DIRECT SOLUTION OF HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS USING ONE-STEP HYBRID BLOCK METHODS WITH GENERALISED OFF-STEP POINTS IN THE PRESENCE OF HIGHER DERIVATIVE

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Abstrak

Sebilangan besar fenomena fizikal dapat diungkapkan sebagai masalah nilai awal atau sempadan persamaan pembeza biasa (PPB) peringkat tinggi yang berkemungkinan tidak mempunyai penyelesaian analitikal. Oleh itu, terdapat keperluan untuk membangunkan kaedah berangka bagi menganggarkan penyelesaian PPB peringkat tinggi. Salah satu kaedah langsung terkenal yang sering digunakan ialah kaedah blok. Walaupun kaedah ini mampu mencari penyelesaian hampir pada beberapa titik secara serentak, namun ianya gagal mengatasi sawar kestabilan-sifar. Justeru itu, kaedah blok hibrid diperkenalkan bagi menangani kelemahan tersebut. Manfaat utama kaedah ini adalah keupayaannya menggunakan data pada titik luar-langkah yang dapat menyumbang kepada kejituan yang lebih baik. Walau bagaimanapun, kebanyakan kaedah blok hibrid sedia ada hanya tertumpu kepada titik luar-langkah khusus kecuali kaedah yang diperkenalkan oleh Abdelrahim pada tahun 2016. Walaupun beliau telah berjaya membangunkan kaedah blok hibrid satu-langkah dengan titik luar-langkah teritlak bagi menyelesaikan PPB peringkat tinggi secara langsung, namun kaedah tersebut hanya terbatas kepada masalah nilai awal sahaja. Tambahan lagi, beliau tidak mempertimbangkan terbitan lebih tinggi semasa membangunkan kaedah tersebut. Oleh itu, kajian ini memperkenalkan kaedah blok hibrid satu- langkah baharu dengan titik luar-langkah teritlak dengan kehadiran terbitan lebih tinggi bagi menyelesaikan PPB peringkat tinggi secara langsung. Dalam pembangunan kaedah ini, siri kuasa telah digunakan sebagai penyelesaian hampir kepada permasalahan PPB peringkat m. Siri kuasa diinterpolasi pada m titik, sementara terbitannya yang ke-m dan ke-$(m+1)$ dikolokasi pada semua titik dalam selang terpilih. Sifat bagi kaedah baharu seperti peringkat, pemalar ralat, kestabilan sifar, ketekalan, penumpuan dan rantau kestabilan mutlak turut dikaji. Beberapa masalah nilai awal dan sempadan PPB peringkat tinggi yang dipertimbangkan dalam literatur kemudiannya diselesaikan dengan menggunakan kaedah baharu yang dibangunkan bagi menyiasat kejituan penyelesaian dari segi ralat. Keputusan berangka mendedahkan, pada umumnya, kaedah baharu berupaya menghasilkan ralat yang lebih kecil berbanding dengan kaedah yang sedia ada dalam menyelesaikan masalah yang sama. Kesimpulannya, kajian ini telah berjaya membangunkan kaedah yang berdaya saing untuk menyelesaikan kedua-dua masalah nilai awal dan sempadan PPB peringkat tinggi secara langsung.

Kata kunci: Penyelesaian langsung, Titik luar-langkah teritlak, Terbitan lebih tinggi, Masalah nilai awal dan sempadan peringkat tinggi, Kaedah blok hibrid satu-langkah.

Abstract

A great number of physical phenomena can be expressed as initial or boundary value problems of higher order ordinary differential equations (ODEs) which may not have analytical solutions. Thus, there is a need to develop numerical methods for approximating the solution of higher order ODEs. One of the well-known direct methods which frequently employed is block method. Even though this method is capable of finding the approximate solutions at several points simultaneously, it fails to overcome the zero-stability barrier. Thus, a hybrid block method was introduced to tackle this drawback. The main benefit of this method is its ability of using data at off-step points which contribute to better accuracy. Most of the existing hybrid block methods, however, only focus on specific off-step point(s) in deriving the methods with the exception of the method proposed by Abdelrahim in 2016. Although he has successfully developed one-step hybrid block methods with generalised off-step point(s) for solving high order ODEs directly, nevertheless, the methods are only confined to initial value problems. Moreover, he did not consider higher derivative in developing those methods. Thus, this study introduced new one-step hybrid block methods with generalised off-step point(s) in the presence of higher derivative for directly solving higher order ODEs. In developing these methods, a power series was used as an approximate solution to the problems of ODEs of order m. The power series was interpolated at m points, while its mth and $(m+1)$th derivatives were collocated at all points in the given interval. Investigations on the properties of the new methods such as order, error constant, zero-stability, consistency, convergence and region of absolute stability were also carried out. Several initial and boundary value problems of higher order ODEs considered in literature were then solved by using the newly developed methods in order to investigate the accuracy of the solution in terms of error. The numerical results revealed that, in general, the new methods were able to produce smaller errors compared to the existing methods in solving the same problems. In conclusion, this study has successfully developed viable methods for directly solving both initial and boundary value problems of higher order ODEs.

Keywords: Direct solution, Generalised off-step point(s), Higher derivative, Higher order initial and boundary value problems, One-step hybrid block method.

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List of Abbreviations

ADM	:	Adomian Decomposition Method (Kelesoglu, 2014)
BVP	:	Boundary Value Problem
BVPs	:	Boundary Value Problems
DMVSS	:	Direct Method Variable Step-Size (Phang, Majid, Suleiman, & Ismail, 2013)
FDM	:	Fourth Derivative Method (Sahi, Jator, & Khan, 2013)
FMAM	:	Fast Multilevel Augmentation Method (Chen, 2011)
FOBM	:	Fourth Order Block Method (Abdullah, Majid, & Senu, 2013)
FSBM	:	Five-Step Block Method (Olabode, 2014)
HAFDS	:	High Accuracy Finite Difference Scheme (Chen & Li, 2012)
HBM	:	Hybrid Block Method
HBMs	:	Hybrid Block Methods
HNHIS	:	Hybrid and Non-Hybrid Implicit Schemes (Gbenga, Olaoluwa, & Olayemi, 2015)
IHBNTM	:	Implicit Hybrid Block Numerov-Type Method (Olabode & Omole, 2015)
IVP	:	Initial Value Problem
IVPs	:	Initial Value Problems
LMM	:	Linear Multistep Method
LMMs	:	Linear Multistep Methods
NIFOBVP	:	Numerical Integrators for Fourth Order Boundary Value Problem (Jator, 2008b)
NITOM	:	Numerical Integration of Third Order Method (Jator, 2008a)
NPSM	:	Non-Polynomial Spline Method (Taiwo & Ogunlaran, 2011)

ODE	:	Ordinary Differential Equation
ODEs	:	Ordinary Differential Equations
OSTHBM	:	One-Step Three Hybrid Block method (Abdelrahim, 2016)
PSM	:	Polynomial Spline Method (Liu, Liu, & Chen, 2011)
THOSBM	:	Three Hybrid One-Step Block Method (Abdelrahim, 2016)
THOSHBM	:	Three Hybrid One-Step Hybrid Block Method (Abdelrahim, 2016)
THOSM	:	Three Hybrid One-Step Method (Abdelrahim, 2016)
TOL	:	Tolerance
TPMNBM	:	Three-Point Modified Numerov Block Method (Sagir, 2013a)
TSBM	:	Three-Step Block Method (Olabode & Yusuph, 2009)
TSHBM	:	Three-Step Hybrid Block Method (Yahaya, Sagir, & Tech, 2013)
TSIHM	:	Three-Step Implicit Hybrid Method (Mohammed & Adeniyi, 2014)
TSYFHM	:	Three-Step Y-Function Hybrid Method (Kayode & Obarhua, 2015)

CHAPTER ONE
INTRODUCTION

1.1 Background of the Study

Some problems in scientific fields such as engineering, economics, operation research, thermodynamics, optics, waves and vibrations involve rate of change which lead to equations that include some derivatives of unknown functions of one or several variables known as differential equations. Differential equations with unknown functions consist of one independent variable is categorised as ordinary differential equations (ODEs). On the other hand, those equations in which unknown functions have at least two independent variables are called partial differential equations.

The general form of an ordinary differential equation (ODE) of m^{th} order is given by an ordinary differential equation (ODE)

$$F(x, y(x), y'(x), y''(x), \ldots, y^{(m-1)}(x), y^{(m)}(x)) = 0 \tag{1.1}$$

where m represents the order of the ODE, F is a continuous function, x is the independent variable in the interval $[a, b]$ and $y(x), y'(x), y''(x), \ldots, y^{(m-1)}(x), y^{(m)}(x)$ are the dependent variables. Equation (1.1) is called linear if it can be expressed in the following form

$$a_m(x)y^{(m)}(x) + a_{m-1}(x)y^{(m-1)}(x) + \ldots + a_1(x)y^{(1)}(x) + a_0(x)y(x) + b(x) = 0 \tag{1.2}$$

where $a_j(x)$ and $b(x)$ are functions of x for $j = 0(1)m$. Otherwise, Equation (1.1) is considered as a nonlinear ODE. Equation (1.1) can also be written as

$$y^{(m)}(x) = f(x, y(x), y'(x), y''(x), \ldots, y^{(m-1)}(x)). \tag{1.3}$$

If the conditions below

$$y^{(i)}(a) = \omega_i, \quad i = 0, 1, \ldots, m-1 \tag{1.4}$$

are imposed on Equation (1.3), then the combination of Equations (1.3) and (1.4) becomes

$$y^{(m)} = f(x, y, y', y'', ..., y^{(m-1)}),\ y^{(i)}(a) = \omega_i,\ i = 0, 1, 2, ..., m-1 \qquad (1.5)$$

which is called an m^{th} order initial value problem (IVP). On the other hand, if the conditions specified at the end points of the integration interval [a,b] are imposed on Equation (1.3), then this combination is called boundary value problem (BVP). Now we shall look at two theorems which guarantee the existence and uniqueness of a solution for m^{th} order ODEs.

1.2 Existence and Uniqueness Theorems

Theorem 1.1 and Theorem 1.2 state the existence and uniqueness of first order and higher order ODEs, respectively.

Theorem 1.1. *(Lambert, 1991): Let $f(x, y)$ be a continuous real function for all points (x, y) in the region Σ defined by $a \leq x \leq b$, $-\infty < y < \infty$ containing initial values (x_1, y_1) where $\{a, b\}$ are finite. Assume L is a constant such that for any $\omega_0 \in \mathbb{R}$ and for any pairs y_a, y_b for which $(x, y_a), (x, y_b)$ are both in Σ, $| f(x, y_a) - f(x, y_b) | \leq L | y_a - y_b |$. Then, for any given number $x \in [a, b]$, the first order IVP has a unique solution and L is called the Lipschitz constant.*

Theorem 1.2. *(Wend, 1969): Let the inequalities $x_0 \leq x \leq x_0 + a$ and $| s_i - \omega_i | \leq b$, $i = 0, 1, ..., m-1$ $(a > 0, b > 0)$ define the region Σ^*. Assume that $f(x, s_0, s_1, ..., s_{m-1})$ is defined in Σ^* and in addition*

1. *f is non-negative and non-decreasing in each of $x, s_0, s_1, ..., s_{m-1}$ in Σ^*,*
2. *$f(x, \omega_0, \omega_1, ..., \omega_{m-1}) > 0$ for $x_0 < x \leq x_0 + a$, and*
3. *$\omega_i \geq 0,\ i = 0(1)m-1$.*

Then, the m^{th} order IVP (1.5) has a unique solution in Σ^*.

There are two approaches for solving higher order ODEs namely reduction method and direct method.

1.3 Reduction Method

In this approach, the m^{th} order ODE (1.5) is transformed into its equivalent first order system of m equations through a simple procedure as follows:

Let
$$\begin{aligned}
u_1(x) &= y(x),\\
u_2(x) &= y'(x),\\
u_3(x) &= y''(x),\\
&\vdots\\
u_{m-1}(x) &= y^{(m-2)}(x),\\
u_m(x) &= y^{(m-1)}(x).
\end{aligned}$$

Differentiating the above equations once yields

$$\begin{aligned}
u_1'(x) &= u_2(x) = y'(x),\\
u_2'(x) &= u_3(x) = y''(x),\\
&\vdots\\
u_{m-1}'(x) &= u_m(x) = y^{(m-1)}(x),\\
u_m'(x) &= y^{(m)}(x) = f(x, u_1, u_2, \ldots, u_m),
\end{aligned}$$

which can be written compactly as

$$\mathbf{u}' = \Phi(x, \mathbf{u})$$

where

$$\Phi(x,\mathbf{u}) = \begin{bmatrix} u_2(x) \\ u_3(x) \\ u_4(x) \\ \vdots \\ u_m(x) \\ f(x, u_1, u_2, \ldots, u_m) \end{bmatrix}, \ \mathbf{u} = \begin{bmatrix} u_1(x) \\ u_2(x) \\ u_3(x) \\ \vdots \\ u_{m-1}(x) \\ u_m(x) \end{bmatrix} \text{ and } \ \mathbf{u}' = \begin{bmatrix} u_1'(x) \\ u_2'(x) \\ u_3'(x) \\ \vdots \\ u_{m-1}'(x) \\ u_m'(x) \end{bmatrix}.$$

Hence, the IVP (1.5) can be represented in the form

$$\mathbf{u}' = \Phi(x,\mathbf{u}),\ \mathbf{u}(a) = \boldsymbol{\omega} \tag{1.6}$$

where $\boldsymbol{\omega} = [\omega_0, \omega_1, ..., \omega_{m-1}]^T$. A suitable numerical method for first order ODE is then used to solve the resultant system (1.6) (Lambert, 1991). However, this approach increases the number of equations to be solved, which may cause computational burden, besides long and complicated computer programmes which may jeopardise the accuracy of the solution.

1.4 Direct Method

Direct methods, on the other hand, a solve higher order ODE without any reduction to a system of first order ODEs. Thus, the number of equations remains the same. This approach was adopted by Jator (2008b), Badmus (2014) and Kuboye and Omar (2015a) to develop numerical methods for solving higher order ODEs.

This study deals not only with IVPs but also BVPs of higher order ODEs. In order to transform BVPs to the equivalent IVPs, the shooting method is employed. For this reason, a brief description on linear and nonlinear shooting methods are presented in the following section.

1.5 Shooting Method

According to Zhang (2012) the shooting method was first proposed by Morrison, Riley, and Zancanaro (1962), and later popularised by Keller (1976). There are two types of shooting methods i.e. linear and nonlinear shooting methods.

1.5.1 Linear Shooting Method

Differential equations where conditions are imposed at different points as mentioned in Section (1.1) can be categorised as BVPs. Initially, we approximate a second order ODE with linear BVP as below

$$f(x,y,y') - q_1(x)y' - q_2(x)y = r(x),\ x \in [a,b],\ y(a) = \alpha,\ y(b) = \beta \tag{1.7}$$

where $f(x,y,y') = y''$. To approximate the unique solution for (1.7), we shall convert it into two IVPs. The first is a non homogeneous IVP of the form

$$f(x,y,y') - q_1(x)y' - q_2(x)y = r(x),\ x \in [a,b],\ y(a) = \alpha,\ y'(a) = 0 \tag{1.8}$$

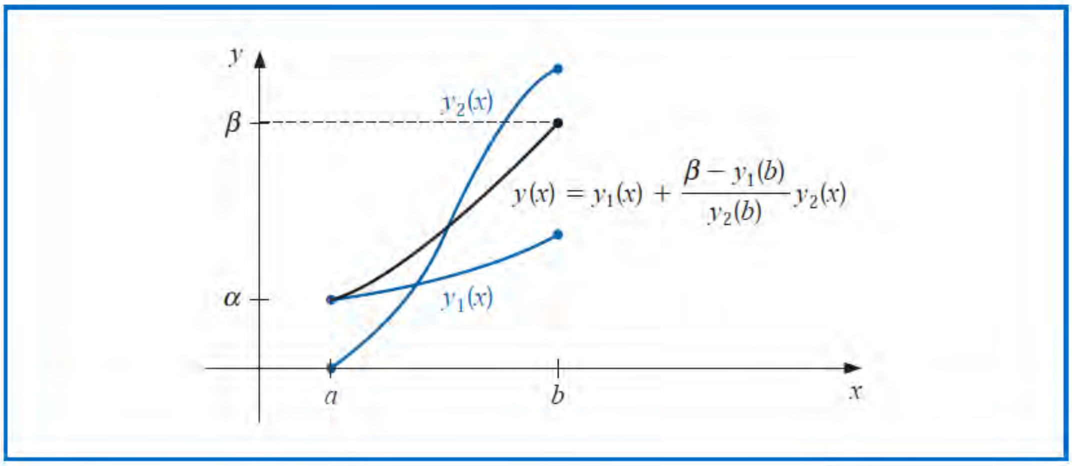
and the other one is a homogeneous IVP of the form

$$f(x,y,y') - q_1(x)y' - q_2(x)y = 0,\ x \in [a,b],\ y(a) = 0,\ y'(a) = 1. \tag{1.9}$$

Both (1.8) and (1.9) have unique solutions, namely $y_1(x)$ and $y_2(x)$ respectively, which can be combined to produce the solution of (1.7) given by

$$y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y_2(x),\ y_2(b) \neq 0 \tag{1.10}$$

as illustrated in Figure 1.1.



Source:(Faires & Burden, 2002)

Figure 1.1. Linear Shooting Method for Second Order BVPs of ODEs

This section discusses the linear case for second order BVPs of ODE only. For the nonlinear second order as well as linear and nonlinear third and fourth order BVPs of ODEs, the nonlinear shooting method that will be described in the following section can be employed.

1.5.2 Nonlinear Shooting Method

Solving nonlinear BVPs is much harder than linear BVPs since we cannot express the solution as linear combination of the solutions of two equivalent IVPs. In this study, we consider the general m^{th} order ODE as stated in (1.3) defined on $x \in [a,b]$ subject to the imposed boundary conditions. Without loss of generality we assume that there is one missing initial condition occurs in the equivalent IVP corresponding to the boundary condition $y(b) = \beta$ in BVP. The procedure of the shooting method can be described as below

Step 1: Choose the parameter $t = \{t_k,\ k = 0,1,2,...\} \in \mathbb{R}$ that satisfies the following condition

$$\lim_{k\to\infty} y(b,t_k) = y(b) = \beta$$

where $y(x,t_k)$ is solution of the IVP and $y(x)$ is solution for the BVP and the initial guess t_0 is given.

Step 2: Solve numerically the obtained IVP and its derivative with respect to t, i.e

$$\begin{aligned} y^{(m)}(x,t) &= f(x,y,y',...,y^{(m-1)}) \\ v^{(m)}(x,t) &= \frac{\partial y^{(m)}(x,t)}{\partial t}. \end{aligned}$$

Step 3: If $\mid F(t_k) \mid = \mid y(b,t_k) - \beta \mid < TOL$ for some small value $TOL > 0$ then the solution is obtained. Otherwise, calculate and update t_{k+1} using the following three-step iterative approach introduced by Yun (2008):

$$t_{k+1} = T_k + U_k + B_k$$

where

$$T_k = t_k - \frac{F(t_k)}{F'(t_k)}, \quad U_k = -\frac{F(T_k)}{F'(t_k)}, \quad B_k = -\frac{F(T_k + U_k)}{F'(t_k)}$$

and go back to Step 2.

For any other missing condition cases, a similar procedure with minor modifications will be used. The shooting method for the nonlinear case will be discussed in details in Chapter Three until Chapter Five.

1.6 Problem Statement

Conventionally, higher order ODEs are solved by reducing them to their equivalent systems of first-order ODEs and then suitable numerical methods are employed to solve the resultant systems (Lambert, 1991). Unfortunately, the major setback for this approach is computational burden besides long and complicated computer codes which may affect the accuracy of the method (Awoyemi, 2003). This approach also does not fully utilise data associated to some ODEs, like oscillatory nature of the solution (Vigo-Aguiar & Ramos, 2006). Direct approach, on the other hand, is an alternative to the rigour reduction approach, where higher order ODEs can be solved without converting them to a system of first order ODEs. Many researchers opted this approach either in predictor-corrector scheme like Kayode (2011) or block scheme such as Omar (2004), Jator (2008b), Badmus (2014) and Kuboye and Omar (2015a). The former scheme, however, requires extra work in computing the prediction values to be used as initial approximations in the corrector step. Not only this scheme consumes more functions evaluations per step, the development of subroutine to supply the needed starting values for predictor may lead to insufficiency in terms of error (Anake, Awoyemi, & Adesanya, 2012a). On the other hand, in a block method, there is no need to develop any predictors. Moreover, the block method can be applied iteratively at the same time, thus reducing the usage of computer memory, computing time and human effort which then contribute to high accuracy of the developed method (Anake, Awoyemi, & Adesanya, 2012b). However, block method fails to satisfy Dahlquist barrier condition which stipulates the order of a k-step linear multistep block method cannot exceed $k+1$ (k is odd) or $k+2$ (k is even) for the method to be zero-stable (Awoyemi, Adebile, Adesanya, & Anake, 2011).

In order to tackle this weakness, Anake et al. (2012b) proposed hybrid block methods (HBMs) involving one-step with specific two off-step points. In the following year, Adeyeye (2013) derived two-step HBM including specific two off-step points for solving second order ODEs. But the accuracy of the proposed methods regarding error can be improved further. Two years later, Omar and Abdelrahim (2015) developed one-step HBM with generalised three off-step using interpolation and collocation for solving second order IVPs. Similarly, Abdelrahim and Omar (2015) derived one-step HBM with generalised two off-step using interpolation and collocation for solving third order IVPs. The following year, Omar and Abdelrahim (2016) proposed one-step HBM including three off-step points for the solution of general fourth order IVPs. In the same year, Abdelrahim (2016) has successfully derived one-step HBMs for solving higher order ODEs directly. However, all these methods were confined to IVPs only.

The introduction of high derivative approach is rarely seen in literature of block methods. Jator and Li (2012) introduced block method for solving second order IVP and BVP directly based on a third derivative method. In addition, Jator, Akinfenwa, Okunuga, and Sofoluwe (2013) derived a continuous third derivative formulas which were then combined to form the block method for solving second order ODEs. In the same year, Sahi et al. (2013) developed a fourth derivative method with continuous coefficients to obtain primary and additional methods that were used to solve third order boundary value problems. Likewise, Adeyeye and Omar (2016) suggested a HBM of order eight with specific off-step points in the presence of third derivative for solving second order IVPs of ODEs. All these methods, however, only considered specific off-step points.

Based on the setbacks of the previous works and also taking the full advantage of the existence of higher derivative, this study attempts to extend and enhance the methods proposed by Abdelrahim (2016) by proposing new methods for solving both IVPs and BVPs of higher order ODEs directly using collocation and interpolation approach in the presence of higher derivative.

1.7 Research Objectives

The aim of this research is to develop new HBMs with generalised off-step point(s) for the direct solution of IVPs and BVPs of higher order ODEs in presence of higher derivative. In order to achieve this main objective, the following sub-objectives need to be accomplished:

1. To derive a continuous implicit hybrid one-step schemes with generalised one, two and three off-step points through interpolating and collocating.
2. To develop new one-step HBMs with generalised one, two and three off-step points using continuous implicit hybrid one-step schemes for solving second, third and fourth order ODEs directly.
3. To establish the basic properties of the new HBMs such as order, error constant, zero-stability, consistency, convergence and region of absolute stability.
4. To compare the performance of the newly developed methods with the existing methods in terms of numerical accuracy.

1.8 Significance of the Study

This research study is aimed at developing one-step HBMs for solving both initial and boundary value problems of higher order ODEs. In order to achieve this objective, new continuous implicit schemes with generalised off-step (hybrid) points are proposed. These schemes are employed to generate a family of one-step HBMs with generalised off-step points for the direct solution of second, third and fourth order ODEs in the presence of higher derivative. This approach has not been attempted by previous researchers. Subsequently, the proposed methods perform better than the existing methods when solving the same problems of second, third and fourth order IVPs and BVPs of ODEs. Thus, this study contributes to the body of knowledge in numerical analysis.

1.9 Limitation of the Study

The generalisation of the proposed method to any m^{th} order ODEs is almost impossible because the points of interpolation and collocation depend on the order of the differential equation. As the order of the differential equation increases, the points of interpolation and collocation also increase. Therefore, our scope of study in this research is limited to second, third and fourth order ODEs and including until three off-step points for the purpose of comparing with the existing methods.

1.10 Guide to the Thesis

Chapter One of this thesis presents two fundamental approaches for solving higher order ODEs namely reduction and direct methods. A brief description on shooting method to transform higher order BVPs into equivalent IVPs beside problem statement, objectives, significant of study and limitation of study are also included in this chapter.

In Chapter Two, some explanation on linear multistep method and its basic properties are provided. This chapter also covers a brief introduction on higher derivative, block and hybrid block methods. In addition, an extensive literature related to this study is carried out at the end of Chapter Two in order to identify the research gaps.

Chapter Three is devoted to the development of the numerical methods for solving both IVPs and BVPs of second order ODEs directly. Three new methods are proposed namely one-step HBMs with generalised one, two and three off-step point(s) in the presence of third derivative.

The development of hybrid block methods in Chapter Three are extended for solving third and fourth order ODEs directly as discussed in Chapters Four and Five, respectively. In Chapter Four, two new HBMs are developed i.e. one-step HBMs with two and three-off step points in the presence of fourth derivative. Meanwhile, one-step HBM with three off-step points in the presence of fifth derivative is proposed in Chapter Five. In those three chapters (Chapter Three until Chapter Five), not only the properties of the developed methods are established, the numerical results are also presented and compared with the relevant existing methods.

Chapter Six concludes our study by summarising the main findings followed by suggestions for future research.

CHAPTER TWO
BASIC CONCEPTS AND LITERATURE REVIEW

2.1 Introduction

Mathematical problems emerged from the fields of engineering and science have inspired scholars to seek approximate solutions for ODE problems which do not have analytical solution. This situation has motivated them to develop new numerical methods for finding more accurate solutions. In this chapter, some basic concepts of the research area are chosen. In addition, literature reviews closely related to this study are presented and discussed.

2.2 Single Step Method

In a single step method, the data obtained from the previous point x_n is used to approximate the numerical solution at x_{n+1}. The approximate value y_{n+1} to the solution $y(x_{n+1})$ at the grid point x_{n+1} is made if only the values of x_n, y_n and the step-size are known. Euler method, Trapezoidal method, Runge-Kutta method and Taylor method are some examples of single step methods (Atkinson & Han, 2004).

2.3 Linear Multistep Method

In a linear multistep method (LMM), the information from several previous points are needed to approximate the solution at the current point. For example, in a k-step method, the previous values of $y_{n+j},\ j = 0(1)k-1$, are used to calculate the current value y_{n+k} (Omar, 1999).

Definition 2.3.1. (Lambert, 1973), A k-step LMM corresponding to the following IVP of first order ODE

$$y' = f(x,y),\ \ y(a) = \eta \tag{2.1}$$

can be defined as

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j}. \tag{2.2}$$

where k is the number of steps, h is the step size and m is the order of ODE.

However, if Equation (2.1) is extended to the IVP of m^{th} order ODE given by (1.5), then the general form of k-step LMM becomes

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h^m \sum_{j=0}^{k} \beta_j f_{n+j}, \tag{2.3}$$

where

$$f_{n+j} \approx f(x_{n+j}, y_{n+j}, y'_{n+j}, y''_{n+j}, \ldots, y^{(m-1)}_{n+j}),$$

$$y_{n+j} \approx y(x_{n+j}), j = 0(1)k$$

and the values of α_j's, β_j's are real constants, α_0, β_0 and are not both zero, and $\alpha_k = 1$. Furthermore, Lambert (1991) defined the characteristic polynomial of Equation (2.3) as

$$\rho(\Psi) = h^m \sigma(\Psi)$$

where

$$\rho(\Psi) = \sum_{j=0}^{k} \alpha_j \Psi^j, \text{ and}$$

$$\sigma(\Psi) = \sum_{j=0}^{k} \beta_j \Psi^j.$$

The functions ρ and σ are known as the first and second characteristic polynomials, respectively.

Definition 2.3.2. (Atkinson, 1989): LMM (2.3) is said to be implicit if $\beta_k \neq 0$, that is, the approximate solution at x_{n+k} which is y_{n+k} appears on both sides of Equation (2.3). On the other hand, (2.3) is said to be explicit if $\beta_k = 0$, that is, the approximate value of y_{n+k} can directly be determined in terms of y_{n+j}, f_{n+j} for $j = 0, 1, 2, ..., k-1$.

In order to employ a LMM and use it as a tool for approximating the numerical solution, it must possess some basic properties which will be discussed in the following section.

2.4 Properties of Linear Multistep Method

This section discusses some basic properties of LMM such as order and error constant, consistency, zero-stability, convergence and region of absolute stability.

2.4.1 Order and Error Constant

The first property associated with LMM is the order and error constant of the method. However, before defining them, the following two definitions are needed.

Definition 2.4.1. (Lambert, 1991) The associated linear difference operator ∇ with (2.3) is defined as

$$\nabla[y(x_n), h] = \sum_{j=0}^{k} \alpha_j y_{n+j} - h^m \sum_{j=0}^{k} \beta_j f_{n+j} \tag{2.4}$$

and expanding the functions y_{n+j} and f_{n+j} in Taylor series about x_n and grouping similar terms yields

$$\nabla[y(x_n), h] = D_0 y(x_n) + D_1 h y^{'}(x_n) + D_2 h^2 y^{''}(x_n) + \cdots + D_l h^l y^{(l)}(x_n) + \cdots \tag{2.5}$$

where the $D_i{}'s$ are constants.

Definition 2.4.2. (Lambert, 1991) The method (2.3) and its associated linear difference operator (2.5) are said to be of order P, if $D_0 = D_1 = D_2 = \cdots = D_{P+m-1} = 0$ and $D_{P+m} \neq 0$ with vector of error constants D_{P+m}.

The second property that is also important for the LMM is zero-stability which shows that as the value of the step size h is getting smaller the numerical solution is approaching the exact solution (Lambert, 1973).

2.4.2 Zero-Stability

The method that satisfies the following definition is said to be zero stable :

Definition 2.4.3. (Fatunla, 1991): A linear multistep method is zero stable, provided the roots $\Psi_j, j = 1,...,k$ of its first characteristic polynomial $\rho(\Psi)$ satisfy $|\Psi_j| \leqq 1, j = 1,...,k$ and for those roots with $|\Psi_j| = 1$, the multiplicity does not exceed the order of its corresponding ODEs

$$\rho(\Psi) = det\left(\Psi_j A^{(0)} - A^{(1)}\right), \quad j = 1,2,\ldots,k. \tag{2.6}$$

where $A^{(0)}$ is a kxk identity matrix and

$$A^{(1)}_{(kxk)} = \begin{pmatrix} 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}.$$

.

The third property that is needed for LMM is the consistency of the method which guarantees the uniqueness of the solution.

2.4.3 Consistency

A LMM is said to be consistent if it satisfies the following definition:

Definition 2.4.4. (Lambert, 1973): A LMM is *consistent* if it has order $P \geq 1$.

The previous two properties mentioned in Sections 2.4.2 - 2.4.3 leads to the following property, i.e. the approximate solution y_n converges to the theoretical solution $y(x_n)$

as the step size h gets smaller, that is

$$\lim_{h \to 0} y_n = y(x_n). \tag{2.7}$$

2.4.4 Convergence

The conditions needed for the LMM to be convergent are stated in the following theorem:

Theorem 2.1. *(Henrici, 1962): Consistency and zero-stability are the necessary and sufficient conditions for a LMM to be convergent.*

The stability property associated with the choice of the step-size h can be illustrated by plotting the region of absolute stability. This region gives the selection of the step-size needed for the convergence of the approximate solution. The following section will be used as a basis in plotting the region of stability for the developed methods in this study.

2.4.5 Region of Absolute Stability

According to Ngwane and Jator (2012), the form of block method corresponding to IVP of first order ODE in (2.1) with the existence of second derivative is given by

$$A^{(0)}Y_i = A^{(1)}Y_{i-1} + h[B^{(0)}F_i + B^{(1)}F_{i-1}] + h^2[C^{(0)}G_i + C^{(1)}G_{i-1}] \tag{2.8}$$

where

$$Y_i = [y_{n+\frac{1}{2}}, y_{n+1}]^T,\ Y_{i-1} = [y_{n-\frac{1}{2}}, y_n]^T,$$

$$F_i = [f_{n+\frac{1}{2}}, f_{n+1}]^T,\ F_{i-1} = [f_{n-\frac{1}{2}}, f_n]^T,$$

$$G_i = [g_{n+\frac{1}{2}}, g_{n+1}]^T,\ G_{i-1} = [g_{n-\frac{1}{2}}, g_n]^T.$$

where $g_i = \frac{df_i}{dx}$ and

$$A^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ A^{(1)} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},\ B^{(0)} = \begin{pmatrix} \frac{4}{15} & \frac{11}{480} \\ \frac{8}{15} & \frac{7}{30} \end{pmatrix},$$

$$B^{(1)} = \begin{pmatrix} 0 & \frac{101}{480} \\ 0 & \frac{7}{30} \end{pmatrix},\ C^{(0)} = \begin{pmatrix} \frac{-1}{24} & \frac{-1}{320} \\ 0 & \frac{-1}{60} \end{pmatrix},\ C^{(1)} = \begin{pmatrix} 0 & \frac{13}{960} \\ 0 & \frac{1}{60} \end{pmatrix}.$$

Now, the test problem of the form $y' = \lambda y$ and $y'' = \lambda^2 y$ are substituted into Equation (2.8) to produce

$$Y_i = M(q)Y_{i-1},\ q = \lambda h$$

where

$$M(q) \ = \ (A^{(0)} - qB^{(0)} - q^2C^{(0)})^{-1}(A^{(1)} + qB^{(1)} + q^2C^{(1)}).$$

The eigenvalues of the matrix $M(q)$ are $\{0, \eta\}$, where the eigenvalue η is a function of q. The region of stability is then obtained by sketching the function $|\eta| < 1$.

The following two sections introduce block method and hybrid block method.

2.5 Block Method

Milne (1953) was the first person who proposed block method (Olabode, 2007). This method can be described as a set of LMM simultaneously combined as a block and then applied to solve IVPs to produce better approximation. In this method, a set of new values obtained by each application of the formulas referred as block. For instance, in a k-step block method, the values $y_{n+1}, y_{n+2}, ..., y_{n+k}$ are computed simultaneously at each iteration. There are two types of block methods, namely one-step and multistep block methods. In one-step block methods, the values of the new block are derived from the previous point x_n. On the other hand, in multistep block methods the previous blocks are used to compute the solution of the next block (Omar &

Suleiman, 1999).

Definition 2.5.1. (Chu & Hamilton, 1987): Let Y_q, Y_{q-j} and F_{q-j} be defined by $Y_q = (y_n, y_{n+1}, \ldots, y_{n+r-1})^T$, $Y_{q-j} = (y_{n-j}, y_{n+1-j}, \ldots, y_{n+r-1-j})^T$ and $F_{q-j} = (f_{n-j}, f_{n+1-j}, \ldots, f_{n+r-1-j})^T$. Then, a general k-block of r-point block method can be represented as a matrix of finite difference equation of the form

$$Y_q = \sum_{j=1}^{k} A_j Y_{q-j} + h \sum_{j=0}^{k} B_j F_{q-j}$$

where $n = qr$ is the first step number of the q^{th} block and r is the proposed block size. A_j's and B_j's are properly chosen $r \times r$ matrix coefficients and $q = 0, 1, 2, ...$ represents number block .

According to Lambert (1973), block methods can be derived in three different ways namely; Taylor series expansion, numerical integration and interpolation. The Taylor series approach as adopted in Lambert was used for deriving LMM for first order ODEs (Omar & Adeyeye, 2016). In light of this, authors usually tend to adopt the other two approaches to derive block methods. However, in a recent work by Omar and Kuboye (2015) it was stated that the integration approach is tedious and complicated in terms of derivation and the resulting methods have low performance in term of numerical accuracy, hence the justification for adopting interpolation approach. This interpolation approach is widely used by researchers such as Kayode and Adeyeye (2011), Badmus (2014) and Kuboye and Omar (2015b) due to its simplicity in developing the block method and also flexibility when writing its computer programs.

2.6 Hybrid Block Method

According to Lambert (1973), hybrid methods were initiated by Gragg and Stetter (1964), Butcher (1965) and Gear (1965) which involved the evaluation of functions at the off-step (non-step) points. These methods are capable of overcoming the zero-stability barrier which implies that the highest order of zero-stability of block methods when the stepnumber k is odd, $k+1$ and $k+2$ when k is even. The main benefit of hybrid methods is that they share the same property with Runge-Kutta methods that is utilising data at the off-step points and have the ability of changing step size. Hybrid methods incorporate off-step points which is helpful in reducing the stepnumber of the method while preserving the zero-stability at the same time.

Definition 2.6.1. (Lambert & Watson, 1976): The general form of k-step hybrid method can be represented as

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} + h\beta_r f_{n+r} \tag{2.9}$$

where $\alpha_k = 1$, α_0 and β_0 are both not zero, $r \notin \{0,1,...,k\}$, and $f_{n+r} = f(x_{n+r}, y_{n+r})$.

The following section presents the advantages of high derivative on LMM methods.

2.7 Introduction of Higher Derivative in Linear Multistep Method

To increase the accuracy of the numerical method further, few authors have introduced higher derivative in deriving their methods. Those authors introduced higher derivative in the derivation process for two reasons. Firstly, the more derivatives included, i.e adding higher derivatives will provide more analytical information into the numerical method (Lambert, 1973). Secondly, to overcome the stability constraint occurred while solving stiff first order ODE, Enright (1974) introduced second derivative for solving first order IVP in deriving his method as stated below

$$y_{n+1} = \sum_{j=1}^{k} \alpha_j y_{n+1-j} + h \sum_{j=0}^{k} \beta_j y'_{n+1-j} + h^2 \sum_{j=0}^{k} \gamma_j y''_{n+1-j}$$

where $\alpha_j, \beta_j, \gamma_j$ are constants. Since then, several researchers followed his step and further extended his work for solving high order ODEs as well. For example, Ngwane and Jator (2012) proposed a continuous HBM with the existence of second derivative for solving first order ODE at specific points $\{y_{n+\frac{1}{2}}, y_{n+1}\}$ given by

$$y_{n+1} = y_n + h(\beta_0 f_n + \beta_1 f_{n+1} + \beta_v f_{n+v}) + h^2(\gamma_0 g_n + \gamma_1 g_{n+1} + \gamma_v g_{n+v})$$
$$y_{n+v} = y_n + h(\hat{\beta}_0 f_n + \hat{\beta}_1 f_{n+1} + \hat{\beta}_v f_{n+v}) + h^2(\hat{\gamma}_0 g_n + \hat{\gamma}_1 g_{n+1} + \hat{\gamma}_v g_{n+v})$$

where $\beta_0, \beta_1, \beta_v, \gamma_0, \gamma_1, \gamma_v, \hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_v, \hat{\gamma}_0, \hat{\gamma}_1$ and $\hat{\gamma}_v$ are constants, y_{n+v} is the numerical approximation of $y(x_{n+v})$, $f_{n+iv} \approx f(x_{n+iv})$ and $g_{n+iv} \approx \frac{df(x,y)}{dx}\Big|^{x_{n+iv}}_{y_{n+iv}}$ for $v \in (0,1)$. It was shown that the method performed better than the existing methods available during that time in terms of accuracy.

Following the same approach, Jator et al. (2013) developed a block method with third derivative for solving IVPs of second order ODEs. Not only the method was able to handle large step-sizes, it still maintained high accuracy. In the same year, Sahi et al. (2013) derived a high-order block method using fourth derivative for solving BVP of third order ODEs as below

$$y_{n+k} = \sum_{j=0}^{k-1} \alpha_j y_{n+j} + h^3 \sum_{j=0}^{k} \beta_j y'''_{n+j} + h^4 \sum_{j=0}^{k} \gamma_j y^{(iv)}_{n+j}.$$

The method was not only of high order but also produced numerical results of high accuracy.

2.8 Literature Review

In this section, recent works related to HBMs for solving second, third and fourth order ODEs are presented.

2.8.1 Hybrid Block Methods for Solving Second Order ODEs

Kayode (2011) was one of the authors who introduced HBMs for the direct solution of second order ODEs. In his work, a zero-stable one-step HBM was proposed through collocation and interpolation approach with one off-step point for solving second order IVPs directly. In the same year, Kayode and Adeyeye (2011) developed three-step HBM with two off-step points for the direct solution of second order IVPs using collocation and interpolation approach. In spite of solving second order ODEs directly, the previous two methods only used specific off-step points in deriving the methods.

In 2012, Adesanya, Odekunle, and Adeyeye derived a continuous two step predictor-corrector method two-step with specific two off-step points. Although this method is capable of solving second order IVPs directly, it still suffers from computation burden due to the associated predictor-corrector mode. Using collocation and interpolation approach, Anake et al. (2012a) derived implicit one-step HBM with two off-step point for the direct solution of general second order ODEs. However, the proposed method only solves IVPs directly.

Adeyeye (2013) suggested a two-step HBM with two off-step points for general second order ODEs. The developed method in predictor-corrector scheme has the advantage of high accuracy in terms of error. However, not only this method is limited to solving IVPs, it also causes computations burden due to the use of the predictor-corrector mode. Subsequently, an attempt to solve general second order IVPs of ODEs has been made by Adesanya, Fasasi, and Anake (2013). In their work, a three-step HBM with three-off-step points was proposed using collocation and interpolation approach. Although the derived method has the ability of solving second order ODEs directly, it only manages to approximate the solution using specific off-step points.

Similarly, using collocation and interpolation approach, Sagir (2013a) proposed two-step modified Numerov block methods with one off-step point for solving second order ODEs. The developed method of order four is capable of solving both IVPs and BVPs of second order ODEs. Unfortunately, the method suffers from low accuracy in terms of error. Following the previous authors, Sagir (2013b) developed a three-step HBM of order five with one-off-step point for solving stiff ODEs. Although this method is capable of solving both IVPs and BVPs of second order ODEs, it is only restricted to a special type of ODEs of the form $y'' = f(x,y)$.

Likewise, James, Adesanya, and Joshua (2013) suggested continuous two-step block method with four off-step points for the solution of second order IVP of ODEs. The collocation approach using two-step and four off-step points produced a block method that can solve second order IVPs directly which overcomes the setback in reduction approach. Still, the method is limited to solving IVPs.

Additionally, Kolawole, Adesanya, Momoh, and Emmanuel (2014) developed a continuous one-step Störmer-Cowell method with two off-step points for the solution of second order ODEs using interpolation and collocation approach. This method is capable of approximating solutions at different grid points without overlapping as in the predictor-corrector scheme. Nevertheless, as seen in most hybrid methods, this method only considered specific off-step points. In the same year, Badmus (2014) proposed an efficient four-step HBM with specific three off-step points for the direct solution of second order ODEs through interpolation and collocation approach. In spite of using seven-point at once to produce simultaneous approximation of the solution in a block directly, the method did not take into account generalised off-step points.

In the following year, Adeniran and Ogundare (2015) developed a one-step HBM with two off-step points for solving general second order IVPs. The method was designed for solving general second order IVPs only through collocation approach. On the other hand, Ramos, Kalogiratou, Monovasilis, and Simos (2015) proposed two-step HBM for solving general second order IVPs where specific two off-step points are collaborated through interpolation and collocation approach. The methods were implemented without the need for starting values or predictors and hence avoided extra computation. Nevertheless, the methods are limited to second order IVPs.

In the same approach, Adeyeye and Omar (2016) investigated a HBM of order eight with the introduction of third derivative for solving second ODEs. In this article, they employed a self starting modified Taylor series method an algorithm for deriving the hybrid block. Still, the method is limited to solving IVPs only.

2.8.2 Hybrid Block Methods for Solving Third Order ODEs

A number of researchers have attempted to develop HBMs to approximate the solution of third order ODE. Adesanya, Udoh, and Ajileye (2013) proposed a new two-step HBM with two off-step points for the solution of general third order IVPs of ODEs. The developed method through collocation approach was derived with constant step-size and implemented in a block form. Despite its ability to solve third order IVPs directly, the method did not generalise the off-step points. Additionally, Kolawole, Fasansi, Remilekun, and Adesanya (2013) derived one-step HBM with specific three off-step points for the solution of $y''' = f(x,y,y',y'')$ through predictor-corrector approach. Even though this method has good convergence and stability properties which make it a suitable candidate of solving linear IVPs of third order ODEs directly, it still suffers from the computational burden due to extra function evaluations needed in predictor-corrector mode.

Subsequently, Adesanya, Abdulqadri, and Ibrahimb (2014) proposed one-step HBM with five off-step points for the solution of third order IVPs of ODEs using collocation approach. Although this method has the ability to solve third order IVP directly, unfortunately, it only caters for specific five off-step points. Mohammed and Adeniyi (2014) continued the effort of solving third order ODEs directly by introducing a three-step implicit HBM. Nevertheless, this method still focuses on using specific one off-step points.

By using five off-step points through collocation scheme,Yap, Ismail, and Senu (2014) proposed an accurate three-step HBM with two off-step points for solving IVPs of third order ODEs. They claimed that their method required less number of total steps compared to the previous methods. However, the method did not consider generalised off-step points in the derivation.

On employing two generalised off-step points, Abdelrahim and Omar (2015) derived a one-step HBM for solving linear third order IVPs through interpolation and collocation approach. In recent year, Hijazi and Abdelrahim (2017) considered a three-step HBM with two off-step points at specific points namely $\{\frac{1}{3}, \frac{3}{5}\}$ for solving third order IVPs through interpolation and collocation approach. Again, these methods are limited to solving linear third order IVPs.

An attempt to solve BVP of third order ODEs was made by Adeyeye and Omar (2017). They introduced linear block method for solving linear third order BVPs using Taylor series expansions approach. However, this method is capable of solving linear third order BVPs only.

2.8.3 Hybrid Block Methods for Solving Fourth Order ODEs

From literature, it was found that effort to solve ODEs using HBM started in 2014. Ademiluyi and Bolarinwa (2014) proposed a modified one-step block method with three off-step points for the direct solution of IVPs of fourth order ODEs using predictor-corrector approach. As explained previously, this approach causes computational burden. In the same year, Kayode, Duromola, and Bolaji (2014) attempted to solve IVPs of fourth order ODEs using modified one-step implicit HBM with four specific off-step points. In deriving this method, interpolation and collocation approach was also employed. But again, they did not attempt to use generalised four off-step points.

In the following year, Yap and Ismail (2015) developed a four-step HBM with three off-step points with application to solve IVPs of fourth order ODEs. This method was also derived using collocation approach and implemented in self-starting mode in order to overcome the computational burden. Although the performance of this method is very promising, nonetheless, it is still limited to specific off-step points and only applicable to IVPs of ODEs.

One year later, Omar and Abdelrahim (2016) proposed one-step HBM using specific three off-step points i.e $\frac{1}{4}, \frac{1}{3}, \frac{2}{3}$ for the solution of the general $y^{(iv)} = f(x, y, y', y'', y''')$ ODEs. In the derivation process, interpolation and collocation approach were also employed. The method, however, was catered for numerical solution of IVPs only.

To generalise the off-step points in deriving HBM, recently, Omar and Abdelrahim (2017) proposed four-step HBM including three generalised off-step points $\{s_1, s_2$ and $s_3\}$ for the solution of the general fourth order ODEs. In deriving the method, they employed interpolation and collocation approach. Unfortunately, this method is also limited to IVPs.

2.9 Summary

This chapter presents some basic concepts needed in this study. An extensive review on HBMs for solving high order ODEs has been carried out to identify the research gaps. The literature shows that most of the previous methods are confined to solve IVPs of ODEs only. It also found that most of the developed methods focus on specific off-step points with the exception of few methods. In addition, the literature also reveals that the presence of higher derivative was rarely considered while deriving numerical methods. Thus, based on the advantages and drawbacks of the existing methods, this study aims to propose new direct methods for solving both IVPs and BVPs of high order ODEs.

CHAPTER THREE

ONE-STEP HYBRID BLOCK METHODS FOR DIRECTLY SOLVING SECOND ORDER ODES IN THE PRESENCE OF THIRD DERIVATIVE

3.1 Introduction

In this chapter, the derivations of one-step HBMs with generalised one, two and three off-step point(s) using interpolation and collocation approach for solving both IVPs and BVPs of second order ODEs in the presence of third derivative are presented.

3.2 Derivation of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

A general second order IVP of ODE is given by

$$y'' = f(x,y,y'), \quad y^{(i)}(a) = \omega_i, \quad i = 0,1. \tag{3.1}$$

The following power series function

$$y(x) = \sum_{j=0}^{2v+u-1} a_j \left(\frac{x-x_n}{h}\right)^j \tag{3.2}$$

is employed as an approximate solution to (3.1) where u and v denote the number of interpolation and collocation points respectively, $n = 0,1,2,...,N$, and $h = x_{n+1} - x_n$ is the constant step size for the partition π_N of the interval $[a,b]$ which is given by $\pi_N = [a = x_0 < x_1 < ... < x_{N-1} < x_N = b]$. In deriving this method, Equation (3.2) is differentiated twice and thrice to produce

$$y''(x) = f(x,y,y') = \sum_{j=2}^{2v+u-1} a_j \frac{j!}{h^2(j-2)!} \left(\frac{x-x_n}{h}\right)^{j-2} \tag{3.3}$$

and

$$y'''(x) = g(x,y,y',y'') = \sum_{j=3}^{2v+u-1} a_j \frac{j!}{h^3(j-3)!} \left(\frac{x-x_n}{h}\right)^{j-3} \tag{3.4}$$

respectively. Then, Equation (3.2) is interpolated at x_n and x_{n+r} where $0 < r < 1$, while Equations (3.3) and (3.4) are collocated at all points, i.e x_n, x_{n+r} and x_{n+1}, in the selected interval as shown in Figure 3.1 below

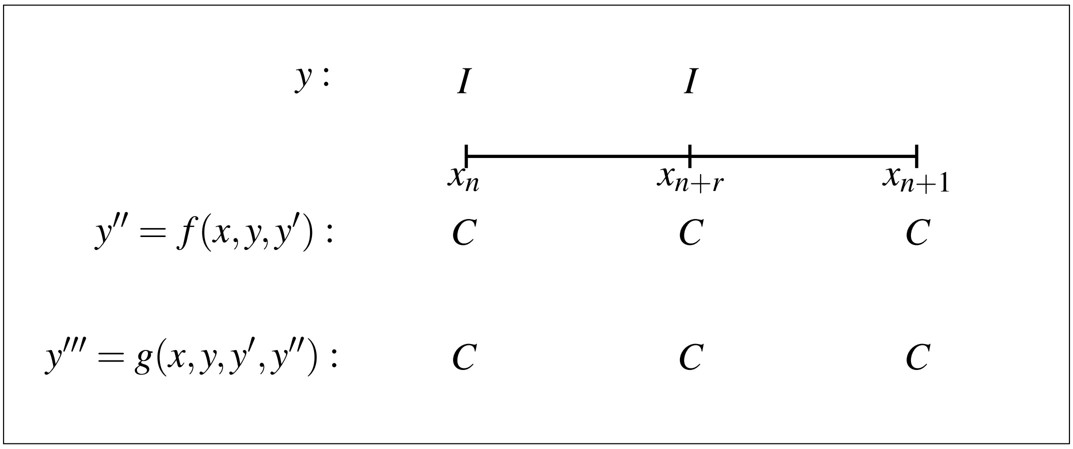


Figure 3.1. Interpolation and Collocation Strategy for One-Step HBM with One Off-Step Point for Solving Second Order ODEs

where I and C represent interpolation and collocation points respectively. Substituting $u = 2$ and $v = 3$ in Equations (3.2)-(3.4) gives

$$y_n = a_0,$$

$$y_{n+r} = a_0 + ra_1 + r^2a_2 + r^3a_3 + r^4a_4 + r^5a_5 + r^6a_6 + r^7a_7,$$

$$f_n = \frac{2}{h^2}a_2,$$

$$f_{n+r} = \frac{2}{h^2}a_2 + \frac{6r}{h^2}a_3 + \frac{12r^2}{h^2}a_4 + \frac{20r^3}{h^2}a_5 + \frac{30r^4}{h^2}a_6 + \frac{42r^5}{h^2}a_7,$$

$$f_{n+1} = \frac{2}{h^2}a_2 + \frac{6}{h^2}a_3 + \frac{12}{h^2}a_4 + \frac{20}{h^2}a_5 + \frac{30}{h^2}a_6 + \frac{42}{h^2}a_7,$$

$$g_n = \frac{6}{h^3}a_3,$$

$$g_{n+r} = \frac{6}{h^3}a_3 + \frac{24r}{h^3}a_4 + \frac{60r^2}{h^3}a_5 + \frac{120r^3}{h^3}a_6 + \frac{210r^4}{h^3}a_7,$$

$$g_{n+1} = \frac{6}{h^3}a_3 + \frac{24}{h^3}a_4 + \frac{60}{h^3}a_5 + \frac{120}{h^3}a_6 + \frac{210}{h^3}a_7,$$

which can be written in a matrix form as below

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 \\ 0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{h^2} & \frac{6r}{h^2} & \frac{12r^2}{h^2} & \frac{20r^3}{h^2} & \frac{30r^4}{h^2} & \frac{42r^5}{h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6}{h^2} & \frac{12}{h^2} & \frac{20}{h^2} & \frac{30}{h^2} & \frac{42}{h^2} \\ 0 & 0 & 0 & \frac{6}{h^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24r}{h^3} & \frac{60r^2}{h^3} & \frac{120r^3}{h^3} & \frac{210r^4}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24}{h^3} & \frac{60}{h^3} & \frac{120}{h^3} & \frac{210}{h^3} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+r} \\ f_n \\ f_{n+r} \\ f_{n+1} \\ g_n \\ g_{n+r} \\ g_{n+1} \end{pmatrix}. \tag{3.5}$$

Solving (3.5) using Gaussian elimination method yields

$a_0 = y_n,$

$a_1 = \frac{-1}{420r(r-1)^3}(420y_{n+r} - 420y_n + 1260ry_n - 1260ry_{n+r} - 1260r^2y_n + 420r^3y_n + 1260r^2y_{n+r} - 420r^3y_{n+r} - 147f_nh^2r^2 + 469f_nh^2r^3 - 510f_nh^2r^4 + 180f_nh^2r^5 + 35f_nh^2r^6 - 33f_nh^2r^7 - 35f_{n+1}h^2r^6 + 6f_nh^2r^8 + 33f_{n+1}h^2r^7 - 6f_{n+1}h^2r^8 - 63f_{n+r}h^2r^2 + 161f_{n+r}h^2r^3 - 120f_{n+r}h^2r^4 + 30f_{n+r}h^2r^5 - 21g_nh^3r^3 + 77g_nh^3r^4 - 108g_nh^3r^5 + 72g_nh^3r^6 - 23g_nh^3r^7 + 7g_{n+1}h^3r^6 + 3g_nh^3r^8 - 10g_{n+1}h^3r^7 + 3g_{n+1}h^3r^8 + 14g_{n+r}h^3r^3 - 28g_{n+r}h^3r^4 + 18g_{n+r}h^3r^5 - 4g_{n+r}h^3r^6),$

$a_2 = \frac{f_nh^2}{2},$

$a_3 = \frac{g_nh^3}{6},$

$a_4 = \frac{h^2}{12r^2(r-1)^3}(3f_n - 3f_{n+r} - 5f_nr + 5f_{n+r}r + 5f_nr^4 - 3f_nr^5 - 5f_{n+1}r^4 + 3f_{n+1}r^5 - 4g_nhr^2 + 4g_nhr^4 - 2g_nhr^5 + g_{n+1}hr^4 - g_{n+1}hr^5 - g_rhr^2 + 2g_nhr + g_{n+r}hr),$

$a_5 = \frac{-h^2}{20r^3(r-1)^3}(2f_n - 2f_{n+r} + 2f_nr - 2f_{n+r}r - 10f_nr^2 + 10f_nr^4 - 2f_nr^5 - 10f_{n+1}r^4 - 2f_nr^6 + 2f_{n+1}r^5 + 2f_{n+1}r^6 + 10f_{n+r}r^2 + g_nhr^2 - 8g_nhr^3 + 8g_nhr^4 - g_nhr^5 + 2g_{n+1}hr^4 - g_nhr^6 - g_{n+1}hr^5 - g_{n+1}hr^6 + g_{n+r}hr^2 - 2g_{n+r}hr^3 + g_nhr + g_{n+r}hr),$

$a_6 = \frac{-h^2}{30r(-r^3+r^2)(r-1)^2}(4f_n - 4f_{n+r} - 5f_n r + 5f_{n+r} r - 5f_n r^2 + 5f_n r^3 + 5f_n r^4 - 5f_{n+1} r^3 - 4f_n r^5 - 5f_{n+1} r^4 + 4f_{n+1} r^5 + 5f_{n+r} r^2 - 4g_n hr^2 + 4g_n hr^4 + g_{n+1} hr^3 - 2g_n hr^5 + g_{n+1} hr^4 - 2g_{n+1} hr^5 - g_{n+r} hr^2 - g_{n+r} hr^3 + 2g_n hr + 2g_{n+r} hr)$,

$a_7 = \frac{-h^2}{42r^3(r-1)(r^2-2r+1)}(2f_n - 2f_{n+r} - 4f_n r + 4f_{n+r} r + 4f_n r^3 - 2f_n r^4 - 4f_{n+1} r^3 + 2f_{n+1} r^4 - 3g_n hr^2 + 3g_n hr^3 - g_n hr^4 + g_{n+1} hr^3 - g_{n+1} hr^4 - g_{n+r} hr^2 + g_n hr + g_{n+r} hr)$.

The values of a_j's, $j = 0(1)7$ are then substituted into Equation (3.2) to produce a continuous implicit scheme of the form

$$y(x) = \sum_{i=0,r} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \beta_r(x) f_{n+r} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} + \gamma_r(x) g_{n+r} \quad (3.6)$$

where

$\alpha_0 = \frac{x_n - x + hr}{hr}$,

$\alpha_r = \frac{x - x_n}{hr}$,

$\beta_0 = \frac{(x-x_n)^2}{2} + \frac{(x-x_n)^5}{10h^3 r^3}(r^3 + 4r^2 + 4r + 1) - \frac{(x-x_n)^4}{12h^2 r^2}(3r^2 + 4r + 3) - \frac{(x-x_n)^6}{30h^4 r^3}(4r^2 + 7r + 4) + \frac{hr(x-x_n)}{420}(-6r^3 + 15r^2 + 28r - 147) + \frac{(x-x_n)^7(r+1)}{21h^5 r^3}$,

$\beta_r = \frac{(5r-3)(x-x_n)^4}{12h^2 r^2 (r-1)^3} - \frac{(4r-2)(x-x_n)^7}{42h^5 r^3 (r-1)^3} + \frac{(x-x_n)^5(-10r^2+2r+2)}{20h^3 r^3 (r-1)^3} + \frac{(x-x_n)^6(5r^2+5r-4)}{30h^4 r^3 (r-1)^3} - \frac{hr(x-x_n)}{420(r-1)^3}(30r^3 - 120r^2 + 161r - 63)$,

$\beta_1 = \frac{r^2(3r-5)}{12h^2(r-1)^3}(x - x_n)^4 - \frac{(x-x_n)^6}{30h^4(r-1)^3}(-4r^2 + 5r + 5) - \frac{(x-x_n)^7}{21h^5(r-1)^3}(r - 2) - \frac{r(x-x_n)^5}{10h^3(r-1)^3}(r^2 + r - 5) + \frac{hr^5(x-x_n)}{420(r-1)^3}(6r^2 - 33r + 35)$,

$\gamma_0 = \frac{(x-x_n)^3}{6} + \frac{(x-x_n)^7}{42h^4 r^2} - \frac{h^2 r^2 (x-x_n)}{420}(3r^2 - 14r + 21) + \frac{(x-x_n)^5}{20h^2 r^2}(r^2 + 4r + 1) - \frac{(x-x_n)^4}{6hr}(r + 1) - \frac{(x-x_n)^6}{15h^3 r^2}(r + 1)$,

$\gamma_r = \frac{(x-x_n)^7}{42h^4 r^2 (r-1)^2} - \frac{(x-x_n)^4}{12hr(r-1)^2} - \frac{(x-x_n)^5}{20h^2 r^2 (r-1)^3}(-2r^2 + r + 1) + \frac{h^2 r^2 (x-x_n)}{210(r-1)^3}(2r^3 - 9r^2 + 14r - 7) - \frac{(x-x_n)^6}{30h^3 r^2 (r-1)^3}(r^2 + r - 2)$,

$\gamma_1 = \frac{(x-x_n)^7}{(42h^4(r-1)^2} - \frac{r^2(x-x_n)^4}{12h(r-1)^2} + \frac{(x-x_n)^6}{30h^3(r-1)^3}(-2r^2 + r + 1) - \frac{h^2 r^5 (x-x_n)}{420(r-1)^3}(3r^2 - 10r + 7) + \frac{r(x-x_n)^5}{20h^2(r-1)^3}(r^2 + r - 2)$.

Now, evaluating (3.6) at the non-interpolating point x_{n+1} yields

$$y_{n+1}-\frac{y_{n+r}}{r}=\frac{y_n(r-1)}{r}-\frac{g_nh^3}{420r^2}(3r^6-14r^5+21r^4-21r^2+14r-3)+\frac{g_{n+1}h^3}{420(r-1)}(-3r^5+4r^4+4r^3+4r^2-10r+4)-\frac{f_nh^2}{420r^3}(6r^7-15r^6-28r^5+147r^4-147r^3+28r^2+15r-6)+\frac{f_{n+1}h^2}{420(r-1)^2}(6r^6-27r^5+8r^4+8r^3+71r^2-90r+30)+\frac{g_{n+r}h^3}{420r^2(r-1)^2}(4r^6-14r^5+14r^4-7r+3)-\frac{f_{n+r}h^2}{420r^3(r-1)^2}(30r^6-90r^5+71r^4+8r^3+8r^2-27r+6). \qquad (3.7)$$

Differentiating (3.6) once gives

$$\begin{aligned} y'(x) &= \frac{d}{dx}\alpha_0(x)y_n+\frac{d}{dx}\alpha_r(x)y_{n+r}+\sum_{i=0}^{1}\frac{d}{dx}\beta_i(x)f_{n+i}+\frac{d}{dx}\beta_r(x)f_{n+r}+\sum_{i=0}^{1}\frac{d}{dx} \\ &\quad \gamma_i(x)g_{n+i}+\frac{d}{dx}\gamma_r(x)g_{n+r}. \qquad (3.8) \end{aligned}$$

Equation (3.8) is then evaluated at all points, i.e x_n, x_{n+r} and x_{n+1}, to get

$$y'_n-\frac{y_{n+r}}{hr}=-\frac{y_n}{hr}+\frac{f_nhr}{420}(-6r^3+15r^2+28r-147)-\frac{g_nh^2r^2}{420}(3r^2-14r+21)-\frac{f_{n+r}hr}{420(r-1)^3}(30r^3-120r^2+161r-63)+\frac{f_{n+1}hr^5}{420(r-1)^3}(6r^2-33r+35)-\frac{g_{n+1}h^2r^5}{420(r-1)^2}(3r-7)+\frac{g_{n+r}h^2r^2}{210(r-1)^2}(2r^2-7r+7), \qquad (3.9)$$

$$y'_{n+r}-\frac{y_{n+r}}{hr}=-\frac{y_n}{hr}-\frac{f_nhr}{420}(-8r^3+13r^2+28r-63)+\frac{g_nh^2r^2}{210}(2r^2-7r+7)+\frac{f_{n+r}hr}{420(r-1)^3}(110r^3-370r^2+413r-147)-\frac{f_{n+1}hr^5}{420(r-1)^3}(8r^2-37r+35)+\frac{g_{n+1}h^2r^5}{420(r-1)^2}(4r-7)-\frac{g_{n+r}h^2r^2}{420(r-1)^2}(10r^2-28r+21), \qquad (3.10)$$

$$y'_{n+1}-\frac{y_{n+r}}{hr}=-\frac{y_n}{hr}-\frac{g_nh^2}{420r^2}(3r^6-14r^5+21r^4-35r^2+28r-7)-\frac{f_nh}{420r^3}(6r^7-15r^6-28r^5+147r^4-210r^3+56r^2+28r-14)-\frac{g_{n+1}h^2}{420(r-1)^2}(3r^6-7r^5+35r^2-42r+14)+\frac{f_{n+1}h}{420(r-1)^3}(6r^7-33r^6+35r^5+210r^3-574r^2+490r-140)+\frac{g_{n+r}h^2}{420r^2(r-1)^2}(4r^6-14r^5+14r^4-14r+7)+\frac{f_{n+r}h}{420r^3(r-1)^3}(-30r^7+120r^6-161r^5+63r^4+70r^2-70r+14). \qquad (3.11)$$

Combining Equations (3.7) and (3.9) produces a block in the form

$$\begin{aligned} H^{[2]_1}Y^{[2]_1}_{n+1} &= M^{[2]_1}_1Y^{[2]_1}_n+M^{[2]_1}_2Y^{[2]_1}_{n-1}+E^{[2]_1}_1F^{[2]_1}_n+E^{[2]_1}_2F^{[2]_1}_{n+1}+K^{[2]_1}_1G^{[2]_1}_n \\ &\quad +K^{[2]_1}_2G^{[2]_1}_{n+1} \qquad (3.12) \end{aligned}$$

where

$$H^{[2]_1} = \begin{pmatrix} \frac{-1}{r} & 1 \\ \frac{-1}{hr} & 0 \end{pmatrix},\ Y_{n+1}^{[2]_1} = \begin{pmatrix} y_{n+r} \\ y_{n+1} \end{pmatrix},\ M_1^{[2]_1} = \begin{pmatrix} 0 & \frac{r-1}{r} \\ 0 & \frac{-1}{hr} \end{pmatrix},$$

$$Y_n^{[2]_1} = \begin{pmatrix} y_{n-1} \\ y_n \end{pmatrix},\quad M_2^{[2]_1} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix},\quad Y_{n-1}^{[2]_1} = \begin{pmatrix} y'_{n-1} \\ y'_n \end{pmatrix},$$

$$E_1^{[2]_1} = \begin{pmatrix} 0 & -\frac{h^2(6r^7-15r^6-28r^5+147r^4-147r^3+28r^2+15r-6)}{420r^3} \\ 0 & \frac{hr(-6r^3+15r^2+28r-147)}{420} \end{pmatrix},\ F_n^{[2]_1} = \begin{pmatrix} f_{n-1} \\ f_n \end{pmatrix},$$

$$E_2^{[2]_1} = \begin{pmatrix} -\frac{h^2(30r^6-90r^5+71r^4+8r^3+8r^2-27r+6)}{420r^3(r-1)^2} & \frac{h^2(6r^6-27r^5+8r^4+8r^3+71r^2-90r+30)}{420(r-1)^2} \\ -\frac{hr(30r^3-120r^2+161r-63)}{420(r-1)^3} & \frac{hr^5(6r^2-33r+35))}{420(r-1)^3} \end{pmatrix},$$

$$F_{n+1}^{[2]_1} = \begin{pmatrix} f_{n+r} \\ f_{n+1} \end{pmatrix},\ K_1^{[2]_1} = \begin{pmatrix} 0 & \frac{-h^3(3r^6-14r^5+21r^4-21r^2+14r-3)}{420r^2} \\ 0 & \frac{-h^2r^2(3r^2-14r+21)}{420} \end{pmatrix},$$

$$K_2^{[2]_1} = \begin{pmatrix} \frac{h^3(4r^6-14r^5+14r^4-7r+3)}{420r^2(r-1)^2} & \frac{h^3(-3r^5+4r^4+4r^3+4r^2-10r+4)}{420(r-1)} \\ \frac{h^2r^2(2r^2-7r+7)}{210(r-1)^2} & \frac{-h^2r^5(3r-7)}{420(r-1)^2} \end{pmatrix},$$

$$G_n^{[2]_1} = \begin{pmatrix} g_{n-1} \\ g_n \end{pmatrix},\ G_{n+1}^{[2]_1} = \begin{pmatrix} g_{n+r} \\ g_{n+1} \end{pmatrix}.$$

Multiplying both sides of Equation (3.12) by the inverse of $H^{[2]_1}$ yields

$$\begin{aligned} I_2 Y_{n+1}^{[2]_1} &= \hat{M}_1^{[2]_1} Y_n^{[2]_1} + h\hat{M}_2^{[2]_1} Y_{n-1}^{[2]_1} + h^2[\hat{E}_1^{[2]_1} F_n^{[2]_1} + \hat{E}_2^{[2]_1} F_{n+1}^{[2]_1}] + h^3[\hat{K}_1^{[2]_1} \\ &\quad G_n^{[2]_1} + \hat{K}_2^{[2]_1} G_{n+1}^{[2]_1}] \end{aligned} \tag{3.13}$$

where

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \hat{M}_1^{[2]_1} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},\ \hat{M}_2^{[2]_1} = \begin{pmatrix} 0 & r \\ 0 & 1 \end{pmatrix},$$

$$\hat{E}_1^{[2]_1} = \begin{pmatrix} 0 & -\frac{r^2(-6r^3+15r^2+28r-147)}{420} \\ 0 & -\frac{-147r^3+28r^2+15r-6}{420r^3} \end{pmatrix},$$

$$\hat{E}_2^{[2]_1} = \begin{pmatrix} \frac{r^2(30r^3-120r^2+161r-63)}{420(r-1)^3} & -\frac{r^6(6r^2-33r+35)}{420(r-1)^3} \\ \frac{35r^2-33r+6}{420r^3(r-1)^3} & \frac{63r^3-161r^2+120r-30}{420(r-1)^3} \end{pmatrix},$$

$$\hat{K}_1^{[2]_1} = \begin{pmatrix} 0 & \frac{r^3(3r^2-14r+21)}{420} \\ 0 & \frac{21r^2-14r+3}{420r^2} \end{pmatrix}, \; \hat{K}_2^{[2]_1} = \begin{pmatrix} -\frac{r^3(2r^2-7r+7)}{210(r-1)^2} & \frac{r^6(3r-7)}{420(r-1)^2} \\ -\frac{7r-3}{420r^2(r-1)^2} & -\frac{7r^2-7r+2}{210(r-1)^2} \end{pmatrix}.$$

From (3.13), the following equations are obtained

$$y_{n+r} = y_n + hry'_n + \frac{h^3r^3(3r^2-14r+21)}{420}g_n - \frac{h^2r^2(-6r^3+15r^2+28r-147)}{420}f_n + \frac{h^2r^2(30r^3-120r^2+161r-63)}{420(r-1)^3}f_{n+r} - \frac{h^2r^6(6r^2-33r+35)}{420(r-1)^3}f_{n+1} + \frac{h^3r^6(3r-7)}{420(r-1)^2}g_{n+1} - \frac{h^3r^3(2r^2-7r+7)}{210(r-1)^2}g_{n+r} \tag{3.14}$$

$$y_{n+1} = y_n + hy'_n + \frac{h^3(21r^2-14r+3)}{420r^2}g_n - \frac{h^2(-147r^3+28r^2+15r-6)}{420r^3}f_n - \frac{h^3(7r-3)}{420r^2(r-1)^2}g_{n+r} + \frac{h^2(35r^2-33r+6)}{420r^3(r-1)^3}f_{n+r} - \frac{h^3(7r^2-7r+2)}{210(r-1)^2}g_{n+1} + \frac{h^2(63r^3-161r^2+120r-30)}{420(r-1)^3}f_{n+1}. \tag{3.15}$$

Substituting the value of y_{n+r} from Equation (3.14) into both Equations (3.10) and (3.11) yields the first derivative of the block as below

$$y'_{n+r} = y'_n - \frac{hr(-r^3+2r^2+4r-15)}{30}f_n - \frac{hr^5(r^2-5r+5)}{30(r-1)^3}f_{n+1} + \frac{h^2r^5(r-2)}{60(r-1)^2}g_{n+1} + \frac{hr(10r^3-35r^2+41r-15)}{30(r-1)^3}f_{n+r} + \frac{h^2r^2(r^2-4r+5)}{60}g_n - \frac{h^2r^2(2r^2-6r+5)}{60(r-1)^2}g_{n+r}, \tag{3.16}$$

$$y'_{n+1} = y'_n - \frac{h^2(5r^2-6r+2)}{60(r-1)^2}g_{n+1} - \frac{h(-15r^3+4r^2+2r-1)}{30r^3}f_n + \frac{h^2(5r^2-4r+1)}{60r^2}g_n + \frac{h(15r^3-41r^2+35r-10)}{30(r-1)^3}f_{n+1} + \frac{h(5r^2-5r+1)}{30r^3(r-1)^3}f_{n+r} - \frac{h^2(2r-1)}{60r^2(r-1)^2}g_{n+r} \tag{3.17}$$

which can be expressed in a block form as follows

$$I_2 Y'^{[2]_1}_{n+1} = M'^{[2]_1}_2 Y^{[2]_1}_{n-1} + h[E'^{[2]_1}_1 F^{[2]_1}_n + E'^{[2]_1}_2 F^{[2]_1}_{n+1}] + h^2[K'^{[2]_1}_1 G^{[2]_1}_n + K'^{[2]_1}_2 G^{[2]_1}_{n+1}] \tag{3.18}$$

where

$$Y_{n+1}^{'[2]_1} = \begin{pmatrix} y'_{n+r} \\ y'_{n+1} \end{pmatrix}, \; M_2^{'[2]_1} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \; E_1^{'[2]_1} = \begin{pmatrix} 0 & -\frac{r(-r^3+2r^2+4r-15)}{30} \\ 0 & -\frac{-15r^3+4r^2+2r-1}{30r^3} \end{pmatrix},$$

$$E_2^{'[2]_1} = \begin{pmatrix} \frac{r(10r^3-35r^2+41r-15)}{30(r-1)^3} & -\frac{r^5(r^2-5r+5)}{30(r-1)^3} \\ \frac{5r^2-5r+1}{30r^3(r-1)^3} & \frac{15r^3-41r^2+35r-10)}{30(r-1)^3} \end{pmatrix}, \; K_1^{'[2]_1} = \begin{pmatrix} 0 & \frac{r^2(r^2-4r+5)}{60} \\ 0 & \frac{5r^2-4r+1}{60r^2} \end{pmatrix},$$

$$K_2^{'[2]_1} = \begin{pmatrix} -\frac{r^2(2r^2-6r+5)}{60(r-1)^2} & \frac{r^5(r-2)}{60(r-1)^2} \\ -\frac{2r-1}{60r^2(r-1)^2} & -\frac{5r^2-6r+2}{60(r-1)^2} \end{pmatrix}.$$

3.2.1 Properties of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

The numerical properties of the one-step HBM with generalised one off-step point which include order, error constant, zero stability, consistency and region of absolute stability are presented in this section.

3.2.1.1 Order of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

Extending (2.3) to one-step HBM for solving IVP of m^{th} order ODE in the presence of $(m+1)^{th}$ derivative gives

$$Y_{n+1}^{[m]_z} = \sum_{i=0}^{m-1}\left(h^i \hat{M}_{i+1}^{[m]_z} Y_{n-i}^{[m]_z}\right) + h^m \sum_{i=0}^{1}\left(\hat{E}_{i+1}^{[m]_z} F_{n+i}^{[m]_z}\right) + h^{m+1} \sum_{i=0}^{1}\left(\hat{K}_{i+1}^{[m]_z} G_{n+i}^{[m]_z}\right) \quad (3.19)$$

where $\hat{M}_{i+1}^{[m]_z}, i = 1, ..., m-1, \hat{E}_{i+1}^{[m]_z}$, and $\hat{K}_{i+1}^{[m]_z}$ are $(z+1)\times(z+1)$ are the coefficient matrices corresponding to $Y_{n-i}^{[m]_z}, F_{n+i}^{[m]_z}$ and $G_{n+i}^{[m]_z}$ respectively, z denotes the number of hybrid (off-step) points, m represents the order of the differential equation where

$$Y_{n+1}^{[m]_z} = [y_{n+r_1}, y_{n+r_2}, \cdots, y_{n+1}]^T,$$

$$Y_n^{[m]_z} = [y_{n-z}, y_{n-(z-1)}, \cdots, y_{n-1}, y_n]^T,$$

$$\vdots$$

$$Y_{n-m+1}^{[m]_z} = [y_{n-z}^{m-1}, y_{n-(z-1)}^{m-1}, \cdots, y_{n-1}^{m-1}, y_n^{m-1}]^T,$$

$$F_n^{[m]_z} = [f_{n-z}, f_{n-(z-1)}, \cdots, f_{n-1}, f_n]^T,$$

$$F_{n+1}^{[m]_z} = [f_{n+r_1}, f_{n+r_2}, \cdots, f_{n+r_z}, f_{n+1}]^T,$$

$$G_n^{[m]_z} = [g_{n-z}, g_{n-(z-1)}, \cdots, g_{n-1}, g_n]^T,$$

$$G_{n+1}^{[m]_z} = [g_{n+r_1}, g_{n+r_2}, \cdots, g_{n+r_z}, g_{n+1}]^T.$$

The linear difference operator ∇ associated with Equation (3.19) is defined as

$$\begin{aligned} \nabla[y(x),h] &= Y_{n+1}^{[m]_z} - \sum_{i=0}^{m-1}\left(h^i \hat{M}_{i+1}^{[m]_z} Y_{n-i}^{[m]_z}\right) - h^m \sum_{i=0}^{1}\left(\hat{E}_{i+1}^{[m]_z} F_{n+i}^{[m]_z}\right) \\ &\quad - h^{m+1}\sum_{i=0}^{1}\left(\hat{K}_{i+1}^{[m]_z} G_{n+i}^{[m]_z}\right). \end{aligned} \tag{3.20}$$

Expanding the elements in vectors $Y_{n+1}^{[m]_z}, F_{n+1}^{[m]_z}$ and $G_{n+1}^{[m]_z}$ about x by using Taylor series and collecting like terms in powers of h gives

$$\nabla[y(x),h] = \bar{D}_0 y(x) + \bar{D}_1 h y'(x) + \bar{D}_2 h^2 y''(x) + \cdots.$$

Definition 3.2.1. One-step HBM (3.19) and its associated linear operator (3.20) are said to be of order P, if $\bar{D}_0 = \bar{D}_1 = \bar{D}_2 = \cdots = \bar{D}_{P+m-1} = 0$ and $\bar{D}_{P+m} \neq 0$ with vector of error constants $\bar{D}_{P+m}$.

Substituting $m = 2$ and $z = 1$ in (3.20) the linear difference operator ∇ associated with Equation (3.13) is represented by

$$\begin{aligned} \nabla[y(x),h] &= Y_{n+1}^{[2]_1} - \hat{M}_1^{[2]_1} Y_n^{[2]_1} - h\hat{M}_2^{[2]_1} Y_{n-1}^{[2]_1} - h^2[\hat{E}_1^{[2]_1} F_n^{[2]_1} + \hat{E}_2^{[2]_1} F_{n+1}^{[2]_1}] - \\ &\quad h^3[\hat{K}_1^{[2]_1} G_n^{[2]_1} + \hat{K}_2^{[2]_1} G_{n+1}^{[2]_1}]. \end{aligned} \tag{3.21}$$

In finding the order of the main block (3.13), we expand each function in $Y_{n+1}^{[2]_1}, F_{n+1}^{[2]_1}$ and $G_{n+1}^{[2]_1}$ about x_n using Taylor series and then set to $\mathbf{0}$ as below

$$
\begin{bmatrix}
\sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n - rhy'_n + \frac{(r^2(-6r^3+15r^2+28r-147))h^2 y_n^{(2)}}{420} \\
-\sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+2)}}{(420(r-1)^3)i!}((r^i r^2(30r^3-120r^2+161r-63))-(r^6(6r^2-33r+35)) \\
-\frac{h^3 y_n^{(3)}(r^3(3r^2-14r+21))}{420} - \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)}}{(420(r-1)^2)i!}(2r^3(2r^2-7r+7)(r)^i - r^6(3r-7)) \\
\sum_{i=0}^{\infty} (\frac{h^i}{i!}) y_n^{(i)} - y_n - hy'_n + \frac{(-147r^3+28r^2+15r-6)h^2 y_n^{(2)}}{420r^3} \\
-\sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+2)}}{(420r^3(r-1)^3)i!}((35r^2-33r+6)r^i + r^3(63r^3-161r^2+120r-30)) \\
-\frac{h^3 y_n^{(3)}(21r^2-14r+3)}{420r^2} + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)}}{(420r^2(r-1)^2)i!}((7r-3)(r)^i + 2r^2(7r^2-7r+2))
\end{bmatrix}
=
\begin{bmatrix} 0 \\ \\ \\ \\ 0 \\ \\ \end{bmatrix}.
$$

Comparing the coefficients of h^i and $y^{(i)}$ yields

$$
\bar{D}_0 = \begin{bmatrix} 1-1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
$$

$$
\bar{D}_1 = \begin{bmatrix} r-r \\ 1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
$$

$$
\bar{D}_2 = \begin{bmatrix}
\frac{r^2}{2} + \frac{r^2(-6r^3+15r^2+28r-147)}{420} + \frac{r^6(6r^2-33r+35)}{420(r-1)^3} - \\
\frac{r^2(30r^3-120r^2+161r-63)}{420(r-1)^3} \\
\frac{1}{2} + \frac{-147r^3+28r^2+15r-6}{420r^3} - \frac{63r^3-161r^2+120r-30}{420(r-1)^3} - \\
\frac{35r^2-33r+6}{420r^3(r-1)^3}
\end{bmatrix}
= \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},
$$

$$
\bar{D}_3 = \begin{bmatrix}
\frac{r^3(2r^2-7r+7}{210(r-1)^2} - \frac{r^6(3r-7)}{420(r-1)^2} - \frac{r^3(3r^2-14r+21)}{420} + \frac{r^6(6r^2-33r+35)}{420(r-1)^3} - \\
\frac{r^3(30r^3-120r^2+161r-63)}{420(r-1)^3} \\
\frac{7r^2-7r+2}{210(r-1)^2} - \frac{21r^2-14r+3}{420r^2} - \frac{63r^3-161r^2+120r-30}{420(r-1)^3} + \frac{7r-3}{420r^2(r-1)^2} - \\
\frac{35r^2-33r+6}{420r^2(r-1)^3}
\end{bmatrix}
= \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},
$$

$$
\bar{D}_4 = \begin{bmatrix}
\frac{r^4}{24} - \frac{r^6(3r-7)}{420(r-1)^2} + \frac{r^4(2r^2-7r+7)}{210(r-1)^2} + \frac{r^6(6r^2-33r+35)}{840(r-1)^3} - \\
\frac{r^4(30r^3-120r^2+161r-63)}{840(r-1)^3} \\
\frac{1}{24} + \frac{7r^2-7r+2}{210(r-1)^2} - \frac{63r^3-161r^2+120r-30}{840(r-1)^3} + \frac{7r-3}{420r(r-1)^2} - \\
\frac{35r^2-33r+6}{840r(r-1)^3}
\end{bmatrix}
= \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},
$$

$$\bar{D}_5 = \begin{bmatrix} \frac{r^5}{120} - \frac{r^6(3r-7)}{840(r-1)^2} + \frac{r^5(2r^2-7r+7)}{420(r-1)^2} + \frac{r^6(6r^2-33r+35)}{2520(r-1)^3} - \frac{r^5(30r^3-120r^2+161r-63)}{2520(r-1)^3} \\ \frac{1}{120} + \frac{7r-3}{840(r-1)^2} + \frac{7r^2-7r+2}{420(r-1)^2} - \frac{35r^2-33r+6}{2520(r-1)^3} - \frac{63r^3-161r^2+120r-30}{2520(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}_6 = \begin{bmatrix} \frac{r^6}{720} - \frac{r^6(3r-7)}{2520(r-1)^2} + \frac{r^6(2r^2-7r+7)}{1260(r-1)^2} + \frac{r^6(6r^2-33r+35)}{10080(r-1)^3} - \frac{r^6(30r^3-120r^2+161r-63)}{10080(r-1)^3} \\ \frac{1}{720} + \frac{7r^2-7r+2}{1260(r-1)^2} - \frac{63r^3-161r^2+120r-30}{10080(r-1)^3} - \frac{r(35r^2-33r+6)}{10080(r-1)^3} + \frac{r(7r-3)}{2520(r-1)^2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}_7 = \begin{bmatrix} \frac{r^7}{5040} - \frac{r^6(3r-7)}{10080(r-1)^2} + \frac{r^7(2r^2-7r+7)}{5040(r-1)^2} + \frac{r^6(6r^2-33r+35)}{50400(r-1)^3} - \frac{r^7(30r^3-120r^2+161r-63)}{50400(r-1)^3} \\ \frac{1}{5040} + \frac{7r^2-7r+2}{5040(r-1)^2} - \frac{63r^3-161r^2+120r-30}{50400(r-1)^3} + \frac{r^2(7r-3)}{10080(r-1)^2} - \frac{r^2(35r^2-33r+6)}{50400(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}_8 = \begin{bmatrix} \frac{r^8}{40320} - \frac{r^6(3r-7)}{50400(r-1)^2} + \frac{r^8(2r^2-7r+7)}{25200(r-1)^2} + \frac{r^6(6r^2-33r+35)}{302400(r-1)^3} - \frac{r^8(30r^3-120r^2+161r-63)}{302400(r-1)^3} \\ \frac{1}{40320} + \frac{7r^2-7r+2}{25200(r-1)^2} - \frac{63r^3-161r^2+120r-30}{302400(r-1)^3} + \frac{r^3(7r-3)}{50400(r-1)^2} - \frac{r^3(35r^2-33r+6)}{302400(r-1)^3} \end{bmatrix} = \begin{bmatrix} \frac{r^6(3r^2-12r+14)}{604800} \\ \frac{14r^2-12r+3}{604800} \end{bmatrix}.$$

By using Definition 3.2.1, the main block method (3.13) has order $[6,6]^T$ together with vector of error constants $\left[\frac{r^6(3r^2-12r+14)}{604800}, \frac{14r^2-12r+3}{604800}\right]^T$ for all $r \in (0,1)$.

