DEVELOPMENT AND MODIFICATION OF *H*-STATISTIC WITH WINSORIZED APPROACH MEANS

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Abstrak

Ujian-*t* pelajar dan ujian-*F* ANOVA adalah ujian statistik klasik untuk membandingkan dua atau lebih kumpulan bebas. Kedua-duanya adalah ujian yang berkuasa apabila data tertabur normal dan mempunyai varians homogen. Walau bagaimanapun, data dengan pencirian tersebut adakalanya sukar untuk dipenuhi dalam kehidupan sebenar dan akan memberi kesan kepada kawalan kadar ralat Jenis I dan mengurangkan kuasa ujian statistik tersebut. Statistik-*H* adalah statistik teguh namun hanya mampu menunjukkan prestasi yang baik hanya pada set data tidak normal. Statistik ini telah diinovasikan dengan penganggar *MOM* dan ditandai sebagai *MOM-H*. Oleh yang demikian, dalam kajian ini, dua statistik-*H* terubah suai dengan min menggunakan pendekatan ter*Winsor* adalah dicadangkan untuk menangani ketidakpatuhan kedua-dua pencirian tersebut. Statistik yang dicadangkan adalah statistik-*H* dengan min ter*Winsor* (*WM*) dan statistik-*H* dengan min ter*Winsor* suai (*AWM*) yang masing-masing ditandai sebagai *WM-H* dan *AWM-H*. Menggunakan pengubahsuaian ini, prestasi ujian lebih baik bukan sahaja pada ketidaknormalan, tetapi juga pada keheterogenan varians. Pendekatan ini menggunakan nilai awal iaitu 15% dan 25% nilai pe*Winsor*an Pendekatan *WM* me*Winsor* secara simetri manakala *AWM* me*Winsor* secara tersuai mengikut bentuk taburan berdasarkan penganggar engsel, *HQ* dan HQ_1. Statistik *WM-H* terdiri daripada 15*WM-H* dan 25*WM-H*, manakala *AWM-H* terdiri daripada 15*WHQ-H*, 25*WHQ-H*, 15WHQ_1-*H* dan 25WHQ_1-*H*. Prestasi ujian yang dicadangkan adalah dinilai dengan menggunakan Kadar Ralat Jenis I dan kuasa ujian berdasarkan kajian simulasi. Semua keputusan daripada ujian yang dicadangkan dibandingkan dengan ujian statistik-*H* yang asal, *MOM-H* dan statistik klasik. Pada taburan terpencong, *WM-H* menunjukkan prestasi lebih baik berbanding dengan yang lain tetapi setanding dengan *MOM-H*. Secara keseluruhan ujian yang dicadangkan dapat memberikan hasil yang lebih baik daripada *MOM-H* dan ujian statistik klasik pada keadaan tertentu. Ujian yang dicadangkan juga ditentusahkan menggunakan set data sebenar.

Kata kunci: Pendekatan ter*Winsor*, Penganggar engsel, Kadar ralat Jenis I, Kuasa ujian, Statistik-*H*

Abstract

Student's *t*-test and ANOVA *F*-test are the classical statistical tests for comparing two or more independent groups. Both are powerful tests when data is normally distributed and variances are homogenous. However, the data with these properties sometime is difficult to be met in real-life will affect the Type I error rates control and reduce statistical power of the tests. *H*-statistic is a robust statistic but performs well only under non-normality dataset. This statistic had been invented with *MOM* estimator denoted as *MOM-H*. Therefore, in this study, two modified *H*-statistic with mean using Winsorizing approach are proposed to handle both violated properties. The proposed statistics are the *H*-statistic with Winsorized mean (*WM*) and the *H*-statistic with adaptive Winsorized mean (*AWM*) which denoted as *WM-H* and *AWM-H*, respectively. Using this modification, the tests perform better not only under non-normality, but also under heterogeneity of variances. The approach use predetermined values of 15% and 25% Winsorization. The *WM* is Winsorizing symmetrically while the *AWM* is Winsorizing adaptively according to the shape of distribution based on hinge estimators, *HQ* and HQ_1. The *WM-H* statistic consists of 15*WM-H* and 25*WM-H*, whereas the *AWM-H* comprises of 15*WHQ-H*, 25*WHQ-H*, 15WHQ_1-*H* and 25WHQ_1-*H*. The performances of the proposed tests are evaluated using Type I error rates and power of test based on simulation study. All the results from the proposed tests are compared with the original *H*-statistic, *MOM-H* and classical statistical tests. The findings indicate that 15*WHQ-H* performs the best for two groups case especially under heavy tailed distribution. Under skewed distribution, *WM-H* has better performance to others but comparable to *MOM-H*. In overall the proposed tests are able to give better results than the *MOM-H* and the classical statistical tests under certain conditions. The proposed tests are also validated using real dataset.

