - 1058158080000\mu^4\lambda_1^5\lambda_2^2 + 14285134080000\mu^5\lambda_1^2\lambda_2^4 + 8465264640000\mu^5\lambda_1^3\lambda_2^3 + 2116316160000\mu^5\lambda_1^4\lambda_2^2 \\
& + 51807419596800\mu^6\lambda_1^2\lambda_2^3 + 26411625676800\mu^6\lambda_1^3\lambda_2^2 + 104224338247680\mu^7\lambda_1^2\lambda_2^2
\end{aligned}
$$

Appendix E: The cases of performance measures for sub-model 1

Case (i):

$$\left(W_S^1\right)_\alpha^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E1}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

$$\left(W_S^1\right)_\alpha^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E2}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

Case (ii)

$$\left(W_S^1\right)_\alpha^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E3}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

$$\left(W_S^1\right)_\alpha^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E4}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

and case (iii)

$$\left(W_S^1\right)_\alpha^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E5}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$$

$$\left(W_S^1\right)_\alpha^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3N_1}{2z\left(6z - x\right)D} \tag{E6}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$$

Case (i):

$$\left(W_S^2\right)_\alpha^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E7)}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$$

$$\left(W_S^2\right)_\alpha^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E8)}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$$

Case (ii):

$$\left(W_S^2\right)_\alpha^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E9)}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$$

$$\left(W_S^2\right)_\alpha^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E10)}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$$

and case (iii)

$$\left(W_S^2\right)_\alpha^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E11)}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$$

$$\left(W_S^2\right)_\alpha^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{N_2}{z(6z-x)(12z-2x-3y)D} \quad \text{(E12)}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$$

Case (i)

$$\left(L_q^1\right)_\alpha^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E13)}$$

s.t. $x_{\alpha}^{LB} \le x \le x_{\alpha}^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$

$$\left(L_q^1\right)_{\alpha}^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E14)}$$

s.t. $x_{\alpha}^{LB} \le x \le x_{\alpha}^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$

Case (ii)

$$\left(L_q^1\right)_{\alpha}^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E15)}$$

s.t. $y_{\alpha}^{LB} \le y \le y_{\alpha}^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$

$$\left(L_q^1\right)_{\alpha}^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E16)}$$

s.t. $y_{\alpha}^{LB} \le y \le y_{\alpha}^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$

and case (iii)

$$\left(L_q^1\right)_{\alpha}^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E17)}$$

s.t. $z_{\alpha}^{LB} \le z \le z_{\alpha}^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$

$$\left(L_q^1\right)_{\alpha}^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{78125x(2x+3y)^{10}}{2z(6z-x)D} \quad \text{(E18)}$$

s.t. $z_{\alpha}^{LB} \le z \le z_{\alpha}^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$

Case (i)

$$\left(L_q^2\right)_{\alpha}^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y(2x+3y)^{10}}{(6z-x)(12z-2x-3y)D} \quad \text{(E19)}$$

s.t. $x_{\alpha}^{LB} \le x \le x_{\alpha}^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$

$$\left(L_q^2\right)_\alpha^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y\left(2x+3y\right)^{10}}{\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E20}$$

$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2\left(\alpha\right),\ z \in \mu\left(\alpha\right)$

Case (ii)

$$\left(L_q^2\right)_\alpha^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y\left(2x+3y\right)^{10}}{\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E21}$$

$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$

$$\left(L_q^2\right)_\alpha^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y\left(2x+3y\right)^{10}}{\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E22}$$

$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$

and case (iii)

$$\left(L_q^2\right)_\alpha^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y\left(2x+3y\right)^{10}}{\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E23}$$

$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$

$$\left(L_q^2\right)_\alpha^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{468750y\left(2x+3y\right)^{10}}{\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E24}$$

$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$

Case (i)

$$\left(L_S^1\right)_\alpha^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z\left(6z-x\right)D} \tag{E25}$$

$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2\left(\alpha\right),\ z \in \mu\left(\alpha\right)$

$$\left(L_S^1\right)_\alpha^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z\left(6z-x\right)D} \tag{E26}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$$

Case (ii)

$$\left(L_S^1\right)_\alpha^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z(6z-x)D} \tag{E27}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$$

$$\left(L_S^1\right)_\alpha^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z(6z-x)D} \tag{E28}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1(\alpha),\ z \in \mu(\alpha)$$

and case (iii)

$$\left(L_S^1\right)_\alpha^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z(6z-x)D} \tag{E29}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$$

$$\left(L_S^1\right)_\alpha^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{3xN_1}{2z(6z-x)D} \tag{E30}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1(\alpha),\ y \in \lambda_2(\alpha)$$

Case (i)

$$\left(L_S^2\right)_\alpha^{LB1} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z(6z-x)(12z-2x-3y)D} \tag{E31}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$$

$$\left(L_S^2\right)_\alpha^{UB1} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z(6z-x)(12z-2x-3y)D} \tag{E32}$$

$$s.t. \quad x_\alpha^{LB} \le x \le x_\alpha^{UB},\ y \in \lambda_2(\alpha),\ z \in \mu(\alpha)$$

Case (ii)

$$\left(L_S^2\right)_\alpha^{LB2} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E33}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

$$\left(L_S^2\right)_\alpha^{UB2} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E34}$$

$$s.t. \quad y_\alpha^{LB} \le y \le y_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ z \in \mu\left(\alpha\right)$$

and case (iii)

$$\left(L_S^2\right)_\alpha^{LB3} = \min_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E35}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$$

$$\left(L_S^2\right)_\alpha^{UB3} = \max_{\substack{x\in X\\ y\in Y\\ z\in Z}} \frac{yN_2}{z\left(6z-x\right)\left(12z-2x-3y\right)D} \tag{E36}$$

$$s.t. \quad z_\alpha^{LB} \le z \le z_\alpha^{UB},\ x \in \lambda_1\left(\alpha\right),\ y \in \lambda_2\left(\alpha\right)$$