Similarly, the linear difference operator ∇ associated with the derivative of main block (3.18) is defined as

$$\begin{aligned} \nabla[y(x),h] &= Y'^{[2]_1}_{n+1} - M'^{[2]_1}_2 Y^{[2]_1}_{n-1} - h[E'^{[2]_1}_1 F^{[2]_1}_n + E'^{[2]_1}_2 F^{[2]_1}_{n+1}] \\ &\quad - h^2[K'^{[2]_1}_1 G^{[2]_1}_n + K'^{[2]_1}_2 G^{[2]_1}_{n+1}]. \end{aligned} \tag{3.22}$$

Now, expanding every function in $Y'^{[2]_1}_{n+1}, F^{[2]_1}_{n+1}$ and $G^{[2]_1}_{n+1}$ about x_n using Taylor series and equating to **0** produces

$$\begin{bmatrix} \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i+1)} - y'_n + \frac{r(-r^3+2r^2+4r-15)hy_n^{(2)}}{30} \\ -\sum_{i=0}^{\infty} \frac{h^{i+1}y_n^{(i+2)}}{(30(r-1)^3)i!}(r(10r^3-35r^2+41r-15)r^i - r^5(r^2-5r+5)) \\ -\frac{h^2y_n^{(3)}(r^2(r^2-4r+5))}{60} + \sum_{i=0}^{\infty} \frac{h^{i+2}y_n^{(i+3)}}{(60(r-1)^2)i!}(r^2(2r^2-6r+5)r^i - r^5(r-2)) \\ \sum_{i=0}^{\infty}(\frac{h^i}{i!})y_n^{(i+1)} - y'_n + \frac{(-15r^3+4r^2+2r-1)hy_n^{(2)}}{30r^3} \\ -\sum_{i=0}^{\infty} \frac{h^{i+1}y_n^{(i+2)}}{(30r^3(r-1)^3)i!}((5r^2-5r+1)r^i + r^3(15r^3-41r^2+35r-10)) \\ -\frac{h^2y_n^{(3)}(5r^2-4r+1)}{(60r^2)} + \sum_{i=0}^{\infty} \frac{h^{i+2}y_n^{(i+3)}}{(60r^2(r-1)^2)i!}((2r-1)r^i + r^2(5r^2-6r+2)) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ \\ \\ 0 \end{bmatrix}.$$

Again, by comparing the coefficients of h^j and $y^{(j)}$, we have

$$\bar{D}'_0 = \begin{bmatrix} 0-0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}'_1 = \begin{bmatrix} 1-1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}'_2 = \begin{bmatrix} r + \frac{r(-r^3+2r^2+4r-15)}{30} + \frac{r^5(r^2-5r+5)}{30(r-1)^3} - \frac{r(10r^3-35r^2+41r-15)}{30(r-1)^3} \\ 1 + \frac{-15r^3+4r^2+2r-1}{30r^3} - \frac{15r^3-41r^2+35r-10}{30(r-1)^3} - \frac{5r^2-5r+1}{30r^3(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\bar{D}'_3 = \begin{bmatrix} \frac{r^5(r^2-5r+5)}{30(r-1)^3} - \frac{r^2(r^2-4r+5)}{60} + \frac{r^2(2r^2-6r+5)}{60(r-1)^2} - \\ \frac{r^2(10r^3-35r^2+41r-15)}{30(r-1)^3} - \frac{r^5(r-2)}{60(r-1)^2} \\ \frac{5r^2-6r+2}{60(r-1)^2} - \frac{5r^2-4r+1}{60r^2} - \frac{15r^3-41r^2+35r-10}{30(r-1)^3} + \\ \frac{2r-1}{60r^2(r-1)^2} - \frac{5r^2-5r+1}{30r^2(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},$$

$$\bar{D}'_4 = \begin{bmatrix} \frac{r^3}{6} + \frac{r^5(r^2-5r+5)}{60(r-1)^3} + \frac{r^3(2r^2-6r+5)}{60(r-1)^2} - \frac{r^3(10r^3-35r^2+41r-15)}{60(r-1)^3} - \\ \frac{r^5(r-2)}{60(r-1)^2} \\ \frac{1}{6} + \frac{5r^2-6r+2}{60(r-1)^2} - \frac{15r^3-41r^2+35r-10}{60(r-1)^3} + \frac{2r-1}{60r(r-1)^2} - \\ \frac{5r^2-5r+1}{60r(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},$$

$$\bar{D}'_5 = \begin{bmatrix} \frac{r^4}{24} + \frac{r^5(r^2-5r+5)}{180(r-1)^3} + \frac{r^4(2r^2-6r+5)}{120(r-1)^2} - \frac{r^4(10r^3-35r^2+41r-15)}{180(r-1)^3} - \\ \frac{r^5(r-2)}{120(r-1)^2} \\ \frac{1}{24} + \frac{2r-1}{120(r-1)^2} - \frac{5r^2-5r+1}{180(r-1)^3} + \frac{5r^2-6r+2}{120(r-1)^2} - \\ \frac{15r^3-41r^2+35r-10}{180(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},$$

$$\bar{D}'_6 = \begin{bmatrix} \frac{r^5}{120} + \frac{r^5(r^2-5r+5)}{720(r-1)^3} + \frac{r^5(2r^2-6r+5)}{360(r-1)^2} - \frac{r^5(10r^3-35r^2+41r-15)}{720(r-1)^3} - \\ \frac{r^5(r-2)}{360(r-1)^2} \\ \frac{1}{120} + \frac{5r^2-6r+2}{360(r-1)^2} - \frac{15r^3-41r^2+35r-10}{720(r-1)^3} - \frac{r(5r^2-5r+1)}{720(r-1)^3} + \\ \frac{r(2r-1)}{360(r-1)^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},$$

$$\bar{D}'_7 = \begin{bmatrix} \frac{r^6}{720} + \frac{r^5(r^2-5r+5)}{3600(r-1)^3} + \frac{r^6(2r^2-6r+5)}{1440(r-1)^2} - \frac{r^6(10r^3-35r^2+41r-15)}{3600(r-1)^3} - \\ \frac{r^5(r-2)}{1440(r-1)^2} \\ \frac{1}{720} + \frac{5r^2-6r+2}{1440(r-1)^2} - \frac{15r^3-41r^2+35r-10}{3600(r-1)^3} + \frac{r^2(2r-1)}{1440(r-1)^2} - \\ \frac{r^2(5r^2-5r+1)}{3600(r-1)^3} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ 0 \end{bmatrix},$$

$$\bar{D}'_8 = \begin{bmatrix} \frac{r^7}{5040} + \frac{r^5(r^2-5r+5)}{21600(r-1)^3} + \frac{r^7(2r^2-6r+5)}{7200(r-1)^2} - \\ \frac{r^7(10r^3-35r^2+41r-15)}{21600(r-1)^3} - \frac{r^5(r-2)}{7200(r-1)^2} \\ \frac{1}{5040} + \frac{5r^2-6r+2}{7200(r-1)^2} - \frac{15r^3-41r^2+35r-10}{21600(r-1)^3} + \\ \frac{r^3(2r-1)}{7200(r-1)^2} - \frac{r^3(5r^2-5r+1)}{21600(r-1)^3} \end{bmatrix} = \begin{bmatrix} \frac{r^5(2r^2-7r+7)}{151200} \\ \\ \\ \frac{7r^2-7r+2}{151200} \end{bmatrix}.$$

Thus, the derivative block (3.18) has order $[6,6]^T$ with vector of error constants $\left[\frac{r^5(2r^2-7r+7)}{151200}, \frac{7r^2-7r+2}{151200}\right]^T$ for all $r \in (0,1)$.

3.2.1.2 Zero-Stability of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

The definition below follows from the extension of Definition 2.4.3.

Definition 3.2.2. One-step HBM (3.19) is said to be zero stable as $h \to 0$ if its first characteristic polynomial

$$\psi^{[m]_z}(q) = |q I_{z+1} - \hat{M}_1^{[m]_z}| \tag{3.23}$$

having all roots q_v such that $|q_v| \leq 1$, and if $|q_v| = 1$ then the multiplicity of q_v must not be greater than m. I_{z+1} is $(z+1) \times (z+1)$ identity matrix and $\hat{M}_1^{[m]_z}$ is $(z+1) \times (z+1)$ coefficients matrix of $Y_n^{[m]_z}$.

To find the zero-stability of the main block method (3.13), we consider the first characteristic polynomial according to Definition 3.2.2 , that is

$$\begin{aligned}
\psi^{[2]_1}(q) &= |q I_2 - \hat{M}_1^{[2]_1}| \\
&= \left| q \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right| \\
&= q(q-1) = 0
\end{aligned}$$

whose roots are $q = \{0, 1\}$. Similarly, the characteristic polynomial for the derivative block (3.18) is given by

$$\begin{aligned}
\psi'^{[2]_1}(q) &= |q I_2 - M_2'^{[2]_1}| \\
&= \left| q \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right| \\
&= q(q-1) = 0
\end{aligned}$$

which also implies the roots are $q = \{0, 1\}$. It follows from Definition 3.2.2 that both main block and its derivative methods are zero stable.

3.2.1.3 Consistency of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

According to Definition 2.4.4, the main block method (3.13) and its derivative (3.18) are consistent.

3.2.1.4 Convergence of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

The main block method (3.13) and its derivative (3.18) are convergent from Theorem (2.1).

3.2.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised One Off-Step Point for Solving Second Order ODEs

Absolute stability region of the hybrid block (3.19) is obtained by using the similar approach adopted by Ngwane and Jator (2012). The test problem of the form $y^{(m)} = \lambda^m y$ and $y^{(m+1)} = \lambda^{m+1} y$ are substituted into (3.19) to produce

$$Y_{n+1}^{[m]_z} = \sum_{i=0}^{m-1} (\lambda h)^i \left(\hat{M}_{i+1}^{[m]_z} Y_n^{[m]_z} \right) + (\lambda h)^m \sum_{i=0}^{1} \left(\hat{E}_{i+1}^{[m]_z} Y_{n+i}^{[m]_z} \right) + (\lambda h)^{m+1} \sum_{i=0}^{1} \left(\hat{K}_{i+1}^{[m]_z} Y_{n+i}^{[m]_z} \right). \tag{3.24}$$

Substituting $\lambda h = q$ in (3.24) yields

$$Y_{n+1}^{[m]_z} = \sum_{i=0}^{m-1} q^i \left(\hat{M}_{i+1}^{[m]_z} Y_n^{[m]_z} \right) + q^m \sum_{i=0}^{1} \left(\hat{E}_{i+1}^{[m]_z} Y_{n+i}^{[m]_z} \right) + q^{m+1} \sum_{i=0}^{1} \left(\hat{K}_{i+1}^{[m]_z} Y_{n+i}^{[m]_z} \right) \tag{3.25}$$

which leads to the following equation

$$Y_{n+1}^{[m]_z} = M^{[m]_z}(q) Y_n^{[m]_z}, \tag{3.26}$$

where the matrix $M^{[m]_z}(q)$ is given by

$$\begin{aligned} M^{[m]_z}(q) &= (I_{z+1} - q^m \hat{E}_2^{[m]_z} - q^{m+1} \hat{K}_2^{[m]_z})^{-1} (\sum_{i=0}^{m-1} q^i \hat{M}_{i+1}^{[m]_z} + q^m \hat{E}_1^{[m]_z} \\ &\quad + q^{m+1} \hat{K}_1^{[m]_z}) \end{aligned} \tag{3.27}$$

for $q = e^{i\theta}$ as $\theta \in [0, 2\pi]$.

Replacing $m = 2$ and $z = 1$ in (3.27) gives

$$M^{[2]_1}(q) = (I_2 - q^2 \hat{E}_2^{[2]_1} - q^3 \hat{K}_2^{[2]_1})^{-1} (\hat{M}_1^{[2]_1} + q \hat{M}_2^{[2]_1} + q^2 \hat{E}_1^{[2]_1} + q^3 \hat{K}_1^{[2]_1}). \tag{3.28}$$

The eigenvalues of the matrix $M^{[2]_1}(q)$ are $\{0, \eta_2^{[2]_1}\}$ where the dominant eigenvalue

$\eta_2^{[2]_1}$ is in terms of q given by

$$\eta_2^{[2]_1} = eig(M^{[2]_1}(q)). \tag{3.29}$$

The region of absolute stability for the specific value $r = \frac{1}{3}$ is obtained by substituting this value into Equation (3.29), this gives

$$\eta_2^{[2]_1} = \frac{\sum_{i=0}^{6} c_i q^i}{\sum_{j=0}^{7} d_j q^j}$$

where the values c_i and d_j are listed in Table 3.1 below

Table 3.1
Coefficients of the Eigenvalue ($\eta_2^{[2]_1}$) for the Matrix $M^{[2]_1}$

c-value	q^i Coefficients	d-value	q^j Coefficients
c_0	2721600	d_0	0
c_1	2721600	d_1	2721600
c_2	1186920	d_2	0
c_3	297960	d_3	- 173880
c_4	46860	d_4	18240
c_5	4500	d_5	2160
c_6	204	d_6	- 480
		d_7	29

By plotting the function ($\eta_2^{[2]_1}$) using Matlab, we obtain the region of absolute stability represented by a dark area as depicted in Figure 3.2.

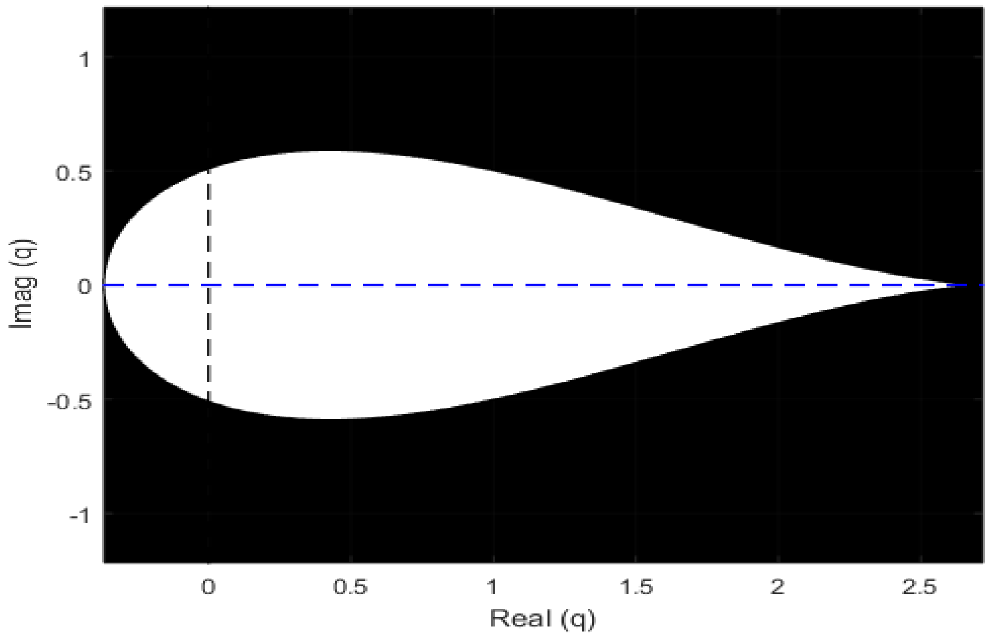


Figure 3.2. Region of Absolute Stability of One-Step HBM with One Off-Step Point $r = \frac{1}{3}$ for Second Order ODEs

A similar procedure can be applied to find the region of absolute stability for the specific value $r = \frac{2}{3}$ whose area is in the dark colour as shown in Figure 3.3.

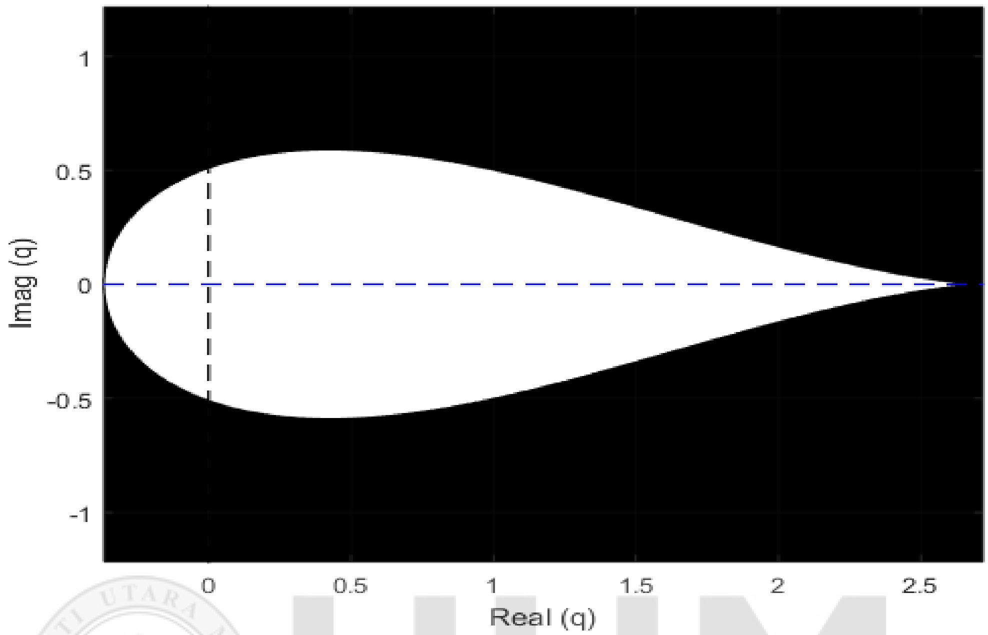


Figure 3.3. Region of Absolute Stability of One-Step HBM with One Off-Step Point $r = \frac{2}{3}$ for Second Order ODEs

The stability region for other specific values r can be found by substituting this value to (3.29) and using the same procedure as mentioned earlier.

3.3 Derivation of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

To derive this method, Equation (3.2) is interpolated at x_n and x_{n+r} $(u = 2)$ while Equations (3.3) and (3.4) are collocated at all points, i.e x_n, x_{n+r}, x_{n+s} and x_{n+1} $(v = 4)$, where $r < s \in (0,1)$ as illustrated in the following Figure 3.4.

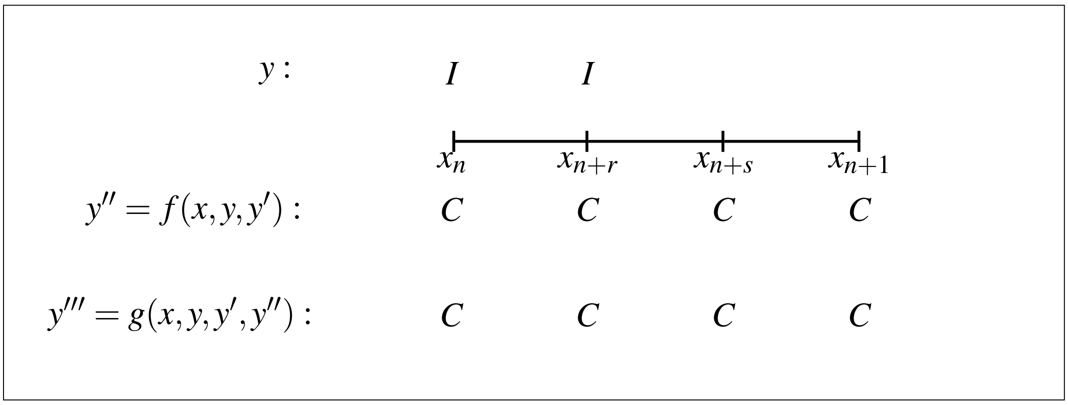


Figure 3.4. Interpolation and Collocation Strategy for One-Step HBM with Two Off-Step Points for Solving Second Order ODEs

Replacing $u = 2$ and $v = 4$ in Equations (3.2)-(3.4) yields

$$
\begin{aligned}
y_n &= a_0, \\
y_{n+r} &= a_0 + ra_1 + r^2a_2 + r^3a_3 + r^4a_4 + r^5a_5 + r^6a_6 + r^7a_7 + r^8a_8 + r^9a_9, \\
f_n &= \frac{2}{h^2}a_2, \\
f_{n+r} &= \frac{2}{h^2}a_2 + \frac{6r}{h^2}a_3 + \frac{12r^2}{h^2}a_4 + \frac{20r^3}{h^2}a_5 + \frac{30r^4}{h^2}a_6 + \frac{42r^5}{h^2}a_7 + \frac{56r^6}{h^2}a_8 + \frac{72r^7}{h^2}a_9, \\
f_{n+s} &= \frac{2}{h^2}a_2 + \frac{6s}{h^2}a_3 + \frac{12s^2}{h^2}a_4 + \frac{20s^3}{h^2}a_5 + \frac{30s^4}{h^2}a_6 + \frac{42s^5}{h^2}a_7 + \frac{56s^6}{h^2}a_8 + \frac{72s^7}{h^2}a_9, \\
f_{n+1} &= \frac{2}{h^2}a_2 + \frac{6}{h^2}a_3 + \frac{12}{h^2}a_4 + \frac{20}{h^2}a_5 + \frac{30}{h^2}a_6 + \frac{42}{h^2}a_7 + \frac{56}{h^2}a_8 + \frac{72}{h^2}a_9, \\
g_n &= \frac{6}{h^3}a_3, \\
g_{n+r} &= \frac{6}{h^3}a_3 + \frac{24r}{h^3}a_4 + \frac{60r^2}{h^3}a_5 + \frac{120r^3}{h^3}a_6 + \frac{210r^4}{h^3}a_7 + \frac{336r^5}{h^3}a_8 + \frac{504r^6}{h^3}a_9, \\
g_{n+s} &= \frac{6}{h^3}a_3 + \frac{24s}{h^3}a_4 + \frac{60s^2}{h^3}a_5 + \frac{120s^3}{h^3}a_6 + \frac{210s^4}{h^3}a_7 + \frac{336s^5}{h^3}a_8 + \frac{504s^6}{h^3}a_9, \\
g_{n+1} &= \frac{6}{h^3}a_3 + \frac{24}{h^3}a_4 + \frac{60}{h^3}a_5 + \frac{120}{h^3}a_6 + \frac{210}{h^3}a_7 + \frac{336}{h^3}a_8 + \frac{504}{h^3}a_9,
\end{aligned}
$$

which can be written in a matrix form as below

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 \\
0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{2}{h^2} & \frac{6r}{h^2} & \frac{12r^2}{h^2} & \frac{20r^3}{h^2} & \frac{30r^4}{h^2} & \frac{42r^5}{h^2} & \frac{56r^6}{h^2} & \frac{72r^7}{h^2} \\
0 & 0 & \frac{2}{h^2} & \frac{6s}{h^2} & \frac{12s^2}{h^2} & \frac{20s^3}{h^2} & \frac{30s^4}{h^2} & \frac{42s^5}{h^2} & \frac{56s^6}{h^2} & \frac{72s^7}{h^2} \\
0 & 0 & \frac{2}{h^2} & \frac{6}{h^2} & \frac{12}{h^2} & \frac{20}{h^2} & \frac{30}{h^2} & \frac{42}{h^2} & \frac{56}{h^2} & \frac{72}{h^2} \\
0 & 0 & 0 & \frac{6}{h^3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24r}{h^3} & \frac{60r^2}{h^3} & \frac{120r^3}{h^3} & \frac{210r^4}{h^3} & \frac{336r^5}{h^3} & \frac{504r^6}{h^3} \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24s}{h^3} & \frac{60s^2}{h^3} & \frac{120s^3}{h^3} & \frac{210s^4}{h^3} & \frac{336s^5}{h^3} & \frac{504s^6}{h^3} \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24}{h^3} & \frac{60}{h^3} & \frac{120}{h^3} & \frac{210}{h^3} & \frac{336}{h^3} & \frac{504}{h^3}
\end{pmatrix}
\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix}
=
\begin{pmatrix} y_n \\ y_{n+r} \\ f_n \\ f_{n+r} \\ f_{n+s} \\ f_{n+1} \\ g_n \\ g_{n+r} \\ g_{n+s} \\ g_{n+1} \end{pmatrix}. \tag{3.30}
$$

Solving for the unknowns in (3.30) and then substituting them into Equation (3.2) produces a continuous implicit scheme of the form

$$y(x) = \sum_{i=0,r} \alpha_i(x) y_{n+i} + \sum_{i=0,r,s,1} \beta_i(x) f_{n+i} + \sum_{i=0,r,s,1} \gamma_i(x) g_{n+i} \tag{3.31}$$

where

$\alpha_0 = \frac{x_n - x + hr}{hr}$,

$\alpha_r = \frac{x - x_n}{hr}$,

$\beta_0 = \frac{(x-x_n)^2}{2} - \frac{(x-x_n)^4}{12h^2r^2s^2}(3r^2s^2 + 4r^2s + 3r^2 + 4rs^2 + 4rs + 3s^2) + \frac{(x-x_n)^9}{36h^7r^3s^3}(r + s + rs) + \frac{(x-x_n)^5}{10h^3r^3s^3}(r^3s^3 + 4r^3s^2 + 4r^3s + r^3 + 4r^2s^3 + 8r^2s^2 + 4r^2s + 4rs^3 + 4rs^2 + s^3) + \frac{hr(x-x_n)}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3) - \frac{(x-x_n)^8}{56h^6r^3s^3}(4r^2s + 4r^2 + 4rs^2 + 11rs + 4r + 4s^2 + 4s) - \frac{(x-x_n)^6}{30h^4r^3s^3}(4r^3s^2 + 7r^3s + 4r^3 + 4r^2s^3 + 20r^2s^2 + 20r^2s + 4r^2 + 7rs^3 + 20rs^2 + 7rs + 4s^3 + 4s^2) + \frac{(x-x_n)^7}{21h^5r^3s^3}(r^3s + r^3 + 4r^2s^2 + 8r^2s + 4r^2 + rs^3 + 8rs^2 + 8rs + r + s^3 + 4s^2 + s)$,

$$\beta_r = \frac{(x-x_n)^8}{56h^6r^3(r-s)^3(r-1)^3}(7r^3 + 7r^2s + 7r^2 - 8rs^2 - 13rs - 8r + 4s^2 + 4s) - \frac{hr(x-x_n)}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3) - \frac{(x-x_n)^6}{30h^4r^3(r-s)^3(r-1)^3}(-7r^3s^2 - 28r^3s - 7r^3 + 5r^2s^3 + 13r^2s^2 + 13r^2s + 5r^2 + 5rs^3 + 4rs^2 + 5rs - 4s^3 - 4s^2) + \frac{(x-x_n)^9}{36h^7r^3(r-s)^3(r-1)^3}(2r - s + 2rs - 3r^2) + \frac{(x-x_n)^7}{21h^5r^3(r-s)^3(r-1)^3}(-7r^3s - 7r^3 + 2r^2s^2 - 2r^2s + 2r^2 + 2rs^3 + 7rs^2 + 7rs + 2r - s^3 - 4s^2 - s) - \frac{s(x-x_n)^5}{10h^3r^3(r-s)^3(r-1)^3}(7r^3s + 7r^3 - 5r^2s^2 - 7r^2s - 5r^2 + rs^2 + rs + s^2) - \frac{s^2(x-x_n)^4}{12h^2r^2(r-s)^3(r-1)^3}(5r - 3s + 5rs - 7r^2),$$

$$\beta_s = \frac{(x-x_n)^7}{21h^5s^3(r-s)^3(s-1)^3}(-2r^3s + r^3 - 2r^2s^2 - 7r^2s + 4r^2 + 7rs^3 + 2rs^2 - 7rs + r + 7s^3 - 2s^2 - 2s) + \frac{(x-x_n)^6}{30h^4s^3(r-s)^3(s-1)^3}(5r^3s^2 + 5r^3s - 4r^3 - 7r^2s^3 + 13r^2s^2 + 4r^2s - 4r^2 - 28rs^3 + 13rs^2 + 5rs - 7s^3 + 5s^2) + \frac{(x-x_n)^9}{36h^7s^3(r-s)^3(s-1)^3}(r - 2s - 2rs + 3s^2) - \frac{(x-x_n)^8}{56h^6s^3(r-s)^3(s-1)^3}(-8r^2s + 4r^2 + 7rs^2 - 13rs + 4r + 7s^3 + 7s^2 - 8s) - \frac{hr^5(x-x_n)}{2520s^3(r-s)^3(s-1)^3}(-20r^4s + 10r^4 + 75r^3s^2 + 25r^3s - 36r^3 - 63r^2s^3 - 243r^2s^2 + 108r^2s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2) + \frac{r(x-x_n)^5}{10h^3s^3(r-s)^3(s-1)^3}(-5r^2s^2 + r^2s + r^2 + 7rs^3 - 7rs^2 + rs + 7s^3 - 5s^2) - \frac{r^2(x-x_n)^4}{12h^2s^2(r-s)^3(s-1)^3}(3r - 5s - 5rs + 7s^2),$$

$$\beta_1 = \frac{(x-x_n)^8}{56h^6(r-1)^3(s-1)^3}(4r^2s - 8r^2 + 4rs^2 - 13rs + 7r - 8s^2 + 7s + 7) - \frac{(x-x_n)^7}{21h^5(r-1)^3(s-1)^3}(r^3s - 2r^3 + 4r^2s^2 - 7r^2s - 2r^2 + rs^3 - 7rs^2 + 2rs + 7r - 2s^3 - 2s^2 + 7s) - \frac{(x-x_n)^6}{30h^4(r-1)^3(s-1)^3}(-4r^3s^2 + 5r^3s + 5r^3 - 4r^2s^3 + 4r^2s^2 + 13r^2s - 7r^2 + 5rs^3 + 13rs^2 - 28rs + 5s^3 - 7s^2) + \frac{(x-x_n)^9}{36h^7(r-1)^3(s-1)^3}(2r + 2s - rs - 3) + \frac{hr^5(x-x_n)}{2520(r-1)^3(s-1)^3}(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2) - \frac{rs(x-x_n)^5}{10h^3(r-1)^3(s-1)^3}(r^2s^2 + r^2s - 5r^2 + rs^2 - 7rs + 7r - 5s^2 + 7s) - \frac{r^2s^2(x-x_n)^4}{12h^2(r-1)^3(s-1)^3}(5r + 5s - 3rs - 7),$$

$$\gamma_0 = \frac{(x-x_n)(x_n-x+hr)}{2520h^6r^2s^2}(18h^7r^6s - 5h^7r^7 + 18h^7r^6 - 18h^7r^5s^2 - 72h^7r^5s - 18h^7r^5 + 84h^7r^4s^2 + 84h^7r^4s - 126h^7r^3s^2 - 5h^6r^6x + 5h^6r^6x_n + 18h^6r^5sx - 18h^6r^5sx_n + 18h^6r^5x - 18h^6r^5x_n - 18h^6r^4s^2x + 18h^6r^4s^2x_n - 72h^6r^4sx + 72h^6r^4sx_n - 18h^6r^4x +$$

$$18h^6r^4x_n + 84h^6r^3s^2x - 84h^6r^3s^2x_n + 84h^6r^3sx - 84h^6r^3sx_n - 126h^6r^2s^2x +$$

$$126h^6r^2s^2x_n - 5h^5r^5x^2 + 10h^5r^5xx_n - 5h^5r^5x_n^2 + 18h^5r^4sx^2 - 36h^5r^4sxx_n +$$

$$18h^5r^4sx_n^2 + 18h^5r^4x^2 - 36h^5r^4xx_n + 18h^5r^4x_n^2 - 18h^5r^3s^2x^2 + 36h^5r^3s^2xx_n -$$

$$18h^5r^3s^2x_n^2 - 72h^5r^3sx^2 + 144h^5r^3sxx_n - 72h^5r^3sx_n^2 - 18h^5r^3x^2 + 36h^5r^3xx_n -$$

$$18h^5r^3x_n^2 + 84h^5r^2s^2x^2 - 168h^5r^2s^2xx_n + 84h^5r^2s^2x_n^2 + 84h^5r^2sx^2 - 168h^5r^2sxx_n +$$

$$84h^5r^2sx_n^2 + 294h^5rs^2x^2 - 588h^5rs^2xx_n + 294h^5rs^2x_n^2 - 5h^4r^4x^3 + 15h^4r^4x^2x_n -$$

$$15h^4r^4xx_n^2 + 5h^4r^4x_n^3 + 18h^4r^3sx^3 - 54h^4r^3sx^2x_n + 54h^4r^3sxx_n^2 - 18h^4r^3sx_n^3 +$$

$$18h^4r^3x^3 - 54h^4r^3x^2x_n + 54h^4r^3xx_n^2 - 18h^4r^3x_n^3 - 18h^4r^2s^2x^3 + 54h^4r^2s^2x^2x_n -$$

$$54h^4r^2s^2xx_n^2 + 18h^4r^2s^2x_n^3 - 72h^4r^2sx^3 + 216h^4r^2sx^2x_n - 216h^4r^2sxx_n^2 + 72h^4r^2sx_n^3 -$$

$$18h^4r^2x^3 + 54h^4r^2x^2x_n - 54h^4r^2xx_n^2 + 18h^4r^2x_n^3 - 336h^4rs^2x^3 + 1008h^4rs^2x^2x_n -$$

$$1008h^4rs^2xx_n^2 + 336h^4rs^2x_n^3 - 336h^4rsx^3 + 1008h^4rsx^2x_n - 1008h^4rsxx_n^2 +$$

$$336h^4rsx_n^3 - 126h^4s^2x^3 + 378h^4s^2x^2x_n - 378h^4s^2xx_n^2 + 126h^4s^2x_n^3 - 5h^3r^3x^4 +$$

$$20h^3r^3x^3x_n - 30h^3r^3x^2x_n^2 + 20h^3r^3xx_n^3 - 5h^3r^3x_n^4 + 18h^3r^2sx^4 - 72h^3r^2sx^3x_n +$$

$$108h^3r^2sx^2x_n^2 - 72h^3r^2sxx_n^3 + 18h^3r^2sx_n^4 + 18h^3r^2x^4 - 72h^3r^2x^3x_n + 108h^3r^2x^2x_n^2 -$$

$$72h^3r^2xx_n^3 + 18h^3r^2x_n^4 + 108h^3rs^2x^4 - 432h^3rs^2x^3x_n + 648h^3rs^2x^2x_n^2 - 432h^3rs^2xx_n^3 +$$

$$108h^3rs^2x_n^4 + 432h^3rsx^4 - 1728h^3rsx^3x_n + 2592h^3rsx^2x_n^2 - 1728h^3rsxx_n^3 +$$

$$432h^3rsx_n^4 + 108h^3rx^4 - 432h^3rx^3x_n + 648h^3rx^2x_n^2 - 432h^3rxx_n^3 + 108h^3rx_n^4 +$$

$$168h^3s^2x^4 - 672h^3s^2x^3x_n + 1008h^3s^2x^2x_n^2 - 672h^3s^2xx_n^3 + 168h^3s^2x_n^4 + 168h^3sx^4 -$$

$$672h^3sx^3x_n + 1008h^3sx^2x_n^2 - 672h^3sxx_n^3 + 168h^3sx_n^4 - 5h^2r^2x^5 + 25h^2r^2x^4x_n -$$

$$50h^2r^2x^3x_n^2 + 50h^2r^2x^2x_n^3 - 25h^2r^2xx_n^4 + 5h^2r^2x_n^5 - 150h^2rsx^5 + 750h^2rsx^4x_n -$$

$$1500h^2rsx^3x_n^2 + 1500h^2rsx^2x_n^3 - 750h^2rsxx_n^4 + 150h^2rsx_n^5 - 150h^2rx^5 + 750h^2rx^4x_n -$$

$$1500h^2rx^3x_n^2 + 1500h^2rx^2x_n^3 - 750h^2rxx_n^4 + 150h^2rx_n^5 - 60h^2s^2x^5 + 300h^2s^2x^4x_n -$$

$$600h^2s^2x^3x_n^2 + 600h^2s^2x^2x_n^3 - 300h^2s^2xx_n^4 + 60h^2s^2x_n^5 - 240h^2sx^5 + 1200h^2sx^4x_n -$$

$$2400h^2sx^3x_n^2 + 2400h^2sx^2x_n^3 - 1200h^2sxx_n^4 + 240h^2sx_n^5 - 60h^2x^5 + 300h^2x^4x_n -$$

$$600h^2x^3x_n^2 + 600h^2x^2x_n^3 - 300h^2xx_n^4 + 60h^2x_n^5 + 55hrx^6 - 330hrx^5x_n + 825hrx^4x_n^2 -$$

$$1100hrx^3x_n^3 + 825hrx^2x_n^4 - 330hrxx_n^5 + 55hrx_n^6 + 90hsx^6 - 540hsx^5x_n + 1350hsx^4x_n^2 -$$

$$1800hsx^3x_n^3 + 1350hsx^2x_n^4 - 540hsxx_n^5 + 90hsx_n^6 + 90hx^6 - 540hx^5x_n + 1350hx^4x_n^2 -$$

$$1800hx^3x_n^3 + 1350hx^2x_n^4 - 540hxx_n^5 + 90hx_n^6 - 35x^7 + 245x^6x_n - 735x^5x_n^2 + 1225x^4x_n^3 -$$

$$1225x^3x_n^4 + 735x^2x_n^5 - 245xx_n^6 + 35x_n^7),$$

$\gamma_r = \frac{(x-x_n)^9}{72h^6r^2(r-s)^2(r-1)^2} - \frac{(x-x_n)^6}{30h^3r^2(r-s)^3(r-1)^2}(r^2s^2 + 4r^2s + r^2 - rs^3 - 2rs^2 + rs - 2s^3 - 2s^2) - \frac{s^2(x-x_n)^4}{12hr(r-s)^2(r-1)^2} + \frac{h^2r^2(x-x_n)}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2) - \frac{(x-x_n)^8}{56h^5r^2(r-s)^3(r-1)^2}(r^2 + rs + 2r - 2s^2 - 2s) + \frac{(x-x_n)^7}{42h^4r^2(r-s)^3(r-1)^2}(2r^2s + 2r^2 - rs^2 + 2rs + r - s^3 - 4s^2 - s) - \frac{s(x-x_n)^5}{20h^2r^2(r-s)^3(r-1)^2}(-2r^2s - 2r^2 + 2rs^2 + rs + s^2),$

$\gamma_s = \frac{(x-x_n)^9}{72h^6s^2(r-s)^2(s-1)^2} + \frac{(x-x_n)^6}{30h^3s^2(r-s)^3(s-1)^2}(-r^3s - 2r^3 + r^2s^2 - 2r^2s - 2r^2 + 4rs^2 + rs + s^2) - \frac{r^2(x-x_n)^4}{12hs(r-s)^2(s-1)^2} + \frac{(x-x_n)^8}{56h^5s^2(r-s)^3(s-1)^2}(-2r^2 + rs - 2r + s^2 + 2s) + \frac{(x-x_n)^7}{42h^4s^2(r-s)^3(s-1)^2}(r^3 + r^2s + 4r^2 - 2rs^2 - 2rs + r - 2s^2 - s) + \frac{r(x-x_n)^5}{20h^2s^2(r-s)^3(s-1)^2}(2r^2s + r^2 - 2rs^2 + rs - 2s^2) - \frac{h^2r^5(x-x_n)}{2520s^2(r-s)^2(s-1)^2}(18r - 42s + 36rs - 9r^2s - 18r^2 + 5r^3),$

$\gamma_1 = \frac{(x-x_n)^9}{72h^6(r-1)^2(s-1)^2} - \frac{(x-x_n)^6}{30h^3(r-1)^2(s-1)^2}(2r^2s + r^2 + 2rs^2 + 4rs + s^2) + \frac{(x-x_n)^7}{42h^4(r-1)^2(s-1)^2}(r^2 + 4rs + 2r + s^2 + 2s) - \frac{(x-x_n)^8}{56h^5(r-1)^2(s-1)^2}(2r + 2s + 1) - \frac{r^2s^2(x-x_n)^4}{12h(r-1)^2(s-1)^2} - \frac{h^2r^5(x-x_n)}{2520(r-1)^2(s-1)^2}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36rs - 42s^2) + \frac{rs(x-x_n)^5}{20h^2(r-1)^2(s-1)^2}(2r + 2s + rs).$

The following equations are obtained after evaluating (3.31) at the non-interpolating points x_{n+s} and x_{n+1}:

$y_{n+s} - \frac{(r-s)y_n}{r} - \frac{sy_{n+r}}{r} = -\frac{h^2f_n}{2520r^3s^2}(10r^9s + 10r^9 - 36r^8s^2 - 53r^8s - 36r^8 + 36r^7s^3 + 108r^7s^2 + 108r^7s + 36r^7 - 90r^6s^3 - 24r^6s^2 - 90r^6s - 168r^5s^3 - 168r^5s^2 + 882r^4s^3 - 36r^3s^7 + 90r^3s^6 + 168r^3s^5 - 882r^3s^4 + 36r^2s^8 - 108r^2s^7 + 24r^2s^6 + 168r^2s^5 - 10rs^9 + 53rs^8 - 108rs^7 + 90rs^6 - 10s^9 + 36s^8 - 36s^7) + \frac{h^3g_n}{2520r^2s}(18r^7s - 5r^8 + 18r^7 - 18r^6s^2 - 72r^6s - 18r^6 + 84r^5s^2 + 84r^5s - 126r^4s^2 + 18r^2s^6 - 84r^2s^5 + 126r^2s^4 - 18rs^7 + 72rs^6 - 84rs^5 + 5s^8 - 18s^7 + 18s^6) + \frac{h^3g_{n+s}}{2520s(r-s)(s-1)^2}(4r^6s - 5r^7 + 18r^6 + 4r^5s^2 - 18r^5s - 18r^5 + 4r^4s^3 - 18r^4s^2 + 24r^4s + 4r^3s^4 - 18r^3s^3 + 24r^3s^2 + 4r^2s^5 - 18r^2s^4 + 24r^2s^3 - 20rs^6 + 66rs^5 - 60rs^4 + 10s^7 - 30s^6 + 24s^5) - \frac{h^3sg_{n+1}}{2520(r-1)^2(s-1)^2}(5r^8 - 18r^7s - 9r^7 + 18r^6s^2 + 36r^6s - 42r^5s^2 - 18r^2s^6 + 42r^2s^5 + 18rs^7 - 36rs^6 - 5s^8 + 9s^7) + \frac{h^2sf_{n+1}}{2520(r-1)^3(s-1)^3}(10r^9s - 20r^9 - 36r^8s^2 + 25r^8s + 75r^8 + 36r^7s^3 + 108r^7s^2 - 243r^7s - 63r^7 - 198r^6s^3 + 138r^6s^2 + 252r^6s + 210r^5s^3 - 294r^5s^2 - 36r^3s^7 + 198r^3s^6 - 210r^3s^5$

$+36r^2s^8 - 108r^2s^7 - 138r^2s^6 + 294r^2s^5 - 10rs^9 - 25rs^8 + 243rs^7 - 252rs^6 + 20s^9 - 75s^8 + 63s^7) - \frac{h^2 f_{n+s}}{2520s^2(r-s)^2(s-1)^3}(10r^8 - 20r^8s + 55r^7s^2 + 35r^7s - 36r^7 - 8r^6s^3 - 208r^6s^2 + 72r^6s + 36r^6 - 8r^5s^4 + 44r^5s^3 + 210r^5s^2 - 162r^5s - 8r^4s^5 + 44r^4s^4 - 84r^4s^3 + 48r^4s^2 - 8r^3s^6 + 44r^3s^5 - 84r^3s^4 + 48r^3s^3 - 188r^2s^7 + 764r^2s^6 - 1050r^2s^5 + 426r^2s^4 + 280rs^8 - 1072rs^7 + 1368rs^6 - 540rs^5 - 105s^9 + 385s^8 - 468s^7 + 180s^6) + \frac{h^2 s f_{n+r}}{2520r^3(r-s)^2(r-1)^3}(280r^8s - 105r^9 + 385r^8 - 188r^7s^2 - 1072r^7s - 468r^7 - 8r^6s^3 + 764r^6s^2 + 1368r^6s + 180r^6 - 8r^5s^4 + 44r^5s^3 - 1050r^5s^2 - 540r^5s - 8r^4s^5 + 44r^4s^4 - 84r^4s^3 + 426r^4s^2 - 8r^3s^6 + 44r^3s^5 - 84r^3s^4 + 48r^3s^3 + 55r^2s^7 - 208r^2s^6 + 210r^2s^5 + 48r^2s^4 - 20rs^8 + 35rs^7 + 72rs^6 - 162rs^5 + 10s^8 - 36s^7 + 36s^6) + \frac{h^3 s g_{n+r}}{2520r^2(r-s)(r-1)^2}(10r^7 - 20r^6s - 30r^6 + 4r^5s^2 + 66r^5s + 24r^5 + 4r^4s^3 - 18r^4s^2 - 60r^4s + 4r^3s^4 - 18r^3s^3 + 24r^3s^2 + 4r^2s^5 - 18r^2s^4 + 24r^2s^3 + 4rs^6 - 18rs^5 + 24rs^4 - 5s^7 + 18s^6 - 18s^5),$ (3.32)

$y_{n+1} - \frac{y_{n+r}}{r} - \frac{(r-1)y_n}{r} = \frac{h^3 g_{n+1}}{2520(r-1)(s-1)^2}(18r^6s - 5r^7 + 4r^6 - 18r^5s^2 - 18r^5s + 4r^5 + 24r^4s^2 - 18r^4s + 4r^4 + 24r^3s^2 - 18r^3s + 4r^3 + 24r^2s^2 - 18r^2s + 4r^2 - 60rs^2 + 66rs - 20r + 24s^2 - 30s + 10) + \frac{h^2 f_{n+1}}{2520(r-1)^2(s-1)^3}(10r^8s - 20r^8 - 36r^7s^2 + 35r^7s + 55r^7 + 36r^6s^3 + 72r^6s^2 - 208r^6s - 8r^6 - 162r^5s^3 + 210r^5s^2 + 44r^5s - 8r^5 + 48r^4s^3 - 84r^4s^2 + 44r^4s - 8r^4 + 48r^3s^3 - 84r^3s^2 + 44r^3s - 8r^3 + 426r^2s^3 - 1050r^2s^2 + 764r^2s - 188r^2 - 540rs^3 + 1368rs^2 - 1072rs + 280r + 180s^3 - 468s^2 + 385s - 105) + \frac{h^3 g_{n+r}}{2520r^2(r-s)^2(r-1)}(10r^7 - 30r^6s - 20r^6 + 24r^5s^2 + 66r^5s + 4r^5 - 60r^4s^2 - 18r^4s + 4r^4 + 24r^3s^2 - 18r^3s + 4r^3 + 24r^2s^2 - 18r^2s + 4r^2 + 24rs^2 - 18rs + 4r - 18s^2 + 18s - 5) - \frac{h^3(r-1)g_n}{2520r^2s^2}(5r^7 - 18r^6s - 13r^6 + 18r^5s^2 + 54r^5s + 5r^5 - 66r^4s^2 - 30r^4s + 5r^4 + 60r^3s^2 - 30r^3s + 5r^3 + 60r^2s^2 - 30r^2s + 5r^2 - 66rs^2 + 54rs - 13r + 18s^2 - 18s + 5) + \frac{h^2 f_{n+r}}{2520r^3(r-s)^3(r-1)^2}(385r^8s - 105r^9 + 280r^8 - 468r^7s^2 - 1072r^7s - 188r^7 + 180r^6s^3 + 1368r^6s^2 + 764r^6s - 8r^6 - 540r^5s^3 - 1050r^5s^2 + 44r^5s - 8r^5 + 426r^4s^3 - 84r^4s^2 + 44r^4s - 8r^4 + 48r^3s^3 - 84r^3s^2 + 44r^3s - 8r^3 + 48r^2s^3 + 210r^2s^2 - 208r^2s + 55r^2 - 162rs^3 + 72rs^2 + 35rs - 20r + 36s^3 - 36s^2 + 10s) - \frac{h^2(r-1)f_n}{2520r^3s^3}(10r^8s + 10r^8 - 36r^7s^2 - 43r^7s - 26r^7 + 36r^6s^3 + 72r^6s^2 + 65r^6s + 10r^6 - 54r^5s^3 + 48r^5s^2 - 25r^5s + 10r^5 - 222r^4s^3 - 120r^4s^2 - 25r^4s + 10r^4 + 660r^3s^3 - 120r^3s^2 - 25r^3s + 10r^3 - 222r^2s^3 +$

$48r^2s^2 + 65r^2s - 26r^2 - 54rs^3 + 72rs^2 - 43rs + 10r + 36s^3 - 36s^2 + 10s) - \frac{h^3(r-1)g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(27rs-9s-13r-15r^2s-15r^3s-15r^4s+27r^5s-9r^6s+5r^2+5r^3+5r^4+5r^5-13r^6+5r^7+5) - \frac{h^2(r-1)f_{n+s}}{2520s^3(r-s)^3(s-1)^3}(10r^8-20r^8s+75r^7s^2+5r^7s-26r^7-63r^6s^3-168r^6s^2+113r^6s+10r^6+189r^5s^3-30r^5s^2-85r^5s+10r^5-105r^4s^3+180r^4s^2-85r^4s+10r^4-105r^3s^3+180r^3s^2-85r^3s+10r^3-105r^2s^3-30r^2s^2+113r^2s-26r^2+189rs^3-168rs^2+5rs+10r-63s^3+75s^2-20s).$ (3.33)

Differentiating (3.31) once we get

$$y'(x) = \sum_{i=0,r}\frac{d}{dx}\alpha_i(x)y_{n+i} + \sum_{i=0,r,s,1}\frac{d}{dx}\beta_i(x)f_{n+i} + \sum_{i=0,r,s,1}\frac{d}{dx}\gamma_i(x)g_{n+i}. \quad (3.34)$$

Now, evaluating (3.34) at all points, i.e x_n, x_{n+r}, x_{n+s} and x_{n+1}, produces

$y'_n - \frac{y_{n+r}}{hr} + \frac{y_n}{hr} = \frac{hrf_n}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3) - \frac{h^2r^2g_n}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2) - \frac{hrf_{n+r}}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3) + \frac{hr^5f_{n+1}}{2520(r-1)^3(s-1)^3}(10r^4s-20r^4-36r^3s^2+25r^3s+75r^3+36r^2s^3+108r^2s^2-243r^2s-63r^2-198rs^3+138rs^2+252rs+210s^3-294s^2) - \frac{h^2r^5g_{n+1}}{2520(r-1)^2(s-1)^2}(5r^3-18r^2s-9r^2+18rs^2+36rs-42s^2) + \frac{h^2r^2g_{n+r}}{1260(r-s)^2(r-1)^2}(5r^4-15r^3s-15r^3+12r^2s^2+48r^2s+12r^2-42rs^2-42rs+42s^2) - \frac{h^2r^5g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(18r-42s+36rs-9r^2s-18r^2+5r^3) - \frac{hr^5f_{n+s}}{2520s^3(r-s)^3(s-1)^3}(-20r^4s+10r^4+75r^3s^2+25r^3s-36r^3-63r^2s^3-243r^2s^2+108r^2s+36r^2+252rs^3+138rs^2-198rs-294s^3+210s^2),$ (3.35)

$y'_{n+r} - \frac{y_{n+r}}{hr} + \frac{y_n}{hr} = \frac{hrf_n}{2520s^3}(20r^5s + 20r^5 - 60r^4s^2 - 85r^4s - 60r^4 + 48r^3s^3 + 132r^3s^2 + 132r^3s + 48r^3 - 78r^2s^3 + 24r^2s^2 - 78r^2s - 168rs^3 - 168rs^2 + 378s^3) + \frac{h^2r^2g_n}{1260s^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2) + \frac{hrf_{n+r}}{2520(r-s)^3(r-1)^3}(525r^6 - 1715r^5s - 1715r^5 + 1860r^4s^2 + 5665r^4s + 1860r^4 - 660r^3s^3-6216r^3s^2-6216r^3s-660r^3+2220r^2s^3+6906r^2s^2+2220r^2s-2478rs^3-2478rs^2+882s^3) - \frac{hr^5f_{n+1}}{2520(r-1)^3(s-1)^3}(20r^4s-40r^4-60r^3s^2+35r^3s+135r^3+48r^2s^3$

$+168r^2s^2 - 345r^2s - 105r^2 - 222rs^3 + 114rs^2 + 336rs + 210s^3 - 294s^2) + \frac{h^2r^5g_{n+1}}{2520(r-1)^2(s-1)^2}(10r^3 - 30r^2s - 15r^2 + 24rs^2 + 48rs - 42s^2) - \frac{h^2r^2g_{n+r}}{2520(r-s)^2(r-1)^2}(35r^4 - 90r^3s - 90r^3 + 60r^2s^2 + 240r^2s + 60r^2 - 168rs^2 - 168rs + 126s^2) + \frac{h^2r^5g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(24r - 42s + 48rs - 15r^2s - 30r^2 + 10r^3) + \frac{hr^5f_{n+r}}{2520s^3(r-s)^3(s-1)^3}(-40r^4s + 20r^4 + 135r^3s^2 + 35r^3s - 60r^3 - 105r^2s^3 - 345r^2s^2 + 168r^2s + 48r^2 + 336rs^3 + 114rs^2 - 222rs - 294s^3 + 210s^2),$ (3.36)

$y'_{n+s} - \frac{y_{n+r}}{hr} + \frac{y_n}{hr} = +\frac{hf_{n+1}}{2520(r-1)^3(s-1)^3}(10r^9s - 20r^9 - 36r^8s^2 + 25r^8s + 75r^8 + 36r^7s^3 + 108r^7s^2 - 243r^7s - 63r^7 - 198r^6s^3 + 138r^6s^2 + 252r^6s + 210r^5s^3 - 294r^5s^2 - 84r^3s^7 + 420r^3s^6 - 420r^3s^5 + 96r^2s^8 - 276r^2s^7 - 252r^2s^6 + 588r^2s^5 - 30rs^9 - 60rs^8 + 588rs^7 - 588rs^6 + 60s^9 - 210s^8 + 168s^7) + \frac{h^2g_n}{2520r^2s^2}(18r^7s - 5r^8 + 18r^7 - 18r^6s^2 - 72r^6s - 18r^6 + 84r^5s^2 + 84r^5s - 126r^4s^2 + 42r^2s^6 - 168r^2s^5 + 210r^2s^4 - 48rs^7 + 168rs^6 - 168rs^5 + 15s^8 - 48s^7 + 42s^6) + \frac{hf_n}{2520r^3s^3}(36r^8s^2 - 10r^9 - 10r^9s + 53r^8s + 36r^8 - 36r^7s^3 - 108r^7s^2 - 108r^7s - 36r^7 + 90r^6s^3 + 24r^6s^2 + 90r^6s + 168r^5s^3 + 168r^5s^2 - 882r^4s^3 + 84r^3s^7 - 168r^3s^6 - 336r^3s^5 + 1260r^3s^4 - 96r^2s^8 + 240r^2s^7 - 336r^2s^5 + 30rs^9 - 138rs^8 + 240rs^7 - 168rs^6 + 30s^9 - 96s^8 + 84s^7)\frac{h^2g_{n+1}}{2520(r-1)^2(s-1)^2}(5r^8 - 18r^7s - 9r^7 + 18r^6s^2 + 36r^6s - 42r^5s^2 - 42r^2s^6 + 84r^2s^5 + 48rs^7 - 84rs^6 - 15s^8 + 24s^7) - \frac{hf_{n+r}}{2520r^3(r-s)^3(r-1)^3}(105r^10 - 385r^9s - 385r^9 + 468r^8s^2 + 1457r^8s + 468r^8 - 180r^7s^3 - 1836r^7s^2 - 1836r^7s - 180r^7 + 720r^6s^3 + 2418r^6s^2 + 720r^6s - 966r^5s^3 - 966r^5s^2 + 378r^4s^3 - 168r^3s^7 + 588r^3s^6 - 588r^3s^5 + 210r^2s^8 - 588r^2s^7 + 252r^2s^6 + 420r^2s^5 - 60rs^9 + 60rs^8 + 276rs^7 - 420rs^6 + 30s^9 - 96s^8 + 84s^7) - \frac{hf_{n+s}}{2520s^3(r-s)^3(s-1)^3}(10r^9 - 20r^9s + 75r^8s^2 + 25r^8s - 36r^8 - 63r^7s^3 - 243r^7s^2 + 108r^7s + 36r^7 + 252r^6s^3 + 138r^6s^2 - 198r^6s - 294r^5s^3 + 210r^5s^2 - 840r^3s^7 + 2940r^3s^6 - 3444r^3s^5 + 1260r^3s^4 + 2328r^2s^8 - 8052r^2s^7 + 9324r^2s^6 - 3444r^2s^5 - 2100rs^9 + 7122rs^8 - 8052rs^7 + 2940rs^6 + 630s^{10} - 2100s^9 + 2328s^8 - 840s^7) + \frac{h^2g_{n+r}}{2520r^2(r-s)^2(r-1)^2}(10r^8 - 30r^7s - 30r^7 + 24r^6s^2 + 96r^6s + 24r^6 - 84r^5s^2 - 84r^5s + 84r^4s^2 - 24rs^7 + 84rs^6 - 84rs^5 + 15s^8 - 48s^7 + 42s^6) - \frac{h^2g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(5r^8 - 9r^7s - 18r^7 + 36r^6s + 18r^6 - 42r^5s + 84r^2s^6 - 252r^2s^5 + 210r^2s^4 - 120rs^7 + 336rs^6 - 252rs^5 + 45s^8 - 120s^7 + 84s^6),$ (3.37)

$y'_{n+1} - \frac{y_{n+r}}{hr} + \frac{y_n}{hr} = \frac{hf_{n+1}}{2520(r-1)^3(s-1)^3}(10r^9s - 20r^9 - 36r^8s^2 + 25r^8s + 75r^8 + 36r^7s^3 + 108r^7s^2 - 243r^7s - 63r^7 - 198r^6s^3 + 138r^6s^2 + 252r^6s + 210r^5s^3 - 294r^5s^2 + 1260r^3s^3 - 3444r^3s^2 + 2940r^3s - 840r^3 - 3444r^2s^3 + 9324r^2s^2 - 8052r^2s + 2328r^2 + 2940rs^3 - 8052rs^2 + 7122rs - 2100r - 840s^3 + 2328s^2 - 2100s + 630) - \frac{h^2g_{n+1}}{2520(r-1)^2(s-1)^2}(5r^8 - 18r^7s - 9r^7 + 18r^6s^2 + 36r^6s - 42r^5s^2 + 210r^2s^2 - 252r^2s + 84r^2 - 252rs^2 + 336rs - 120r + 84s^2 - 120s + 45) - \frac{h^2g_n}{2520r^2s^2}(5r^8 - 18r^7s - 18r^7 + 18r^6s^2 + 72r^6s + 18r^6 - 84r^5s^2 - 84r^5s + 126r^4s^2 - 210r^2s^2 + 168r^2s - 42r^2 + 168rs^2 - 168rs + 48r - 42s^2 + 48s - 15) + \frac{hf_n}{2520r^3s^3}(36r^8s^2 - 10r^9 - 10r^9s + 53r^8s + 36r^8 - 36r^7s^3 - 108r^7s^2 - 108r^7s - 36r^7 + 90r^6s^3 + 24r^6s^2 + 90r^6s + 168r^5s^3 + 168r^5s^2 - 882r^4s^3 + 1260r^3s^3 - 336r^3s^2 - 168r^3s + 84r^3 - 336r^2s^3 + 240r^2s - 96r^2 - 168rs^3 + 240rs^2 - 138rs + 30r + 84s^3 - 96s^2 + 30s) + \frac{hf_{n+s}}{2520s^3(r-s)^3(s-1)^3}(20r^9s - 10r^9 - 75r^8s^2 - 25r^8s + 36r^8 + 63r^7s^3 + 243r^7s^2 - 108r^7s - 36r^7 - 252r^6s^3 - 138r^6s^2 + 198r^6s + 294r^5s^3 - 210r^5s^2 + 420r^3s^2 - 420r^3s + 84r^3 - 588r^2s^3 + 252r^2s^2 + 276r^2s - 96r^2 + 588rs^3 - 588rs^2 + 60rs + 30r - 168s^3 + 210s^2 - 60s) - \frac{h^2g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(48r + 24s - 84rs + 84r^2s - 42r^5s + 36r^6s - 9r^7s - 42r^2 + 18r^6 - 18r^7 + 5r^8 - 15) + \frac{h^2g_{n+r}}{2520r^2(r-s)^2(r-1)^2}(10r^8 - 30r^7s - 30r^7 + 24r^6s^2 + 96r^6s + 24r^6 - 84r^5s^2 - 84r^5s + 84r^4s^2 - 84rs^2 + 84rs - 24r + 42s^2 - 48s + 15) - \frac{hf_{n+r}}{2520r^3(r-s)^3(r-1)^3}(105r^{10} - 385r^9s - 385r^9 + 468r^8s^2 + 1457r^8s + 468r^8 - 180r^7s^3 - 1836r^7s^2 - 1836r^7s - 180r^7 + 720r^6s^3 + 2418r^6s^2 + 720r^6s - 966r^5s^3 - 966r^5s^2 + 378r^4s^3 - 588r^3s^2 + 588r^3s - 168r^3 + 420r^2s^3 + 252r^2s^2 - 588r^2s + 210r^2 - 420rs^3 + 276rs^2 + 60rs - 60r + 84s^3 - 96s^2 + 30s). \quad (3.38)$

Following the same procedure in Section 3.2, the main block form below is obtained by combining Equations (3.32)-(3.33) and (3.35):

$$\begin{aligned} H^{[2]_2}Y_{n+1}^{[2]_2} &= M_1^{[2]_2}Y_n^{[2]_2} + M_2^{[2]_2}Y_{n-1}^{[2]_2} + E_1^{[2]_2}F_n^{[2]_2} + E_2^{[2]_2}F_{n+1}^{[2]_2} + K_1^{[2]_2}G_n^{[2]_2} \\ &\quad + K_2^{[2]_2}G_{n+1}^{[2]_2} \end{aligned} \quad (3.39)$$

where

$$H^{[2]_2} = \begin{pmatrix} \frac{-s}{r} & 1 & 0 \\ \frac{-1}{r} & 0 & 1 \\ \frac{-1}{hr} & 0 & 0 \end{pmatrix}, Y_{n+1}^{[2]_2} = \begin{pmatrix} y_{n+r} \\ y_{n+s} \\ y_{n+1} \end{pmatrix}, M_1^{[2]_2} = \begin{pmatrix} 0 & 0 & \frac{r-s}{r} \\ 0 & 0 & \frac{r-1}{r} \\ 0 & 0 & \frac{-1}{hr} \end{pmatrix},$$

$$Y_n^{[2]_2} = \begin{pmatrix} y_{n-s} \\ y_{n-r} \\ y_n \end{pmatrix}, M_2^{[2]_2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, Y_{n-1}^{[2]_2} = \begin{pmatrix} y'_{n-s} \\ y'_{n-r} \\ y'_n \end{pmatrix},$$

$$F_n^{[2]_2} = \begin{pmatrix} f_{n-s} \\ f_{n-r} \\ f_n \end{pmatrix}, \quad F_{n+1}^{[2]_2} = \begin{pmatrix} f_{n+r} \\ f_{n+s} \\ f_{n+1} \end{pmatrix}, \quad G_n^{[2]_2} = \begin{pmatrix} g_{n-s} \\ g_{n-r} \\ g_n \end{pmatrix},$$

$$G_{n+1}^{[2]_2} = \begin{pmatrix} g_{n+r} \\ g_{n+s} \\ g_{n+1} \end{pmatrix}.$$

Multiplying both sides of Equation (3.39) by the inverse of $H^{[2]_2}$ yields

$$\begin{aligned} I_3 Y_{n+1}^{[2]_2} \;=\; & \hat{M}_1^{[2]_2} Y_n^{[2]_2} + h\hat{M}_2^{[2]_2} Y_{n-1}^{[2]_2} + h^2[\hat{E}_1^{[2]_2} F_n^{[2]_2} + \hat{E}_2^{[2]_2} F_{n+1}^{[2]_2}] + h^3 \\ & [\hat{K}_1^{[2]_2} G_n^{[2]_2} + \hat{K}_2^{[2]_2} G_{n+1}^{[2]_2}] \end{aligned} \tag{3.40}$$

where

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \hat{M}_1^{[2]_2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \hat{M}_2^{[2]_2} = \begin{pmatrix} 0 & 0 & r \\ 0 & 0 & s \\ 0 & 0 & 1 \end{pmatrix},$$

$$\hat{E}_1^{[2]_2} = \begin{pmatrix} 0 & 0 & \hat{E}_{1_{13}}^{[2]_2} \\ 0 & 0 & \hat{E}_{1_{23}}^{[2]_2} \\ 0 & 0 & \hat{E}_{1_{33}}^{[2]_2} \end{pmatrix}, \hat{E}_2^{[2]_2} = \begin{pmatrix} \hat{E}_{2_{11}}^{[2]_2} & \hat{E}_{2_{12}}^{[2]_2} & \hat{E}_{2_{13}}^{[2]_2} \\ \hat{E}_{2_{21}}^{[2]_2} & \hat{E}_{2_{22}}^{[2]_2} & \hat{E}_{2_{23}}^{[2]_2} \\ \hat{E}_{2_{31}}^{[2]_2} & \hat{E}_{2_{32}}^{[2]_2} & \hat{E}_{2_{33}}^{[2]_2} \end{pmatrix},$$

$$\hat{K}_1^{[2]_2} = \begin{pmatrix} 0 & 0 & \hat{K}_{1_{13}}^{[2]_2} \\ 0 & 0 & \hat{K}_{1_{23}}^{[2]_2} \\ 0 & 0 & \hat{K}_{1_{33}}^{[2]_2} \end{pmatrix}, \hat{K}_2^{[2]_2} = \begin{pmatrix} \hat{K}_{2_{11}}^{[2]_2} & \hat{K}_{2_{12}}^{[2]_2} & \hat{K}_{2_{13}}^{[2]_2} \\ \hat{K}_{2_{21}}^{[2]_2} & \hat{K}_{2_{22}}^{[2]_2} & \hat{K}_{2_{23}}^{[2]_2} \\ \hat{K}_{2_{31}}^{[2]_2} & \hat{K}_{2_{32}}^{[2]_2} & \hat{K}_{2_{33}}^{[2]_2} \end{pmatrix}.$$

whose elements are given below :

$\hat{E}_{1_{13}}^{[2]_2} = -\frac{r^2}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3)$,

$\hat{E}_{1_{23}}^{[2]_2} = -\frac{s^2}{2520r^3}(-36r^3s^3 + 90r^3s^2 + 168r^3s - 882r^3 + 36r^2s^4 - 108r^2s^3 + 24r^2s^2 + 168r^2s - 10rs^5 + 53rs^4 - 108rs^3 + 90rs^2 - 10s^5 + 36s^4 - 36s^3)$,

$\hat{E}_{1_{33}}^{[2]_2} = -\frac{1}{2520r^3s^3}(-882r^3s^3 + 168r^3s^2 + 90r^3s - 36r^3 + 168r^2s^3 + 24r^2s^2 - 108r^2s + 36r^2 + 90rs^3 - 108rs^2 + 53rs - 10r - 36s^3 + 36s^2 - 10s)$.

$\hat{E}_{2_{11}}^{[2]_2} = \frac{r^2}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3)$,

$\hat{E}_{2_{21}}^{[2]_2} = -\frac{s^6}{2520r^3(r-s)^3(r-1)^3}(-63r^3s^2 + 252r^3s - 294r^3 + 75r^2s^3 - 243r^2s^2 + 138r^2s + 210r^2 - 20rs^4 + 25rs^3 + 108rs^2 - 198rs + 10s^4 - 36s^3 + 36s^2)$,

$\hat{E}_{2_{31}}^{[2]_2} = -\frac{1}{2520r^3(r-s)^3(r-1)^3}(-294r^3s^2 + 252r^3s - 63r^3 + 210r^2s^3 + 138r^2s^2 - 243r^2s + 75r^2 - 198rs^3 + 108rs^2 + 25rs - 20r + 36s^3 - 36s^2 + 10s)$,

$\hat{E}_{2_{12}}^{[2]_2} = \frac{r^6}{2520s^3(r-s)^3(s-1)^3}(-20r^4s + 10r^4 + 75r^3s^2 + 25r^3s - 36r^3 - 63r^2s^3 - 243r^2s^2 + 108r^2s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2)$,

$\hat{E}_{2_{22}}^{[2]_2} = -\frac{s^2}{2520(r-s)^3(s-1)^3}(-180r^3s^3 + 720r^3s^2 - 966r^3s + 378r^3 + 468r^2s^4 - 1836r^2s^3 + 2418r^2s^2 - 966r^2s - 385rs^5 + 1457rs^4 - 1836rs^3 + 720rs^2 + 105s^6 - 385s^5 + 468s^4 - 180s^3)$,

$\hat{E}^{[2]_2}_{2_{32}} = \frac{1}{2520s^3(r-s)^3(s-1)^3}(210r^3s^2 - 198r^3s + 36r^3 - 294r^2s^3 + 138r^2s^2 + 108r^2s - 36r^2 + 252rs^3 - 243rs^2 + 25rs + 10r - 63s^3 + 75s^2 - 20s),$

$\hat{E}^{[2]_2}_{2_{13}} = -\frac{r^6}{2520(r-1)^3(s-1)^3}(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2),$

$\hat{E}^{[2]_2}_{2_{23}} = -\frac{s^6}{2520(r-1)^3(s-1)^3}(36r^3s^2 - 198r^3s + 210r^3 - 36r^2s^3 + 108r^2s^2 + 138r^2s - 294r^2 + 10rs^4 + 25rs^3 - 243rs^2 + 252rs - 20s^4 + 75s^3 - 63s^2),$

$\hat{E}^{[2]_2}_{2_{33}} = -\frac{1}{2520(r-1)^3(s-1)^3}(-378r^3s^3 + 966r^3s^2 - 720r^3s + 180r^3 + 966r^2s^3 - 2418r^2s^2 + 1836r^2s - 468r^2 - 720rs^3 + 1836rs^2 - 1457rs + 385r + 180s^3 - 468s^2 + 385s - 105),$

$\hat{K}^{[2]_2}_{1_{13}} = \frac{r^3}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2),$

$\hat{K}^{[2]_2}_{1_{23}} = \frac{s^3}{2520r^2}(18r^2s^2 - 84r^2s + 126r^2 - 18rs^3 + 72rs^2 - 84rs + 5s^4 - 18s^3 + 18s^2),$

$\hat{K}^{[2]_2}_{1_{33}} = \frac{1}{2520r^2s^2}(126r^2s^2 - 84r^2s + 18r^2 - 84rs^2 + 72rs - 18r + 18s^2 - 18s + 5),$

$\hat{K}^{[2]_2}_{2_{11}} = -\frac{r^3}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2),$

$\hat{K}^{[2]_2}_{2_{21}} = -\frac{s^6}{2520r^2(r-s)^2(r-1)^2}(42r - 18s - 36rs + 9rs^2 + 18s^2 - 5s^3),$

$\hat{K}^{[2]_2}_{2_{31}} = -\frac{1}{2520r^2(r-s)^2(r-1)^2}(9r + 18s - 36rs + 42rs^2 - 18s^2 - 5),$

$\hat{K}^{[2]_2}_{2_{12}} = \frac{r^6}{2520s^2(r-s)^2(s-1)^2}(18r - 42s + 36rs - 9r^2s - 18r^2 + 5r^3),$

$\hat{K}^{[2]_2}_{2_{22}} = -\frac{s^3}{1260(r-s)^2(s-1)^2}(12r^2s^2 - 42r^2s + 42r^2 - 15rs^3 + 48rs^2 - 42rs + 5s^4 - 15s^3 + 12s^2),$

$\hat{K}^{[2]_2}_{2_{32}} = -\frac{1}{2520s^2(r-s)^2(s-1)^2}(18r + 9s - 36rs + 42r^2s - 18r^2 - 5),$

$$\hat{K}^{[2]_2}_{2_{13}} = \frac{r^6}{2520(r-1)^2(s-1)^2}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36rs - 42s^2),$$

$$\hat{K}^{[2]_2}_{2_{23}} = \frac{s^6}{2520(r-1)^2(s-1)^2}(18r^2s - 42r^2 - 18rs^2 + 36rs + 5s^3 - 9s^2),$$

$$\hat{K}^{[2]_2}_{2_{33}} = -\frac{1}{1260(r-1)^2(s-1)^2}(42r^2s^2 - 42r^2s + 12r^2 - 42rs^2 + 48rs - 15r + 12s^2 - 15s + 5).$$

From (3.40), the following equations are obtained:

$$\begin{aligned} y_{n+r} &= y_n + hry'_n - \frac{h^2r^2f_n}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3) + \frac{h^3r^3g_n}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2) + \frac{h^3r^6g_{n+1}}{2520(r-1)^2(s-1)^2}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36rs - 42s^2) - \frac{h^2r^6f_{n+1}}{2520(r-1)^3(s-1)^3}(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2) + \frac{h^2r^2f_{n+r}}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3) - \frac{h^3r^3g_{n+r}}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2) + \frac{h^2r^6f_{n+s}}{2520s^3(r-s)^3(s-1)^3}(-20r^4s + 10r^4 + 75r^3s^2 + 25r^3s - 36r^3 - 63r^2s^3 - 243r^2s^2 + 108r^2s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2) + \frac{h^3r^6g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(18r - 42s + 36rs - 9r^2s - 18r^2 + 5r^3), \end{aligned} \tag{3.41}$$

$$y_{n+s} = y_n + hsy'_n - \frac{h^2s^2f_n}{2520r^3}(-36r^3s^3 + 90r^3s^2 + 168r^3s - 882r^3 + 36r^2s^4 - 108r^2s^3 + 24r^2s^2 + 168r^2s - 10rs^5 + 53rs^4 - 108rs^3 + 90rs^2 - 10s^5 + 36s^4 - 36s^3) + \frac{h^3s^3g_n}{2520r^2}(18r^2s^2 - 84r^2s + 126r^2 - 18rs^3 + 72rs^2 - 84rs + 5s^4 - 18s^3 + 18s^2) + \frac{h^3s^6g_{n+1}}{2520(r-1)^2(s-1)^2}(18r^2s - 42r^2 - 18rs^2 + 36rs + 5s^3 - 9s^2) - \frac{h^2s^6f_{n+1}}{2520(r-1)^3(s-1)^3}(36r^3s^2 - 198r^3s + 210r^3 - 36r^2s^3 + 108r^2s^2 + 138r^2s - 294r^2 + 10rs^4 + 25rs^3 - 243rs^2 + 252rs - 20s^4 + 75s^3 - 63s^2) - \frac{h^2s^2f_{n+s}}{2520(r-s)^3(s-1)^3}(-180r^3s^3 + 720r^3s^2 - 966r^3s + 378r^3 + 468r^2s^4 - 1836r^2s^3 + 2418r^2s^2 - 966r^2s - 385rs^5 + 1457rs^4 - 1836rs^3 + 720rs^2 + 105s^6 - 385s^5 + 468s^4 - 180s^3) - \frac{h^3s^3g_{n+s}}{1260(r-s)^2(s-1)^2}(12r^2s^2 - 42r^2s + 42r^2 - 15rs^3 + 48rs^2 - 42rs + 5s^4 - 15s^3 + 12s^2) - \frac{h^2s^6f_{n+r}}{2520r^3(r-s)^3(r-1)^3}(-63r^3s^2 + 252r^3s -$$

$294r^3+75r^2s^3-243r^2s^2+138r^2s+210r^2-20rs^4+25rs^3+108rs^2-198rs+10s^4-36s^3+36s^2)-\frac{h^3s^6g_{n+r}}{2520r^2(r-s)^2(r-1)^2}(42r-18s-36rs+9rs^2+18s^2-5s^3),\qquad(3.42)$

$$y_{n+1}=y_n+hy'_n+\frac{h^3g_n}{2520r^2s^2}(126r^2s^2-84r^2s+18r^2-84rs^2+72rs-18r+18s^2-18s+5)-\frac{h^2f_{n+1}}{2520(r-1)^3(s-1)^3}(-378r^3s^3+966r^3s^2-720r^3s+180r^3+966r^2s^3-2418r^2s^2+1836r^2s-468r^2-720rs^3+1836rs^2-1457rs+385r+180s^3-468s^2+385s-105)-\frac{h^2f_n}{2520r^3s^3}(-882r^3s^3+168r^3s^2+90r^3s-36r^3+168r^2s^3+24r^2s^2-108r^2s+36r^2+90rs^3-108rs^2+53rs-10r-36s^3+36s^2-10s)-\frac{h^3g_{n+1}}{1260(r-1)^2(s-1)^2}(42r^2s^2-42r^2s+12r^2-42rs^2+48rs-15r+12s^2-15s+5)-\frac{h^2f_{n+r}}{2520r^3(r-s)^3(r-1)^3}(-294r^3s^2+252r^3s-63r^3+210r^2s^3+138r^2s^2-243r^2s+75r^2-198rs^3+108rs^2+25rs-20r+36s^3-36s^2+10s)+\frac{h^2f_{n+s}}{2520s^3(r-s)^3(s-1)^3}(210r^3s^2-198r^3s+36r^3-294r^2s^3+138r^2s^2+108r^2s-36r^2+252rs^3-243rs^2+25rs+10r-63s^3+75s^2-20s)-\frac{h^3g_{n+r}}{2520r^2(r-s)^2(r-1)^2}(9r+18s-36rs+42rs^2-18s^2-5)-\frac{h^3g_{n+s}}{2520s^2(r-s)^2(s-1)^2}(18r+9s-36rs+42r^2s-18r^2-5).\qquad(3.43)$$

To get the derivative of the main block, we substitute (3.41) into (3.36) – (3.38) which leads to the following equations:

$$y'_{n+r}=y'_n-\frac{hrf_n}{420s^3}(-5r^5s-5r^5+16r^4s^2+23r^4s+16r^4-14r^3s^3-40r^3s^2-40r^3s-14r^3+28r^2s^3+28r^2s+56rs^3+56rs^2-210s^3)+\frac{h^2r^2g_n}{840s^2}(5r^4-16r^3s-16r^3+14r^2s^2+56r^2s+14r^2-56rs^2-56rs+70s^2)+\frac{hrf_{n+r}}{420(r-s)^3(r-1)^3}(105r^6-350r^5s-350r^5+388r^4s^2+1187r^4s+388r^4-140r^3s^3-1342r^3s^2-1342r^3s-140r^3+490r^2s^3+1554r^2s^2+490r^2s-574rs^3-574rs^2+210s^3)-\frac{hr^5f_{n+1}}{420(r-1)^3(s-1)^3}(5r^4s-10r^4-16r^3s^2+10r^3s+35r^3+14r^2s^3+46r^2s^2-98r^2s-28r^2-70rs^3+42rs^2+98rs+70s^3-98s^2)+\frac{h^2r^5g_{n+1}}{840(r-1)^2(s-1)^2}(5r^3-16r^2s-8r^2+14rs^2+28rs-28s^2)-\frac{h^2r^2g_{n+r}}{840(r-s)^2(r-1)^2}(15r^4-40r^3s-40r^3+28r^2s^2+112r^2s+28r^2-84rs^2-84rs+70s^2)+\frac{h^2r^5g_{n+s}}{840s^2(r-s)^2(s-1)^2}(14r-28s+28rs-8r^2s-16r^2+5r^3)+\frac{hr^5f_{n+s}}{420s^3(r-s)^3(s-1)^3}(-10r^4s+5r^4+35r^3s^2+10r^3s-16r^3-28r^2s^3-98r^2s^2+46r^2s+14r^2+98rs^3+42rs^2-70rs-98s^3+70s^2),\qquad(3.44)$$

$$y'_{n+s} = y'_n + \frac{hsf_n}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - 56r^2s + 5rs^5 - 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3) + \frac{h^2s^2g_n}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2) - \frac{hsf_{n+s}}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3) - \frac{hs^5f_{n+1}}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2) + \frac{h^2s^5g_{n+1}}{840(r-1)^2(s-1)^2}(14r^2s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2) - \frac{h^2s^2g_{n+s}}{840(r-s)^2(s-1)^2}(28r^2s^2 - 84r^2s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4 - 40s^3 + 28s^2) - \frac{h^2s^5g_{n+r}}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3) - \frac{hs^5f_{n+r}}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2), \tag{3.45}$$

$$y'_{n+1} = y'_n + \frac{h^2g_n}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5) - \frac{hf_{n+1}}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105) + \frac{hf_n}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s) - \frac{h^2g_{n+1}}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15) - \frac{h^2g_{n+r}}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5) - \frac{h^2g_{n+s}}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5) - \frac{hf_{n+r}}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s) + \frac{hf_{n+s}}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s). \tag{3.46}$$

Equations (3.44)-(3.46) can be expressed in the following block form

$$\begin{aligned} I_3Y'^{[2]_2}_{n+1} &= M'^{[2]_2}_2 Y^{[2]_2}_{n-1} + h[E'^{[2]_2}_1 F^{[2]_2}_n + E'^{[2]_2}_2 F^{[2]_2}_{n+1}] + h^2[K'^{[2]_2}_1 G^{[2]_2}_n \\ &\quad + K'^{[2]_2}_2 G^{[2]_2}_{n+1}] \end{aligned} \tag{3.47}$$

where

$$Y_{n+1}^{'[2]_2} = \begin{pmatrix} y_{n+r}^{'} \\ y_{n+s}^{'} \\ y_{n+1}^{'} \end{pmatrix},\ M_2^{'[2]_2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_1^{'[2]_2} = \begin{pmatrix} 0 & 0 & E_{1_{13}}^{'[2]_2} \\ 0 & 0 & E_{1_{23}}^{'[2]_2} \\ 0 & 0 & E_{1_{33}}^{'[2]_2} \end{pmatrix},\ K_1^{'[2]_2} = \begin{pmatrix} 0 & 0 & K_{1_{13}}^{'[2]_2} \\ 0 & 0 & K_{1_{23}}^{'[2]_2} \\ 0 & 0 & K_{1_{33}}^{'[2]_2} \end{pmatrix},$$

$$E_2^{'[2]_2} = \begin{pmatrix} E_{2_{11}}^{'[2]_2} & E_{2_{12}}^{'[2]_2} & E_{2_{13}}^{'[2]_2} \\ E_{2_{21}}^{'[2]_2} & E_{2_{22}}^{'[2]_2} & E_{2_{23}}^{'[2]_2} \\ E_{2_{31}}^{'[2]_2} & E_{2_{32}}^{'[2]_2} & E_{2_{33}}^{'[2]_2} \end{pmatrix},\ K_2^{'[2]_2} = \begin{pmatrix} K_{2_{11}}^{'[2]_2} & K_{2_{12}}^{'[2]_2} & K_{2_{13}}^{'[2]_2} \\ K_{2_{21}}^{'[2]_2} & K_{2_{22}}^{'[2]_2} & K_{2_{23}}^{'[2]_2} \\ K_{2_{31}}^{'[2]_2} & K_{2_{32}}^{'[2]_2} & K_{2_{33}}^{'[2]_2} \end{pmatrix}.$$

whose entries of $E_1^{'[2]_2}$, $E_2^{'[2]_2}$, $K_1^{'[2]_2}$ and $K_2^{'[2]_2}$ given as below :

$E_{1_{13}}^{'[2]_2} = -\frac{hrf_n}{420s^3}(-5r^5s - 5r^5 + 16r^4s^2 + 23r^4s + 16r^4 - 14r^3s^3 - 40r^3s^2 - 40r^3s - 14r^3 + 28r^2s^3 + 28r^2s + 56rs^3 + 56rs^2 - 210s^3)$,

$E_{1_{23}}^{'[2]_2} = \frac{hsf_n}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - 56r^2s + 5rs^5 - 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3)$,

$E_{1_{33}}^{'[2]_2} = \frac{hf_n}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s)$,

$E_{2_{11}}^{'[2]_2} = \frac{hrf_{n+r}}{420(r-s)^3(r-1)^3}(105r^6 - 350r^5s - 350r^5 + 388r^4s^2 + 1187r^4s + 388r^4 - 140r^3s^3 - 1342r^3s^2 - 1342r^3s - 140r^3 + 490r^2s^3 + 1554r^2s^2 + 490r^2s - 574rs^3 - 574rs^2 + 210s^3)$,

$E_{2_{21}}^{'[2]_2} = -\frac{hs^5f_{n+r}}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2)$,

$E_{2_{31}}^{'[2]_2} = -\frac{hf_{n+r}}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s)$,

$E_{2_{12}}^{'[2]_2} = \frac{hr^5f_{n+s}}{420s^3(r-s)^3(s-1)^3}(-10r^4s + 5r^4 + 35r^3s^2 + 10r^3s - 16r^3 - 28r^2s^3 - 98r^2s^2 + 46r^2s + 14r^2 + 98rs^3 + 42rs^2 - 70rs - 98s^3 + 70s^2)$,

$E_{2_{22}}^{'[2]_2} = -\frac{hsf_{n+s}}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3)$,

$E_{2_{32}}^{'[2]_2} = \frac{hf_{n+s}}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s)$,

$E_{2_{13}}^{'[2]_2} = -\frac{hr^5f_{n+1}}{420(r-1)^3(s-1)^3}(5r^4s - 10r^4 - 16r^3s^2 + 10r^3s + 35r^3 + 14r^2s^3 + 46r^2s^2 - 98r^2s - 28r^2 - 70rs^3 + 42rs^2 + 98rs + 70s^3 - 98s^2)$,

$E_{2_{23}}^{'[2]_2} = -\frac{hs^5f_{n+1}}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2)$,

$E_{2_{33}}^{'[2]_2} = -\frac{hf_{n+1}}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105)$,

$K_{1_{13}}^{'[2]_2} = \frac{h^2r^2g_n}{840s^2}(5r^4 - 16r^3s - 16r^3 + 14r^2s^2 + 56r^2s + 14r^2 - 56rs^2 - 56rs + 70s^2)$,

$K_{1_{23}}^{'[2]_2} = \frac{h^2s^2g_n}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2)$,

$K_{1_{33}}^{'[2]_2} = \frac{h^2g_n}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5)$.