Keywords: Winsorizing approach, Hinge estimator, Type I error rates, Power of test, *H*-statistic

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CHAPTER ONE
INTRODUCTION

1.1 Background

In the case of employing classical procedures in comparing independent groups, the normality of distribution and the homogeneity of variances among the groups are the primary concerns that will affect the analysis results. The devastating effect on controlling Type I errors rate and reducing the statistical power will happen when dispersion in these criteria occurs. (Syed Yahaya, 2005; Syed Yahaya, Othman, & Keselman, 2006; Keselman, Algina, Lix, Wilcox, & Deering, 2008). In order to deal with these violation of assumptions, the alternative procedures such as non-parametric procedure may be employed. However, the use of this procedure may cause loss of information as this procedure is testing on the ranking value rather than on the original parametric value (Siegel, 1957).

Besides the non-parametric procedure, another common method used to deal with the violation of normality is simple data transformation. In other words, each observation of the data is transformed by taking inverse, logarithms, square roots, or other transformations, before performing test analysis (Rasmussen, 1989; Wilcox & Keselman, 2003a). Based on Rasmussen's study in 1989, an accurate transformation may provide better control of Type I error rate and increase the statistical power under more non-normal distribution. However, for the mildly skewed data or the data that have groups with skewed data in opposite directions, it may not be advantageous. Furthermore, the transformations are complicated to perform and wrong or inaccurate transformation being chosen will affect the accuracy of the analysis results.

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APPENDIX A

The statistical power for $J = 2$

Type of Distribution	Sample Size		Variance		Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	Student's *t*-test
						Small, $f = 0.20$	9.04	8.96	10.08	9.86	9.76	9.64	6.96	9.16
	20	20	1	1		Medium, $f = 0.50$	31.16	29.08	34.64	33.40	33.84	33.00	23.50	33.62
						Large, $f = 0.80$	64.92*	60.86*	68.84*	66.88*	68.90*	66.54*	53.44*	68.66*
						Small, $f = 0.20$	10.64	10.34	14.12	14.52	12.28	14.36	8.52	11.16
	20	20	1	36		Medium, $f = 0.50$	32.88	30.16	41.86	42.20	37.74	41.36	25.84	35.34
						Large, $f = 0.80$	64.68*	61.56*	74.48*	74.38*	70.54*	73.54*	54.16*	68.54*
						Small, $f = 0.20$	9.06	8.28	9.90	10.58	9.92	9.88	6.70	9.12
	15	25	1	1		Medium, $f = 0.50$	29.66	27.82	32.14	32.52	32.12	31.20	21.38	31.52
g=0; h=0						Large, $f = 0.80$	63.36*	60.44*	66.50*	66.64*	66.52*	65.04*	50.02*	66.52*
						Small, $f = 0.20$	12.90	12.32	14.90	18.64	14.88	16.76	9.68	5.46
	15	25	1	36	+	Medium, $f = 0.50$	46.56	43.08	51.96*	57.56*	51.98*	53.88*	35.88	29.28
						Large, $f = 0.80$	**84.68***	**80.94***	**87.90***	**90.46***	**87.88***	**88.82***	73.26*	70.26*
						Small, $f = 0.20$	7.96	7.66	8.58	7.14	8.58	7.14	5.94	16.84
	15	25	36	1	-	Medium, $f = 0.50$	19.96	19.44	21.40	18.72	21.40	18.54	15.10	36.60
						Large, $f = 0.80$	43.02	40.58	44.58	40.58	44.56	40.72	31.72	62.86*
					AVERAGE		35.37	33.43	38.79	38.94	38.06	38.03	28.14	37.00