Appendix F: Codes for validation of the sub-model 1 and sub-model 2 using Matlab

```
 %%%%%%%%%% Zena Simulation Program %%%
clc
clear all
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n=1000;
lambda1=23;
lambda2=20;
mu=0.997592;
lam1L=3;
lam1H=70;
lam2L=2;
lam2H=48;
muL=0.5;
muH=2.583333;
%%%%%%%%%%
c1=6;
c2=4;
L=1000;
alp=1;
for q=1:L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x=poissrnd(lambda1,1,n);
x111=(x>=lam1L&x<=lambda1);
x11=x(x111);
x1=x11(1:192);
x112=(x>=lambda1&x<=lam1H);
x12=x(x112);
x2=x12(1:161);
X1=[x1 x2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply1 for lambda1
%%%%%%%%%%%%%%%%%%%%%%%%%%%
y=poissrnd(lambda2,1,n);
y111=(y>=lam2L&y<=lambda2);
y11=y(y111);
y1=y11(1:178);
y112=(y>=lambda2&y<=lam2H);
y12=y(y112);
y2=y12(1:175);
X2=[y1 y2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply2 for lambda2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t1= exprnd(mu,n,1); %
t111=(t1>=muL&t1<=mu);
t11=t1(t111);
ty1=t11(1:178);
t112=(t1>=mu&t1<=muH);
t12=t1(t112);
ty2=t12(1:175);
EX=[ty1' ty2'];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  services time
exponential
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
X1s=sort(X1);
avgmX1s=mean(X1s);
avgmX1s1=(X1s<avgmX1s);
```

```
avgL_lambda1=mean(X1s(avgmX1s1))/24;
avgmX1s2=(X1s>avgmX1s);
avgH_lambda1=mean(X1s(avgmX1s2))/24;
%%%%%%%%%%%%%
X2s=sort(X2);
avgmX2s=mean(X2s);
avgmX2s1=(X2s<avgmX2s);
avgL_lambda2=mean(X2s(avgmX2s1))/24;
avgmX2s2=(X2s>avgmX2s);
avgH_lambda2=mean(X2s(avgmX2s2))/24;
%%%%%%%%%%
EXs=sort(EX);
avgmEXs=mean(EX);
avgmEXs1=(EXs<avgmEXs);
avgL_mu=mean(EXs(avgmEXs1));
avgmEXs2=(EXs>avgmEXs);
avgH_mu=mean(EXs(avgmEXs2));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%
lambda1Low(q,:)=[lam1L avgL_lambda1 avgmX1s];
lambda1Mediam(q,:)=[avgL_lambda1 avgmX1s avgH_lambda1];
lambda1High(q,:)=[avgmX1s avgH_lambda1 lam1H];

lambda2Low(q,:)=[lam2L avgL_lambda2 avgmX2s];
lambda2Mediam(q,:)=[avgL_lambda2 avgmX2s avgH_lambda2];
lambda2High(q,:)=[avgmX2s avgH_lambda2 lam2H];

muLow(q,:)=[muL avgL_mu avgmEXs];
muMediam(q,:)=[avgL_mu avgmEXs avgH_mu];
muHigh(q,:)=[avgmEXs avgH_mu muH];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rlambda1L=(0.5*(lambda1Low(q,1)+lambda1Low(q,3))+0.25*(2*lambda1Low(q,2)-
lambda1Low(q,1)-lambda1Low(q,3)))/24;
Rlambda2L=(0.5*(lambda2Low(q,1)+lambda2Low(q,3))+0.25*(2*lambda2Low(q,2)-
lambda2Low(q,1)-lambda2Low(q,3)))/24;
RmuL=1/(0.5*(muLow(q,1)+muLow(q,3))+0.25*(2*muLow(q,2)-muLow(q,1)-muLow(q,3)));
Rho1L(q)=Rlambda1L/(c1*RmuL);
Rho2L(q)=Rlambda2L/(c2*RmuL);
c=c1+c2;
RhoL(q)=Rho1L(q)+Rho2L(q);
ERL=RmuL;
nn=0:(c-1);
pWL(q)=(((c*RhoL(q))^c)/factorial(c))*((1-
RhoL(q))*sum(((c*RhoL(q)).^nn)./factorial(nn))+(((c*RhoL(q))^c)/factorial(c)))^(-1);
Wq1L(q)=(pWL(q)/(1-Rho1L(q)))*(ERL/c);
Wq2L(q)=(pWL(q)/((1-RhoL(q))*(1-Rho1L(q))))*(ERL/c);
WqL(q)=Wq1L(q)+Wq2L(q);
Ws1L(q)=Wq1L(q)+ERL;
Ws2L(q)=Wq2L(q)+ERL;
WsL(q)=Ws1L(q)+Ws2L(q);
Lq1L(q)=Rlambda1L*Wq1L(q);
Lq2L(q)=Rlambda2L*Wq2L(q);
LqL(q)=Lq1L(q)+Lq2L(q);
Ls1L(q)=Rlambda1L*Ws1L(q);
Ls2L(q)=Rlambda2L*Ws2L(q);
LsL(q)=Ls1L(q)+Ls2L(q);
```

```
Rlambda1M=(0.5*(lambda1Mediam(q,1)+lambda1Mediam(q,3))+0.25*(2*lambda1Mediam(q,2)-
lambda1Mediam(q,1)-lambda1Mediam(q,3)))/24;
Rlambda2M=(0.5*(lambda2Mediam(q,1)+lambda2Mediam(q,3))+0.25*(2*lambda2Mediam(q,2)-
lambda2Mediam(q,1)-lambda2Mediam(q,3)))/24;
RmuM=1/(0.5*(muMediam(q,1)+muMediam(q,3))+0.25*(2*muMediam(q,2)-muMediam(q,1)-
muMediam(q,3)));
Rho1M(q)=Rlambda1M/(c1*RmuM);