$K_{2_{11}}^{'[2]_2} = -\frac{h^2r^2g_{n+r}}{840(r-s)^2(r-1)^2}(15r^4 - 40r^3s - 40r^3 + 28r^2s^2 + 112r^2s + 28r^2 - 84rs^2 - 84rs + 70s^2)$,

$K_{2_{21}}^{'[2]_2} = -\frac{h^2 s^5 g_{n+r}}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3),$

$K_{2_{31}}^{'[2]_2} = -\frac{h^2 g_{n+r}}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5),$

$K_{2_{12}}^{'[2]_2} = \frac{h^2 r^5 g_{n+s}}{840s^2(r-s)^2(s-1)^2}(14r - 28s + 28rs - 8r^2 s - 16r^2 + 5r^3),$

$K_{2_{22}}^{'[2]_2} = -\frac{h^2 s^2 g_{n+s}}{840(r-s)^2(s-1)^2}(28r^2 s^2 - 84r^2 s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4 - 40s^3 + 28s^2),$

$K_{2_{32}}^{'[2]_2} = -\frac{h^2 g_{n+s}}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2 s - 14r^2 - 5),$

$K_{2_{13}}^{'[2]_2} = \frac{h^2 r^5 g_{n+1}}{840(r-1)^2(s-1)^2}(5r^3 - 16r^2 s - 8r^2 + 14rs^2 + 28rs - 28s^2),$

$K_{2_{23}}^{'[2]_2} = \frac{h^2 s^5 g_{n+1}}{840(r-1)^2(s-1)^2}(14r^2 s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2),$

$K_{2_{33}}^{'[2]_2} = -\frac{h^2 g_{n+1}}{840(r-1)^2(s-1)^2}(70r^2 s^2 - 84r^2 s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15).$

3.3.1 Properties of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

The same basic properties of the developed method as discussed in Section (3.2.1) are also investigated in this section.

3.3.1.1 Order of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

The linear difference operator ∇ associated with Equation (3.40) is defined as

$$\begin{aligned}\nabla[y(x),h] \quad = \quad & Y_{n+1}^{[2]_2} - \hat{M}_1^{[2]_2} Y_n^{[2]_2} - h\hat{M}_2^{[2]_2} Y_{n-1}^{[2]_2} - h^2[\hat{E}_1^{[2]_2} F_n^{[2]_2} + \hat{E}_2^{[2]_2} F_{n+1}^{[2]_2}] - \\ & h^3[\hat{K}_1^{[2]_2} G_n^{[2]_2} + \hat{K}_2^{[2]_2} G_{n+1}^{[2]_2}]. \qquad (3.48)\end{aligned}$$

The same strategy used in Section (3.2.1.1) is employed for finding the order of HBM given in Equation (3.40). Expanding each function in $Y^{[2]_2}_{n+1}$, $F^{[2]_2}_{n+1}$ and $G^{[2]_2}_{n+1}$ about x_n and set to $\mathbf{0}$, we have

$$\left[Q^{[2]_2}_{11}\ Q^{[2]_2}_{21}\ Q^{[2]_2}_{31}\right]^T = \left[0\ 0\ 0\right]^T$$

where

$Q^{[2]_2}_{11} = \sum_{i=0}^{\infty}\frac{(rh)^i}{i!}y_n^{(i)} - y_n - rhy_n' + \frac{h^2r^2y_n''}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3) - \sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+2)}}{2520(r-s)^3(r-1)^3i!}(r^2(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3) - \sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+2)}}{2520s^3(r-s)^3(s-1)^3i!}(h^2r^6(-20r^4s + 10r^4 + 75r^3s^2 + 25r^3s - 36r^3 - 63r^2s^3 - 243r^2s^2 + 108r^2s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2) + \sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+2)}}{2520(r-1)^3(s-1)^3i!}(h^2r^6(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2) - \frac{h^3r^3y_n'''}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2) + \sum_{i=0}^{\infty}\frac{r^ih^{i+3}y_n^{(i+3)}r^3}{1260(r-s)^2(r-1)^2i!}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2) - \sum_{i=0}^{\infty}\frac{s^ih^{i+3}y_n^{(i+3)}r^6}{2520s^2(r-s)^2(s-1)^2i!}(18r - 42s + 36rs - 9r^2s - 18r^2 + 5r^3) - \sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+3)}r^6}{2520(r-1)^2(s-1)^2i!}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36r - 42s^2),$

$Q^{[2]_2}_{21} = \sum_{i=0}^{\infty}\frac{(sh)^i}{i!}y_n^{(i)} - y_n - shy_n' + \frac{h^2s^2y_n''}{2520r^3}(-36r^3s^3 + 90r^3s^2 + 168r^3s - 882r^3 + 36r^2s^4 - 108r^2s^3 + 24r^2s^2 + 168r^2s - 10rs^5 + 53rs^4 - 108rs^3 + 90rs^2 - 10s^5 + 36s^4 - 36s^3) + \sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+2)}s^6}{2520r^3(r-s)^3(r-1)^3i!}(-63r^3s^2 + 252r^3s - 294r^3 + 75r^2s^3 - 243r^2s^2 + 138r^2s + 210r^2 - 20rs^4 + 25rs^3 + 108rs^2 - 198rs + 10s^4 - 36s^3 + 36s^2) + \sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+2)}s^2}{2520(r-s)^3(s-1)^3i!}(-180r^3s^3 + 720r^3s^2 - 966r^3s + 378r^3 + 468r^2s^4 - 1836r^2s^3 + 2418r^2s^2 - 966r^2s - 385rs^5 + 1457rs^4 - 1836rs^3 + 720rs^2 + 105s^6 - 385s^5 + 468s^4 - 180s^3) + \sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+2)}s^6}{2520(r-1)^3(s-1)^3}(36r^3s^2 - 198r^3s + 210r^3 - 36r^2s^3 + 108r^2s^2 + 138r^2s - 294r^2 + 10rs^4 + 25rs^3 - 243rs^2 + 252rs - 20s^4 + 75s^3 - 63s^2) - \frac{h^3s^3y_n'''}{2520r^2}(18r^2s^2 - 84r^2s + 126r^2 - 18rs^3 + 72rs^2 - 84rs + 5s^4 - 18s^3 + 18s^2)$

$+\sum_{i=0}^{\infty}\frac{r^ih^{i+3}y_n^{(i+3)}s^6}{2520r^2(r-s)^2(r-1)^2i!}(42r - 18s - 36rs + 9rs^2 + 18s^2 - 5s^3) + \sum_{i=0}^{\infty}\frac{s^ih^{i+3}y_n^{(i+3)}s^3}{1260(r-s)^2(s-1)^2i!}(12r^2s^2 - 42r^2s + 42r^2 - 15rs^3 + 48rs^2 - 42rs + 5s^4 - 15s^3 + 12s^2) - \sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+3)}s^6}{2520(r-1)^2(s-1)^2i!}(18r^2s - 42r^2 - 18rs^2 + 36rs + 5s^3 - 9s^2),$

$Q_{31}^{[2]_2} = \sum_{i=0}^{\infty}\frac{(h)^i}{i!}y_n^{(i)} - y_n - hy'_n + \frac{h^2y''_n}{2520r^3s^3}(-882r^3s^3 + 168r^3s^2 + 90r^3s - 36r^3 + 168r^2s^3 + 24r^2s^2 - 108r^2s + 36r^2 + 90rs^3 - 108rs^2 + 53rs - 10r - 36s^3 + 36s^2 - 10s) + \sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+2)}}{2520r^3(r-s)^3(r-1)^3i!}(-294r^3s^2 + 252r^3s - 63r^3 + 210r^2s^3 + 138r^2s^2 - 243r^2s + 75r^2 - 198rs^3 + 108rs^2 + 25rs - 20r + 36s^3 - 36s^2 + 10s) - \sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+2)}}{2520s^3(r-s)^3(s-1)^3i!}(210r^3s^2 - 198r^3s + 36r^3 - 294r^2s^3 + 138r^2s^2 + 108r^2s - 36r^2 + 252rs^3 - 243rs^2 + 25rs + 10r - 63s^3 + 75s^2 - 20s) + \sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+2)}}{2520(r-1)^3(s-1)^3i!}(-378r^3s^3 + 966r^3s^2 - 720r^3s + 180r^3 + 966r^2s^3 - 2418r^2s^2 + 1836r^2s - 468r^2 - 720rs^3 + 1836rs^2 - 1457rs + 385r + 180s^3 - 468s^2 + 385s - 105) - \frac{h^3y'''_n}{2520r^2s^2}(126r^2s^2 - 84r^2s + 18r^2 - 84rs^2 + 72rs - 18r + 18s^2 - 18s + 5) + \sum_{i=0}^{\infty}\frac{r^ih^{i+3}y_n^{(i+3)}}{2520r^2(r-s)^2(r-1)^2i!}(9r + 18s - 36rs + 42rs^2 - 18s^2 - 5) + \sum_{i=0}^{\infty}\frac{s^ih^{i+3}y_n^{(i+3)}}{2520s^2(r-s)^2(s-1)^2i!}(18r + 9s - 36rs + 42r^2s - 18r^2 - 5) + \sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+3)}}{1260(r-1)^2(s-1)^2i!}(42r^2s^2 - 42r^2s + 12r^2 - 42rs^2 + 48rs - 15r + 12s^2 - 15s + 5),$

which leads to $\bar{D}_0 = \bar{D}_1 = \bar{D}_2 = \bar{D}_3 = \bar{D}_4 = \bar{D}_5 = \bar{D}_6 = \bar{D}_7 = \bar{D}_8 = \bar{D}_9 = 0$ and $\bar{D}_{10} \neq 0$ after comparing the coefficients of h^j and $y^{(j)}$. As a result, the order of the method is found to be $[8,8,8]^T$ with vector of error constants

$$\bar{D}_{10} = \begin{bmatrix} \frac{r^6(3r^4-10r^3s-10r^3+9r^2s^2+36r^2s+9r^2-36rs^2-36rs+42s^2)}{101606400} \\ \frac{s^6(9r^2s^2-36r^2s+42r^2-10rs^3+36rs^2-36rs+3s^4-10s^3+9s^2)}{101606400} \\ \frac{(42r^2s^2-36r^2s+9r^2-36rs^2+36rs-10r+9s^2-10s+3)}{101606400} \end{bmatrix}.$$

Repeat the same procedure as mentioned earlier to determine the order of (3.47). The linear difference operator ∇ associated with (3.47) is given by

$$\nabla[y(x),h] = Y'^{[2]_2}_{n+1} - M'^{[2]_2}_2 Y^{[2]_2}_{n-1} - h[E'^{[2]_2}_1 F^{[2]_2}_n + E'^{[2]_2}_2 F^{[2]_2}_{n+1}] - h^2[K'^{[2]_2}_1 G^{[2]_2}_n + K'^{[2]_2}_2 G^{[2]_2}_{n+1}]. \quad (3.49)$$

Expanding (3.49) in Taylor series and equating to **0** yields

$$\left[Q_{11}^{'[2]_2}\ Q_{21}^{'[2]_2}\ Q_{31}^{'[2]_2}\right]^T = \left[0\ 0\ 0\right]^T$$

where

$Q_{11}^{'[2]_2} = \sum_{j=0}^{\infty}\frac{(r)^j h^j}{j!}y_n^{j+1} - y_n' + \frac{hry_n''}{420s^3}(-5r^5s - 5r^5 + 16r^4s^2 + 23r^4s + 16r^4 - 14r^3s^3 - 40r^3s^2 - 40r^3s - 14r^3 + 28r^2s^3 + 28r^2s + 56rs^3 + 56rs^2 - 210s^3) - \frac{hrf_{n+r}}{420(r-s)^3(r-1)^3}(105r^6 - 350r^5s - 350r^5 + 388r^4s^2 + 1187r^4s + 388r^4 - 140r^3s^3 - 1342r^3s^2 - 1342r^3s - 140r^3 + 490r^2s^3 + 1554r^2s^2 + 490r^2s - 574rs^3 - 574rs^2 + 210s^3) - \frac{hr^5f_{n+s}}{420s^3(r-s)^3(s-1)^3}(-10r^4s + 5r^4 + 35r^3s^2 + 10r^3s - 16r^3 - 28r^2s^3 - 98r^2s^2 + 46r^2s + 14r^2 + 98rs^3 + 42rs^2 - 70rs - 98s^3 + 70s^2) + \frac{hr^5f_{n+1}}{420(r-1)^3(s-1)^3}(5r^4s - 10r^4 - 16r^3s^2 + 10r^3s + 35r^3 + 14r^2s^3 + 46r^2s^2 - 98r^2s - 28r^2 - 70rs^3 + 42rs^2 + 98rs + 70s^3 - 98s^2) - \frac{h^2r^2y_n'''}{840s^2}(5r^4 - 16r^3s - 16r^3 + 14r^2s^2 + 56r^2s + 14r^2 - 56rs^2 - 56rs + 70s^2) + \frac{h^2r^2g_{n+r}}{840(r-s)^2(r-1)^2}(15r^4 - 40r^3s - 40r^3 + 28r^2s^2 + 112r^2s + 28r^2 - 84rs^2 - 84rs + 70s^2) - \frac{h^2r^5g_{n+s}}{840s^2(r-s)^2(s-1)^2}(14r - 28s + 28rs - 8r^2s - 16r^2 + 5r^3) - \frac{h^2r^5g_{n+1}}{840(r-1)^2(s-1)^2}(5r^3 - 16r^2s - 8r^2 + 14rs^2 + 28rs - 28s^2),$

$Q_{21}^{'[2]_2} = \sum_{j=0}^{\infty}\frac{(s)^j h^j}{j!}y_n^{j+1} - y_n' - \frac{hsy_n''}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - 56r^2s + 5rs^5 - 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3) + \frac{hs^5f_{n+r}}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2)\frac{hsf_{n+s}}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3) + \frac{hs^5f_{n+1}}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2) - \frac{h^2s^2y_n'''}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2) + \frac{h^2s^5g_{n+r}}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3) + \frac{h^2s^2g_{n+s}}{840(r-s)^2(s-1)^2}(28r^2s^2 - 84r^2s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4 - 40s^3 + 28s^2) - \frac{h^2s^5g_{n+1}}{840(r-1)^2(s-1)^2}(14r^2s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2),$

$Q_{31}^{'[2]_2} = \sum_{j=0}^{\infty}\frac{h^j}{j!}y_n^{j+1} - y_n' - \frac{hy_n''}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s) + \frac{hf_{n+r}}{420r^3(r-s)^3(r-1)^3}(-98$

$r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s) - \frac{hf_{n+s}}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s) + \frac{hf_{n+1}}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105) - \frac{h^2y_n'''}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5) + \frac{h^2g_{n+r}}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5) + \frac{h^2g_{n+s}}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5) + \frac{h^2g_{n+1}}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15).$

Using a similar argument as before, comparing the coefficients of h^j and $y^{(j)}$ gives $\bar{D}'_0 = \bar{D}'_1 = \bar{D}'_2 = \bar{D}'_3 = \bar{D}'_4 = \bar{D}'_5 = \bar{D}'_6 = \bar{D}'_7 = \bar{D}'_8 = \bar{D}'_9 = 0$ and $\bar{D}'_{10} \neq 0$. Thus, the order is $[8,8,8]^T$ for the derivative of the method with vector of error constants

$$\bar{D}'_{10} = \begin{bmatrix} \frac{r^5(5r^4-15r^3s-15r^3+12r^2s^2+48r^2s+12r^2-42rs^2-42rs+42s^2)}{50803200} \\ \frac{s^5(12r^2s^2-42r^2s+42r^2-15rs^3+48rs^2-42rs+5s^4-15s^3+12s^2)}{50803200} \\ \frac{(42r^2s^2-42r^2s+12r^2-42rs^2+48rs-15r+12s^2-15s+5)}{50803200} \end{bmatrix}.$$

3.3.1.2 Zero-Stability of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

The first characteristic polynomial corresponding to (3.40) is

$$\begin{aligned} \psi^{[2]_2}(q) &= |qI_3 - \hat{M}_1^{[2]_2}| \\ &= \left| q\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| \\ &= q^2(q-1) \end{aligned}$$

whose solutions are $q = \{0,0,1\}$ if $\psi^{[2]_2}(q)$ is set to 0. Meanwhile, the characteristic polynomial for the derivative block (3.47) is given by

$$\begin{aligned}
\psi'^{[2]_2}(q) &= |qI_3 - M_2'^{[2]_2}| \\
&= \left| q\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^2(q-1)
\end{aligned}$$

which also implies $q = \{0,0,1\}$ when $\psi'^{[2]_2}(q) = 0$. Therefore, the main block method and its derivative are zero stable by Definition 3.2.2.

3.3.1.3 Consistency of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

It follows from Definition 2.4.4 that both main block method (3.40) and its derivatives (3.47) are consistent.

3.3.1.4 Convergence of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

By Theorem (2.1) both the main block method (3.40) and its derivatives (3.47) are convergent.

3.3.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Second Order ODEs

Substituting $m = z = 2$ in (3.27) leads to

$$M^{[2]_2}(q) = (I_3 - q^2\hat{E}_2^{[2]_2} - q^3\hat{K}_2^{[2]_2})^{-1}(\hat{M}_1^{[2]_2} + q\hat{M}_2^{[2]_2} + q^2\hat{E}_1^{[2]_2} + q^3\hat{K}_1^{[2]_2}) \quad (3.50)$$

whose eigenvalues are $\{0,0,\eta_3^{[2]_2}\}$. The largest eigenvalue $\eta_3^{[2]_2}$ is a function of q given by

$$\eta_3^{[2]_2} = eig(M^{[2]_2}(q)). \quad (3.51)$$

Let us consider two hybrid points, i.e $r = \frac{1}{5}$ and $s = \frac{3}{5}$, for the purpose of illustration. Substituting these two values in Equation (3.51), we have

$$\eta_3^{[2]_2} \quad = \frac{\sum_{i=0}^{9} c_i q^i}{\sum_{j=0}^{10} d_j q^j}$$

where the values c_i and d_j are listed in Table 3.2 below

Table 3.2
Coefficients of the Eigenvalue ($\eta_3^{[2]_2}$) *for the Matrix* $M^{[2]_2}$

c-value	q^i Coefficients	d-value	q^j Coefficients
c_0	4798226578050472120156160000000	d_0	0
c_1	4798226578050471176437760000000	d_1	4798226578050472120156160000000
c_2	2181667466407547324421505024000	d_2	0
c_3	600728923295631640688433168384	d_3	- 2174458226176943298095349760000
c_4	112023036822335583730613878784	d_4	184703162382455079436841123840
c_5	14891066904641980421390821120	d_5	23495244741672972547391488000
c_6	1432502656635986777761382192	d_6	- 4378667361195926931908928000
c_7	97515205598253810354554672	d_7	132601444092681681436925920
c_8	4283472008687434715176712	d_8	20239458282669595984754560
c_9	91594264723102607522982	d_9	- 204460145926302866020840
		d_{10}	7303423107746833984375

With the help of Matlab, the function ($\eta_3^{[2]_2}$) is plotted to obtain the region of absolute stability represented by the dark area in Figure 3.5.

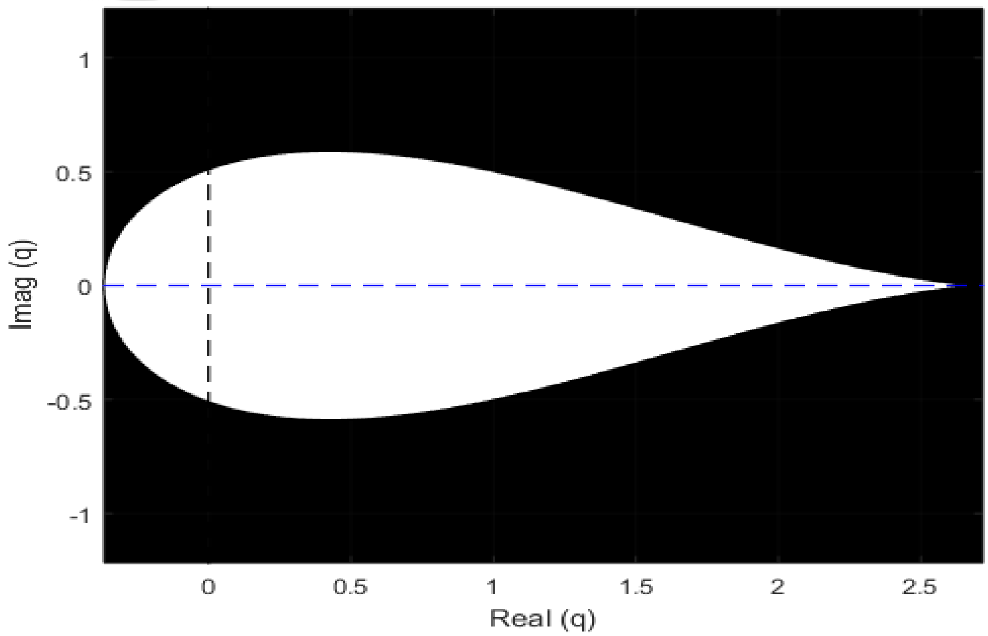


Figure 3.5. Region of Absolute Stability of One-Step HBM with Two Off-Step Points $r = \frac{1}{5}$ and $s = \frac{3}{5}$ for Second Order ODEs

In a similar manner, we can plot the region of absolute stability for the function $(\eta_3^{[2]_2})$ when $r = \frac{2}{5}$ and $s = \frac{4}{5}$ as shown in Figure 3.6 below.

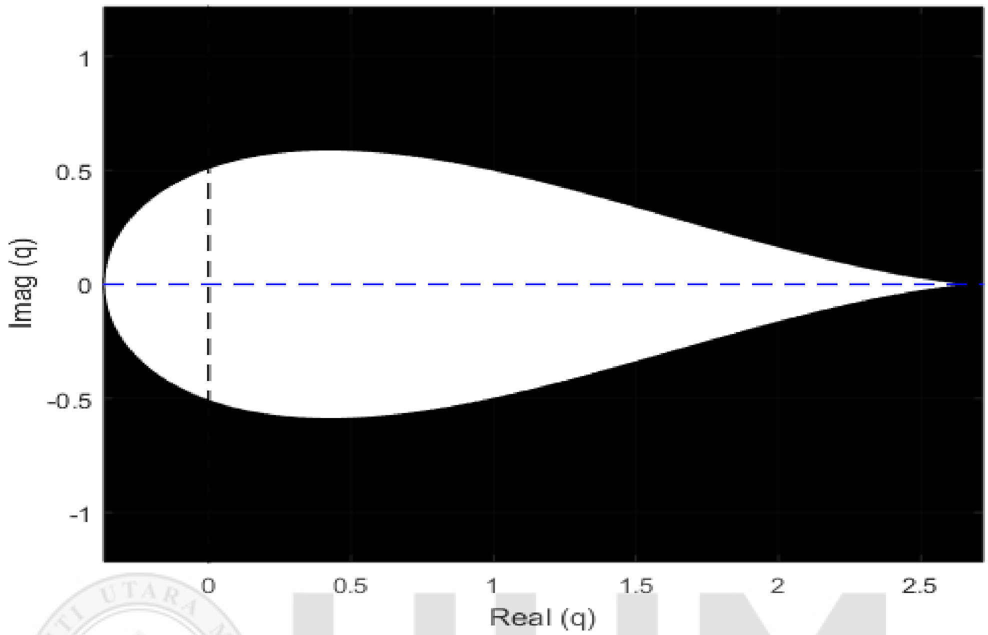


Figure 3.6. Region of Absolute Stability of One-Step HBM with Two Off-Step Points $r = \frac{2}{5}$ and $s = \frac{4}{5}$ for Second Order ODEs

Using the same procedure, we can plot the region of absolute stability for any value of r and s.

Next section explains the derivation procedure of HBM with generalised three off-step points in the presence of third derivative.

3.4 Derivation of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

In deriving this method, (3.2) is interpolated at the points x_n and x_{n+r} while (3.3) and (3.4) are collocated at all points, i.e $x_n, x_{n+r}, x_{n+s}, x_{n+t}$ and x_{n+1}, where $0 < r < s < t < 1$ as depicted in Figure 3.7 below

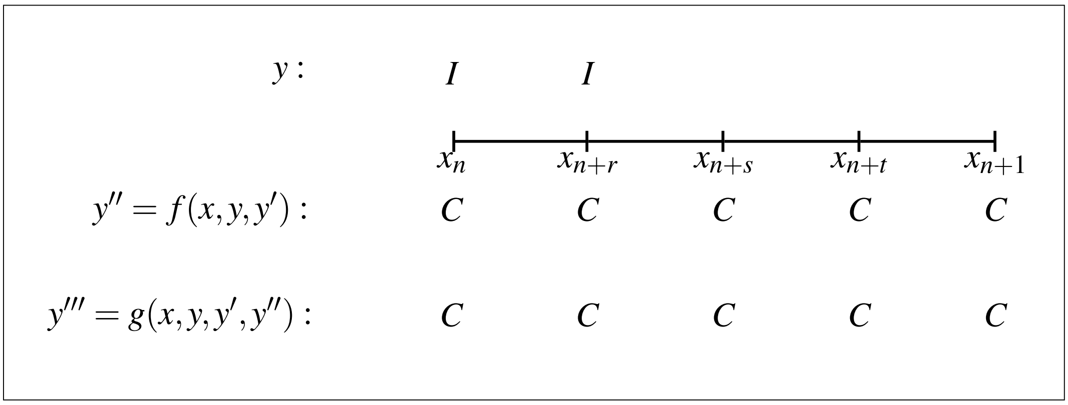


Figure 3.7. Interpolation and Collocation Strategy for One-Step HBM with Three Off-Step Points for Solving Second Order ODEs

Now, substituting $u = 2$ and $v = 5$ in Equation (3.2) gives

$$y(x) = \sum_{j=0}^{11} a_j (\frac{x - x_n}{h})^j$$

Interpolating the above equation and collocating Equations (3.3) and (3.4) yields

$$\begin{aligned}
y_n &= a_0, \\
y_{n+r} &= a_0 + ra_1 + r^2 a_2 + r^3 a_3 + r^4 a_4 + r^5 a_5 + r^6 a_6 + r^7 a_7 + r^8 a_8 + r^9 a_9 \\
&\quad + r^{10} a_{10} + r^{11} a_{10}, \\
f_n &= \frac{2}{h^2} a_2, \\
f_{n+r} &= \frac{2}{h^2} a_2 + \frac{6r}{h^2} a_3 + \frac{12r^2}{h^2} a_4 + \frac{20r^3}{h^2} a_5 + \frac{30r^4}{h^2} a_6 + \frac{42r^5}{h^2} a_7 + \frac{56r^6}{h^2} a_8 + \frac{72r^7}{h^2} a_9 \\
&\quad + \frac{90r^8}{h^2} a_{10} + \frac{110r^9}{h^2} a_{11}, \\
f_{n+s} &= \frac{2}{h^2} a_2 + \frac{6s}{h^2} a_3 + \frac{12s^2}{h^2} a_4 + \frac{20s^3}{h^2} a_5 + \frac{30s^4}{h^2} a_6 + \frac{42s^5}{h^2} a_7 + \frac{56s^6}{h^2} a_8 + \frac{72s^7}{h^2} a_9 \\
&\quad + \frac{90s^8}{h^2} a_{10} + \frac{110s^9}{h^2} a_{11}, \\
f_{n+t} &= \frac{2}{h^2} a_2 + \frac{6t}{h^2} a_3 + \frac{12t^2}{h^2} a_4 + \frac{20t^3}{h^2} a_5 + \frac{30t^4}{h^2} a_6 + \frac{42t^5}{h^2} a_7 + \frac{56t^6}{h^2} a_8 + \frac{72t^7}{h^2} a_9 \\
&\quad + \frac{90t^8}{h^2} a_{10} + \frac{110t^9}{h^2} a_{11},
\end{aligned}$$

$$
\begin{aligned}
f_{n+1} &= \frac{2}{h^2}a_2+\frac{6}{h^2}a_3+\frac{12}{h^2}a_4+\frac{20}{h^2}a_5+\frac{30}{h^2}a_6+\frac{42}{h^2}a_7+\frac{56}{h^2}a_8+\frac{72}{h^2}a_9+\frac{90}{h^2}a_{10}+ \\
&\quad \frac{110}{h^2}a_{11}, \\
g_n &= \frac{6}{h^3}a_3, \\
g_{n+r} &= \frac{6}{h^3}a_3+\frac{24r}{h^3}a_4+\frac{60r^2}{h^3}a_5+\frac{120r^3}{h^3}a_6+\frac{210r^4}{h^3}a_7+\frac{336r^5}{h^3}a_8+\frac{504r^6}{h^3}a_9 \\
&\quad +\frac{720r^7}{h^3}a_{10}+\frac{990r^8}{h^3}a_{11}, \\
g_{n+s} &= \frac{6}{h^3}a_3+\frac{24s}{h^3}a_4+\frac{60s^2}{h^3}a_5+\frac{120s^3}{h^3}a_6+\frac{210s^4}{h^3}a_7+\frac{336s^5}{h^3}a_8+\frac{504s^6}{h^3}a_9 \\
&\quad +\frac{720s^7}{h^3}a_{10}+\frac{990s^8}{h^3}a_{11}, \\
g_{n+t} &= \frac{6}{h^3}a_3+\frac{24t}{h^3}a_4+\frac{60t^2}{h^3}a_5+\frac{120t^3}{h^3}a_6+\frac{210t^4}{h^3}a_7+\frac{336t^5}{h^3}a_8+\frac{504t^6}{h^3}a_9 \\
&\quad +\frac{720t^7}{h^3}a_{10}+\frac{990t^8}{h^3}a_{11}, \\
g_{n+1} &= \frac{6}{h^3}a_3+\frac{24}{h^3}a_4+\frac{60}{h^3}a_5+\frac{120}{h^3}a_6+\frac{210}{h^3}a_7+\frac{336}{h^3}a_8+\frac{504}{h^3}a_9+\frac{720}{h^3}a_{10} \\
&\quad +\frac{990}{h^3}a_{11}.
\end{aligned}
$$

which can be converted into matrix form $AX = B$, where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} \\ 0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{h^2} & \frac{6r}{h^2} & \frac{12r^2}{h^2} & \frac{20r^3}{h^2} & \frac{30r^4}{h^2} & \frac{42r^5}{h^2} & \frac{56r^6}{h^2} & \frac{72r^7}{h^2} & \frac{90r^8}{h^2} & \frac{110r^9}{h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6s}{h^2} & \frac{12s^2}{h^2} & \frac{20s^3}{h^2} & \frac{30s^4}{h^2} & \frac{42s^5}{h^2} & \frac{56s^6}{h^2} & \frac{72s^7}{h^2} & \frac{90s^8}{h^2} & \frac{110s^9}{h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6t}{h^2} & \frac{12t^2}{h^2} & \frac{20t^3}{h^2} & \frac{30t^4}{h^2} & \frac{42t^5}{h^2} & \frac{56t^6}{h^2} & \frac{72t^7}{h^2} & \frac{90t^8}{h^2} & \frac{110t^9}{h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6}{h^2} & \frac{12}{h^2} & \frac{20}{h^2} & \frac{30}{h^2} & \frac{42}{h^2} & \frac{56}{h^2} & \frac{72}{h^2} & \frac{90}{h^2} & \frac{110}{h^2} \\ 0 & 0 & 0 & \frac{6}{h^3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24r}{h^3} & \frac{60r^2}{h^3} & \frac{120r^3}{h^3} & \frac{210r^4}{h^3} & \frac{336r^5}{h^3} & \frac{504r^6}{h^3} & \frac{720r^7}{h^3} & \frac{990r^8}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24s}{h^3} & \frac{60s^2}{h^3} & \frac{120s^3}{h^3} & \frac{210s^4}{h^3} & \frac{336s^5}{h^3} & \frac{504s^6}{h^3} & \frac{720s^7}{h^3} & \frac{990s^8}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24t}{h^3} & \frac{60t^2}{h^3} & \frac{120t^3}{h^3} & \frac{210t^4}{h^3} & \frac{336t^5}{h^3} & \frac{504t^6}{h^3} & \frac{720t^7}{h^3} & \frac{990t^8}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24}{h^3} & \frac{60}{h^3} & \frac{120}{h^3} & \frac{210}{h^3} & \frac{336}{h^3} & \frac{504}{h^3} & \frac{720}{h^3} & \frac{990}{h^3} \end{pmatrix},$$

$X = [a_0,\, a_1,\, a_2,\, a_3,\, a_4,\, a_5,\, a_6,\, a_7,\, a_8,\, a_9,\, a_{10},\, a_{11}]^T,$

$B = [y_n,\, y_{n+r},\, f_n,\, f_{n+r},\, f_{n+s},\, f_{n+t},\, f_{n+1},\, g_n,\, g_{n+r},\, g_{n+s},\, g_{n+t},\, g_{n+1}]^T.$

Solving for the unknown values a_i's; $i = 0(1)11$ and then substituting them into Equation (3.2) produces the following continuous implicit scheme

$$y(x) = \sum_{i=0,r} \alpha_i(x) y_{n+i} + \sum_{i=0,r,s,t,1} \beta_i(x) f_{n+i} + \sum_{i=0,r,s,t,1} \gamma_i(x) g_{n+i} \tag{3.52}$$

where

$\alpha_0 = \frac{x_n - x + hr}{hr},$

$\alpha_r = \frac{x - x_n}{hr},$

$\beta_0 = \frac{(x-x_n)^2}{2} + \frac{h(x-x_n)}{27720 s^3 t^3}(132r^7s^2t - 42r^8s - 42r^8t - 42r^8st + 132r^7s^2 + 132r^7st^2 + 321r^7st + 132r^7s + 132r^7t^2 + 132r^7t - 110r^6s^3t - 110r^6s^3 - 440r^6s^2t^2 - 748r^6s^2t - 440r^6s^2 - 110r^6st^3 - 748r^6st^2 - 748r^6st - 110r^6s - 110r^6t^3 - 440r^6t^2 - 110r^6t + 396r^5s^3t^2 + 583r^5s^3t + 396r^5s^3 + 396r^5s^2t^3 + 1540r^5s^2t^2 + 1540r^5s^2t + 396r^5s^2 + 583r^5st^3 + 1540r^5st^2 + 583r^5st + 396r^5t^3 + 396r^5t^2 - 396r^4s^3t^3 - 1188r^4s^3t^2 - 1188$

$r^4s^3t - 396r^4s^3 - 1188r^4s^2t^3 - 1584r^4s^2t^2 - 1188r^4s^2t - 1188r^4st^3 - 1188r^4st^2 - 396r^4t^3 + 990r^3s^3t^3 + 264r^3s^3t^2 + 990r^3s^3t + 264r^3s^2t^3 + 264r^3s^2t^2 + 990r^3st^3 + 1848r^2s^3t^3 + 1848r^2s^3t^2 + 1848r^2s^2t^3 - 9702rs^3t^3) + \frac{(x-x_n)^5}{10h^3r^3s^3t^3}(r^3s^3t^3 + 4r^3s^3t^2 + 4r^3s^3t + r^3s^3 + 4r^3s^2t^3 + 8r^3s^2t^2 + 4r^3s^2t + 4r^3st^3 + 4r^3st^2 + r^3t^3 + 4r^2s^3t^3 + 8r^2s^3t^2 + 4r^2s^3t + 8r^2s^2t^3 + 8r^2s^2t^2 + 4r^2st^3 + 4rs^3t^3 + 4rs^3t^2 + 4rs^2t^3 + s^3t^3) - \frac{(x-x_n)^8}{56h^6r^3s^3t^3}(4r^3s^2t + 4r^3s^2 + 4r^3st^2 + 11r^3st + 4r^3s + 4r^3t^2 + 4r^3t + 4r^2s^3t + 4r^2s^3 + 16r^2s^2t^2 + 36r^2s^2t + 16r^2s^2 + 4r^2st^3 + 36r^2st^2 + 36r^2st + 4r^2s + 4r^2t^3 + 16r^2t^2 + 4r^2t + 4rs^3t^2 + 11rs^3t + 4rs^3 + 4rs^2t^3 + 36rs^2t^2 + 36rs^2t + 4rs^2 + 11rst^3 + 36rst^2 + 11rst + 4rt^3 + 4rt^2 + 4s^3t^2 + 4s^3t + 4s^2t^3 + 16s^2t^2 + 4s^2t + 4st^3 + 4st^2) + \frac{(x-x_n)^9}{36h^7r^3s^3t^3}(r^3st + r^3s + r^3t + 4r^2s^2t + 4r^2s^2 + 4r^2st^2 + 12r^2st + 4r^2s + 4r^2t^2 + 4r^2t + rs^3t + rs^3 + 4rs^2t^2 + 12rs^2t + 4rs^2 + rst^3 + 12rst^2 + 12rst + rs + rt^3 + 4rt^2 + rt + s^3t + 4s^2t^2 + 4s^2t + st^3 + 4st^2 + st) + \frac{(x-x_n)^{11}}{55h^9r^3s^3t^3}(rs + rt + st + rst) - \frac{(x-x_n)^4}{12h^2r^2s^2t^2}(3r^2s^2t^2 + 4r^2s^2t + 3r^2s^2 + 4r^2st^2 + 4r^2st + 3r^2t^2 + 4rs^2t^2 + 4rs^2t + 4rst^2 + 3s^2t^2) + \frac{(x-x_n)^7}{21h^5r^3s^3t^3}(r^3s^3t + r^3s^3 + 4r^3s^2t^2 + 8r^3s^2t + 4r^3s^2 + r^3st^3 + 8r^3st^2 + 8r^3st + r^3s + r^3t^3 + 4r^3t^2 + r^3t + 4r^2s^3t^2 + 8r^2s^3t + 4r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 + 24r^2s^2t + 4r^2s^2 + 8r^2st^3 + 24r^2st^2 + 8r^2st + 4r^2t^3 + 4r^2t^2 + rs^3t^3 + 8rs^3t^2 + 8rs^3t + rs^3 + 8rs^2t^3 + 24rs^2t^2 + 8rs^2t + 8rst^3 + 8rst^2 + rt^3 + s^3t^3 + 4s^3t^2 + s^3t + 4s^2t^3 + 4s^2t^2 + st^3) - \frac{(x-x_n)^{10}}{90h^8r^3s^3t^3}(4r^2st + 4r^2s + 4r^2t + 4rs^2t + 4rs^2 + 4rst^2 + 15rst + 4rs + 4rt^2 + 4rt + 4s^2t + 4st^2 + 4st) - \frac{(x-x_n)^6}{30h^4r^3s^3t^3}(4r^3s^3t^2 + 7r^3s^3t + 4r^3s^3 + 4r^3s^2t^3 + 20r^3s^2t^2 + 20r^3s^2t + 4r^3s^2 + 7r^3st^3 + 20r^3st^2 + 7r^3st + 4r^3t^3 + 4r^3t^2 + 4r^2s^3t^3 + 20r^2s^3t^2 + 20r^2s^3t + 4r^2s^3 + 20r^2s^2t^3 + 48r^2s^2t^2 + 20r^2s^2t + 20r^2st^3 + 20r^2st^2 + 4r^2t^3 + 7rs^3t^3 + 20rs^3t^2 + 7rs^3t + 20rs^2t^3 + 20rs^2t^2 + 7rst^3 + 4s^3t^3 + 4s^3t^2 + 4s^2t^3),$

$\beta_r = \frac{(x-x_n)^{10}}{90h^8r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4 + 9r^3s + 9r^3t + 9r^3 - 12r^2s^2 - 19r^2st - 19r^2s - 12r^2t^2 - 19r^2t - 12r^2 + 8rs^2t + 8rs^2 + 8rst^2 + 21rst + 8rs + 8rt^2 + 8rt - 4s^2t - 4st^2 - 4st) - \frac{h(x-x_n)}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^{10} - 2646r^9s - 2646r^9t - 2646r^9 + 3069r^8s^2 + 9420r^8st + 9420r^8s + 3069r^8t^2 + 9420r^8t + 3069r^8 - 1155r^7s^3 - 11121r^7s^2t - 11121r^7s^2 - 11121r^7st^2 - 34278r^7st - 11121r^7s - 1155r^7t^3 - 11121r^7t^2 - 11121r^7t - 1155r^7 + 4235r^6s^3t + 4235r^6s^3 + 13376r^6s^2t^2 + 41437r^6s^2t + 13376$

$$
\begin{aligned}
& r^6s^2 + 4235r^6st^3 + 41437r^6st^2 + 41437r^6st + 4235r^6s + 4235r^6t^3 + 13376r^6t^2 + \\
& 4235r^6t - 5148r^5s^3t^2 - 16027r^5s^3t - 5148r^5s^3 - 5148r^5s^2t^3 - 51436r^5s^2t^2 - \\
& 51436r^5s^2t - 5148r^5s^2 - 16027r^5st^3 - 51436r^5st^2 - 16027r^5st - 5148r^5t^3 - \\
& 5148r^5t^2 + 1980r^4s^3t^3 + 20196r^4s^3t^2 + 20196r^4s^3t + 1980r^4s^3 + 20196r^4s^2t^3 + \\
& 66330r^4s^2t^2 + 20196r^4s^2t + 20196r^4st^3 + 20196r^4st^2 + 1980r^4t^3 - 7920r^3s^3t^3 - \\
& 26598r^3s^3t^2 - 7920r^3s^3t - 26598r^3s^2t^3 - 26598r^3s^2t^2 - 7920r^3st^3 + 10626r^2s^3t^3 + \\
& 10626r^2s^3t^2 + 10626r^2s^2t^3 - 4158rs^3t^3) + \tfrac{(x-x_n)^6}{30h^4r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4s^2t^2 + 36r^4s^2t + \\
& 9r^4s^2 + 36r^4st^2 + 36r^4st + 9r^4t^2 - 7r^3s^3t^2 - 28r^3s^3t - 7r^3s^3 - 7r^3s^2t^3 - 47r^3s^2t^2 - \\
& 47r^3s^2t - 7r^3s^2 - 28r^3st^3 - 47r^3st^2 - 28r^3st - 7r^3t^3 - 7r^3t^2 + 5r^2s^3t^3 + 13r^2s^3t^2 + \\
& 13r^2s^3t + 5r^2s^3 + 13r^2s^2t^3 + 24r^2s^2t^2 + 13r^2s^2t + 13r^2st^3 + 13r^2st^2 + 5r^2t^3 + \\
& 5rs^3t^3 + 4rs^3t^2 + 5rs^3t + 4rs^2t^3 + 4rs^2t^2 + 5rst^3 - 4s^3t^3 - 4s^3t^2 - 4s^2t^3) - \\
& \tfrac{(x-x_n)^7}{21h^5r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4s^2t + 9r^4s^2 + 9r^4st^2 + 36r^4st + 9r^4s + 9r^4t^2 + 9r^4t - \\
& 7r^3s^3t - 7r^3s^3 - 10r^3s^2t^2 - 26r^3s^2t - 10r^3s^2 - 7r^3st^3 - 26r^3st^2 - 26r^3st - 7r^3s - \\
& 7r^3t^3 - 10r^3t^2 - 7r^3t + 2r^2s^3t^2 - 2r^2s^3t + 2r^2s^3 + 2r^2s^2t^3 + 3r^2s^2t^2 + 3r^2s^2t + \\
& 2r^2s^2 - 2r^2st^3 + 3r^2st^2 - 2r^2st + 2r^2t^3 + 2r^2t^2 + 2rs^3t^3 + 7rs^3t^2 + 7rs^3t + 2rs^3 + \\
& 7rs^2t^3 + 12rs^2t^2 + 7rs^2t + 7rst^3 + 7rst^2 + 2rt^3 - s^3t^3 - 4s^3t^2 - s^3t - 4s^2t^3 - 4s^2t^2 - \\
& st^3) - \tfrac{(x-x_n)^8}{56h^6r^3(r-s)^3(r-t)^3(r-1)^3}(7r^3s^3 - 36r^4st - 36r^4s - 9r^4t^2 - 36r^4t - 9r^4 - 9r^4s^2 + \\
& 19r^3s^2t + 19r^3s^2 + 19r^3st^2 + 20r^3st + 19r^3s + 7r^3t^3 + 19r^3t^2 + 19r^3t + 7r^3 + 7r^2s^3t + \\
& 7r^2s^3 + 4r^2s^2t^2 + 27r^2s^2t + 4r^2s^2 + 7r^2st^3 + 27r^2st^2 + 27r^2st + 7r^2s + 7r^2t^3 + \\
& 4r^2t^2 + 7r^2t - 8rs^3t^2 - 13rs^3t - 8rs^3 - 8rs^2t^3 - 36rs^2t^2 - 36rs^2t - 8rs^2 - 13rst^3 - \\
& 36rst^2 - 13rst - 8rt^3 - 8rt^2 + 4s^3t^2 + 4s^3t + 4s^2t^3 + 16s^2t^2 + 4s^2t + 4st^3 + 4st^2) + \\
& \tfrac{(x-x_n)^9}{36h^7r^3(r-s)^3(r-t)^3(r-1)^3}(3r^3s^2 - 9r^4t - 9r^4 - 9r^4s - 2r^3st - 2r^3s + 3r^3t^2 - 2r^3t + 3r^3 + \\
& 3r^2s^3 + 10r^2s^2t + 10r^2s^2 + 10r^2st^2 + 21r^2st + 10r^2s + 3r^2t^3 + 10r^2t^2 + 10r^2t + 3r^2 - \\
& 2rs^3t - 2rs^3 - 8rs^2t^2 - 15rs^2t - 8rs^2 - 2rst^3 - 15rst^2 - 15rst - 2rs - 2rt^3 - 8rt^2 - \\
& 2rt + s^3t + 4s^2t^2 + 4s^2t + st^3 + 4st^2 + st) - \tfrac{(x-x_n)^{11}}{55h^9r^3(r-s)^3(r-t)^3(r-1)^3}(2rs + 2rt - st - \\
& 3r^2s - 3r^2t - 3r^2 + 4r^3 + 2rst) + \tfrac{s^2t^2(x-x_n)^4}{12h^2r^2(r-s)^3(r-t)^3(r-1)^3}(5rs + 5rt - 3st - 7r^2s - \\
& 7r^2t - 7r^2 + 9r^3 + 5rst) + \tfrac{st(x-x_n)^5}{10h^3r^3(r-s)^3(r-t)^3(r-1)^3}(7r^3s^2t - 9r^4 - 9r^4t - 9r^4st + 7r^3s^2 + \\
& 7r^3st^2 + 17r^3st + 7r^3s + 7r^3t^2 + 7r^3t - 5r^2s^2t^2 - 7r^2s^2t - 5r^2s^2 - 7r^2st^2 - 7r^2st - \\
& 5r^2t^2 + rs^2t^2 + rs^2t + rst^2 + s^2t^2),
\end{aligned}
$$

$$
\begin{aligned}
&\beta_s = \frac{(x-x_n)^7}{21h^5s^3(r-s)^3(s-t)^3(s-1)^3}(2r^3s^2t^2 - 7r^3s^3 - 7r^3s^3t - 2r^3s^2t + 2r^3s^2 + 2r^3st^3 + \\
&7r^3st^2 + 7r^3st + 2r^3s - r^3t^3 - 4r^3t^2 - r^3t + 9r^2s^4t + 9r^2s^4 - 10r^2s^3t^2 - 26r^2s^3t - \\
&10r^2s^3 + 2r^2s^2t^3 + 3r^2s^2t^2 + 3r^2s^2t + 2r^2s^2 + 7r^2st^3 + 12r^2st^2 + 7r^2st - 4r^2t^3 - \\
&4r^2t^2 + 9rs^4t^2 + 36rs^4t + 9rs^4 - 7rs^3t^3 - 26rs^3t^2 - 26rs^3t - 7rs^3 - 2rs^2t^3 + \\
&3rs^2t^2 - 2rs^2t + 7rst^3 + 7rst^2 - rt^3 + 9s^4t^2 + 9s^4t - 7s^3t^3 - 10s^3t^2 - 7s^3t + \\
&2s^2t^3 + 2s^2t^2 + 2st^3) - \frac{h(x-x_n)}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(84r^{11}st - 126r^{11}s^2 + 84r^{11}s - \\
&42r^{11}t + 399r^{10}s^3 + 105r^{10}s^2t + 105r^{10}s^2 - 264r^{10}st^2 - 345r^{10}st - 264r^{10}s + \\
&132r^{10}t^2 + 132r^{10}t - 297r^9s^4 - 1067r^9s^3t - 1067r^9s^3 + 616r^9s^2t^2 + 407r^9s^2t + \\
&616r^9s^2 + 220r^9st^3 + 506r^9st^2 + 506r^9st + 220r^9s - 110r^9t^3 - 440r^9t^2 - 110r^9t + \\
&990r^8s^4t + 990r^8s^4 + 363r^8s^3t^2 + 2992r^8s^3t + 363r^8s^3 - 825r^8s^2t^3 - 2387r^8s^2t^2 - \\
&2387r^8s^2t - 825r^8s^2 - 275r^8st^3 + 484r^8st^2 - 275r^8st + 396r^8t^3 + 396r^8t^2 - \\
&891r^7s^4t^2 - 3564r^7s^4t - 891r^7s^4 + 693r^7s^3t^3 - 891r^7s^3t^2 - 891r^7s^3t + 693r^7s^3 + \\
&2673r^7s^2t^3 + 3168r^7s^2t^2 + 2673r^7s^2t - 1188r^7st^3 - 1188r^7st^2 - 396r^7t^3 + \\
&3564r^6s^4t^2 + 3564r^6s^4t - 2772r^6s^3t^3 - 726r^6s^3t^2 - 2772r^6s^3t - 1518r^6s^2t^3 - \\
&1518r^6s^2t^2 + 2178r^6st^3 - 4158r^5s^4t^2 + 3234r^5s^3t^3 + 3234r^5s^3t^2 - 2310r^5s^2t^3) - \\
&\frac{(x-x_n)^6}{30h^4s^3(r-s)^3(s-t)^3(s-1)^3}(5r^3s^2t^3 - 28r^3s^3t - 7r^3s^3 - 7r^3s^3t^2 + 13r^3s^2t^2 + 13r^3s^2t + \\
&5r^3s^2 + 5r^3st^3 + 4r^3st^2 + 5r^3st - 4r^3t^3 - 4r^3t^2 + 9r^2s^4t^2 + 36r^2s^4t + 9r^2s^4 - 7r^2s^3t^3 - \\
&47r^2s^3t^2 - 47r^2s^3t - 7r^2s^3 + 13r^2s^2t^3 + 24r^2s^2t^2 + 13r^2s^2t + 4r^2st^3 + 4r^2st^2 - 4r^2t^3 + \\
&36rs^4t^2 + 36rs^4t - 28rs^3t^3 - 47rs^3t^2 - 28rs^3t + 13rs^2t^3 + 13rs^2t^2 + 5rst^3 + 9s^4t^2 - \\
&7s^3t^3 - 7s^3t^2 + 5s^2t^3) - \frac{(x-x_n)^{10}}{90h^8s^3(r-s)^3(s-t)^3(s-1)^3}(8r^2st - 12r^2s^2 + 8r^2s - 4r^2t + 9rs^3 - \\
&19rs^2t - 19rs^2 + 8rst^2 + 21rst + 8rs - 4rt^2 - 4rt + 9s^4 + 9s^3t + 9s^3 - 12s^2t^2 - 19s^2t - \\
&12s^2 + 8st^2 + 8st) + \frac{(x-x_n)^8}{56h^6s^3(r-s)^3(s-t)^3(s-1)^3}(7r^3s^3 + 7r^3s^2t + 7r^3s^2 - 8r^3st^2 - 13r^3st - \\
&8r^3s + 4r^3t^2 + 4r^3t - 9r^2s^4 + 19r^2s^3t + 19r^2s^3 + 4r^2s^2t^2 + 27r^2s^2t + 4r^2s^2 - 8r^2st^3 - \\
&36r^2st^2 - 36r^2st - 8r^2s + 4r^2t^3 + 16r^2t^2 + 4r^2t - 36rs^4t - 36rs^4 + 19rs^3t^2 + 20rs^3t + \\
&19rs^3 + 7rs^2t^3 + 27rs^2t^2 + 27rs^2t + 7rs^2 - 13rst^3 - 36rst^2 - 13rst + 4rt^3 + 4rt^2 - \\
&9s^4t^2 - 36s^4t - 9s^4 + 7s^3t^3 + 19s^3t^2 + 19s^3t + 7s^3 + 7s^2t^3 + 4s^2t^2 + 7s^2t - 8st^3 - \\
&8st^2) - \frac{(x-x_n)^9}{36h^7s^3(r-s)^3(s-t)^3(s-1)^3}(3r^3s^2 - 2r^3st - 2r^3s + r^3t + 3r^2s^3 + 10r^2s^2t + 10r^2s^2 - \\
&8r^2st^2 - 15r^2st - 8r^2s + 4r^2t^2 + 4r^2t - 9rs^4 - 2rs^3t - 2rs^3 + 10rs^2t^2 + 21rs^2t + \\
&10rs^2 - 2rst^3 - 15rst^2 - 15rst - 2rs + rt^3 + 4rt^2 + rt - 9s^4t - 9s^4 + 3s^3t^2 - 2s^3t + 3s^3 +
\end{aligned}
$$

$3s^2t^3+10s^2t^2+10s^2t+3s^2-2st^3-8st^2-2st)+\frac{(x-x_n)^{11}}{55h^9s^3(r-s)^3(s-t)^3(s-1)^3}(2rs-rt+2st-3rs^2-3s^2t-3s^2+4s^3+2rst)-\frac{r^2t^2(x-x_n)^4}{12h^2s^2(r-s)^3(s-t)^3(s-1)^3}(5rs-3rt+5st-7rs^2-7s^2t-7s^2+9s^3+5rst)-\frac{rt(x-x_n)^5}{10h^3s^3(r-s)^3(s-t)^3(s-1)^3}(7r^2s^3t+7r^2s^3-5r^2s^2t^2-7r^2s^2t-5r^2s^2+r^2st^2+r^2st+r^2t^2-9rs^4t-9rs^4+7rs^3t^2+17rs^3t+7rs^3-7rs^2t^2-7rs^2t+rst^2-9s^4t+7s^3t^2+7s^3t-5s^2t^2),$

$\beta_t=\frac{(x-x_n)^{10}}{90h^8t^3(r-t)^3(s-t)^3(t-1)^3}(8r^2st-4r^2s-12r^2t^2+8r^2t+8rs^2t-4rs^2-19rst^2+21rst-4rs+9rt^3-19rt^2+8rt-12s^2t^2+8s^2t+9st^3-19st^2+8st+9t^4+9t^3-12t^2)-\frac{h(x-x_n)}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(42r^{11}s-84r^{11}st+126r^{11}t^2-84r^{11}t+264r^{10}s^2t-132r^{10}s^2-105r^{10}st^2+345r^{10}st-132r^{10}s-399r^{10}t^3-105r^{10}t^2+264r^{10}t-220r^9s^3t+110r^9s^3-616r^9s^2t^2-506r^9s^2t+440r^9s^2+1067r^9st^3-407r^9st^2-506r^9st+110r^9s+297r^9t^4+1067r^9t^3-616r^9t^2-220r^9t+825r^8s^3t^2+275r^8s^3t-396r^8s^3-363r^8s^2t^3+2387r^8s^2t^2-484r^8s^2t-396r^8s^2-990r^8st^4-2992r^8st^3+2387r^8st^2+275r^8st-990r^8t^4-363r^8t^3+825r^8t^2-693r^7s^3t^3-2673r^7s^3t^2+1188r^7s^3t+396r^7s^3+891r^7s^2t^4+891r^7s^2t^3-3168r^7s^2t^2+1188r^7s^2t+3564r^7st^4+891r^7st^3-2673r^7st^2+891r^7t^4-693r^7t^3+2772r^6s^3t^3+1518r^6s^3t^2-2178r^6s^3t-3564r^6s^2t^4+726r^6s^2t^3+1518r^6s^2t^2-3564r^6st^4+2772r^6st^3-3234r^5s^3t^3+2310r^5s^3t^2+4158r^5s^2t^4-3234r^5s^2t^3)+\frac{(x-x_n)^6}{30h^4t^3(r-t)^3(s-t)^3(t-1)^3}(5r^3s^3t^2+5r^3s^3t-4r^3s^3-7r^3s^2t^3+13r^3s^2t^2+4r^3s^2t-4r^3s^2-28r^3st^3+13r^3st^2+5r^3st-7r^3t^3+5r^3t^2-7r^2s^3t^3+13r^2s^3t^2+4r^2s^3t-4r^2s^3+9r^2s^2t^4-47r^2s^2t^3+24r^2s^2t^2+4r^2s^2t+36r^2st^4-47r^2st^3+13r^2st^2+9r^2t^4-7r^2t^3-28rs^3t^3+13rs^3t^2+5rs^3t+36rs^2t^4-47rs^2t^3+13rs^2t^2+36rst^4-28rst^3-7s^3t^3+5s^3t^2+9s^2t^4-7s^2t^3)-\frac{(x-x_n)^7}{21h^5t^3(r-t)^3(s-t)^3(t-1)^3}(2r^3s^3t-r^3s^3+2r^3s^2t^2+7r^3s^2t-4r^3s^2-7r^3st^3-2r^3st^2+7r^3st-r^3s-7r^3t^3+2r^3t^2+2r^3t+2r^2s^3t^2+7r^2s^3t-4r^2s^3-10r^2s^2t^3+3r^2s^2t^2+12r^2s^2t-4r^2s^2+9r^2st^4-26r^2st^3+3r^2st^2+7r^2st+9r^2t^4-10r^2t^3+2r^2t^2-7rs^3t^3-2rs^3t^2+7rs^3t-rs^3+9rs^2t^4-26rs^2t^3+3rs^2t^2+7rs^2t+36rst^4-26rst^3-2rst^2+9rt^4-7rt^3-7s^3t^3+2s^3t^2+2s^3t+9s^2t^4-10s^2t^3+2s^2t^2+9st^4-7st^3)-\frac{(x-x_n)^8}{56h^6t^3(r-t)^3(s-t)^3(t-1)^3}(4r^3s^2-8r^3s^2t+7r^3st^2-13r^3st+4r^3s+7r^3t^3+7r^3t^2-8r^3t-8r^2s^3t+4r^2s^3+4r^2s^2t^2-36r^2s^2t+16r^2s^2+19r^2st^3+27r^2st^2-36r^2st+4r^2s-$

$9r^2t^4 + 19r^2t^3 + 4r^2t^2 - 8r^2t + 7rs^3t^2 - 13rs^3t + 4rs^3 + 19rs^2t^3 + 27rs^2t^2 -$
$36rs^2t + 4rs^2 - 36rst^4 + 20rst^3 + 27rst^2 - 13rst - 36rt^4 + 19rt^3 + 7rt^2 + 7s^3t^3 +$
$7s^3t^2 - 8s^3t - 9s^2t^4 + 19s^2t^3 + 4s^2t^2 - 8s^2t - 36st^4 + 19st^3 + 7st^2 - 9t^4 +$
$7t^3) + \frac{(x-x_n)^{11}}{55h^9t^3(r-t)^3(s-t)^3(t-1)^3}(rs - 2rt - 2st + 3rt^2 + 3st^2 + 3t^2 - 4t^3 - 2rst) +$
$\frac{(x-x_n)^9}{36h^7t^3(r-t)^3(s-t)^3(t-1)^3}(r^3s - 2r^3st + 3r^3t^2 - 2r^3t - 8r^2s^2t + 4r^2s^2 + 10r^2st^2 - 15r^2st +$
$4r^2s + 3r^2t^3 + 10r^2t^2 - 8r^2t - 2rs^3t + rs^3 + 10rs^2t^2 - 15rs^2t + 4rs^2 - 2rst^3 + 21rst^2 -$
$15rst + rs - 9rt^4 - 2rt^3 + 10rt^2 - 2rt + 3s^3t^2 - 2s^3t + 3s^2t^3 + 10s^2t^2 - 8s^2t - 9st^4 -$
$2st^3 + 10st^2 - 2st - 9t^4 + 3t^3 + 3t^2) - \frac{r^2s^2(x-x_n)^4}{12h^2t^2(r-t)^3(s-t)^3(t-1)^3}(3rs - 5rt - 5st + 7rt^2 +$
$7st^2 + 7t^2 - 9t^3 - 5rst) + \frac{rs(x-x_n)^5}{10h^3t^3(r-t)^3(s-t)^3(t-1)^3}(r^2s^2t - 5r^2s^2t^2 + r^2s^2 + 7r^2st^3 -$
$7r^2st^2 + r^2st + 7r^2t^3 - 5r^2t^2 + 7rs^2t^3 - 7rs^2t^2 + rs^2t - 9rst^4 + 17rst^3 - 7rst^2 - 9rt^4 +$
$7rt^3 + 7s^2t^3 - 5s^2t^2 - 9st^4 + 7st^3),$

$\beta_1 = \frac{(x-x_n)^7}{21h^5(r-1)^3(s-1)^3(t-1)^3}(2r^3s^3 - r^3s^3t - 4r^3s^2t^2 + 7r^3s^2t + 2r^3s^2 - r^3st^3 +$
$7r^3st^2 - 2r^3st - 7r^3s + 2r^3t^3 + 2r^3t^2 - 7r^3t - 4r^2s^3t^2 + 7r^2s^3t + 2r^2s^3 - 4r^2s^2t^3 +$
$12r^2s^2t^2 + 3r^2s^2t - 10r^2s^2 + 7r^2st^3 + 3r^2st^2 - 26r^2st + 9r^2s + 2r^2t^3 - 10r^2t^2 +$
$9r^2t - rs^3t^3 + 7rs^3t^2 - 2rs^3t - 7rs^3 + 7rs^2t^3 + 3rs^2t^2 - 26rs^2t + 9rs^2 - 2rst^3 -$
$26rst^2 + 36rst - 7rt^3 + 9rt^2 + 2s^3t^3 + 2s^3t^2 - 7s^3t + 2s^2t^3 - 10s^2t^2 + 9s^2t -$
$7st^3 + 9st^2) - \frac{(x-x_n)^{11}}{55h^9(r-1)^3(s-1)^3(t-1)^3}(3r + 3s + 3t - 2rs - 2rt - 2st + rst - 4) -$
$\frac{(x-x_n)^9}{36h^7(r-1)^3(s-1)^3(t-1)^3}(r^3st - 2r^3s - 2r^3t + 3r^3 + 4r^2s^2t - 8r^2s^2 + 4r^2st^2 - 15r^2st +$
$10r^2s - 8r^2t^2 + 10r^2t + 3r^2 + rs^3t - 2rs^3 + 4rs^2t^2 - 15rs^2t + 10rs^2 + rst^3 - 15rst^2 +$
$21rst - 2rs - 2rt^3 + 10rt^2 - 2rt - 9r - 2s^3t + 3s^3 - 8s^2t^2 + 10s^2t + 3s^2 - 2st^3 +$
$10st^2 - 2st - 9s + 3t^3 + 3t^2 - 9t) + \frac{(x-x_n)^8}{56h^6(r-1)^3(s-1)^3(t-1)^3}(4r^3s^2t - 8r^3s^2 + 4r^3st^2 -$
$13r^3st + 7r^3s - 8r^3t^2 + 7r^3t + 7r^3 + 4r^2s^3t - 8r^2s^3 + 16r^2s^2t^2 - 36r^2s^2t + 4r^2s^2 +$
$4r^2st^3 - 36r^2st^2 + 27r^2st + 19r^2s - 8r^2t^3 + 4r^2t^2 + 19r^2t - 9r^2 + 4rs^3t^2 - 13rs^3t +$
$7rs^3 + 4rs^2t^3 - 36rs^2t^2 + 27rs^2t + 19rs^2 - 13rst^3 + 27rst^2 + 20rst - 36rs + 7rt^3 +$
$19rt^2 - 36rt - 8s^3t^2 + 7s^3t + 7s^3 - 8s^2t^3 + 4s^2t^2 + 19s^2t - 9s^2 + 7st^3 + 19st^2 - 36st +$
$7t^3 - 9t^2) - \frac{(x-x_n)^6}{30h^4(r-1)^3(s-1)^3(t-1)^3}(5r^3s^3t - 4r^3s^3t^2 + 5r^3s^3 - 4r^3s^2t^3 + 4r^3s^2t^2 +$
$13r^3s^2t - 7r^3s^2 + 5r^3st^3 + 13r^3st^2 - 28r^3st + 5r^3t^3 - 7r^3t^2 - 4r^2s^3t^3 + 4r^2s^3t^2 +$
$13r^2s^3t - 7r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 - 47r^2s^2t + 9r^2s^2 + 13r^2st^3 - 47r^2st^2 + 36r^2st$

$$-7r^2t^3+9r^2t^2+5rs^3t^3+13rs^3t^2-28rs^3t+13rs^2t^3-47rs^2t^2+36rs^2t-28rst^3+36rst^2+5s^3t^3-7s^3t^2-7s^2t^3+9s^2t^2)+\frac{h(x-x_n)}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^{11}st-84r^{11}s-84r^{11}t+126r^{11}-132r^{10}s^2t+264r^{10}s^2-132r^{10}st^2+345r^{10}st-105r^{10}s+264r^{10}t^2-105r^{10}t-399r^{10}+110r^9s^3t-220r^9s^3+440r^9s^2t^2-506r^9s^2t-616r^9s^2+110r^9st^3-506r^9st^2-407r^9st+1067r^9s-220r^9t^3-616r^9t^2+1067r^9t+297r^9-396r^8s^3t^2+275r^8s^3t+825r^8s^3-396r^8s^2t^3-484r^8s^2t^2+2387r^8s^2t-363r^8s^2+275r^8st^3+2387r^8st^2-2992r^8st-990r^8s+825r^8t^3-363r^8t^2-990r^8t+396r^7s^3t^3+1188r^7s^3t^2-2673r^7s^3t-693r^7s^3+1188r^7s^2t^3-3168r^7s^2t^2+891r^7s^2t+891r^7s^2-2673r^7st^3+891r^7st^2+3564r^7st-693r^7t^3+891r^7t^2-2178r^6s^3t^3+1518r^6s^3t^2+2772r^6s^3t+1518r^6s^2t^3+726r^6s^2t^2-3564r^6s^2t+2772r^6st^3-3564r^6st^2+2310r^5s^3t^3-3234r^5s^3t^2-3234r^5s^2t^3+4158r^5s^2t^2)-\frac{(x-x_n)^{10}}{90h^8(r-1)^3(s-1)^3(t-1)^3}(8r^2s-4r^2st+8r^2t-12r^2-4rs^2t+8rs^2-4rst^2+21rst-19rs+8rt^2-19rt+9r+8s^2t-12s^2+8st^2-19st+9s-12t^2+9t+9)-\frac{rst(x-x_n)^5}{10h^3(r-1)^3(s-1)^3(t-1)^3}(r^2s^2t^2+r^2s^2t-5r^2s^2+r^2st^2-7r^2st+7r^2s-5r^2t^2+7r^2t+rs^2t^2-7rs^2t+7rs^2-7rst^2+17rst-9rs+7rt^2-9rt-5s^2t^2+7s^2t+7st^2-9st)+\frac{r^2s^2t^2(x-x_n)^4}{12h^2(r-1)^3(s-1)^3(t-1)^3}(7r+7s+7t-5rs-5rt-5st+3rst-9),$$

$$\gamma_0=\frac{(x-x_n)^3}{6}+\frac{(x-x_n)^{11}}{110h^8r^2s^2t^2}-\frac{h^2(x-x_n)}{27720s^2t^2}(21r^8-66r^7s-66r^7t-66r^7+55r^6s^2+220r^6st+220r^6s+55r^6t^2+220r^6t+55r^6-198r^5s^2t-198r^5s^2-198r^5st^2-792r^5st-198r^5s-198r^5t^2-198r^5t+198r^4s^2t^2+792r^4s^2t+198r^4s^2+792r^4st^2+792r^4st+198r^4t^2-924r^3s^2t^2-924r^3s^2t-924r^3st^2+1386r^2s^2t^2)+\frac{(x-x_n)^7}{42h^4r^2s^2t^2}(r^2s^2+4r^2st+4r^2s+r^2t^2+4r^2t+r^2+4rs^2t+4rs^2+4rst^2+16rst+4rs+4rt^2+4rt+s^2t^2+4s^2t+s^2+4st^2+4st+t^2)-\frac{(x-x_n)^8}{28h^5r^2s^2t^2}(r^2s+r^2t+r^2+rs^2+4rst+4rs+rt^2+4rt+r+s^2t+s^2+st^2+4st+s+t^2+t)-\frac{(x-x_n)^6}{15h^3r^2s^2t^2}(r^2s^2t+r^2s^2+r^2st^2+4r^2st+r^2s+r^2t^2+r^2t+rs^2t^2+4rs^2t+rs^2+4rst^2+4rst+rt^2+s^2t^2+s^2t+st^2)-\frac{(x-x_n)^{10}}{45h^7r^2s^2t^2}(r+s+t+1)-\frac{(x-x_n)^4}{6hrst}(rs+rt+st+rst)+\frac{(x-x_n)^5}{20h^2r^2s^2t^2}(r^2s^2t^2+4r^2s^2t+r^2s^2+4r^2st^2+4r^2st+r^2t^2+4rs^2t^2+4rs^2t+4rst^2+s^2t^2)+\frac{(x-x_n)^9}{72h^6r^2s^2t^2}(r^2+4rs+4rt+4r+s^2+4st+4s+t^2+4t+1),$$

$$\gamma_r = \frac{(x-x_n)^{11}}{(110h^8r^2(r-s)^2(r-t)^2(r-1)^2} + \frac{h^2(x-x_n)}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^8 - 77r^7s - 77r^7t - 77r^7 + 55r^6s^2 + 220r^6st + 220r^6s + 55r^6t^2 + 220r^6t + 55r^6 - 165r^5s^2t - 165r^5s^2 - 165r^5st^2 - 660r^5st - 165r^5s - 165r^5t^2 - 165r^5t + 132r^4s^2t^2 + 528r^4s^2t + 132r^4s^2 + 528r^4st^2 + 528r^4st + 132r^4t^2 - 462r^3s^2t^2 - 462r^3s^2t - 462r^3st^2 + 462r^2s^2t^2) - \frac{(x-x_n)^8}{56h^5r^2(r-s)^2(r-t)^2(r-1)^2}(r + 2s + 2t + 4rs + 4rt + 8st + rs^2 + rt^2 + 2st^2 + 2s^2t + 2s^2 + 2t^2 + 4rst) + \frac{(x-x_n)^9}{72h^6r^2(r-s)^2(r-t)^2(r-1)^2}(2r + 4s + 4t + 2rs + 2rt + 4st + s^2 + t^2 + 1) + \frac{(x-x_n)^7}{42h^4r^2(r-s)^2(r-t)^2(r-1)^2}(s^2t^2 + 2rs + 2rt + 4st + 2rs^2 + 2rt^2 + 4st^2 + 4s^2t + s^2 + t^2 + 2rst^2 + 2rs^2t + 8rst) - \frac{(x-x_n)^6}{30h^3r^2(r-s)^2(r-t)^2(r-1)^2}(2s^2t^2 + rs^2 + rt^2 + 2st^2 + 2s^2t + 4rst^2 + 4rs^2t + rs^2t^2 + 4rst) - \frac{(x-x_n)^{10}}{90h^7r^2(r-s)^2(r-t)^2(r-1)^2}(r + 2s + 2t + 2) - \frac{s^2t^2(x-x_n)^4)}{(12hr(r-s)^2(r-t)^2(r-1)^2} + \frac{st(x-x_n)^5}{20h^2r^2(r-s)^2(r-t)^2(r-1)^2}(2rs + 2rt + st + 2rst),$$

$$\gamma_s = \frac{(x-x_n)^{11}}{(110h^8s^2(r-s)^2(s-t)^2(s-1)^2} - \frac{(x-x_n)^8}{56h^5s^2(r-s)^2(s-t)^2(s-1)^2}(2r + s + 2t + 4rs + 8rt + 4st + r^2s + 2rt^2 + 2r^2t + st^2 + 2r^2 + 2t^2 + 4rst) + \frac{(x-x_n)^9}{72h^6s^2(r-s)^2(s-t)^2(s-1)^2}(4r + 2s + 4t + 2rs + 4rt + 2st + r^2 + t^2 + 1) + \frac{(x-x_n)^7}{42h^4s^2(r-s)^2(s-t)^2(s-1)^2}(r^2t^2 + 2rs + 4rt + 2st + 2r^2s + 4rt^2 + 4r^2t + 2st^2 + r^2 + t^2 + 2rst^2 + 2r^2st + 8rst) - \frac{(x-x_n)^6}{30h^3s^2(r-s)^2(s-t)^2(s-1)^2}(2r^2t^2 + r^2s + 2rt^2 + 2r^2t + st^2 + 4rst^2 + 4r^2st + r^2st^2 + 4rst) - \frac{(x-x_n)^{10}}{90h^7s^2(r-s)^2(s-t)^2(s-1)^2}(2r + s + 2t + 2) + \frac{h^2r^5(x-x_n)}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 + 396r^2st - 110r^3st + 99r^2st^2 - 396rst) - \frac{r^2t^2(x-x_n)^4}{12hs(r-s)^2(s-t)^2(s-1)^2} + \frac{rt(x-x_n)^5}{20h^2s^2(r-s)^2(s-t)^2(s-1)^2}(2rs + rt + 2st + 2rst),$$

$$\gamma_t = \frac{(x-x_n)^{11}}{110h^8t^2(r-t)^2(s-t)^2(t-1)^2} - \frac{(x-x_n)^8}{56h^5t^2(r-t)^2(s-t)^2(t-1)^2}(2r + 2s + t + 8rs + 4rt + 4st + 2rs^2 + 2r^2s + r^2t + s^2t + 2r^2 + 2s^2 + 4rst) + \frac{(x-x_n)^9}{72h^6t^2(r-t)^2(s-t)^2(t-1)^2}(4r + 4s + 2t + 4rs + 2rt + 2st + r^2 + s^2 + 1) + \frac{(x-x_n)^7}{42h^4t^2(r-t)^2(s-t)^2(t-1)^2}(r^2s^2 + 4rs + 2rt + 2st + 4rs^2 + 4r^2s + 2r^2t + 2s^2t + r^2 + s^2 + 2rs^2t + 2r^2st + 8rst) - \frac{(x-x_n)^6}{30h^3t^2(r-t)^2(s-t)^2(t-1)^2}(2r^2s^2 + 2rs^2 + 2r^2s + r^2t + s^2t + 4rs^2t + 4r^2st + r^2s^2t + 4rst) - \frac{(x-x_n)^{10}}{90h^7t^2(r-t)^2(s-t)^2(t-1)^2}(2r + 2s + t + 2) + \frac{h^2r^5(x-x_n)}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst) - \frac{r^2s^2(x-x_n)^4}{12ht(r-t)^2(s-t)^2(t-1)^2} + \frac{rs(x-x_n)^5}{20h^2t^2(r-t)^2(s-t)^2(t-1)^2}(rs + 2rt + 2st + 2rst),$$

$$\gamma_1 = \frac{(x-x_n)^{11}}{110h^8(r-1)^2(s-1)^2(t-1)^2} - \frac{(x-x_n)^8}{56h^5(r-1)^2(s-1)^2(t-1)^2}(2r^2s + 2r^2t + r^2 + 2rs^2 + 8rst + 4rs + 2rt^2 + 4rt + 2s^2t + s^2 + 2st^2 + 4st + t^2) + \frac{(x-x_n)^7}{42h^4(r-1)^2(s-1)^2(t-1)^2}(r^2s^2 + 4r^2st + 2r^2s + r^2t^2 + 2r^2t + 4rs^2t + 2rs^2 + 4rst^2 + 8rst + 2rt^2 + s^2t^2 + 2s^2t + 2st^2) - \frac{(x-x_n)^{10}}{90h^7(r-1)^2(s-1)^2(t-1)^2}(2r+2s+2t+1) - \frac{(x-x_n)^6}{30h^3(r-1)^2(s-1)^2(t-1)^2}(2r^2s^2t + r^2s^2 + 2r^2st^2 + 4r^2st + r^2t^2 + 2rs^2t^2 + 4rs^2t + 4rst^2 + s^2t^2) + \frac{(x-x_n)^9}{72h^6(r-1)^2(s-1)^2(t-1)^2}(r^2 + 4rs + 4rt + 2r + s^2 + 4st + 2s + t^2 + 2t) + \frac{h^2r^5(x-x_n)}{27720(r-1)^2(s-1)^2(t-1)^2}(-21r^5 + 66r^4s + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st - 110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 - 198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2) - \frac{r^2s^2t^2(x-x_n)^4}{12h(r-1)^2(s-1)^2(t-1)^2} + \frac{rst(x-x_n)^5}{20h^2(r-1)^2(s-1)^2(t-1)^2}(2rs + 2rt + 2st + rst).$$