APPENDIX A

Type of Distribution	Sample Size		Variance		Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	Student's *t*-test
	20	20	1	1		Small, $f = 0.20$	3.04	4.18	3.58	3.12	2.56	2.92	4.50	4.34
						Medium, $f = 0.50$	8.14	13.06	7.94	7.30	5.72	7.00	14.34	9.72
						Large, $f = 0.80$	19.78	31.62	16.60	17.44	12.94	16.38	35.02	19.24
	20	20	1	36		Small, $f = 0.20$	4.38	4.96	7.18	7.26	4.56	5.78	5.58	5.94
						Medium, $f = 0.50$	12.24	16.58	16.94	17.82	10.74	14.50	17.76	12.24
						Large, $f = 0.80$	27.08	36.28	31.56	35.92	22.06	29.74	38.42	24.80
	15	25	1	1		Small, $f = 0.20$	1.84	3.30	4.34	2.94	2.96	2.36	3.60	4.68
						Medium, $f = 0.50$	6.20	10.24	10.20	6.88	6.52	6.74	12.72	9.58
g=0; h=0.5						Large, $f = 0.80$	15.56	26.38	20.28	15.52	13.94	15.92	31.04	18.42
	15	25	1	36	+	Small, $f = 0.20$	4.20	6.00	8.30	7.90	4.52	5.70	6.54	2.10
						Medium, $f = 0.50$	14.32	10.24	22.30	22.36	13.18	17.04	25.14	7.12
						Large, $f = 0.80$	32.84	50.22*	41.96	43.30	27.44	35.54	54.54*	17.40
	15	25	36	1	-	Small, $f = 0.20$	2.56	3.78	4.18	3.20	3.98	2.68	3.92	11.64
						Medium, $f = 0.50$	5.82	8.88	7.60	5.00	6.78	5.34	9.82	18.38
						Large, $f = 0.80$	11.90	19.00	14.40	9.90	13.06	11.16	22.06	28.62
					AVERAGE		11.33	16.31	14.49	13.72	10.06	11.92	19.00	12.95

APPENDIX A

Type of Distribution	Sample Size		Variance		Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	Student's *t*-test
	20	20	1	1		Small, $f = 0.20$	4.42	4.82	7.16	4.82	4.84	4.62	6.02	5.50
						Medium, $f = 0.50$	13.92	17.18	17.42	12.78	12.74	12.44	18.16	15.14
						Large, $f = 0.80$	31.78	39.26	35.06	28.28	28.00	27.64	38.84	32.16
	20	20	1	36		Small, $f = 0.20$	12.46	8.14	33.18	21.12	19.56	20.26	5.88	25.48
						Medium, $f = 0.50$	34.48	27.24	66.72*	48.90	45.84	46.86	18.80	57.92*
						Large, $f = 0.80$	64.36*	59.82*	**90.24***	75.76*	70.48*	71.04*	46.44	**85.68***
	15	25	1	1		Small, $f = 0.20$	5.04	4.94	7.66	6.36	6.40	5.02	6.12	4.90
						Medium, $f = 0.50$	14.94	15.28	19.38	15.62	15.80	13.64	17.70	14.50
g=1; h=0						Large, $f = 0.80$	31.32	35.50	35.46	30.28	30.74	28.68	37.62	31.84
	15	25	1	36	+	Small, $f = 0.20$	18.80	10.46	37.32	2882	26.66	20.54	6.54	12.10
						Medium, $f = 0.50$	52.32*	42.50	75.90*	61.14*	59.00*	49.68	27.52	46.16
						Large, $f = 0.80$	79.74*	78.72*	**95.04***	**81.22***	**80.56***	73.42*	63.90*	79.74*
	15	25	36	1	-	Small, $f = 0.20$	3.82	5.18	8.40	6.44	8.46	5.12	8.88	15.50
						Medium, $f = 0.50$	7.22	10.64	8.68	7.34	8.46	6.88	18.00	13.28
						Large, $f = 0.80$	13.58	20.42	13.58	12.60	13.26	12.60	30.00	18.24
					AVERAGE		25.88	25.34	36.75	29.43	28.72	26.56	23.36	30.54