Rho2M(q)=Rlambda2M/(c2*RmuM);
c=c1+c2;
RhoM(q)=Rho1M(q)+Rho2M(q);
ERM=RmuM;
nn=0:(c-1);
pWM(q)=(((c*RhoM(q))^c)/factorial(c))*((1-
RhoM(q))*sum(((c*RhoM(q)).^nn)./factorial(nn))+(((c*RhoM(q))^c)/factorial(c)))^(-1);
Wq1M(q)=(pWM(q)/(1-Rho1M(q)))*(ERM/c);
Wq2M(q)=(pWM(q)/((1-RhoM(q))*(1-Rho1M(q))))*(ERM/c);
WqM(q)=Wq1M(q)+Wq2M(q);
Ws1M(q)=Wq1M(q)+ERM;
Ws2M(q)=Wq2M(q)+ERM;
WsM(q)=Ws1M(q)+Ws2M(q);
Lq1M(q)=Rlambda1M*Wq1M(q);
Lq2M(q)=Rlambda2M*Wq2M(q);
LqM(q)=Lq1M(q)+Lq2M(q);
Ls1M(q)=Rlambda1M*Ws1M(q);
Ls2M(q)=Rlambda2M*Ws2M(q);
LsM(q)=Ls1M(q)+Ls2M(q);
Rlambda1H=(0.5*(lambda1High(q,1)+lambda1High(q,3))+0.25*(2*lambda1High(q,2)-
lambda1High(q,1)-lambda1High(q,3)))/24;
Rlambda2H=(0.5*(lambda2High(q,1)+lambda2High(q,3))+0.25*(2*lambda2High(q,2)-
lambda2High(q,1)-lambda2High(q,3)))/24;
RmuH=1/(0.5*(muHigh(q,1)+muHigh(q,3))+0.25*(2*muHigh(q,2)-muHigh(q,1)-muHigh(q,3)));
Rho1H(q)=Rlambda1H/(c1*RmuH);
Rho2H(q)=Rlambda2H/(c2*RmuH);
c=c1+c2;
RhoH(q)=Rho1H(q)+Rho2H(q);
ERH=RmuH;
nn=0:(c-1);
pWH(q)=(((c*RhoH(q))^c)/factorial(c))*((1-
RhoH(q))*sum(((c*RhoH(q)).^nn)./factorial(nn))+(((c*RhoH(q))^c)/factorial(c)))^(-1);
Wq1H(q)=(pWH(q)/(1-Rho1H(q)))*(ERH/c);
Wq2H(q)=(pWH(q)/((1-RhoH(q))*(1-Rho1H(q))))*(ERH/c);
WqH(q)=Wq1H(q)+Wq2H(q);
Ws1H(q)=Wq1H(q)+ERH;
Ws2H(q)=Wq2H(q)+ERH;
WsH(q)=Ws1H(q)+Ws2H(q);
Lq1H(q)=Rlambda1H*Wq1H(q);
Lq2H(q)=Rlambda2H*Wq2H(q);
LqH(q)=Lq1H(q)+Lq2H(q);
Ls1H(q)=Rlambda1H*Ws1H(q);
Ls2H(q)=Rlambda2H*Ws2H(q);
LsH(q)=Ls1H(q)+Ls2H(q);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PNLP

lambda1LUBLow=[lam1L+(avgL_lambda1-lam1L)*alp avgmX1s-(avgmX1s-avgL_lambda1)*alp];
lambda1LUBMediam=[avgL_lambda1+(avgmX1s-avgL_lambda1)*alp avgH_lambda1-
(avgH_lambda1-avgmX1s)*alp];
lambda1LUBHigh=[avgmX1s+(avgH_lambda1-avgmX1s)*alp lam1H-(lam1H-avgH_lambda1)*alp];
```

```
lambda2LUBLow=[lam2L+(avgL_lambda2-lam2L)*alp avgmX2s-(avgmX2s-avgL_lambda2)*alp];
lambda2LUBMediam=[avgL_lambda2+(avgmX2s-avgL_lambda2)*alp avgH_lambda2-
(avgH_lambda2-avgmX2s)*alp];
lambda2LUBHigh=[avgmX2s+(avgH_lambda2-avgmX2s)*alp lam2H-(lam2H-avgH_lambda2)*alp];

muLUBLow=[muL+(avgL_mu-muL)*alp avgmEXs-(avgmEXs-avgL_mu)*alp];
muLUBMediam=[avgL_mu+(avgmEXs-avgL_mu)*alp avgH_mu-(avgH_mu-avgmEXs)*alp];
muLUBHigh=[avgmEXs+(avgH_mu-avgmEXs1)*alp muH-(muH-avgH_mu)*alp];

Rho1LowL=lambda1LUBLow(1,1)/(c1*muLUBLow(1,2));
Rho2LowL=lambda2LUBLow(1,1)/(c2*muLUBLow(1,2));
Rho1LowU=lambda1LUBLow(1,2)/(c1*muLUBLow(1,1));
Rho2LowU=lambda2LUBLow(1,2)/(c2*muLUBLow(1,1));
RhoLowL=Rho1LowL+Rho2LowL;
RhoLowU=Rho1LowU+Rho2LowU;
ERLowL=1/muLUBLow(1,2);
ERLowU=1/muLUBLow(1,1);
c=c1+c2;
nn=0:(c-1);
pWLowL=(((c*RhoLowL)^c)/factorial(c))*((1-
RhoLowL)*sum(((c*RhoLowL).^nn)./factorial(nn))+(((c*RhoLowL)^c)/factorial(c)))^(-1);
pWLowU=(((c*RhoLowU)^c)/factorial(c))*((1-
RhoLowU)*sum(((c*RhoLowU).^nn)./factorial(nn))+(((c*RhoLowU)^c)/factorial(c)))^(-1);
Wq1Low=[(pWLowL/(1-Rho1LowL))*(ERLowL/c) (pWLowU/(1-Rho1LowU))*(ERLowU/c)];
Wq2Low=[(pWLowL/((1-RhoLowL)*(1-Rho1LowL)))*(ERLowL/c) (pWLowU/((1-RhoLowU)*(1-
Rho1LowU)))*(ERLowU/c)];
WqLow=Wq1Low+Wq2Low;
Ws1Low=[Wq1Low(1,1)+ERLowL Wq1Low(1,2)+ERLowU];
Ws2Low=[Wq2Low(1,1)+ERLowL Wq2Low(1,2)+ERLowU];
WsLow=Ws1Low+Ws2Low;
Lq1Low=[lambda1LUBLow(1,1)*Wq1Low(1,1) lambda1LUBLow(1,2)*Wq1Low(1,2)];
Lq2Low=[lambda2LUBLow(1,1)*Wq2Low(1,1) lambda2LUBLow(1,2)*Wq2Low(1,2)];
LqLow=Lq1Low+Lq2Low;
Ls1Low=[lambda1LUBLow(1,1)*Ws1Low(1,1) lambda1LUBLow(1,2)*Ws1Low(1,2)];
Ls2Low=[lambda2LUBLow(1,1)*Ws2Low(1,1) lambda2LUBLow(1,2)*Ws2Low(1,2)];
LsLow=Ls1Low+Ls2Low;