Evaluating (3.52) at the non-interpolated points, i.e x_{n+s}, x_{n+t} and x_{n+1}, produces the following equations

$$y_{n+s} - \frac{s}{r}y_{n+r} + \frac{(s-r)}{r}y_n = -\frac{h^2(r-s)}{27720r^3s^2t^3}f_n(110r^3s^6 - 286r^2s^7 + 110r^4s^5 + 110r^5s^4 + 110r^6s^3 - 286r^7s^2 + 308r^2s^8 - 88r^3s^7 - 88r^4s^6 - 88r^5s^5 - 88r^6s^4 - 88r^7s^3 + 308r^8s^2 - 90r^2s^9 + 20r^3s^8 + 20r^4s^7 + 20r^5s^6 + 20r^6s^5 + 20r^7s^4 + 20r^8s^3 - 90r^9s^2 + 396r^6t^3 - 396r^7t^2 - 396r^7t^3 + 440r^8t^2 + 110r^8t^3 - 132r^9t^2 + 396s^6t^3 - 396s^7t^2 - 396s^7t^3 + 440s^8t^2 + 110s^8t^3 - 132s^9t^2 + 110rs^8 + 110r^8s - 132rs^9 - 132r^9s + 42rs^{10} + 42r^{10}s + 110r^8t - 132r^9t + 42r^{10}t + 110s^8t - 132s^9t + 42s^{10}t - 473rs^7t - 473r^7st + 616rs^8t + 616r^8st - 279rs^9t - 279r^9st + 42rs^{10}t + 42r^{10}st - 594rs^5t^3 + 792rs^6t^2 + 715r^2s^6t - 275r^3s^5t - 275r^4s^4t - 594r^5st^3 - 275r^5s^3t + 792r^6st^2 + 715r^6s^2t + 792rs^6t^3 - 1100rs^7t^2 - 924r^2s^7t + 264r^3s^6t + 264r^4s^5t + 264r^5s^4t + 792r^6st^3 + 264r^6s^3t - 1100r^7st^2 - 924r^7s^2t - 473rs^7t^3 + 616rs^8t^2 + 469r^2s^8t - 114r^3s^7t - 114r^4s^6t - 114r^5s^5t - 114r^6s^4t - 473r^7st^3 - 114r^7s^3t + 616r^8st^2 + 469r^8s^2t + 110rs^8t^3 - 132rs^9t^2 - 90r^2s^9t + 20r^3s^8t + 20r^4s^7t + 20r^5s^6t + 20r^6s^5t + 20r^7s^4t + 110r^8st^3 + 20r^8s^3t - 132r^9st^2 - 90r^9s^2t - 2442r^2s^4t^3 + 528r^2s^5t^2 + 7260r^3s^3t^3 - 1320r^3s^4t^2 - 2442r^4s^2t^3 - 1320r^4s^3t^2 + 528r^5s^2t^2 + 528r^2s^5t^3 + 484r^2s^6t^2 - 1320r^3s^4t^3 + 220r^3s^5t^2 - 1320r^4s^3t^3 + 220r^4s^4t^2 + 528r^5s^2t^3 + 220r^5s^3t^2 + 484r^6s^2t^2 + 715r^2s^6t^3 - 924r^2s^7t^2 - 275r^3s^5t^3 + 264r^3s^6t^2 - 275r^4s^4t^3 +$$

$$
264r^4s^5t^2 - 275r^5s^3t^3 + 264r^5s^4t^2 + 715r^6s^2t^3 + 264r^6s^3t^2 - 924r^7s^2t^2 -
286r^2s^7t^3 + 308r^2s^8t^2 + 110r^3s^6t^3 - 88r^3s^7t^2 + 110r^4s^5t^3 - 88r^4s^6t^2 +
110r^5s^4t^3 - 88r^5s^5t^2 + 110r^6s^3t^3 - 88r^6s^4t^2 - 286r^7s^2t^3 - 88r^7s^3t^2 + 308r^8s^2t^2) +
\frac{h^2 s f_{n+r}}{27720r^3(r-s)^2(r-t)^3(r-1)^3}(88r^3s^6 - 605r^2s^7 + 88r^4s^5 + 88r^5s^4 + 88r^6s^3 + 2068r^7s^2 +
352r^2s^8 + 715r^3s^7 - 176r^4s^6 - 176r^5s^5 - 176r^6s^4 - 176r^7s^3 - 5324r^8s^2 + 189r^2s^9 -
878r^3s^8 + 112r^4s^7 + 112r^5s^6 + 112r^6s^5 + 112r^7s^4 + 112r^8s^3 + 4347r^9s^2 -
126r^2s^{10} + 273r^3s^9 - 24r^4s^8 - 24r^5s^7 - 24r^6s^6 - 24r^7s^5 - 24r^8s^4 - 24r^9s^3 -
1179r^{10}s^2 - 1980r^6t^3 + 5148r^7t^2 + 5148r^7t^3 - 13376r^8t^2 - 4235r^8t^3 + 11121r^9t^2 +
1155r^9t^3 - 3069r^{10}t^2 - 396s^6t^3 + 396s^7t^2 + 396s^7t^3 - 440s^8t^2 - 110s^8t^3 + 132s^9t^2 +
220rs^8 - 3080r^8s - 264rs^9 + 8052r^9s + 84rs^{10} - 6774r^{10}s + 1890r^{11}s - 4235r^8t +
11121r^9t - 9420r^{10}t + 2646r^{11}t - 110s^8t + 132s^9t - 42s^{10}t + 1155r^9 - 3069r^{10} +
2646r^{11} - 756r^{1}2 - 385rs^7t + 11792r^7st + 638rs^8t - 30316r^8st - 387rs^9t +
24858r^9st + 84rs^{10}t - 6774r^{10}st + 1782rs^5t^3 - 792rs^6t^2 + 2288r^2s^6t - 484r^3s^5t -
484r^4s^4t + 5940r^5st^3 - 484r^5s^3t - 15048r^6st^2 - 8404r^6s^2t - 792rs^6t^3 + 44rs^7t^2 -
1749r^2s^7t - 2640r^3s^6t + 924r^4s^5t + 924r^5s^4t - 15048r^6st^3 + 924r^6s^3t + 38060r^7st^2 +
21120r^7s^2t - 385rs^7t^3 + 638rs^8t^2 + 20r^2s^8t + 3012r^3s^7t - 552r^4s^6t - 552r^5s^5t -
552r^6s^4t + 11792r^7st^3 - 552r^7s^3t - 30316r^8st^2 - 16579r^8s^2t + 220rs^8t^3 - 264rs^9t^2 +
189r^2s^9t - 878r^3s^8t + 112r^4s^7t + 112r^5s^6t + 112r^6s^5t + 112r^7s^4t - 3080r^8st^3 +
112r^8s^3t + 8052r^9st^2 + 4347r^9s^2t - 528r^2s^4t^3 - 2310r^2s^5t^2 - 528r^3s^3t^3 +
924r^3s^4t^2 - 4686r^4s^2t^3 + 924r^4s^3t^2 + 11550r^5s^2t^2 - 2310r^2s^5t^3 + 3212r^2s^6t^2 +
924r^3s^4t^3 + 2486r^3s^5t^2 + 924r^4s^3t^3 - 1672r^4s^4t^2 + 11550r^5s^2t^3 - 1672r^5s^3t^2 -
28270r^6s^2t^2 + 2288r^2s^6t^3 - 1749r^2s^7t^2 - 484r^3s^5t^3 - 2640r^3s^6t^2 - 484r^4s^4t^3 +
924r^4s^5t^2 - 484r^5s^3t^3 + 924r^5s^4t^2 - 8404r^6s^2t^3 + 924r^6s^3t^2 + 21120r^7s^2t^2 -
605r^2s^7t^3 + 352r^2s^8t^2 + 88r^3s^6t^3 + 715r^3s^7t^2 + 88r^4s^5t^3 - 176r^4s^6t^2 + 88r^5s^4t^3 -
176r^5s^5t^2 + 88r^6s^3t^3 - 176r^6s^4t^2 + 2068r^7s^2t^3 - 176r^7s^3t^2 - 5324r^8s^2t^2) -
\frac{h^2 f_{n+s}}{27720s^2(r-s)^2(s-t)^3(s-1)^3}(2068r^2s^7 + 88r^3s^6 + 88r^4s^5 + 88r^5s^4 + 88r^6s^3 - 605r^7s^2 -
5324r^2s^8 - 176r^3s^7 - 176r^4s^6 - 176r^5s^5 - 176r^6s^4 + 715r^7s^3 + 352r^8s^2 + 4347r^2s^9 +
112r^3s^8 + 112r^4s^7 + 112r^5s^6 + 112r^6s^5 + 112r^7s^4 - 878r^8s^3 + 189r^9s^2 - 1179r^2s^{10} -
24r^3s^9 - 24r^4s^8 - 24r^5s^7 - 24r^6s^6 - 24r^7s^5 - 24r^8s^4 + 273r^9s^3 - 126r^{10}s^2 - 396r^6t^3
$$

$$+396r^7t^2 + 396r^7t^3 - 440r^8t^2 - 110r^8t^3 + 132r^9t^2 - 1980s^6t^3 + 5148s^7t^2 +$$

$$5148s^7t^3 - 13376s^8t^2 - 4235s^8t^3 + 11121s^9t^2 + 1155s^9t^3 - 3069s^{10}t^2 - 3080rs^8 +$$

$$220r^8s + 8052rs^9 - 264r^9s - 6774rs^{10} + 84r^{10}s + 1890rs^{11} - 110r^8t + 132r^9t -$$

$$42r^{10}t - 4235s^8t + 11121s^9t - 9420s^{10}t + 2646s^{11}t + 1155s^9 - 3069s^{10} + 2646s^{11} -$$

$$756s^12 + 11792rs^7t - 385r^7st - 30316rs^8t + 638r^8st + 24858rs^9t - 387r^9st -$$

$$6774rs^{10}t + 84r^{10}st + 5940rs^5t^3 - 15048rs^6t^2 - 8404r^2s^6t - 484r^3s^5t - 484r^4s^4t +$$

$$1782r^5st^3 - 484r^5s^3t - 792r^6st^2 + 2288r^6s^2t - 15048rs^6t^3 + 38060rs^7t^2 +$$

$$21120r^2s^7t + 924r^3s^6t + 924r^4s^5t + 924r^5s^4t - 792r^6st^3 - 2640r^6s^3t + 44r^7st^2 -$$

$$1749r^7s^2t + 11792rs^7t^3 - 30316rs^8t^2 - 16579r^2s^8t - 552r^3s^7t - 552r^4s^6t -$$

$$552r^5s^5t - 552r^6s^4t - 385r^7st^3 + 3012r^7s^3t + 638r^8st^2 + 20r^8s^2t - 3080rs^8t^3 +$$

$$8052rs^9t^2 + 4347r^2s^9t + 112r^3s^8t + 112r^4s^7t + 112r^5s^6t + 112r^6s^5t + 112r^7s^4t +$$

$$220r^8st^3 - 878r^8s^3t - 264r^9st^2 + 189r^9s^2t - 4686r^2s^4t^3 + 11550r^2s^5t^2 - 528r^3s^3t^3 +$$

$$924r^3s^4t^2 - 528r^4s^2t^3 + 924r^4s^3t^2 - 2310r^5s^2t^2 + 11550r^2s^5t^3 - 28270r^2s^6t^2 +$$

$$924r^3s^4t^3 - 1672r^3s^5t^2 + 924r^4s^3t^3 - 1672r^4s^4t^2 - 2310r^5s^2t^3 + 2486r^5s^3t^2 +$$

$$3212r^6s^2t^2 - 8404r^2s^6t^3 + 21120r^2s^7t^2 - 484r^3s^5t^3 + 924r^3s^6t^2 - 484r^4s^4t^3 +$$

$$924r^4s^5t^2 - 484r^5s^3t^3 + 924r^5s^4t^2 + 2288r^6s^2t^3 - 2640r^6s^3t^2 - 1749r^7s^2t^2 +$$

$$2068r^2s^7t^3 - 5324r^2s^8t^2 + 88r^3s^6t^3 - 176r^3s^7t^2 + 88r^4s^5t^3 - 176r^4s^6t^2 +$$

$$88r^5s^4t^3 - 176r^5s^5t^2 + 88r^6s^3t^3 - 176r^6s^4t^2 - 605r^7s^2t^3 + 715r^7s^3t^2 + 352r^8s^2t^2) -$$

$$\frac{h^2s(r-s)f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(110r^3s^6 - 286r^2s^7 + 110r^4s^5 + 110r^5s^4 + 110r^6s^3 - 286r^7s^2 +$$

$$308r^2s^8 - 88r^3s^7 - 88r^4s^6 - 88r^5s^5 - 88r^6s^4 - 88r^7s^3 + 308r^8s^2 - 90r^2s^9 + 20r^3s^8 +$$

$$20r^4s^7 + 20r^5s^6 + 20r^6s^5 + 20r^7s^4 + 20r^8s^3 - 90r^9s^2 - 693r^6t^3 + 825r^7t^2 + 891r^6t^4 -$$

$$363r^7t^3 - 616r^8t^2 - 990r^7t^4 + 1067r^8t^3 - 105r^9t^2 + 297r^8t^4 - 399r^9t^3 + 126r^{10}t^2 -$$

$$693s^6t^3 + 825s^7t^2 + 891s^6t^4 - 363s^7t^3 - 616s^8t^2 - 990s^7t^4 + 1067s^8t^3 - 105s^9t^2 +$$

$$297s^8t^4 - 399s^9t^3 + 126s^{10}t^2 + 110rs^8 + 110r^8s - 132rs^9 - 132r^9s + 42rs^{10} +$$

$$42r^{10}s - 220r^8t + 264r^9t - 84r^{10}t - 220s^8t + 264s^9t - 84s^{10}t + 55rs^7t + 55r^7st -$$

$$242rs^8t - 242r^8st + 261rs^9t + 261r^9st - 84rs^{10}t - 84r^{10}st + 2079rs^5t^3 - 1848rs^6t^2 +$$

$$1243r^2s^6t - 935r^3s^5t - 935r^4s^4t + 2079r^5st^3 - 935r^5s^3t - 1848r^6st^2 + 1243r^6s^2t -$$

$$2673rs^5t^4 + 528rs^6t^3 + 1771rs^7t^2 - 726r^2s^7t + 462r^3s^6t + 462r^4s^5t - 2673r^5st^4 +$$

$$462r^5s^4t + 528r^6st^3 + 462r^6s^3t + 1771r^7st^2 - 726r^7s^2t + 2574rs^6t^4 - 1925rs^7t^3 -$$

$512rs^8t^2 - 245r^2s^8t + 30r^3s^7t + 30r^4s^6t + 30r^5s^5t + 2574r^6st^4 + 30r^6s^4t -$
$1925r^7st^3 + 30r^7s^3t - 512r^8st^2 - 245r^8s^2t - 693rs^7t^4 + 668rs^8t^3 + 21rs^9t^2 +$
$180r^2s^9t - 40r^3s^8t - 40r^4s^7t - 40r^5s^6t - 40r^6s^5t - 693r^7st^4 - 40r^7s^4t + 668r^8st^3 -$
$40r^8s^3t + 21r^9st^2 + 180r^9s^2t - 1155r^2s^4t^3 - 330r^2s^5t^2 - 1155r^3s^3t^3 + 1980r^3s^4t^2 -$
$1155r^4s^2t^3 + 1980r^4s^3t^2 - 330r^5s^2t^2 + 1485r^2s^4t^4 + 1254r^2s^5t^3 - 1397r^2s^6t^2 +$
$1485r^3s^3t^4 - 1980r^3s^4t^3 + 121r^3s^5t^2 + 1485r^4s^2t^4 - 1980r^4s^3t^3 + 121r^4s^4t^2 +$
$1254r^5s^2t^3 + 121r^5s^3t^2 - 1397r^6s^2t^2 - 990r^2s^5t^4 - 1034r^2s^6t^3 + 1875r^2s^7t^2 -$
$990r^3s^4t^4 + 1738r^3s^5t^3 - 798r^3s^6t^2 - 990r^4s^3t^4 + 1738r^4s^4t^3 - 798r^4s^5t^2 -$
$990r^5s^2t^4 + 1738r^5s^3t^3 - 798r^5s^4t^2 - 1034r^6s^2t^3 - 798r^6s^3t^2 + 1875r^7s^2t^2 +$
$198r^2s^6t^4 + 305r^2s^7t^3 - 595r^2s^8t^2 + 198r^3s^5t^4 - 388r^3s^6t^3 + 230r^3s^7t^2 + 198r^4s^4t^4 -$
$388r^4s^5t^3 + 230r^4s^6t^2 + 198r^5s^3t^4 - 388r^5s^4t^3 + 230r^5s^5t^2 + 198r^6s^2t^4 - 388r^6s^3t^3 +$
$230r^6s^4t^2 + 305r^7s^2t^3 + 230r^7s^3t^2 - 595r^8s^2t^2) + \frac{h^2s(r-s)f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(198r^2s^6 +$
$198r^3s^5 + 198r^4s^4 + 198r^5s^3 + 198r^6s^2 + 305r^2s^7 - 388r^3s^6 - 388r^4s^5 - 388r^5s^4 -$
$388r^6s^3 + 305r^7s^2 - 595r^2s^8 + 230r^3s^7 + 230r^4s^6 + 230r^5s^5 + 230r^6s^4 + 230r^7s^3 -$
$595r^8s^2 + 180r^2s^9 - 40r^3s^8 - 40r^4s^7 - 40r^5s^6 - 40r^6s^5 - 40r^7s^4 - 40r^8s^3 +$
$180r^9s^2 + 891r^6t^2 - 693r^6t^3 - 363r^7t^2 + 825r^7t^3 - 616r^8t^2 - 220r^8t^3 + 264r^9t^2 +$
$891s^6t^2 - 693s^6t^3 - 363s^7t^2 + 825s^7t^3 - 616s^8t^2 - 220s^8t^3 + 264s^9t^2 - 693rs^7 -$
$693r^7s + 668rs^8 + 668r^8s + 21rs^9 + 21r^9s - 84rs^{10} - 84r^{10}s - 990r^7t + 1067r^8t -$
$105r^9t - 84r^{10}t - 990s^7t + 1067s^8t - 105s^9t - 84s^{10}t + 297r^8 - 399r^9 + 126r^{10} +$
$297s^8 - 399s^9 + 126s^{10} + 2574rs^6t + 2574r^6st - 1925rs^7t - 1925r^7st - 512rs^8t -$
$512r^8st + 261rs^9t + 261r^9st + 42rs^{10}t + 42r^{10}st - 2673rs^5t^2 - 990r^2s^5t - 990r^3s^4t -$
$990r^4s^3t - 2673r^5st^2 - 990r^5s^2t + 2079rs^5t^3 + 528rs^6t^2 - 1034r^2s^6t + 1738r^3s^5t +$
$1738r^4s^4t + 2079r^5st^3 + 1738r^5s^3t + 528r^6st^2 - 1034r^6s^2t - 1848rs^6t^3 + 1771rs^7t^2 +$
$1875r^2s^7t - 798r^3s^6t - 798r^4s^5t - 798r^5s^4t - 1848r^6st^3 - 798r^6s^3t + 1771r^7st^2 +$
$1875r^7s^2t + 55rs^7t^3 - 242rs^8t^2 - 245r^2s^8t + 30r^3s^7t + 30r^4s^6t + 30r^5s^5t + 30r^6s^4t +$
$55r^7st^3 + 30r^7s^3t - 242r^8st^2 - 245r^8s^2t + 110rs^8t^3 - 132rs^9t^2 - 90r^2s^9t + 20r^3s^8t +$
$20r^4s^7t + 20r^5s^6t + 20r^6s^5t + 20r^7s^4t + 110r^8st^3 + 20r^8s^3t - 132r^9st^2 - 90r^9s^2t +$
$1485r^2s^4t^2 + 1485r^3s^3t^2 + 1485r^4s^2t^2 - 1155r^2s^4t^3 + 1254r^2s^5t^2 - 1155r^3s^3t^3 -$
$1980r^3s^4t^2 - 1155r^4s^2t^3 - 1980r^4s^3t^2 + 1254r^5s^2t^2 - 330r^2s^5t^3 - 1397r^2s^6t^2 +$

$1980r^3s^4t^3 + 121r^3s^5t^2 + 1980r^4s^3t^3 + 121r^4s^4t^2 - 330r^5s^2t^3 + 121r^5s^3t^2 - 1397r^6s^2t^2 + 1243r^2s^6t^3 - 726r^2s^7t^2 - 935r^3s^5t^3 + 462r^3s^6t^2 - 935r^4s^4t^3 + 462r^4s^5t^2 - 935r^5s^3t^3 + 462r^5s^4t^2 + 1243r^6s^2t^3 + 462r^6s^3t^2 - 726r^7s^2t^2 - 286r^2s^7t^3 + 308r^2s^8t^2 + 110r^3s^6t^3 - 88r^3s^7t^2 + 110r^4s^5t^3 - 88r^4s^6t^2 + 110r^5s^4t^3 - 88r^5s^5t^2 + 110r^6s^3t^3 - 88r^6s^4t^2 - 286r^7s^2t^3 - 88r^7s^3t^2 + 308r^8s^2t^2) - \frac{g_n}{27720r^2st^2}(55h^3r^8 - 66h^3r^9 + 21h^3r^{10} - 55h^3s^8 + 66h^3s^9 - 21h^3s^{10} + 198h^3rs^7 - 198h^3r^7s - 220h^3rs^8 + 220h^3r^8s + 66h^3rs^9 - 66h^3r^9s - 198h^3r^7t + 220h^3r^8t - 66h^3r^9t + 198h^3s^7t - 220h^3s^8t + 66h^3s^9t - 198h^3r^2s^6 + 198h^3r^6s^2 + 198h^3r^2s^7 - 198h^3r^7s^2 - 55h^3r^2s^8 + 55h^3r^8s^2 + 198h^3r^6t^2 - 198h^3r^7t^2 + 55h^3r^8t^2 - 198h^3s^6t^2 + 198h^3s^7t^2 - 55h^3s^8t^2 + 924h^3rs^5t^2 + 924h^3r^2s^5t - 924h^3r^5st^2 - 924h^3r^5s^2t - 792h^3rs^6t^2 - 792h^3r^2s^6t + 792h^3r^6st^2 + 792h^3r^6s^2t + 198h^3rs^7t^2 + 198h^3r^2s^7t - 198h^3r^7st^2 - 198h^3r^7s^2t - 1386h^3r^2s^4t^2 + 1386h^3r^4s^2t^2 + 924h^3r^2s^5t^2 - 924h^3r^5s^2t^2 - 198h^3r^2s^6t^2 + 198h^3r^6s^2t^2 - 792h^3rs^6t + 792h^3r^6st + 792h^3rs^7t - 792h^3r^7st - 220h^3rs^8t + 220h^3r^8st) + \frac{h^3sg_{n+r}}{27720r^2(r-s)(r-t)^2(r-1)^2}(44r^2s^5 + 44r^3s^4 + 44r^4s^3 + 44r^5s^2 - 44r^2s^6 - 44r^3s^5 - 44r^4s^4 - 44r^5s^3 - 44r^6s^2 + 12r^2s^7 + 12r^3s^6 + 12r^4s^5 + 12r^5s^4 + 12r^6s^3 + 12r^7s^2 + 264r^5t^2 - 330r^6t^2 + 110r^7t^2 - 198s^5t^2 + 198s^6t^2 - 55s^7t^2 + 44rs^6 - 220r^6s - 44rs^7 + 286r^7s + 12rs^8 - 98r^8s - 330r^6t + 440r^7t - 154r^8t + 198s^6t - 220s^7t + 66s^8t + 110r^7 - 154r^8 + 56r^9 - 55s^7 + 66s^8 - 21s^9 - 198rs^5t + 726r^5st + 176rs^6t - 880r^6st - 44rs^7t + 286r^7st + 264rs^4t^2 - 198r^2s^4t - 198r^3s^3t - 660r^4st^2 - 198r^4s^2t - 198rs^5t^2 + 176r^2s^5t + 176r^3s^4t + 176r^4s^3t + 726r^5st^2 + 176r^5s^2t + 44rs^6t^2 - 44r^2s^6t - 44r^3s^5t - 44r^4s^4t - 44r^5s^3t - 220r^6st^2 - 44r^6s^2t + 264r^2s^3t^2 + 264r^3s^2t^2 - 198r^2s^4t^2 - 198r^3s^3t^2 - 198r^4s^2t^2 + 44r^2s^5t^2 + 44r^3s^4t^2 + 44r^4s^3t^2 + 44r^5s^2t^2) + \frac{h^3g_{n+s}}{27720s(r-s)(s-t)^2(s-1)^2}(44r^2s^5 + 44r^3s^4 + 44r^4s^3 + 44r^5s^2 - 44r^2s^6 - 44r^3s^5 - 44r^4s^4 - 44r^5s^3 - 44r^6s^2 + 12r^2s^7 + 12r^3s^6 + 12r^4s^5 + 12r^5s^4 + 12r^6s^3 + 12r^7s^2 - 198r^5t^2 + 198r^6t^2 - 55r^7t^2 + 264s^5t^2 - 330s^6t^2 + 110s^7t^2 - 220rs^6 + 44r^6s + 286rs^7 - 44r^7s - 98rs^8 + 12r^8s + 198r^6t - 220r^7t + 66r^8t - 330s^6t + 440s^7t - 154s^8t - 55r^7 + 66r^8 - 21r^9 + 110s^7 - 154s^8 + 56s^9 + 726rs^5t - 198r^5st - 880rs^6t + 176r^6st + 286rs^7t - 44r^7st - 660rs^4t^2 - 198r^2s^4t - 198r^3s^3t + 264r^4st^2 - 198r^4s^2t + 726rs^5t^2 + 176r^2s^5t + 176r^3s^4t +$

$$
\begin{aligned}
&176r^4s^3t - 198r^5st^2 + 176r^5s^2t - 220rs^6t^2 - 44r^2s^6t - 44r^3s^5t - 44r^4s^4t - \\
&44r^5s^3t + 44r^6st^2 - 44r^6s^2t + 264r^2s^3t^2 + 264r^3s^2t^2 - 198r^2s^4t^2 - 198r^3s^3t^2 - \\
&198r^4s^2t^2 + 44r^2s^5t^2 + 44r^3s^4t^2 + 44r^4s^3t^2 + 44r^5s^2t^2) - \frac{h^3sg_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(r- \\
&s)(55r^2s^5 + 55r^3s^4 + 55r^4s^3 + 55r^5s^2 - 44r^2s^6 - 44r^3s^5 - 44r^4s^4 - 44r^5s^3 - \\
&44r^6s^2 + 10r^2s^7 + 10r^3s^6 + 10r^4s^5 + 10r^5s^4 + 10r^6s^3 + 10r^7s^2 - 143rs^6 - \\
&143r^6s + 154rs^7 + 154r^7s - 45rs^8 - 45r^8s - 99r^6t + 110r^7t - 33r^8t - 99s^6t + \\
&110s^7t - 33s^8t + 55r^7 - 66r^8 + 21r^9 + 55s^7 - 66s^8 + 21s^9 + 297rs^5t + 297r^5st - \\
&286rs^6t - 286r^6st + 77rs^7t + 77r^7st - 165r^2s^4t - 165r^3s^3t - 165r^4s^2t + 110r^2s^5t + \\
&110r^3s^4t + 110r^4s^3t + 110r^5s^2t - 22r^2s^6t - 22r^3s^5t - 22r^4s^4t - 22r^5s^3t - 22r^6s^2t) - \\
&\frac{h^3sg_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(r-s)(10r^2s^7 - 22r^3s^5 - 22r^4s^4 - 22r^5s^3 - 22r^6s^2 - 22r^2s^6 + \\
&10r^3s^6 + 10r^4s^5 + 10r^5s^4 + 10r^6s^3 + 10r^7s^2 - 99r^6t^2 + 55r^7t^2 - 99s^6t^2 + 55s^7t^2 + \\
&77rs^7 + 77r^7s - 45rs^8 - 45r^8s + 110r^7t - 66r^8t + 110s^7t - 66s^8t - 33r^8 + 21r^9 - \\
&33s^8 + 21s^9 - 286rs^6t - 286r^6st + 154rs^7t + 154r^7st + 297rs^5t^2 + 110r^2s^5t + \\
&110r^3s^4t + 110r^4s^3t + 297r^5st^2 + 110r^5s^2t - 143rs^6t^2 - 44r^2s^6t - 44r^3s^5t - 44r^4s^4t - \\
&44r^5s^3t - 143r^6st^2 - 44r^6s^2t - 165r^2s^4t^2 - 165r^3s^3t^2 - 165r^4s^2t^2 + 55r^2s^5t^2 + 55r^3 \\
&s^4t^2 + 55r^4s^3t^2 + 55r^5s^2t^2), \qquad (3.53)
\end{aligned}
$$

$$
\begin{aligned}
&y_{n+t} - \frac{t}{r}y_{n+r} + \frac{t-r}{r}y_n = -\frac{g_n}{27720r^2s^2t}(55h^3r^8 - 66h^3r^9 + 21h^3r^{10} - 55h^3t^8 + 66h^3t^9 - \\
&21h^3t^{10} - 198h^3r^7s + 220h^3r^8s - 66h^3r^9s + 198h^3rt^7 - 198h^3r^7t - 220h^3rt^8 + \\
&220h^3r^8t + 66h^3rt^9 - 66h^3r^9t + 198h^3st^7 - 220h^3st^8 + 66h^3st^9 + 198h^3r^6s^2 - \\
&198h^3r^7s^2 + 55h^3r^8s^2 - 198h^3r^2t^6 + 198h^3r^6t^2 + 198h^3r^2t^7 - 198h^3r^7t^2 - \\
&55h^3r^2t^8 + 55h^3r^8t^2 - 198h^3s^2t^6 + 198h^3s^2t^7 - 55h^3s^2t^8 + 924h^3rs^2t^5 + 924h^3r^2st^5 - \\
&924h^3r^5st^2 - 924h^3r^5s^2t - 792h^3rs^2t^6 - 792h^3r^2st^6 + 792h^3r^6st^2 + 792h^3r^6s^2t + \\
&198h^3rs^2t^7 + 198h^3r^2st^7 - 198h^3r^7st^2 - 198h^3r^7s^2t - 1386h^3r^2s^2t^4 + 1386h^3r^4s^2t^2 + \\
&924h^3r^2s^2t^5 - 924h^3r^5s^2t^2 - 198h^3r^2s^2t^6 + 198h^3r^6s^2t^2 - 792h^3rst^6 + 792h^3r^6st + \\
&792h^3rst^7 - 792h^3r^7st - 220h^3rst^8 + 220h^3r^8st) - \frac{h^2(r-t)fn}{27720r^3s^3t^2}(396r^6s^3 - 396r^7s^2 - \\
&396r^7s^3 + 440r^8s^2 + 110r^8s^3 - 132r^9s^2 - 286r^2t^7 + 110r^3t^6 + 110r^4t^5 + 110r^5t^4 + \\
&110r^6t^3 - 286r^7t^2 + 308r^2t^8 - 88r^3t^7 - 88r^4t^6 - 88r^5t^5 - 88r^6t^4 - 88r^7t^3 + 308r^8t^2 - \\
&90r^2t^9 + 20r^3t^8 + 20r^4t^7 + 20r^5t^6 + 20r^6t^5 + 20r^7t^4 + 20r^8t^3 - 90r^9t^2 - 396s^2t^7
\end{aligned}
$$

$$+396s^3t^6 + 440s^2t^8 - 396s^3t^7 - 132s^2t^9 + 110s^3t^8 + 110r^8s - 132r^9s + 42r^{10}s +$$

$$110rt^8 + 110r^8t - 132rt^9 - 132r^9t + 42rt^{10} + 42r^{10}t + 110st^8 - 132st^9 + 42st^{10} -$$

$$473rst^7 - 473r^7st + 616rst^8 + 616r^8st - 279rst^9 - 279r^9st + 42rst^{10} + 42r^{10}st +$$

$$792rs^2t^6 - 594rs^3t^5 + 715r^2st^6 - 275r^3st^5 - 275r^4st^4 - 275r^5st^3 - 594r^5s^3t +$$

$$715r^6st^2 + 792r^6s^2t - 1100rs^2t^7 + 792rs^3t^6 - 924r^2st^7 + 264r^3st^6 + 264r^4st^5 +$$

$$264r^5st^4 + 264r^6st^3 + 792r^6s^3t - 924r^7st^2 - 1100r^7s^2t + 616rs^2t^8 - 473rs^3t^7 +$$

$$469r^2st^8 - 114r^3st^7 - 114r^4st^6 - 114r^5st^5 - 114r^6st^4 - 114r^7st^3 - 473r^7s^3t +$$

$$469r^8st^2 + 616r^8s^2t - 132rs^2t^9 + 110rs^3t^8 - 90r^2st^9 + 20r^3st^8 + 20r^4st^7 +$$

$$20r^5st^6 + 20r^6st^5 + 20r^7st^4 + 20r^8st^3 + 110r^8s^3t - 90r^9st^2 - 132r^9s^2t + 528r^2s^2t^5 -$$

$$2442r^2s^3t^4 - 1320r^3s^2t^4 + 7260r^3s^3t^3 - 1320r^4s^2t^3 - 2442r^4s^3t^2 + 528r^5s^2t^2 +$$

$$484r^2s^2t^6 + 528r^2s^3t^5 + 220r^3s^2t^5 - 1320r^3s^3t^4 + 220r^4s^2t^4 - 1320r^4s^3t^3 +$$

$$220r^5s^2t^3 + 528r^5s^3t^2 + 484r^6s^2t^2 - 924r^2s^2t^7 + 715r^2s^3t^6 + 264r^3s^2t^6 -$$

$$275r^3s^3t^5 + 264r^4s^2t^5 - 275r^4s^3t^4 + 264r^5s^2t^4 - 275r^5s^3t^3 + 264r^6s^2t^3 + 715r^6s^3t^2 -$$

$$924r^7s^2t^2 + 308r^2s^2t^8 - 286r^2s^3t^7 - 88r^3s^2t^7 + 110r^3s^3t^6 - 88r^4s^2t^6 + 110r^4s^3t^5 -$$

$$88r^5s^2t^5 + 110r^5s^3t^4 - 88r^6s^2t^4 + 110r^6s^3t^3 - 88r^7s^2t^3 - 286r^7s^3t^2 + 308r^8s^2t^2) +$$

$$\frac{h^2 f_{n+t}}{27720t^2(r-t)^2(s-t)^3(t-1)^3}(396r^7s^2 - 396r^6s^3 + 396r^7s^3 - 440r^8s^2 - 110r^8s^3 + 132r^9s^2 +$$

$$2068r^2t^7 + 88r^3t^6 + 88r^4t^5 + 88r^5t^4 + 88r^6t^3 - 605r^7t^2 - 5324r^2t^8 - 176r^3t^7 -$$

$$176r^4t^6 - 176r^5t^5 - 176r^6t^4 + 715r^7t^3 + 352r^8t^2 + 4347r^2t^9 + 112r^3t^8 + 112r^4t^7 +$$

$$112r^5t^6 + 112r^6t^5 + 112r^7t^4 - 878r^8t^3 + 189r^9t^2 - 1179r^2t^{10} - 24r^3t^9 - 24r^4t^8 -$$

$$24r^5t^7 - 24r^6t^6 - 24r^7t^5 - 24r^8t^4 + 273r^9t^3 - 126r^{10}t^2 + 5148s^2t^7 - 1980s^3t^6 -$$

$$13376s^2t^8 + 5148s^3t^7 + 11121s^2t^9 - 4235s^3t^8 - 3069s^2t^{10} + 1155s^3t^9 - 110r^8s +$$

$$132r^9s - 42r^{10}s - 3080rt^8 + 220r^8t + 8052rt^9 - 264r^9t - 6774rt^{10} + 84r^{10}t +$$

$$1890rt^{11} - 4235st^8 + 11121st^9 - 9420st^{10} + 2646st^{11} + 1155t^9 - 3069t^{10} +$$

$$2646t^{11} - 756t^12 + 11792rst^7 - 385r^7st - 30316rst^8 + 638r^8st + 24858rst^9 -$$

$$387r^9st - 6774rst^{10} + 84r^{10}st - 15048rs^2t^6 + 5940rs^3t^5 - 8404r^2st^6 - 484r^3st^5 -$$

$$484r^4st^4 - 484r^5st^3 + 1782r^5s^3t + 2288r^6st^2 - 792r^6s^2t + 38060rs^2t^7 - 15048rs^3t^6 +$$

$$21120r^2st^7 + 924r^3st^6 + 924r^4st^5 + 924r^5st^4 - 2640r^6st^3 - 792r^6s^3t - 1749r^7st^2 +$$

$$44r^7s^2t - 30316rs^2t^8 + 11792rs^3t^7 - 16579r^2st^8 - 552r^3st^7 - 552r^4st^6 - 552r^5st^5 -$$

$$552r^6st^4 + 3012r^7st^3 - 385r^7s^3t + 20r^8st^2 + 638r^8s^2t + 8052rs^2t^9 - 3080rs^3t^8 +$$

$4347r^2st^9 + 112r^3st^8 + 112r^4st^7 + 112r^5st^6 + 112r^6st^5 + 112r^7st^4 - 878r^8st^3 + 220r^8s^3t + 189r^9st^2 - 264r^9s^2t + 11550r^2s^2t^5 - 4686r^2s^3t^4 + 924r^3s^2t^4 - 528r^3s^3t^3 + 924r^4s^2t^3 - 528r^4s^3t^2 - 2310r^5s^2t^2 - 28270r^2s^2t^6 + 11550r^2s^3t^5 - 1672r^3s^2t^5 + 924r^3s^3t^4 - 1672r^4s^2t^4 + 924r^4s^3t^3 + 2486r^5s^2t^3 - 2310r^5s^3t^2 + 3212r^6s^2t^2 + 21120r^2s^2t^7 - 8404r^2s^3t^6 + 924r^3s^2t^6 - 484r^3s^3t^5 + 924r^4s^2t^5 - 484r^4s^3t^4 + 924r^5s^2t^4 - 484r^5s^3t^3 - 2640r^6s^2t^3 + 2288r^6s^3t^2 - 1749r^7s^2t^2 - 5324r^2s^2t^8 + 2068r^2s^3t^7 - 176r^3s^2t^7 + 88r^3s^3t^6 - 176r^4s^2t^6 + 88r^4s^3t^5 - 176r^5s^2t^5 + 88r^5s^3t^4 - 176r^6s^2t^4 + 88r^6s^3t^3 + 715r^7s^2t^3 - 605r^7s^3t^2 + 352r^8s^2t^2) + \frac{g_{n+t}h^3}{27720t(r-t)(s-t)^2(t-1)^2}(198r^6s^2 - 198r^5s^2 - 55r^7s^2 + 44r^2t^5 + 44r^3t^4 + 44r^4t^3 + 44r^5t^2 - 44r^2t^6 - 44r^3t^5 - 44r^4t^4 - 44r^5t^3 - 44r^6t^2 + 12r^2t^7 + 12r^3t^6 + 12r^4t^5 + 12r^5t^4 + 12r^6t^3 + 12r^7t^2 + 264s^2t^5 - 330s^2t^6 + 110s^2t^7 + 198r^6s - 220r^7s + 66r^8s - 220rt^6 + 44r^6t + 286rt^7 - 44r^7t - 98rt^8 + 12r^8t - 330st^6 + 440st^7 - 154st^8 - 55r^7 + 66r^8 - 21r^9 + 110t^7 - 154t^8 + 56t^9 + 726rst^5 - 198r^5st - 880rst^6 + 176r^6st + 286rst^7 - 44r^7st - 660rs^2t^4 - 198r^2st^4 - 198r^3st^3 - 198r^4st^2 + 264r^4s^2t + 726rs^2t^5 + 176r^2st^5 + 176r^3st^4 + 176r^4st^3 + 176r^5st^2 - 198r^5s^2t - 220rs^2t^6 - 44r^2st^6 - 44r^3st^5 - 44r^4st^4 - 44r^5st^3 - 44r^6st^2 + 44r^6s^2t + 264r^2s^2t^3 + 264r^3s^2t^2 - 198r^2s^2t^4 - 198r^3s^2t^3 - 198r^4s^2t^2 + 44r^2s^2t^5 + 44r^3s^2t^4 + 44r^4s^2t^3 + 44r^5s^2t^2) - \frac{g_{n+1}h^3t(r-t)}{27720(r-1)^2(s-1)^2(t-1)^2}(55r^7s^2 - 99r^6s^2 - 22r^2t^6 - 22r^3t^5 - 22r^4t^4 - 22r^5t^3 - 22r^6t^2 + 10r^2t^7 + 10r^3t^6 + 10r^4t^5 + 10r^5t^4 + 10r^6t^3 + 10r^7t^2 - 99s^2t^6 + 55s^2t^7 + 110r^7s - 66r^8s + 77rt^7 + 77r^7t - 45rt^8 - 45r^8t + 110st^7 - 66st^8 - 33r^8 + 21r^9 - 33t^8 + 21t^9 - 286rst^6 - 286r^6st + 154rst^7 + 154r^7st + 297rs^2t^5 + 110r^2st^5 + 110r^3st^4 + 110r^4st^3 + 110r^5st^2 + 297r^5s^2t - 143rs^2t^6 - 44r^2st^6 - 44r^3st^5 - 44r^4st^4 - 44r^5st^3 - 44r^6st^2 - 143r^6s^2t - 165r^2s^2t^4 - 165r^3s^2t^3 - 165r^4s^2t^2 + 55r^2s^2t^5 + 55r^3s^2t^4 + 55r^4s^2t^3 + 55r^5s^2t^2) + \frac{h^2t(r-t)f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(891r^6s^2 - 693r^6s^3 - 363r^7s^2 + 825r^7s^3 - 616r^8s^2 - 220r^8s^3 + 264r^9s^2 + 198r^2t^6 + 198r^3t^5 + 198r^4t^4 + 198r^5t^3 + 198r^6t^2 + 305r^2t^7 - 388r^3t^6 - 388r^4t^5 - 388r^5t^4 - 388r^6t^3 + 305r^7t^2 - 595r^2t^8 + 230r^3t^7 + 230r^4t^6 + 230r^5t^5 + 230r^6t^4 + 230r^7t^3 - 595r^8t^2 + 180r^2t^9 - 40r^3t^8 - 40r^4t^7 - 40r^5t^6 - 40r^6t^5 - 40r^7t^4 - 40r^8t^3 + 180r^9t^2 + 891s^2t^6 - 363s^2t^7 - 693s^3t^6 - 616s^2t^8 + 825s^3t^7 + 264s^2t^9 - 220s^3t^8 - 990r^7s + 1067r^8s - 105r^9s - 84r^{10}s - 693r$

$$t^7 - 693r^7t + 668rt^8 + 668r^8t + 21rt^9 + 21r^9t - 84rt^{10} - 84r^{10}t - 990st^7 + 1067st^8 -$$

$$105st^9 - 84st^{10} + 297r^8 - 399r^9 + 126r^{10} + 297t^8 - 399t^9 + 126t^{10} + 2574rst^6 +$$

$$2574r^6st - 1925rst^7 - 1925r^7st - 512rst^8 - 512r^8st + 261rst^9 + 261r^9st + 42rst^{10} +$$

$$42r^{10}st - 2673rs^2t^5 - 990r^2st^5 - 990r^3st^4 - 990r^4st^3 - 990r^5st^2 - 2673r^5s^2t +$$

$$528rs^2t^6 + 2079rs^3t^5 - 1034r^2st^6 + 1738r^3st^5 + 1738r^4st^4 + 1738r^5st^3 + 2079r^5s^3t -$$

$$1034r^6st^2 + 528r^6s^2t + 1771rs^2t^7 - 1848rs^3t^6 + 1875r^2st^7 - 798r^3st^6 - 798r^4st^5 -$$

$$798r^5st^4 - 798r^6st^3 - 1848r^6s^3t + 1875r^7st^2 + 1771r^7s^2t - 242rs^2t^8 + 55rs^3t^7 -$$

$$245r^2st^8 + 30r^3st^7 + 30r^4st^6 + 30r^5st^5 + 30r^6st^4 + 30r^7st^3 + 55r^7s^3t - 245r^8st^2 -$$

$$242r^8s^2t - 132rs^2t^9 + 110rs^3t^8 - 90r^2st^9 + 20r^3st^8 + 20r^4st^7 + 20r^5st^6 + 20r^6st^5 +$$

$$20r^7st^4 + 20r^8st^3 + 110r^8s^3t - 90r^9st^2 - 132r^9s^2t + 1485r^2s^2t^4 + 1485r^3s^2t^3 +$$

$$1485r^4s^2t^2 + 1254r^2s^2t^5 - 1155r^2s^3t^4 - 1980r^3s^2t^4 - 1155r^3s^3t^3 - 1980r^4s^2t^3 -$$

$$1155r^4s^3t^2 + 1254r^5s^2t^2 - 1397r^2s^2t^6 - 330r^2s^3t^5 + 121r^3s^2t^5 + 1980r^3s^3t^4 +$$

$$121r^4s^2t^4 + 1980r^4s^3t^3 + 121r^5s^2t^3 - 330r^5s^3t^2 - 1397r^6s^2t^2 - 726r^2s^2t^7 +$$

$$1243r^2s^3t^6 + 462r^3s^2t^6 - 935r^3s^3t^5 + 462r^4s^2t^5 - 935r^4s^3t^4 + 462r^5s^2t^4 -$$

$$935r^5s^3t^3 + 462r^6s^2t^3 + 1243r^6s^3t^2 - 726r^7s^2t^2 + 308r^2s^2t^8 - 286r^2s^3t^7 - 88r^3s^2t^7 +$$

$$110r^3s^3t^6 - 88r^4s^2t^6 + 110r^4s^3t^5 - 88r^5s^2t^5 + 110r^5s^3t^4 - 88r^6s^2t^4 + 110r^6s^3t^3 -$$

$$88r^7s^2t^3 - 286r^7s^3t^2 + 308r^8s^2t^2) + \frac{h^3tg_{n+r}}{27720r^2(r-s)^2(r-t)(r-1)^2}(264r^5s^2 - 330r^6s^2 +$$

$$110r^7s^2 + 44r^2t^5 + 44r^3t^4 + 44r^4t^3 + 44r^5t^2 - 44r^2t^6 - 44r^3t^5 - 44r^4t^4 - 44r^5t^3 -$$

$$44r^6t^2 + 12r^2t^7 + 12r^3t^6 + 12r^4t^5 + 12r^5t^4 + 12r^6t^3 + 12r^7t^2 - 198s^2t^5 + 198s^2t^6 -$$

$$55s^2t^7 - 330r^6s + 440r^7s - 154r^8s + 44rt^6 - 220r^6t - 44rt^7 + 286r^7t + 12rt^8 -$$

$$98r^8t + 198st^6 - 220st^7 + 66st^8 + 110r^7 - 154r^8 + 56r^9 - 55t^7 + 66t^8 - 21t^9 -$$

$$198rst^5 + 726r^5st + 176rst^6 - 880r^6st - 44rst^7 + 286r^7st + 264rs^2t^4 - 198r^2st^4 -$$

$$198r^3st^3 - 198r^4st^2 - 660r^4s^2t - 198rs^2t^5 + 176r^2st^5 + 176r^3st^4 + 176r^4st^3 +$$

$$176r^5st^2 + 726r^5s^2t + 44rs^2t^6 - 44r^2st^6 - 44r^3st^5 - 44r^4st^4 - 44r^5st^3 - 44r^6st^2 -$$

$$220r^6s^2t + 264r^2s^2t^3 + 264r^3s^2t^2 - 198r^2s^2t^4 - 198r^3s^2t^3 - 198r^4s^2t^2 + 44r^2s^2t^5 +$$

$$44r^3s^2t^4 + 44r^4s^2t^3 + 44r^5s^2t^2) + \frac{h^2tf_{n+r}}{27720r^3(r-s)^3(r-t)^2(r-1)^3}(5148r^7s^2 - 1980r^6s^3 +$$

$$5148r^7s^3 - 13376r^8s^2 - 4235r^8s^3 + 11121r^9s^2 + 1155r^9s^3 - 3069r^{10}s^2 - 605r^2t^7 +$$

$$88r^3t^6 + 88r^4t^5 + 88r^5t^4 + 88r^6t^3 + 2068r^7t^2 + 352r^2t^8 + 715r^3t^7 - 176r^4t^6 -$$

$$176r^5t^5 - 176r^6t^4 - 176r^7t^3 - 5324r^8t^2 + 189r^2t^9 - 878r^3t^8 + 112r^4t^7 + 112r^5t^6$$

$+112r^6t^5+112r^7t^4+112r^8t^3+4347r^9t^2-126r^2t^{10}+273r^3t^9-24r^4t^8-24r^5t^7-24r^6t^6-24r^7t^5-24r^8t^4-24r^9t^3-1179r^{10}t^2+396s^2t^7-396s^3t^6-440s^2t^8+396s^3t^7+132s^2t^9-110s^3t^8-4235r^8s+11121r^9s-9420r^{10}s+2646r^{11}s+220rt^8-3080r^8t-264rt^9+8052r^9t+84rt^{10}-6774r^{10}t+1890r^{11}t-110st^8+132st^9-42st^{10}+1155r^9-3069r^{10}+2646r^{11}-756r^12-385rst^7+11792r^7st+638rst^8-30316r^8st-387rst^9+24858r^9st+84rst^{10}-6774r^{10}st-792rs^2t^6+1782rs^3t^5+2288r^2st^6-484r^3st^5-484r^4st^4-484r^5st^3+5940r^5s^3t-8404r^6st^2-15048r^6s^2t+44rs^2t^7-792rs^3t^6-1749r^2st^7-2640r^3st^6+924r^4st^5+924r^5st^4+924r^6st^3-15048r^6s^3t+21120r^7st^2+38060r^7s^2t+638rs^2t^8-385rs^3t^7+20r^2st^8+3012r^3st^7-552r^4st^6-552r^5st^5-552r^6st^4-552r^7st^3+11792r^7s^3t-16579r^8st^2-30316r^8s^2t-264rs^2t^9+220rs^3t^8+189r^2st^9-878r^3st^8+112r^4st^7+112r^5st^6+112r^6st^5+112r^7st^4+112r^8st^3-3080r^8s^3t+4347r^9st^2+8052r^9s^2t-2310r^2s^2t^5-528r^2s^3t^4+924r^3s^2t^4-528r^3s^3t^3+924r^4s^2t^3-4686r^4s^3t^2+11550r^5s^2t^2+3212r^2s^2t^6-2310r^2s^3t^5+2486r^3s^2t^5+924r^3s^3t^4-1672r^4s^2t^4+924r^4s^3t^3-1672r^5s^2t^3+11550r^5s^3t^2-28270r^6s^2t^2-1749r^2s^2t^7+2288r^2s^3t^6-2640r^3s^2t^6-484r^3s^3t^5+924r^4s^2t^5-484r^4s^3t^4+924r^5s^2t^4-484r^5s^3t^3+924r^6s^2t^3-8404r^6s^3t^2+21120r^7s^2t^2+352r^2s^2t^8-605r^2s^3t^7+715r^3s^2t^7+88r^3s^3t^6-176r^4s^2t^6+88r^4s^3t^5-176r^5s^2t^5+88r^5s^3t^4-176r^6s^2t^4+88r^6s^3t^3-176r^7s^2t^3+2068r^7s^3t^2-5324r^8s^2t^2)+\frac{h^2t(r-t)f_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(825r^7s^2-693r^6s^3+891r^6s^4-363r^7s^3-616r^8s^2-990r^7s^4+1067r^8s^3-105r^9s^2+297r^8s^4-399r^9s^3+126r^{10}s^2-286r^2t^7+110r^3t^6+110r^4t^5+110r^5t^4+110r^6t^3-286r^7t^2+308r^2t^8-88r^3t^7-88r^4t^6-88r^5t^5-88r^6t^4-88r^7t^3+308r^8t^2-90r^2t^9+20r^3t^8+20r^4t^7+20r^5t^6+20r^6t^5+20r^7t^4+20r^8t^3-90r^9t^2+825s^2t^7-693s^3t^6-616s^2t^8-363s^3t^7+891s^4t^6-105s^2t^9+1067s^3t^8-990s^4t^7+126s^2t^{10}-399s^3t^9+297s^4t^8-220r^8s+264r^9s-84r^{10}s+110rt^8+110r^8t-132rt^9-132r^9t+42rt^{10}+42r^{10}t-220st^8+264st^9-84st^{10}+55rst^7+55r^7st-242rst^8-242r^8st+261rst^9+261r^9st-84rst^{10}-84r^{10}st-1848rs^2t^6+2079rs^3t^5+1243r^2st^6-935r^3st^5-935r^4st^4-935r^5st^3+2079r^5s^3t+1243r^6st^2-1848r^6s^2t+1771rs^2t^7+528rs^3t^6-2673rs^4t^5-726r^2st^7+462r^3st^6+462r^4st^5+462r^5st^4-2673r^5s^4t+462r^6st^3+528r^6s^3t-$

$$726r^7st^2 + 1771r^7s^2t - 512rs^2t^8 - 1925rs^3t^7 + 2574rs^4t^6 - 245r^2st^8 + 30r^3st^7 + 30r^4st^6 + 30r^5st^5 + 30r^6st^4 + 2574r^6s^4t + 30r^7st^3 - 1925r^7s^3t - 245r^8st^2 - 512r^8s^2t + 21rs^2t^9 + 668rs^3t^8 - 693rs^4t^7 + 180r^2st^9 - 40r^3st^8 - 40r^4st^7 - 40r^5st^6 - 40r^6st^5 - 40r^7st^4 - 693r^7s^4t - 40r^8st^3 + 668r^8s^3t + 180r^9st^2 + 21r^9s^2t - 330r^2s^2t^5 - 1155r^2s^3t^4 + 1980r^3s^2t^4 - 1155r^3s^3t^3 + 1980r^4s^2t^3 - 1155r^4s^3t^2 - 330r^5s^2t^2 - 1397r^2s^2t^6 + 1254r^2s^3t^5 + 1485r^2s^4t^4 + 121r^3s^2t^5 - 1980r^3s^3t^4 + 1485r^3s^4t^3 + 121r^4s^2t^4 - 1980r^4s^3t^3 + 1485r^4s^4t^2 + 121r^5s^2t^3 + 1254r^5s^3t^2 - 1397r^6s^2t^2 + 1875r^2s^2t^7 - 1034r^2s^3t^6 - 990r^2s^4t^5 - 798r^3s^2t^6 + 1738r^3s^3t^5 - 990r^3s^4t^4 - 798r^4s^2t^5 + 1738r^4s^3t^4 - 990r^4s^4t^3 - 798r^5s^2t^4 + 1738r^5s^3t^3 - 990r^5s^4t^2 - 798r^6s^2t^3 - 1034r^6s^3t^2 + 1875r^7s^2t^2 - 595r^2s^2t^8 + 305r^2s^3t^7 + 198r^2s^4t^6 + 230r^3s^2t^7 - 388r^3s^3t^6 + 198r^3s^4t^5 + 230r^4s^2t^6 - 388r^4s^3t^5 + 198r^4s^4t^4 + 230r^5s^2t^5 - 388r^5s^3t^4 + 198r^5s^4t^3 + 230r^6s^2t^4 - 388r^6s^3t^3 + 198r^6s^4t^2 + 230r^7s^2t^3 + 305r^7s^3t^2 - 595r^8s^2t^2) - \frac{h^3t(r-t)g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(55r^2t^5 + 55r^3t^4 + 55r^4t^3 + 55r^5t^2 - 44r^2t^6 - 44r^3t^5 - 44r^4t^4 - 44r^5t^3 - 44r^6t^2 + 10r^2t^7 + 10r^3t^6 + 10r^4t^5 + 10r^5t^4 + 10r^6t^3 + 10r^7t^2 - 99r^6s + 110r^7s - 33r^8s - 143rt^6 - 143r^6t + 154rt^7 + 154r^7t - 45rt^8 - 45r^8t - 99st^6 + 110st^7 - 33st^8 + 55r^7 - 66r^8 + 21r^9 + 55t^7 - 66t^8 + 21t^9 + 297rst^5 + 297r^5st - 286rst^6 - 286r^6st + 77rst^7 + 77r^7st - 165r^2st^4 - 165r^3st^3 - 165r^4st^2 + 110r^2st^5 + 110r^3st^4 + 110r^4st^3 + 110r^5st^2 - 22r^2st^6 - 22r^3st^5 - 22r^4st^4 - 22r^5st^3 - 22r^6st^2), \tag{3.54}$$

$$y_{n+1} - \frac{1}{r}y_{n+r} + \frac{1-r}{r}y_n = \frac{h^3g_{n+1}}{27720(r-1)(s-1)^2(t-1)^2}(44r^2s^2 - 154s - 154t - 98r + 44r^3s^2 + 44r^4s^2 + 44r^5s^2 + 44r^6s^2 - 55r^7s^2 + 44r^2t^2 + 44r^3t^2 + 44r^4t^2 + 44r^5t^2 + 44r^6t^2 - 55r^7t^2 + 264s^2t^2 + 286rs + 286rt + 440st - 220rs^2 - 44r^2s - 44r^3s - 44r^4s - 44r^5s - 44r^6s - 44r^7s + 66r^8s - 220rt^2 - 44r^2t - 44r^3t - 44r^4t - 44r^5t - 44r^6t - 44r^7t + 66r^8t - 330st^2 - 330s^2t + 12r^2 + 12r^3 + 12r^4 + 12r^5 + 12r^6 + 12r^7 + 12r^8 - 21r^9 + 110s^2 + 110t^2 + 726rst^2 + 726rs^2t + 176r^2st + 176r^3st + 176r^4st + 176r^5st + 176r^6st - 220r^7st - 660rs^2t^2 - 198r^2st^2 - 198r^2s^2t - 198r^3st^2 - 198r^3s^2t - 198r^4st^2 - 198r^4s^2t - 198r^5st^2 - 198r^5s^2t + 198r^6st^2 + 198r^6s^2t - 880rst + 264r^2s^2t^2 + 264r^3s^2t^2 + 264r^4s^2t^2 - 198r^5s^2t^2 + 56) - \frac{h^2f_{n+1}}{27720(r-1)^2(s-1)^3(t-1)^3}(1890r+$$

$2646s + 2646t - 5324r^2s^2 + 2068r^2s^3 - 176r^3s^2 + 88r^3s^3 - 176r^4s^2 + 88r^4s^3 - 176r^5s^2 + 88r^5s^3 - 176r^6s^2 + 88r^6s^3 + 715r^7s^2 - 605r^7s^3 + 352r^8s^2 + 220r^8s^3 - 264r^9s^2 - 5324r^2t^2 + 2068r^2t^3 - 176r^3t^2 + 88r^3t^3 - 176r^4t^2 + 88r^4t^3 - 176r^5t^2 + 88r^5t^3 - 176r^6t^2 + 88r^6t^3 + 715r^7t^2 - 605r^7t^3 + 352r^8t^2 + 220r^8t^3 - 264r^9t^2 - 13376s^2t^2 + 5148s^2t^3 + 5148s^3t^2 - 1980s^3t^3 - 6774rs - 6774rt - 9420st + 8052rs^2 + 4347r^2s - 3080rs^3 + 112r^3s + 112r^4s + 112r^5s + 112r^6s + 112r^7s - 878r^8s + 189r^9s + 84r^{10}s + 8052rt^2 + 4347r^2t - 3080rt^3 + 112r^3t + 112r^4t + 112r^5t + 112r^6t + 112r^7t - 878r^8t + 189r^9t + 84r^{10}t + 11121st^2 + 11121s^2t - 4235st^3 - 4235s^3t - 1179r^2 - 24r^3 - 24r^4 - 24r^5 - 24r^6 - 24r^7 - 24r^8 + 273r^9 - 126r^{10} - 3069s^2 + 1155s^3 - 3069t^2 + 1155t^3 - 30316rst^2 - 30316rs^2t - 16579r^2st + 11792rst^3 + 11792rs^3t - 552r^3st - 552r^4st - 552r^5st - 552r^6st + 3012r^7st + 20r^8st - 387r^9st - 42r^{10}st + 38060rs^2t^2 + 21120r^2st^2 + 21120r^2s^2t - 15048rs^2t^3 - 15048rs^3t^2 - 8404r^2st^3 - 8404r^2s^3t + 924r^3st^2 + 924r^3s^2t + 5940rs^3t^3 - 484r^3st^3 - 484r^3s^3t + 924r^4st^2 + 924r^4s^2t - 484r^4st^3 - 484r^4s^3t + 924r^5st^2 + 924r^5s^2t - 484r^5st^3 - 484r^5s^3t - 2640r^6st^2 - 2640r^6s^2t + 2288r^6st^3 + 2288r^6s^3t - 1749r^7st^2 - 1749r^7s^2t - 385r^7st^3 - 385r^7s^3t + 638r^8st^2 + 638r^8s^2t - 110r^8st^3 - 110r^8s^3t + 132r^9st^2 + 132r^9s^2t + 24858rst - 28270r^2s^2t^2 + 11550r^2s^2t^3 + 11550r^2s^3t^2 - 1672r^3s^2t^2 - 4686r^2s^3t^3 + 924r^3s^2t^3 + 924r^3s^3t^2 - 1672r^4s^2t^2 - 528r^3s^3t^3 + 924r^4s^2t^3 + 924r^4s^3t^2 + 2486r^5s^2t^2 - 528r^4s^3t^3 - 2310r^5s^2t^3 - 2310r^5s^3t^2 + 3212r^6s^2t^2 + 1782r^5s^3t^3 - 792r^6s^2t^3 - 792r^6s^3t^2 + 44r^7s^2t^2 - 396r^6s^3t^3 + 396r^7s^2t^3 + 396r^7s^3t^2 - 440r^8s^2t^2 - 756) - \frac{h^3(r-1)g_n}{27720r^2s^2t^2}(55r^2s^2 - 66s - 66t - 45r + 55r^3s^2 + 55r^4s^2 + 55r^5s^2 - 143r^6s^2 + 55r^7s^2 + 55r^2t^2 + 55r^3t^2 + 55r^4t^2 + 55r^5t^2 - 143r^6t^2 + 55r^7t^2 + 198s^2t^2 + 154rs + 154rt + 220st - 143rs^2 - 44r^2s - 44r^3s - 44r^4s - 44r^5s - 44r^6s + 154r^7s - 66r^8s - 143rt^2 - 44r^2t - 44r^3t - 44r^4t - 44r^5t - 44r^6t + 154r^7t - 66r^8t - 198st^2 - 198s^2t + 10r^2 + 10r^3 + 10r^4 + 10r^5 + 10r^6 + 10r^7 - 45r^8 + 21r^9 + 55s^2 + 55t^2 + 594rst^2 + 594rs^2t + 220r^2st + 220r^3st + 220r^4st + 220r^5st - 572r^6st + 220r^7st - 726rs^2t^2 - 330r^2st^2 - 330r^2s^2t - 330r^3st^2 - 330r^3s^2t - 330r^4st^2 - 330r^4s^2t + 594r^5st^2 + 594r^5s^2t - 198r^6st^2 - 198r^6s^2t - 572rst + 660r^2s^2t^2 + 660r^3s^2t^2 - 726r^4s^2t^2 + 198r^5s^2t^2 + 21) - \frac{h^2(r-1)f_n}{27720r^3s^3t^3}(308r^2s^2 - 286r^2s^3 - 88r^3s^2 + 110r^3s^3 -$

$$88r^4s^2 + 110r^4s^3 - 88r^5s^2 + 110r^5s^3 - 88r^6s^2 + 110r^6s^3 - 88r^7s^2 - 286r^7s^3 +$$
$$308r^8s^2 + 110r^8s^3 - 132r^9s^2 + 308r^2t^2 - 286r^2t^3 - 88r^3t^2 + 110r^3t^3 - 88r^4t^2 +$$
$$110r^4t^3 - 88r^5t^2 + 110r^5t^3 - 88r^6t^2 + 110r^6t^3 - 88r^7t^2 - 286r^7t^3 + 308r^8t^2 +$$
$$110r^8t^3 - 132r^9t^2 + 440s^2t^2 - 396s^2t^3 - 396s^3t^2 + 396s^3t^3 + 42rs + 42rt + 42st -$$
$$132rs^2 - 90r^2s + 110rs^3 + 20r^3s + 20r^4s + 20r^5s + 20r^6s + 20r^7s + 20r^8s - 90r^9s +$$
$$42r^{10}s - 132rt^2 - 90r^2t + 110rt^3 + 20r^3t + 20r^4t + 20r^5t + 20r^6t + 20r^7t + 20r^8t -$$
$$90r^9t + 42r^{10}t - 132st^2 - 132s^2t + 110st^3 + 110s^3t + 616rst^2 + 616rs^2t + 469r^2st -$$
$$473rst^3 - 473rs^3t - 114r^3st - 114r^4st - 114r^5st - 114r^6st - 114r^7st + 469r^8st -$$
$$279r^9st + 42r^{10}st - 1100rs^2t^2 - 924r^2st^2 - 924r^2s^2t + 792rs^2t^3 + 792rs^3t^2 +$$
$$715r^2st^3 + 715r^2s^3t + 264r^3st^2 + 264r^3s^2t - 594rs^3t^3 - 275r^3st^3 - 275r^3s^3t +$$
$$264r^4st^2 + 264r^4s^2t - 275r^4st^3 - 275r^4s^3t + 264r^5st^2 + 264r^5s^2t - 275r^5st^3 -$$
$$275r^5s^3t + 264r^6st^2 + 264r^6s^2t + 715r^6st^3 + 715r^6s^3t - 924r^7st^2 - 924r^7s^2t -$$
$$473r^7st^3 - 473r^7s^3t + 616r^8st^2 + 616r^8s^2t + 110r^8st^3 + 110r^8s^3t - 132r^9st^2 -$$
$$132r^9s^2t - 279rst + 484r^2s^2t^2 + 528r^2s^2t^3 + 528r^2s^3t^2 + 220r^3s^2t^2 - 2442r^2s^3t^3 -$$
$$1320r^3s^2t^3 - 1320r^3s^3t^2 + 220r^4s^2t^2 + 7260r^3s^3t^3 - 1320r^4s^2t^3 - 1320r^4s^3t^2 +$$
$$220r^5s^2t^2 - 2442r^4s^3t^3 + 528r^5s^2t^3 + 528r^5s^3t^2 + 484r^6s^2t^2 - 594r^5s^3t^3 +$$
$$792r^6s^2t^3 + 792r^6s^3t^2 - 1100r^7s^2t^2 + 396r^6s^3t^3 - 396r^7s^2t^3 - 396r^7s^3t^2 +$$
$$440r^8s^2t^2) + \frac{h^2 f_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^2}(352r^2s^2 - 605r^2s^3 + 715r^3s^2 + 88r^3s^3 -$$
$$176r^4s^2 + 88r^4s^3 - 176r^5s^2 + 88r^5s^3 - 176r^6s^2 + 88r^6s^3 - 176r^7s^2 + 2068r^7s^3 -$$
$$5324r^8s^2 - 3080r^8s^3 + 8052r^9s^2 + 1155r^9s^3 - 3069r^{10}s^2 + 352r^2t^2 - 605r^2t^3 +$$
$$715r^3t^2 + 88r^3t^3 - 176r^4t^2 + 88r^4t^3 - 176r^5t^2 + 88r^5t^3 - 176r^6t^2 + 88r^6t^3 -$$
$$176r^7t^2 + 2068r^7t^3 - 5324r^8t^2 - 3080r^8t^3 + 8052r^9t^2 + 1155r^9t^3 - 3069r^{10}t^2 -$$
$$440s^2t^2 + 396s^2t^3 + 396s^3t^2 - 396s^3t^3 + 84rs + 84rt - 42st - 264rs^2 + 189r^2s +$$
$$220rs^3 - 878r^3s + 112r^4s + 112r^5s + 112r^6s + 112r^7s + 112r^8s + 4347r^9s -$$
$$6774r^{10}s + 2646r^{11}s - 264rt^2 + 189r^2t + 220rt^3 - 878r^3t + 112r^4t + 112r^5t +$$
$$112r^6t + 112r^7t + 112r^8t + 4347r^9t - 6774r^{10}t + 2646r^{11}t + 132st^2 + 132s^2t -$$
$$110st^3 - 110s^3t - 126r^2 + 273r^3 - 24r^4 - 24r^5 - 24r^6 - 24r^7 - 24r^8 - 24r^9 -$$
$$1179r^{10} + 1890r^{11} - 756r^12 + 638rst^2 + 638rs^2t + 20r^2st - 385rst^3 - 385rs^3t +$$
$$3012r^3st - 552r^4st - 552r^5st - 552r^6st - 552r^7st - 16579r^8st + 24858r^9st - 9420$$

$$r^{10}st + 44rs^2t^2 - 1749r^2st^2 - 1749r^2s^2t - 792rs^2t^3 - 792rs^3t^2 + 2288r^2st^3 +$$
$$2288r^2s^3t - 2640r^3st^2 - 2640r^3s^2t + 1782rs^3t^3 - 484r^3st^3 - 484r^3s^3t + 924r^4st^2 +$$
$$924r^4s^2t - 484r^4st^3 - 484r^4s^3t + 924r^5st^2 + 924r^5s^2t - 484r^5st^3 - 484r^5s^3t +$$
$$924r^6st^2 + 924r^6s^2t - 8404r^6st^3 - 8404r^6s^3t + 21120r^7st^2 + 21120r^7s^2t +$$
$$11792r^7st^3 + 11792r^7s^3t - 30316r^8st^2 - 30316r^8s^2t - 4235r^8st^3 - 4235r^8s^3t +$$
$$11121r^9st^2 + 11121r^9s^2t - 387rst + 3212r^2s^2t^2 - 2310r^2s^2t^3 - 2310r^2s^3t^2 +$$
$$2486r^3s^2t^2 - 528r^2s^3t^3 + 924r^3s^2t^3 + 924r^3s^3t^2 - 1672r^4s^2t^2 - 528r^3s^3t^3 +$$
$$924r^4s^2t^3 + 924r^4s^3t^2 - 1672r^5s^2t^2 - 4686r^4s^3t^3 + 11550r^5s^2t^3 + 11550r^5s^3t^2 -$$
$$28270r^6s^2t^2 + 5940r^5s^3t^3 - 15048r^6s^2t^3 - 15048r^6s^3t^2 + 38060r^7s^2t^2 -$$
$$1980r^6s^3t^3 + 5148r^7s^2t^3 + 5148r^7s^3t^2 - 13376r^8s^2t^2) + \frac{h^3 g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)}(12r +$$
$$66s + 66t + 44r^2s^2 + 44r^3s^2 + 44r^4s^2 + 44r^5s^2 - 220r^6s^2 + 110r^7s^2 + 44r^2t^2 +$$
$$44r^3t^2 + 44r^4t^2 + 44r^5t^2 - 220r^6t^2 + 110r^7t^2 - 198s^2t^2 - 44rs - 44rt - 220st +$$
$$44rs^2 - 44r^2s - 44r^3s - 44r^4s - 44r^5s - 44r^6s + 286r^7s - 154r^8s + 44rt^2 - 44r^2t -$$
$$44r^3t - 44r^4t - 44r^5t - 44r^6t + 286r^7t - 154r^8t + 198st^2 + 198s^2t + 12r^2 + 12r^3 +$$
$$12r^4 + 12r^5 + 12r^6 + 12r^7 - 98r^8 + 56r^9 - 55s^2 - 55t^2 - 198rst^2 - 198rs^2t +$$
$$176r^2st + 176r^3st + 176r^4st + 176r^5st - 880r^6st + 440r^7st + 264rs^2t^2 - 198r^2st^2 -$$
$$198r^2s^2t - 198r^3st^2 - 198r^3s^2t - 198r^4st^2 - 198r^4s^2t + 726r^5st^2 + 726r^5s^2t -$$
$$330r^6st^2 - 330r^6s^2t + 176rst + 264r^2s^2t^2 + 264r^3s^2t^2 - 660r^4s^2t^2 + 264r^5s^2t^2 -$$
$$21) + \frac{h^2(r-1) f_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(305r^2s^3 - 595r^2s^2 + 230r^3s^2 + 198r^2s^4 - 388r^3s^3 +$$
$$230r^4s^2 + 198r^3s^4 - 388r^4s^3 + 230r^5s^2 + 198r^4s^4 - 388r^5s^3 + 230r^6s^2 + 198r^5s^4 -$$
$$388r^6s^3 + 230r^7s^2 + 198r^6s^4 + 305r^7s^3 - 595r^8s^2 - 693r^7s^4 + 668r^8s^3 + 21r^9s^2 +$$
$$297r^8s^4 - 399r^9s^3 + 126r^{10}s^2 + 308r^2t^2 - 286r^2t^3 - 88r^3t^2 + 110r^3t^3 - 88r^4t^2 +$$
$$110r^4t^3 - 88r^5t^2 + 110r^5t^3 - 88r^6t^2 + 110r^6t^3 - 88r^7t^2 - 286r^7t^3 + 308r^8t^2 +$$
$$110r^8t^3 - 132r^9t^2 - 616s^2t^2 + 825s^2t^3 - 363s^3t^2 - 693s^3t^3 + 891s^4t^2 - 84rs + 42rt -$$
$$84st + 21rs^2 + 180r^2s + 668rs^3 - 40r^3s - 693rs^4 - 40r^4s - 40r^5s - 40r^6s - 40r^7s -$$
$$40r^8s + 180r^9s - 84r^{10}s - 132rt^2 - 90r^2t + 110rt^3 + 20r^3t + 20r^4t + 20r^5t + 20r^6t +$$
$$20r^7t + 20r^8t - 90r^9t + 42r^{10}t + 264st^2 - 105s^2t - 220st^3 + 1067s^3t - 990s^4t +$$
$$126s^2 - 399s^3 + 297s^4 - 242rst^2 - 512rs^2t - 245r^2st + 55rst^3 - 1925rs^3t + 30r^3st +$$
$$2574rs^4t + 30r^4st + 30r^5st + 30r^6st + 30r^7st - 245r^8st + 261r^9st - 84r^{10}st + 1771$$

$rs^2t^2 - 726r^2st^2 + 1875r^2s^2t - 1848rs^2t^3 + 528rs^3t^2 + 1243r^2st^3 - 1034r^2s^3t + 462r^3st^2 - 798r^3s^2t + 2079rs^3t^3 - 2673rs^4t^2 - 990r^2s^4t - 935r^3st^3 + 1738r^3s^3t + 462r^4st^2 - 798r^4s^2t - 990r^3s^4t - 935r^4st^3 + 1738r^4s^3t + 462r^5st^2 - 798r^5s^2t - 990r^4s^4t - 935r^5st^3 + 1738r^5s^3t + 462r^6st^2 - 798r^6s^2t - 990r^5s^4t + 1243r^6st^3 - 1034r^6s^3t - 726r^7st^2 + 1875r^7s^2t + 2574r^6s^4t + 55r^7st^3 - 1925r^7s^3t - 242r^8st^2 - 512r^8s^2t - 990r^7s^4t - 220r^8st^3 + 1067r^8s^3t + 264r^9st^2 - 105r^9s^2t + 261rst - 1397r^2s^2t^2 - 330r^2s^2t^3 + 1254r^2s^3t^2 + 121r^3s^2t^2 - 1155r^2s^3t^3 + 1485r^2s^4t^2 + 1980r^3s^2t^3 - 1980r^3s^3t^2 + 121r^4s^2t^2 - 1155r^3s^3t^3 + 1485r^3s^4t^2 + 1980r^4s^2t^3 - 1980r^4s^3t^2 + 121r^5s^2t^2 - 1155r^4s^3t^3 + 1485r^4s^4t^2 - 330r^5s^2t^3 + 1254r^5s^3t^2 - 1397r^6s^2t^2 + 2079r^5s^3t^3 - 2673r^5s^4t^2 - 1848r^6s^2t^3 + 528r^6s^3t^2 + 1771r^7s^2t^2 - 693r^6s^3t^3 + 891r^6s^4t^2 + 825r^7s^2t^3 - 363r^7s^3t^2 - 616r^8s^2t^2) - \frac{h^2(r-1)f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(308r^2s^2 - 286r^2s^3 - 88r^3s^2 + 110r^3s^3 - 88r^4s^2 + 110r^4s^3 - 88r^5s^2 + 110r^5s^3 - 88r^6s^2 + 110r^6s^3 - 88r^7s^2 - 286r^7s^3 + 308r^8s^2 + 110r^8s^3 - 132r^9s^2 - 595r^2t^2 + 305r^2t^3 + 230r^3t^2 + 198r^2t^4 - 388r^3t^3 + 230r^4t^2 + 198r^3t^4 - 388r^4t^3 + 230r^5t^2 + 198r^4t^4 - 388r^5t^3 + 230r^6t^2 + 198r^5t^4 - 388r^6t^3 + 230r^7t^2 + 198r^6t^4 + 305r^7t^3 - 595r^8t^2 - 693r^7t^4 + 668r^8t^3 + 21r^9t^2 + 297r^8t^4 - 399r^9t^3 + 126r^{10}t^2 - 616s^2t^2 - 363s^2t^3 + 825s^3t^2 + 891s^2t^4 - 693s^3t^3 + 42rs - 84rt - 84st - 132rs^2 - 90r^2s + 110rs^3 + 20r^3s + 20r^4s + 20r^5s + 20r^6s + 20r^7s + 20r^8s - 90r^9s + 42r^{10}s + 21rt^2 + 180r^2t + 668rt^3 - 40r^3t - 693rt^4 - 40r^4t - 40r^5t - 40r^6t - 40r^7t - 40r^8t + 180r^9t - 84r^{10}t - 105st^2 + 264s^2t + 1067st^3 - 220s^3t - 990st^4 + 126t^2 - 399t^3 + 297t^4 - 512rst^2 - 242rs^2t - 245r^2st - 1925rst^3 + 55rs^3t + 30r^3st + 2574rst^4 + 30r^4st + 30r^5st + 30r^6st + 30r^7st - 245r^8st + 261r^9st - 84r^{10}st + 1771rs^2t^2 + 1875r^2st^2 - 726r^2s^2t + 528rs^2t^3 - 1848rs^3t^2 - 1034r^2st^3 + 1243r^2s^3t - 798r^3st^2 + 462r^3s^2t - 2673rs^2t^4 + 2079rs^3t^3 - 990r^2st^4 + 1738r^3st^3 - 935r^3s^3t - 798r^4st^2 + 462r^4s^2t - 990r^3st^4 + 1738r^4st^3 - 935r^4s^3t - 798r^5st^2 + 462r^5s^2t - 990r^4st^4 + 1738r^5st^3 - 935r^5s^3t - 798r^6st^2 + 462r^6s^2t - 990r^5st^4 - 1034r^6st^3 + 1243r^6s^3t + 1875r^7st^2 - 726r^7s^2t + 2574r^6st^4 - 1925r^7st^3 + 55r^7s^3t - 512r^8st^2 - 242r^8s^2t - 990r^7st^4 + 1067r^8st^3 - 220r^8s^3t - 105r^9st^2 + 264r^9s^2t + 261rst - 1397r^2s^2t^2 + 1254r^2s^2t^3 - 330r^2s^3t^2 + 121r^3s^2t^2 + 1485r^2s^2t^4 - 1155r^2s^3t^3 -$

$1980r^3s^2t^3 + 1980r^3s^3t^2 + 121r^4s^2t^2 + 1485r^3s^2t^4 - 1155r^3s^3t^3 - 1980r^4s^2t^3 + 1980r^4s^3t^2 + 121r^5s^2t^2 + 1485r^4s^2t^4 - 1155r^4s^3t^3 + 1254r^5s^2t^3 - 330r^5s^3t^2 - 1397r^6s^2t^2 - 2673r^5s^2t^4 + 2079r^5s^3t^3 + 528r^6s^2t^3 - 1848r^6s^3t^2 + 1771r^7s^2t^2 + 891r^6s^2t^4 - 693r^6s^3t^3 - 363r^7s^2t^3 + 825r^7s^3t^2 - 616r^8s^2t^2) - \frac{h^3(r-1)g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(55r^2s^2 - 66s - 33t - 45r + 55r^3s^2 + 55r^4s^2 + 55r^5s^2 - 143r^6s^2 + 55r^7s^2 + 154rs + 77rt + 110st - 143rs^2 - 44r^2s - 44r^3s - 44r^4s - 44r^5s - 44r^6s + 154r^7s - 66r^8s - 22r^2t - 22r^3t - 22r^4t - 22r^5t - 22r^6t + 77r^7t - 33r^8t - 99s^2t + 10r^2 + 10r^3 + 10r^4 + 10r^5 + 10r^6 + 10r^7 - 45r^8 + 21r^9 + 55s^2 + 297rs^2t + 110r^2st + 110r^3st + 110r^4st + 110r^5st - 286r^6st + 110r^7st - 165r^2s^2t - 165r^3s^2t - 165r^4s^2t + 297r^5s^2t - 99r^6s^2t - 286rst + 21) - \frac{h^3(r-1)g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(55r^2t^2 - 33s - 66t - 45r + 55r^3t^2 + 55r^4t^2 + 55r^5t^2 - 143r^6t^2 + 55r^7t^2 + 77rs + 154rt + 110st - 22r^2s - 22r^3s - 22r^4s - 22r^5s - 22r^6s + 77r^7s - 33r^8s - 143rt^2 - 44r^2t - 44r^3t - 44r^4t - 44r^5t - 44r^6t + 154r^7t - 66r^8t - 99st^2 + 10r^2 + 10r^3 + 10r^4 + 10r^5 + 10r^6 + 10r^7 - 45r^8 + 21r^9 + 55t^2 + 297rst^2 + 110r^2st + 110r^3st + 110r^4st + 110r^5st - 286r^6st + 110r^7st - 165r^2st^2 - 165r^3st^2 - 165r^4st^2 + 297r^5st^2 - 99r^6st^2 - 286rst + 21). \quad (3.55)$

Now, differentiating (3.52) once gives

$$y^{'}(x) = \sum_{i=0,r} \frac{d}{dx}\alpha_i(x)y_{n+i} + \sum_{i=0,r,s,t,1} \frac{d}{dx}\beta_i(x)f_{n+i} + \sum_{i=0,r,s,t,1} \frac{d}{dx}\gamma_i(x)g_{n+i}. \quad (3.56)$$