APPENDIX A

Type of Distribution	Sample Size		Variance		Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-H	25 WHQ_1-H	*MOM-H*	Student's *t*-test
.*g*=1; *h*=0.5	20	20	1	1		Small, *f* = 0.20	1.78	2.46	3.54	2.12	1.68	1.80	3.58	2.60
						Medium, *f* = 0.50	4.48	7.98	6.22	4.54	3.52	3.80	13.30	4.80
						Large, *f* = 0.80	10.40	19.72	11.76	8.74	6.80	7.30	31.78	9.32
	20	20	1	36		Small, *f* = 0.20	3.60	3.54	11.92	7.92	5.70	6.30	3.32	6.72
						Medium, *f* = 0.50	10.96	12.80	23.32	18.04	12.64	13.94	11.88	14.44
						Large, *f* = 0.80	24.12	30.74	39.00	32.88	22.48	24.48	33.06	26.84
	15	25	1	1		Small, *f* = 0.20	1.78	2.30	5.22	2.58	2.62	1.72	3.94	2.78
						Medium, *f* = 0.50	4.66	7.42	9.50	4.56	4.90	3.60	12.78	4.50
						Large, *f* = 0.80	9.84	17.26	16.06	8.82	9.00	7.72	30.26	8.54
	15	25	1	36	+	Small, *f* = 0.20	5.30	4.54	17.34	9.82	7.74	5.34	3.74	2.08
						Medium, *f* = 0.50	15.00	18.56	34.36	20.52	16.66	13.40	17.00	7.10
						Large, *f* = 0.80	30.74	43.94	50.74*	32.78	27.92	24.76	48.38	18.04
	15	25	36	1	-	Small, *f* = 0.20	1.82	3.18	4.84	3.92	4.44	2.32	6.58	9.22
						Medium, *f* = 0.50	3.40	6.40	5.70	3.96	4.60	3.24	15.46	9.88
						Large, *f* = 0.80	6.44	12.82	8.16	5.52	6.60	5.54	27.94	11.92
						AVERAGE	8.95	12.91	16.51	11.11	9.15	8.35	17.53	9.25
						GRAND AVERAGE	20.38	22.00	26.64	23.30	21.50	21.22	22.01	22.43