Rho1MediamL=lambda1LUBMediam(1,1)/(c1*muLUBMediam(1,2));
Rho2MediamL=lambda2LUBMediam(1,1)/(c2*muLUBMediam(1,2));
Rho1MediamU=lambda1LUBMediam(1,2)/(c1*muLUBMediam(1,1));
Rho2MediamU=lambda2LUBMediam(1,2)/(c2*muLUBMediam(1,1));
RhoMediamL=Rho1MediamL+Rho2MediamL;
RhoMediamU=Rho1MediamU+Rho2MediamU;
ERMediamL=1/muLUBMediam(1,2);
ERMediamU=1/muLUBMediam(1,1);
c=c1+c2;
nn=0:(c-1);
pWMediamL=(((c*RhoMediamL)^c)/factorial(c))*((1-
RhoMediamL)*sum(((c*RhoMediamL).^nn)./factorial(nn))+(((c*RhoMediamL)^c)/factorial(c)))^(-
1);
pWMediamU=(((c*RhoMediamU)^c)/factorial(c))*((1-
RhoMediamU)*sum(((c*RhoMediamU).^nn)./factorial(nn))+(((c*RhoMediamU)^c)/factorial(c)))^(-
1);
Wq1Mediam=[(pWMediamL/(1-Rho1MediamL))*(ERMediamL/c) (pWMediamU/(1-
Rho1MediamU))*(ERMediamU/c)];
Wq2Mediam=[(pWMediamL/((1-RhoMediamL)*(1-Rho1MediamL)))*(ERMediamL/c)
(pWMediamU/((1-RhoMediamU)*(1-Rho1MediamU)))*(ERMediamU/c)];
```

```
WqMediam=Wq1Mediam+Wq2Mediam;
Ws1Mediam=[Wq1Mediam(1,1)+ERMediamL Wq1Mediam(1,2)+ERMediamU];
Ws2Mediam=[Wq2Mediam(1,1)+ERMediamL Wq2Mediam(1,2)+ERMediamU];
WsMediam=Ws1Mediam+Ws2Mediam;
Lq1Mediam=[lambda1LUBMediam(1,1)*Wq1Mediam(1,1)
lambda1LUBMediam(1,2)*Wq1Mediam(1,2)];
Lq2Mediam=[lambda2LUBMediam(1,1)*Wq2Mediam(1,1)
lambda2LUBMediam(1,2)*Wq2Mediam(1,2)];
LqMediam=Lq1Mediam+Lq2Mediam;
Ls1Mediam=[lambda1LUBMediam(1,1)*Ws1Mediam(1,1)
lambda1LUBMediam(1,2)*Ws1Mediam(1,2)];
Ls2Mediam=[lambda2LUBMediam(1,1)*Ws2Mediam(1,1)
lambda2LUBMediam(1,2)*Ws2Mediam(1,2)];
LsMediam=Ls1Mediam+Ls2Mediam;
Rho1HighL=lambda1LUBHigh(1,1)/(c1*muLUBHigh(1,2));
Rho2HighL=lambda2LUBHigh(1,1)/(c2*muLUBHigh(1,2));
Rho1HighU=lambda1LUBHigh(1,2)/(c1*muLUBHigh(1,1));
Rho2HighU=lambda2LUBHigh(1,2)/(c2*muLUBHigh(1,1));
RhoHighL=Rho1HighL+Rho2HighL;
RhoHighU=Rho1HighU+Rho2HighU;
ERHighL=1/muLUBHigh(1,2);
ERHighU=1/muLUBHigh(1,1);
c=c1+c2;
nn=0:(c-1);
pWHighL=(((c*RhoHighL)^c)/factorial(c))*((1-
RhoHighL)*sum(((c*RhoHighL).^nn)./factorial(nn))+(((c*RhoHighL)^c)/factorial(c)))^(-1);
pWHighU=(((c*RhoHighU)^c)/factorial(c))*((1-
RhoHighU)*sum(((c*RhoHighU).^nn)./factorial(nn))+(((c*RhoHighU)^c)/factorial(c)))^(-1);
Wq1High=[(pWHighL/(1-Rho1HighL))*(ERHighL/c) (pWHighU/(1-Rho1HighU))*(ERHighU/c)];
Wq2High=[(pWHighL/((1-RhoHighL)*(1-Rho1HighL)))*(ERHighL/c) (pWHighU/((1-
RhoHighU)*(1-Rho1HighU)))*(ERHighU/c)];
WqHigh=Wq1High+Wq2High;
Ws1High=[Wq1High(1,1)+ERHighL Wq1High(1,2)+ERHighU];
Ws2High=[Wq2High(1,1)+ERHighL Wq2High(1,2)+ERHighU];
WsHigh=Ws1High+Ws2High;
Lq1High=[lambda1LUBHigh(1,1)*Wq1High(1,1) lambda1LUBHigh(1,2)*Wq1High(1,2)];
Lq2High=[lambda2LUBHigh(1,1)*Wq2High(1,1) lambda2LUBHigh(1,2)*Wq2High(1,2)];
LqHigh=Lq1High+Lq2High;
Ls1High=[lambda1LUBHigh(1,1)*Ws1High(1,1) lambda1LUBHigh(1,2)*Ws1High(1,2)];
Ls2High=[lambda2LUBHigh(1,1)*Ws2High(1,1) lambda2LUBHigh(1,2)*Ws2High(1,2)];
LsHigh=Ls1High+Ls2High;

end

Result_Lower=[mean(Wq1L) mean(Wq2L) mean(Ws1L) mean(Ws2L) mean(Lq1L) mean(Lq2L)
mean(Ls1L) mean(Ls2L);mean(Wq1Low) mean(Wq2Low) mean(Ws1Low) mean(Ws2Low)
mean(Lq1Low) mean(Lq2Low) mean(Ls1Low) mean(Ls2Low)]
Resultg_Mediam=[mean(Wq1M) mean(Wq2M) mean(Ws1M) mean(Ws2M) mean(Lq1M)
mean(Lq2M) mean(Ls1M) mean(Ls2M);mean(Wq1Mediam) mean(Wq2Mediam)
mean(Ws1Mediam) mean(Ws2Mediam) mean(Lq1Mediam) mean(Lq2Mediam) mean(Ls1Mediam)
mean(Ls2Mediam)]
Resultg_High=[mean(Wq1H) mean(Wq2H) mean(Ws1H) mean(Ws2H) mean(Lq1H) mean(Lq2H)
mean(Ls1H) mean(Ls2H);mean(Wq1High) mean(Wq2High) mean(Ws1High) mean(Ws2High)
mean(Lq1High) mean(Lq2High) mean(Ls1High) mean(Ls2High)]

%%%%
```