Then, evaluating (3.56) at all points, i.e $x_n, x_{n+r}, x_{n+s}, x_{n+t}$ and x_{n+1}, we get

$y'_n = \frac{1}{hr}y_{n+r} - \frac{1}{hr}y_n + \frac{hf_n}{27720s^3t^3}(132r^7s^2t - 42r^8s - 42r^8t - 42r^8st + 132r^7s^2 + 132r^7st^2 + 321r^7st + 132r^7s + 132r^7t^2 + 132r^7t - 110r^6s^3t - 110r^6s^3 - 440r^6s^2t^2 - 748r^6s^2t - 440r^6s^2 - 110r^6st^3 - 748r^6st^2 - 748r^6st - 110r^6s - 110r^6t^3 - 440r^6t^2 - 110r^6t + 396r^5s^3t^2 + 583r^5s^3t + 396r^5s^3 + 396r^5s^2t^3 + 1540r^5s^2t^2 + 1540r^5s^2t + 396r^5s^2 + 583r^5st^3 + 1540r^5st^2 + 583r^5st + 396r^5t^3 + 396r^5t^2 - 396r^4s^3t^3 - 1188r^4s^3t^2 - 1188r^4s^3t - 396r^4s^3 - 1188r^4s^2t^3 - 1584r^4s^2t^2 - 1188r^4s^2t - 1188r^4st^3 - 1188r^4st^2 - 396r^4t^3 + 990r^3s^3t^3 + 264r^3s^3t^2 + 990r^3s^3t + 264r^3s^2t^3 + 264r^3s^2t^2 + 990r^3st^3 + 1848r^2s^3t^3 + 1848r^2s^3t^2 + 1848r^2s^2t^3 - 9702rs^3t^3) - \frac{h^2g_n}{27720s^2t^2}$

$$(21r^8-66r^7s-66r^7t-66r^7+55r^6s^2+220r^6st+220r^6s+55r^6t^2+220r^6t+55r^6-$$
$$198r^5s^2t-198r^5s^2-198r^5st^2-792r^5st-198r^5s-198r^5t^2-198r^5t+198r^4s^2t^2+$$
$$792r^4s^2t+198r^4s^2+792r^4st^2+792r^4st+198r^4t^2-924r^3s^2t^2-924r^3s^2t-$$
$$924r^3st^2+1386r^2s^2t^2)+\frac{h^2g_{n+r}}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^8-77r^7s-77r^7t-77r^7+$$
$$55r^6s^2+220r^6st+220r^6s+55r^6t^2+220r^6t+55r^6-165r^5s^2t-165r^5s^2-$$
$$165r^5st^2-660r^5st-165r^5s-165r^5t^2-165r^5t+132r^4s^2t^2+528r^4s^2t+$$
$$132r^4s^2+528r^4st^2+528r^4st+132r^4t^2-462r^3s^2t^2-462r^3s^2t-462r^3st^2+$$
$$462r^2s^2t^2)+\frac{hf_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^{11}st-84r^{11}s-84r^{11}t+126r^{11}-132r^{10}s^2t+$$
$$264r^{10}s^2-132r^{10}st^2+345r^{10}st-105r^{10}s+264r^{10}t^2-105r^{10}t-399r^{10}+$$
$$110r^9s^3t-220r^9s^3+440r^9s^2t^2-506r^9s^2t-616r^9s^2+110r^9st^3-506r^9st^2-$$
$$407r^9st+1067r^9s-220r^9t^3-616r^9t^2+1067r^9t+297r^9-396r^8s^3t^2+275r^8s^3t+$$
$$825r^8s^3-396r^8s^2t^3-484r^8s^2t^2+2387r^8s^2t-363r^8s^2+275r^8st^3+2387r^8st^2-$$
$$2992r^8st-990r^8s+825r^8t^3-363r^8t^2-990r^8t+396r^7s^3t^3+1188r^7s^3t^2-$$
$$2673r^7s^3t-693r^7s^3+1188r^7s^2t^3-3168r^7s^2t^2+891r^7s^2t+891r^7s^2-2673r^7st^3+$$
$$891r^7st^2+3564r^7st-693r^7t^3+891r^7t^2-2178r^6s^3t^3+1518r^6s^3t^2+2772r^6s^3t+$$
$$1518r^6s^2t^3+726r^6s^2t^2-3564r^6s^2t+2772r^6st^3-3564r^6st^2+2310r^5s^3t^3-$$
$$3234r^5s^3t^2-3234r^5s^2t^3+4158r^5s^2t^2)-\frac{hf_{n+r}}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^{10}-2646r^9s-$$
$$2646r^9t-2646r^9+3069r^8s^2+9420r^8st+9420r^8s+3069r^8t^2+9420r^8t+$$
$$3069r^8-1155r^7s^3-11121r^7s^2t-11121r^7s^2-11121r^7st^2-34278r^7st-$$
$$11121r^7s-1155r^7t^3-11121r^7t^2-11121r^7t-1155r^7+4235r^6s^3t+4235r^6s^3+$$
$$13376r^6s^2t^2+41437r^6s^2t+13376r^6s^2+4235r^6st^3+41437r^6st^2+41437r^6st+$$
$$4235r^6s+4235r^6t^3+13376r^6t^2+4235r^6t-5148r^5s^3t^2-16027r^5s^3t-$$
$$5148r^5s^3-5148r^5s^2t^3-51436r^5s^2t^2-51436r^5s^2t-5148r^5s^2-16027r^5st^3-$$
$$51436r^5st^2-16027r^5st-5148r^5t^3-5148r^5t^2+1980r^4s^3t^3+20196r^4s^3t^2+$$
$$20196r^4s^3t+1980r^4s^3+20196r^4s^2t^3+66330r^4s^2t^2+20196r^4s^2t+20196r^4st^3+$$
$$20196r^4st^2+1980r^4t^3-7920r^3s^3t^3-26598r^3s^3t^2-7920r^3s^3t-26598r^3s^2t^3-$$
$$26598r^3s^2t^2-7920r^3st^3+10626r^2s^3t^3+10626r^2s^3t^2+10626r^2s^2t^3-4158rs^3t^3)-$$
$$\frac{hf_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(84r^{11}st-126r^{11}s^2+84r^{11}s-42r^{11}t+399r^{10}s^3+105r^{10}$$

$$
\begin{aligned}
&s^2t + 105r^{10}s^2 - 264r^{10}st^2 - 345r^{10}st - 264r^{10}s + 132r^{10}t^2 + 132r^{10}t - 297r^9s^4 - \\
&1067r^9s^3t - 1067r^9s^3 + 616r^9s^2t^2 + 407r^9s^2t + 616r^9s^2 + 220r^9st^3 + 506r^9st^2 + \\
&506r^9st + 220r^9s - 110r^9t^3 - 440r^9t^2 - 110r^9t + 990r^8s^4t + 990r^8s^4 + 363r^8s^3t^2 + \\
&2992r^8s^3t + 363r^8s^3 - 825r^8s^2t^3 - 2387r^8s^2t^2 - 2387r^8s^2t - 825r^8s^2 - 275r^8st^3 + \\
&484r^8st^2 - 275r^8st + 396r^8t^3 + 396r^8t^2 - 891r^7s^4t^2 - 3564r^7s^4t - 891r^7s^4 + \\
&693r^7s^3t^3 - 891r^7s^3t^2 - 891r^7s^3t + 693r^7s^3 + 2673r^7s^2t^3 + 3168r^7s^2t^2 + 2673r^7s^2t - \\
&1188r^7st^3 - 1188r^7st^2 - 396r^7t^3 + 3564r^6s^4t^2 + 3564r^6s^4t - 2772r^6s^3t^3 - \\
&726r^6s^3t^2 - 2772r^6s^3t - 1518r^6s^2t^3 - 1518r^6s^2t^2 + 2178r^6st^3 - 4158r^5s^4t^2 + \\
&3234r^5s^3t^3 + 3234r^5s^3t^2 - 2310r^5s^2t^3) - \tfrac{hf_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(42r^{11}s - 84r^{11}st + \\
&126r^{11}t^2 - 84r^{11}t + 264r^{10}s^2t - 132r^{10}s^2 - 105r^{10}st^2 + 345r^{10}st - 132r^{10}s - \\
&399r^{10}t^3 - 105r^{10}t^2 + 264r^{10}t - 220r^9s^3t + 110r^9s^3 - 616r^9s^2t^2 - 506r^9s^2t + \\
&440r^9s^2 + 1067r^9st^3 - 407r^9st^2 - 506r^9st + 110r^9s + 297r^9t^4 + 1067r^9t^3 - 616r^9t^2 - \\
&220r^9t + 825r^8s^3t^2 + 275r^8s^3t - 396r^8s^3 - 363r^8s^2t^3 + 2387r^8s^2t^2 - 484r^8s^2t - \\
&396r^8s^2 - 990r^8st^4 - 2992r^8st^3 + 2387r^8st^2 + 275r^8st - 990r^8t^4 - 363r^8t^3 + \\
&825r^8t^2 - 693r^7s^3t^3 - 2673r^7s^3t^2 + 1188r^7s^3t + 396r^7s^3 + 891r^7s^2t^4 + 891r^7s^2t^3 - \\
&3168r^7s^2t^2 + 1188r^7s^2t + 3564r^7st^4 + 891r^7st^3 - 2673r^7st^2 + 891r^7t^4 - 693r^7t^3 + \\
&2772r^6s^3t^3 + 1518r^6s^3t^2 - 2178r^6s^3t - 3564r^6s^2t^4 + 726r^6s^2t^3 + 1518r^6s^2t^2 - \\
&3564r^6st^4 + 2772r^6st^3 - 3234r^5s^3t^3 + 2310r^5s^3t^2 + 4158r^5s^2t^4 - 3234r^5s^2t^3) + \\
&\tfrac{h^2r^5g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(66r^4s - 21r^5 + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st - 110r^3s - \\
&55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 - 198rs^2t^2 - \\
&396rs^2t - 396rst^2 + 462s^2t^2) + \tfrac{h^2r^5g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - \\
&110r^3s + 33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - \\
&396rst^2 + 396r^2st - 110r^3st + 99r^2st^2 - 396rst) + \tfrac{h^2r^5g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - \\
&55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - \\
&55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst), \qquad (3.57)
\end{aligned}
$$

$$
\begin{aligned}
&y'_{n+r} = \tfrac{1}{hr}y_{n+r} - \tfrac{1}{hr}y_n + \tfrac{h^2r^2g_n}{13860s^2t^2}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + \\
&220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - \\
&165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st +
\end{aligned}
$$

$$132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2) + \frac{hrf_n}{27720s^3t^3}(112r^7st + 112r^7s +$$
$$112r^7t - 308r^6s^2t - 308r^6s^2 - 308r^6st^2 - 735r^6st - 308r^6s - 308r^6t^2 - 308r^6t +$$
$$220r^5s^3t + 220r^5s^3 + 880r^5s^2t^2 + 1452r^5s^2t + 880r^5s^2 + 220r^5st^3 + 1452r^5st^2 +$$
$$1452r^5st + 220r^5s + 220r^5t^3 + 880r^5t^2 + 220r^5t - 660r^4s^3t^2 - 935r^4s^3t - 660r^4s^3 -$$
$$660r^4s^2t^3 - 2420r^4s^2t^2 - 2420r^4s^2t - 660r^4s^2 - 935r^4st^3 - 2420r^4st^2 - 935r^4st -$$
$$660r^4t^3 - 660r^4t^2 + 528r^3s^3t^3 + 1452r^3s^3t^2 + 1452r^3s^3t + 528r^3s^3 + 1452r^3s^2t^3 +$$
$$1584r^3s^2t^2 + 1452r^3s^2t + 1452r^3st^3 + 1452r^3st^2 + 528r^3t^3 - 858r^2s^3t^3 + 264r^2s^3t^2 -$$
$$858r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 - 858r^2st^3 - 1848rs^3t^3 - 1848rs^3t^2 - 1848rs^2t^3 +$$
$$4158s^3t^3) - \frac{h^2r^5g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(154r^4s - 56r^5 + 154r^4t + 77r^4 - 110r^3s^2 -$$
$$440r^3st - 220r^3s - 110r^3t^2 - 220r^3t + 330r^2s^2t + 165r^2s^2 + 330r^2st^2 + 660r^2st +$$
$$165r^2t^2 - 264rs^2t^2 - 528rs^2t - 528rst^2 + 462s^2t^2) - \frac{h^2r^2g_{n+r}}{27720(r-s)^2(r-t)^2(r-1)^2}(252r^6 -$$
$$616r^5s - 616r^5t - 616r^5 + 385r^4s^2 + 1540r^4st + 1540r^4s + 385r^4t^2 + 1540r^4t +$$
$$385r^4 - 990r^3s^2t - 990r^3s^2 - 990r^3st^2 - 3960r^3st - 990r^3s - 990r^3t^2 - 990r^3t +$$
$$660r^2s^2t^2 + 2640r^2s^2t + 660r^2s^2 + 2640r^2st^2 + 2640r^2st + 660r^2t^2 - 1848rs^2t^2 -$$
$$1848rs^2t - 1848rst^2 + 1386s^2t^2) + \frac{hrf_{n+r}}{27720(r-s)^3(r-t)^3(r-1)^3}(4788r^9 - 15372r^8s -$$
$$15372r^8t - 15372r^8 + 16401r^7s^2 + 49672r^7st + 49672r^7s + 16401r^7t^2 + 49672r^7t +$$
$$16401r^7 - 5775r^6s^3 - 53339r^6s^2t - 53339r^6s^2 - 53339r^6st^2 - 161808r^6st -$$
$$53339r^6s - 5775r^6t^3 - 53339r^6t^2 - 53339r^6t - 5775r^6 + 18865r^5s^3t + 18865r^5s^3 +$$
$$57640r^5s^2t^2 + 175197r^5s^2t + 57640r^5s^2 + 18865r^5st^3 + 175197r^5st^2 + 175197r^5st +$$
$$18865r^5s + 18865r^5t^3 + 57640r^5t^2 + 18865r^5t - 20460r^4s^3t^2 - 62315r^4s^3t -$$
$$20460r^4s^3 - 20460r^4s^2t^3 - 191312r^4s^2t^2 - 191312r^4s^2t - 20460r^4s^2 - 62315r^4st^3 -$$
$$191312r^4st^2 - 62315r^4st - 20460r^4t^3 - 20460r^4t^2 + 7260r^3s^3t^3 + 68376r^3s^3t^2 +$$
$$68376r^3s^3t + 7260r^3s^3 + 68376r^3s^2t^3 + 211266r^3s^2t^2 + 68376r^3s^2t + 68376r^3st^3 +$$
$$68376r^3st^2 + 7260r^3t^3 - 24420r^2s^3t^3 - 75966r^2s^3t^2 - 24420r^2s^3t - 75966r^2s^2t^3 -$$
$$75966r^2s^2t^2 - 24420r^2st^3 + 27258rs^3t^3 + 27258rs^3t^2 + 27258rs^2t^3 - 9702s^3t^3) -$$
$$\frac{hr^5f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(112r^6st - 224r^6s - 224r^6t + 336r^6 - 308r^5s^2t + 616r^5s^2 -$$
$$308r^5st^2 + 777r^5st - 203r^5s + 616r^5t^2 - 203r^5t - 987r^5 + 220r^4s^3t - 440r^4s^3 +$$
$$880r^4s^2t^2 - 924r^4s^2t - 1364r^4s^2 + 220r^4st^3 - 924r^4st^2 - 1023r^4st + 2233r^4s -$$
$$440r^4t^3 - 1364r^4t^2 + 2233r^4t + 693r^4 - 660r^3s^3t^2 + 385r^3s^3t + 1485r^3s^3 - 660$$

$$
\begin{aligned}
& r^3s^2t^3 - 1100r^3s^2t^2 + 4345r^3s^2t - 495r^3s^2 + 385r^3st^3 + 4345r^3st^2 - 5060r^3st - \\
& 1980r^3s + 1485r^3t^3 - 495r^3t^2 - 1980r^3t + 528r^2s^3t^3 + 1848r^2s^3t^2 - 3795r^2s^3t - \\
& 1155r^2s^3 + 1848r^2s^2t^3 - 4356r^2s^2t^2 + 561r^2s^2t + 1485r^2s^2 - 3795r^2st^3 + 561r^2st^2 + \\
& 5940r^2st - 1155r^2t^3 + 1485r^2t^2 - 2442rs^3t^3 + 1254rs^3t^2 + 3696rs^3t + 1254rs^2t^3 + \\
& 2046rs^2t^2 - 4752rs^2t + 3696rst^3 - 4752rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + \\
& 4158s^2t^2) - \frac{h^2r^5g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(330r^2t^2 - 110r^3t^2 + 165r^2s - 220r^3s + 77r^4s - \\
& 264rt^2 + 330r^2t - 440r^3t + 154r^4t + 462st^2 - 110r^3 + 154r^4 - 56r^5 - 528rst^2 + \\
& 660r^2st - 220r^3st + 165r^2st^2 - 528rst) - \frac{h^2r^5g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(330r^2s^2 - \\
& 110r^3s^2 - 264rs^2 + 330r^2s - 440r^3s + 154r^4s + 165r^2t - 220r^3t + 77r^4t + 462s^2t - \\
& 110r^3 + 154r^4 - 56r^5 - 528rs^2t + 660r^2st - 220r^3st + 165r^2s^2t - 528rst) + \\
& \frac{hr^5f_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(224r^6st - 336r^6s^2 + 224r^6s - 112r^6t + 987r^5s^3 + 203r^5s^2t + \\
& 203r^5s^2 - 616r^5st^2 - 777r^5st - 616r^5s + 308r^5t^2 + 308r^5t - 693r^4s^4 - 2233r^4s^3t - \\
& 2233r^4s^3 + 1364r^4s^2t^2 + 1023r^4s^2t + 1364r^4s^2 + 440r^4st^3 + 924r^4st^2 + 924r^4st + \\
& 440r^4s - 220r^4t^3 - 880r^4t^2 - 220r^4t + 1980r^3s^4t + 1980r^3s^4 + 495r^3s^3t^2 + \\
& 5060r^3s^3t + 495r^3s^3 - 1485r^3s^2t^3 - 4345r^3s^2t^2 - 4345r^3s^2t - 1485r^3s^2 - 385r^3st^3 + \\
& 1100r^3st^2 - 385r^3st + 660r^3t^3 + 660r^3t^2 - 1485r^2s^4t^2 - 5940r^2s^4t - 1485r^2s^4 + \\
& 1155r^2s^3t^3 - 561r^2s^3t^2 - 561r^2s^3t + 1155r^2s^3 + 3795r^2s^2t^3 + 4356r^2s^2t^2 + \\
& 3795r^2s^2t - 1848r^2st^3 - 1848r^2st^2 - 528r^2t^3 + 4752rs^4t^2 + 4752rs^4t - 3696rs^3t^3 - \\
& 2046rs^3t^2 - 3696rs^3t - 1254rs^2t^3 - 1254rs^2t^2 + 2442rst^3 - 4158s^4t^2 + 3234s^3t^3 + \\
& 3234s^3t^2 - 2310s^2t^3) + \frac{hr^5f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(112r^6s - 224r^6st + 336r^6t^2 - 224r^6t + \\
& 616r^5s^2t - 308r^5s^2 - 203r^5st^2 + 777r^5st - 308r^5s - 987r^5t^3 - 203r^5t^2 + 616r^5t - \\
& 440r^4s^3t + 220r^4s^3 - 1364r^4s^2t^2 - 924r^4s^2t + 880r^4s^2 + 2233r^4st^3 - 1023r^4st^2 - \\
& 924r^4st + 220r^4s + 693r^4t^4 + 2233r^4t^3 - 1364r^4t^2 - 440r^4t + 1485r^3s^3t^2 + \\
& 385r^3s^3t - 660r^3s^3 - 495r^3s^2t^3 + 4345r^3s^2t^2 - 1100r^3s^2t - 660r^3s^2 - 1980r^3st^4 - \\
& 5060r^3st^3 + 4345r^3st^2 + 385r^3st - 1980r^3t^4 - 495r^3t^3 + 1485r^3t^2 - 1155r^2s^3t^3 - \\
& 3795r^2s^3t^2 + 1848r^2s^3t + 528r^2s^3 + 1485r^2s^2t^4 + 561r^2s^2t^3 - 4356r^2s^2t^2 + \\
& 1848r^2s^2t + 5940r^2st^4 + 561r^2st^3 - 3795r^2st^2 + 1485r^2t^4 - 1155r^2t^3 + 3696rs^3t^3 + \\
& 1254rs^3t^2 - 2442rs^3t - 4752rs^2t^4 + 2046rs^2t^3 + 1254rs^2t^2 - 4752rst^4 + 3696rst^3 - \\
& 3234s^3t^3 + 2310s^3t^2 + 4158s^2t^4 - 3234s^2t^3),
\end{aligned}
\tag{3.58}
$$

$$
\begin{aligned}
&y'_{n+s} = \frac{1}{hr}y_{n+r} - \frac{1}{hr}y_n - \frac{h^2 g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(264r^2s^7 - 99r^7s^2 - 165r^2s^8 + 55r^8s^2 - \\
&99r^7t^2 + 55r^8t^2 + 264s^7t^2 - 165s^8t^2 - 330rs^8 + 110r^8s + 220rs^9 - 66r^9s + 110r^8t - \\
&66r^9t - 330s^8t + 220s^9t - 33r^9 + 21r^{10} + 110s^9 - 77s^{10} + 1056rs^7t - 396r^7st - \\
&660rs^8t + 220r^8st - 924rs^6t^2 - 924r^2s^6t + 396r^6st^2 + 396r^6s^2t + 528rs^7t^2 + \\
&528r^2s^7t - 198r^7st^2 - 198r^7s^2t + 924r^2s^5t^2 - 462r^5s^2t^2 - 462r^2s^6t^2 + 198r^6s^2t^2) + \\
&\frac{hf_n}{27720r^3s^3t^3}(924r^3s^7 - 1056r^2s^8 - 396r^7s^3 + 396r^8s^2 + 1320r^2s^9 - 1056r^3s^8 + \\
&396r^8s^3 - 440r^9s^2 - 440r^2s^{10} + 330r^3s^9 - 110r^9s^3 + 132r^{10}s^2 - 396r^7t^3 + \\
&396r^8t^2 + 396r^8t^3 - 440r^9t^2 - 110r^9t^3 + 132r^{10}t^2 + 924s^7t^3 - 1056s^8t^2 - 1056s^8t^3 + \\
&1320s^9t^2 + 330s^9t^3 - 440s^{10}t^2 + 330rs^9 - 110r^9s - 440rs^{10} + 132r^{10}s + 154rs^{11} - \\
&42r^{11}s - 110r^9t + 132r^{10}t - 42r^{11}t + 330s^9t - 440s^{10}t + 154s^{11}t - 1518rs^8t + \\
&583r^8st + 2200rs^9t - 748r^9st - 1056rs^{10}t + 321r^{10}st + 154rs^{11}t - 42r^{11}st - \\
&1848rs^6t^3 + 2640rs^7t^2 + 2640r^2s^7t - 1848r^3s^6t + 990r^6st^3 + 990r^6s^3t - 1188r^7st^2 - \\
&1188r^7s^2t + 2640rs^7t^3 - 3960rs^8t^2 - 3960r^2s^8t + 2640r^3s^7t - 1188r^7st^3 - \\
&1188r^7s^3t + 1540r^8st^2 + 1540r^8s^2t - 1518rs^8t^3 + 2200rs^9t^2 + 2200r^2s^9t - \\
&1518r^3s^8t + 583r^8st^3 + 583r^8s^3t - 748r^9st^2 - 748r^9s^2t + 330rs^9t^3 - 440rs^{10}t^2 - \\
&440r^2s^{10}t + 330r^3s^9t - 110r^9st^3 - 110r^9s^3t + 132r^{10}st^2 + 132r^{10}s^2t - 3696r^2s^5t^3 + \\
&13860r^3s^4t^3 - 3696r^3s^5t^2 - 9702r^4s^3t^3 + 1848r^5s^2t^3 + 1848r^5s^3t^2 + 264r^6s^2t^2 + \\
&3168r^2s^7t^2 - 3696r^3s^5t^3 + 1848r^5s^3t^3 + 264r^6s^2t^3 + 264r^6s^3t^2 - 1584r^7s^2t^2 + \\
&2640r^2s^7t^3 - 3960r^2s^8t^2 - 1848r^3s^6t^3 + 2640r^3s^7t^2 + 990r^6s^3t^3 - 1188r^7s^2t^3 - \\
&1188r^7s^3t^2 + 1540r^8s^2t^2 - 1056r^2s^8t^3 + 1320r^2s^9t^2 + 924r^3s^7t^3 - 1056r^3s^8t^2 - \\
&396r^7s^3t^3 + 396r^8s^2t^3 + 396r^8s^3t^2 - 440r^9s^2t^2) - \frac{hf_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(2376r^2s^7 - \\
&891r^7s^2 - 858r^2s^8 - 1848r^3s^7 + 693r^7s^3 + 363r^8s^2 - 1980r^2s^9 + 2310r^3s^8 - \\
&825r^8s^3 + 616r^9s^2 + 880r^2s^{10} - 660r^3s^9 + 220r^9s^3 - 264r^{10}s^2 - 891r^7t^2 + 693r^7t^3 + \\
&363r^8t^2 - 825r^8t^3 + 616r^9t^2 + 220r^9t^3 - 264r^{10}t^2 + 2376s^7t^2 - 1848s^7t^3 - 858s^8t^2 + \\
&2310s^8t^3 - 1980s^9t^2 - 660s^9t^3 + 880s^{10}t^2 - 2970rs^8 + 990r^8s + 3300rs^9 - 1067r^9s - \\
&308rs^{10} + 105r^{10}s - 308rs^{11} + 84r^{11}s + 990r^8t - 1067r^9t + 105r^{10}t + 84r^{11}t - \\
&2970s^8t + 3300s^9t - 308s^{10}t - 308s^{11}t - 297r^9 + 399r^{10} - 126r^{11} + 990s^9 - \\
&1386s^{10} + 462s^{11} + 9504rs^7t - 3564r^7st - 8052rs^8t + 2992r^8st - 1430rs^9t + \\
&407r^9st + 1122rs^{10}t - 345r^{10}st + 154rs^{11}t - 42r^{11}st - 8316rs^6t^2 - 8316r^2s^6t + 3564
\end{aligned}
$$

$r^6st^2 + 3564r^6s^2t + 6468rs^6t^3 + 1452rs^7t^2 + 1452r^2s^7t + 6468r^3s^6t - 2772r^6st^3 - 2772r^6s^3t - 891r^7st^2 - 891r^7s^2t - 6468rs^7t^3 + 6732rs^8t^2 + 6732r^2s^8t - 6468r^3s^7t + 2673r^7st^3 + 2673r^7s^3t - 2387r^8st^2 - 2387r^8s^2t + 660rs^8t^3 - 1430rs^9t^2 - 1430r^2s^9t + 660r^3s^8t - 275r^8st^3 - 275r^8s^3t + 506r^9st^2 + 506r^9s^2t + 330rs^9t^3 - 440rs^{10}t^2 - 440r^2s^{10}t + 330r^3s^9t - 110r^9st^3 - 110r^9s^3t + 132r^{10}st^2 + 132r^{10}s^2t + 8316r^2s^5t^2 - 4158r^5s^2t^2 - 6468r^2s^5t^3 + 2772r^2s^6t^2 - 6468r^3s^5t^2 + 3234r^5s^2t^3 + 3234r^5s^3t^2 - 726r^6s^2t^2 + 2772r^2s^6t^3 - 7524r^2s^7t^2 + 4620r^3s^5t^3 + 2772r^3s^6t^2 - 2310r^5s^3t^3 - 1518r^6s^2t^3 - 1518r^6s^3t^2 + 3168r^7s^2t^2 + 3036r^2s^7t^3 - 1584r^2s^8t^2 - 4620r^3s^6t^3 + 3036r^3s^7t^2 + 2178r^6s^3t^3 - 1188r^7s^2t^3 - 1188r^7s^3t^2 + 484r^8s^2t^2 - 1056r^2s^8t^3 + 1320r^2s^9t^2 + 924r^3s^7t^3 - 1056r^3s^8t^2 - 396r^7s^3t^3 + 396r^8s^2t^3 + 396r^8s^3t^2 - 440r^9s^2t^2) + \frac{h^2g_n}{27720r^2s^2t^2}(462r^2s^6 - 198r^6s^2 - 528r^2s^7 + 198r^7s^2 + 165r^2s^8 - 55r^8s^2 - 198r^6t^2 + 198r^7t^2 - 55r^8t^2 + 462s^6t^2 - 528s^7t^2 + 165s^8t^2 - 528rs^7 + 198r^7s + 660rs^8 - 220r^8s - 220rs^9 + 66r^9s + 198r^7t - 220r^8t + 66r^9t - 528s^7t + 660s^8t - 220s^9t - 55r^8 + 66r^9 - 21r^{10} + 165s^8 - 220s^9 + 77s^{10} + 1848rs^6t - 792r^6st - 2112rs^7t + 792r^7st + 660rs^8t - 220r^8st - 1848rs^5t^2 - 1848r^2s^5t + 924r^5st^2 + 924r^5s^2t + 1848rs^6t^2 + 1848r^2s^6t - 792r^6st^2 - 792r^6s^2t - 528rs^7t^2 - 528r^2s^7t + 198r^7st^2 + 198r^7s^2t + 2310r^2s^4t^2 - 1386r^4s^2t^2 - 1848r^2s^5t^2 + 924r^5s^2t^2 + 462r^2s^6t^2 - 198r^6s^2t^2) + \frac{h^2g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(264r^6s^2 - 330r^7s^2 + 110r^8s^2 + 264r^6t^2 - 330r^7t^2 + 110r^8t^2 + 462s^6t^2 - 528s^7t^2 + 165s^8t^2 - 264rs^7 - 330r^7s + 330rs^8 + 440r^8s - 110rs^9 - 154r^9s - 330r^7t + 440r^8t - 154r^9t - 528s^7t + 660s^8t - 220s^9t + 110r^8 - 154r^9 + 56r^{10} + 165s^8 - 220s^9 + 77s^{10} + 924rs^6t + 1056r^6st - 1056rs^7t - 1320r^7st + 330rs^8t + 440r^8st - 924rs^5t^2 - 924r^5st^2 - 924r^5s^2t + 924rs^6t^2 + 1056r^6st^2 + 1056r^6s^2t - 264rs^7t^2 - 330r^7st^2 - 330r^7s^2t + 924r^4s^2t^2 - 924r^5s^2t^2 + 264r^6s^2t^2) - \frac{h^2g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(924r^2s^6 - 1320r^2s^7 + 495r^2s^8 + 198r^6t^2 - 198r^7t^2 + 55r^8t^2 + 924s^6t^2 - 1320s^7t^2 + 495s^8t^2 - 1320rs^7 - 99r^7s + 1980rs^8 + 110r^8s - 770rs^9 - 33r^9s - 198r^7t + 220r^8t - 66r^9t - 1320s^7t + 1980s^8t - 770s^9t + 55r^8 - 66r^9 + 21r^{10} + 495s^8 - 770s^9 + 308s^{10} + 3696rs^6t + 396r^6st - 5280rs^7t - 396r^7st + 1980rs^8t + 110r^8st - 2772rs^5t^2 - 2772r^2s^5t - 462r^5st^2 + 3696rs^6t^2 + 3696r^2s^6t + 396r^6st^2 - 1320rs^7t^2 - 1320r^2s^7t - 99r^7st^2 + 2310r^2s^4t^2$

$-2772r^2s^5t^2+924r^2s^6t^2)+\frac{h^2g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(462r^2s^6-198r^6s^2-528r^2s^7+198r^7s^2+165r^2s^8-55r^8s^2-528rs^7+198r^7s+660rs^8-220r^8s-220rs^9+66r^9s+99r^7t-110r^8t+33r^9t-264s^7t+330s^8t-110s^9t-55r^8+66r^9-21r^{10}+165s^8-220s^9+77s^{10}+924rs^6t-396r^6st-1056rs^7t+396r^7st+330rs^8t-110r^8st-924r^2s^5t+462r^5s^2t+924r^2s^6t-396r^6s^2t-264r^2s^7t+99r^7s^2t)+\frac{hf_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(2310r^2s^8-1848r^3s^7-1980r^7s^3+5148r^8s^2-1980r^2s^9-858r^3s^8+2376r^4s^7+5148r^8s^3-13376r^9s^2-308r^2s^{10}+3300r^3s^9-2970r^4s^8-4235r^9s^3+11121r^{10}s^2+462r^2s^{11}-1386r^3s^{10}+990r^4s^9+1155r^{10}s^3-3069r^{11}s^2-1980r^7t^3+5148r^8t^2+5148r^8t^3-13376r^9t^2-4235r^9t^3+11121r^{10}t^2+1155r^{10}t^3-3069r^{11}t^2+924s^7t^3-1056s^8t^2-1056s^8t^3+1320s^9t^2+330s^9t^3-440s^{10}t^2-660rs^9-4235r^9s+880rs^{10}+11121r^{10}s-308rs^{11}-9420r^{11}s+2646r^12s-4235r^9t+11121r^{10}t-9420r^{11}t+2646r^12t+330s^9t-440s^{10}t+154s^{11}t+1155r^{10}-3069r^{11}+2646r^12-756r^13+660rs^8t+16027r^8st-1430rs^9t-41437r^9st+1122rs^{10}t+34278r^{10}st-308rs^{11}t-9420r^{11}st-4620rs^6t^3+3036rs^7t^2-6468r^2s^7t+6468r^3s^6t+7920r^6st^3+7920r^6s^3t-20196r^7st^2-20196r^7s^2t+3036rs^7t^3-1584rs^8t^2+6732r^2s^8t+1452r^3s^7t-8316r^4s^6t-20196r^7st^3-20196r^7s^3t+51436r^8st^2+51436r^8s^2t+660rs^8t^3-1430rs^9t^2-1430r^2s^9t-8052r^3s^8t+9504r^4s^7t+16027r^8st^3+16027r^8s^3t-41437r^9st^2-41437r^9s^2t-660rs^9t^3+880rs^{10}t^2-308r^2s^{10}t+3300r^3s^9t-2970r^4s^8t-4235r^9st^3-4235r^9s^3t+11121r^{10}st^2+11121r^{10}s^2t+4620r^2s^5t^3+2772r^2s^6t^2-6468r^3s^5t^2+4158r^4s^3t^3-10626r^5s^2t^3-10626r^5s^3t^2+26598r^6s^2t^2+2772r^2s^6t^3-7524r^2s^7t^2-6468r^3s^5t^3+2772r^3s^6t^2+8316r^4s^5t^2-10626r^5s^3t^3+26598r^6s^2t^3+26598r^6s^3t^2-66330r^7s^2t^2-6468r^2s^7t^3+6732r^2s^8t^2+6468r^3s^6t^3+1452r^3s^7t^2-8316r^4s^6t^2+7920r^6s^3t^3-20196r^7s^2t^3-20196r^7s^3t^2+51436r^8s^2t^2+2310r^2s^8t^3-1980r^2s^9t^2-1848r^3s^7t^3-858r^3s^8t^2+2376r^4s^7t^2-1980r^7s^3t^3+5148r^8s^2t^3+5148r^8s^3t^2-13376r^9s^2t^2)+\frac{hf_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(25608r^2s^8-9240r^3s^7-693r^7s^3+825r^8s^2-71016r^2s^9+25608r^3s^8+891r^7s^4-363r^8s^3-616r^9s^2+64460r^2s^{10}-23100r^3s^9-990r^8s^4+1067r^9s^3-105r^{10}s^2-19470r^2s^{11}+6930r^3s^{10}+297r^9s^4-399r^{10}s^3+126r^{11}s^2+396r^7t^3-396r^8t^2-396r^8t^3+440r^9t^2+110r^9t^3-132r^{10}t^2-9240s^7t^3$

$+25608s^8t^2 + 25608s^8t^3 - 71016s^9t^2 - 23100s^9t^3 + 64460s^{10}t^2 + 6930s^{10}t^3 -$
$19470s^{11}t^2 - 23100rs^9 - 220r^9s + 64460rs^{10} + 264r^{10}s - 59092rs^{11} - 84r^{11}s +$
$18018rs^12 + 110r^9t - 132r^{10}t + 42r^{11}t - 23100s^9t + 64460s^{10}t - 59092s^{11}t +$
$18018s^12t + 6930s^{10} - 19470s^{11} + 18018s^12 - 5544s^13 + 78342rs^8t + 275r^8st -$
$216634rs^9t - 506r^9st + 196086rs^{10}t + 345r^{10}st - 59092rs^{11}t - 84r^{11}st +$
$32340rs^6t^3 - 88572rs^7t^2 - 88572r^2s^7t + 32340r^3s^6t - 2178r^6st^3 + 2772r^6s^3t +$
$1188r^7st^2 - 2673r^7s^2t - 88572rs^7t^3 + 242748rs^8t^2 + 242748r^2s^8t - 88572r^3s^7t -$
$3564r^6s^4t + 1188r^7st^3 + 891r^7s^3t - 484r^8st^2 + 2387r^8s^2t + 78342rs^8t^3 -$
$216634rs^9t^2 - 216634r^2s^9t + 78342r^3s^8t + 3564r^7s^4t + 275r^8st^3 - 2992r^8s^3t -$
$506r^9st^2 - 407r^9s^2t - 23100rs^9t^3 + 64460rs^{10}t^2 + 64460r^2s^{10}t - 23100r^3s^9t -$
$990r^8s^4t - 220r^9st^3 + 1067r^9s^3t + 264r^{10}st^2 - 105r^{10}s^2t - 37884r^2s^5t^3 +$
$102564r^2s^6t^2 + 13860r^3s^4t^3 - 37884r^3s^5t^2 + 2310r^5s^2t^3 - 3234r^5s^3t^2 +$
$1518r^6s^2t^2 + 102564r^2s^6t^3 - 277596r^2s^7t^2 - 37884r^3s^5t^3 + 102564r^3s^6t^2 -$
$3234r^5s^3t^3 + 4158r^5s^4t^2 + 1518r^6s^2t^3 + 726r^6s^3t^2 - 3168r^7s^2t^2 - 88572r^2s^7t^3 +$
$242748r^2s^8t^2 + 32340r^3s^6t^3 - 88572r^3s^7t^2 + 2772r^6s^3t^3 - 3564r^6s^4t^2 -$
$2673r^7s^2t^3 + 891r^7s^3t^2 + 2387r^8s^2t^2 + 25608r^2s^8t^3 - 71016r^2s^9t^2 - 9240r^3s^7t^3 +$
$25608r^3s^8t^2 - 693r^7s^3t^3 + 891r^7s^4t^2 + 825r^8s^2t^3 - 363r^8s^3t^2 - 616r^9s^2t^2) -$
$\frac{hf_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(1056r^2s^8 - 924r^3s^7 + 396r^7s^3 - 396r^8s^2 - 1320r^2s^9 +$
$1056r^3s^8 - 396r^8s^3 + 440r^9s^2 + 440r^2s^{10} - 330r^3s^9 + 110r^9s^3 - 132r^{10}s^2 -$
$693r^7t^3 + 825r^8t^2 + 891r^7t^4 - 363r^8t^3 - 616r^9t^2 - 990r^8t^4 + 1067r^9t^3 - 105r^{10}t^2 +$
$297r^9t^4 - 399r^{10}t^3 + 126r^{11}t^2 + 1848s^7t^3 - 2310s^8t^2 - 2376s^7t^4 + 858s^8t^3 +$
$1980s^9t^2 + 2970s^8t^4 - 3300s^9t^3 + 308s^{10}t^2 - 990s^9t^4 + 1386s^{10}t^3 - 462s^{11}t^2 -$
$330rs^9 + 110r^9s + 440rs^{10} - 132r^{10}s - 154rs^{11} + 42r^{11}s - 220r^9t + 264r^{10}t -$
$84r^{11}t + 660s^9t - 880s^{10}t + 308s^{11}t - 660rs^8t + 275r^8st + 1430rs^9t - 506r^9st -$
$1122rs^{10}t + 345r^{10}st + 308rs^{11}t - 84r^{11}st - 6468rs^6t^3 + 6468rs^7t^2 - 3036r^2s^7t +$
$4620r^3s^6t + 2772r^6st^3 - 2178r^6s^3t - 2673r^7st^2 + 1188r^7s^2t + 8316rs^6t^4 -$
$1452rs^7t^3 - 6732rs^8t^2 + 1584r^2s^8t - 3036r^3s^7t - 3564r^6st^4 + 891r^7st^3 + 1188r^7s^3t +$
$2387r^8st^2 - 484r^8s^2t - 9504rs^7t^4 + 8052rs^8t^3 + 1430rs^9t^2 + 1430r^2s^9t - 660r^3s^8t +$
$3564r^7st^4 - 2992r^8st^3 + 275r^8s^3t - 407r^9st^2 - 506r^9s^2t + 2970rs^8t^4 - 3300rs^9t^3 +$

$$308rs^{10}t^2 - 880r^2s^{10}t + 660r^3s^9t - 990r^8st^4 + 1067r^9st^3 - 220r^9s^3t - 105r^{10}st^2 +$$
$$264r^{10}s^2t + 6468r^2s^5t^3 - 2772r^2s^6t^2 - 4620r^3s^5t^2 - 3234r^5s^2t^3 + 2310r^5s^3t^2 +$$
$$1518r^6s^2t^2 - 8316r^2s^5t^4 - 2772r^2s^6t^3 + 7524r^2s^7t^2 + 6468r^3s^5t^3 - 2772r^3s^6t^2 +$$
$$4158r^5s^2t^4 - 3234r^5s^3t^3 + 726r^6s^2t^3 + 1518r^6s^3t^2 - 3168r^7s^2t^2 + 8316r^2s^6t^4 -$$
$$1452r^2s^7t^3 - 6732r^2s^8t^2 - 6468r^3s^6t^3 + 6468r^3s^7t^2 - 3564r^6s^2t^4 + 2772r^6s^3t^3 +$$
$$891r^7s^2t^3 - 2673r^7s^3t^2 + 2387r^8s^2t^2 - 2376r^2s^7t^4 + 858r^2s^8t^3 + 1980r^2s^9t^2 + 1848$$
$$r^3s^7t^3 - 2310r^3s^8t^2 + 891r^7s^2t^4 - 693r^7s^3t^3 - 363r^8s^2t^3 + 825r^8s^3t^2 - 616r^9s^2t^2), \tag{3.59}$$

$$y'_{n+t} = \frac{1}{hr}y_{n+r} - \frac{1}{hr}y_n + \frac{h^2g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(99r^7s^2 - 55r^8s^2 - 264r^2t^7 + 99r^7t^2 +$$
$$165r^2t^8 - 55r^8t^2 - 264s^2t^7 + 165s^2t^8 - 110r^8s + 66r^9s + 330rt^8 - 110r^8t - 220rt^9 +$$
$$66r^9t + 330st^8 - 220st^9 + 33r^9 - 21r^{10} - 110t^9 + 77t^{10} - 1056rst^7 + 396r^7st +$$
$$660rst^8 - 220r^8st + 924rs^2t^6 + 924r^2st^6 - 396r^6st^2 - 396r^6s^2t - 528rs^2t^7 -$$
$$528r^2st^7 + 198r^7st^2 + 198r^7s^2t - 924r^2s^2t^5 + 462r^5s^2t^2 + 462r^2s^2t^6 - 198r^6s^2t^2) +$$
$$\frac{hf_n}{27720r^3s^3t^3}(396r^8s^2 - 396r^7s^3 + 396r^8s^3 - 440r^9s^2 - 110r^9s^3 + 132r^{10}s^2 - 1056r^2t^8 +$$
$$924r^3t^7 - 396r^7t^3 + 396r^8t^2 + 1320r^2t^9 - 1056r^3t^8 + 396r^8t^3 - 440r^9t^2 - 440r^2t^{10} +$$
$$330r^3t^9 - 110r^9t^3 + 132r^{10}t^2 - 1056s^2t^8 + 924s^3t^7 + 1320s^2t^9 - 1056s^3t^8 -$$
$$440s^2t^{10} + 330s^3t^9 - 110r^9s + 132r^{10}s - 42r^{11}s + 330rt^9 - 110r^9t - 440rt^{10} +$$
$$132r^{10}t + 154rt^{11} - 42r^{11}t + 330st^9 - 440st^{10} + 154st^{11} - 1518rst^8 + 583r^8st +$$
$$2200rst^9 - 748r^9st - 1056rst^{10} + 321r^{10}st + 154rst^{11} - 42r^{11}st + 2640rs^2t^7 -$$
$$1848rs^3t^6 + 2640r^2st^7 - 1848r^3st^6 + 990r^6st^3 + 990r^6s^3t - 1188r^7st^2 - 1188r^7s^2t -$$
$$3960rs^2t^8 + 2640rs^3t^7 - 3960r^2st^8 + 2640r^3st^7 - 1188r^7st^3 - 1188r^7s^3t +$$
$$1540r^8st^2 + 1540r^8s^2t + 2200rs^2t^9 - 1518rs^3t^8 + 2200r^2st^9 - 1518r^3st^8 +$$
$$583r^8st^3 + 583r^8s^3t - 748r^9st^2 - 748r^9s^2t - 440rs^2t^{10} + 330rs^3t^9 - 440r^2st^{10} +$$
$$330r^3st^9 - 110r^9st^3 - 110r^9s^3t + 132r^{10}st^2 + 132r^{10}s^2t - 3696r^2s^3t^5 - 3696r^3s^2t^5 +$$
$$13860r^3s^3t^4 - 9702r^4s^3t^3 + 1848r^5s^2t^3 + 1848r^5s^3t^2 + 264r^6s^2t^2 + 3168r^2s^2t^7 -$$
$$3696r^3s^3t^5 + 1848r^5s^3t^3 + 264r^6s^2t^3 + 264r^6s^3t^2 - 1584r^7s^2t^2 - 3960r^2s^2t^8 +$$
$$2640r^2s^3t^7 + 2640r^3s^2t^7 - 1848r^3s^3t^6 + 990r^6s^3t^3 - 1188r^7s^2t^3 - 1188r^7s^3t^2 +$$
$$1540r^8s^2t^2 + 1320r^2s^2t^9 - 1056r^2s^3t^8 - 1056r^3s^2t^8 + 924r^3s^3t^7 - 396r^7s^3t^3 + 396$$

$$r^8s^2t^3 + 396r^8s^3t^2 - 440r^9s^2t^2) + \frac{hf_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(891r^7s^2 - 693r^7s^3 - 363r^8s^2 + 825r^8s^3 - 616r^9s^2 - 220r^9s^3 + 264r^{10}s^2 - 2376r^2t^7 + 891r^7t^2 + 858r^2t^8 + 1848r^3t^7 - 693r^7t^3 - 363r^8t^2 + 1980r^2t^9 - 2310r^3t^8 + 825r^8t^3 - 616r^9t^2 - 880r^2t^{10} + 660r^3t^9 - 220r^9t^3 + 264r^{10}t^2 - 2376s^2t^7 + 858s^2t^8 + 1848s^3t^7 + 1980s^2t^9 - 2310s^3t^8 - 880s^2t^{10} + 660s^3t^9 - 990r^8s + 1067r^9s - 105r^{10}s - 84r^{11}s + 2970rt^8 - 990r^8t - 3300rt^9 + 1067r^9t + 308rt^{10} - 105r^{10}t + 308rt^{11} - 84r^{11}t + 2970st^8 - 3300st^9 + 308st^{10} + 308st^{11} + 297r^9 - 399r^{10} + 126r^{11} - 990t^9 + 1386t^{10} - 462t^{11} - 9504rst^7 + 3564r^7st + 8052rst^8 - 2992r^8st + 1430rst^9 - 407r^9st - 1122rst^{10} + 345r^{10}st - 154rst^{11} + 42r^{11}st + 8316rs^2t^6 + 8316r^2st^6 - 3564r^6st^2 - 3564r^6s^2t - 1452rs^2t^7 - 6468rs^3t^6 - 1452r^2st^7 - 6468r^3st^6 + 2772r^6st^3 + 2772r^6s^3t + 891r^7st^2 + 891r^7s^2t - 6732rs^2t^8 + 6468rs^3t^7 - 6732r^2st^8 + 6468r^3st^7 - 2673r^7st^3 - 2673r^7s^3t + 2387r^8st^2 + 2387r^8s^2t + 1430rs^2t^9 - 660rs^3t^8 + 1430r^2st^9 - 660r^3st^8 + 275r^8st^3 + 275r^8s^3t - 506r^9st^2 - 506r^9s^2t + 440rs^2t^{10} - 330rs^3t^9 + 440r^2st^{10} - 330r^3st^9 + 110r^9st^3 + 110r^9s^3t - 132r^{10}st^2 - 132r^{10}s^2t - 8316r^2s^2t^5 + 4158r^5s^2t^2 - 2772r^2s^2t^6 + 6468r^2s^3t^5 + 6468r^3s^2t^5 - 3234r^5s^2t^3 - 3234r^5s^3t^2 + 726r^6s^2t^2 + 7524r^2s^2t^7 - 2772r^2s^3t^6 - 2772r^3s^2t^6 - 4620r^3s^3t^5 + 2310r^5s^3t^3 + 1518r^6s^2t^3 + 1518r^6s^3t^2 - 3168r^7s^2t^2 + 1584r^2s^2t^8 - 3036r^2s^3t^7 - 3036r^3s^2t^7 + 4620r^3s^3t^6 - 2178r^6s^3t^3 + 1188r^7s^2t^3 + 1188r^7s^3t^2 - 484r^8s^2t^2 - 1320r^2s^2t^9 + 1056r^2s^3t^8 + 1056r^3s^2t^8 - 924r^3s^3t^7 + 396r^7s^3t^3 - 396r^8s^2t^3 - 396r^8s^3t^2 + 440r^9s^2t^2) - \frac{h^2g_n}{27720r^2s^2t^2}(198r^6s^2 - 198r^7s^2 + 55r^8s^2 - 462r^2t^6 + 198r^6t^2 + 528r^2t^7 - 198r^7t^2 - 165r^2t^8 + 55r^8t^2 - 462s^2t^6 + 528s^2t^7 - 165s^2t^8 - 198r^7s + 220r^8s - 66r^9s + 528rt^7 - 198r^7t - 660rt^8 + 220r^8t + 220rt^9 - 66r^9t + 528st^7 - 660st^8 + 220st^9 + 55r^8 - 66r^9 + 21r^{10} - 165t^8 + 220t^9 - 77t^{10} - 1848rst^6 + 792r^6st + 2112rst^7 - 792r^7st - 660rst^8 + 220r^8st + 1848rs^2t^5 + 1848r^2st^5 - 924r^5st^2 - 924r^5s^2t - 1848rs^2t^6 - 1848r^2st^6 + 792r^6st^2 + 792r^6s^2t + 528rs^2t^7 + 528r^2st^7 - 198r^7st^2 - 198r^7s^2t - 2310r^2s^2t^4 + 1386r^4s^2t^2 + 1848r^2s^2t^5 - 924r^5s^2t^2 - 462r^2s^2t^6 + 198r^6s^2t^2) + \frac{h^2g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(264r^6s^2 - 330r^7s^2 + 110r^8s^2 + 264r^6t^2 - 330r^7t^2 + 110r^8t^2 + 462s^2t^6 - 528s^2t^7 + 165s^2t^8 - 330r^7s + 440r^8s - 154r^9s - 264rt^7 - 330r^7t + 330rt^8 + 440r^8t - 110rt^9 - 154r^9t - 528st^7 +$$

$$660st^8 - 220st^9 + 110r^8 - 154r^9 + 56r^{10} + 165t^8 - 220t^9 + 77t^{10} + 924rst^6 +$$

$$1056r^6st - 1056rst^7 - 1320r^7st + 330rst^8 + 440r^8st - 924rs^2t^5 - 924r^5st^2 -$$

$$924r^5s^2t + 924rs^2t^6 + 1056r^6st^2 + 1056r^6s^2t - 264rs^2t^7 - 330r^7st^2 - 330r^7s^2t +$$

$$924r^4s^2t^2 - 924r^5s^2t^2 + 264r^6s^2t^2) - \frac{h^2g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^6s^2 - 198r^7s^2 +$$

$$55r^8s^2 + 924r^2t^6 - 1320r^2t^7 + 495r^2t^8 + 924s^2t^6 - 1320s^2t^7 + 495s^2t^8 - 198r^7s +$$

$$220r^8s - 66r^9s - 1320rt^7 - 99r^7t + 1980rt^8 + 110r^8t - 770rt^9 - 33r^9t - 1320st^7 +$$

$$1980st^8 - 770st^9 + 55r^8 - 66r^9 + 21r^{10} + 495t^8 - 770t^9 + 308t^{10} + 3696rst^6 +$$

$$396r^6st - 5280rst^7 - 396r^7st + 1980rst^8 + 110r^8st - 2772rs^2t^5 - 2772r^2st^5 -$$

$$462r^5s^2t + 3696rs^2t^6 + 3696r^2st^6 + 396r^6s^2t - 1320rs^2t^7 - 1320r^2st^7 - 99r^7s^2t +$$

$$2310r^2s^2t^4 - 2772r^2s^2t^5 + 924r^2s^2t^6) + \frac{h^2g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(462r^2t^6 - 198r^6t^2 -$$

$$528r^2t^7 + 198r^7t^2 + 165r^2t^8 - 55r^8t^2 + 99r^7s - 110r^8s + 33r^9s - 528rt^7 + 198r^7t +$$

$$660rt^8 - 220r^8t - 220rt^9 + 66r^9t - 264st^7 + 330st^8 - 110st^9 - 55r^8 + 66r^9 -$$

$$21r^{10} + 165t^8 - 220t^9 + 77t^{10} + 924rst^6 - 396r^6st - 1056rst^7 + 396r^7st + 330rst^8 -$$

$$110r^8st - 924r^2st^5 + 462r^5st^2 + 924r^2st^6 - 396r^6st^2 - 264r^2st^7 + 99r^7st^2) -$$

$$\frac{hf_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(1980r^7s^3 - 5148r^8s^2 - 5148r^8s^3 + 13376r^9s^2 + 4235r^9s^3 -$$

$$11121r^{10}s^2 - 1155r^{10}s^3 + 3069r^{11}s^2 - 2310r^2t^8 + 1848r^3t^7 + 1980r^7t^3 - 5148r^8t^2 +$$

$$1980r^2t^9 + 858r^3t^8 - 2376r^4t^7 - 5148r^8t^3 + 13376r^9t^2 + 308r^2t^{10} - 3300r^3t^9 +$$

$$2970r^4t^8 + 4235r^9t^3 - 11121r^{10}t^2 - 462r^2t^{11} + 1386r^3t^{10} - 990r^4t^9 - 1155r^{10}t^3 +$$

$$3069r^{11}t^2 + 1056s^2t^8 - 924s^3t^7 - 1320s^2t^9 + 1056s^3t^8 + 440s^2t^{10} - 330s^3t^9 +$$

$$4235r^9s - 11121r^{10}s + 9420r^{11}s - 2646r^{12}s + 660rt^9 + 4235r^9t - 880rt^{10} -$$

$$11121r^{10}t + 308rt^{11} + 9420r^{11}t - 2646r^{12}t - 330st^9 + 440st^{10} - 154st^{11} - 1155r^{10} +$$

$$3069r^{11} - 2646r^12 + 756r^{13} - 660rst^8 - 16027r^8st + 1430rst^9 + 41437r^9st -$$

$$1122rst^{10} - 34278r^{10}st + 308rst^{11} + 9420r^{11}st - 3036rs^2t^7 + 4620rs^3t^6 +$$

$$6468r^2st^7 - 6468r^3st^6 - 7920r^6st^3 - 7920r^6s^3t + 20196r^7st^2 + 20196r^7s^2t +$$

$$1584rs^2t^8 - 3036rs^3t^7 - 6732r^2st^8 - 1452r^3st^7 + 8316r^4st^6 + 20196r^7st^3 +$$

$$20196r^7s^3t - 51436r^8st^2 - 51436r^8s^2t + 1430rs^2t^9 - 660rs^3t^8 + 1430r^2st^9 +$$

$$8052r^3st^8 - 9504r^4st^7 - 16027r^8st^3 - 16027r^8s^3t + 41437r^9st^2 + 41437r^9s^2t -$$

$$880rs^2t^{10} + 660rs^3t^9 + 308r^2st^{10} - 3300r^3st^9 + 2970r^4st^8 + 4235r^9st^3 + 4235r^9s^3t -$$

$$11121r^{10}st^2 - 11121r^{10}s^2t - 2772r^2s^2t^6 - 4620r^2s^3t^5 + 6468r^3s^2t^5 - 4158r^4s^3t^3 +$$

$$10626r^5s^2t^3 + 10626r^5s^3t^2 - 26598r^6s^2t^2 + 7524r^2s^2t^7 - 2772r^2s^3t^6 -$$
$$2772r^3s^2t^6 + 6468r^3s^3t^5 - 8316r^4s^2t^5 + 10626r^5s^3t^3 - 26598r^6s^2t^3 - 26598r^6s^3t^2 +$$
$$66330r^7s^2t^2 - 6732r^2s^2t^8 + 6468r^2s^3t^7 - 1452r^3s^2t^7 - 6468r^3s^3t^6 + 8316r^4s^2t^6 -$$
$$7920r^6s^3t^3 + 20196r^7s^2t^3 + 20196r^7s^3t^2 - 51436r^8s^2t^2 + 1980r^2s^2t^9 - 2310r^2s^3t^8 +$$
$$858r^3s^2t^8 + 1848r^3s^3t^7 - 2376r^4s^2t^7 + 1980r^7s^3t^3 - 5148r^8s^2t^3 - 5148r^8s^3t^2 +$$
$$13376r^9s^2t^2) - \frac{hf_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^7s^3 - 825r^8s^2 - 891r^7s^4 + 363r^8s^3 +$$
$$616r^9s^2 + 990r^8s^4 - 1067r^9s^3 + 105r^{10}s^2 - 297r^9s^4 + 399r^{10}s^3 - 126r^{11}s^2 -$$
$$1056r^2t^8 + 924r^3t^7 - 396r^7t^3 + 396r^8t^2 + 1320r^2t^9 - 1056r^3t^8 + 396r^8t^3 - 440r^9t^2 -$$
$$440r^2t^{10} + 330r^3t^9 - 110r^9t^3 + 132r^{10}t^2 + 2310s^2t^8 - 1848s^3t^7 - 1980s^2t^9 -$$
$$858s^3t^8 + 2376s^4t^7 - 308s^2t^{10} + 3300s^3t^9 - 2970s^4t^8 + 462s^2t^{11} - 1386s^3t^{10} +$$
$$990s^4t^9 + 220r^9s - 264r^{10}s + 84r^{11}s + 330rt^9 - 110r^9t - 440rt^{10} + 132r^{10}t +$$
$$154rt^{11} - 42r^{11}t - 660st^9 + 880st^{10} - 308st^{11} + 660rst^8 - 275r^8st - 1430rst^9 +$$
$$506r^9st + 1122rst^{10} - 345r^{10}st - 308rst^{11} + 84r^{11}st - 6468rs^2t^7 + 6468rs^3t^6 +$$
$$3036r^2st^7 - 4620r^3st^6 + 2178r^6st^3 - 2772r^6s^3t - 1188r^7st^2 + 2673r^7s^2t +$$
$$6732rs^2t^8 + 1452rs^3t^7 - 8316rs^4t^6 - 1584r^2st^8 + 3036r^3st^7 + 3564r^6s^4t -$$
$$1188r^7st^3 - 891r^7s^3t + 484r^8st^2 - 2387r^8s^2t - 1430rs^2t^9 - 8052rs^3t^8 + 9504rs^4t^7 -$$
$$1430r^2st^9 + 660r^3st^8 - 3564r^7s^4t - 275r^8st^3 + 2992r^8s^3t + 506r^9st^2 + 407r^9s^2t -$$
$$308rs^2t^{10} + 3300rs^3t^9 - 2970rs^4t^8 + 880r^2st^{10} - 660r^3st^9 + 990r^8s^4t + 220r^9st^3 -$$
$$1067r^9s^3t - 264r^{10}st^2 + 105r^{10}s^2t + 2772r^2s^2t^6 - 6468r^2s^3t^5 + 4620r^3s^2t^5 -$$
$$2310r^5s^2t^3 + 3234r^5s^3t^2 - 1518r^6s^2t^2 - 7524r^2s^2t^7 + 2772r^2s^3t^6 + 8316r^2s^4t^5 +$$
$$2772r^3s^2t^6 - 6468r^3s^3t^5 + 3234r^5s^3t^3 - 4158r^5s^4t^2 - 1518r^6s^2t^3 - 726r^6s^3t^2 +$$
$$3168r^7s^2t^2 + 6732r^2s^2t^8 + 1452r^2s^3t^7 - 8316r^2s^4t^6 - 6468r^3s^2t^7 + 6468r^3s^3t^6 -$$
$$2772r^6s^3t^3 + 3564r^6s^4t^2 + 2673r^7s^2t^3 - 891r^7s^3t^2 - 2387r^8s^2t^2 - 1980r^2s^2t^9 -$$
$$858r^2s^3t^8 + 2376r^2s^4t^7 + 2310r^3s^2t^8 - 1848r^3s^3t^7 + 693r^7s^3t^3 - 891r^7s^4t^2 -$$
$$825r^8s^2t^3 + 363r^8s^3t^2 + 616r^9s^2t^2) - \frac{hf_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(396r^7s^3 - 396r^8s^2 -$$
$$396r^8s^3 + 440r^9s^2 + 110r^9s^3 - 132r^{10}s^2 + 25608r^2t^8 - 9240r^3t^7 - 693r^7t^3 +$$
$$825r^8t^2 - 71016r^2t^9 + 25608r^3t^8 + 891r^7t^4 - 363r^8t^3 - 616r^9t^2 + 64460r^2t^{10} -$$
$$23100r^3t^9 - 990r^8t^4 + 1067r^9t^3 - 105r^{10}t^2 - 19470r^2t^{11} + 6930r^3t^{10} + 297r^9t^4 -$$
$$399r^{10}t^3 + 126r^{11}t^2 + 25608s^2t^8 - 9240s^3t^7 - 71016s^2t^9 + 25608s^3t^8 + 64460s^2t^{10} -$$

$23100s^3t^9 - 19470s^2t^{11} + 6930s^3t^{10} + 110r^9s - 132r^{10}s + 42r^{11}s - 23100rt^9 - 220r^9t + 64460rt^{10} + 264r^{10}t - 59092rt^{11} - 84r^{11}t + 18018rt^12 - 23100st^9 + 64460st^{10} - 59092st^{11} + 18018st^12 + 6930t^{10} - 19470t^{11} + 18018t^{12} - 5544t^13 + 78342rst^8 + 275r^8st - 216634rst^9 - 506r^9st + 196086rst^{10} + 345r^{10}st - 59092rst^{11} - 84r^{11}st - 88572rs^2t^7 + 32340rs^3t^6 - 88572r^2st^7 + 32340r^3st^6 + 2772r^6st^3 - 2178r^6s^3t - 2673r^7st^2 + 1188r^7s^2t + 242748rs^2t^8 - 88572rs^3t^7 + 242748r^2st^8 - 88572r^3st^7 - 3564r^6st^4 + 891r^7st^3 + 1188r^7s^3t + 2387r^8st^2 - 484r^8s^2t - 216634rs^2t^9 + 78342rs^3t^8 - 216634r^2st^9 + 78342r^3st^8 + 3564r^7st^4 - 2992r^8st^3 + 275r^8s^3t - 407r^9st^2 - 506r^9s^2t + 64460rs^2t^{10} - 23100rs^3t^9 + 64460r^2st^{10} - 23100r^3st^9 - 990r^8st^4 + 1067r^9st^3 - 220r^9s^3t - 105r^{10}st^2 + 264r^{10}s^2t + 102564r^2s^2t^6 - 37884r^2s^3t^5 - 37884r^3s^2t^5 + 13860r^3s^3t^4 - 3234r^5s^2t^3 + 2310r^5s^3t^2 + 1518r^6s^2t^2 - 277596r^2s^2t^7 + 102564r^2s^3t^6 + 102564r^3s^2t^6 - 37884r^3s^3t^5 + 4158r^5s^2t^4 - 3234r^5s^3t^3 + 726r^6s^2t^3 + 1518r^6s^3t^2 - 3168r^7s^2t^2 + 242748r^2s^2t^8 - 88572r^2s^3t^7 - 88572r^3s^2t^7 + 32340r^3s^3t^6 - 3564r^6s^2t^4 + 2772r^6s^3t^3 + 891r^7s^2t^3 - 2673r^7s^3t^2 + 2387r^8s^2t^2 - 71016r^2s^2t^9 + 25608r^2s^3t^8 + 25608r^3s^2t^8 - 9240r^3s^3t^7 + 891r^7s^2t^4 - 693r^7s^3t^3 - 363r^8s^2t^3 + 825r^8s^3t^2 - 616r^9s^2t^2), \quad (3.60)$

$y'_{n+1} = \frac{1}{hr}y_{n+r} - \frac{1}{hr}y_n + \frac{hf_n}{27720r^3s^3t^3}(1320r^2s^2 - 1056r^2s^3 - 1056r^3s^2 + 924r^3s^3 - 396r^7s^3 + 396r^8s^2 + 396r^8s^3 - 440r^9s^2 - 110r^9s^3 + 132r^{10}s^2 + 1320r^2t^2 - 1056r^2t^3 - 1056r^3t^2 + 924r^3t^3 - 396r^7t^3 + 396r^8t^2 + 396r^8t^3 - 440r^9t^2 - 110r^9t^3 + 132r^{10}t^2 + 1320s^2t^2 - 1056s^2t^3 - 1056s^3t^2 + 924s^3t^3 + 154rs + 154rt + 154st - 440rs^2 - 440r^2s + 330rs^3 + 330r^3s - 110r^9s + 132r^{10}s - 42r^{11}s - 440rt^2 - 440r^2t + 330rt^3 + 330r^3t - 110r^9t + 132r^{10}t - 42r^{11}t - 440st^2 - 440s^2t + 330st^3 + 330s^3t + 2200rst^2 + 2200rs^2t + 2200r^2st - 1518rst^3 - 1518rs^3t - 1518r^3st + 583r^8st - 748r^9st + 321r^{10}st - 42r^{11}st - 3960rs^2t^2 - 3960r^2st^2 - 3960r^2s^2t + 2640rs^2t^3 + 2640rs^3t^2 + 2640r^2st^3 + 2640r^2s^3t + 2640r^3st^2 + 2640r^3s^2t - 1848rs^3t^3 - 1848r^3st^3 - 1848r^3s^3t + 990r^6st^3 + 990r^6s^3t - 1188r^7st^2 - 1188r^7s^2t - 1188r^7st^3 - 1188r^7s^3t + 1540r^8st^2 + 1540r^8s^2t + 583r^8st^3 + 583r^8s^3t - 748r^9st^2 - 748r^9s^2t - 110r^9st^3 -$

$$110r^9s^3t + 132r^{10}st^2 + 132r^{10}s^2t - 1056rst + 3168r^2s^2t^2 - 3696r^2s^3t^3 - 3696r^3s^2t^3 - 3696r^3s^3t^2 + 13860r^3s^3t^3 - 9702r^4s^3t^3 + 1848r^5s^2t^3 + 1848r^5s^3t^2 + 264r^6s^2t^2 + 1848r^5s^3t^3 + 264r^6s^2t^3 + 264r^6s^3t^2 - 1584r^7s^2t^2 + 990r^6s^3t^3 - 1188r^7s^2t^3 - 1188r^7s^3t^2 + 1540r^8s^2t^2 - 396r^7s^3t^3 + 396r^8s^2t^3 + 396r^8s^3t^2 - 440r^9s^2t^2) - \frac{h^2g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(924r^2s^2 - 770s - 770t - 770r - 99r^7s^2 + 55r^8s^2 + 924r^2t^2 - 99r^7t^2 + 55r^8t^2 + 924s^2t^2 + 1980rs + 1980rt + 1980st - 1320rs^2 - 1320r^2s + 110r^8s - 66r^9s - 1320rt^2 - 1320r^2t + 110r^8t - 66r^9t - 1320st^2 - 1320s^2t + 495r^2 - 33r^9 + 21r^{10} + 495s^2 + 495t^2 + 3696rst^2 + 3696rs^2t + 3696r^2st - 396r^7st + 220r^8st - 2772rs^2t^2 - 2772r^2st^2 - 2772r^2s^2t + 396r^6st^2 + 396r^6s^2t - 198r^7st^2 - 198r^7s^2t - 5280rst + 2310r^2s^2t^2 - 462r^5s^2t^2 + 198r^6s^2t^2 + 308) + \frac{hf_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(18018r + 18018s + 18018t - 71016r^2s^2 + 25608r^2s^3 + 25608r^3s^2 - 9240r^3s^3 + 891r^7s^2 - 693r^7s^3 - 363r^8s^2 + 825r^8s^3 - 616r^9s^2 - 220r^9s^3 + 264r^{10}s^2 - 71016r^2t^2 + 25608r^2t^3 + 25608r^3t^2 - 9240r^3t^3 + 891r^7t^2 - 693r^7t^3 - 363r^8t^2 + 825r^8t^3 - 616r^9t^2 - 220r^9t^3 + 264r^{10}t^2 - 71016s^2t^2 + 25608s^2t^3 + 25608s^3t^2 - 9240s^3t^3 - 59092rs - 59092rt - 59092st + 64460rs^2 + 64460r^2s - 23100rs^3 - 23100r^3s - 990r^8s + 1067r^9s - 105r^{10}s - 84r^{11}s + 64460rt^2 + 64460r^2t - 23100rt^3 - 23100r^3t - 990r^8t + 1067r^9t - 105r^{10}t - 84r^{11}t + 64460st^2 + 64460s^2t - 23100st^3 - 23100s^3t - 19470r^2 + 6930r^3 + 297r^9 - 399r^{10} + 126r^{11} - 19470s^2 + 6930s^3 - 19470t^2 + 6930t^3 - 216634rst^2 - 216634rs^2t - 216634r^2st + 78342rst^3 + 78342rs^3t + 78342r^3st + 3564r^7st - 2992r^8st - 407r^9st + 345r^{10}st + 42r^{11}st + 242748rs^2t^2 + 242748r^2st^2 + 242748r^2s^2t - 88572rs^2t^3 - 88572rs^3t^2 - 88572r^2st^3 - 88572r^2s^3t - 88572r^3st^2 - 88572r^3s^2t + 32340rs^3t^3 + 32340r^3st^3 + 32340r^3s^3t - 3564r^6st^2 - 3564r^6s^2t + 2772r^6st^3 + 2772r^6s^3t + 891r^7st^2 + 891r^7s^2t - 2673r^7st^3 - 2673r^7s^3t + 2387r^8st^2 + 2387r^8s^2t + 275r^8st^3 + 275r^8s^3t - 506r^9st^2 - 506r^9s^2t + 110r^9st^3 + 110r^9s^3t - 132r^{10}st^2 - 132r^{10}s^2t + 196086rst - 277596r^2s^2t^2 + 102564r^2s^2t^3 + 102564r^2s^3t^2 + 102564r^3s^2t^2 - 37884r^2s^3t^3 - 37884r^3s^2t^3 - 37884r^3s^3t^2 + 13860r^3s^3t^3 + 4158r^5s^2t^2 - 3234r^5s^2t^3 - 3234r^5s^3t^2 + 726r^6s^2t^2 + 2310r^5s^3t^3 + 1518r^6s^2t^3 + 1518r^6s^3t^2 - 3168r^7s^2t^2 - 2178r^6s^3t^3 + 1188r^7s^2t^3 + 1188r^7s^3t^2 - 484r^8s^2t^2 + 396r^7s^3t^3 - 396$$

$r^8s^2t^3 - 396r^8s^3t^2 + 440r^9s^2t^2 - 5544) - \frac{h^2 g_n}{27720r^2s^2t^2}(220r + 220s + 220t - 462r^2s^2 + 198r^6s^2 - 198r^7s^2 + 55r^8s^2 - 462r^2t^2 + 198r^6t^2 - 198r^7t^2 + 55r^8t^2 - 462s^2t^2 - 660rs - 660rt - 660st + 528rs^2 + 528r^2s - 198r^7s + 220r^8s - 66r^9s + 528rt^2 + 528r^2t - 198r^7t + 220r^8t - 66r^9t + 528st^2 + 528s^2t - 165r^2 + 55r^8 - 66r^9 + 21r^{10} - 165s^2 - 165t^2 - 1848rst^2 - 1848rs^2t - 1848r^2st + 792r^6st - 792r^7st + 220r^8st + 1848rs^2t^2 + 1848r^2st^2 + 1848r^2s^2t - 924r^5st^2 - 924r^5s^2t + 792r^6st^2 + 792r^6s^2t - 198r^7st^2 - 198r^7s^2t + 2112rst - 2310r^2s^2t^2 + 1386r^4s^2t^2 - 924r^5s^2t^2 + 198r^6s^2t^2 - 77) - \frac{hf_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(1980r^2s^2 - 2310r^2s^3 + 858r^3s^2 + 1848r^3s^3 - 2376r^4s^2 + 1980r^7s^3 - 5148r^8s^2 - 5148r^8s^3 + 13376r^9s^2 + 4235r^9s^3 - 11121r^{10}s^2 - 1155r^{10}s^3 + 3069r^{11}s^2 + 1980r^2t^2 - 2310r^2t^3 + 858r^3t^2 + 1848r^3t^3 - 2376r^4t^2 + 1980r^7t^3 - 5148r^8t^2 - 5148r^8t^3 + 13376r^9t^2 + 4235r^9t^3 - 11121r^{10}t^2 - 1155r^{10}t^3 + 3069r^{11}t^2 - 1320s^2t^2 + 1056s^2t^3 + 1056s^3t^2 - 924s^3t^3 + 308rs + 308rt - 154st - 880rs^2 + 308r^2s + 660rs^3 - 3300r^3s + 2970r^4s + 4235r^9s - 11121r^{10}s + 9420r^{11}s - 2646r^12s - 880rt^2 + 308r^2t + 660rt^3 - 3300r^3t + 2970r^4t + 4235r^9t - 11121r^{10}t + 9420r^{11}t - 2646r^12t + 440st^2 + 440s^2t - 330st^3 - 330s^3t - 462r^2 + 1386r^3 - 990r^4 - 1155r^{10} + 3069r^{11} - 2646r^12 + 756r^13 + 1430rst^2 + 1430rs^2t + 1430r^2st - 660rst^3 - 660rs^3t + 8052r^3st - 9504r^4st - 16027r^8st + 41437r^9st - 34278r^{10}st + 9420r^{11}st + 1584rs^2t^2 - 6732r^2st^2 - 6732r^2s^2t - 3036rs^2t^3 - 3036rs^3t^2 + 6468r^2st^3 + 6468r^2s^3t - 1452r^3st^2 - 1452r^3s^2t + 4620rs^3t^3 - 6468r^3st^3 - 6468r^3s^3t + 8316r^4st^2 + 8316r^4s^2t - 7920r^6st^3 - 7920r^6s^3t + 20196r^7st^2 + 20196r^7s^2t + 20196r^7st^3 + 20196r^7s^3t - 51436r^8st^2 - 51436r^8s^2t - 16027r^8st^3 - 16027r^8s^3t + 41437r^9st^2 + 41437r^9s^2t + 4235r^9st^3 + 4235r^9s^3t - 11121r^{10}st^2 - 11121r^{10}s^2t - 1122rst + 7524r^2s^2t^2 - 2772r^2s^2t^3 - 2772r^2s^3t^2 - 2772r^3s^2t^2 - 4620r^2s^3t^3 + 6468r^3s^2t^3 + 6468r^3s^3t^2 - 8316r^4s^2t^2 - 4158r^4s^3t^3 + 10626r^5s^2t^3 + 10626r^5s^3t^2 - 26598r^6s^2t^2 + 10626r^5s^3t^3 - 26598r^6s^2t^3 - 26598r^6s^3t^2 + 66330r^7s^2t^2 - 7920r^6s^3t^3 + 20196r^7s^2t^3 + 20196r^7s^3t^2 - 51436r^8s^2t^2 + 1980r^7s^3t^3 - 5148r^8s^2t^3 - 5148r^8s^3t^2 + 13376r^9s^2t^2 + \frac{hf_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(1980r^2s^2 + 858r^2s^3 - 2310r^3s^2 - 2376r^2s^4 + 1848r^3s^3 - 693r^7s^3 + 825r^8s^2 + 891r^7s^4 - 363r^8s^3 - 616r^9s^2 - 990r^8s^4 + 1067r^9s^3 - 105r^{10}s^2 + 297r^9s^4 - 399r^{10}s^3 + 126r^{11}s^2 - 1320r^2t^2 + 1056r^2t^3 +$

$1056r^3t^2 - 924r^3t^3 + 396r^7t^3 - 396r^8t^2 - 396r^8t^3 + 440r^9t^2 + 110r^9t^3 - 132r^{10}t^2 +$

$1980s^2t^2 - 2310s^2t^3 + 858s^3t^2 + 1848s^3t^3 - 2376s^4t^2 + 308rs - 154rt + 308st +$

$308rs^2 - 880r^2s - 3300rs^3 + 660r^3s + 2970rs^4 - 220r^9s + 264r^{10}s - 84r^{11}s +$

$440rt^2 + 440r^2t - 330rt^3 - 330r^3t + 110r^9t - 132r^{10}t + 42r^{11}t - 880st^2 + 308s^2t +$

$660st^3 - 3300s^3t + 2970s^4t - 462s^2 + 1386s^3 - 990s^4 + 1430rst^2 + 1430rs^2t +$

$1430r^2st - 660rst^3 + 8052rs^3t - 660r^3st - 9504rs^4t + 275r^8st - 506r^9st + 345r^{10}st -$

$84r^{11}st - 6732rs^2t^2 + 1584r^2st^2 - 6732r^2s^2t + 6468rs^2t^3 - 1452rs^3t^2 - 3036r^2st^3 -$

$1452r^2s^3t - 3036r^3st^2 + 6468r^3s^2t - 6468rs^3t^3 + 8316rs^4t^2 + 8316r^2s^4t +$

$4620r^3st^3 - 6468r^3s^3t - 2178r^6st^3 + 2772r^6s^3t + 1188r^7st^2 - 2673r^7s^2t -$

$3564r^6s^4t + 1188r^7st^3 + 891r^7s^3t - 484r^8st^2 + 2387r^8s^2t + 3564r^7s^4t + 275r^8st^3 -$

$2992r^8s^3t - 506r^9st^2 - 407r^9s^2t - 990r^8s^4t - 220r^9st^3 + 1067r^9s^3t + 264r^{10}st^2 -$

$105r^{10}s^2t - 1122rst + 7524r^2s^2t^2 - 2772r^2s^2t^3 - 2772r^2s^3t^2 - 2772r^3s^2t^2 +$

$6468r^2s^3t^3 - 8316r^2s^4t^2 - 4620r^3s^2t^3 + 6468r^3s^3t^2 + 2310r^5s^2t^3 - 3234r^5s^3t^2 +$

$1518r^6s^2t^2 - 3234r^5s^3t^3 + 4158r^5s^4t^2 + 1518r^6s^2t^3 + 726r^6s^3t^2 - 3168r^7s^2t^2 +$

$2772r^6s^3t^3 - 3564r^6s^4t^2 - 2673r^7s^2t^3 + 891r^7s^3t^2 + 2387r^8s^2t^2 - 693r^7s^3t^3 +$

$891r^7s^4t^2 + 825r^8s^2t^3 - 363r^8s^3t^2 - 616r^9s^2t^2) + \frac{hf_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(1320r^2s^2 -$

$1056r^2s^3 - 1056r^3s^2 + 924r^3s^3 - 396r^7s^3 + 396r^8s^2 + 396r^8s^3 - 440r^9s^2 - 110r^9s^3 +$

$132r^{10}s^2 - 1980r^2t^2 - 858r^2t^3 + 2310r^3t^2 + 2376r^2t^4 - 1848r^3t^3 + 693r^7t^3 -$

$825r^8t^2 - 891r^7t^4 + 363r^8t^3 + 616r^9t^2 + 990r^8t^4 - 1067r^9t^3 + 105r^{10}t^2 - 297r^9t^4 +$

$399r^{10}t^3 - 126r^{11}t^2 - 1980s^2t^2 - 858s^2t^3 + 2310s^3t^2 + 2376s^2t^4 - 1848s^3t^3 +$

$154rs - 308rt - 308st - 440rs^2 - 440r^2s + 330rs^3 + 330r^3s - 110r^9s + 132r^{10}s -$

$42r^{11}s - 308rt^2 + 880r^2t + 3300rt^3 - 660r^3t - 2970rt^4 + 220r^9t - 264r^{10}t +$

$84r^{11}t - 308st^2 + 880s^2t + 3300st^3 - 660s^3t - 2970st^4 + 462t^2 - 1386t^3 + 990t^4 -$

$1430rst^2 - 1430rs^2t - 1430r^2st - 8052rst^3 + 660rs^3t + 660r^3st + 9504rst^4 -$

$275r^8st + 506r^9st - 345r^{10}st + 84r^{11}st + 6732rs^2t^2 + 6732r^2st^2 - 1584r^2s^2t +$

$1452rs^2t^3 - 6468rs^3t^2 + 1452r^2st^3 + 3036r^2s^3t - 6468r^3st^2 + 3036r^3s^2t -$

$8316rs^2t^4 + 6468rs^3t^3 - 8316r^2st^4 + 6468r^3st^3 - 4620r^3s^3t - 2772r^6st^3 +$

$2178r^6s^3t + 2673r^7st^2 - 1188r^7s^2t + 3564r^6st^4 - 891r^7st^3 - 1188r^7s^3t - 2387r^8st^2 +$

$484r^8s^2t - 3564r^7st^4 + 2992r^8st^3 - 275r^8s^3t + 407r^9st^2 + 506r^9s^2t + 990r^8st^4 -$