Notes: (*) more than 50%; (**bold**) more than 80%

APPENDIX B

The statistical power rate for $J = 4$

Type of Distribution	Sample Size				Variance				Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	ANOVA *F*-test
	20	20	20	20	1	1	1	1		Small, f = 0.10	8.28	7.28	9.62	9.20	9.66	9.04	4.90	9.78
										Medium, f = 0.25	35.48	31.54	40.38	38.42	40.02	38.08	23.82	41.76
										Large, f = 0.40	78.30*	73.66*	**82.48***	**80.54***	**82.46***	**80.24***	**62.60***	**83.64***
	20	20	20	20	1	1	1	36		Small, f = 0.10	7.42	7.06	9.80	9.96	8.80	9.76	5.62	14.12
										Medium, f = 0.25	20.56	19.16	27.16	27.20	24.46	26.72	15.04	37.14
										Large, f = 0.40	58.86*	53.50*	69.18*	67.56*	65.78*	66.92*	43.12	**85.64***
	10	15	25	30	1	1	1	1		Small, f = 0.10	9.64	8.12	11.10	11.68	11.80	10.56	5.34	9.80
										Medium, f = 0.25	40.72	35.06	43.00	44.72	44.58	41.96	24.42	41.38
g=0; h=0										Large, f = 0.40	**84.12***	79.08*	**85.80***	**86.52***	**86.54***	**85.40***	65.62*	**85.64***
	10	15	25	30	1	1	1	36	+	Small, f = 0.10	9.32	8.58	11.90	13.10	11.42	11.52	7.14	5.58
										Medium, f = 0.25	43.18	39.34	50.02*	53.48*	50.06*	49.74	31.12	29.16
										Large, f = 0.40	**96.38***	**92.54***	**98.00***	**97.80***	**98.22***	**97.40***	**85.88***	**91.38***
	10	15	25	30	36	1	1	1	-	Small, f = 0.10	8.04	6.66	9.00	9.00	9.76	8.56	6.10	32.22
										Medium, f = 0.25	14.98	12.32	15.16	15.42	16.90	14.92	9.74	52.44*
										Large, f = 0.40	31.04	24.76	31.18	31.70	34.00	30.64	20.16	**86.02***
					AVERAGE						36.42	33.24	39.59	39.75	39.63	38.76	27.37	47.05

PPENDIX B

Type of Distribution	Sample Size				Variance				Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	ANOVA *F*-test
	20	20	20	20	1	1	1	1		Small, f = 0.10	0.82	1.62	1.50	0.84	0.72	0.82	1.62	3.78
										Medium, f = 0.25	3.00	7.74	3.82	3.06	2.00	3.12	8.96	7.68
										Large, f = 0.40	10.56	25.44	9.74	9.12	5.72	9.00	29.64	17.26
	20	20	20	20	1	1	1	36		Small, f = 0.10	2.44	3.28	4.68	4.10	2.86	3.54	3.82	8.30
										Medium, f = 0.25	5.64	8.68	8.96	8.48	5.18	6.96	9.78	13.06
										Large, f = 0.40	15.04	23.88	18.90	20.08	11.98	16.98	26.50	24.74
	10	15	25	30	1	1	1	1		Small, f = 0.10	0.96	1.38	1.98	g=0; h=0.5	1.44	1.06	1.66	4.58
										Medium, f = 0.25	3.22	5.60	5.86	4.92	3.90	3.76	8.38	8.78
										Large, f = 0.40	11.10	19.34	15.42	13.02	10.50	11.08	29.78	18.38
	10	15	25	30	1	1	1	36	+	Small, f = 0.10	3.14	3.74	5.76	6.26	3.84	4.20	4.30	2.60
										Medium, f = 0.25	4.24	6.24	14.86	15.56	9.70	10.86	17.56	5.76
										Large, f = 0.40	27.76	46.04	34.36	35.20	24.46	28.44	56.20*	14.64
	10	15	25	30	36	1	1	1	-	Small, f = 0.10	3.04	2.04	4.26	4.28	4.14	4.06	3.20	25.56
										Medium, f = 0.25	5.16	3.88	6.02	5.62	5.68	5.38	5.52	31.14
										Large, f = 0.40	9.56	8.14	10.16	10.12	9.48	9.44	11.98	40.56
					AVERAGE						7.05	11.14	9.75	9.48	6.77	7.91	14.59	15.12