MSE_Lower=[mean((Wq1L-mean(Wq1L)).^2) mean((Wq2L-mean(Wq2L)).^2) mean((Ws1L-mean(Ws1L)).^2) mean((Ws2L-mean(Ws2L)).^2) mean((Lq1L-mean(Lq1L)).^2) mean((Lq2L-mean(Lq2L)).^2) mean((Ls1L-mean(Ls1L)).^2) mean((Ls2L-mean(Ls2L)).^2);mean((Wq1Low-mean(Wq1Low)).^2) mean((Wq2Low-mean(Wq2Low)).^2) mean((Ws1Low-mean(Ws1Low)).^2) mean((Ws2Low-mean(Ws2Low)).^2) mean((Lq1Low-mean(Lq1Low)).^2) mean((Lq2Low-mean(Lq2Low)).^2) mean((Ls1Low-mean(Ls1Low)).^2) mean((Ls2Low-mean(Ls2Low)).^2)]
MSE_Mediam=[mean((Wq1M-mean(Wq1M)).^2) mean((Wq2M-mean(Wq2M)).^2) mean((Ws1M-mean(Ws1M)).^2) mean((Ws2M-mean(Ws2M)).^2) mean((Lq1M-mean(Lq1M)).^2) mean((Lq2M-mean(Lq2M)).^2) mean((Ls1M-mean(Ls1M)).^2) mean((Ls2M-mean(Ls2M)).^2);mean((Wq1Mediam-mean(Wq1Mediam)).^2) mean((Wq2Mediam-mean(Wq2Mediam)).^2) mean((Ws1Mediam-mean(Ws1Mediam)).^2) mean((Ws2Mediam-mean(Ws2Mediam)).^2) mean((Lq1Mediam-mean(Lq1Mediam)).^2) mean((Lq2Mediam-mean(Lq2Mediam)).^2) mean((Ls1Mediam-mean(Ls1Mediam)).^2) mean((Ls2Mediam-mean(Ls2Mediam)).^2)]
MSE_High=[mean((Wq1H-mean(Wq1H)).^2) mean((Wq2H-mean(Wq2H)).^2) mean((Ws1H-mean(Ws1H)).^2) mean((Ws2H-mean(Ws2H)).^2) mean((Lq1H-mean(Lq1H)).^2) mean((Lq2H-mean(Lq2H)).^2) mean((Ls1H-mean(Ls1H)).^2) mean((Ls2H-mean(Ls2H)).^2);mean((Wq1High-mean(Wq1High)).^2) mean((Wq2High-mean(Wq2High)).^2) mean((Ws1High-mean(Ws1High)).^2) mean((Ws2High-mean(Ws2High)).^2) mean((Lq1High-mean(Lq1High)).^2) mean((Lq2High-mean(Lq2High)).^2) mean((Ls1High-mean(Ls1High)).^2) mean((Ls2High-mean(Ls2High)).^2)]