$1067r^9st^3+220r^9s^3t+105r^{10}st^2-264r^{10}s^2t+1122rst-7524r^2s^2t^2+2772r^2s^2t^3+$
$2772r^2s^3t^2+2772r^3s^2t^2+8316r^2s^2t^4-6468r^2s^3t^3-6468r^3s^2t^3+4620r^3s^3t^2+$
$3234r^5s^2t^3-2310r^5s^3t^2-1518r^6s^2t^2-4158r^5s^2t^4+3234r^5s^3t^3-726r^6s^2t^3-$
$1518r^6s^3t^2+3168r^7s^2t^2+3564r^6s^2t^4-2772r^6s^3t^3-891r^7s^2t^3+2673r^7s^3t^2-$
$2387r^8s^2t^2-891r^7s^2t^4+693r^7s^3t^3+363r^8s^2t^3-825r^8s^3t^2+616r^9s^2t^2)+$
$\frac{h^2g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(264r^6s^2-220s-220t-110r-330r^7s^2+110r^8s^2+$
$264r^6t^2-330r^7t^2+110r^8t^2+462s^2t^2+330rs+330rt+660st-264rs^2-330r^7s+$
$440r^8s-154r^9s-264rt^2-330r^7t+440r^8t-154r^9t-528st^2-528s^2t+110r^8-$
$154r^9+56r^{10}+165s^2+165t^2+924rst^2+924rs^2t+1056r^6st-1320r^7st+440r^8st-$
$924rs^2t^2-924r^5st^2-924r^5s^2t+1056r^6st^2+1056r^6s^2t-330r^7st^2-330r^7s^2t-$
$1056rst+924r^4s^2t^2-924r^5s^2t^2+264r^6s^2t^2+77)-\frac{h^2g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(220r+$
$220s+110t-462r^2s^2+198r^6s^2-198r^7s^2+55r^8s^2-660rs-330rt-330st+$
$528rs^2+528r^2s-198r^7s+220r^8s-66r^9s+264r^2t-99r^7t+110r^8t-33r^9t+$
$264s^2t-165r^2+55r^8-66r^9+21r^{10}-165s^2-924rs^2t-924r^2st+396r^6st-$
$396r^7st+110r^8st+924r^2s^2t-462r^5s^2t+396r^6s^2t-99r^7s^2t+1056rst-77)-$
$\frac{h^2g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(220r+110s+220t-462r^2t^2+198r^6t^2-198r^7t^2+55r^8t^2-$
$330rs-660rt-330st+264r^2s-99r^7s+110r^8s-33r^9s+528rt^2+528r^2t-$
$198r^7t+220r^8t-66r^9t+264st^2-165r^2+55r^8-66r^9+21r^{10}-165t^2-924rst^2-$
$924r^2st+396r^6st-396r^7st+110r^8st+924r^2st^2-462r^5st^2+396r^6st^2-99r^7st^2+$
$1056rst-77).$ (3.61)

Combining Equations (3.53) - (3.55) and (3.57) gives a block in the form

$$\begin{aligned} H^{[2]_3}Y^{[2]_3}_{n+1} &= M^{[2]_3}_1Y^{[2]_3}_n+M^{[2]_3}_2Y^{[2]_3}_{n-1}+E^{[2]_3}_1F^{[2]_3}_n+E^{[2]_3}_2F^{[2]_3}_{n+1}+K^{[2]_3}_1G^{[2]_3}_n \\ &\quad +K^{[2]_3}_2G^{[2]_3}_{n+1} \end{aligned} \tag{3.62}$$

where

$$H^{[2]_3}=\begin{pmatrix} -\frac{s}{r} & 1 & 0 & 0 \\ -\frac{t}{r} & 0 & 1 & 0 \\ -\frac{1}{r} & 0 & 0 & 1 \\ -\frac{1}{hr} & 0 & 0 & 0 \end{pmatrix},\ Y^{[2]_3}_{n+1}=\begin{pmatrix} y_{n+r} \\ y_{n+s} \\ y_{n+t} \\ y_{n+1} \end{pmatrix},\ M^{[2]_3}_1=\begin{pmatrix} 0 & 0 & 0 & \frac{r-s}{r} \\ 0 & 0 & 0 & \frac{r-t}{r} \\ 0 & 0 & 0 & \frac{r-1}{r} \\ 0 & 0 & 0 & \frac{-1}{hr} \end{pmatrix},$$

$$Y_n^{[2]_3} = \begin{pmatrix} y_{n-t} \\ y_{n-s} \\ y_{n-r} \\ y_n \end{pmatrix}, M_2^{[2]_3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, Y_{n-1}^{[2]_3} = \begin{pmatrix} y'_{n-t} \\ y'_{n-s} \\ y'_{n-r} \\ y'_n \end{pmatrix},$$

$$F_n^{[2]_3} = \begin{pmatrix} f_{n-t} \\ f_{n-s} \\ f_{n-r} \\ f_n \end{pmatrix}, \quad F_{n+1}^{[2]_3} = \begin{pmatrix} f_{n+r} \\ f_{n+s} \\ f_{n+t} \\ f_{n+1} \end{pmatrix}, \quad G_n^{[2]_3} = \begin{pmatrix} g_{n-t} \\ g_{n-s} \\ g_{n-r} \\ g_n \end{pmatrix},$$

$$G_{n+1}^{[2]_3} = \begin{pmatrix} g_{n+r} \\ g_{n+s} \\ g_{n+t} \\ g_{n+1} \end{pmatrix}.$$

Multiplying both sides of Equation (3.62) by the inverse of $H^{[2]_3}$ yields

$$\begin{aligned} I_4 Y_{n+1}^{[2]_3} &= \hat{M}_1^{[2]_3} Y_n^{[2]_3} + h\hat{M}_2^{[2]_3} Y_{n-1}^{[2]_3} + h^2[\hat{E}_1^{[2]_3} F_n^{[2]_3} + \hat{E}_2^{[2]_3} F_{n+1}^{[2]_3}] + h^3 \\ &\quad [\hat{K}_1^{[2]_3} G_n^{[2]_3} + \hat{K}_2^{[2]_3} G_{n+1}^{[2]_3}] \end{aligned} \tag{3.63}$$

where

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{M}_1^{[2]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{M}_2^{[2]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\hat{E}_1^{[2]3} = \begin{pmatrix} 0 & 0 & 0 & \hat{E}_{1_{14}}^{[2]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{24}}^{[2]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{34}}^{[2]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{44}}^{[2]_3} \end{pmatrix}, \hat{E}_2^{[2]_3} = \begin{pmatrix} \hat{E}_{2_{11}}^{[2]_2} & \hat{E}_{2_{12}}^{[2]_2} & \hat{E}_{2_{13}}^{[2]_2} & \hat{E}_{2_{14}}^{[2]_2} \\ \hat{E}_{2_{21}}^{[2]_2} & \hat{E}_{2_{22}}^{[2]_2} & \hat{E}_{2_{23}}^{[2]_2} & \hat{E}_{2_{24}}^{[2]_2} \\ \hat{E}_{2_{31}}^{[2]_2} & \hat{E}_{2_{32}}^{[2]_2} & \hat{E}_{2_{33}}^{[2]_2} & \hat{E}_{2_{34}}^{[2]_2} \\ \hat{E}_{2_{41}}^{[2]_2} & \hat{E}_{2_{42}}^{[2]_2} & \hat{E}_{2_{43}}^{[2]_2} & \hat{E}_{2_{44}}^{[2]_2} \end{pmatrix},$$

$$\hat{K}_1^{[2]_3} = \begin{pmatrix} 0 & 0 & 0 & \hat{K}_{1_{14}}^{[2]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{24}}^{[2]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{34}}^{[2]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{44}}^{[2]_3} \end{pmatrix}, \ \hat{K}_2^{[2]_3} = \begin{pmatrix} \hat{K}_{2_{11}}^{[2]_2} & \hat{K}_{2_{12}}^{[2]_2} & \hat{K}_{2_{13}}^{[2]_2} & \hat{K}_{2_{14}}^{[2]_2} \\ \hat{K}_{2_{21}}^{[2]_2} & \hat{K}_{2_{22}}^{[2]_2} & \hat{K}_{2_{23}}^{[2]_2} & \hat{K}_{2_{24}}^{[2]_2} \\ \hat{K}_{2_{31}}^{[2]_2} & \hat{K}_{2_{32}}^{[2]_2} & \hat{K}_{2_{33}}^{[2]_2} & \hat{K}_{2_{34}}^{[2]_2} \\ \hat{K}_{2_{41}}^{[2]_2} & \hat{K}_{2_{42}}^{[2]_2} & \hat{K}_{2_{43}}^{[2]_2} & \hat{K}_{2_{44}}^{[2]_2} \end{pmatrix}.$$

with entries of the matrices $\hat{E}_1^{[2]_3}, \hat{E}_2^{[2]_3}, \hat{K}_1^{[2]_3}$ and $\hat{K}_2^{[2]_3}$ are as follows

$\hat{E}_{1_{14}}^{[2]_3} = \frac{-r^2}{27720s^3t^3}(-42r^7st - 42r^7s - 42r^7t + 132r^6s^2t + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 - 1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 - 9702s^3t^3),$

$\hat{E}_{1_{24}}^{[2]_3} = \frac{-s^2}{27720r^3t^3}(-110r^3s^5t - 110r^3s^5 + 396r^3s^4t^2 + 583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 + 264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t + 132r^2s^6 - 440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 - 1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t - 110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t + 132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3),$

$\hat{E}_{1_{34}}^{[2]_3} = \frac{-t^2}{27720r^3s^3}(-396r^3s^3t^3 + 990r^3s^3t^2 + 1848r^3s^3t - 9702r^3s^3 + 396r^3s^2t^4 - 1188r^3s^2t^3 + 264r^3s^2t^2 + 1848r^3s^2t - 110r^3st^5 + 583r^3st^4 - 1188r^3st^3 + 990r^3st^2 - 110r^3t^5 + 396r^3t^4 - 396r^3t^3 + 396r^2s^3t^4 - 1188r^2s^3t^3 + 264r^2s^3t^2 + 1848r^2s^3t - 440r^2s^2t^5 + 1540r^2s^2t^4 - 1584r^2s^2t^3 + 264r^2s^2t^2 + 132r^2st^6 - 748r^2st^5 + 1540r^2st^4 - 1188r^2st^3 + 132r^2t^6 - 440r^2t^5 + 396r^2t^4 - 110rs^3t^5 + 583rs^3t^4 - 1188$

$rs^3t^3+990rs^3t^2+132rs^2t^6-748rs^2t^5+1540rs^2t^4-1188rs^2t^3-42rst^7+321rst^6-748rst^5+583rst^4-42rt^7+132rt^6-110rt^5-110s^3t^5+396s^3t^4-396s^3t^3+132s^2t^6-440s^2t^5+396s^2t^4-42st^7+132st^6-110st^5),$

$\hat{E}^{[2]_3}_{1_{44}} = \frac{-1}{27720r^3s^3t^3}(-9702r^3s^3t^3+1848r^3s^3t^2+990r^3s^3t-396r^3s^3+1848r^3s^2t^3+264r^3s^2t^2-1188r^3s^2t+396r^3s^2+990r^3st^3-1188r^3st^2+583r^3st-110r^3s-396r^3t^3+396r^3t^2-110r^3t+1848r^2s^3t^3+264r^2s^3t^2-1188r^2s^3t+396r^2s^3+264r^2s^2t^3-1584r^2s^2t^2+1540r^2s^2t-440r^2s^2-1188r^2st^3+1540r^2st^2-748r^2st+132r^2s+396r^2t^3-440r^2t^2+132r^2t+990rs^3t^3-1188rs^3t^2+583rs^3t-110rs^3-1188rs^2t^3+1540rs^2t^2-748rs^2t+132rs^2+583rst^3-748rst^2+321rst-42rs-110rt^3+132rt^2-42rt-396s^3t^3+396s^3t^2-110s^3t+396s^2t^3-440s^2t^2+132s^2t-110st^3+132st^2-42st),$

$\hat{E}^{[2]_3}_{2_{11}} = \frac{r^2}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^9-2646r^8s-2646r^8t-2646r^8+3069r^7s^2+9420r^7st+9420r^7s+3069r^7t^2+9420r^7t+3069r^7-1155r^6s^3-11121r^6s^2t-11121r^6s^2-11121r^6st^2-34278r^6st-11121r^6s-1155r^6t^3-11121r^6t^2-11121r^6t-1155r^6+4235r^5s^3t+4235r^5s^3+13376r^5s^2t^2+41437r^5s^2t+13376r^5s^2+4235r^5st^3+41437r^5st^2+41437r^5st+4235r^5s+4235r^5t^3+13376r^5t^2+4235r^5t-5148r^4s^3t^2-16027r^4s^3t-5148r^4s^3-5148r^4s^2t^3-51436r^4s^2t^2-51436r^4s^2t-5148r^4s^2-16027r^4st^3-51436r^4st^2-16027r^4st-5148r^4t^3-5148r^4t^2+1980r^3s^3t^3+20196r^3s^3t^2+20196r^3s^3t+1980r^3s^3+20196r^3s^2t^3+66330r^3s^2t^2+20196r^3s^2t+20196r^3st^3+20196r^3st^2+1980r^3t^3-7920r^2s^3t^3-26598r^2s^3t^2-7920r^2s^3t-26598r^2s^2t^3-26598r^2s^2t^2-7920r^2st^3+10626rs^3t^3+10626rs^3t^2+10626rs^2t^3-4158s^3t^3),$

$\hat{E}^{[2]_3}_{2_{21}} = \frac{s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(297r^4s^4-990r^4s^3t-990r^4s^3+891r^4s^2t^2+3564r^4s^2t+891r^4s^2-3564r^4st^2-3564r^4st+4158r^4t^2-399r^3s^5+1067r^3s^4t+1067r^3s^4-363r^3s^3t^2-2992r^3s^3t-363r^3s^3-693r^3s^2t^3+891r^3s^2t^2+891r^3s^2t-693r^3s^2+2772r^3st^3+726r^3st^2+2772r^3st-3234r^3t^3-3234r^3t^2+126r^2s^6-105r^2s^5t-105r^2s^5-616r^2s^4t^2-407r^2s^4t-616r^2s^4+825r^2s^3t^3+2387r^2s^3t^2+$

$2387r^2s^3t + 825r^2s^3 - 2673r^2s^2t^3 - 3168r^2s^2t^2 - 2673r^2s^2t + 1518r^2st^3 + 1518r^2st^2 + 2310r^2t^3 - 84rs^6t - 84rs^6 + 264rs^5t^2 + 345rs^5t + 264rs^5 - 220rs^4t^3 - 506rs^4t^2 - 506rs^4t - 220rs^4 + 275rs^3t^3 - 484rs^3t^2 + 275rs^3t + 1188rs^2t^3 + 1188rs^2t^2 - 2178rst^3 + 42s^6t - 132s^5t^2 - 132s^5t + 110s^4t^3 + 440s^4t^2 + 110s^4t - 396s^3t^3 - 396s^3t^2 + 396s^2t^3)$,

$\hat{E}_{2_{31}}^{[2]_3} = \frac{t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2 - 3564r^4s^2t + 4158r^4s^2 - 990r^4st^3 + 3564r^4st^2 - 3564r^4st + 297r^4t^4 - 990r^4t^3 + 891r^4t^2 - 693r^3s^3t^2 + 2772r^3s^3t - 3234r^3s^3 - 363r^3s^2t^3 + 891r^3s^2t^2 + 726r^3s^2t - 3234r^3s^2 + 1067r^3st^4 - 2992r^3st^3 + 891r^3st^2 + 2772r^3st - 399r^3t^5 + 1067r^3t^4 - 363r^3t^3 - 693r^3t^2 + 825r^2s^3t^3 - 2673r^2s^3t^2 + 1518r^2s^3t + 2310r^2s^3 - 616r^2s^2t^4 + 2387r^2s^2t^3 - 3168r^2s^2t^2 + 1518r^2s^2t - 105r^2st^5 - 407r^2st^4 + 2387r^2st^3 - 2673r^2st^2 + 126r^2t^6 - 105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t + 264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 - 84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 - 396s^2t^3 + 42st^6 - 132st^5 + 110st^4)$,

$\hat{E}_{2_{41}}^{[2]_3} = \frac{-1}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-4158r^4s^2t^2 + 3564r^4s^2t - 891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 + 3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - 891r^3st^2 + 2992r^3st - 1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - 1518r^2s^3t^2 + 2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 + 2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t - 126r^2 + 2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + 506rs^2t - 264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st)$,

$\hat{E}_{2_{12}}^{[2]_3} = \frac{r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-126r^6s^2 + 84r^6st + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 - 345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 + 616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 +$

$506r^4st+220r^4s-110r^4t^3-440r^4t^2-110r^4t+990r^3s^4t+990r^3s^4+363r^3s^3t^2+ 2992r^3s^3t+363r^3s^3-825r^3s^2t^3-2387r^3s^2t^2-2387r^3s^2t-825r^3s^2-275r^3st^3+ 484r^3st^2-275r^3st+396r^3t^3+396r^3t^2-891r^2s^4t^2-3564r^2s^4t-891r^2s^4+ 693r^2s^3t^3-891r^2s^3t^2-891r^2s^3t+693r^2s^3+2673r^2s^2t^3+3168r^2s^2t^2+2673r^2s^2t- 1188r^2st^3-1188r^2st^2-396r^2t^3+3564rs^4t^2+3564rs^4t-2772rs^3t^3-726rs^3t^2- 2772rs^3t-1518rs^2t^3-1518rs^2t^2+2178rst^3-4158s^4t^2+3234s^3t^3+3234s^3t^2- 2310s^2t^3)$,

$\hat{E}^{[2]_3}_{2_{22}} = \frac{s^2}{27720(r-s)^3(s-t)^3(s-1)^3}(1155r^3s^6-4235r^3s^5t-4235r^3s^5+5148r^3s^4t^2+ 16027r^3s^4t+5148r^3s^4-1980r^3s^3t^3-20196r^3s^3t^2-20196r^3s^3t-1980r^3s^3+ 7920r^3s^2t^3+26598r^3s^2t^2+7920r^3s^2t-10626r^3st^3-10626r^3st^2+4158r^3t^3- 3069r^2s^7+11121r^2s^6t+11121r^2s^6-13376r^2s^5t^2-41437r^2s^5t-13376r^2s^5+ 5148r^2s^4t^3+51436r^2s^4t^2+51436r^2s^4t+5148r^2s^4-20196r^2s^3t^3-66330r^2s^3t^2- 20196r^2s^3t+26598r^2s^2t^3+26598r^2s^2t^2-10626r^2st^3+2646rs^8-9420rs^7t- 9420rs^7+11121rs^6t^2+34278rs^6t+11121rs^6-4235rs^5t^3-41437rs^5t^2- 41437rs^5t-4235rs^5+16027rs^4t^3+51436rs^4t^2+16027rs^4t-20196rs^3t^3- 20196rs^3t^2+7920rs^2t^3-756s^9+2646s^8t+2646s^8-3069s^7t^2-9420s^7t- 3069s^7+1155s^6t^3+11121s^6t^2+11121s^6t+1155s^6-4235s^5t^3-13376s^5t^2- 4235s^5t+5148s^4t^3+5148s^4t^2-1980s^3t^3)$,

$\hat{E}^{[2]_3}_{2_{32}} = \frac{t^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^3s^3t^2-2772r^3s^3t+3234r^3s^3-825r^3s^2t^3+ 2673r^3s^2t^2-1518r^3s^2t-2310r^3s^2+220r^3st^4-275r^3st^3-1188r^3st^2+2178r^3st- 110r^3t^4+396r^3t^3-396r^3t^2-891r^2s^4t^2+3564r^2s^4t-4158r^2s^4+363r^2s^3t^3- 891r^2s^3t^2-726r^2s^3t+3234r^2s^3+616r^2s^2t^4-2387r^2s^2t^3+3168r^2s^2t^2-$

$1518r^2s^2t-264r^2st^5+506r^2st^4+484r^2st^3-1188r^2st^2+132r^2t^5-440r^2t^4+ 396r^2t^3+990rs^4t^3-3564rs^4t^2+3564rs^4t-1067rs^3t^4+2992rs^3t^3-891rs^3t^2- 2772rs^3t+105rs^2t^5+407rs^2t^4-2387rs^2t^3+2673rs^2t^2+84rst^6-345rst^5+ 506rst^4-275rst^3-42rt^6+132rt^5-110rt^4-297s^4t^4+990s^4t^3-891s^4t^2+ 399s^3t^5-1067s^3t^4+363s^3t^3+693s^3t^2-126s^2t^6+105s^2t^5+616s^2t^4-825s^2t^3+ 84st^6-264st^5+220st^4)$,

$$\hat{E}^{[2]_3}_{2_{42}} = \frac{1}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - 726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st),$$

$$\hat{E}^{[2]_3}_{2_{13}} = \frac{r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-84r^6st + 42r^6s + 126r^6t^2 - 84r^6t + 264r^5s^2t - 132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t + 110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st + 110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 - 363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 + 275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t + 396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 + 891r^2st^3 - 2673r^2st^2 + 891r^2t^4 - 693r^2t^3 + 2772rs^3t^3 + 1518rs^3t^2 - 2178rs^3t - 3564rs^2t^4 + 726rs^2t^3 + 1518rs^2t^2 - 3564rst^4 + 2772rst^3 - 3234s^3t^3 + 2310s^3t^2 + 4158s^2t^4 - 3234s^2t^3),$$

$$\hat{E}^{[2]_3}_{2_{23}} = \frac{-s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t - 110r^3s^4 - 825r^3s^3t^2 - 275r^3s^3t + 396r^3s^3 + 693r^3s^2t^3 + 2673r^3s^2t^2 - 1188r^3s^2t - 396r^3s^2 - 2772r^3st^3 - 1518r^3st^2 + 2178r^3st + 3234r^3t^3 - 2310r^3t^2 - 264r^2s^5t + 132r^2s^5 + 616r^2s^4t^2 + 506r^2s^4t - 440r^2s^4 + 363r^2s^3t^3 - 2387r^2s^3t^2 + 484r^2s^3t + 396r^2s^3 - 891r^2s^2t^4 - 891r^2s^2t^3 + 3168r^2s^2t^2 - 1188r^2s^2t + 3564r^2st^4 - 726r^2st^3 - 1518r^2st^2 - 4158r^2t^4 + 3234r^2t^3 + 84rs^6t - 42rs^6 + 105rs^5t^2 - 345rs^5t + 132rs^5 - 1067rs^4t^3 + 407rs^4t^2 + 506rs^4t - 110rs^4 + 990rs^3t^4 + 2992rs^3t^3 - 2387rs^3t^2 - 275rs^3t - 3564rs^2t^4 - 891rs^2t^3 + 2673rs^2t^2 + 3564rst^4 - 2772rst^3 - 126s^6t^2 + 84s^6t + 399s^5t^3 + 105s^5t^2 - 264s^5t - 297s^4t^4 - 1067s^4t^3 + 616s^4t^2 + 220s^4t + 990s^3t^4 + 363s^3t^3 - 825s^3t^2 - 891s^2t^4$$

$+693s^2t^3)$,

$$\hat{E}^{[2]_3}_{2_{33}} = \frac{-t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(-1980r^3s^3t^3+7920r^3s^3t^2-10626r^3s^3t+4158r^3s^3+5148r^3s^2t^4-20196r^3s^2t^3+26598r^3s^2t^2-10626r^3s^2t-4235r^3st^5+16027r^3st^4-20196r^3st^3+7920r^3st^2+1155r^3t^6-4235r^3t^5+5148r^3t^4-1980r^3t^3+5148r^2s^3t^4-20196r^2s^3t^3+26598r^2s^3t^2-10626r^2s^3t-13376r^2s^2t^5+51436r^2s^2t^4-66330r^2s^2t^3+26598r^2s^2t^2+11121r^2st^6-41437r^2st^5+51436r^2st^4-20196r^2st^3-3069r^2t^7+11121r^2t^6-13376r^2t^5+5148r^2t^4-4235rs^3t^5+16027rs^3t^4-20196rs^3t^3+7920rs^3t^2+11121rs^2t^6-41437rs^2t^5+51436rs^2t^4-20196rs^2t^3-9420rst^7+34278rst^6-41437rst^5+16027rst^4+2646rt^8-9420rt^7+11121rt^6-4235rt^5+1155s^3t^6-4235s^3t^5+5148s^3t^4-1980s^3t^3-3069s^2t^7+11121s^2t^6-13376s^2t^5+5148s^2t^4+2646st^8-9420st^7+11121st^6-4235st^5-756t^9+2646t^8-3069t^7+1155t^6),$$

$$\hat{E}^{[2]_3}_{2_{43}} = \frac{1}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2-2178r^3s^3t+396r^3s^3-3234r^3s^2t^3+1518r^3s^2t^2+1188r^3s^2t-396r^3s^2+2772r^3st^3-2673r^3st^2+275r^3st+110r^3s-693r^3t^3+825r^3t^2-220r^3t-3234r^2s^3t^3+1518r^2s^3t^2+1188r^2s^3t-396r^2s^3+4158r^2s^2t^4+726r^2s^2t^3-3168r^2s^2t^2-484r^2s^2t+440r^2s^2-3564r^2st^4+891r^2st^3+2387r^2st^2-506r^2st-132r^2s+891r^2t^4-363r^2t^3-616r^2t^2+264r^2t+2772rs^3t^3-2673rs^3t^2+275rs^3t+110rs^3-3564rs^2t^4+891rs^2t^3+2387rs^2t^2-506rs^2t-132rs^2+3564rst^4-2992rst^3-407rst^2+345rst+42rs-990rt^4+1067rt^3-105rt^2-84rt-693s^3t^3+825s^3t^2-220s^3t+891s^2t^4-363s^2t^3-616s^2t^2+264s^2t-990st^4+1067st^3-105st^2-84st+297t^4-399t^3+126t^2),$$

$$\hat{E}^{[2]_3}_{2_{14}} = \frac{-r^6}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^6st-84r^6s-84r^6t+126r^6-132r^5s^2t+264r^5s^2-132r^5st^2+345r^5st-105r^5s+264r^5t^2-105r^5t-399r^5+110r^4s^3t-220r^4s^3+440r^4s^2t^2-506r^4s^2t-616r^4s^2+110r^4st^3-506r^4st^2-407r^4st+1067r^4s-220r^4t^3-616r^4t^2+1067r^4t+297r^4-396r^3s^3t^2+275r^3s^3t+825r^3s^3-396r^3s^2t^3-484r^3s^2t^2+2387r^3s^2t-363r^3s^2+275r^3st^3+2387r^3st^2-2992r^3st-990r^3s+825r^3t^3-363r^3t^2-990r^3t+396r^2s^3t^3+1188r^2s^3t^2-2673r^2s^3t-693r^2s^3+1188$$

$r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 + 726rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2)$,

$\hat{E}^{[2]_3}_{2_{24}} = \frac{-s^6}{27720(r-1)^3(s-1)^3(t-1)^3}(110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + 440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 + 2387rs^3t^2 - 2992rs^3t - 990rs^3 - 2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - 616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2)$,

$\hat{E}^{[2]_3}_{2_{34}} = \frac{-t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 + 1188r^3s^2t^2 + 1518r^3s^2t - 3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st - 220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + 1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 + 440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - 506r^2st^4 + 2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 + 275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 - 3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4)$,

$\hat{E}^{[2]_3}_{2_{44}} = \frac{1}{27720(r-1)^3(s-1)^3(t-1)^3}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - 10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 -$

$$20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - 41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756),$$

$$\hat{K}^{[2]_3}_{1_{14}} = \frac{r^3}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2),$$

$$\hat{K}^{[2]_3}_{1_{24}} = \frac{s^3}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st + 1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 + 21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2),$$

$$\hat{K}^{[2]_3}_{1_{34}} = \frac{t^3}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4),$$

$$\hat{K}^{[2]_3}_{1_{44}} = \frac{1}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21).$$

$$\hat{K}^{[2]_3}_{2_{11}} = \frac{-r^3}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2),$$

$\hat{K}^{[2]_3}_{2_{21}} = \frac{-s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^3t^2 + 99rs^2 - 110rs^3 + 33rs^4 + 462rt^2 - 198st^2 + 198s^2t - 220s^3t + 66s^4t - 55s^3 + 66s^4 - 21s^5 - 396rst^2 + 396rs^2t - 110rs^3t + 99rs^2t^2 - 396rst)$,

$\hat{K}^{[2]_3}_{2_{31}} = \frac{-t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst)$,

$\hat{K}^{[2]_3}_{2_{41}} = \frac{-1}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21)$,

$\hat{K}^{[2]_3}_{2_{12}} = \frac{-r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 + 396r^2st - 110r^3st + 99r^2st^2 - 396rst)$,

$\hat{K}^{[2]_3}_{2_{22}} = \frac{-s^3}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2)$,

$\hat{K}^{[2]_3}_{2_{32}} = \frac{-t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + 66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + 99r^2st^2 - 396rst)$,

$\hat{K}^{[2]_3}_{2_{42}} = \frac{-1}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + 99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + 396rst - 21)$,

$\hat{K}^{[2]_3}_{2_{13}} = \frac{-r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst)$,

$\hat{K}^{[2]_3}_{2_{23}} = \frac{-s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^2s^3 + 198rs^2 - 198r^2s - 220rs^3 + 66rs^4 + 462r^2t + 99s^2t - 110s^3t + 33s^4t - 55s^3 + 66s^4 - 21s^5 + 396rs^2t - 396r^2st - 110rs^3t + 99r^2s^2t - 396rst)$,

$\hat{K}^{[2]_3}_{2_{33}} = \frac{-t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)$,

$\hat{K}^{[2]_3}_{2_{43}} = \frac{-1}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t + 396rst - 21)$,

$\hat{K}^{[2]_3}_{2_{14}} = \frac{-r^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-21r^5 + 66r^4s + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st - 110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 - 198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$,

$\hat{K}^{[2]_3}_{2_{24}} = \frac{-s^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-55r^2s^3 + 198r^2s^2t + 99r^2s^2 - 198r^2st^2 - 396r^2st + 462r^2t^2 + 66rs^4 - 220rs^3t - 110rs^3 + 198rs^2t^2 + 396rs^2t - 396rst^2 - 21s^5 + 66s^4t + 33s^4 - 55s^3t^2 - 110s^3t + 99s^2t^2)$,

$\hat{K}^{[2]_3}_{2_{34}} = \frac{-t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-198r^2s^2t + 462r^2s^2 + 198r^2st^2 - 396r^2st - 55r^2t^3 + 99r^2t^2 + 198rs^2t^2 - 396rs^2t - 220rst^3 + 396rst^2 + 66rt^4 - 110rt^3 - 55s^2t^3 + 99s^2t^2 + 66st^4 - 110st^3 - 21t^5 + 33t^4)$,

$\hat{K}^{[2]_3}_{2_{44}} = \frac{-1}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$.

From (3.63), the following equations are obtained

$$y_{n+r}-y_n-hry'_n=\frac{h^3r^3g_n}{27720s^2t^2}(21r^6-66r^5s-66r^5t-66r^5+55r^4s^2+220r^4st+$$
$$220r^4s+55r^4t^2+220r^4t+55r^4-198r^3s^2t-198r^3s^2-198r^3st^2-792r^3st-$$
$$198r^3s-198r^3t^2-198r^3t+198r^2s^2t^2+792r^2s^2t+198r^2s^2+792r^2st^2+792r^2st+$$
$$198r^2t^2-924rs^2t^2-924rs^2t-924rst^2+1386s^2t^2)-\frac{h^2r^2f_n}{27720s^3t^3}(132r^6s^2t-42r^7s-$$
$$42r^7t-42r^7st+132r^6s^2+132r^6st^2+321r^6st+132r^6s+132r^6t^2+132r^6t-$$
$$110r^5s^3t-110r^5s^3-440r^5s^2t^2-748r^5s^2t-440r^5s^2-110r^5st^3-748r^5st^2-$$
$$748r^5st-110r^5s-110r^5t^3-440r^5t^2-110r^5t+396r^4s^3t^2+583r^4s^3t+396r^4s^3+$$
$$396r^4s^2t^3+1540r^4s^2t^2+1540r^4s^2t+396r^4s^2+583r^4st^3+1540r^4st^2+583r^4st+$$
$$396r^4t^3+396r^4t^2-396r^3s^3t^3-1188r^3s^3t^2-1188r^3s^3t-396r^3s^3-1188r^3s^2t^3-$$
$$1584r^3s^2t^2-1188r^3s^2t-1188r^3st^3-1188r^3st^2-396r^3t^3+990r^2s^3t^3+264r^2s^3t^2+$$
$$990r^2s^3t+264r^2s^2t^3+264r^2s^2t^2+990r^2st^3+1848rs^3t^3+1848rs^3t^2+1848rs^2t^3-$$
$$9702s^3t^3)-\frac{h^3r^6g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(66r^4s-21r^5+66r^4t+33r^4-55r^3s^2-220r^3st-$$
$$110r^3s-55r^3t^2-110r^3t+198r^2s^2t+99r^2s^2+198r^2st^2+396r^2st+99r^2t^2-$$
$$198rs^2t^2-396rs^2t-396rst^2+462s^2t^2)-\frac{h^3r^3g_{n+r}}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6-77r^5s-$$
$$77r^5t-77r^5+55r^4s^2+220r^4st+220r^4s+55r^4t^2+220r^4t+55r^4-165r^3s^2t-$$
$$165r^3s^2-165r^3st^2-660r^3st-165r^3s-165r^3t^2-165r^3t+132r^2s^2t^2+528r^2s^2t+$$
$$132r^2s^2+528r^2st^2+528r^2st+132r^2t^2-462rs^2t^2-462rs^2t-462rst^2+462s^2t^2)-$$
$$\frac{h^2r^6f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^6st-84r^6s-84r^6t+126r^6-132r^5s^2t+264r^5s^2-$$
$$132r^5st^2+345r^5st-105r^5s+264r^5t^2-105r^5t-399r^5+110r^4s^3t-220r^4s^3+$$
$$440r^4s^2t^2-506r^4s^2t-616r^4s^2+110r^4st^3-506r^4st^2-407r^4st+1067r^4s-$$
$$220r^4t^3-616r^4t^2+1067r^4t+297r^4-396r^3s^3t^2+275r^3s^3t+825r^3s^3-396r^3s^2t^3-$$
$$484r^3s^2t^2+2387r^3s^2t-363r^3s^2+275r^3st^3+2387r^3st^2-2992r^3st-990r^3s+$$
$$825r^3t^3-363r^3t^2-990r^3t+396r^2s^3t^3+1188r^2s^3t^2-2673r^2s^3t-693r^2s^3+$$
$$1188r^2s^2t^3-3168r^2s^2t^2+891r^2s^2t+891r^2s^2-2673r^2st^3+891r^2st^2+3564r^2st-$$
$$693r^2t^3+891r^2t^2-2178rs^3t^3+1518rs^3t^2+2772rs^3t+1518rs^2t^3+726rs^2t^2-$$
$$3564rs^2t+2772rst^3-3564rst^2+2310s^3t^3-3234s^3t^2-3234s^2t^3+4158s^2t^2)+$$
$$\frac{h^2r^2f_{n+r}}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^9-2646r^8s-2646r^8t-2646r^8+3069r^7s^2+9420r^7st$$

$$+9420r^7s + 3069r^7t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 -$$

$$11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 - 11121r^6t^2 - 11121r^6t - 1155r^6 +$$

$$4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 +$$

$$41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t - 5148r^4s^3t^2 -$$

$$16027r^4s^3t - 5148r^4s^3 - 5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 -$$

$$16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 - 5148r^4t^2 + 1980r^3s^3t^3 +$$

$$20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t +$$

$$20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t -$$

$$26598r^2s^2t^3 - 26598r^2s^2t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 -$$

$$4158s^3t^3) - \frac{h^3r^6g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s -$$

$$198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 +$$

$$396r^2st - 110r^3st + 99r^2st^2 - 396rst) - \frac{h^3r^6g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 -$$

$$198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 -$$

$$21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst) + \frac{h^2r^6f_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}$$

$$(84r^6st - 126r^6s^2 + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 -$$

$$345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 +$$

$$616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s -$$

$$110r^4t^3 - 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 + 2992r^3s^3t +$$

$$363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3 + 484r^3st^2 -$$

$$275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 + 693r^2s^3t^3 -$$

$$891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t - 1188r^2st^3 -$$

$$1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 - 2772rs^3t -$$

$$1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 - 2310s^2t^3) +$$

$$\frac{h^2r^6f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(42r^6s - 84r^6st + 126r^6t^2 - 84r^6t + 264r^5s^2t - 132r^5s^2 -$$

$$105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t + 110r^4s^3 -$$

$$616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st + 110r^4s +$$

$$297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 -$$

$$363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 +$$

$$275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t +$$

$396r^2s^3+891r^2s^2t^4+891r^2s^2t^3-3168r^2s^2t^2+1188r^2s^2t+3564r^2st^4+891r^2st^3-2673r^2st^2+891r^2t^4-693r^2t^3+2772rs^3t^3+1518rs^3t^2-2178rs^3t-3564rs^2t^4+726rs^2t^3+1518rs^2t^2-3564rst^4+2772rst^3-3234s^3t^3+2310s^3t^2+4158s^2t^4-3234s^2t^3),$ (3.64)

$$y_{n+s}-y_n-hsy'_n=\frac{h^3s^3g_n}{27720r^2t^2}(55r^2s^4-198r^2s^3t-198r^2s^3+198r^2s^2t^2+792r^2s^2t+198r^2s^2-924r^2st^2-924r^2st+1386r^2t^2-66rs^5+220rs^4t+220rs^4-198rs^3t^2-792rs^3t-198rs^3+792rs^2t^2+792rs^2t-924rst^2+21s^6-66s^5t-66s^5+55s^4t^2+220s^4t+55s^4-198s^3t^2-198s^3t+198s^2t^2)-\frac{h^2s^2f_n}{27720r^3t^3}(396r^3s^4t^2-110r^3s^5-110r^3s^5t+583r^3s^4t+396r^3s^4-396r^3s^3t^3-1188r^3s^3t^2-1188r^3s^3t-396r^3s^3+990r^3s^2t^3+264r^3s^2t^2+990r^3s^2t+1848r^3st^3+1848r^3st^2-9702r^3t^3+132r^2s^6t+132r^2s^6-440r^2s^5t^2-748r^2s^5t-440r^2s^5+396r^2s^4t^3+1540r^2s^4t^2+1540r^2s^4t+396r^2s^4-1188r^2s^3t^3-1584r^2s^3t^2-1188r^2s^3t+264r^2s^2t^3+264r^2s^2t^2+1848r^2st^3-42rs^7t-42rs^7+132rs^6t^2+321rs^6t+132rs^6-110rs^5t^3-748rs^5t^2-748rs^5t-110rs^5+583rs^4t^3+1540rs^4t^2+583rs^4t-1188rs^3t^3-1188rs^3t^2+990rs^2t^3-42s^7t+132s^6t^2+132s^6t-110s^5t^3-440s^5t^2-110s^5t+396s^4t^3+396s^4t^2-396s^3t^3)-\frac{h^3s^6g_{n+1}}{27720(r-1)^2(s-1)^2(t-1)^2}(198r^2s^2t-55r^2s^3+99r^2s^2-198r^2st^2-396r^2st+462r^2t^2+66rs^4-220rs^3t-110rs^3+198rs^2t^2+396rs^2t-396rst^2-21s^5+66s^4t+33s^4-55s^3t^2-110s^3t+99s^2t^2)-\frac{h^3s^3g_{n+s}}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4-165r^2s^3t-165r^2s^3+132r^2s^2t^2+528r^2s^2t+132r^2s^2-462r^2st^2-462r^2st+462r^2t^2-77rs^5+220rs^4t+220rs^4-165rs^3t^2-660rs^3t-165rs^3+528rs^2t^2+528rs^2t-462rst^2+28s^6-77s^5t-77s^5+55s^4t^2+220s^4t+55s^4-165s^3t^2-165s^3t+132s^2t^2)-\frac{h^2s^6f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(110r^3s^4t-220r^3s^4-396r^3s^3t^2+275r^3s^3t+825r^3s^3+396r^3s^2t^3+1188r^3s^2t^2-2673r^3s^2t-693r^3s^2-2178r^3st^3+1518r^3st^2+2772r^3st+2310r^3t^3-3234r^3t^2-132r^2s^5t+264r^2s^5+440r^2s^4t^2-506r^2s^4t-616r^2s^4-396r^2s^3t^3-484r^2s^3t^2+2387r^2s^3t-363r^2s^3+1188r^2s^2t^3-3168r^2s^2t^2+891r^2s^2t+891r^2s^2+1518r^2st^3+726r^2st^2-3564r^2st-3234r^2t^3+4158r^2t^2+42rs^6t-84rs^6-132rs^5t^2+345rs^5t-105rs^5+110rs^4t^3-506rs^4t^2-407rs^4t+1067rs^4+275rs^3t^3+2387rs^3t^2-2992rs^3t-990rs^3-2673rs^2t^3+891$$

$$rs^2t^2+3564rs^2t+2772rst^3-3564rst^2-84s^6t+126s^6+264s^5t^2-105s^5t-399s^5-220s^4t^3-616s^4t^2+1067s^4t+297s^4+825s^3t^3-363s^3t^2-990s^3t-693s^2t^3+891s^2t^2)+\frac{h^2s^2f_{n+s}}{27720(r-s)^3(s-t)^3(s-1)^3}(1155r^3s^6-4235r^3s^5t-4235r^3s^5+5148r^3s^4t^2+16027r^3s^4t+5148r^3s^4-1980r^3s^3t^3-20196r^3s^3t^2-20196r^3s^3t-1980r^3s^3+7920r^3s^2t^3+26598r^3s^2t^2+7920r^3s^2t-10626r^3st^3-10626r^3st^2+4158r^3t^3-3069r^2s^7+11121r^2s^6t+11121r^2s^6-13376r^2s^5t^2-41437r^2s^5t-13376r^2s^5+5148r^2s^4t^3+51436r^2s^4t^2+51436r^2s^4t+5148r^2s^4-20196r^2s^3t^3-66330r^2s^3t^2-20196r^2s^3t+26598r^2s^2t^3+26598r^2s^2t^2-10626r^2st^3+2646rs^8-9420rs^7t-9420rs^7+11121rs^6t^2+34278rs^6t+11121rs^6-4235rs^5t^3-41437rs^5t^2-41437rs^5t-4235rs^5+16027rs^4t^3+51436rs^4t^2+16027rs^4t-20196rs^3t^3-20196rs^3t^2+7920rs^2t^3-756s^9+2646s^8t+2646s^8-3069s^7t^2-9420s^7t-3069s^7+1155s^6t^3+11121s^6t^2+11121s^6t+1155s^6-4235s^5t^3-13376s^5t^2-4235s^5t+5148s^4t^3+5148s^4t^2-1980s^3t^3)-\frac{h^2s^6f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t-110r^3s^4-825r^3s^3t^2-275r^3s^3t+396r^3s^3+693r^3s^2t^3+2673r^3s^2t^2-1188r^3s^2t-396r^3s^2-2772r^3st^3-1518r^3st^2+2178r^3st+3234r^3t^3-2310r^3t^2-264r^2s^5t+132r^2s^5+616r^2s^4t^2+506r^2s^4t-440r^2s^4+363r^2s^3t^3-2387r^2s^3t^2+484r^2s^3t+396r^2s^3-891r^2s^2t^4-891r^2s^2t^3+3168r^2s^2t^2-1188r^2s^2t+3564r^2st^4-726r^2st^3-1518r^2st^2-4158r^2t^4+3234r^2t^3+84rs^6t-42rs^6+105rs^5t^2-345rs^5t+132rs^5-1067rs^4t^3+407rs^4t^2+506rs^4t-110rs^4+990rs^3t^4+2992rs^3t^3-2387rs^3t^2-275rs^3t-3564rs^2t^4-891rs^2t^3+2673rs^2t^2+3564rst^4-2772rst^3-126s^6t^2+84s^6t+399s^5t^3+105s^5t^2-264s^5t-297s^4t^4-1067s^4t^3+616s^4t^2+220s^4t+990s^3t^4+363s^3t^3-825s^3t^2-891s^2t^4+693s^2t^3)-\frac{h^3s^6g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2-55s^3t^2+99rs^2-110rs^3+33rs^4+462rt^2-198st^2+198s^2t-220s^3t+66s^4t-55s^3+66s^4-21s^5-396rst^2+396rs^2t-110rs^3t+99rs^2t^2-396rst)-\frac{h^3s^6g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2-55r^2s^3+198rs^2-198r^2s-220rs^3+66rs^4+462r^2t+99s^2t-110s^3t+33s^4t-55s^3+66s^4-21s^5+396rs^2t-396r^2st-110rs^3t+99r^2s^2t-396rst)+\frac{h^2s^6f_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(297r^4s^4-990r^4s^3t-990r^4s^3+891r^4s^2t^2+3564r^4s^2t+891r^4s^2-3564r^4st^2-3564r^4st+4158r^4t^2-399r^3s^5+1067r^3s^4t+1067r^3s^4-363r^3s^3t^2-2992r^3s^3t-363r^3s^3-693r^3s^2t^3+891r^3s^2t^2$$

$$+891r^3s^2t-693r^3s^2+2772r^3st^3+726r^3st^2+2772r^3st-3234r^3t^3-3234r^3t^2+$$
$$126r^2s^6-105r^2s^5t-105r^2s^5-616r^2s^4t^2-407r^2s^4t-616r^2s^4+825r^2s^3t^3+$$
$$2387r^2s^3t^2+2387r^2s^3t+825r^2s^3-2673r^2s^2t^3-3168r^2s^2t^2-2673r^2s^2t+$$
$$1518r^2st^3+1518r^2st^2+2310r^2t^3-84rs^6t-84rs^6+264rs^5t^2+345rs^5t+$$
$$264rs^5-220rs^4t^3-506rs^4t^2-506rs^4t-220rs^4+275rs^3t^3-484rs^3t^2+275rs^3t+$$
$$1188rs^2t^3+1188rs^2t^2-2178rst^3+42s^6t-132s^5t^2-132s^5t+110s^4t^3+440s^4t^2+$$
$$110s^4t-396s^3t^3-396s^3t^2+396s^2t^3), \tag{3.65}$$

$$y_{n+t}-y_n-hty'_n=\frac{g_nh^3t^3}{27720r^2s^2}(198r^2s^2t^2-924r^2s^2t+1386r^2s^2-198r^2st^3+792r^2st^2$$
$$-924r^2st+55r^2t^4-198r^2t^3+198r^2t^2-198rs^2t^3+792rs^2t^2-924rs^2t+220rst^4-$$
$$792rst^3+792rst^2-66rt^5+220rt^4-198rt^3+55s^2t^4-198s^2t^3+198s^2t^2-$$
$$66st^5+220st^4-198st^3+21t^6-66t^5+55t^4)-\frac{f_nh^2t^2}{27720r^3s^3}(990r^3s^3t^2-396r^3s^3t^3+$$
$$1848r^3s^3t-9702r^3s^3+396r^3s^2t^4-1188r^3s^2t^3+264r^3s^2t^2+1848r^3s^2t-110r^3st^5+$$
$$583r^3st^4-1188r^3st^3+990r^3st^2-110r^3t^5+396r^3t^4-396r^3t^3+396r^2s^3t^4-$$
$$1188r^2s^3t^3+264r^2s^3t^2+1848r^2s^3t-440r^2s^2t^5+1540r^2s^2t^4-1584r^2s^2t^3+$$
$$264r^2s^2t^2+132r^2st^6-748r^2st^5+1540r^2st^4-1188r^2st^3+132r^2t^6-440r^2t^5+$$
$$396r^2t^4-110rs^3t^5+583rs^3t^4-1188rs^3t^3+990rs^3t^2+132rs^2t^6-748rs^2t^5+$$
$$1540rs^2t^4-1188rs^2t^3-42rst^7+321rst^6-748rst^5+583rst^4-42rt^7+132rt^6-$$
$$110rt^5-110s^3t^5+396s^3t^4-396s^3t^3+132s^2t^6-440s^2t^5+396s^2t^4-42st^7+$$
$$132st^6-110st^5)-\frac{g_{n+1}h^3t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2-198r^2s^2t+198r^2st^2-396r^2st-$$
$$55r^2t^3+99r^2t^2+198rs^2t^2-396rs^2t-220rst^3+396rst^2+66rt^4-110rt^3-55s^2t^3+$$
$$99s^2t^2+66st^4-110st^3-21t^5+33t^4)-\frac{g_{n+t}h^3t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2-462r^2s^2t+$$
$$462r^2s^2-165r^2st^3+528r^2st^2-462r^2st+55r^2t^4-165r^2t^3+132r^2t^2-165rs^2t^3+$$
$$528rs^2t^2-462rs^2t+220rst^4-660rst^3+528rst^2-77rt^5+220rt^4-165rt^3+$$
$$55s^2t^4-165s^2t^3+132s^2t^2-77st^5+220st^4-165st^3+28t^6-77t^5+55t^4)-$$
$$\frac{f_{n+1}h^2t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(396r^3s^3t^2-2178r^3s^3t+2310r^3s^3-396r^3s^2t^3+1188r^3s^2t^2+$$
$$1518r^3s^2t-3234r^3s^2+110r^3st^4+275r^3st^3-2673r^3st^2+2772r^3st-220r^3t^4+$$
$$825r^3t^3-693r^3t^2-396r^2s^3t^3+1188r^2s^3t^2+1518r^2s^3t-3234r^2s^3+440r^2s^2t^4-$$
$$484r^2s^2t^3-3168r^2s^2t^2+726r^2s^2t+4158r^2s^2-132r^2st^5-506r^2st^4+2387r^2st^3$$

$+891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 +$
$275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 -$
$3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 +$
$1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 +$
$891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4) -$
$\frac{f_{n+t}h^2t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(7920r^3s^3t^2 - 1980r^3s^3t^3 - 10626r^3s^3t + 4158r^3s^3 +$
$5148r^3s^2t^4 - 20196r^3s^2t^3 + 26598r^3s^2t^2 - 10626r^3s^2t - 4235r^3st^5 + 16027r^3st^4 -$
$20196r^3st^3 + 7920r^3st^2 + 1155r^3t^6 - 4235r^3t^5 + 5148r^3t^4 - 1980r^3t^3 + 5148r^2s^3t^4 -$
$20196r^2s^3t^3 + 26598r^2s^3t^2 - 10626r^2s^3t - 13376r^2s^2t^5 + 51436r^2s^2t^4 -$
$66330r^2s^2t^3 + 26598r^2s^2t^2 + 11121r^2st^6 - 41437r^2st^5 + 51436r^2st^4 - 20196r^2st^3 -$
$3069r^2t^7 + 11121r^2t^6 - 13376r^2t^5 + 5148r^2t^4 - 4235rs^3t^5 + 16027rs^3t^4 -$
$20196rs^3t^3 + 7920rs^3t^2 + 11121rs^2t^6 - 41437rs^2t^5 + 51436rs^2t^4 - 20196rs^2t^3 -$
$9420rst^7 + 34278rst^6 - 41437rst^5 + 16027rst^4 + 2646rt^8 - 9420rt^7 + 11121rt^6 -$
$4235rt^5 + 1155s^3t^6 - 4235s^3t^5 + 5148s^3t^4 - 1980s^3t^3 - 3069s^2t^7 + 11121s^2t^6 -$
$13376s^2t^5 + 5148s^2t^4 + 2646st^8 - 9420st^7 + 11121st^6 - 4235st^5 - 756t^9 + 2646t^8 -$
$3069t^7 + 1155t^6) + \frac{f_{n+r}h^2t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2 - 3564r^4s^2t + 4158r^4s^2 -$
$990r^4st^3 + 3564r^4st^2 - 3564r^4st + 297r^4t^4 - 990r^4t^3 + 891r^4t^2 - 693r^3s^3t^2 +$
$2772r^3s^3t - 3234r^3s^3 - 363r^3s^2t^3 + 891r^3s^2t^2 + 726r^3s^2t - 3234r^3s^2 + 1067r^3st^4 -$
$2992r^3st^3 + 891r^3st^2 + 2772r^3st - 399r^3t^5 + 1067r^3t^4 - 363r^3t^3 - 693r^3t^2 +$
$825r^2s^3t^3 - 2673r^2s^3t^2 + 1518r^2s^3t + 2310r^2s^3 - 616r^2s^2t^4 + 2387r^2s^2t^3 -$
$3168r^2s^2t^2 + 1518r^2s^2t - 105r^2st^5 - 407r^2st^4 + 2387r^2st^3 - 2673r^2st^2 + 126r^2t^6 -$
$105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t +$
$264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 -$
$84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 -$
$396s^2t^3 + 42st^6 - 132st^5 + 110st^4) + \frac{f_{n+s}h^2t^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^3s^3t^2 - 2772r^3s^3t +$
$3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 - 275r^3st^3 -$
$1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 + 3564r^2s^4t -$
$4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 - 2387r^2s^2t^3 +$
$3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 + 132r^2t^5 -$

$$
\begin{aligned}
&440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 + 2992rs^3t^3 - \\
&891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 + 84rst^6 - \\
&345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 - \\
&891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + \\
&616s^2t^4 - 825s^2t^3 + 84st^6 - 264st^5 + 220st^4) - \tfrac{g_{n+r}h^3t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - \\
&55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - \\
&55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst) - \\
&\tfrac{g_{n+s}h^3t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + \\
&66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + \\
&99r^2st^2 - 396rst),
\end{aligned}
\tag{3.66}
$$

$$
\begin{aligned}
&y_{n+1} - y_n - hy'_n = \tfrac{g_nh^3}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - \\
&198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + \\
&220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + \\
&55t^2 - 66t + 21) + \tfrac{h^2f_{n+1}}{27720(r-1)^3(s-1)^3(t-1)^3}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - \\
&1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - \\
&20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - \\
&10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + \\
&51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + \\
&5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - \\
&4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - \\
&41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - \\
&1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - \\
&3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - \\
&756) - \tfrac{h^3g_{n+1}}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - \\
&165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + \\
&220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + \\
&55t^2 - 77t + 28) - \tfrac{h^2f_n}{27720r^3s^3t^3}(1848r^3s^3t^2 - 9702r^3s^3t^3 + 990r^3s^3t - 396r^3s^3 + \\
&1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 + 990r^3st^3 - 1188r^3st^2 + 583r^3st -
\end{aligned}
$$

$110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t + 396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - 748r^2st + 132r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - 110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 + 583rst^3 - 748rst^2 + 321rst - 42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) - \frac{h^2 f_{n+r}}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(3564r^4s^2t - 4158r^4s^2t^2 - 891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 + 3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - 891r^3st^2 + 2992r^3st - 1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - 1518r^2s^3t^2 + 2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 + 2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t - 126r^2 + 2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + 506rs^2t - 264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) + \frac{h^2 f_{n+s}}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - 726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st) + \frac{h^2 f_{n+t}}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2 - 2178r^3s^3t + 396r^3s^3 - 3234r^3s^2t^3 + 1518r^3s^2t^2 + 1188r^3s^2t - 396r^3s^2 + 2772r^3st^3 - 2673r^3st^2 + 275r^3st + 110r^3s - 693r^3t^3 + 825r^3t^2 - 220r^3t - 3234r^2s^3t^3 + 1518r^2s^3t^2 + 1188r^2s^3t - 396r^2s^3 + 4158r^2s^2t^4 + 726r^2s^2t^3 - 3168r^2s^2t^2 - 484r^2s^2t + 440r^2s^2 - 3564r^2st^4 + 891r^2st^3 + 2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t + 2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 -$

$506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 + 1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 - 616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2) - \frac{h^3g_{n+r}}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21) - \frac{h^3g_{n+s}}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + 99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + 396rst - 21) - \frac{h^3g_{n+t}}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462 r^2s^2t + 396rst - 21).$ (3.67)

Now, substituting Equation (3.64) into (3.58) – (3.61) produces the following first derivative block:

$y'_{n+r} - y'_n = \frac{hrf_n}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 - 20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t + 60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 - 69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 - 69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 + 144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 - 168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3) + \frac{h^2r^2g_n}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2) - \frac{h^2r^5g_{n+1}}{2520(r-1)^2(s-1)^2(t-1)^2}(20r^4s - 7r^5 + 20r^4t + 10r^4 - 15r^3s^2 - 60r^3st - 30r^3s - 15r^3t^2 - 30r^3t + 48r^2s^2t + 24r^2s^2 + 48r^2st^2 + 96r^2st + 24r^2t^2 - 42rs^2t^2 - 84rs^2t - 84rst^2 + 84s^2t^2) - \frac{h^2r^2g_{n+r}}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 70r^5s - 70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t - 120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2) + \frac{hrf_{n+r}}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9 - 819$

$r^8s-819r^8t-819r^8+885r^7s^2+2686r^7st+2686r^7s+885r^7t^2+2686r^7t+885r^7-$

$315r^6s^3-2930r^6s^2t-2930r^6s^2-2930r^6st^2-8913r^6st-2930r^6s-315r^6t^3-$

$2930r^6t^2-2930r^6t-315r^6+1050r^5s^3t+1050r^5s^3+3228r^5s^2t^2+9847r^5s^2t+$

$3228r^5s^2+1050r^5st^3+9847r^5st^2+9847r^5st+1050r^5s+1050r^5t^3+3228r^5t^2+$

$1050r^5t-1164r^4s^3t^2-3561r^4s^3t-1164r^4s^3-1164r^4s^2t^3-11034r^4s^2t^2-$

$11034r^4s^2t-1164r^4s^2-3561r^4st^3-11034r^4st^2-3561r^4st-1164r^4t^3-1164r^4t^2+$

$420r^3s^3t^3+4026r^3s^3t^2+4026r^3s^3t+420r^3s^3+4026r^3s^2t^3+12618r^3s^2t^2+$

$4026r^3s^2t+4026r^3st^3+4026r^3st^2+420r^3t^3-1470r^2s^3t^3-4662r^2s^3t^2-$

$1470r^2s^3t-4662r^2s^2t^3-4662r^2s^2t^2-1470r^2st^3+1722rs^3t^3+1722rs^3t^2+$

$1722rs^2t^3-630s^3t^3)-\frac{hr^5f_{n+1}}{1260(r-1)^3(s-1)^3(t-1)^3}(7r^6st-14r^6s-14r^6t+21r^6-20r^5s^2t+$

$40r^5s^2-20r^5st^2+51r^5st-14r^5s+40r^5t^2-14r^5t-63r^5+15r^4s^3t-30r^4s^3+$

$60r^4s^2t^2-65r^4s^2t-90r^4s^2+15r^4st^3-65r^4st^2-65r^4st+150r^4s-30r^4t^3-90r^4t^2+$

$150r^4t+45r^4-48r^3s^3t^2+30r^3s^3t+105r^3s^3-48r^3s^2t^3-72r^3s^2t^2+306r^3s^2t-$

$39r^3s^2+30r^3st^3+306r^3st^2-366r^3st-135r^3s+105r^3t^3-39r^3t^2-135r^3t+$

$42r^2s^3t^3+138r^2s^3t^2-294r^2s^3t-84r^2s^3+138r^2s^2t^3-342r^2s^2t^2+66r^2s^2t+$

$108r^2s^2-294r^2st^3+66r^2st^2+432r^2st-84r^2t^3+108r^2t^2-210rs^3t^3+126rs^3t^2+$

$294rs^3t+126rs^2t^3+126rs^2t^2-378rs^2t+294rst^3-378rst^2+210s^3t^3-294s^3t^2-$

$294s^2t^3+378s^2t^2)-\frac{h^2r^5g_{n+s}}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2-15r^3t^2+24r^2s-30r^3s+$

$10r^4s-42rt^2+48r^2t-60r^3t+20r^4t+84st^2-15r^3+20r^4-7r^5-84rst^2+96r^2st-$

$30r^3st+24r^2st^2-84rst)-\frac{h^2r^5g_{n+t}}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2-15r^3s^2-42rs^2+48r^2s-$

$60r^3s+20r^4s+24r^2t-30r^3t+10r^4t+84s^2t-15r^3+20r^4-7r^5-84rs^2t+96r^2st-$

$30r^3st+24r^2s^2t-84rst)+\frac{hr^5f_{n+s}}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(14r^6st-21r^6s^2+14r^6s-7r^6t+$

$63r^5s^3+14r^5s^2t+14r^5s^2-40r^5st^2-51r^5st-40r^5s+20r^5t^2+20r^5t-45r^4s^4-$

$150r^4s^3t-150r^4s^3+90r^4s^2t^2+65r^4s^2t+90r^4s^2+30r^4st^3+65r^4st^2+65r^4st+$

$30r^4s-15r^4t^3-60r^4t^2-15r^4t+135r^3s^4t+135r^3s^4+39r^3s^3t^2+366r^3s^3t+$

$39r^3s^3-105r^3s^2t^3-306r^3s^2t^2-306r^3s^2t-105r^3s^2-30r^3st^3+72r^3st^2-30r^3st+$

$48r^3t^3+48r^3t^2-108r^2s^4t^2-432r^2s^4t-108r^2s^4+84r^2s^3t^3-66r^2s^3t^2-66r^2s^3t+$

$84r^2s^3+294r^2s^2t^3+342r^2s^2t^2+294r^2s^2t-138r^2st^3-138r^2st^2-42r^2t^3+$

$378rs^4t^2+378rs^4t-294rs^3t^3-126rs^3t^2-294rs^3t-126rs^2t^3-126rs^2t^2+210rst^3$

$$-378s^4t^2+294s^3t^3+294s^3t^2-210s^2t^3)+\frac{hr^5f_{n+t}}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(7r^6s-14r^6st+
21r^6t^2-14r^6t+40r^5s^2t-20r^5s^2-14r^5st^2+51r^5st-20r^5s-63r^5t^3-14r^5t^2+
40r^5t-30r^4s^3t+15r^4s^3-90r^4s^2t^2-65r^4s^2t+60r^4s^2+150r^4st^3-65r^4st^2-
65r^4st+15r^4s+45r^4t^4+150r^4t^3-90r^4t^2-30r^4t+105r^3s^3t^2+30r^3s^3t-48r^3s^3-
39r^3s^2t^3+306r^3s^2t^2-72r^3s^2t-48r^3s^2-135r^3st^4-366r^3st^3+306r^3st^2+30r^3st-
135r^3t^4-39r^3t^3+105r^3t^2-84r^2s^3t^3-294r^2s^3t^2+138r^2s^3t+42r^2s^3+108r^2s^2t^4+
66r^2s^2t^3-342r^2s^2t^2+138r^2s^2t+432r^2st^4+66r^2st^3-294r^2st^2+108r^2t^4-84r^2t^3+
294rs^3t^3+126rs^3t^2-210rs^3t-378rs^2t^4+126rs^2t^3+126rs^2t^2-378rst^4+294rst^3-
294s^3t^3+210s^3t^2+378s^2t^4-294s^2t^3), \quad (3.68)$$

$$y'_{n+s}-y'_n=\frac{hsf_n}{1260r^3t^3}(15r^3s^5t+15r^3s^5-48r^3s^4t^2-69r^3s^4t-48r^3s^4+42r^3s^3t^3+
120r^3s^3t^2+120r^3s^3t+42r^3s^3-84r^3s^2t^3-84r^3s^2t-168r^3st^3-168r^3st^2+
630r^3t^3-20r^2s^6t-20r^2s^6+60r^2s^5t^2+100r^2s^5t+60r^2s^5-48r^2s^4t^3-180r^2s^4t^2-
180r^2s^4t-48r^2s^4+120r^2s^3t^3+144r^2s^3t^2+120r^2s^3t-168r^2st^3+7rs^7t+7rs^7-
20rs^6t^2-48rs^6t-20rs^6+15rs^5t^3+100rs^5t^2+100rs^5t+15rs^5-69rs^4t^3-
180rs^4t^2-69rs^4t+120rs^3t^3+120rs^3t^2-84rs^2t^3+7s^7t-20s^6t^2-20s^6t+
15s^5t^3+60s^5t^2+15s^5t-48s^4t^3-48s^4t^2+42s^3t^3)+\frac{h^2s^2g_n}{2520r^2t^2}(15r^2s^4-48r^2s^3t-
48r^2s^3+42r^2s^2t^2+168r^2s^2t+42r^2s^2-168r^2st^2-168r^2st+210r^2t^2-20rs^5+
60rs^4t+60rs^4-48rs^3t^2-192rs^3t-48rs^3+168rs^2t^2+168rs^2t-168rst^2+
7s^6-20s^5t-20s^5+15s^4t^2+60s^4t+15s^4-48s^3t^2-48s^3t+42s^2t^2)-
\frac{h^2s^5g_{n+1}}{2520(r-1)^2(s-1)^2(t-1)^2}(48r^2s^2t-15r^2s^3+24r^2s^2-42r^2st^2-84r^2st+84r^2t^2+
20rs^4-60rs^3t-30rs^3+48rs^2t^2+96rs^2t-84rst^2-7s^5+20s^4t+10s^4-15s^3t^2-
30s^3t+24s^2t^2)-\frac{h^2s^2g_{n+s}}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4-120r^2s^3t-120r^2s^3+84r^2s^2t^2+
336r^2s^2t+84r^2s^2-252r^2st^2-252r^2st+210r^2t^2-70rs^5+180rs^4t+180rs^4-
120rs^3t^2-480rs^3t-120rs^3+336rs^2t^2+336rs^2t-252rst^2+28s^6-70s^5t-70s^5+
45s^4t^2+180s^4t+45s^4-120s^3t^2-120s^3t+84s^2t^2)+\frac{f_{n+s}hs}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6-
1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-
4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-
1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-$$

$9847r^2s^5t - 3228r^2s^5 + 1164r^2s^4t^3 + 11034r^2s^4t^2 + 11034r^2s^4t + 1164r^2s^4 - 4026r^2s^3t^3 - 12618r^2s^3t^2 - 4026r^2s^3t + 4662r^2s^2t^3 + 4662r^2s^2t^2 - 1722r^2st^3 + 819rs^8 - 2686rs^7t - 2686rs^7 + 2930rs^6t^2 + 8913rs^6t + 2930rs^6 - 1050rs^5t^3 - 9847rs^5t^2 - 9847rs^5t - 1050rs^5 + 3561rs^4t^3 + 11034rs^4t^2 + 3561rs^4t - 4026rs^3t^3 - 4026rs^3t^2 + 1470rs^2t^3 - 252s^9 + 819s^8t + 819s^8 - 885s^7t^2 - 2686s^7t - 885s^7 + 315s^6t^3 + 2930s^6t^2 + 2930s^6t + 315s^6 - 1050s^5t^3 - 3228s^5t^2 - 1050s^5t + 1164s^4t^3 + 1164s^4t^2 - 420s^3t^3) - \frac{hs^5f_{n+1}}{1260(r-1)^3(s-1)^3(t-1)^3}(15r^3s^4t - 30r^3s^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - 84r^3s^2 - 210r^3st^3 + 126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 - 48r^2s^3t^3 - 72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 + 126r^2st^3 + 126r^2st^2 - 378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + 40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2) - \frac{h^2s^5g_{n+r}}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^3t^2 + 24rs^2 - 30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 + 96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst) - \frac{h^2s^5g_{n+t}}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 + 84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t + 24r^2s^2t - 84rst) + \frac{hs^5f_{n+r}}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4 - 135r^4s^3t - 135r^4s^3 + 108r^4s^2t^2 + 432r^4s^2t + 108r^4s^2 - 378r^4st^2 - 378r^4st + 378r^4t^2 - 63r^3s^5 + 150r^3s^4t + 150r^3s^4 - 39r^3s^3t^2 - 366r^3s^3t - 39r^3s^3 - 84r^3s^2t^3 + 66r^3s^2t^2 + 66r^3s^2t - 84r^3s^2 + 294r^3st^3 + 126r^3st^2 + 294r^3st - 294r^3t^3 - 294r^3t^2 + 21r^2s^6 - 14r^2s^5t - 14r^2s^5 - 90r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 + 105r^2s^3t^3 + 306r^2s^3t^2 + 306r^2s^3t + 105r^2s^3 - 294r^2s^2t^3 - 342r^2s^2t^2 - 294r^2s^2t + 126r^2st^3 + 126r^2st^2 + 210r^2t^3 - 14rs^6t - 14rs^6 + 40rs^5t^2 + 51rs^5t + 40rs^5 - 30rs^4t^3 - 65rs^4t^2 - 65rs^4t - 30rs^4 + 30rs^3t^3 - 72rs^3t^2 + 30rs^3t + 138rs^2t^3 + 138rs^2t^2 - 210rst^3 + 7s^6t - 20s^5t^2 - 20s^5t + 15s^4t^3 + 60s^4t^2 + 15s^4t - 48s^3t^3 - 48s^3t^2 + 42s^2t^3) - \frac{hs^5f_{n+t}}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t - 15r^3s^4 - 105r^3s^3t^2 - 30r^3s^3t + 48r^3s^3 + 84r^3s^2t^3 + 294r^3s^2t^2 - 138r^3s^2t - 42r^3s^2 - 294r^3st^3 - 126r^3st^2 + 210r^3st + 294r^3t^3 - 210r^3t^2 - 40r^2s^5t + 20r^2s^5 + 90r^2s^4t^2 + 65r^2s^4t - 60r^2s^4 +$

$39r^2s^3t^3 - 306r^2s^3t^2 + 72r^2s^3t + 48r^2s^3 - 108r^2s^2t^4 - 66r^2s^2t^3 + 342r^2s^2t^2 - 138r^2s^2t + 378r^2st^4 - 126r^2st^3 - 126r^2st^2 - 378r^2t^4 + 294r^2t^3 + 14rs^6t - 7rs^6 + 14rs^5t^2 - 51rs^5t + 20rs^5 - 150rs^4t^3 + 65rs^4t^2 + 65rs^4t - 15rs^4 + 135rs^3t^4 + 366rs^3t^3 - 306rs^3t^2 - 30rs^3t - 432rs^2t^4 - 66rs^2t^3 + 294rs^2t^2 + 378rst^4 - 294rst^3 - 21s^6t^2 + 14s^6t + 63s^5t^3 + 14s^5t^2 - 40s^5t - 45s^4t^4 - 150s^4t^3 + 90s^4t^2 + 30s^4t + 135s^3t^4 + 39s^3t^3 - 105s^3t^2 - 108s^2t^4 + 84s^2t^3),$ (3.69)

$y'_{n+t} - y'_n = \frac{htf_n}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5) + \frac{h^2t^2g_n}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4) - \frac{h^2t^5g_{n+1}}{2520(r-1)^2(s-1)^2(t-1)^2}(84r^2s^2 - 42r^2s^2t + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4) - \frac{h^2t^2g_{n+t}}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4) - \frac{htf_{n+t}}{1260(r-t)^3(s-t)^3(t-1)^3}(1470r^3s^3t^2 - 420r^3s^3t^3 - 1722r^3s^3t + 630r^3s^3 + 1164r^3s^2t^4 - 4026r^3s^2t^3 + 4662r^3s^2t^2 - 1722r^3s^2t - 1050r^3st^5 + 3561r^3st^4 - 4026r^3st^3 + 1470r^3st^2 + 315r^3t^6 - 1050r^3t^5 + 1164r^3t^4 - 420r^3t^3 + 1164r^2s^3t^4 - 4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686r$

$st^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 +$

$315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 +$

$1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6) -$

$\frac{ht^5 f_{n+1}}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 +$

$126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 -$

$84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 -$

$342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st +$

$40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t -$

$20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 -$

$366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 +$

$40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 +$

$45t^4) - \frac{h^2t^5 g_{n+r}}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 +$

$48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 +$

$24rs^2t^2 - 84rst) - \frac{h^2t^5 g_{n+s}}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^2t^3 + 84r^2s + 48rt^2 - 42r^2t -$

$60rt^3 + 20rt^4 + 24st^2 - 30st^3 + 10st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84r^2st -$

$30rst^3 + 24r^2st^2 - 84rst) + \frac{ht^5 f_{n+r}}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 -$

$135r^4st^3 + 432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t -$

$294r^3s^3 - 39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 +$

$66r^3st^2 + 294r^3st - 63r^3t^5 + 150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 +$

$126r^2s^3t + 210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 -$

$65r^2st^4 + 306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 +$

$30rs^3t^3 + 138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 +$

$51rst^5 - 65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 -$

$20s^2t^5 + 60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4) + \frac{ht^5 f_{n+s}}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2 -$

$294r^3s^3t + 294r^3s^3 - 105r^3s^2t^3 + 294r^3s^2t^2 - 126r^3s^2t - 210r^3s^2 + 30r^3st^4 -$

$30r^3st^3 - 138r^3st^2 + 210r^3st - 15r^3t^4 + 48r^3t^3 - 42r^3t^2 - 108r^2s^4t^2 + 378r^2s^4t -$

$378r^2s^4 + 39r^2s^3t^3 - 66r^2s^3t^2 - 126r^2s^3t + 294r^2s^3 + 90r^2s^2t^4 - 306r^2s^2t^3 +$

$342r^2s^2t^2 - 126r^2s^2t - 40r^2st^5 + 65r^2st^4 + 72r^2st^3 - 138r^2st^2 + 20r^2t^5 - 60r^2t^4 +$

$48r^2t^3 + 135rs^4t^3 - 432rs^4t^2 + 378rs^4t - 150rs^3t^4 + 366rs^3t^3 - 66rs^3t^2 - 294rs^3t +$

$$14rs^2t^5+65rs^2t^4-306rs^2t^3+294rs^2t^2+14rst^6-51rst^5+65rst^4-30rst^3-7rt^6+$$
$$20rt^5-15rt^4-45s^4t^4+135s^4t^3-108s^4t^2+63s^3t^5-150s^3t^4+39s^3t^3+84s^3t^2-$$
$$21s^2t^6+14s^2t^5+90s^2t^4-105s^2t^3+14st^6-40st^5+30st^4), \tag{3.70}$$

$$y'_{n+1}-y'_n=\frac{hf_{n+1}}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3-1722r^3s^3t^2+1470r^3s^3t-420r^3s^3-$$
$$1722r^3s^2t^3+4662r^3s^2t^2-4026r^3s^2t+1164r^3s^2+1470r^3st^3-4026r^3st^2+$$
$$3561r^3st-1050r^3s-420r^3t^3+1164r^3t^2-1050r^3t+315r^3-1722r^2s^3t^3+$$
$$4662r^2s^3t^2-4026r^2s^3t+1164r^2s^3+4662r^2s^2t^3-12618r^2s^2t^2+11034r^2s^2t-$$
$$3228r^2s^2-4026r^2st^3+11034r^2st^2-9847r^2st+2930r^2s+1164r^2t^3-3228r^2t^2+$$
$$2930r^2t-885r^2+1470rs^3t^3-4026rs^3t^2+3561rs^3t-1050rs^3-4026rs^2t^3+$$
$$11034rs^2t^2-9847rs^2t+2930rs^2+3561rst^3-9847rst^2+8913rst-2686rs-$$
$$1050rt^3+2930rt^2-2686rt+819r-420s^3t^3+1164s^3t^2-1050s^3t+315s^3+$$
$$1164s^2t^3-3228s^2t^2+2930s^2t-885s^2-1050st^3+2930st^2-2686st+819s+$$
$$315t^3-885t^2+819t-252)+\frac{g_nh^2}{2520r^2s^2t^2}(210r^2s^2t^2-168r^2s^2t+42r^2s^2-168r^2st^2+$$
$$168r^2st-48r^2s+42r^2t^2-48r^2t+15r^2-168rs^2t^2+168rs^2t-48rs^2+168rst^2-$$
$$192rst+60rs-48rt^2+60rt-20r+42s^2t^2-48s^2t+15s^2-48st^2+60st-20s+$$
$$15t^2-20t+7)-\frac{h^2g_{n+1}}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2-252r^2s^2t+84r^2s^2-252r^2st^2+$$
$$336r^2st-120r^2s+84r^2t^2-120r^2t+45r^2-252rs^2t^2+336rs^2t-120rs^2+336rst^2-$$
$$480rst+180rs-120rt^2+180rt-70r+84s^2t^2-120s^2t+45s^2-120st^2+180st-$$
$$70s+45t^2-70t+28)+\frac{hf_n}{1260r^3s^3t^3}(630r^3s^3t^3-168r^3s^3t^2-84r^3s^3t+42r^3s^3-$$
$$168r^3s^2t^3+120r^3s^2t-48r^3s^2-84r^3st^3+120r^3st^2-69r^3st+15r^3s+42r^3t^3-$$
$$48r^3t^2+15r^3t-168r^2s^3t^3+120r^2s^3t-48r^2s^3+144r^2s^2t^2-180r^2s^2t+60r^2s^2+$$
$$120r^2st^3-180r^2st^2+100r^2st-20r^2s-48r^2t^3+60r^2t^2-20r^2t-84rs^3t^3+$$
$$120rs^3t^2-69rs^3t+15rs^3+120rs^2t^3-180rs^2t^2+100rs^2t-20rs^2-69rst^3+$$
$$100rst^2-48rst+7rs+15rt^3-20rt^2+7rt+42s^3t^3-48s^3t^2+15s^3t-48s^2t^3+$$
$$60s^2t^2-20s^2t+15st^3-20st^2+7st)-\frac{h^2g_{n+r}}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r+20s+20t-$$
$$42s^2t^2-30rs-30rt-60st+24rs^2+24rt^2+48st^2+48s^2t-15s^2-15t^2-84rst^2-$$
$$84rs^2t+84rs^2t^2+96rst-7)-\frac{h^2g_{n+s}}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r+10s+20t-42r^2t^2-$$
$$30rs-60rt-30st+24r^2s+48rt^2+48r^2t+24st^2-15r^2-15t^2-84rst^2-84r^2st+$$

$$
\begin{aligned}
&84r^2st^2 + 96rst - 7) - \frac{h^2 g_{n+t}}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r + 20s + 10t - 42r^2s^2 - 60rs - \\
&30rt - 30st + 48rs^2 + 48r^2s + 24r^2t + 24s^2t - 15r^2 - 15s^2 - 84rs^2t - 84r^2st + \\
&84r^2s^2t + 96rst - 7) - \frac{hf_{n+r}}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(378r^4s^2t - 378r^4s^2t^2 - 108r^4s^2 + \\
&378r^4st^2 - 432r^4st + 135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + 294r^3s^3t^2 - 294r^3s^3t + \\
&84r^3s^3 + 294r^3s^2t^3 - 126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - 294r^3st^3 - 66r^3st^2 + 366r^3st - \\
&150r^3s + 84r^3t^3 + 39r^3t^2 - 150r^3t + 63r^3 - 210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - \\
&105r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 + 294r^2st^3 - 306r^2st^2 + \\
&65r^2st + 14r^2s - 105r^2t^3 + 90r^2t^2 + 14r^2t - 21r^2 + 210rs^3t^3 - 138rs^3t^2 - 30rs^3t + \\
&30rs^3 - 138rs^2t^3 + 72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs + \\
&30rt^3 - 40rt^2 + 14rt - 42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - 60s^2t^2 + 20s^2t - 15st^3 + \\
&20st^2 - 7st) + \frac{hf_{n+s}}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 - 210r^3s^2t^3 - \\
&126r^3s^2t^2 + 294r^3s^2t - 105r^3s^2 + 210r^3st^3 - 138r^3st^2 - 30r^3st + 30r^3s - 42r^3t^3 + \\
&48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - 108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + \\
&39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - \\
&40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + 378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - \\
&66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - 306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + \\
&65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - 108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + \\
&39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + 14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st) + \\
&\frac{hf_{n+t}}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2 - 210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + \\
&138r^3s^2t - 48r^3s^2 + 294r^3st^3 - 294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - \\
&30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + 138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - \\
&342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - 378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + \\
&108r^2t^4 - 39r^2t^3 - 90r^2t^2 + 40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - \\
&378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - 65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + \\
&7rs - 135rt^4 + 150rt^3 - 14rt^2 - 14rt - 84s^3t^3 + 105s^3t^2 - 30s^3t + 108s^2t^4 - 39s^2t^3 - \\
&90s^2t^2 + 40s^2t - 135st^4 + 150st^3 - 14st^2 - 14st + 45t^4 - 63t^3 + 21t^2).
\end{aligned}
\tag{3.71}
$$

The first derivative of the method in (3.63) can be expressed in following block form

$$I_4 Y_{n+1}^{'[2]_3} = M_2^{'[2]_3} Y_{n-1}^{[2]_3} + h[E_1^{'[2]_3} F_n^{[2]_3} + E_2^{'[2]_3} F_{n+1}^{[2]_3}] + h^2[K_1^{'[2]_3} G_n^{[2]_3} + K_2^{'[2]_3} G_{n+1}^{[2]_3}] \quad (3.72)$$

where

$$Y_{n+1}^{'[2]_3} = \begin{pmatrix} y_{n+r}^{'} \\ y_{n+s}^{'} \\ y_{n+t}^{'} \\ y_{n+1}^{'} \end{pmatrix}, \; M_2^{'[2]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; E_1^{'[2]_3} = \begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{'[2]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{'[2]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{'[2]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{'[2]_3} \end{pmatrix},$$

$$E_2^{'[2]_3} = \begin{pmatrix} E_{2_{11}}^{'[2]_3} & E_{2_{12}}^{'[2]_3} & E_{2_{13}}^{'[2]_3} & E_{2_{14}}^{'[2]_3} \\ E_{2_{21}}^{'[2]_3} & E_{2_{22}}^{'[2]_3} & E_{2_{23}}^{'[2]_3} & E_{2_{24}}^{'[2]_3} \\ E_{2_{31}}^{'[2]_3} & E_{2_{32}}^{'[2]_3} & E_{2_{33}}^{'[2]_3} & E_{2_{34}}^{'[2]_3} \\ E_{2_{41}}^{'[2]_3} & E_{2_{42}}^{'[2]_3} & E_{2_{43}}^{'[2]_3} & E_{2_{44}}^{'[2]_3} \end{pmatrix}, \; K_1^{'[2]_3} = \begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{'[2]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{'[2]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{'[2]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{'[2]_3} \end{pmatrix},$$

$$K_2^{'[2]_2} = \begin{pmatrix} K_{2_{11}}^{'[2]_3} & K_{2_{12}}^{'[2]_3} & K_{2_{13}}^{'[2]_3} & K_{2_{14}}^{'[2]_3} \\ K_{2_{21}}^{'[2]_3} & K_{2_{22}}^{'[2]_3} & K_{2_{23}}^{'[2]_3} & K_{2_{24}}^{'[2]_3} \\ K_{2_{31}}^{'[2]_3} & K_{2_{32}}^{'[2]_3} & K_{2_{33}}^{'[2]_3} & K_{2_{34}}^{'[2]_3} \\ K_{2_{41}}^{'[2]_3} & K_{2_{42}}^{'[2]_3} & K_{2_{43}}^{'[2]_3} & K_{2_{44}}^{'[2]_3} \end{pmatrix},$$

with the entries of the matrices $E_1^{'[2]_3}, E_2^{'[2]_3}, K_1^{'[2]_3}$ and $K_2^{'[2]_3}$ are as follows

$E_{1_{14}}^{'[2]_3} = \frac{r}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 - 20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t + 60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 - 69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 - 69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 + 144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 - 168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3)$,