APPENDIX B

Type of Distribution	Sample Size				Variance				Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	ANOVA *F*-test
g=1; h=0	20	20	20	20	1	1	1	1		Small, f = 0.10	2.00	2.26	5.08	2.28	2.44	2.26	2.50	5.78
										Medium, f = 0.25	7.76	11.32	14.04	8.12	8.34	8.10	11.34	14.86
										Large, f = 0.40	24.76	33.84	32.38	21.40	21.70	21.42	33.40	33.14
	20	20	20	20	1	1	1	36		Small, f = 0.10	7.64	5.12	24.72	14.02	13.42	13.76	5.24	33.08
										Medium, f = 0.25	19.36	15.10	47.46	29.54	28.84	29.28	12.28	58.82*
										Large, f = 0.40	48.10	44.82	78.42*	57.48*	56.20*	565.8*	36.76	**86.10***
	10	15	25	30	1	1	1	1		Small, f = 0.10	2.54	1.96	4.42	3.40	3.94	2.60	2.44	5.06
										Medium, f = 0.25	10.14	10.04	14.96	11.92	13.02	9.34	11.22	13.80
										Large, f = 0.40	27.48	31.36	33.98	28.32	29.74	24.90	35.40	34.10
	10	15	25	30	1	1	1	36	+	Small, f = 0.10	12.92	7.68	32.84	24.58	25.08	17.82	5.30	20.86
										Medium, f = 0.25	39.34	32.44	65.54*	52.90*	52.00*	41.52	22.54	48.70
										Large, f = 0.40	75.42*	76.96*	**92.54***	79.12*	78.14*	69.06*	66.42*	**82.52***
	10	15	25	30	36	1	1	1	-	Small, f = 0.10	5.40	4.12	9.60	9.56	9.78	8.84	7.96	35.80
										Medium, f = 0.25	8.26	7.06	11.24	10.78	11.16	10.12	13.62	40.24
										Large, f = 0.40	14.50	13.26	17.18	16.90	17.28	15.96	22.00	52.78*
					AVERAGE						20.37	19.82	32.29	24.69	24.74	22.10	19.23	37.71

APPENDIX B

Type of Distribution	Sample Size				Variance				Natural Pairing	Effect Size	15 *WM-H*	25 *WM-H*	15 *WHQ-H*	25 *WHQ-H*	15 WHQ_1-*H*	25 WHQ_1-*H*	*MOM-H*	ANOVA *F*-test
	20	20	20	20	1	1	1	1		Small, f = 0.10	0.32	0.54	1.26	0.48	0.46	0.48	1.38	2.34
										Medium, f = 0.25	1.38	2.64	2.60	1.04	0.88	0.96	6.14	3.84
										Large, f = 0.40	3036	9.60	5.04	2.32	2.10	2.26	21.92	6.46
	20	20	20	20	1	1	1	36		Small, f = 0.10	2.02	2.10	7.98	4.68	3.42	3.88	2.98	10.92
										Medium, f = 0.25	4.16	5.18	11.84	7.48	5.34	6.04	6.36	15.32
										Large, f = 0.40	9.94	16.60	19.94	13.20	9.62	10.80	22.84	24.18
	10	15	25	30	1	1	1	1		Small, f = 0.10	0.42	0.52	1.78	0.96	1.06	0.64	0.92	3.52
										Medium, f = 0.25	1.30	2.00	3.84	2.00	2.02	1.32	5.84	4.20
g=1; h=0.5										Large, f = 0.40	3.92	7.76	8.14	4.12	4.04	2.62	21.66	6.18
	10	15	25	30	1	1	1	36	+	Small, f = 0.10	2.78	2.42	13.16	7.72	5.98	3.56	3.08	3.68
										Medium, f = 0.25	6.90	9.76	21.66	12.92	10.40	7.10	11.48	6.86
										Large, f = 0.40	17.74	31.04	35.42	21.68	19.16	13.88	48.82	14.48
	10	15	25	30	36	1	1	1	-	Small, f = 0.10	2.00	1.58	3.80	3.28	3.42	3.14	4.56	21.32
										Medium, f = 0.25	2.94	3.14	4.64	3.38	3.60	3.14	8.68	21.30
										Large, f = 0.40	4.92	5.84	7.06	5.08	5.36	4.46	15.58	24.28
					AVERAGE						4.27	6.71	9.88	6.02	5.12	04.29	12.15	11.26
					GRAND AVERAGE						17.03	17.73	22.88	19.99	19.07	18.27	18.34	27.78

Notes: (*) more than 50%; (**bold**) more than 80%