Appendix G: Codes for validation of the sub-model 2 and sub-model 3 using Matlab

```
clc
clear all
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n=1000;
lambda1=23;
lambda2=20;
mu=0.997592;
alfa=60.06;
beta=0.016;
lam1L=3;
lam1H=70;
lam2L=2;
lam2H=48;
muL=0.5;
muH=2.583333;
%%%%%%%%%%
c1=6;
c2=4;
L=1000;
for q=1:L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x=poissrnd(lambda1,1,n);
x111=(x>=lam1L&x<=lambda1);
x11=x(x111);
x1=x11(1:192);
x112=(x>=lambda1&x<=lam1H);
x12=x(x112);
x2=x12(1:161);
X1=[x1 x2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply1 for
lambda1
%%%%%%%%%%%%%%%%%%%%%%%%%%%
y=poissrnd(lambda2,1,n);
y111=(y>=lam2L&y<=lambda2);
y11=y(y111);
y1=y11(1:178);
y112=(y>=lambda2&y<=lam2H);
y12=y(y112);
y2=y12(1:175);
X2=[y1 y2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply2 for
lambda2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t1= exprnd(mu,n,1); %
t111=(t1>=muL&t1<=mu);
t11=t1(t111);
ty1=t11(1:178);
t112=(t1>=mu&t1<=muH);
t12=t1(t112);
ty2=t12(1:175);
```

```
EX=[ty1' ty2'];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  services
time Exponential
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
u1=gamrnd(elf,beta,n,1);
mug=(elf*beta);
u111=(u1>=muL&u1<=mug);
u11=u1(u111);
uy1=u11(1:178);
u112=(u1>=mug&u1<=muH);
u12=u1(u112);
uy2=u12(1:175);
EXg=[uy1' uy2'];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  services
time Gamma
X1s=sort(X1);
avgmX1s=mean(X1s);
avgmX1s1=(X1s<avgmX1s);
avgL_lambda1=mean(X1s(avgmX1s1))/24;
avgmX1s2=(X1s>avgmX1s);
avgH_lambda1=mean(X1s(avgmX1s2))/24;
%%%%%%%%%%%%%
X2s=sort(X2);
avgmX2s=mean(X2s);
avgmX2s1=(X2s<avgmX2s);
avgL_lambda2=mean(X2s(avgmX2s1))/24;
avgmX2s2=(X2s>avgmX2s);
avgH_lambda2=mean(X2s(avgmX2s2))/24;
%%%%%%%%%%
EXs=sort(EX);
avgmEXs=mean(EX);
avgmEXs1=(EXs<avgmEXs);
avgL_mu=mean(EXs(avgmEXs1));
avgmEXs2=(EXs>avgmEXs);
avgH_mu=mean(EXs(avgmEXs2));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%
EXsg=sort(EXg);
avgmEXsg=mean(EXg);
avgmEXs1g=(EXsg<avgmEXsg);
avgL_mug=mean(EXsg(avgmEXs1g));
avgmEXs2g=(EXsg>avgmEXsg);
avgH_mug=mean(EXsg(avgmEXs2g));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%
lambda1Low(q,:)=[lam1L avgL_lambda1 avgmX1s];
lambda1Mediam(q,:)=[avgL_lambda1 avgmX1s avgH_lambda1];
lambda1High(q,:)=[avgmX1s avgH_lambda1 lam1H];

lambda2Low(q,:)=[lam2L avgL_lambda2 avgmX2s];
lambda2Mediam(q,:)=[avgL_lambda2 avgmX2s avgH_lambda2];
lambda2High(q,:)=[avgmX2s avgH_lambda2 lam2H];
```

```
muLow(q,:)=[muL avgL_mu avgmEXs];
muMediam(q,:)=[avgL_mu avgmEXs avgH_mu];
muHigh(q,:)=[avgmEXs avgH_mu muH];


mugLow(q,:)=[muL avgL_mug avgmEXsg];
mugMediam(q,:)=[avgL_mug avgmEXsg avgH_mug];
mugHigh(q,:)=[avgmEXsg avgH_mug muH];


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% EXPONENTIAL
FM1,FM2/ESR/10-2Pr
Rlambda1L=(0.5*(lambda1Low(q,1)+lambda1Low(q,3))+0.25*(2*lambda1Low(q,2)-
lambda1Low(q,1)-lambda1Low(q,3)))/24;
Rlambda2L=(0.5*(lambda2Low(q,1)+lambda2Low(q,3))+0.25*(2*lambda2Low(q,2)-
lambda2Low(q,1)-lambda2Low(q,3)))/24;
RmuL=1/(0.5*(muLow(q,1)+muLow(q,3))+0.25*(2*muLow(q,2)-muLow(q,1)-
muLow(q,3)));
Rho1L(q)=Rlambda1L/(c1*RmuL);
Rho2L(q)=Rlambda2L/(c2*RmuL);
c=c1+c2;
RhoL(q)=Rho1L(q)+Rho2L(q);
ERL=RmuL;
nn=0:(c-1);
pWL(q)=(((c*RhoL(q))^c)/factorial(c))*((1-
RhoL(q))*sum(((c*RhoL(q)).^nn)./factorial(nn))+(((c*RhoL(q))^c)/factorial(c)))^(-1);
Wq1L(q)=(pWL(q)/(1-Rho1L(q)))*(ERL/c);
Wq2L(q)=(pWL(q)/((1-RhoL(q))*(1-Rho1L(q))))*(ERL/c);
WqL(q)=Wq1L(q)+Wq2L(q);
Ws1L(q)=Wq1L(q)+ERL;
Ws2L(q)=Wq2L(q)+ERL;
WsL(q)=Ws1L(q)+Ws2L(q);
Lq1L(q)=Rlambda1L*Wq1L(q);
Lq2L(q)=Rlambda2L*Wq2L(q);
LqL(q)=Lq1L(q)+Lq2L(q);
Ls1L(q)=Rlambda1L*Ws1L(q);
Ls2L(q)=Rlambda2L*Ws2L(q);
LsL(q)=Ls1L(q)+Ls2L(q);

Rlambda1M=(0.5*(lambda1Mediam(q,1)+lambda1Mediam(q,3))+0.25*(2*lambda1Mediam
(q,2)-lambda1Mediam(q,1)-lambda1Mediam(q,3)))/24;
Rlambda2M=(0.5*(lambda2Mediam(q,1)+lambda2Mediam(q,3))+0.25*(2*lambda2Mediam
(q,2)-lambda2Mediam(q,1)-lambda2Mediam(q,3)))/24;
RmuM=1/(0.5*(muMediam(q,1)+muMediam(q,3))+0.25*(2*muMediam(q,2)-
muMediam(q,1)-muMediam(q,3)));
Rho1M(q)=Rlambda1M/(c1*RmuM);
Rho2M(q)=Rlambda2M/(c2*RmuM);
c=c1+c2;
RhoM(q)=Rho1M(q)+Rho2M(q);
ERM=RmuM;
nn=0:(c-1);
```

```
pWM(q)=(((c*RhoM(q))^c)/factorial(c))*((1-
RhoM(q))*sum(((c*RhoM(q)).^nn)./factorial(nn))+(((c*RhoM(q))^c)/factorial(c)))^(-1);
Wq1M(q)=(pWM(q)/(1-Rho1M(q)))*(ERM/c);
Wq2M(q)=(pWM(q)/((1-RhoM(q))*(1-Rho1M(q))))*(ERM/c);
WqM(q)=Wq1M(q)+Wq2M(q);
Ws1M(q)=Wq1M(q)+ERM;
Ws2M(q)=Wq2M(q)+ERM;
WsM(q)=Ws1M(q)+Ws2M(q);
Lq1M(q)=Rlambda1M*Wq1M(q);
Lq2M(q)=Rlambda2M*Wq2M(q);
LqM(q)=Lq1M(q)+Lq2M(q);
Ls1M(q)=Rlambda1M*Ws1M(q);
Ls2M(q)=Rlambda2M*Ws2M(q);
LsM(q)=Ls1M(q)+Ls2M(q);

Rlambda1H=(0.5*(lambda1High(q,1)+lambda1High(q,3))+0.25*(2*lambda1High(q,2)-
lambda1High(q,1)-lambda1High(q,3)))/24;
Rlambda2H=(0.5*(lambda2High(q,1)+lambda2High(q,3))+0.25*(2*lambda2High(q,2)-