$E_{1_{24}}^{'[2]_3} = \frac{s}{1260r^3t^3}(15r^3s^5t + 15r^3s^5 - 48r^3s^4t^2 - 69r^3s^4t - 48r^3s^4 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 - 84r^3s^2t^3 - 84r^3s^2t - 168r^3st^3 - 168r^3st^2 + 630r^3t^3$

$$-20r^2s^6t - 20r^2s^6 + 60r^2s^5t^2 + 100r^2s^5t + 60r^2s^5 - 48r^2s^4t^3 - 180r^2s^4t^2 - 180r^2s^4t - 48r^2s^4 + 120r^2s^3t^3 + 144r^2s^3t^2 + 120r^2s^3t - 168r^2st^3 + 7rs^7t + 7rs^7 - 20rs^6t^2 - 48rs^6t - 20rs^6 + 15rs^5t^3 + 100rs^5t^2 + 100rs^5t + 15rs^5 - 69rs^4t^3 - 180rs^4t^2 - 69rs^4t + 120rs^3t^3 + 120rs^3t^2 - 84rs^2t^3 + 7s^7t - 20s^6t^2 - 20s^6t + 15s^5t^3 + 60s^5t^2 + 15s^5t - 48s^4t^3 - 48s^4t^2 + 42s^3t^3),$$

$$E_{1_{34}}^{'[2]_3} = \frac{t}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5),$$

$$E_{1_{44}}^{'[2]_3} = \frac{1}{1260r^3s^3t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - 168r^3s^2t^3 + 120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - 48r^3t^2 + 15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + 120r^2st^3 - 180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t - 84rs^3t^3 + 120rs^3t^2 - 69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + 100rst^2 - 48rst + 7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + 60s^2t^2 - 20s^2t + 15st^3 - 20st^2 + 7st),$$

$$\hat{E}_{2_{11}}^{[2]_3} = \frac{r}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9 - 819r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t + 885r^7 - 315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s - 315r^6t^3 - 2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 + 9847r^5s^2t + 3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 + 3228r^5t^2 + 1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 - 11034r^4s^2t^2 - 11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st - 1164r^4t^3 - 1164r^4t^2 + 420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 + 12618r^3s^2t^2 + 4026r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 - 4662r^2s^3t^2 - 1470r^2s^3t - 4662r^2s^2t^3$$

$$-4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 + 1722rs^3t^2 + 1722rs^2t^3 - 630s^3t^3),$$

$$\hat{E}^{[2]_3}_{2_{21}} = \frac{s^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4 - 135r^4s^3t - 135r^4s^3 + 108r^4s^2t^2 + 432r^4s^2t +
108r^4s^2 - 378r^4st^2 - 378r^4st + 378r^4t^2 - 63r^3s^5 + 150r^3s^4t + 150r^3s^4 - 39r^3s^3t^2 -
366r^3s^3t - 39r^3s^3 - 84r^3s^2t^3 + 66r^3s^2t^2 + 66r^3s^2t - 84r^3s^2 + 294r^3st^3 + 126r^3st^2 +
294r^3st - 294r^3t^3 - 294r^3t^2 + 21r^2s^6 - 14r^2s^5t - 14r^2s^5 - 90r^2s^4t^2 - 65r^2s^4t -
90r^2s^4 + 105r^2s^3t^3 + 306r^2s^3t^2 + 306r^2s^3t + 105r^2s^3 - 294r^2s^2t^3 - 342r^2s^2t^2 -
294r^2s^2t + 126r^2st^3 + 126r^2st^2 + 210r^2t^3 - 14rs^6t - 14rs^6 + 40rs^5t^2 + 51rs^5t +
40rs^5 - 30rs^4t^3 - 65rs^4t^2 - 65rs^4t - 30rs^4 + 30rs^3t^3 - 72rs^3t^2 + 30rs^3t + 138rs^2t^3 +
138rs^2t^2 - 210rst^3 + 7s^6t - 20s^5t^2 - 20s^5t + 15s^4t^3 + 60s^4t^2 + 15s^4t - 48s^3t^3 -
48s^3t^2 + 42s^2t^3),$$

$$\hat{E}^{[2]_3}_{2_{31}} = \frac{t^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 - 135r^4st^3 +
432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t - 294r^3s^3 -
39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 + 66r^3st^2 +
294r^3st - 63r^3t^5 + 150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 + 126r^2s^3t +
210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 - 65r^2st^4 +
306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 + 30rs^3t^3 +
138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 + 51rst^5 -
65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 - 20s^2t^5 +
60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4),$$

$$\hat{E}^{[2]_3}_{2_{41}} = \frac{1}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(-378r^4s^2t^2 + 378r^4s^2t - 108r^4s^2 + 378r^4st^2 -
432r^4st + 135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + 294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 +
294r^3s^2t^3 - 126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - 294r^3st^3 - 66r^3st^2 + 366r^3st - 150r^3s +
84r^3t^3 + 39r^3t^2 - 150r^3t + 63r^3 - 210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - 105r^2s^3 -
126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 + 294r^2st^3 - 306r^2st^2 + 65r^2st + 14r^2s -
105r^2t^3 + 90r^2t^2 + 14r^2t - 21r^2 + 210rs^3t^3 - 138rs^3t^2 - 30rs^3t + 30rs^3 - 138rs^2t^3 +
72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs + 30rt^3 - 40rt^2 + 14rt -
42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - 60s^2t^2 + 20s^2t - 15st^3 + 20st^2 - 7st),$$

$\hat{E}^{[2]_3}_{2_{12}} = \frac{r^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2+14r^6st+14r^6s-7r^6t+63r^5s^3+14r^5s^2t+14r^5s^2-40r^5st^2-51r^5st-40r^5s+20r^5t^2+20r^5t-45r^4s^4-150r^4s^3t-150r^4s^3+90r^4s^2t^2+65r^4s^2t+90r^4s^2+30r^4st^3+65r^4st^2+65r^4st+30r^4s-15r^4t^3-60r^4t^2-15r^4t+135r^3s^4t+135r^3s^4+39r^3s^3t^2+366r^3s^3t+39r^3s^3-105r^3s^2t^3-306r^3s^2t^2-306r^3s^2t-105r^3s^2-30r^3st^3+72r^3st^2-30r^3st+48r^3t^3+48r^3t^2-108r^2s^4t^2-432r^2s^4t-108r^2s^4+84r^2s^3t^3-66r^2s^3t^2-66r^2s^3t+84r^2s^3+294r^2s^2t^3+342r^2s^2t^2+294r^2s^2t-138r^2st^3-138r^2st^2-42r^2t^3+378rs^4t^2+378rs^4t-294rs^3t^3-126rs^3t^2-294rs^3t-126rs^2t^3-126rs^2t^2+210rst^3-378s^4t^2+294s^3t^3+294s^3t^2-210s^2t^3)$,

$\hat{E}^{[2]_3}_{2_{22}} = \frac{s}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6-1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-9847r^2s^5t-3228r^2s^5+1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-4026r^2s^3t^3-12618r^2s^3t^2-4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+819rs^8-2686rs^7t-2686rs^7+2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-9847rs^5t^2-9847rs^5t-1050rs^5+3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-4026rs^3t^2+1470rs^2t^3-252s^9+819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+315s^6t^3+2930s^6t^2+2930s^6t+315s^6-1050s^5t^3-3228s^5t^2-1050s^5t+1164s^4t^3+1164s^4t^2-420s^3t^3)$,

$\hat{E}^{[2]_3}_{2_{32}} = \frac{t^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2-294r^3s^3t+294r^3s^3-105r^3s^2t^3+294r^3s^2t^2-126r^3s^2t-210r^3s^2+30r^3st^4-30r^3st^3-138r^3st^2+210r^3st-15r^3t^4+48r^3t^3-42r^3t^2-108r^2s^4t^2+378r^2s^4t-378r^2s^4+39r^2s^3t^3-66r^2s^3t^2-126r^2s^3t+294r^2s^3+90r^2s^2t^4-306r^2s^2t^3+342r^2s^2t^2-126r^2s^2t-40r^2st^5+65r^2st^4+72r^2st^3-138r^2st^2+20r^2t^5-60r^2t^4+48r^2t^3+135rs^4t^3-432rs^4t^2+378rs^4t-150rs^3t^4+366rs^3t^3-66rs^3t^2-294rs^3t+14rs^2t^5+65rs^2t^4-306rs^2t^3+294rs^2t^2+14rst^6-51rst^5+65rst^4-30rst^3-7rt^6+20rt^5-15rt^4-45s^4t^4+135s^4t^3-108s^4t^2+63s^3t^5-150s^3t^4+39s^3t^3+84s^3t^2-21s^2t^6+14s^2t^5+90s^2t^4-105s^2t^3+14st^6-40st^5+30st^4)$,

$\hat{E}_{2_{42}}^{[2]_3} = \frac{1}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 - 210r^3s^2t^3 - 126r^3s^2t^2 + 294r^3s^2t - 105r^3s^2 + 210r^3st^3 - 138r^3st^2 - 30r^3st + 30r^3s - 42r^3t^3 + 48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - 108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + 39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - 40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + 378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - 66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - 306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - 108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + 39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + 14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st)$,

$\hat{E}_{2_{13}}^{[2]_3} = \frac{r^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st + 7r^6s + 21r^6t^2 - 14r^6t + 40r^5s^2t - 20r^5s^2 - 14r^5st^2 + 51r^5st - 20r^5s - 63r^5t^3 - 14r^5t^2 + 40r^5t - 30r^4s^3t + 15r^4s^3 - 90r^4s^2t^2 - 65r^4s^2t + 60r^4s^2 + 150r^4st^3 - 65r^4st^2 - 65r^4st + 15r^4s + 45r^4t^4 + 150r^4t^3 - 90r^4t^2 - 30r^4t + 105r^3s^3t^2 + 30r^3s^3t - 48r^3s^3 - 39r^3s^2t^3 + 306r^3s^2t^2 - 72r^3s^2t - 48r^3s^2 - 135r^3st^4 - 366r^3st^3 + 306r^3st^2 + 30r^3st - 135r^3t^4 - 39r^3t^3 + 105r^3t^2 - 84r^2s^3t^3 - 294r^2s^3t^2 + 138r^2s^3t + 42r^2s^3 + 108r^2s^2t^4 + 66r^2s^2t^3 - 342r^2s^2t^2 + 138r^2s^2t + 432r^2st^4 + 66r^2st^3 - 294r^2st^2 + 108r^2t^4 - 84r^2t^3 + 294rs^3t^3 + 126rs^3t^2 - 210rs^3t - 378rs^2t^4 + 126rs^2t^3 + 126rs^2t^2 - 378rst^4 + 294rst^3 - 294s^3t^3 + 210s^3t^2 + 378s^2t^4 - 294s^2t^3)$,

$\hat{E}_{2_{23}}^{[2]_3} = \frac{-s^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t - 15r^3s^4 - 105r^3s^3t^2 - 30r^3s^3t + 48r^3s^3 + 84r^3s^2t^3 + 294r^3s^2t^2 - 138r^3s^2t - 42r^3s^2 - 294r^3st^3 - 126r^3st^2 + 210r^3st + 294r^3t^3 - 210r^3t^2 - 40r^2s^5t + 20r^2s^5 + 90r^2s^4t^2 + 65r^2s^4t - 60r^2s^4 + 39r^2s^3t^3 - 306r^2s^3t^2 + 72r^2s^3t + 48r^2s^3 - 108r^2s^2t^4 - 66r^2s^2t^3 + 342r^2s^2t^2 - 138r^2s^2t + 378r^2st^4 - 126r^2st^3 - 126r^2st^2 - 378r^2t^4 + 294r^2t^3 + 14rs^6t - 7rs^6 + 14rs^5t^2 - 51rs^5t + 20rs^5 - 150rs^4t^3 + 65rs^4t^2 + 65rs^4t - 15rs^4 + 135rs^3t^4 + 366rs^3t^3 - 306rs^3t^2 - 30rs^3t - 432rs^2t^4 - 66rs^2t^3 + 294rs^2t^2 + 378rst^4 - 294rst^3 - 21s^6t^2 + 14s^6t + 63s^5t^3 + 14s^5t^2 - 40s^5t - 45s^4t^4 - 150s^4t^3 + 90s^4t^2 + 30s^4t + 135s^3t^4 + 39s^3t^3 - 105s^3t^2 - 108s^2t^4 + 84s^2t^3)$,

$\hat{E}_{2_{33}}^{[2]_3} = \frac{-t}{1260(r-t)^3(s-t)^3(t-1)^3}(-420r^3s^3t^3 + 1470r^3s^3t^2 - 1722r^3s^3t + 630r^3s^3 + 1164 r^3s^2t^4 - 4026r^3s^2t^3 + 4662r^3s^2t^2 - 1722r^3s^2t - 1050r^3st^5 + 3561r^3st^4 - 4026r^3st^3 + 1470r^3st^2 + 315r^3t^6 - 1050r^3t^5 + 1164r^3t^4 - 420r^3t^3 + 1164r^2s^3t^4 - 4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686rst^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 + 315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 + 1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6),$

$\hat{E}_{2_{43}}^{[2]_3} = \frac{1}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2 - 210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + 138r^3s^2t - 48r^3s^2 + 294r^3st^3 - 294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - 30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + 138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - 342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - 378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + 108r^2t^4 - 39r^2t^3 - 90r^2t^2 + 40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - 378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - 65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + 7rs - 135rt^4 + 150rt^3 - 14rt^2 - 14rt - 84s^3t^3 + 105s^3t^2 - 30s^3t + 108s^2t^4 - 39s^2t^3 - 90s^2t^2 + 40s^2t - 135st^4 + 150st^3 - 14st^2 - 14st + 45t^4 - 63t^3 + 21t^2),$

$\hat{E}_{2_{14}}^{[2]_3} = \frac{-r^5}{1260(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 20r^5s^2t + 40r^5s^2 - 20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t - 63r^5 + 15r^4s^3t - 30r^4s^3 + 60r^4s^2t^2 - 65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st + 150r^4s - 30r^4t^3 - 90r^4t^2 + 150r^4t + 45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3 - 72r^3s^2t^2 + 306r^3s^2t - 39r^3s^2 + 30r^3st^3 + 306r^3st^2 - 366r^3st - 135r^3s + 105r^3t^3 - 39r^3t^2 - 135r^3t + 42r^2s^3t^3 + 138r^2s^3t^2 - 294r^2s^3t - 84r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 - 294r^2st^3 + 66r^2st^2 + 432r^2st - 84r^2t^3 + 108r^2t^2 - 210rs^3t^3 + 126rs^3t^2 + 294rs^3t + 126rs^2t^3 + 126rs^2t^2 - 378rs^2t + 294rst^3 - 378rst^2 + 210s^3t^3 - 294s^3t^2 - 294s^2t^3 + 378s^2t^2),$

$\hat{E}^{[2]_3}_{2_{24}} = \frac{-s^5}{1260(r-1)^3(s-1)^3(t-1)^3}(15r^3s^4t - 30r^3s^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - 84r^3s^2 - 210r^3st^3 + 126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 - 48r^2s^3t^3 - 72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 + 126r^2st^3 + 126r^2st^2 - 378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + 40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2)$,

$\hat{E}^{[2]_3}_{2_{34}} = \frac{-t^5}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 - 366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 + 40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 + 45t^4)$,

$\hat{E}^{[2]_3}_{2_{44}} = \frac{1}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3 - 1722r^3s^3t^2 + 1470r^3s^3t - 420r^3s^3 - 1722r^3s^2t^3 + 4662r^3s^2t^2 - 4026r^3s^2t + 1164r^3s^2 + 1470r^3st^3 - 4026r^3st^2 + 3561r^3st - 1050r^3s - 420r^3t^3 + 1164r^3t^2 - 1050r^3t + 315r^3 - 1722r^2s^3t^3 + 4662r^2s^3t^2 - 4026r^2s^3t + 1164r^2s^3 + 4662r^2s^2t^3 - 12618r^2s^2t^2 + 11034r^2s^2t - 3228r^2s^2 - 4026r^2st^3 + 11034r^2st^2 - 9847r^2st + 2930r^2s + 1164r^2t^3 - 3228r^2t^2 + 2930r^2t - 885r^2 + 1470rs^3t^3 - 4026rs^3t^2 + 3561rs^3t - 1050rs^3 - 4026rs^2t^3 + 11034rs^2t^2 - 9847rs^2t + 2930rs^2 + 3561rst^3 - 9847rst^2 + 8913rst - 2686rs - 1050rt^3 + 2930rt^2 - 2686rt + 819r - 420s^3t^3 + 1164s^3t^2 - 1050s^3t + 315s^3 + 1164s^2t^3 - 3228s^2t^2 + 2930s^2t - 885s^2 - 1050st^3 + 2930st^2 - 2686st + 819s + 315t^3 - 885t^2 + 819t - 252)$,

$\hat{K}^{[2]_3}_{1_{14}} = \frac{r^2}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2)$,

$\hat{K}^{[2]_3}_{1_{24}} = \frac{s^2}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t - 48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 - 48s^3t^2 - 48s^3t + 42s^2t^2)$,

$\hat{K}^{[2]_3}_{1_{34}} = \frac{t^2}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4)$,

$\hat{K}^{[2]_3}_{1_{44}} = \frac{1}{2520r^2s^2t^2}(210r^2s^2t^2 - 168r^2s^2t + 42r^2s^2 - 168r^2st^2 + 168r^2st - 48r^2s + 42r^2t^2 - 48r^2t + 15r^2 - 168rs^2t^2 + 168rs^2t - 48rs^2 + 168rst^2 - 192rst + 60rs - 48rt^2 + 60rt - 20r + 42s^2t^2 - 48s^2t + 15s^2 - 48st^2 + 60st - 20s + 15t^2 - 20t + 7)$,

$\hat{K}^{[2]_3}_{2_{11}} = \frac{-r^2}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 70r^5s - 70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t - 120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2)$,

$\hat{K}^{[2]_3}_{2_{21}} = \frac{-s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^3t^2 + 24rs^2 - 30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 + 96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst)$,

$\hat{K}^{[2]_3}_{2_{31}} = \frac{-t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 + 48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 + 24rs^2t^2 - 84rst)$,

$\hat{K}^{[2]_3}_{2_{41}} = \frac{-1}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r+20s+20t-42s^2t^2-30rs-30rt-60st+24rs^2+24rt^2+48st^2+48s^2t-15s^2-15t^2-84rst^2-84rs^2t+84rs^2t^2+96rst-7)$,

$\hat{K}^{[2]_3}_{2_{12}} = \frac{-r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2-15r^3t^2+24r^2s-30r^3s+10r^4s-42rt^2+48r^2t-60r^3t+20r^4t+84st^2-15r^3+20r^4-7r^5-84rst^2+96r^2st-30r^3st+24r^2st^2-84rst)$,

$\hat{K}^{[2]_3}_{2_{22}} = \frac{-s^2}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4-120r^2s^3t-120r^2s^3+84r^2s^2t^2+336r^2s^2t+84r^2s^2-252r^2st^2-252r^2st+210r^2t^2-70rs^5+180rs^4t+180rs^4-120rs^3t^2-480rs^3t-120rs^3+336rs^2t^2+336rs^2t-252rst^2+28s^6-70s^5t-70s^5+45s^4t^2+180s^4t+45s^4-120s^3t^2-120s^3t+84s^2t^2)$,

$\hat{K}^{[2]_3}_{2_{32}} = \frac{-t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2-15r^2t^3+84r^2s+48rt^2-42r^2t-60rt^3+20rt^4+24st^2-30st^3+10st^4-15t^3+20t^4-7t^5+96rst^2-84r^2st-30rst^3+24r^2st^2-84rst)$,

$\hat{K}^{[2]_3}_{2_{42}} = \frac{-1}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r+10s+20t-42r^2t^2-30rs-60rt-30st+24r^2s+48rt^2+48r^2t+24st^2-15r^2-15t^2-84rst^2-84r^2st+84r^2st^2+96rst-7)$,

$\hat{K}^{[2]_3}_{2_{13}} = \frac{-r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2-15r^3s^2-42rs^2+48r^2s-60r^3s+20r^4s+24r^2t-30r^3t+10r^4t+84s^2t-15r^3+20r^4-7r^5-84rs^2t+96r^2st-30r^3st+24r^2s^2t-84rst)$,

$\hat{K}^{[2]_3}_{2_{23}} = \frac{-s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2-15r^2s^3+48rs^2-42r^2s-60rs^3+20rs^4+84r^2t+24s^2t-30s^3t+10s^4t-15s^3+20s^4-7s^5+96rs^2t-84r^2st-30rs^3t+24r^2s^2t-84rst)$,

$\hat{K}^{[2]_3}_{2_{33}} = \frac{-t^2}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2-252r^2s^2t+210r^2s^2-120r^2st^3+336r^2st^2-252r^2st+45r^2t^4-120r^2t^3+84r^2t^2-120rs^2t^3+336rs^2t^2-252rs^2t+180rst^4-480rst^3+336rst^2-70rt^5+180rt^4-120rt^3+45s^2t^4-120s^2t^3+84s^2t^2-70st^5+180st^4-120st^3+28t^6-70t^5+45t^4)$,

$\hat{K}^{[2]_3}_{2_{43}} = \frac{-1}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r+20s+10t-42r^2s^2-60rs-30rt-30st+48rs^2+48r^2s+24r^2t+24s^2t-15r^2-15s^2-84rs^2t-84r^2st+84r^2s^2t+96rst-7),$

$\hat{K}^{[2]_3}_{2_{14}} = \frac{-r^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-7r^5+20r^4s+20r^4t+10r^4-15r^3s^2-60r^3st-30r^3s-15r^3t^2-30r^3t+48r^2s^2t+24r^2s^2+48r^2st^2+96r^2st+24r^2t^2-42rs^2t^2-84rs^2t-84rst^2+84s^2t^2),$

$\hat{K}^{[2]_3}_{2_{24}} = \frac{-s^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-15r^2s^3+48r^2s^2t+24r^2s^2-42r^2st^2-84r^2st+84r^2t^2+20rs^4-60rs^3t-30rs^3+48rs^2t^2+96rs^2t-84rst^2-7s^5+20s^4t+10s^4-15s^3t^2-30s^3t+24s^2t^2),$

$\hat{K}^{[2]_3}_{2_{34}} = \frac{-t^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-42r^2s^2t+84r^2s^2+48r^2st^2-84r^2st-15r^2t^3+24r^2t^2+48rs^2t^2-84rs^2t-60rst^3+96rst^2+20rt^4-30rt^3-15s^2t^3+24s^2t^2+20st^4-30st^3-7t^5+10t^4),$

$\hat{K}^{[2]_3}_{2_{44}} = \frac{-1}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2-252r^2s^2t+84r^2s^2-252r^2st^2+336r^2st-120r^2s+84r^2t^2-120r^2t+45r^2-252rs^2t^2+336rs^2t-120rs^2+336rst^2-480rst+180rs-120rt^2+180rt-70r+84s^2t^2-120s^2t+45s^2-120st^2+180st-70s+45t^2-70t+28).$

3.4.1 Properties of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

This section investigates some basic properties of the proposed method as mentioned previously.

3.4.1.1 Order of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

A similar procedure as adopted in Section (3.2.1.1) will be used in finding the order of HBM with generalised three off-step points (3.63). The linear difference operator

∇ corresponding to (3.63) below is obtained by substituting $m = 2$ and $z = 3$ in (3.20):

$$\begin{aligned}\nabla[y(x),h] &= Y_{n+1}^{[2]_3} - \hat{M}_1^{[2]_3} Y_n^{[2]_3} - h\hat{M}_2^{[2]_3} Y_{n-1}^{[2]_3} - h^2[\hat{E}_1^{[2]_3} F_n^{[2]_3} + \hat{E}_2^{[2]_3} F_{n+1}^{[2]_3}] \\ &\quad - h^3[\hat{K}_1^{[2]_3} G_n^{[2]_3} + \hat{K}_2^{[2]_3} G_{n+1}^{[2]_3}] \qquad (3.73)\end{aligned}$$

By expanding each function y and its derivative in $Y_{n+1}^{[2]_3}$, $F_{n+1}^{[2]_3}$ and $G_{n+1}^{[2]_3}$ about x_n using Taylor series expansion, collecting all terms and then comparing similar terms of h^i and $y^{(i)}$, we obtain $\bar{D}_0 = \bar{D}_1 = ... = \bar{D}_{11} = 0$ and $\bar{D}_{12} \neq 0$ after equating to $\mathbf{0}$. Hence, the order of the main method is $[10, 10, 10, 10]^T$ with vector of error constants

$$\bar{D}_{12} = \begin{bmatrix} \frac{r^6}{100590336000}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s \\ +33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st \\ -110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 \\ +396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2) \\ \\ \frac{s^6}{100590336000}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t \\ +99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + \\ 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - \\ 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110 \\ s^3t^2 - 110s^3t + 99s^2t^2) \\ \\ \frac{t^6}{100590336000}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 \\ -396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 \\ -396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 \\ +33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 \\ -42t^5 + 33t^4) \\ \\ \frac{1}{100590336000}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st \\ -110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 \\ +396rst^2 - 440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 \\ -110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14) \end{bmatrix}.$$

In order to find the order of the derivative block (3.72), we first define the linear difference operator ∇ as follows

$$\begin{aligned}\nabla[y(x),h] &= Y'^{[2]_3}_{n+1} - M'^{[2]_3}_{2} Y^{[2]_3}_{n-1} - h[E'^{[2]_3}_{1} F^{[2]_3}_{n} + E'^{[2]_3}_{2} F^{[2]_3}_{n+1}] \\ &\quad -h^2[K'^{[2]_3}_{1} G^{[2]_3}_{n} + K'^{[2]_3}_{2} G^{[2]_3}_{n+1}]. \qquad (3.74)\end{aligned}$$

Then, again every function y and its derivatives in $Y'^{[2]_3}_{n+1}, F^{[2]_3}_{n+1}$ and $G^{[2]_3}_{n+1}$ are expanded using Taylor series expansion. After comparing the coefficients of h^j and $y^{(j)}$ in (3.74) and equating to $\mathbf{0}$, we have $\bar{D}'_0 = \bar{D}'_1 = ... = \bar{D}'_{11} = 0$ and $\bar{D}'_{12} \neq 0$. As a result, the order of the derivative of the block is found to be $[10,10,10,10]^T$ with vector of error constants

$$\bar{D}'_{12} = \begin{bmatrix} \frac{r^5}{50295168000}(28r^6 - 77r^5 s - 77r^5 t - 77r^5 + 55r^4 s^2 + 220r^4 st + 220r^4 s \\ +55r^4 t^2 + 220r^4 t + 55r^4 - 165r^3 s^2 t - 165r^3 s^2 - 165r^3 st^2 - 660r^3 st - \\ 165r^3 s - 165r^3 t^2 - 165r^3 t + 132r^2 s^2 t^2 + 528r^2 s^2 t + 132r^2 s^2 + 528r^2 \\ st^2 + 528r^2 st + 132r^2 t^2 - 462rs^2 t^2 - 462rs^2 t - 462rst^2 + 462s^2 t^2) \\ \frac{s^5}{50295168000}(55r^2 s^4 - 165r^2 s^3 t - 165r^2 s^3 + 132r^2 s^2 t^2 + 528r^2 s^2 t + 132r^2 s^2 \\ -462r^2 st^2 - 462r^2 st + 462r^2 t^2 - 77rs^5 + 220rs^4 t + 220rs^4 - 165rs^3 t^2 - \\ 660rs^3 t - 165rs^3 + 528rs^2 t^2 + 528rs^2 t - 462rst^2 + 28s^6 - 77s^5 t - 77s^5 \\ +55s^4 t^2 + 220s^4 t + 55s^4 - 165s^3 t^2 - 165s^3 t + 132s^2 t^2) \\ \frac{t^5}{50295168000}(132r^2 s^2 t^2 - 462r^2 s^2 t + 462r^2 s^2 - 165r^2 st^3 + 528r^2 st^2 - \\ 462r^2 st + 55r^2 t^4 - 165r^2 t^3 + 132r^2 t^2 - 165rs^2 t^3 + 528rs^2 t^2 - 462rs^2 t \\ +220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2 t^4 - \\ 165s^2 t^3 + 132s^2 t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4) \\ \frac{1}{50295168000}(462r^2 s^2 t^2 - 462r^2 s^2 t + 132r^2 s^2 - 462r^2 st^2 + 528r^2 st - \\ 165r^2 s + 132r^2 t^2 - 165r^2 t + 55r^2 - 462rs^2 t^2 + 528rs^2 t - 165rs^2 + \\ 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2 t^2 - 165s^2 t \\ +55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28) \end{bmatrix}$$

.

3.4.1.2 Zero-Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

In investigating the zero-stability for the block (3.63), we first formulate its first characteristic polynomial, as below

$$
\begin{aligned}
\psi^{[2]_3}(q) &= |qI_4 - \hat{M}_1^{[2]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1)
\end{aligned}
$$

which leads to $q = \{0,0,0,1\}$ when $\psi^{[2]_3}(q)$ is set to 0. In the same manner, the characteristic polynomial for the derivative block (3.72) is given by

$$
\begin{aligned}
\psi'^{[2]_3}(q) &= |qI_4 - M_2'^{[2]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1).
\end{aligned}
$$

By setting $\psi'^{[2]_3}(q) = 0$, we have $q = \{0,0,0,1\}$. As a result, the block method and its derivative are zero stable since Definition 3.2.2 is fulfilled.

3.4.1.3 Consistency of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

The main block method (3.63) and its derivative (3.72) are consistent according to Definition 2.4.4.

3.4.1.4 Convergence of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

The block method (3.63) and its derivative (3.72) are convergent by Theorem (2.1).

3.4.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Second Order ODEs

The following equation is obtained after substituting $m = 2$ and $z = 3$ in (3.27):

$$M^{[2]_3}(q) = (I_4 - q^2\hat{E}_2^{[2]_3} - q^3\hat{K}_2^{[2]_3})^{-1}(\hat{M}_1^{[2]_3} + q\hat{M}_2^{[2]_3} + q^2\hat{E}_1^{[2]_3} + q^3\hat{K}_1^{[2]_3}) \quad (3.75)$$

whose eigenvalues are $\{0, 0, 0, \eta_4^{[2]_3}\}$. The dominant eigenvalue $\eta_4^{[2]_3}$ is a function of q given by

$$\eta_4^{[2]_3} = eig(M^{[2]_3}(q)). \quad (3.76)$$

In order to sketch the region of absolute stability, specific values $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ are substituted into Equation (3.76) which leads to

$$\eta_4^{[2]_3} \;\; = \frac{\sum_{i=0}^{12} c_i q^i}{K \sum_{j=0}^{13} d_j q^j}$$

where $K = 2696539284087339220992$ and the values c_i and d_j are given in the following Table 3.3.

Table 3.3

Coefficients of the Eigenvalue $(\eta_4^{[2]_3})$ *for the Matrix* $M^{[2]_3}$

c-value	q^i Coefficients	d-value	q^j Coefficients
c_0	-2666454639806495839868323757870284800 0	d_0	0
c_1	-26664546398064958398683237578702848000	d_1	- 9888432390144000
c_2	-11415221184170897879283369573875712000	d_2	0
c_3	-2901120791073837650442285073249075200	d_3	710930497536000
c_4	-499826226840493860518360961672806400	d_4	- 138726604800000
c_5	-14891066904641980421390821120	d_5	9920965017600
c_6	-59320532319692785524330335934873 60	d_6	150080716800
c_7	-4368926778396425009052496587366 40	d_7	- 77509051200
c_8	-254196988795461212135314442649 60	d_8	4799692800
c_9	-117666522571306366894313937232 8	d_9	41441960
c_{10}	-42683493362425639112056135680	d_{10}	- 19475680
c_{11}	-1130119613961003703223224857	d_{11}	632070
c_{12}	-134206760169026912683368 36	d_{12}	42570
		d_{13}	- 4977

Plotting the function $(\eta_4^{[2]_3})$ gives the region of absolute stability as shown in the dark area as depicted in Figure 3.8 below .

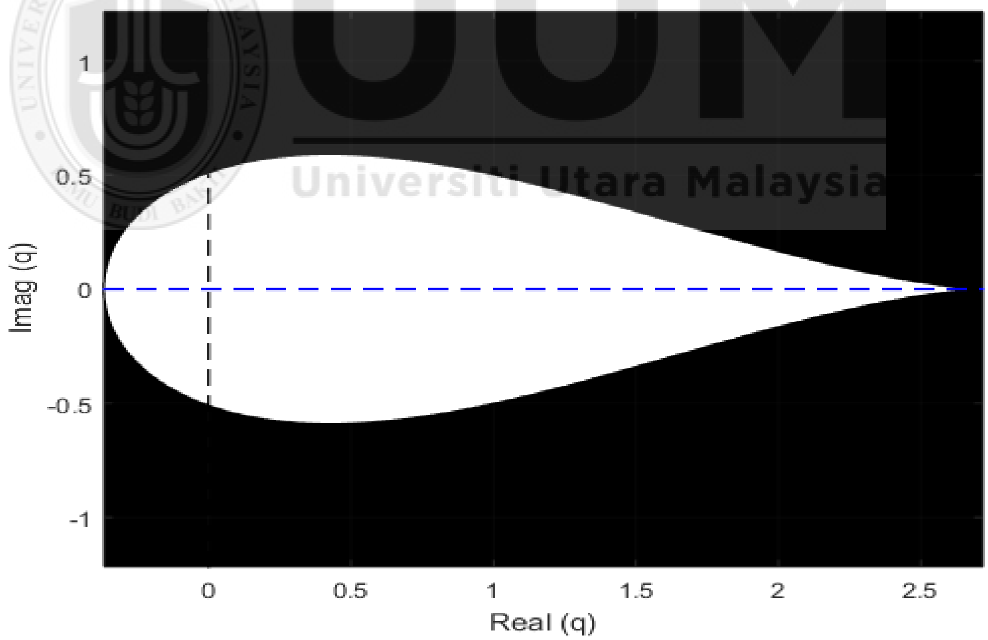


Figure 3.8. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ for Second Order ODEs

Similarly, plotting the function $(\eta_4^{[2]_3})$ subject to the values $r = \frac{1}{5}$, $s = \frac{2}{5}$ and $t = \frac{3}{5}$ gives the region of absolute stability represented by a dark area in Figure 3.9. This region is quite similar to the region in Figure 3.8.

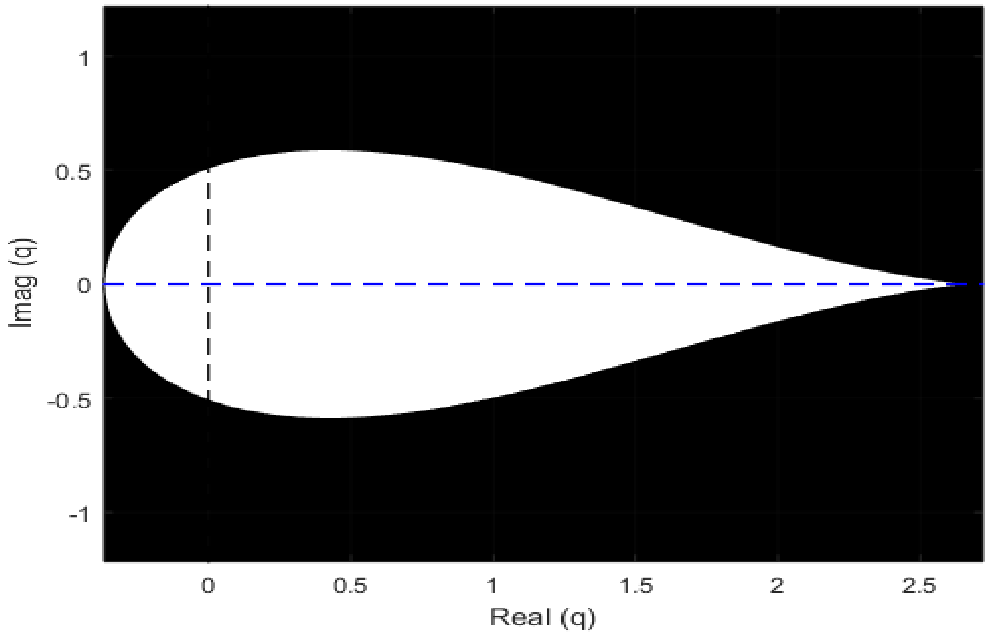


Figure 3.9. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{5}$, $s = \frac{2}{5}$ and $t = \frac{3}{5}$ for Second Order ODEs

A detailed explanation on how to transform BVPs of second order ODEs to their equivalent IVPs is covered in the following section.

3.5 Transforming Boundary Value Problems of Second Order ODEs to the Equivalent Initial Value Problems Using Shooting Method

Following the shooting strategy as in Section 1.5.2 for the nonlinear second order BVP

$$y'' = f(x,y,y'), \quad x \in [a,b], \quad y(a) = \alpha, \; y(b) = \beta. \tag{3.77}$$

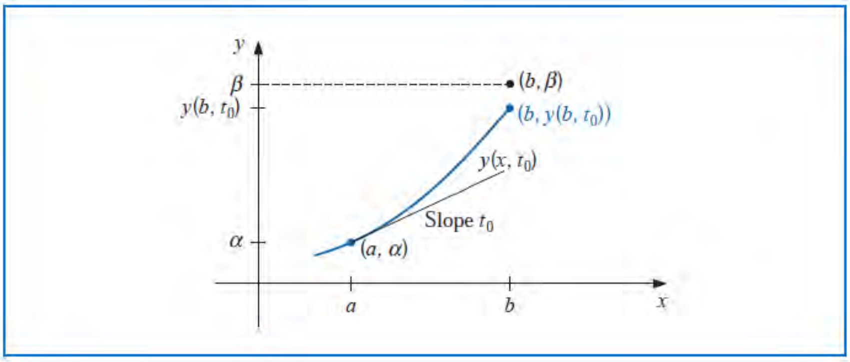
To approximate the solution to the BVP (3.77), we first assume that $y'(a) = t$ as given below

$$y'' = f(x,y,y'), \; x \in [a,b], \; y(a) = \alpha, \; y'(a) = t \tag{3.78}$$

using the solutions to a sequence of IVPs depending on the variable t, where t takes the values t_k for $k = 0,1,2...$ which satisfies the following condition

$$\lim_{k\to\infty} y(b,t_k) = y(b) = \beta. \tag{3.79}$$

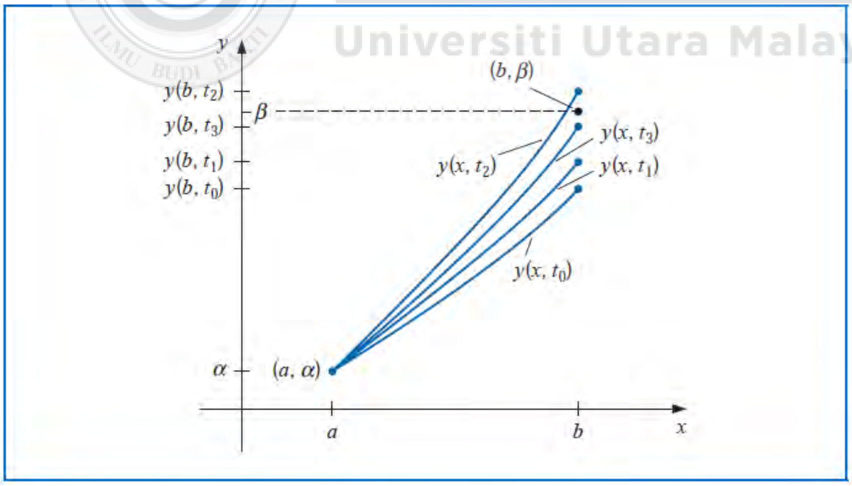
To start with this technique, choose an initial guess t_0 that produces $y(b,t_0)$ as depicted in Figure 3.10.



Source:(Burden & Faires, 2011)

Figure 3.10. Initial Guess t_0 to Approximate (b,β)

If the value of $y(b,t_0)$ is not sufficiently close to β then we correct our approximation by selecting different guesses t_1, t_2, t_3,..., which will produce successive solutions $y(x,t_1)$ $y(x,t_2)$, $y(x,t_3)$, ... until $y(b,t_k)$ is sufficiently close to β where $y(x,t_k)$ denotes the solution to (3.78) and $y(x)$ denotes the solution to (3.77) as depicted in Figure 3.11



Source:(Burden & Faires, 2011)

Figure 3.11. Multiple Guesses $t_0, t_1, t_2, \ldots$ to Approximate (b,β)

Geometrically speaking, the parameter t_0 prescribes the initial slope of the solution curve. If we assume that f is continuous and satisfies a Lipschitz condition with respect to y and y', then by Theorem 1.1, for each $t \in \mathbb{R}$, there exists a unique solution

$y(x,t)$ of (3.78). To arrive at a solution to the BVP (3.77), the parameter t has to be chosen such that $y(b,t) \approx \beta$, i.e. we need to solve the equation

$$F(t) = 0$$

where the function $F : \mathbb{R} \to \mathbb{R}$ is defined by

$$F(t) = y(b,t) - \beta.$$

For each t, the value $F(t)$ can be computed approximately by one of the numerical methods for finding the roots of functions such as Secant method, Bisection method or Newton Raphson method. Note that for a nonlinear differential equation, the equation $F(t) = 0$ is also nonlinear. For finding the zeroes of F, the three-step iterative method proposed by Yun (2008) is employed instead of Newton method because the former method converges faster. For the computation of the derivative $F'(t)$ which is required for the three-step iterative method, we assume that the solution $y(x)$ to (3.78) depends on a continuously differentiable manner on the parameters t. By setting

$$v(x,t) = \frac{\partial y(x,t)}{\partial t}, \tag{3.80}$$

Equation (3.78) can now be written in the form

$$y''(x,t) = f(x, y(x,t), y'(x,t)),\ x \in [a,b],\ y(a,t) = \alpha,\ y'(a,t) = t. \tag{3.81}$$

Differentiating the above equation once with respect to t, we obtain

$$\frac{\partial y''(x,t)}{\partial t} = f_x(x,y,y')\frac{\partial x}{\partial t} + f_y(x,y,y')\frac{\partial y(x,t)}{\partial t} + f_{y'}(x,y,y')\frac{\partial y'(x,t)}{\partial t}$$

which leads to

$$v''(x,t) = f_y(x,y,y')v(x,t) + f_{y'}(x,y,y')v'(x,t) \tag{3.82}$$

with initial conditions

$$v(a,t) = 0,\ v'(a,t) = 1. \tag{3.83}$$

Since $F'(t) = v(b,t)$, finding the derivative of $F(t)$ requires solving the additional initial value problem given by (3.82) and (3.83), where $y(x,t)$ is known from solving (3.81). Note that from a numerical approximation, $y(x,t)$ is known only at grid points.

As explained in the introduction the algorithm for the shooting method with the three-step iterative method for second order BVPs consists of the following steps:

Step 1: Choose an initial slope $t \in \mathbb{R}$ for the second order BVP

$$y'' = f(x,y,y')$$

subject to the following boundary conditions

$$y(a) = \alpha,\ y(b) = \beta. \tag{3.84}$$

Step 2: Solve numerically the following IVPs

$$\textbf{i.} \qquad y''(x,t) = f(x,y,y'), \tag{3.85a}$$

$$\textbf{ii.} \qquad \frac{\partial y''(x,t)}{\partial t} = v''(x,t), \tag{3.85b}$$

(3.85a) is solved subject to the following initial conditions

$$y(a) = \alpha,\ y'(a) = t, \tag{3.86}$$

and its derivative (3.85b) with respect to t, i.e

$$v''(x,t) = f_y(x,y,y')v(x,t) + f_{y'}(x,y,y')v'(x,t)$$

with initial conditions

$$v(a,t) = 0,\ v'(a,t) = 1.$$

Step 3: If $\mid F(t) \mid = \mid y(b,t) - \beta \mid < TOL$ for some small value $TOL > 0$ then the solution is obtained. Otherwise, update t_{k+1} using the following equation

$$t_{k+1} = T_k + U_k + B_k$$

where

$$T_k = t_k - \frac{F(t_k)}{F'(t_k)}, \quad U_k = -\frac{F(T_k)}{F'(t_k)}, \quad B_k = -\frac{F(T_k + U_k)}{F'(t_k)}.$$

and go back to Step 2.

3.6 Numerical Results for Solving Second Order ODEs

In order to compare the performance of the developed one-step HBMs with the introduction of a third derivative for solving second order ODEs with the existing methods, we have randomly chosen specific one, two and three off-step points. By letting $r = \frac{1}{3}$ in Equations (3.14)-(3.17), the following one-step HBM with specific one off-step point and its derivative are obtained as follows

$$
\begin{aligned}
y_{n+\frac{1}{3}} &= y_n + \frac{hy_n'}{3} + h^2\left[\frac{613}{17010}f_n + \frac{37}{136080}f_{n+1} + \frac{97}{5040}f_{n+\frac{1}{3}}\right] \\
&\quad + h^3\left[\frac{5}{3402}g_n - \frac{1}{22680}g_{n+1} - \frac{11}{5670}g_{n+\frac{1}{3}}\right], \\
y_{n+1} &= y_n + hy_n' + h^2\left[\frac{3}{14}f_n + \frac{5}{112}f_{n+1} + \frac{27}{112}f_{n+\frac{1}{3}}\right] \\
&\quad + h^3\left[\frac{1}{70}g_n - \frac{1}{210}g_{n+1} + \frac{9}{280}g_{n+\frac{1}{3}}\right], \\
y'_{n+\frac{1}{3}} &= y_n' + h\left[\frac{182}{1215}f_n + \frac{31}{19440}f_{n+1} + \frac{131}{720}f_{n+\frac{1}{3}}\right] \\
&\quad + h^2\left[\frac{17}{2430}g_n - \frac{1}{3888}g_{n+1} - \frac{29}{2160}g_{n+\frac{1}{3}}\right], \\
y'_{n+1} &= y_n' + h\left[\frac{2}{5}f_n + \frac{21}{80}f_{n+1} + \frac{27}{80}f_{n+\frac{1}{3}}\right] \\
&\quad + h^2\left[\frac{1}{30}g_n - \frac{1}{48}g_{n+1} + \frac{9}{80}g_{n+\frac{1}{3}}\right].
\end{aligned} \qquad (3.87)
$$

Similarly, substituting $r = \frac{1}{5}$ and $s = \frac{3}{5}$ into Equations (3.41)-(3.46) produces HBM with specific two-off step points as given below

$$
\begin{aligned}
y_{n+\frac{1}{5}} &= y_n + \frac{h}{5}y_n' + h^2\left[\frac{247655101483925}{19949538974367744}f_n + \frac{2445274453677084}{90792568487789234375}f_{n+1}\right. \\
&\quad \left. + \frac{1059751035515804}{145268109580462725}f_{n+\frac{1}{5}} \frac{6468069784829506352}{245139934917030810546875}f_{n+\frac{3}{5}}\right] + \\
&\quad h^3\left[\frac{19823}{70875000}g_n - \frac{641}{252000000}g_{n+1} - \frac{5423}{10080000}g_{n+\frac{1}{5}} - \frac{71}{1417500}g_{n+\frac{3}{5}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{3}{5}} &= y_n+\frac{3}{5}hy'_n+h^2\left[\frac{51201}{875000}f_n+\frac{217917364019436453888}{2837267765243413574218750}f_{n+1}+\right. \\
&\quad \left.\frac{288389623434535564738 56}{2905362191609255371093750}f_{n+\frac{1}{5}}+\frac{2403}{112000}f_{n+\frac{3}{5}}\right]+ \\
&\quad h^3\left[\frac{1593}{875000}g_n-\frac{243}{3500000}g_{n+1}+\frac{243}{80000}g_{n+\frac{1}{5}}-\frac{639}{280000}g_{n+\frac{3}{5}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} &= y_n+hy'_n+h^2\left[\frac{9299}{68040}f_n+\frac{1504202275541745664}{907925684877892343750}f_{n+1}+\right. \\
&\quad \left.\frac{2882303761517117744}{143033215586917187500}f_{n+\frac{1}{5}}+\frac{227882141144947097 6}{1568895583468997187500}f_{n+\frac{3}{5}}\right]+ \\
&\quad h^3\left[\frac{17}{3240}g_n-\frac{83}{80640}g_{n+1}+\frac{275}{16128}g_{n+\frac{1}{5}}+\frac{25}{18144}g_{n+\frac{3}{5}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{1}{5}} &= y'_n+h\left[\frac{13505146864579135}{159596311794941952}f_nh+\frac{97375866085941875}{864691128455135232}f_{n+\frac{1}{5}}\right. \\
&\quad \left.+\frac{164363372000513622016}{6537064931120821289062 5}f_{n+\frac{3}{5}}+\frac{16903}{67200000}f_{n+1}\right] \\
&\quad +h^2\left[\frac{10223}{4725000}g_n-\frac{797}{33600000}g_{n+1}-\frac{7997}{1344000}g_{n+\frac{1}{5}}-\frac{1429}{3024000}g_{n+\frac{3}{5}}\right] \\
y'_{n+\frac{3}{5}} &= y'_n+h\left[\frac{12597}{87500}f_n+\frac{344946908418964822425 6}{1291272085159668945312 5}f_{n+\frac{1}{5}}\right. \\
&\quad \left.+\frac{23339905068846875}{126100789566373888}f_{n+\frac{3}{5}}+\frac{85293}{22400000}f_{n+1}\right] \\
&\quad +h^2\left[\frac{957}{175000}g_n-\frac{3807}{11200000}g_{n+1}+\frac{9153}{448000}g_{n+\frac{1}{5}}-\frac{1551}{112000}g_{n+\frac{3}{5}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+1} &= y'_n+h\left[\frac{593}{2268}f_n+\frac{29543613555550453 76}{1239621201753282187 5}f_{n+\frac{1}{5}}\right. \\
&\quad \left.+\frac{7277816997830721536}{20918607779586628125}f_{n+\frac{3}{5}}+\frac{3275}{21504}f_{n+1}\right] \\
&\quad +h^2\left[\frac{19}{1512}g_n-\frac{73}{10752}g_{n+1}+\frac{575}{10752}g_{n+\frac{1}{5}}+\frac{775}{24192}g_{n+\frac{3}{5}}\right].
\end{aligned}
$$

Finally, the following one-step HBM with specific three off-step points are obtained when the values $r=\frac{1}{4}$, $s=\frac{1}{2}$ and $t=\frac{3}{4}$ are substituted into Equations (3.64)- (3.71):

$$
\begin{aligned}
y_{n+\frac{1}{4}} &= y_n+\frac{h}{4}y'_n+h^2\left[\frac{2602339}{153280512}f_n+\frac{382169}{766402560}f_{n+1}+\frac{148231}{47900160}f_{n+\frac{1}{4}}\right.\\
&\left.+\frac{1807}{322560}f_{n+\frac{1}{2}}+\frac{243193}{47900160}f_{n+\frac{3}{4}}\right]+h^3\left[\frac{28343}{72990720}g_n-\frac{14339}{510935040}g_{n+1}\right.\\
&\left.-\frac{551}{199584}g_{n+\frac{1}{4}}-\frac{32027}{14192640}g_{n+\frac{1}{2}}-\frac{3959}{6386688}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{1}{2}} &= y_n+\frac{h}{2}y'_n+h^2\left[\frac{35}{891}f_n+\frac{13}{8910}f_{n+1}+\frac{196}{4455}f_{n+\frac{1}{4}}+\frac{1}{40}f_{n+\frac{1}{2}}+\right.\\
&\left.\frac{68}{4455}f_{n+\frac{3}{4}}\right]+h^3\left[\frac{1277}{1330560}g_n-\frac{109}{1330560}g_{n+1}-\frac{41}{5544}g_{n+\frac{1}{4}}\right.\\
&\left.-\frac{5}{693}g_{n+\frac{1}{2}}-\frac{17}{9240}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{3}{4}} &= y_n+\frac{3}{4}hy'_n+h^2\left[\frac{39015}{630784}f_n+\frac{8469}{3153920}f_{n+1}+\frac{18531}{197120}f_{n+\frac{1}{4}}+\right.\\
&\left.\frac{3159}{35840}f_{n+\frac{1}{2}}+\frac{6813}{197120}f_{n+\frac{3}{4}}\right]+h^3\left[\frac{9747}{6307840}g_n-\frac{27}{180224}g_{n+1}-\right.\\
&\left.\frac{4509}{394240}g_{n+\frac{1}{4}}-\frac{19197}{1576960}g_{n+\frac{1}{2}}-\frac{9}{2464}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} &= y_n+hy'_n+h^2\left[\frac{6353}{74844}f_n+\frac{3457}{374220}f_{n+1}+\frac{13952}{93555}f_{n+\frac{1}{4}}+\right.\\
&\left.\frac{52}{315}f_{n+\frac{1}{2}}+\frac{8576}{93555}f_{n+\frac{3}{4}}\right]+h^3\left[\frac{269}{124740}g_n-\frac{5}{12474}g_{n+1}-\right.\\
&\left.\frac{464}{31185}g_{n+\frac{1}{4}}-\frac{10}{693}g_{n+\frac{1}{2}}-\frac{16}{4455}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{1}{4}} &= y'_n+h\left[\frac{1539551}{17418240}f_nh+\frac{59681}{17418240}f_{n+1}+\frac{89371}{1088640}f_{n+\frac{1}{4}}+\right.\\
&\left.\frac{103}{2520}f_{n+\frac{1}{2}}+\frac{38341}{1088640}f_{n+\frac{3}{4}}\right]+h^2\left[\frac{26051}{11612160}g_n-\frac{2237}{11612160}g_{n+1}-\right.\\
&\left.\frac{31207}{1451520}g_{n+\frac{1}{4}}-\frac{81}{5120}g_{n+\frac{1}{2}}-\frac{1243}{290304}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{1}{2}} &= y'_n+h\left[\frac{24463}{272160}f_nh+\frac{1153}{272160}f_{n+1}+\frac{1654}{8505}f_{n+\frac{1}{4}}+\frac{52}{315}f_{n+\frac{1}{2}}\right.\\
&\left.+\frac{394}{8505}f_{n+\frac{3}{4}}\right]+h^2\left[\frac{421}{181440}g_n-\frac{43}{181440}g_{n+1}-\frac{19}{1134}g_{n+\frac{1}{4}}-\right.\\
&\left.\frac{1}{40}g_{n+\frac{1}{2}}-\frac{31}{5670}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{3}{4}} &= y'_n + h\left[\frac{6501}{71680}f_n + \frac{411}{71680}f_{n+1} + \frac{921}{4480}f_{n+\frac{1}{4}} + \frac{81}{280}f_{n+\frac{1}{2}} + \right. \\
&\left. \frac{711}{4480}f_{n+\frac{3}{4}}\right] + h^2\left[\frac{339}{143360}g_n - \frac{9}{28672}g_{n+1} - \frac{279}{17920}g_{n+\frac{1}{4}} - \right. \\
&\left. \frac{81}{5120}g_{n+\frac{1}{2}} - \frac{183}{17920}g_{n+\frac{3}{4}}\right]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+1} &= y'_n + h\left[\frac{1601}{17010}f_n + \frac{1601}{17010}f_{n+1} + \frac{2048}{8505}f_{n+\frac{1}{4}} + \frac{104}{315}f_{n+\frac{1}{2}} + \frac{2048}{8505}f_{n+\frac{3}{4}}\right] \\
&+h^2\left[\frac{29}{11340}g_n - \frac{29}{11340}g_{n+1} - \frac{32}{2835}g_{n+\frac{1}{4}} + \frac{32}{2835}g_{n+\frac{3}{4}}\right].
\end{aligned}
$$

The HBMs obtained in this section are the examples of HBMs with one, two and three specific off-step points obtained by substituting $r = \frac{1}{3}$ in (3.14)-(3.17), $r = \frac{1}{5}$ and $s = \frac{3}{5}$ in (3.41)-(3.46), $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ in (3.64)-(3.71) respectively. By replacing different values of r, s and t in the corresponding equations gives HBMs with different specific off-step point(s).

3.7 Implementation of the Developed Methods

To implement the developed one-step HBMs for directly solving both IVPs and BVPs of higher ODEs simultaneously, we first combine all derived hybrid integrators. Then, the initial conditions at $x_n; n = 0, 1, ...N-1$ are obtained using the computed value $y_{n+1} = y(x_{n+1})$ over the subintervals $[x_0, x_1], ..., [x_{N-1}, x_N]$. For example, using Equation (3.87), at $n = 0$ we have $\{y_{\frac{1}{3}}, y_1, y'_{\frac{1}{3}}, y'_1\}$ where $y_0^i(x_0)$ for $i = 0, 1$ are given over the subinterval $[x_0, x_1]$ and $\{y''_{\frac{1}{3}} = f_{\frac{1}{3}}, y''_1 = f_1, y'''_{\frac{1}{3}} = g_{\frac{1}{3}}, y'''_1 = g_1\}$ are obtained by substituting the values of the initial conditions in the corresponding functions of f and g. Similarly, for $n = 1$, we obtain $\{y_{\frac{4}{3}}, y_2, y'_{\frac{4}{3}} = y'_2\}$ over the subinterval $[x_1, x_2]$ where $y_1^i(x_1)$ for $i = 0, 1$ are obtained from the previous block. This process is repeated until we reach the last interval $[x_{N-1}, x_N]$ to obtain $\{y_{N-\frac{2}{3}}, y_N, y'_{N-\frac{2}{3}}, y'_N\}$ without the need of using predictors.

3.8 Test Problems and Numerical Results

The following different types of IVPs and BVPs were tested using the developed HBMs in this chapter:

Problem 1: $y'' + xy = (3 - x - x^2 + x^3)\sin(x) + 4\cos(x),$

$y'(0) = -1,\ y'(1) = 2\sin(1),\ x \in [0,1].$

Exact solution: $y(x) = (x^2 - 1)\sin(x)$

Source: (Liu et al., 2011)

Problem 2: $y'' - x(y')^2 = 0, \quad y(0) = 1, \quad y'(0) = \frac{1}{2},\ x \in [0,1].$

Exact solution: $y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right)$

Source: (Kayode & Obarhua, 2015)

Problem 3: $y'' = 2y^3, \quad y(1) = 1, \quad y'(1) = -1,\ x \in [1,2].$

Exact solution: $y(x) = \frac{1}{x}$

Source: (Yahaya et al., 2013)

Problem 4: $y'' - y' = 0, \quad y(0) = 0, \quad y'(0) = -1,\ x \in [0,1].$

Exact solution: $y(x) = 1 - e^x$

Source: (Adeniyi & Adeyefa, 2013)

Problem 5: $y'' = y + 4x - 5, \quad y(0) = 0, \quad y(1) = 0,\ x \in [0,1].$

Exact solution: $y(x) = \frac{1}{e^{-1} - e}[(5e-1)e^{-x} - (5e^{-1} - 1)e^x] - 4x + 5$

Source: (Sagir, 2013a)

Problem 6: $y'' = -e^{-2y}, \quad y'(0) = 1, \quad y'(1) = \frac{1}{2},\ x \in [0,1].$

Exact solution: $y(x) = \ln(1+x)$

Source: (Phang et al., 2013)

Problem 7: $y^{''} = -y^2 + (\sin(\pi x))^2 - \pi^2 \sin(\pi x), \quad y(0) = 0,$

$y(1) = 0,\ x \in [0,1].$

Exact solution: $y(x) = \sin(\pi x)$

Source: (Chen, 2011)

Even though the test problems considered in this section can be solved using the developed HBMs for larger intervals, for the sake of comparisons, only the numerical results on the same intervals as solved by the existing methods are reported. The tables below display the numerical results for Problem 1 until Problem 7 when solved using HBMs with one, two and three specific off-step point(s) in the presence of third derivative. The numerical results obtained using the existing methods for solving the same problems are also given for the purpose of comparison in terms of errors. The following notations are used in the tables:

h	:	Step size used
TOL	:	Tolerance
N/A	:	Numerical results of the previous methods are not available
Error	:	Absolute error for the employed method
Max Error	:	Maximum absolute error for the employed method

The absolute and maximum errors are defined as below :

$$\text{Error} = \mid (y(x_i) - y_i) \mid$$

$$\text{Max Error} = \text{maximum}(\mid (y(x_i) - y_i) \mid)$$

where $y(x_i)$ and y_i are the exact and approximate values at x_i respectively.

Table 3.4
Comparison of One-Step HBMs with PSM of order 6 for Solving Problem 1

h		HBM with $r = \frac{1}{2}$	HBM with $r = \frac{1}{3}, s = \frac{2}{3}$	HBM with $r = \frac{1}{4}, s = \frac{1}{2}, t = \frac{3}{4}$	PSM
$\frac{1}{8}$	Exact solution	0.000000000000000000	-0.066765667575886617	-0.363632245398905480	N/A
	Computed solution	0.000000000406772995	-0.066765667575976129	-0.363632245398904990	N/A
	Max Error	4.067E(-10)	8.951E(-14)	4.996E(-16)	2.224E(-04)
$\frac{1}{16}$	Exact solution	0.000000000000000000	-0.066765667575886617	-0.025825598463905121	N/A
	Computed Solution	0.000000000006351082	-0.066765667575964763	-0.025825598463905176	N/A
	Max Error	6.351E(-12)	7.814E(-14)	5.551E(-17)	5.045E(-06)
$\frac{1}{32}$	Exact solution	0.000000000000000000	-0.017321753633322140	-0.280693164920194970	N/A
	Computed solution	0.000000000000099668	-0.017321753633398950	-0.280693164920195080	N/A
	Max Error	9.966E(-14)	7.681E(-14)	1.110E(-16)	1.625E(-07)
$\frac{1}{64}$	Exact solution	-0.352401702248770600	0.000000000000000000	-0.294841112992959560	N/A
	Computed solution	-0.352401702248768710	-0.000000000000076281	-0.294841112992959950	N/A
	Max Error	1.887E(-15)	7.628E(-14)	3.885E(-16)	5.577E(-09)
$\frac{1}{128}$	Exact Solution	0.000000000000000000	-0.004369621178635001	-0.363497882513640510	N/A
	Computed solution	0.000000000000000158	-0.004369621178710887	-0.363497882513641070	N/A
	Max Error	1.581E(-16)	7.588E(-14)	5.551E(-16)	1.892E(-10)

Table 3.5

Comparison of One-Step HBMs with TSYFHM of order 7 for Solving Problem 2

x		HBM with $r=\frac{1}{5}$	HBM with $r=\frac{2}{7}, s=\frac{4}{7}$	HBM with $r=\frac{1}{4}, s=\frac{1}{2}, t=\frac{3}{4}$	TSYFHM
0.1	Exact solution	1.050041729278	1.050041729278	1.050041729278	1.050041729278
	Computed solution	1.050041729278	1.050041729278	1.050041729278	1.050041729281
	Error	4.440892E(-16)	4.440892E(-16)	4.440892E(-16)	2.312595E(-12)
0.2	Exact solution	1.100335347731	1.100335347731	1.100335347731	1.100335347731
	Computed Solution	1.100335347731	1.100335347731	1.100335347731	1.100335347742
	Error	2.220446E(-16)	2.220446E(-16)	4.440892E(-16)	1.088329E(-11)
0.3	Exact solution	1.151140435936	1.151140435936	1.151140435936	1.151140435936
	Computed solution	1.151140435936	1.151140435936	1.151140435936	1.151140435961
	Error	4.440892E(-16)	4.440892E(-16)	6.661338E(-16)	2.430833E(-11)
0.4	Exact solution	1.202732554054	1.202732554054	1.202732554054	1.202732554054
	Computed solution	1.202732554054	1.202732554054	1.202732554054	1.202732554094
	Error	4.440892E(-16)	4.440892E(-16)	8.881784E(-16)	4.018186E(-11)
0.5	Exact Solution	1.255412811883	1.255412811883	1.255412811883	1.255412811883
	Computed solution	1.255412811882	1.255412811882	1.255412811882	1.255412811937
	Error	6.661338E(-16)	6.661338E(-16)	1.332268E(-15)	5.422818E(-11)
0.6	Exact Solution	1.309519604203	1.309519604203	1.309519604203	1.309519604203
	Computed solution	1.309519604203	1.309519604203	1.309519604203	1.309519604262
	Error	4.440892E(-16)	4.440892E(-16)	2.220446E(-15)	5.901679E(-11)
0.7	Exact solution	1.365443754271	1.365443754271	1.365443754271	1.365443754271
	Computed solution	1.365443754271	1.365443754271	1.365443754271	1.365443754313
	Error	4.440892E(-16)	4.440892E(-16)	3.330669E(-15)	4.161738E(-11)
0.8	Exact solution	1.423648930193	1.423648930193	1.423648930193	1.423648930193
	Computed solution	1.423648930193	1.423648930193	1.423648930193	1.423648930173
	Error	6.661338E(-16)	6.661338E(-16)	5.995204E(-15)	2.077827E(-11)
0.9	Exact solution	1.484700278594	1.484700278594	1.484700278594	1.484700278594
	Computed Solution	1.484700278594	1.484700278594	1.484700278594	1.484700278425
	Error	6.661338E(-16)	6.661338E(-16)	8.215650E(-15)	1.692806E(-10)
1.0	Exact solution	1.549306144334	1.549306144334	1.549306144334	1.549306144334
	Computed solution	1.549306144334	1.549306144334	1.549306144334	1.549306143854
	Error	8.881784E(-16)	6.661338E(-16)	1.265654E(-14)	4.802496E(-10)

Table 3.6

Comparison of One-Step HBMs with TSIHM of order 5 for Solving Problem 3

x		HBM with $r=\frac{11}{20}$	HBM with $r=\frac{4}{9}$, $s=\frac{7}{9}$	HBM with $r=\frac{1}{5}$, $s=\frac{2}{5}$, $t=\frac{3}{5}$	TSIHM
	Exact solution	0.909090909	0.909090909	0.909090909	0.909090109
1.1	Computed solution	0.909090908	0.909090909	0.909090909	0.9090914826
	Error	2.80456E(-10)	5.28466E(-13)	5.88418E(-15)	1.37360E(-6)
	Exact solution	0.833333333	0.833333333	0.833333333	0.833333333
1.2	Computed Solution	0.833333332	0.833333333	0.833333333	0.8333348875
	Error	8.82586E(-10)	1.42497E(-12)	7.10542E(-15)	1.55450E(-6)
	Exact solution	0.769230769	0.769230769	0.769230769	0.769230769
1.3	Computed solution	0.769230767	0.769230769	0.769230769	0.7692330259
	Error	1.66058E(-9)	2.51276E(-12)	4.44089E(-15)	2.25690E(-6)
	Exact solution	0.714285714	0.714285714	0.714285714	0.714285714
1.4	Computed solution	0.714285711	0.714285714	0.714285714	0.7142880945
	Error	2.56463E(-9)	3.74489E(-12)	1.33226E(-15)	2.38050E(-6)
	Exact Solution	0.666666667	0.666666667	0.666666667	0.666666667
1.5	Computed solution	0.666666663	0.666666666	0.666666666	0.6666693006
	Error	3.58136E(-9)	5.11646E(-12)	9.65894E(-15)	2.63360E(-6)
	Exact Solution	0.625	0.625	0.625	0.625
1.6	Computed solution	0.624999995	0.624999999	0.625000000	0.625002904
	Error	4.71185E(-9)	6.63602E(-12)	2.04281E(-14)	2.90400E(-6)
	Exact solution	0.588235294	0.588235294	0.588235294	0.588235294
1.7	Computed solution	0.588235288	0.588235294	0.588235294	0.5882382492
	Error	5.96292E(-9)	8.31712E(-12)	3.33066E(-14)	2.95520E(-6)
	Exact solution	0.555555556	0.555555556	0.555555556	0.555555556
1.8	Computed solution	0.555555548	0.555555555	0.555555555	0.5555586357
	Error	7.34364E(-9)	1.01739E(-11)	4.82947E(-14)	3.07970E(-6)
	Exact solution	0.526315789	0.526315789	0.526315789	0.526315789
1.9	Computed Solution	0.526315780	0.526315789	0.526315789	0.5263190397
	Error	8.86383E(-9)	1.22210E(-11)	6.55031E(-14)	3.25070E(-6)
	Exact solution	0.5	0.5	0.5	0.5
2.0	Computed solution	0.499999989	0.499999999	0.500000000	0.5000032814
	Error	1.05334E(-8)	1.44722E(-11)	8.47100E(-14)	3.28140E(-6)

Table 3.7

Comparison of One-Step HBMs with THOSBM of order 5 for Solving Problem 4

x		HBM with $r=\frac{2}{5}$	HBM with $r=\frac{1}{3}$, $s=\frac{2}{3}$	HBM with $r=\frac{1}{4}$, $s=\frac{1}{2}$, $t=\frac{3}{4}$	THOSBM
0.1	Exact solution	-0.105170918075647710	-0.105170918075647710	-0.105170918075647710	-0.105170918075647710
	Computed solution	-0.105170918075639820	-0.105170918075647550	-0.105170918075647620	-0.105170918075555580
	Error	7.896461E(-15)	1.665335E(-16)	9.714451E(-17)	9.213463E(-14)
0.2	Exact solution	-0.221402758160169850	-0.221402758160169850	-0.221402758160169850	-0.221402758160169850
	Computed Solution	-0.221402758160129250	-0.221402758160169550	-0.221402758160169800	-0.221402758011792180
	Error	4.060641E(-14)	3.053113E(-16)	5.551115E(-17)	1.483777E(-10)
0.3	Exact solution	-0.349858807576003180	-0.349858807576003180	-0.349858807576003180	-0.349858807576003180
	Computed solution	-0.349858807575899600	-0.349858807576002460	-0.349858807576003020	-0.349858807099976080
	Error	1.035838E(-13)	7.216450E(-16)	1.665335E(-16)	4.760271E(-10)
0.4	Exact solution	-0.491824697641270350	-0.491824697641270350	-0.491824697641270350	-0.491824697641270350
	Computed solution	-0.648721270699783580	-0.491824697641269130	-0.491824697641270240	-0.491824696622142730
	Error	3.446132E(-13)	1.221245E(-15)	1.110223E(-16)	1.019128E(-9)
0.5	Exact Solution	-0.648721270700128190	-0.648721270700128190	-0.648721270700128190	-0.648721270700128190
	Computed solution	-0.648721270699783580	-0.648721270700126200	-0.648721270700127970	-0.648721268880754320
	Error	3.446132E(-13)	1.998401E(-15)	2.220446E(-16)	1.819374E(-9)
0.6	Exact Solution	-0.822118800390508890	-0.822118800390508890	-0.822118800390508890	-0.822118800390508890
	Computed solution	-0.822118800389971430	-0.822118800390506000	-0.822118800390508770	-0.822118797465661970
	Error	5.374590E(-13)	2.886580E(-15)	1.110223E(-16)	2.924847E(-9)
0.7	Exact solution	-1.013752707470476600	-1.013752707470476600	-1.013752707470476600	-1.013752707470476600
	Computed solution	-1.013752707469685900	-1.013752707470472200	-1.013752707470476400	-1.013752703079578500
	Error	7.907008E(-13)	4.440892E(-15)	2.220446E(-16)	4.390898E(-9)
0.8	Exact solution	-1.225540928492467400	-1.225540928492467400	-1.225540928492467400	-1.225540928492467400
	Computed solution	-1.225540928491353200	-1.225540928492461700	-1.225540928492467400	-1.225540922211324500
	Error	1.114220E(-12)	5.773160E(-15)	0.000000E(+00)	6.281143E(-9)
0.9	Exact solution	-1.459603111156949400	-1.459603111156949400	-1.459603111156949400	-1.459603111156949400
	Computed Solution	-1.459603111155428800	-1.459603111156941400	-1.459603111156949400	-1.459603102488357100
	Error	1.520561E(-12)	7.993606E(-15)	0.000000E(+00)	8.668592E(-9)
1.0	Exact solution	-1.718281828459045100	-1.718281828459045100	-1.718281828459045100	-1.718281828459045100
	Computed solution	-1.718281828457021600	-1.718281828459034400	-1.718281828459044900	-1.718281816822123900
	Error	2.023492E(-12)	1.065814E(-14)	2.220446E(-16)	1.163692E(-8)

Table 3.8

Comparison of One-Step HBMs with TPMNBM of order 4 for Solving Problem 5

x		HBM with $r=\frac{1}{2}$	HBM with $r=\frac{1}{3}$, $s=\frac{2}{3}$	HBM with $r=\frac{1}{4}$, $s=\frac{1}{2}$, $t=\frac{3}{4}$	TPMNBM
0.1	Exact solution	0.14735784233	0.14735784233	0.14735784233	0.14735784233
	Computed solution	0.14735784233	0.14735784233	0.14735784233	0.1473578176
	Error	2.263704E(-13)	1.295649E(-15)	1.725558E(-16)	2.470000E(-08)
0.2	Exact solution	0.25015214536	0.25015214536	0.25015214536	0.25015214536
	Computed Solution	0.25015214536	0.25015214536	0.25015214536	0.25015214536
	Error	3.810540E(-13)	2.060034E(-15)	1.728474E(-16)	6.820000E(-08)
0.3	Exact solution	0.31341504347	0.31341504347	0.31341504347	0.31341504347
	Computed solution	0.31341504347	0.31341504347	0.31341504347	0.3134149552
	Error	4.742214E(-13)	2.634997E(-15)	2.924532E(-16)	8.820000E(-08)
0.4	Exact solution	0.34178302746	0.34178302746	0.34178302746	0.34178302746
	Computed solution	0.34178302746	0.34178302746	0.34178302746	0.3417829037
	Error	5.139031E(-13)	2.488207E(-15)	4.526000E(-17)	1.237000E(-07)
0.5	Exact Solution	0.33954334808	0.33954334808	0.33954334808	0.33954334808
	Computed solution	0.33954334808	0.33954334808	0.33954334808	0.3395432096
	Error	5.084271E(-13)	2.487851E(-15)	1.088123E(-17)	1.385000E(-07)
0.6	Exact Solution	0.31067692433	0.31067692433	0.31067692433	0.31067692433
	Computed solution	0.31067692433	0.31067692433	0.31067692433	0.3106767591
	Error	4.632974E(-13)	1.636319E(-15)	6.360239E(-16)	1.652000E(-07)
0.7	Exact solution	0.25889818576	0.25889818576	0.25889818576	0.25889818576
	Computed solution	0.25889818576	0.25889818576	0.25889818576	0.2588980094
	Error	3.870152E(-13)	2.760667E(-15)	8.764485E(-16)	1.763000E(-07)
0.8	Exact solution	0.18769224781	0.18769224781	0.18769224781	0.18769224781
	Computed solution	0.18769224781	0.18769224781	0.18769224781	0.18769224781
	Error	2.802208E(-13)	1.536332E(-15)	1.755215E(-16)	1.966000E(-07)
0.9	Exact solution	0.10034979196	0.10034979196	0.10034979196	0.10034979196
	Computed Solution	0.10034979196	0.10034979196	0.10034979196	0.10034979196
	Error	1.501644E(-13)	8.508733E(-16)	1.251965E(-16)	2.045000E(-07)
1.0	Exact solution	0.00000000000	0.00000000000	0.00000000000	0.00000000000
	Computed solution	0.00000000000	0.00000000000	0.00000000000	-0.0000002194
	Error	0.000000E(+00)	0.000000E(+00)	0.000000E(+00)	2.194000E(-07)

Table 3.9

Comparison of One-Step HBMs with DMVSS for Solving Problem 6

TOL		HBM with $r=\frac{7}{9}$	HBM with $r=\frac{2}{5}, s=\frac{4}{5}$	HBM with $r=\frac{1}{7}, s=\frac{3}{7}, t=\frac{5}{7}$	DMVSS
10^{-3}	Exact solution	0.693147180559945180	0.693147180559945180	0.693147180559945180	N/A
	Computed solution	0.693147138726289860	0.693147164557371350	0.693147164572091800	N/A
	Max Error	4.18E(-08)	1.60E(-08)	1.55E(-08)	3.90E(-05)
10^{-5}	Exact solution	0.693147180559945180	0.693147180559945180	0.693147180559945180	N/A
	Computed Solution	0.693147138726289860	0.693147164557371350	0.693147164572091800	N/A
	Max Error	4.18E(-08)	1.60E(-08)	1.55E(-08)	4.70E(-06)
10^{-7}	Exact solution	0.693147180559945180	0.693147180559945180	0.693147180559945180	N/A
	Computed solution	0.693147138726289860	0.693147164557371350	0.693147164572091800	N/A
	Max Error	4.18E(-08)	1.60E(-08)	1.55E(-08)	2.50E(-07)

Table 3.10

Comparison of One-Step HBMs with FMAM for Solving Problem 7

TOL		HBM with $r=\frac{3}{4}$	HBM with $r=\frac{1}{7}, s=\frac{5}{7}$	HBM with $r=\frac{3}{8}, s=\frac{5}{8}, t=\frac{7}{8}$	FMAM
10^{-2}	Exact solution	-0.000000000000000766	-0.000000000000000766	-0.000000000000000766	N/A
	Computed solution	-0.000000158557741449	-0.000000158557777923	-0.000000158557778328	N/A
	Max Error	1.58E(-07)	1.58E(-07)	1.58E(-07)	2.46E(-04)
10^{-4}	Exact solution	0.000000000000000122	0.000000000000000122	0.000000000000000122	N/A
	Computed Solution	-0.000000158557773683	-0.000000158557778257	-0.000000158557777220	N/A
	Max Error	1.58E(-07)	1.58E(-07)	1.58E(-07)	3.94E(-05)
10^{-6}	Exact solution	0.000000000000001455	0.000000000000001455	0.000000000000001455	N/A
	Computed solution	-0.000000158557777384	-0.000000158557777511	-0.000000158557776918	N/A
	Max Error	1.58E(-07)	1.58E(-07)	1.58E(-07)	4.94E(-06)
10^{-8}	Exact solution	1.000000000000000000	1.000000000000000000	0.195090322016128610	N/A
	Computed Solution	1.000000000004933200	0.999999999999993340	0.195090322016128300	N/A
	Max Error	4.93E(-12)	6.66E(-15)	3.05E(-16)	6.17E(-07)
10^{-10}	Exact solution	1.000000000000000000	1.000000000000000000	0.000000000000003675	N/A
	Computed solution	1.000000000000214300	0.999999999999993890	-0.000000000000000274	N/A
	Max Error	2.14E(-13)	6.10E(-15)	3.94E(-15)	5.00E(-08)

3.9 Comments on the Results

In Problem 1, three new one-step HBMs in the presence of third derivative were employed to solve a linear BVP of second order ODE using five different step sizes as shown in Table 3.4. The numerical results indicate that as the step size h gets smaller, the errors produced by all three HBMs also get smaller. It should be mentioned that the maximum error for the methods considered may happen at different points throughout the entire interval, therefore the exact solution obtained need not be the same for all methods. In general, it can be seen that among the three developed methods, HBM with three off-step points performs the best with the least error. Furthermore, the results also suggest that all HBMs outperform the existing method in terms of maximum error.

One-step HBMs with different one, two and three specific off-step points were employed to solve a non-linear IVP of second order ODE in Problem 2. This time, the value of step size h was fixed at $\frac{1}{100}$ while the errors were recorded at every increment value of x from 0 to 1.

Based on the results displayed in Table 3.5, although all HBMs produce smaller errors compared to the existing TSYFHM method, the results show that HBMs with one and two off-step point(s) perform the best and their accuracies are comparable. Surprisingly, the accuracy produced by HBM with three off-step points starts to drop when $x = 0.5$, though it still performs better than the existing method considered. It should be mentioned that we only displayed the results for $x = 0.1(0.1)1$ in Table 3.5 for comparison reasons.

The numerical results for solving two IVPs of second order ODEs in Problems 3-4 using $h = \frac{1}{10}$ reveal the superiority of one-step HBM with three off-points in terms of accuracy. Even though one-step HBMs with one and two off-step point(s) produce larger errors than their three off-step points counterpart, their performances are still better than the existing method.

Table 3.8 clearly demonstrates the advantages of new HBMs over the existing method for solving linear BVP of second order ODEs directly. All new HBMs are capable of approximating the numerical solutions more accurate than TPMNBM method. It is also discovered that among the HBMs, the performance of one-step HBM with three off-step points is the best.

Numerical results shown in Tables 3.9-3.10 confirm the superiority of the new HBMs over the existing methods when solving the two nonlinear BVP corresponding to different TOL values. As observed, the maximum errors for the new HBMs remain the same regardless of the tolerance when solving Problem 6. However, in Problem 7 the maximum errors for HBMs are the same for the TOL values 10^{-2}, 10^{-4} and 10^{-6}, but decrease drastically for the TOL values 10^{-8} and 10^{-10}.

3.10 Summary

One-step HBMs with generalized one, two and three off-step point(s) in the presence of third derivative using interpolation and collocation approach have been successfully developed. Besides possessing good properties of numerical methods, the new methods are capable of solving both IVPs and BVPs of second order ODEs directly with better accuracy if compared to the existing methods.

CHAPTER FOUR

ONE-STEP HYBRID BLOCK METHODS FOR DIRECTLY SOLVING THIRD ORDER ODES IN THE PRESENCE OF FOURTH DERIVATIVE

4.1 Introduction

In this chapter, the development of one-step HBMs with generalised two and three off-step points using collocation and interpolation method for solving third order IVPs BVPs of ODEs is considered.