lambda2High(q,1)-lambda2High(q,3)))/24;
RmuH=1/(0.5*(muHigh(q,1)+muHigh(q,3))+0.25*(2*muHigh(q,2)-muHigh(q,1)-
muHigh(q,3)));
Rho1H(q)=Rlambda1H/(c1*RmuH);
Rho2H(q)=Rlambda2H/(c2*RmuH);
c=c1+c2;
RhoH(q)=Rho1H(q)+Rho2H(q);
ERH=RmuH;
nn=0:(c-1);
pWH(q)=(((c*RhoH(q))^c)/factorial(c))*((1-
RhoH(q))*sum(((c*RhoH(q)).^nn)./factorial(nn))+(((c*RhoH(q))^c)/factorial(c)))^(-1);
Wq1H(q)=(pWH(q)/(1-Rho1H(q)))*(ERH/c);
Wq2H(q)=(pWH(q)/((1-RhoH(q))*(1-Rho1H(q))))*(ERH/c);
WqH(q)=Wq1H(q)+Wq2H(q);
Ws1H(q)=Wq1H(q)+ERH;
Ws2H(q)=Wq2H(q)+ERH;
WsH(q)=Ws1H(q)+Ws2H(q);
Lq1H(q)=Rlambda1H*Wq1H(q);
Lq2H(q)=Rlambda2H*Wq2H(q);
LqH(q)=Lq1H(q)+Lq2H(q);
Ls1H(q)=Rlambda1H*Ws1H(q);
Ls2H(q)=Rlambda2H*Ws2H(q);
LsH(q)=Ls1H(q)+Ls2H(q);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Gamma FM1,FM2/GSR/10-2Pr

Rlambda1gL=(0.5*(lambda1Low(q,1)+lambda1Low(q,3))+0.25*(2*lambda1Low(q,2)-
lambda1Low(q,1)-lambda1Low(q,3)))/24;
Rlambda2gL=(0.5*(lambda2Low(q,1)+lambda2Low(q,3))+0.25*(2*lambda2Low(q,2)-
lambda2Low(q,1)-lambda2Low(q,3)))/24;
RmugL=1/(0.5*(mugLow(q,1)+mugLow(q,3))+0.25*(2*mugLow(q,2)-mugLow(q,1)-
mugLow(q,3)));
```

```
EbgL=RmugL;
Rho1gL(q)=Rlambda1gL/(c1*EbgL);
Rho2gL(q)=Rlambda2gL/(c2*EbgL);
c=c1+c2;
RhogL(q)=Rho1gL(q)+Rho2gL(q);
ERgL(q)=(elf*(elf+1)*beta^2)/(2*elf*beta);
nn=0:(c-1);
pWgL(q)=(((c*RhogL(q))^c)/factorial(c))*((1-
RhogL(q))*sum(((c*RhogL(q)).^nn)./factorial(nn))+(((c*RhogL(q))^c)/factorial(c)))^(-1);
Wq1gL(q)=(pWgL(q)/(1-Rho1gL(q)))*(ERgL(q)/c);
Wq2gL(q)=(pWgL(q)/((1-RhogL(q))*(1-Rho1gL(q))))*(ERgL(q)/c);
WqgL(q)=Wq1gL(q)+Wq2gL(q);
Ws1gL(q)=Wq1gL(q)+RmugL;
Ws2gL(q)=Wq2gL(q)+RmugL;
WsgL(q)=Ws1gL(q)+Ws2gL(q);
Lq1gL(q)=Rlambda1gL*Wq1gL(q);
Lq2gL(q)=Rlambda2gL*Wq2gL(q);
LqgL(q)=Lq1gL(q)+Lq2gL(q);
Ls1gL(q)=Rlambda1gL*Ws1gL(q);
Ls2gL(q)=Rlambda2gL*Ws2gL(q);
LsgL(q)=Ls1gL(q)+Ls2gL(q);

Rlambda1gM=(0.5*(lambda1Mediam(q,1)+lambda1Mediam(q,3))+0.25*(2*lambda1Media
m(q,2)-lambda1Mediam(q,1)-lambda1Mediam(q,3)))/24;
Rlambda2gM=(0.5*(lambda2Mediam(q,1)+lambda2Mediam(q,3))+0.25*(2*lambda2Media
m(q,2)-lambda2Mediam(q,1)-lambda2Mediam(q,3)))/24;
RmugM=1/(0.5*(mugMediam(q,1)+mugMediam(q,3))+0.25*(2*mugMediam(q,2)-
mugMediam(q,1)-mugMediam(q,3)));
EbgM=RmugM;
Rho1gM(q)=Rlambda1gM/(c1*EbgM);
Rho2gM(q)=Rlambda2gM/(c2*EbgM);
c=c1+c2;
RhogM(q)=Rho1gM(q)+Rho2gM(q);
ERgM(q)=(elf*(elf+1)*beta^2)/(2*elf*beta);
nn=0:(c-1);
pWgM(q)=(((c*RhogM(q))^c)/factorial(c))*((1-
RhogM(q))*sum(((c*RhogM(q)).^nn)./factorial(nn))+(((c*RhogM(q))^c)/factorial(c)))^(-1);
Wq1gM(q)=(pWgM(q)/(1-Rho1gM(q)))*(ERgM(q)/c);
Wq2gM(q)=(pWgM(q)/((1-RhogM(q))*(1-Rho1gM(q))))*(ERgM(q)/c);
WqgM(q)=Wq1gM(q)+Wq2gM(q);
Ws1gM(q)=Wq1gM(q)+RmugM;
Ws2gM(q)=Wq2gM(q)+RmugM;
WsgM(q)=Ws1gM(q)+Ws2gM(q);
Lq1gM(q)=Rlambda1gM*Wq1gM(q);
Lq2gM(q)=Rlambda2gM*Wq2gM(q);
LqgM(q)=Lq1gM(q)+Lq2gM(q);
Ls1gM(q)=Rlambda1gM*Ws1gM(q);
Ls2gM(q)=Rlambda2gM*Ws2gM(q);
LsgM(q)=Ls1gM(q)+Ls2gM(q);

Rlambda1gH=(0.5*(lambda1High(q,1)+lambda1High(q,3))+0.25*(2*lambda1High(q,2)-
lambda1High(q,1)-lambda1High(q,3)))/24;
```

```
Rlambda2gH=(0.5*(lambda2High(q,1)+lambda2High(q,3))+0.25*(2*lambda2High(q,2)-
lambda2High(q,1)-lambda2High(q,3)))/24;
RmugH=1/(0.5*(mugHigh(q,1)+mugHigh(q,3))+0.25*(2*mugHigh(q,2)-mugHigh(q,1)-
mugHigh(q,3)));
EbgH=RmugH;
Rho1gH(q)=Rlambda1gH/(c1*EbgH);
Rho2gH(q)=Rlambda2gH/(c2*EbgH);
c=c1+c2;
RhogH(q)=Rho1gH(q)+Rho2gH(q);
ERgH(q)=(elf*(elf+1)*beta^2)/(2*elf*beta);
nn=0:(c-1);
pWgH(q)=(((c*RhogH(q))^c)/factorial(c))*((1-
RhogH(q))*sum(((c*RhogH(q)).^nn)./factorial(nn))+(((c*RhogH(q))^c)/factorial(c)))^(-1);
Wq1gH(q)=(pWgH(q)/(1-Rho1gH(q)))*(ERgH(q)/c);
Wq2gH(q)=(pWgH(q)/((1-RhogH(q))*(1-Rho1gH(q))))*(ERgH(q)/c);
WqgH(q)=Wq1gH(q)+Wq2gH(q);
Ws1gH(q)=Wq1gH(q)+RmugH;
Ws2gH(q)=Wq2gH(q)+RmugH;
WsgH(q)=Ws1gH(q)+Ws2gH(q);
Lq1gH(q)=Rlambda1gH*Wq1gH(q);
Lq2gH(q)=Rlambda2gH*Wq2gH(q);
LqgH(q)=Lq1gH(q)+Lq2gH(q);
Ls1gH(q)=Rlambda1gH*Ws1gH(q);
Ls2gH(q)=Rlambda2gH*Ws2gH(q);
LsgH(q)=Ls1gH(q)+Ls2gH(q);

end

Result_Lower=[mean(Rho1L) mean(Wq1L) mean(Ws1L) mean(Lq1L)
mean(Ls1L);mean(Rho2L) mean(Wq2L) mean(Ws2L) mean(Lq2L)
mean(Ls2L);mean(RhoL) mean(WqL) mean(WsL) mean(LqL) mean(LsL)]
Result_Mediam=[mean(Rho1M) mean(Wq1M) mean(Ws1M) mean(Lq1M)
mean(Ls1M);mean(Rho2M) mean(Wq2M) mean(Ws2M) mean(Lq2M)
mean(Ls2M);mean(RhoM) mean(WqM) mean(WsM) mean(LqM) mean(LsM)]
Result_High=[mean(Rho1H) mean(Wq1H) mean(Ws1H) mean(Lq1H)
mean(Ls1H);mean(Rho2H) mean(Wq2H) mean(Ws2H) mean(Lq2H)
mean(Ls2H);mean(RhoH) mean(WqH) mean(WsH) mean(LqH) mean(LsH)]

Resultg_Lower=[mean(Rho1gL) mean(Wq1gL) mean(Ws1gL) mean(Lq1gL)
mean(Ls1gL);mean(Rho2gL) mean(Wq2gL) mean(Ws2gL) mean(Lq2gL)
mean(Ls2gL);mean(RhogL) mean(WqgL) mean(WsgL) mean(LqgL) mean(LsgL)]
Resultg_Mediam=[mean(Rho1gM) mean(Wq1gM) mean(Ws1gM) mean(Lq1gM)
mean(Ls1gM);mean(Rho2gM) mean(Wq2gM) mean(Ws2gM) mean(Lq2gM)
```

Appendix H: Codes for simulation experiments of the proposed TrMF-UF model using Matlab

```
%%%%%%%%%% Program Rho %%%
clc
clear all
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n=1000;
lambda1=23;
lambda2=20;
mu=0.99;
lam1L=3;
lam1H=70;
lam2L=2;
lam2H=48;
muL=0.5;
muH=2.583333333;
%%%%%%%%%%
k1=6;
k2=4;
c1=6;
c2=4;
L=1000;
for q=1:L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x=poissrnd(lambda1,1,n);
x111=(x>=lam1L&x<=lambda1);
x11=x(x111);
x1=x11(1:192);
x112=(x>=lambda1&x<=lam1H);
x12=x(x112);
x2=x12(1:161);
X1=[x1 x2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply1 for
lambda1
%%%%%%%%%%%%%%%%%%%%%%%%%%%
y=poissrnd(lambda2,1,n);
y111=(y>=lam2L&y<=lambda2);
y11=y(y111);
y1=y11(1:178);
y112=(y>=lambda2&y<=lam2H);
y12=y(y112);
y2=y12(1:178);
X2=[y1 y2];  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  supply2 for
lambda2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t1= exprnd(mu,n,1); %
t111=(t1>=muL&t1<=mu);
t11=t1(t111);
ty1=t11(1:175);
t112=(t1>=mu&t1<=muH);
t12=t1(t112);
```

```
ty2=t12(1:178);
EX=[ty1' ty2'];  %%%%%%%%%%%%%%%%%%%%%%%%%%%  services
time exponential
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
EX1=mean(X1);
EX2=mean(X2);
lambda1av=[lam1L EX1 lam1H];
lambda2av=[lam2L EX2 lam2H];
muav=[muL EX muH];
lambdadf1=mean(lambda1av)/24;
lambdadf2=mean(lambda2av)/24;
muavdf=mean(muav);
rho1=(lambdadf1/muavdf)^k1;
rho2=(lambdadf2/muavdf)^k2;
Rho(q)=rho1+rho2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
X1s=sort(X1);
avgmX1s=mean(X1s);
avgmX1s1=(X1s<avgmX1s);
avgL_lambda1=mean(X1s(avgmX1s1))/24;
avgmX1s2=(X1s>avgmX1s);
avgH_lambda1=mean(X1s(avgmX1s2))/24;
%%%%%%%%%%%%
X2s=sort(X2);
avgmX2s=mean(X2s);
avgmX2s1=(X2s<avgmX2s);
avgL_lambda2=mean(X2s(avgmX2s1))/24;
avgmX2s2=(X2s>avgmX2s);
avgH_lambda2=mean(X2s(avgmX2s2))/24;
%%%%%%%%%%
EXs=sort(EX);
avgmEXs=mean(EX);
avgmEXs1=(EXs<avgmEXs);
avgL_mu=mean(EXs(avgmEXs1));
avgmEXs2=(EXs>avgmEXs);
avgH_mu=mean(EXs(avgmEXs2));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%
lambda1Low=[lam1L avgL_lambda1 avgmX1s];
lambda1Mediam=[avgL_lambda1 avgmX1s avgH_lambda1];
lambda1High=[avgmX1s avgH_lambda1 lam1H];
lambda2Low=[lam2L avgL_lambda2 avgmX2s];
lambda2Mediam=[avgL_lambda2 avgmX2s avgH_lambda2];
lambda2High=[avgmX2s avgH_lambda2 lam2H];
muLow=[muL avgL_mu avgmEXs];
muMediam=[avgL_mu avgmEXs avgH_mu];
muHigh=[avgmEXs avgH_mu muH];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Rlambd1L=(0.5)*((lambda1Low(1)+lambda1Low(3))+0.5*(2*lambda1Low(2)-
lambda1Low(1)-lambda1Low(3)));
```

```
Rlambd2L=(0.5)*((lambda2Low(1)+lambda2Low(3))+0.5*(2*lambda2Low(2)-lambda2Low(1)-lambda2Low(3)));
RmuL=(0.5)*((muLow(1)+muLow(3))+0.5*(2*muLow(2)-muLow(1)-muLow(3)));
L_ambda1LowRH=Rlambd1L/24;
L_ambda2LowRH=Rlambd2L/24;
muRHL=(1/RmuL);
Rho1Low=L_ambda1LowRH/(c1*muRHL);
Rho2Low=L_ambda2LowRH/(c2*muRHL);
RhoLow(q)=Rho1Low+Rho2Low;
%%%%%%%%
Rlambd1Mediam=(0.5)*((lambda1Mediam(1)+lambda1Mediam(3))+0.5*(2*lambda1Mediam(2)-lambda1Mediam(1)-lambda1Mediam(3)));
Rlambd2Mediam=(0.5)*((lambda2Mediam(1)+lambda2Mediam(3))+0.5*(2*lambda2Mediam(2)-lambda2Mediam(1)-lambda2Mediam(3)));
RmuMediam=(0.5)*((muMediam(1)+muMediam(3))+0.5*(2*muMediam(2)-muMediam(1)-muMediam(3)));
L_ambda1MediamRH=Rlambd1Mediam/24;
L_ambda2MediamRH=Rlambd2Mediam/24;
muRHMediam=(1/RmuMediam);
Rho1Mediam=L_ambda1MediamRH/(c1*muRHMediam);
Rho2Mediam=L_ambda2MediamRH/(c2*muRHMediam);
RhoMediam(q)=Rho1Mediam+Rho2Mediam;
%%%%%%
Rlambd1High=(0.5)*((lambda1High(1)+lambda1High(3))+0.5*(2*lambda1High(2)-lambda1High(1)-lambda1High(3)));
Rlambd2High=(0.5)*((lambda2High(1)+lambda2High(3))+0.5*(2*lambda2High(2)-lambda2High(1)-lambda2High(3)));
RmuHigh=(0.5)*((muHigh(1)+muHigh(3))+0.5*(2*muHigh(2)-muHigh(1)-muHigh(3)));
L_ambda1HighRH=Rlambd1High/24;
L_ambda2HighRH=Rlambd2High/24;
muRHHigh=(1/RmuHigh);
Rho1High=L_ambda1HighRH/(c1*muRHHigh);
Rho2High=L_ambda2HighRH/(c2*muRHHigh);
RhoHigh(q)=Rho1High+Rho2High;
end
RHO=[mean(Rho) mean(RhoLow) mean(RhoMediam) mean(RhoHigh)]%%%
```