4.2 Derivation of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

Equation (3.2) is also used to approximate the solution of the general third order ODEs of the form

$$y^{'''}(x) = f(x, y, y^{'}, y^{''}), \quad y^{(i)}(a) = \omega_i, \quad i = 0, 1, 2. \tag{4.1}$$

The third and fourth derivatives of Equation (3.2) are

$$y'''(x) = \sum_{j=3}^{2v+u-1} a_j \frac{j!}{h^3(j-3)!} (\frac{x - x_n}{h})^{j-3}, \tag{4.2}$$

$$y^{(iv)}(x) = \sum_{j=4}^{2v+u-1} a_j \frac{j!}{h^4(j-4)!} (\frac{x - x_n}{h})^{j-4}. \tag{4.3}$$

Substituting Equation (4.1) into Equation (4.2) yields

$$\sum_{j=3}^{2v+u-1} a_j \frac{j!}{h^3(j-3)!} (\frac{x - x_n}{h})^{j-3} = f(x, y, y^{'}, y^{''}), \tag{4.4}$$

while its derivative is

$$\sum_{j=4}^{2v+u-1} a_j \frac{j!}{h^4(j-4)!} (\frac{x - x_n}{h})^{j-4} = g(x, y, y^{'}, y^{''}). \tag{4.5}$$

In deriving the continuous scheme for third order ODEs, the approximate solution (3.2) is interpolated at three points, i.e x_n, x_{n+r} and x_{n+s} where $0 < r < s < 1$, while (4.4) and (4.5) are collocated at all points as depicted in Figure 4.1 below :

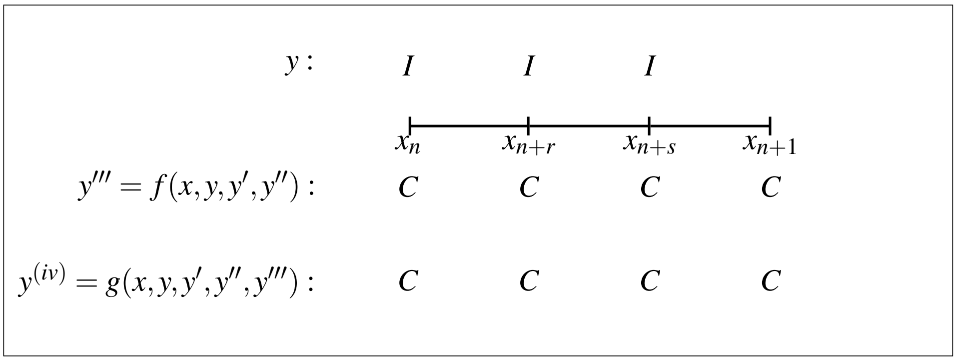


Figure 4.1. Interpolation and Collocation Strategy for One-Step HBM with Two Off-Step Points for Solving Third Order ODEs

From Figure 4.1, $u = 3$ and $v = 4$. This interpolation and collocation approach produces the following equations

$$
\begin{aligned}
y_n &= a_0, \\
y_{n+r} &= a_0 + ra_1 + r^2a_2 + r^3a_3 + r^4a_4 + r^5a_5 + r^6a_6 + r^7a_7 + r^8a_8 + r^9a_9 + r^{10}a_{10}, \\
y_{n+s} &= a_0 + sa_1 + s^2a_2 + s^3a_3 + s^4a_4 + s^5a_5 + s^6a_6 + s^7a_7 + s^8a_8 + s^9a_9 + s^{10}a_{10}, \\
f_n &= \frac{6}{h^3}a_3, \\
f_{n+r} &= \frac{6}{h^3}a_3 + \frac{24r}{h^3}a_4 + \frac{60r^2}{h^3}a_5 + \frac{120r^3}{h^3}a_6 + \frac{210r^4}{h^3}a_7 + \frac{336r^5}{h^3}a_8 + \frac{504r^6}{h^3}a_9 \\
&\quad + \frac{720r^7}{h^3}a_{10}, \\
f_{n+s} &= \frac{6}{h^3}a_3 + \frac{24s}{h^3}a_4 + \frac{60s^2}{h^3}a_5 + \frac{120s^3}{h^3}a_6 + \frac{210s^4}{h^3}a_7 + \frac{336s^5}{h^3}a_8 + \frac{504s^6}{h^3}a_9 \\
&\quad + \frac{720s^7}{h^3}a_{10}, \\
f_{n+1} &= \frac{6}{h^3}a_3 + \frac{24}{h^3}a_4 + \frac{60}{h^3}a_5 + \frac{120}{h^3}a_6 + \frac{210}{h^3}a_7 + \frac{336}{h^3}a_8 + \frac{504}{h^3}a_9 + \frac{720}{h^3}a_{10}, \\
g_n &= \frac{24}{h^4}a_4,
\end{aligned}
$$

$$
\begin{aligned}
g_{n+r} &= \frac{24}{h^4}a_4+\frac{120r}{h^4}a_5+\frac{360r^2}{h^4}a_6+\frac{840r^3}{h^4}a_7+\frac{1680r^4}{h^4}a_8+\frac{3024r^5}{h^4}a_9 \\
&\quad +\frac{5040r^6}{h^4}a_{10}, \\
g_{n+s} &= \frac{24}{h^4}a_4+\frac{120s}{h^4}a_5+\frac{360s^2}{h^4}a_6+\frac{840s^3}{h^4}a_7+\frac{1680s^4}{h^4}a_8+\frac{3024s^5}{h^4}a_9 \\
&\quad +\frac{5040s^6}{h^4}a_{10}, \\
g_{n+1} &= \frac{24}{h^4}a_4+\frac{120}{h^4}a_5+\frac{360}{h^4}a_6+\frac{840}{h^4}a_7+\frac{1680}{h^4}a_8+\frac{3024}{h^4}a_9+\frac{5040}{h^4}a_{10},
\end{aligned}
$$

which can be written in matrix form $AX = B$, where

$$
A = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} \\
1 & s & s^2 & s^3 & s^4 & s^5 & s^6 & s^7 & s^8 & s^9 & s^{10} \\
0 & 0 & 0 & \frac{6}{h^3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24r}{h^3} & \frac{60r^2}{h^3} & \frac{120r^3}{h^3} & \frac{210r^4}{h^3} & \frac{336r^5}{h^3} & \frac{504r^6}{h^3} & \frac{720r^7}{h^3} \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24s}{h^3} & \frac{60s^2}{h^3} & \frac{120s^3}{h^3} & \frac{210s^4}{h^3} & \frac{336s^5}{h^3} & \frac{504s^6}{h^3} & \frac{720s^7}{h^3} \\
0 & 0 & 0 & \frac{6}{h^3} & \frac{24}{h^3} & \frac{60}{h^3} & \frac{120}{h^3} & \frac{210}{h^3} & \frac{336}{h^3} & \frac{504}{h^3} & \frac{720}{h^3} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120r}{h^4} & \frac{360r^2}{h^4} & \frac{840r^3}{h^4} & \frac{1680r^4}{h^4} & \frac{3024r^5}{h^4} & \frac{5040r^6}{h^4} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120s}{h^4} & \frac{360s^2}{h^4} & \frac{840s^3}{h^4} & \frac{1680s^4}{h^4} & \frac{3024s^5}{h^4} & \frac{5040s^6}{h^4} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120}{h^4} & \frac{360}{h^4} & \frac{840}{h^4} & \frac{1680}{h^4} & \frac{3024}{h^4} & \frac{5040}{h^4}
\end{pmatrix},
$$

$X = [a_0,\, a_1,\, a_2,\, a_3,\, a_4,\, a_5,\, a_6,\, a_7,\, a_8,\, a_9,\, a_{10}]^T$,

$B = [y_n,\, y_{n+r},\, y_{n+s},\, f_n,\, f_{n+r},\, f_{n+s},\, f_{n+1},\, g_n,\, g_{n+r},\, g_{n+s},\, g_{n+1}]^T$.

Solving the system $AX = B$ using Gaussian elimination method. Then, the values of a_j's, $j = 0(1)10$ are substituted into Equation (3.2) to produce a continuous implicit scheme of the form

$$
y(x) = \sum_{i=0,r,s}\alpha_i(x)y_{n+i}+\sum_{i=0}^{1}\beta_i(x)f_{n+i}+\sum_{i=r,s}\beta_i(x)f_{n+i}+\sum_{i=0}^{1}\gamma_i(x)g_{n+i}+\sum_{i=r,s}\gamma_i(x)g_{n+i}. \tag{4.6}
$$

Calculating the first and second derivatives of Equation (4.6) gives

$$
\begin{aligned}
y^{'}(x) &= \frac{d}{dx}[\sum_{i=0,r,s}\alpha_i(x)y_{n+i}+\sum_{i=0}^{1}\beta_i(x)f_{n+i}+\sum_{i=r,s}\beta_i(x)f_{n+i}+\sum_{i=0}^{1}\gamma_i(x)g_{n+i} \\
&\quad +\sum_{i=r,s}\gamma_i(x)g_{n+i}] \qquad (4.7) \\
y^{''}(x) &= \frac{d^2}{dx^2}[\sum_{i=0,r,s}\alpha_i(x)y_{n+i}+\sum_{i=0}^{1}\beta_i(x)f_{n+i}+\sum_{i=r,s}\beta_i(x)f_{n+i}+\sum_{i=0}^{1}\gamma_i(x)g_{n+i} \\
&\quad +\sum_{i=r,s}\gamma_i(x)g_{n+i}] \qquad (4.8)
\end{aligned}
$$

where

$\alpha_0=\frac{(x_n-x+hr)(x_n-x+hs)}{h^2rs}$,

$\alpha_r=\frac{-(x-x_n)(x_n-x+hs)}{h^2r(r-s)}$,

$\alpha_s=\frac{(x-x_n)(x_n-x+hr)}{h^2s(r-s)}$,

$\beta_0 = -\frac{(x-x_n)(x_n-x+hr)(x_n-x+hs)}{2520h^7r^3s^3}(6h^7r^7s^2 - 2h^7r^8 - 2h^7r^8s + 10h^7r^7s + 8h^7r^7 -$
$3h^7r^6s^3 - 18h^7r^6s^2 - 20h^7r^6s - 9h^7r^6 - 3h^7r^5s^4 + 9h^7r^5s^3 - 8h^7r^5s^2 + 18h^7r^5s -$
$3h^7r^4s^5 + 9h^7r^4s^4 + 40h^7r^4s^3 + 66h^7r^4s^2 - 3h^7r^3s^6 + 9h^7r^3s^5 + 40h^7r^3s^4 -$
$270h^7r^3s^3 + 6h^7r^2s^7 - 18h^7r^2s^6 - 8h^7r^2s^5 + 66h^7r^2s^4 - 2h^7rs^8 + 10h^7rs^7 -$
$20h^7rs^6 + 18h^7rs^5 - 2h^7s^8 + 8h^7s^7 - 9h^7s^6 - 2h^6r^7sx + 2h^6r^7sx_n - 2h^6r^7x +$
$2h^6r^7x_n + 6h^6r^6s^2x - 6h^6r^6s^2x_n + 10h^6r^6sx - 10h^6r^6sx_n + 8h^6r^6x - 8h^6r^6x_n -$
$3h^6r^5s^3x + 3h^6r^5s^3x_n - 18h^6r^5s^2x + 18h^6r^5s^2x_n - 20h^6r^5sx + 20h^6r^5sx_n - 9h^6r^5x +$
$9h^6r^5x_n - 3h^6r^4s^4x + 3h^6r^4s^4x_n + 9h^6r^4s^3x - 9h^6r^4s^3x_n - 8h^6r^4s^2x + 8h^6r^4s^2x_n +$
$18h^6r^4sx - 18h^6r^4sx_n - 3h^6r^3s^5x + 3h^6r^3s^5x_n + 9h^6r^3s^4x - 9h^6r^3s^4x_n + 40h^6r^3s^3x -$
$40h^6r^3s^3x_n + 66h^6r^3s^2x - 66h^6r^3s^2x_n + 6h^6r^2s^6x - 6h^6r^2s^6x_n - 18h^6r^2s^5x +$
$18h^6r^2s^5x_n - 8h^6r^2s^4x + 8h^6r^2s^4x_n + 66h^6r^2s^3x - 66h^6r^2s^3x_n - 2h^6rs^7x + 2h^6rs^7x_n +$
$10h^6rs^6x - 10h^6rs^6x_n - 20h^6rs^5x + 20h^6rs^5x_n + 18h^6rs^4x - 18h^6rs^4x_n - 2h^6s^7x +$
$2h^6s^7x_n + 8h^6s^6x - 8h^6s^6x_n - 9h^6s^5x + 9h^6s^5x_n - 2h^5r^6sx^2 + 4h^5r^6sxx_n - 2h^5r^6sx_n^2 -$
$2h^5r^6x^2 + 4h^5r^6xx_n - 2h^5r^6x_n^2 + 6h^5r^5s^2x^2 - 12h^5r^5s^2xx_n + 6h^5r^5s^2x_n^2 + 10h^5r^5sx^2 -$
$20h^5r^5sxx_n + 10h^5r^5sx_n^2 + 8h^5r^5x^2 - 16h^5r^5xx_n + 8h^5r^5x_n^2 - 3h^5r^4s^3x^2 + 6h^5r^4s^3xx_n -$
$3h^5r^4s^3x_n^2 - 18h^5r^4s^2x^2 + 36h^5r^4s^2xx_n - 18h^5r^4s^2x_n^2 - 20h^5r^4sx^2 + 40h^5r^4sxx_n -$
$20h^5r^4sx_n^2 - 9h^5r^4x^2 + 18h^5r^4xx_n - 9h^5r^4x_n^2 - 3h^5r^3s^4x^2 + 6h^5r^3s^4xx_n - 3h^5r^3s^4x_n^2 +$

$9h^5r^3s^3x^2 - 18h^5r^3s^3xx_n + 9h^5r^3s^3x_n^2 - 8h^5r^3s^2x^2 + 16h^5r^3s^2xx_n - 8h^5r^3s^2x_n^2 + 18h^5r^3sx^2 - 36h^5r^3sxx_n + 18h^5r^3sx_n^2 + 6h^5r^2s^5x^2 - 12h^5r^2s^5xx_n + 6h^5r^2s^5x_n^2 - 18h^5r^2s^4x^2 + 36h^5r^2s^4xx_n - 18h^5r^2s^4x_n^2 - 8h^5r^2s^3x^2 + 16h^5r^2s^3xx_n - 8h^5r^2s^3x_n^2 - 18h^5r^2s^2x^2 + 36h^5r^2s^2xx_n - 18h^5r^2s^2x_n^2 - 2h^5rs^6x^2 + 4h^5rs^6xx_n - 2h^5rs^6x_n^2 + 10h^5rs^5x^2 - 20h^5rs^5xx_n + 10h^5rs^5x_n^2 - 20h^5rs^4x^2 + 40h^5rs^4xx_n - 20h^5rs^4x_n^2 + 18h^5rs^3x^2 - 36h^5rs^3xx_n + 18h^5rs^3x_n^2 - 2h^5s^6x^2 + 4h^5s^6xx_n - 2h^5s^6x_n^2 + 8h^5s^5x^2 - 16h^5s^5xx_n + 8h^5s^5x_n^2 - 9h^5s^4x^2 + 18h^5s^4xx_n - 9h^5s^4x_n^2 - 2h^4r^5sx^3 + 6h^4r^5sx^2x_n - 6h^4r^5sxx_n^2 + 2h^4r^5sx_n^3 - 2h^4r^5x^3 + 6h^4r^5x^2x_n - 6h^4r^5xx_n^2 + 2h^4r^5x_n^3 + 6h^4r^4s^2x^3 - 18h^4r^4s^2x^2x_n + 18h^4r^4s^2xx_n^2 - 6h^4r^4s^2x_n^3 + 10h^4r^4sx^3 - 30h^4r^4sx^2x_n + 30h^4r^4sxx_n^2 - 10h^4r^4sx_n^3 + 8h^4r^4x^3 - 24h^4r^4x^2x_n + 24h^4r^4xx_n^2 - 8h^4r^4x_n^3 - 3h^4r^3s^3x^3 + 9h^4r^3s^3x^2x_n - 9h^4r^3s^3xx_n^2 + 3h^4r^3s^3x_n^3 - 18h^4r^3s^2x^3 + 54h^4r^3s^2x^2x_n - 54h^4r^3s^2xx_n^2 + 18h^4r^3s^2x_n^3 - 20h^4r^3sx^3 + 60h^4r^3sx^2x_n - 60h^4r^3sxx_n^2 + 20h^4r^3sx_n^3 - 9h^4r^3x^3 + 27h^4r^3x^2x_n - 27h^4r^3xx_n^2 + 9h^4r^3x_n^3 + 6h^4r^2s^4x^3 - 18h^4r^2s^4x^2x_n + 18h^4r^2s^4xx_n^2 - 6h^4r^2s^4x_n^3 - 18h^4r^2s^3x^3 + 54h^4r^2s^3x^2x_n - 54h^4r^2s^3xx_n^2 + 18h^4r^2s^3x_n^3 - 56h^4r^2s^2x^3 + 168h^4r^2s^2x^2x_n - 168h^4r^2s^2xx_n^2 + 56h^4r^2s^2x_n^3 - 66h^4r^2sx^3 + 198h^4r^2sx^2x_n - 198h^4r^2sxx_n^2 + 66h^4r^2sx_n^3 - 2h^4rs^5x^3 + 6h^4rs^5x^2x_n - 6h^4rs^5xx_n^2 + 2h^4rs^5x_n^3 + 10h^4rs^4x^3 - 30h^4rs^4x^2x_n + 30h^4rs^4xx_n^2 - 10h^4rs^4x_n^3 - 20h^4rs^3x^3 + 60h^4rs^3x^2x_n - 60h^4rs^3xx_n^2 + 20h^4rs^3x_n^3 - 66h^4rs^2x^3 + 198h^4rs^2x^2x_n - 198h^4rs^2xx_n^2 + 66h^4rs^2x_n^3 - 2h^4s^5x^3 + 6h^4s^5x^2x_n - 6h^4s^5xx_n^2 + 2h^4s^5x_n^3 + 8h^4s^4x^3 - 24h^4s^4x^2x_n + 24h^4s^4xx_n^2 - 8h^4s^4x_n^3 - 9h^4s^3x^3 + 27h^4s^3x^2x_n - 27h^4s^3xx_n^2 + 9h^4s^3x_n^3 - 2h^3r^4sx^4 + 8h^3r^4sx^3x_n - 12h^3r^4sx^2x_n^2 + 8h^3r^4sxx_n^3 - 2h^3r^4sx_n^4 - 2h^3r^4x^4 + 8h^3r^4x^3x_n - 12h^3r^4x^2x_n^2 + 8h^3r^4xx_n^3 - 2h^3r^4x_n^4 + 6h^3r^3s^2x^4 - 24h^3r^3s^2x^3x_n + 36h^3r^3s^2x^2x_n^2 - 24h^3r^3s^2xx_n^3 + 6h^3r^3s^2x_n^4 + 10h^3r^3sx^4 - 40h^3r^3sx^3x_n + 60h^3r^3sx^2x_n^2 - 40h^3r^3sxx_n^3 + 10h^3r^3sx_n^4 + 8h^3r^3x^4 - 32h^3r^3x^3x_n + 48h^3r^3x^2x_n^2 - 32h^3r^3xx_n^3 + 8h^3r^3x_n^4 + 6h^3r^2s^3x^4 - 24h^3r^2s^3x^3x_n + 36h^3r^2s^3x^2x_n^2 - 24h^3r^2s^3xx_n^3 + 6h^3r^2s^3x_n^4 + 81h^3r^2s^2x^4 - 324h^3r^2s^2x^3x_n + 486h^3r^2s^2x^2x_n^2 - 324h^3r^2s^2xx_n^3 + 81h^3r^2s^2x_n^4 + 100h^3r^2sx^4 - 400h^3r^2sx^3x_n + 600h^3r^2sx^2x_n^2 - 400h^3r^2sxx_n^3 + 100h^3r^2sx_n^4 + 33h^3r^2x^4 - 132h^3r^2x^3x_n + 198h^3r^2x^2x_n^2 - 132h^3r^2xx_n^3 + 33h^3r^2x_n^4 - 2h^3rs^4x^4 + 8h^3rs^4x^3x_n - 12h^3rs^4x^2x_n^2 + 8h^3rs^4xx_n^3 - 2h^3rs^4x_n^4 + 10h^3rs^3x^4 - 40h^3rs^3x^3x_n + 60h^3rs^3x^2x_n^2 -$

$40h^3rs^3xx_n^3 + 10h^3rs^3x_n^4 + 100h^3rs^2x^4 - 400h^3rs^2x^3x_n + 600h^3rs^2x^2x_n^2 - 400h^3rs^2xx_n^3 + 100h^3rs^2x_n^4 + 54h^3rsx^4 - 216h^3rsx^3x_n + 324h^3rsx^2x_n^2 - 216h^3rsxx_n^3 + 54h^3rsx_n^4 - 2h^3s^4x^4 + 8h^3s^4x^3x_n - 12h^3s^4x^2x_n^2 + 8h^3s^4xx_n^3 - 2h^3s^4x_n^4 + 8h^3s^3x^4 - 32h^3s^3x^3x_n + 48h^3s^3x^2x_n^2 - 32h^3s^3xx_n^3 + 8h^3s^3x_n^4 + 33h^3s^2x^4 - 132h^3s^2x^3x_n + 198h^3s^2x^2x_n^2 - 132h^3s^2xx_n^3 + 33h^3s^2x_n^4 - 2h^2r^3sx^5 + 10h^2r^3sx^4x_n - 20h^2r^3sx^3x_n^2 + 20h^2r^3sx^2x_n^3 - 10h^2r^3sxx_n^4 + 2h^2r^3sx_n^5 - 2h^2r^3x^5 + 10h^2r^3x^4x_n - 20h^2r^3x^3x_n^2 + 20h^2r^3x^2x_n^3 - 10h^2r^3xx_n^4 + 2h^2r^3x_n^5 - 27h^2r^2s^2x^5 + 135h^2r^2s^2x^4x_n - 270h^2r^2s^2x^3x_n^2 + 270h^2r^2s^2x^2x_n^3 - 135h^2r^2s^2xx_n^4 + 27h^2r^2s^2x_n^5 - 59h^2r^2sx^5 + 295h^2r^2sx^4x_n - 590h^2r^2sx^3x_n^2 + 590h^2r^2sx^2x_n^3 - 295h^2r^2sxx_n^4 + 59h^2r^2sx_n^5 - 40h^2r^2x^5 + 200h^2r^2x^4x_n - 400h^2r^2x^3x_n^2 + 400h^2r^2x^2x_n^3 - 200h^2r^2xx_n^4 + 40h^2r^2x_n^5 - 2h^2rs^3x^5 + 10h^2rs^3x^4x_n - 20h^2rs^3x^3x_n^2 + 20h^2rs^3x^2x_n^3 - 10h^2rs^3xx_n^4 + 2h^2rs^3x_n^5 - 59h^2rs^2x^5 + 295h^2rs^2x^4x_n - 590h^2rs^2x^3x_n^2 + 590h^2rs^2x^2x_n^3 - 295h^2rs^2xx_n^4 + 59h^2rs^2x_n^5 - 80h^2rsx^5 + 400h^2rsx^4x_n - 800h^2rsx^3x_n^2 + 800h^2rsx^2x_n^3 - 400h^2rsxx_n^4 + 80h^2rsx_n^5 - 15h^2rx^5 + 75h^2rx^4x_n - 150h^2rx^3x_n^2 + 150h^2rx^2x_n^3 - 75h^2rxx_n^4 + 15h^2rx_n^5 - 2h^2s^3x^5 + 10h^2s^3x^4x_n - 20h^2s^3x^3x_n^2 + 20h^2s^3x^2x_n^3 - 10h^2s^3xx_n^4 + 2h^2s^3x_n^5 - 40h^2s^2x^5 + 200h^2s^2x^4x_n - 400h^2s^2x^3x_n^2 + 400h^2s^2x^2x_n^3 - 200h^2s^2xx_n^4 + 40h^2s^2x_n^5 - 15h^2sx^5 + 75h^2sx^4x_n - 150h^2sx^3x_n^2 + 150h^2sx^2x_n^3 - 75h^2sxx_n^4 + 15h^2sx_n^5 + 13hr^2sx^6 - 78hr^2sx^5x_n + 195hr^2sx^4x_n^2 - 260hr^2sx^3x_n^3 + 195hr^2sx^2x_n^4 - 78hr^2sxx_n^5 + 13hr^2sx_n^6 + 13hr^2x^6 - 78hr^2x^5x_n + 195hr^2x^4x_n^2 - 260hr^2x^3x_n^3 + 195hr^2x^2x_n^4 - 78hr^2xx_n^5 + 13hr^2x_n^6 + 13hrs^2x^6 - 78hrs^2x^5x_n + 195hrs^2x^4x_n^2 - 260hrs^2x^3x_n^3 + 195hrs^2x^2x_n^4 - 78hrs^2xx_n^5 + 13hrs^2x_n^6 + 41hrsx^6 - 246hrsx^5x_n + 615hrsx^4x_n^2 - 820hrsx^3x_n^3 + 615hrsx^2x_n^4 - 246hrsxx_n^5 + 41hrsx_n^6 + 20hrx^6 - 120hrx^5x_n + 300hrx^4x_n^2 - 400hrx^3x_n^3 + 300hrx^2x_n^4 - 120hrxx_n^5 + 20hrx_n^6 + 13hs^2x^6 - 78hs^2x^5x_n + 195hs^2x^4x_n^2 - 260hs^2x^3x_n^3 + 195hs^2x^2x_n^4 - 78hs^2xx_n^5 + 13hs^2x_n^6 + 20hsx^6 - 120hsx^5x_n + 300hsx^4x_n^2 - 400hsx^3x_n^3 + 300hsx^2x_n^4 - 120hsxx_n^5 + 20hsx_n^6 - 7rsx^7 + 49rsx^6x_n - 147rsx^5x_n^2 + 245rsx^4x_n^3 - 245rsx^3x_n^4 + 147rsx^2x_n^5 - 49rsxx_n^6 + 7rsx_n^7 - 7rx^7 + 49rx^6x_n - 147rx^5x_n^2 + 245rx^4x_n^3 - 245rx^3x_n^4 + 147rx^2x_n^5 - 49rxx_n^6 + 7rx_n^7 - 7sx^7 + 49sx^6x_n - 147sx^5x_n^2 + 245sx^4x_n^3 - 245sx^3x_n^4 + 147sx^2x_n^5 - 49sxx_n^6 + 7sx_n^7),$

$$\beta_r = \frac{(x-x_n)^9}{504h^6r^3(r-s)^3(r-1)^3}(7r^3+7r^2s+7r^2-8rs^2-13rs-8r+4s^2+4s) - \frac{(x-x_n)^7}{210h^4r^3(r-s)^3(r-1)^3}(5r^2s^3-28r^3s-7r^3-7r^3s^2+13r^2s^2+13r^2s+5r^2+5rs^3+4rs^2+5rs-4s^3-4s^2) + \frac{(x-x_n)^{10}}{360h^7r^3(r-s)^3(r-1)^3}(2r-s+2rs-3r^2) + \frac{(x-x_n)^8}{168h^5r^3(r-s)^3(r-1)^3}(2r^2s^2-7r^3-7r^3s-2r^2s+2r^2+2rs^3+7rs^2+7rs+2r-s^3-4s^2-s) + \frac{h(x-x_n)^2}{2520r^3(r-s)^3(r-1)^3}(42r^9s-14r^{10}+56r^9-32r^8s^2-178r^8s-74r^8-2r^7s^3+147r^7s^2+251r^7s+30r^7-2r^6s^4+12r^6s^3-229r^6s^2-105r^6s-2r^5s^5+12r^5s^4-25r^5s^3+99r^5s^2-2r^4s^6+12r^4s^5-25r^4s^4+15r^4s^3-2r^3s^7+12r^3s^6-25r^3s^5+15r^3s^4+12r^2s^8-51r^2s^7+59r^2s^6+15r^2s^5-4rs^9+8rs^8+17rs^7-45rs^6+2s^9-8s^8+9s^7) - \frac{h^2s(x-x_n)}{2520r^2(r-s)^3(r-1)^3}(42r^8s-14r^9+56r^8-32r^7s^2-178r^7s-74r^7-2r^6s^3+147r^6s^2+251r^6s+30r^6-2r^5s^4+12r^5s^3-229r^5s^2-105r^5s-2r^4s^5+12r^4s^4-25r^4s^3+99r^4s^2-2r^3s^6+12r^3s^5-25r^3s^4+15r^3s^3+12r^2s^7-51r^2s^6+59r^2s^5+15r^2s^4-4rs^8+8rs^7+17rs^6-45rs^5+2s^8-8s^7+9s^6) - \frac{s(x-x_n)^6}{60h^3r^3(r-s)^3(r-1)^3}(7r^3s+7r^3-5r^2s^2-7r^2s-5r^2+rs^2+rs+s^2) - \frac{s^2(x-x_n)^5}{60h^2r^2(r-s)^3(r-1)^3}(5r-3s+5rs-7r^2),$$

$$\beta_s = \frac{(x-x_n)^8}{168h^5s^3(r-s)^3(s-1)^3}(r^3-2r^3s-2r^2s^2-7r^2s+4r^2+7rs^3+2rs^2-7rs+r+7s^3-2s^2-2s) + \frac{(x-x_n)^7}{210h^4s^3(r-s)^3(s-1)^3}(5r^3s^2+5r^3s-4r^3-7r^2s^3+13r^2s^2+4r^2s-4r^2-28rs^3+13rs^2+5rs-7s^3+5s^2) + \frac{(x-x_n)^{10}}{360h^7s^3(r-s)^3(s-1)^3}(r-2s-2rs+3s^2) - \frac{(x-x_n)^9}{504h^6s^3(r-s)^3(s-1)^3}(4r^2-8r^2s+7rs^2-13rs+4r+7s^3+7s^2-8s) - \frac{h(x-x_n)^2}{2520s^3(r-s)^3(s-1)^3}(2r^9-4r^9s+12r^8s^2+8r^8s-8r^8-2r^7s^3-51r^7s^2+17r^7s+9r^7-2r^6s^4+12r^6s^3+59r^6s^2-45r^6s-2r^5s^5+12r^5s^4-25r^5s^3+15r^5s^2-2r^4s^6+12r^4s^5-25r^4s^4+15r^4s^3-2r^3s^7+12r^3s^6-25r^3s^5+15r^3s^4-32r^2s^8+147r^2s^7-229r^2s^6+99r^2s^5+42rs^9-178rs^8+251rs^7-105rs^6-14s^10+56s^9-74s^8+30s^7) + \frac{h^2r(x-x_n)}{2520s^2(r-s)^3(s-1)^3}(2r^8-4r^8s+12r^7s^2+8r^7s-8r^7-2r^6s^3-51r^6s^2+17r^6s+9r^6-2r^5s^4+12r^5s^3+59r^5s^2-45r^5s-2r^4s^5+12r^4s^4-25r^4s^3+15r^4s^2-2r^3s^6+12r^3s^5-25r^3s^4+15r^3s^3-32r^2s^7+147r^2s^6-229r^2s^5+99r^2s^4+42rs^8-178rs^7+251rs^6-105rs^5-14s^9+56s^8-74s^7+30s^6) + \frac{r(x-x_n)^6}{60h^3s^3(r-s)^3(s-1)^3}(r^2s-5r^2s^2+r^2+7rs^3-7rs^2+rs+7s^3-5s^2) - \frac{r^2(x-x_n)^5}{60h^2s^2(r-s)^3(s-1)^3}(3r-5s-5rs+7s^2),$$

$$\beta_1 = \frac{(x-x_n)^9}{504h^6(r-1)^3(s-1)^3}(4r^2s-8r^2+4rs^2-13rs+7r-8s^2+7s+7) - \frac{(x-x_n)^8}{168h^5(r-1)^3(s-1)^3}(r^3s-2r^3+4r^2s^2-7r^2s-2r^2+rs^3-7rs^2+2rs+7r-2s^3-2s^2+$$

$$7s) - \frac{h(x-x_n)^2}{2520(r-1)^3(s-1)^3}(4r^9 - 2r^9s + 6r^8s^2 - 2r^8s - 16r^8 - 3r^7s^3 - 27r^7s^2 + 43r^7s +$$
$$14r^7 - 3r^6s^4 + 27r^6s^3 + r^6s^2 - 49r^6s - 3r^5s^5 + 27r^5s^4 - 59r^5s^3 + 35r^5s^2 - 3r^4s^6 +$$
$$27r^4s^5 - 59r^4s^4 + 35r^4s^3 - 3r^3s^7 + 27r^3s^6 - 59r^3s^5 + 35r^3s^4 + 6r^2s^8 - 27r^2s^7 + r^2s^6 +$$
$$35r^2s^5 - 2rs^9 - 2rs^8 + 43rs^7 - 49rs^6 + 4s^9 - 16s^8 + 14s^7) - \frac{(x-x_n)^7}{210h^4(r-1)^3(s-1)^3}(5r^3s -$$
$$4r^3s^2 + 5r^3 - 4r^2s^3 + 4r^2s^2 + 13r^2s - 7r^2 + 5rs^3 + 13rs^2 - 28rs + 5s^3 - 7s^2) +$$
$$\frac{(x-x_n)^{10}}{360h^7(r-1)^3(s-1)^3}(2r + 2s - rs - 3) + \frac{h^2rs(x-x_n)}{2520(r-1)^3(s-1)^3}(4r^8 - 2r^8s + 6r^7s^2 - 2r^7s -$$
$$16r^7 - 3r^6s^3 - 27r^6s^2 + 43r^6s + 14r^6 - 3r^5s^4 + 27r^5s^3 + r^5s^2 - 49r^5s - 3r^4s^5 +$$
$$27r^4s^4 - 59r^4s^3 + 35r^4s^2 - 3r^3s^6 + 27r^3s^5 - 59r^3s^4 + 35r^3s^3 + 6r^2s^7 - 27r^2s^6 + r^2s^5 +$$
$$35r^2s^4 - 2rs^8 - 2rs^7 + 43rs^6 - 49rs^5 + 4s^8 - 16s^7 + 14s^6) - \frac{rs(x-x_n)^6}{60h^3(r-1)^3(s-1)^3}(r^2s^2 +$$
$$r^2s - 5r^2 + rs^2 - 7rs + 7r - 5s^2 + 7s) - \frac{r^2s^2(x-x_n)^5}{60h^2(r-1)^3(s-1)^3}(5r + 5s - 3rs - 7),$$

$$\gamma_0 = -\frac{(x-x_n)(x_n-x+hr)(x_n-x+hs)}{5040h^6r^2s^2}(6h^7r^6s - 2h^7r^7 + 8h^7r^6 - 3h^7r^5s^2 - 28h^7r^5s$$
$$-9h^7r^5 - 3h^7r^4s^3 + 20h^7r^4s^2 + 39h^7r^4s - 3h^7r^3s^4 + 20h^7r^3s^3 - 45h^7r^3s^2 -$$
$$3h^7r^2s^5 + 20h^7r^2s^4 - 45h^7r^2s^3 + 6h^7rs^6 - 28h^7rs^5 + 39h^7rs^4 - 2h^7s^7 + 8h^7s^6 -$$
$$9h^7s^5 - 2h^6r^6x + 2h^6r^6x_n + 6h^6r^5sx - 6h^6r^5sx_n + 8h^6r^5x - 8h^6r^5x_n - 3h^6r^4s^2x +$$
$$3h^6r^4s^2x_n - 28h^6r^4sx + 28h^6r^4sx_n - 9h^6r^4x + 9h^6r^4x_n - 3h^6r^3s^3x + 3h^6r^3s^3x_n +$$
$$20h^6r^3s^2x - 20h^6r^3s^2x_n + 39h^6r^3sx - 39h^6r^3sx_n - 3h^6r^2s^4x + 3h^6r^2s^4x_n +$$
$$20h^6r^2s^3x - 20h^6r^2s^3x_n - 45h^6r^2s^2x + 45h^6r^2s^2x_n + 6h^6rs^5x - 6h^6rs^5x_n - 28h^6rs^4x +$$
$$28h^6rs^4x_n + 39h^6rs^3x - 39h^6rs^3x_n - 2h^6s^6x + 2h^6s^6x_n + 8h^6s^5x - 8h^6s^5x_n - 9h^6s^4x +$$
$$9h^6s^4x_n - 2h^5r^5x^2 + 4h^5r^5xx_n - 2h^5r^5x_n^2 + 6h^5r^4sx^2 - 12h^5r^4sxx_n + 6h^5r^4sx_n^2 +$$
$$8h^5r^4x^2 - 16h^5r^4xx_n + 8h^5r^4x_n^2 - 3h^5r^3s^2x^2 + 6h^5r^3s^2xx_n - 3h^5r^3s^2x_n^2 - 28h^5r^3sx^2 +$$
$$56h^5r^3sxx_n - 28h^5r^3sx_n^2 - 9h^5r^3x^2 + 18h^5r^3xx_n - 9h^5r^3x_n^2 - 3h^5r^2s^3x^2 + 6h^5r^2s^3xx_n -$$
$$3h^5r^2s^3x_n^2 + 20h^5r^2s^2x^2 - 40h^5r^2s^2xx_n + 20h^5r^2s^2x_n^2 + 39h^5r^2sx^2 - 78h^5r^2sxx_n +$$
$$39h^5r^2sx_n^2 + 6h^5rs^4x^2 - 12h^5rs^4xx_n + 6h^5rs^4x_n^2 - 28h^5rs^3x^2 + 56h^5rs^3xx_n -$$
$$28h^5rs^3x_n^2 + 39h^5rs^2x^2 - 78h^5rs^2xx_n + 39h^5rs^2x_n^2 - 2h^5s^5x^2 + 4h^5s^5xx_n -$$
$$2h^5s^5x_n^2 + 8h^5s^4x^2 - 16h^5s^4xx_n + 8h^5s^4x_n^2 - 9h^5s^3x^2 + 18h^5s^3xx_n - 9h^5s^3x_n^2 -$$
$$2h^4r^4x^3 + 6h^4r^4x^2x_n - 6h^4r^4xx_n^2 + 2h^4r^4x_n^3 + 6h^4r^3sx^3 - 18h^4r^3sx^2x_n + 18h^4r^3sxx_n^2 -$$
$$6h^4r^3sx_n^3 + 8h^4r^3x^3 - 24h^4r^3x^2x_n + 24h^4r^3xx_n^2 - 8h^4r^3x_n^3 - 3h^4r^2s^2x^3 + 9h^4r^2s^2x^2x_n -$$
$$9h^4r^2s^2xx_n^2 + 3h^4r^2s^2x_n^3 - 28h^4r^2sx^3 + 84h^4r^2sx^2x_n - 84h^4r^2sxx_n^2 + 28h^4r^2sx_n^3 - 9h^4$$

$9h^4r^2x^3 + 27h^4r^2x^2x_n - 27h^4r^2xx_n^2 + 9h^4r^2x_n^3 + 6h^4rs^3x^3 - 18h^4rs^3x^2x_n + 18h^4rs^3xx_n^2 - 6h^4rs^3x_n^3 - 28h^4rs^2x^3 + 84h^4rs^2x^2x_n - 84h^4rs^2xx_n^2 + 28h^4rs^2x_n^3 - 87h^4rsx^3 + 261h^4rsx^2x_n - 261h^4rsxx_n^2 + 87h^4rsx_n^3 - 2h^4s^4x^3 + 6h^4s^4x^2x_n - 6h^4s^4xx_n^2 + 2h^4s^4x_n^3 + 8h^4s^3x^3 - 24h^4s^3x^2x_n + 24h^4s^3xx_n^2 - 8h^4s^3x_n^3 - 9h^4s^2x^3 + 27h^4s^2x^2x_n - 27h^4s^2xx_n^2 + 9h^4s^2x_n^3 - 2h^3r^3x^4 + 8h^3r^3x^3x_n - 12h^3r^3x^2x_n^2 + 8h^3r^3xx_n^3 - 2h^3r^3x_n^4 + 6h^3r^2sx^4 - 24h^3r^2sx^3x_n + 36h^3r^2sx^2x_n^2 - 24h^3r^2sxx_n^3 + 6h^3r^2sx_n^4 + 8h^3r^2x^4 - 32h^3r^2x^3x_n + 48h^3r^2x^2x_n^2 - 32h^3r^2xx_n^3 + 8h^3r^2x_n^4 + 6h^3rs^2x^4 - 24h^3rs^2x^3x_n + 36h^3rs^2x^2x_n^2 - 24h^3rs^2xx_n^3 + 6h^3rs^2x_n^4 + 92h^3rsx^4 - 368h^3rsx^3x_n + 552h^3rsx^2x_n^2 - 368h^3rsxx_n^3 + 92h^3rsx_n^4 + 33h^3rx^4 - 132h^3rx^3x_n + 198h^3rx^2x_n^2 - 132h^3rxx_n^3 + 33h^3rx_n^4 - 2h^3s^3x^4 + 8h^3s^3x^3x_n - 12h^3s^3x^2x_n^2 + 8h^3s^3xx_n^3 - 2h^3s^3x_n^4 + 8h^3s^2x^4 - 32h^3s^2x^3x_n + 48h^3s^2x^2x_n^2 - 32h^3s^2xx_n^3 + 8h^3s^2x_n^4 + 33h^3sx^4 - 132h^3sx^3x_n + 198h^3sx^2x_n^2 - 132h^3sxx_n^3 + 33h^3sx_n^4 - 2h^2r^2x^5 + 10h^2r^2x^4x_n - 20h^2r^2x^3x_n^2 + 20h^2r^2x^2x_n^3 - 10h^2r^2xx_n^4 + 2h^2r^2x_n^5 - 27h^2rsx^5 + 135h^2rsx^4x_n - 270h^2rsx^3x_n^2 + 270h^2rsx^2x_n^3 - 135h^2rsxx_n^4 + 27h^2rsx_n^5 - 40h^2rx^5 + 200h^2rx^4x_n - 400h^2rx^3x_n^2 + 400h^2rx^2x_n^3 - 200h^2rxx_n^4 + 40h^2rx_n^5 - 2h^2s^2x^5 + 10h^2s^2x^4x_n - 20h^2s^2x^3x_n^2 + 20h^2s^2x^2x_n^3 - 10h^2s^2xx_n^4 + 2h^2s^2x_n^5 - 40h^2sx^5 + 200h^2sx^4x_n - 400h^2sx^3x_n^2 + 400h^2sx^2x_n^3 - 200h^2sxx_n^4 + 40h^2sx_n^5 - 15h^2x^5 + 75h^2x^4x_n - 150h^2x^3x_n^2 + 150h^2x^2x_n^3 - 75h^2xx_n^4 + 15h^2x_n^5 + 13hrx^6 - 78hrx^5x_n + 195hrx^4x_n^2 - 260hrx^3x_n^3 + 195hrx^2x_n^4 - 78hrxx_n^5 + 13hrx_n^6 + 13hsx^6 - 78hsx^5x_n + 195hsx^4x_n^2 - 260hsx^3x_n^3 + 195hsx^2x_n^4 - 78hsxx_n^5 + 13hsx_n^6 + 20hx^6 - 120hx^5x_n + 300hx^4x_n^2 - 400hx^3x_n^3 + 300hx^2x_n^4 - 120hxx_n^5 + 20hx_n^6 - 7x^7 + 49x^6x_n - 147x^5x_n^2 + 245x^4x_n^3 - 245x^3x_n^4 + 147x^2x_n^5 - 49xx_n^6 + 7x_n^7),$

$\gamma_r = \frac{(x-x_n)^{10}}{720h^6r^2(r-s)^2(r-1)^2} - \frac{(x-x_n)^7}{210h^3r^2(r-s)^3(r-1)^2}(r^2s^2 + 4r^2s + r^2 - rs^3 - 2rs^2 + rs - 2s^3 - 2s^2) - \frac{s^2(x-x_n)^5}{60hr(r-s)^2(r-1)^2} - \frac{(x-x_n)^9}{504h^5r^2(r-s)^3(r-1)^2}(r^2 + rs + 2r - 2s^2 - 2s) + \frac{h^2(x-x_n)^2}{5040r^2(r-s)^3(r-1)^2}(3r^9 - 10r^8s - 10r^8 + 9r^7s^2 + 36r^7s + 9r^7 - 36r^6s^2 - 36r^6s + 42r^5s^2 - 4rs^8 + 18rs^7 - 24rs^6 + 2s^9 - 8s^8 + 9s^7) + \frac{(x-x_n)^8}{336h^4r^2(r-s)^3(r-1)^2}(2r^2s + 2r^2 - rs^2 + 2rs + r - s^3 - 4s^2 - s) - \frac{s(x-x_n)^6}{120h^2r^2(r-s)^3(r-1)^2}(-2r^2s - 2r^2 + 2rs^2 + rs + s^2) - \frac{h^3s(x-x_n)}{5040r(r-s)^2(r-1)^2}(3r^7 - 7r^6s - 10r^6 + 2r^5s^2 + 26r^5s + 9r^5 + 2r^4s^3 - 10r^4s^2 - 27r^4s + 2r^3s^4 - 10r^3s^3 + 15r^3s^2 +$

$2r^2s^5-10r^2s^4+15r^2s^3+2rs^6-10rs^5+15rs^4-2s^7+8s^6-9s^5)$,

$\gamma_s = \frac{(x-x_n)^{10}}{720h^6s^2(r-s)^2(s-1)^2} + \frac{(x-x_n)^7}{210h^3s^2(r-s)^3(s-1)^2}(-r^3s-2r^3+r^2s^2-2r^2s-2r^2+4rs^2+rs+s^2) - \frac{r^2(x-x_n)^5}{60hs(r-s)^2(s-1)^2} + \frac{(x-x_n)^9}{504h^5s^2(r-s)^3(s-1)^2}(-2r^2+rs-2r+s^2+2s) + \frac{(x-x_n)^8}{336h^4s^2(r-s)^3(s-1)^2}(r^3+r^2s+4r^2-2rs^2-2rs+r-2s^2-s) - \frac{h^2(x-x_n)^2}{5040s^2(r-s)^3(s-1)^2}(2r^9-4r^8s-8r^8+18r^7s+9r^7-24r^6s+9r^2s^7-36r^2s^6+42r^2s^5-10rs^8+36rs^7-36rs^6+3s^9-10s^8+9s^7) + \frac{r(x-x_n)^6}{120h^2s^2(r-s)^3(s-1)^2}(2r^2s+r^2-2rs^2+rs-2s^2) - \frac{h^3r(x-x_n)}{5040s(r-s)^2(s-1)^2}(-2r^7+2r^6s+8r^6+2r^5s^2-10r^5s-9r^5+2r^4s^3-10r^4s^2+15r^4s+2r^3s^4-10r^3s^3+15r^3s^2+2r^2s^5-10r^2s^4+15r^2s^3-7rs^6+26rs^5-27rs^4+3s^7-10s^6+9s^5)$,

$\gamma_1 = -\frac{(x-x_n)(x_n-x+hr)(x_n-x+hs)}{5040h^6(r-1)^2(s-1)^2}(6h^7r^6s-2h^7r^7+4h^7r^6-3h^7r^5s^2-14h^7r^5s-3h^7r^4s^3+10h^7r^4s^2-3h^7r^3s^4+10h^7r^3s^3-3h^7r^2s^5+10h^7r^2s^4+6h^7rs^6-14h^7rs^5-2h^7s^7+4h^7s^6-2h^6r^6x+2h^6r^6x_n+6h^6r^5sx-6h^6r^5sx_n+4h^6r^5x-4h^6r^5x_n-3h^6r^4s^2x+3h^6r^4s^2x_n-14h^6r^4sx+14h^6r^4sx_n-3h^6r^3s^3x+3h^6r^3s^3x_n+10h^6r^3s^2x-10h^6r^3s^2x_n-3h^6r^2s^4x+3h^6r^2s^4x_n+10h^6r^2s^3x-10h^6r^2s^3x_n+6h^6rs^5x-6h^6rs^5x_n-14h^6rs^4x+14h^6rs^4x_n-2h^6s^6x+2h^6s^6x_n+4h^6s^5x-4h^6s^5x_n-2h^5r^5x^2+4h^5r^5xx_n-2h^5r^5x_n^2+6h^5r^4sx^2-12h^5r^4sxx_n+6h^5r^4sx_n^2+4h^5r^4x^2-8h^5r^4xx_n+4h^5r^4x_n^2-3h^5r^3s^2x^2+6h^5r^3s^2xx_n-3h^5r^3s^2x_n^2-14h^5r^3sx^2+28h^5r^3sxx_n-14h^5r^3sx_n^2-3h^5r^2s^3x^2+6h^5r^2s^3xx_n-3h^5r^2s^3x_n^2+10h^5r^2s^2x^2-20h^5r^2s^2xx_n+10h^5r^2s^2x_n^2+6h^5rs^4x^2-12h^5rs^4xx_n+6h^5rs^4x_n^2-14h^5rs^3x^2+28h^5rs^3xx_n-14h^5rs^3x_n^2-2h^5s^5x^2+4h^5s^5xx_n-2h^5s^5x_n^2+4h^5s^4x^2-8h^5s^4xx_n+4h^5s^4x_n^2-2h^4r^4x^3+6h^4r^4x^2x_n-6h^4r^4xx_n^2+2h^4r^4x_n^3+6h^4r^3sx^3-18h^4r^3sx^2x_n+18h^4r^3sxx_n^2-6h^4r^3sx_n^3+4h^4r^3x^3-12h^4r^3x^2x_n+12h^4r^3xx_n^2-4h^4r^3x_n^3-3h^4r^2s^2x^3+9h^4r^2s^2x^2x_n-9h^4r^2s^2xx_n^2+3h^4r^2s^2x_n^3-14h^4r^2sx^3+42h^4r^2sx^2x_n-42h^4r^2sxx_n^2+14h^4r^2sx_n^3+6h^4rs^3x^3-18h^4rs^3x^2x_n+18h^4rs^3xx_n^2-6h^4rs^3x_n^3-14h^4rs^2x^3+42h^4rs^2x^2x_n-42h^4rs^2xx_n^2+14h^4rs^2x_n^3-2h^4s^4x^3+6h^4s^4x^2x_n-6h^4s^4xx_n^2+2h^4s^4x_n^3+4h^4s^3x^3-12h^4s^3x^2x_n+12h^4s^3xx_n^2-4h^4s^3x_n^3-2h^3r^3x^4+8h^3r^3x^3x_n-12h^3r^3x^2x_n^2+8h^3r^3xx_n^3-2h^3r^3x_n^4+6h^3r^2sx^4-24h^3r^2sx^3x_n+36h^3r^2sx^2x_n^2-24h^3r^2sxx_n^3+6h^3r^2sx_n^4+4h^3r^2x^4-16h^3r^2x^3x_n+24h^3r^2x^2x_n^2-16h^3r^2xx_n^3+4h^3r^2x_n^4+6h^3rs^2x^4-24h^3rs^2x^3x_n+36h^3$

$rs^2x^2x_n^2 - 24h^3rs^2xx_n^3 + 6h^3rs^2x_n^4 + 46h^3rsx^4 - 184h^3rsx^3x_n + 276h^3rsx^2x_n^2 - 184h^3rsxx_n^3 + 46h^3rsx_n^4 - 2h^3s^3x^4 + 8h^3s^3x^3x_n - 12h^3s^3x^2x_n^2 + 8h^3s^3xx_n^3 - 2h^3s^3x_n^4 + 4h^3s^2x^4 - 16h^3s^2x^3x_n + 24h^3s^2x^2x_n^2 - 16h^3s^2xx_n^3 + 4h^3s^2x_n^4 - 2h^2r^2x^5 + 10h^2r^2x^4x_n - 20h^2r^2x^3x_n^2 + 20h^2r^2x^2x_n^3 - 10h^2r^2xx_n^4 + 2h^2r^2x_n^5 - 27h^2rsx^5 + 135h^2rsx^4x_n - 270h^2rsx^3x_n^2 + 270h^2rsx^2x_n^3 - 135h^2rsxx_n^4 + 27h^2rsx_n^5 - 20h^2rx^5 + 100h^2rx^4x_n - 200h^2rx^3x_n^2 + 200h^2rx^2x_n^3 - 100h^2rxx_n^4 + 20h^2rx_n^5 - 2h^2s^2x^5 + 10h^2s^2x^4x_n - 20h^2s^2x^3x_n^2 + 20h^2s^2x^2x_n^3 - 10h^2s^2xx_n^4 + 2h^2s^2x_n^5 - 20h^2sx^5 + 100h^2sx^4x_n - 200h^2sx^3x_n^2 + 200h^2sx^2x_n^3 - 100h^2sxx_n^4 + 20h^2sx_n^5 + 13hrx^6 - 78hrx^5x_n + 195hrx^4x_n^2 - 260hrx^3x_n^3 + 195hrx^2x_n^4 - 78hrxx_n^5 + 13hrx_n^6 + 13hsx^6 - 78hsx^5x_n + 195hsx^4x_n^2 - 260hsx^3x_n^3 + 195hsx^2x_n^4 - 78hsxx_n^5 + 13hsx_n^6 + 10hx^6 - 60hx^5x_n + 150hx^4x_n^2 - 200hx^3x_n^3 + 150hx^2x_n^4 - 60hxx_n^5 + 10hx_n^6 - 7x^7 + 49x^6x_n - 147x^5x_n^2 + 245x^4x_n^3 - 245x^3x_n^4 + 147x^2x_n^5 - 49xx_n^6 + 7x_n^7).$

Now, evaluating Equation (4.6) at the non-interpolating point x_{n+1} yields

$y_{n+1} = \frac{y_{n+s}(r-1)}{s(r-s)} - \frac{y_{n+r}(s-1)}{r(r-s)} + \frac{y_n(r-1)(s-1)}{rs} - \frac{g_{n+1}h^4}{5040(r-1)(s-1)}(6r^6s - 2r^7 + 2r^6 - 3r^5s^2 - 8r^5s + 2r^5 - 3r^4s^3 + 7r^4s^2 - 8r^4s + 2r^4 - 3r^3s^4 + 7r^3s^3 + 7r^3s^2 - 8r^3s + 2r^3 - 3r^2s^5 + 7r^2s^4 + 7r^2s^3 + 7r^2s^2 - 8r^2s + 2r^2 + 6rs^6 - 8rs^5 - 8rs^4 - 8rs^3 - 8rs^2 + 19rs - 7r - 2s^7 + 2s^6 + 2s^5 + 2s^4 + 2s^3 + 2s^2 - 7s + 3) - \frac{f_{n+1}h^3}{2520(r-1)^2(s-1)^2}(2r^8s - 4r^8 - 6r^7s^2 + 4r^7s + 12r^7 + 3r^6s^3 + 21r^6s^2 - 39r^6s - 2r^6 + 3r^5s^4 - 24r^5s^3 + 20r^5s^2 + 10r^5s - 2r^5 + 3r^4s^5 - 24r^4s^4 + 35r^4s^3 - 15r^4s^2 + 10r^4s - 2r^4 + 3r^3s^6 - 24r^3s^5 + 35r^3s^4 - 15r^3s^2 + 10r^3s - 2r^3 - 6r^2s^7 + 21r^2s^6 + 20r^2s^5 - 15r^2s^4 - 15r^2s^3 - 114r^2s^2 + 115r^2s - 32r^2 + 2rs^8 + 4rs^7 - 39rs^6 + 10rs^5 + 10rs^4 + 10rs^3 + 115rs^2 - 136rs + 42r - 4s^8 + 12s^7 - 2s^6 - 2s^5 - 2s^4 - 2s^3 - 32s^2 + 42s - 14) + \frac{f_nh^3(r-1)(s-1)}{2520r^3s^3}(2r^8s + 2r^8 - 6r^7s^2 - 8r^7s - 6r^7 + 3r^6s^3 + 12r^6s^2 + 12r^6s + 3r^6 + 3r^5s^4 - 6r^5s^3 + 20r^5s^2 - 6r^5s + 3r^5 + 3r^4s^5 - 6r^4s^4 - 46r^4s^3 - 46r^4s^2 - 6r^4s + 3r^4 + 3r^3s^6 - 6r^3s^5 - 46r^3s^4 + 224r^3s^3 - 46r^3s^2 - 6r^3s + 3r^3 - 6r^2s^7 + 12r^2s^6 + 20r^2s^5 - 46r^2s^4 - 46r^2s^3 + 20r^2s^2 + 12r^2s - 6r^2 + 2rs^8 - 8rs^7 + 12rs^6 - 6rs^5 - 6rs^4 - 6rs^3 + 12rs^2 - 8rs + 2r + 2s^8 - 6s^7 + 3s^6 + 3s^5 + 3s^4 + 3s^3 - 6s^2 + 2s) - \frac{g_{n+r}h^4(s-1)}{5040r^2(r-s)^2(r-1)}(3r^7 - 7r^6s - 7r^6 + 2r^5s^2 + 19r^5s + 2r^5 + 2r^4s^3 - 8r^4s^2 - 8r^4s + 2r^4 + 2r^3s^4 - 8r^3s^3 + 7r^3s^2 - 8r^3s + 2r^3 + 2r^2s^5 - 8r^2s^4 +$

$7r^2s^3+7r^2s^2-8r^2s+2r^2+2rs^6-8rs^5+7rs^4+7rs^3+7rs^2-8rs+2r-2s^7+6s^6-3s^5-3s^4-3s^3-3s^2+6s-2)-\frac{g_{n+s}h^4(r-1)}{5040s^2(r-s)^2(s-1)}(2r^6s-2r^7+6r^6+2r^5s^2-8r^5s-3r^5+2r^4s^3-8r^4s^2+7r^4s-3r^4+2r^3s^4-8r^3s^3+7r^3s^2+7r^3s-3r^3+2r^2s^5-8r^2s^4+7r^2s^3+7r^2s^2+7r^2s-3r^2-7rs^6+19rs^5-8rs^4-8rs^3-8rs^2-8rs+6r+3s^7-7s^6+2s^5+2s^4+2s^3+2s^2+2s-2)+\frac{f_{n+r}h^3(s-1)}{2520r^3(r-s)^3(r-1)^2}(14r^9-42r^8s-42r^8+32r^7s^2+136r^7s+32r^7+2r^6s^3-115r^6s^2-115r^6s+2r^6+2r^5s^4-10r^5s^3+114r^5s^2-10r^5s+2r^5+2r^4s^5-10r^4s^4+15r^4s^3+15r^4s^2-10r^4s+2r^4+2r^3s^6-10r^3s^5+15r^3s^4+15r^3s^2-10r^3s+2r^3-12r^2s^7+39r^2s^6-20r^2s^5-35r^2s^4-35r^2s^3-20r^2s^2+39r^2s-12r^2+4rs^8-4rs^7-21rs^6+24rs^5+24rs^4+24rs^3-21rs^2-4rs+4r-2s^8+6s^7-3s^6-3s^5-3s^4-3s^3+6s^2-2s)+\frac{f_{n+s}h^3(r-1)}{2520s^3(r-s)^3(s-1)^2}(2r^8-4r^8s+12r^7s^2+4r^7s-6r^7-2r^6s^3-39r^6s^2+21r^6s+3r^6-2r^5s^4+10r^5s^3+20r^5s^2-24r^5s+3r^5-2r^4s^5+10r^4s^4-15r^4s^3+35r^4s^2-24r^4s+3r^4-2r^3s^6+10r^3s^5-15r^3s^4+35r^3s^2-24r^3s+3r^3-32r^2s^7+115r^2s^6-114r^2s^5-15r^2s^4-15r^2s^3+20r^2s^2+21r^2s-6r^2+42rs^8-136rs^7+115rs^6+10rs^5+10rs^4+10rs^3-39rs^2+4rs+2r-14s^9+42s^8-32s^7-2s^6-2s^5-2s^4-2s^3+12s^2-4s)+\frac{g_nh^4(r-1)(s-1)}{5040r^2s^2}(2r^7-6r^6s-6r^6+3r^5s^2+22r^5s+3r^5+3r^4s^3-17r^4s^2-17r^4s+3r^4+3r^3s^4-17r^3s^3+28r^3s^2-17r^3s+3r^3+3r^2s^5-17r^2s^4+28r^2s^3+28r^2s^2-17r^2s+3r^2-6rs^6+22rs^5-17rs^4-17rs^3-17rs^2+22rs-6r+2s^7-6s^6+3s^5+3s^4+3s^3+3s^2-6s+2) \quad (4.9)$

Similarly, evaluating (4.7) and (4.8) at all points, i.e x_n, x_{n+r}, x_{n+s} and x_{n+1}, produces the following equations

$y'_n = \frac{1}{5040hr^2s^2(r-s)^3(r-1)^3(s-1)^3}(2f_{n+s}h^3r^3(r-1)^3(2r^8-4r^8s+12r^7s^2+8r^7s-8r^7-2r^6s^3-51r^6s^2+17r^6s+9r^6-2r^5s^4+12r^5s^3+59r^5s^2-45r^5s-2r^4s^5+12r^4s^4-25r^4s^3+15r^4s^2-2r^3s^6+12r^3s^5-25r^3s^4+15r^3s^3-32r^2s^7+147r^2s^6-229r^2s^5+99r^2s^4+42rs^8-178rs^7+251rs^6-105rs^5-14s^9+56s^8-74s^7+30s^6)-2f_{n+r}h^3s^3(s-1)^3(42r^8s-14r^9+56r^8-32r^7s^2-178r^7s-74r^7-2r^6s^3+147r^6s^2+251r^6s+30r^6-2r^5s^4+12r^5s^3-229r^5s^2-105r^5s-2r^4s^5+12r^4s^4-25r^4s^3+99r^4s^2-2r^3s^6+12r^3s^5-25r^3s^4+15r^3s^3+12r^2s^7-51r^2s^6+59r^2s^5+15r^2s^4-4rs^8+8rs^7+17rs^6-45rs^5+2s^8-8s^7+9s^6)+2f_{n+1}h^3r^3s^3(r-s)^3(4r^8-2r^8s+$

$$6r^7s^2 - 2r^7s - 16r^7 - 3r^6s^3 - 27r^6s^2 + 43r^6s + 14r^6 - 3r^5s^4 + 27r^5s^3 + r^5s^2 -$$
$$49r^5s - 3r^4s^5 + 27r^4s^4 - 59r^4s^3 + 35r^4s^2 - 3r^3s^6 + 27r^3s^5 - 59r^3s^4 + 35r^3s^3 +$$
$$6r^2s^7 - 27r^2s^6 + r^2s^5 + 35r^2s^4 - 2rs^8 - 2rs^7 + 43rs^6 - 49rs^5 + 4s^8 - 16s^7 + 14s^6) -$$
$$5040rs^3y_{n+r}(r-s)^2(r-1)^3(s-1)^3 + 5040r^3sy_{n+s}(r-s)^2(r-1)^3(s-1)^3 + 2f_nh^3(r-$$
$$s)^3(r-1)^3(s-1)^3(2r^8s + 2r^8 - 6r^7s^2 - 10r^7s - 8r^7 + 3r^6s^3 + 18r^6s^2 + 20r^6s +$$
$$9r^6 + 3r^5s^4 - 9r^5s^3 + 8r^5s^2 - 18r^5s + 3r^4s^5 - 9r^4s^4 - 40r^4s^3 - 66r^4s^2 + 3r^3s^6 -$$
$$9r^3s^5 - 40r^3s^4 + 270r^3s^3 - 6r^2s^7 + 18r^2s^6 + 8r^2s^5 - 66r^2s^4 + 2rs^8 - 10rs^7 + 20rs^6 -$$
$$18rs^5 + 2s^8 - 8s^7 + 9s^6) - g_{n+r}h^4rs^3(r-1)(s-1)^3(3r^8 - 10r^7s - 10r^7 + 9r^6s^2 +$$
$$36r^6s + 9r^6 - 36r^5s^2 - 36r^5s + 42r^4s^2 - 4rs^7 + 18rs^6 - 24rs^5 + 2s^8 - 8s^7 + 9s^6) +$$
$$g_{n+s}h^4r^3s(r-1)^3(s-1)(2r^8 - 4r^7s - 8r^7 + 18r^6s + 9r^6 - 24r^5s + 9r^2s^6 - 36r^2s^5 +$$
$$42r^2s^4 - 10rs^7 + 36rs^6 - 36rs^5 + 3s^8 - 10s^7 + 9s^6) - 5040rsy_n(r+s)(r-s)^3(r-$$
$$1)^3(s-1)^3 + g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(2r^7 - 6r^6s - 8r^6 + 3r^5s^2 + 28r^5s + 9r^5 +$$
$$3r^4s^3 - 20r^4s^2 - 39r^4s + 3r^3s^4 - 20r^3s^3 + 45r^3s^2 + 3r^2s^5 - 20r^2s^4 + 45r^2s^3 - 6rs^6 +$$
$$28rs^5 - 39rs^4 + 2s^7 - 8s^6 + 9s^5) + g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(2r^7 - 6r^6s - 4r^6 +$$
$$3r^5s^2 + 14r^5s + 3r^4s^3 - 10r^4s^2 + 3r^3s^4 - 10r^3s^3 + 3r^2s^5 - 10r^2s^4 - 6rs^6 + 14rs^5 +$$
$$2s^7 - 4s^6)) \tag{4.10}$$

$$y'_{n+r} = \frac{1}{5040hr^2s^3(r-s)^2(r-1)^3(s-1)^3}(2f_{n+s}h^3r^3(r-1)^3(6r^8 - 12r^8s + 35r^7s^2 + 17r^7s$$
$$- 20r^7 - 12r^6s^3 - 116r^6s^2 + 46r^6s + 18r^6 - 10r^5s^4 + 61r^5s^3 + 83r^5s^2 - 81r^5s -$$
$$8r^4s^5 + 49r^4s^4 - 102r^4s^3 + 54r^4s^2 - 6r^3s^6 + 37r^3s^5 - 77r^3s^4 + 39r^3s^3 - 4r^2s^7 +$$
$$25r^2s^6 - 52r^2s^5 + 24r^2s^4 + 28rs^8 - 122rs^7 + 177rs^6 - 75rs^5 - 14s^9 + 56s^8 - 74s^7 +$$
$$30s^6) - 2f_{n+r}h^3s^3(s-1)^3(210r^8s - 77r^9 + 273r^8 - 152r^7s^2 - 772r^7s - 320r^7 +$$
$$592r^6s^2 + 940r^6s + 120r^6 + 2r^5s^4 - 5r^5s^3 - 769r^5s^2 - 360r^5s + 4r^4s^5 - 17r^4s^4 +$$
$$18r^4s^3 + 303r^4s^2 + 6r^3s^6 - 29r^3s^5 + 43r^3s^4 - 6r^3s^3 + 8r^2s^7 - 41r^2s^6 + 68r^2s^5 -$$
$$21r^2s^4 - 4rs^8 + 10rs^7 + 9rs^6 - 36rs^5 + 2s^8 - 8s^7 + 9s^6) + 2f_{n+1}h^3r^3s^3(r-s)^3(12r^8 -$$
$$6r^8s + 16r^7s^2 - 5r^7s - 43r^7 - 8r^6s^3 - 61r^6s^2 + 98r^6s + 35r^6 - 5r^5s^4 + 56r^5s^3 + r^5s^2 -$$
$$105r^5s - 2r^4s^5 + 29r^4s^4 - 90r^4s^3 + 70r^4s^2 + r^3s^6 + 2r^3s^5 - 31r^3s^4 + 35r^3s^3 + 4r^2s^7 -$$
$$25r^2s^6 + 28r^2s^5 - 2rs^8 + 2rs^7 + 27rs^6 - 35rs^5 + 4s^8 - 16s^7 + 14s^6) + 5040rs^3y_{n+r}(r-$$
$$1)^3(s-1)^3(2r^2 - 3rs + s^2) + 5040rs^2y_n(r-s)^3(r-1)^3(s-1)^3$$

$-5040r^3s^2y_{n+s}(r-s)(r-1)^3(s-1)^3+2f_nh^3(r-s)^3(r-1)^3(s-1)^3(6r^8s+6r^8-16r^7s^2-25r^7s-20r^7+8r^6s^3+37r^6s^2+40r^6s+18r^6+5r^5s^4-17r^5s^3+20r^5s^2-27r^5s+2r^4s^5-8r^4s^4-60r^4s^3-81r^4s^2-r^3s^6+r^3s^5-20r^3s^4+195r^3s^3-4r^2s^7+10r^2s^6+20r^2s^5-75r^2s^4+2rs^8-8rs^7+12rs^6-9rs^5+2s^8-8s^7+9s^6)+g_{n+s}h^4r^3s(r-1)^3(s-1)(6r^8-12r^7s-20r^7+2r^6s^2+44r^6s+18r^6+2r^5s^3-10r^5s^2-45r^5s+2r^4s^4-10r^4s^3+15r^4s^2+2r^3s^5-10r^3s^4+15r^3s^3+2r^2s^6-10r^2s^5+15r^2s^4-7rs^7+26rs^6-27rs^5+3s^8-10s^7+9s^6)+g_{n+r}h^4rs^3(r-1)(s-1)^3(43r^7s-14r^8+40r^7-37r^6s^2-130r^6s-30r^6+2r^5s^3+122r^5s^2+105r^5s+2r^4s^4-10r^4s^3-111r^4s^2+2r^3s^5-10r^3s^4+15r^3s^3+2r^2s^6-10r^2s^5+15r^2s^4+2rs^7-10rs^6+15rs^5-2s^8+8s^7-9s^6)+g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(6r^7-16r^6s-20r^6+8r^5s^2+60r^5s+18r^5+5r^4s^3-40r^4s^2-63r^4s+2r^3s^4-20r^3s^3+60r^3s^2-r^2s^5+15r^2s^3-4rs^6+20rs^5-30rs^4+2s^7-8s^6+9s^5)-g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(16r^6s-6r^7+10r^6-8r^5s^2-30r^5s-5r^4s^3+20r^4s^2-2r^3s^4+10r^3s^3+r^2s^5+4rs^6-10rs^5-2s^7+4s^6))$ (4.11)

$y'_{n+s}=\frac{1}{5040hr^3s^2(r-s)^2(r-1)^3(s-1)^3}(2f_{n+r}h^3s^3(s-1)^3(28r^8s-14r^9+56r^8-4r^7s^2-122r^7s-74r^7-6r^6s^3+25r^6s^2+177r^6s+30r^6-8r^5s^4+37r^5s^3-52r^5s^2-75r^5s-10r^4s^5+49r^4s^4-77r^4s^3+24r^4s^2-12r^3s^6+61r^3s^5-102r^3s^4+39r^3s^3+35r^2s^7-116r^2s^6+83r^2s^5+54r^2s^4-12rs^8+17rs^7+46rs^6-81rs^5+6s^8-20s^7+18s^6)-2f_{n+s}h^3r^3(r-1)^3(2r^8-4r^8s+8r^7s^2+10r^7s-8r^7+6r^6s^3-41r^6s^2+9r^6s+9r^6+4r^5s^4-29r^5s^3+68r^5s^2-36r^5s+2r^4s^5-17r^4s^4+43r^4s^3-21r^4s^2-5r^3s^5+18r^3s^4-6r^3s^3-152r^2s^7+592r^2s^6-769r^2s^5+303r^2s^4+210rs^8-772rs^7+940rs^6-360rs^5-77s^9+273s^8-320s^7+120s^6)-2f_{n+1}h^3r^3s^3(r-s)^3(4r^8-2r^8s+4r^7s^2+2r^7s-16r^7+r^6s^3-25r^6s^2+27r^6s+14r^6-2r^5s^4+2r^5s^3+28r^5s^2-35r^5s-5r^4s^5+29r^4s^4-31r^4s^3-8r^3s^6+56r^3s^5-90r^3s^4+35r^3s^3+16r^2s^7-61r^2s^6+r^2s^5+70r^2s^4-6rs^8-5rs^7+98rs^6-105rs^5+12s^8-43s^7+35s^6)+5040r^3sy_{n+s}(r-1)^3(s-1)^3(r^2-3rs+2s^2)-5040r^2sy_n(r-s)^3(r-1)^3(s-1)^3+5040r^2s^3y_{n+r}(r-s)(r-1)^3(s-1)^3-2f_nh^3(r-s)^3(r-1)^3(s-1)^3(2r^8s+2r^8-4r^7s^2-8r^7s-8r^7-r^6s^3+10r^6s^2+12r^6s+9r^6+2r^5s^4+r^5s^3+20r^5s^2-9r^5s+5r^4s^5-8r^4s^4-20r^4s^3-$

$75r^4s^2+8r^3s^6-17r^3s^5-60r^3s^4+195r^3s^3-16r^2s^7+37r^2s^6+20r^2s^5-81r^2s^4+6rs^8-25rs^7+40rs^6-27rs^5+6s^8-20s^7+18s^6)+g_{n+r}h^4rs^3(r-1)(s-1)^3(3r^8-7r^7s-10r^7+2r^6s^2+26r^6s+9r^6+2r^5s^3-10r^5s^2-27r^5s+2r^4s^4-10r^4s^3+15r^4s^2+2r^3s^5-10r^3s^4+15r^3s^3+2r^2s^6-10r^2s^5+15r^2s^4-12rs^7+44rs^6-45rs^5+6s^8-20s^7+18s^6)+g_{n+s}h^4r^3s(r-1)^3(s-1)(2r^7s-2r^8+8r^7+2r^6s^2-10r^6s-9r^6+2r^5s^3-10r^5s^2+15r^5s+2r^4s^4-10r^4s^3+15r^4s^2+2r^3s^5-10r^3s^4+15r^3s^3-37r^2s^6+122r^2s^5-111r^2s^4+43rs^7-130rs^6+105rs^5-14s^8+40s^7-30s^6)-g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(2r^7-4r^6s-8r^6-r^5s^2+20r^5s+9r^5+2r^4s^3-30r^4s+5r^3s^4-20r^3s^3+15r^3s^2+8r^2s^5-40r^2s^4+60r^2s^3-16rs^6+60rs^5-63rs^4+6s^7-20s^6+18s^5)+g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(4r^6s-2r^7+4r^6+r^5s^2-10r^5s-2r^4s^3-5r^3s^4+10r^3s^3-8r^2s^5+20r^2s^4+16rs^6-30rs^5-6s^7+10s^6))$ (4.12)

$y'_{n+1}=\frac{1}{5040hr^3s^3(r-s)^3(r-1)^3(s-1)^3}(2f_{n+r}h^3s^3(s-1)^3(14r^{10}s-28r^{10}-42r^9s^2+28r^9s+112r^9+32r^8s^3+114r^8s^2-282r^8s-148r^8+2r^7s^4-151r^7s^3+43r^7s^2+472r^7s+60r^7+2r^6s^5-16r^6s^4+253r^6s^3-353r^6s^2-210r^6s+2r^5s^6-16r^5s^5+49r^5s^4-149r^5s^3+198r^5s^2+2r^4s^7-16r^4s^6+49r^4s^5-65r^4s^4+30r^4s^3-12r^3s^8+47r^3s^7-35r^3s^6-65r^3s^5+30r^3s^4+294r^3s^2-252r^3s+63r^3+4r^2s^9+16r^2s^8-119r^2s^7+163r^2s^6+30r^2s^5-210r^2s^3-138r^2s^2+243r^2s-75r^2-10rs^9+24rs^8+25rs^7-90rs^6+198rs^3-108rs^2-25rs+20r+4s^9-16s^8+18s^7-36s^3+36s^2-10s)-2f_{n+s}h^3r^3(r-1)^3(4r^9s^2-10r^9s+4r^9-12r^8s^3+16r^8s^2+24r^8s-16r^8+2r^7s^4+47r^7s^3-119r^7s^2+25r^7s+18r^7+2r^6s^5-16r^6s^4-35r^6s^3+163r^6s^2-90r^6s+2r^5s^6-16r^5s^5+49r^5s^4-65r^5s^3+30r^5s^2+2r^4s^7-16r^4s^6+49r^4s^5-65r^4s^4+30r^4s^3+32r^3s^8-151r^3s^7+253r^3s^6-149r^3s^5+30r^3s^4-210r^3s^2+198r^3s-36r^3-42r^2s^9+114r^2s^8+43r^2s^7-353r^2s^6+198r^2s^5+294r^2s^3-138r^2s^2-108r^2s+36r^2+14rs^{10}+28rs^9-282rs^8+472rs^7-210rs^6-252rs^3+243rs^2-25rs-10r-28s^{10}+112s^9-148s^8+60s^7+63s^3-75s^2+20s)+2f_nh^3(r-s)^3(r-1)^3(s-1)^3(2r^9s^2-2r^9s-4r^9-6r^8s^3+2r^8s^2+12r^8s+16r^8+3r^7s^4+12r^7s^3-16r^7s^2-31r^7s-18r^7+3r^6s^5-15r^6s^4+26r^6s^3-34r^6s^2+36r^6s+3r^5s^6-15r^5s^5-22r^5s^4+14r^5s^3+132r^5s^2+3r^4s^7-15r^4s^6-22r^4s^5+350r^4s^4-540r^4s^3-$

$$
\begin{aligned}
&6r^3s^8+12r^3s^7+26r^3s^6+14r^3s^5-540r^3s^4+882r^3s^3-168r^3s^2-90r^3s+36r^3+\\
&2r^2s^9+2r^2s^8-16r^2s^7-34r^2s^6+132r^2s^5-168r^2s^3-24r^2s^2+108r^2s-36r^2-\\
&2rs^9+12rs^8-31rs^7+36rs^6-90rs^3+108rs^2-53rs+10r-4s^9+16s^8-18s^7+\\
&36s^3-36s^2+10s)-2f_{n+1}h^3r^3s^3(r-s)^3(2r^9s^2-8r^9s+8r^9-6r^8s^3+14r^8s^2+\\
&12r^8s-32r^8+3r^7s^4+21r^7s^3-97r^7s^2+72r^7s+28r^7+3r^6s^5-33r^6s^4+53r^6s^3+\\
&51r^6s^2-98r^6s+3r^5s^6-33r^5s^5+113r^5s^4-153r^5s^3+70r^5s^2+3r^4s^7-33r^4s^6+\\
&113r^4s^5-153r^4s^4+70r^4s^3-6r^3s^8+21r^3s^7+53r^3s^6-153r^3s^5+70r^3s^4-378r^3s^3+\\
&966r^3s^2-720r^3s+180r^3+2r^2s^9+14r^2s^8-97r^2s^7+51r^2s^6+70r^2s^5+966r^2s^3-\\
&2418r^2s^2+1836r^2s-468r^2-8rs^9+12rs^8+72rs^7-98rs^6-720rs^3+1836rs^2-\\
&1457rs+385r+8s^9-32s^8+28s^7+180s^3-468s^2+385s-105)-5040r^2s^2y_n(r-\\
&s)^3(r-1)^3(s-1)^3(r+s-2)+g_{n+s}h^4r^3s(r-1)^3(s-1)(2r^9s-4r^9-4r^8s^2+16r^8+\\
&18r^7s^2-27r^7s-18r^7-24r^6s^2+48r^6s+9r^3s^7-36r^3s^6+42r^3s^5-84r^3s+36r^3-\\
&10r^2s^8+18r^2s^7+36r^2s^6-84r^2s^5+84r^2s^2+36r^2s-36r^2+3rs^9+10rs^8-63rs^7+\\
&72rs^6-72rs^2+18rs+10r-6s^9+20s^8-18s^7+18s^2-10s)-g_{n+r}h^4rs^3(r-1)(s-\\
&1)^3(3r^9s-6r^9-10r^8s^2+10r^8s+20r^8+9r^7s^3+18r^7s^2-63r^7s-18r^7-36r^6s^3+\\
&36r^6s^2+72r^6s+42r^5s^3-84r^5s^2-4r^2s^8+18r^2s^7-24r^2s^6+84r^2s^2-72r^2s+18r^2+\\
&2rs^9-27rs^7+48rs^6-84rs^3+36rs^2+18rs-10r-4s^9+16s^8-18s^7+36s^3-\\
&36s^2+10s)+5040r^3s^2y_{n+s}(r-s)^2(r-1)^3(r-2)(s-1)^3-5040r^2s^3y_{n+r}(r-s)^2(r-\\
&1)^3(s-1)^3(s-2)+g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(2r^8s-4r^8-6r^7s^2+4r^7s+16r^7+\\
&3r^6s^3+22r^6s^2-47r^6s-18r^6+3r^5s^4-26r^5s^3+r^5s^2+78r^5s+3r^4s^5-26r^4s^4+\\
&85r^4s^3-90r^4s^2+3r^3s^6-26r^3s^5+85r^3s^4-90r^3s^3-6r^2s^7+22r^2s^6+r^2s^5-90r^2s^4+\\
&252r^2s^2-168r^2s+36r^2+2rs^8+4rs^7-47rs^6+78rs^5-168rs^2+144rs-36r-\\
&4s^8+16s^7-18s^6+36s^2-36s+10)+g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(2r^8s-\\
&4r^8-6r^7s^2+8r^7s+8r^7+3r^6s^3+8r^6s^2-28r^6s+3r^5s^4-16r^5s^3+20r^5s^2+3r^4s^5-\\
&16r^4s^4+20r^4s^3+3r^3s^6-16r^3s^5+20r^3s^4-6r^2s^7+8r^2s^6+20r^2s^5-168r^2s^2+\\
&168r^2s-48r^2+2rs^8+8rs^7-28rs^6+168rs^2-192rs+60r-4s^8+8s^7-48s^2+\\
&60s-20))
\end{aligned}
\tag{4.13}
$$

$$y_n'' = \frac{1}{2520h^2r^3s^3(r-s)^3(r-1)^3(s-1)^3}(2f_{n+r}h^3s^3(s-1)^3(42r^9s-14r^{10}+56r^9-32r^8s^2-178r^8s-74r^8-2r^7s^3+147r^7s^2+251r^7s+30r^7-2r^6s^4+12r^6s^3-229r^6s^2-105r^6s-2r^5s^5+12r^5s^4-25r^5s^3+99r^5s^2-2r^4s^6+12r^4s^5-25r^4s^4+15r^4s^3-2r^3s^7+12r^3s^6-25r^3s^5+15r^3s^4+12r^2s^8-51r^2s^7+59r^2s^6+15r^2s^5-4rs^9+8rs^8+17rs^7-45rs^6+2s^9-8s^8+9s^7)-2f_{n+s}h^3r^3(r-1)^3(2r^9-4r^9s+12r^8s^2+8r^8s-8r^8-2r^7s^3-51r^7s^2+17r^7s+9r^7-2r^6s^4+12r^6s^3+59r^6s^2-45r^6s-2r^5s^5+12r^5s^4-25r^5s^3+15r^5s^2-2r^4s^6+12r^4s^5-25r^4s^4+15r^4s^3-2r^3s^7+12r^3s^6-25r^3s^5+15r^3s^4-32r^2s^8+147r^2s^7-229r^2s^6+99r^2s^5+42rs^9-178rs^8+251rs^7-105rs^6-14s^10+56s^9-74s^8+30s^7)-2f_{n+1}h^3r^3s^3(r-s)^3(4r^9-2r^9s+6r^8s^2-2r^8s-16r^8-3r^7s^3-27r^7s^2+43r^7s+14r^7-3r^6s^4+27r^6s^3+r^6s^2-49r^6s-3r^5s^5+27r^5s^4-59r^5s^3+35r^5s^2-3r^4s^6+27r^4s^5-59r^4s^4+35r^4s^3-3r^3s^7+27r^3s^6-59r^3s^5+35r^3s^4+6r^2s^8-27r^2s^7+r^2s^6+35r^2s^5-2rs^9-2rs^8+43rs^7-49rs^6+4s^9-16s^8+14s^7)-2f_nh^3(r-s)^3(r-1)^3(s-1)^3(2r^9s+2r^9-6r^8s^2-10r^8s-8r^8+3r^7s^3+18r^7s^2+20r^7s+9r^7+3r^6s^4-9r^6s^3+8r^6s^2-18r^6s+3r^5s^5-9r^5s^4-40r^5s^3-66r^5s^2+3r^4s^6-9r^4s^5-40r^4s^4+270r^4s^3+3r^3s^7-9r^3s^6-40r^3s^5+270r^3s^4-6r^2s^8+18r^2s^7+8r^2s^6-66r^2s^5+2rs^9-10rs^8+20rs^7-18rs^6+2s^9-8s^8+9s^7)+5040r^2s^2y_n(r-s)^3(r-1)^3(s-1)^3+5040r^2s^3y_{n+r}(r-s)^2(r-1)^3(s-1)^3-5040r^3s^2y_{n+s}(r-s)^2(r-1)^3(s-1)^3+g_{n+r}h^4rs^3(r-1)(s-1)^3(3r^9-10r^8s-10r^8+9r^7s^2+36r^7s+9r^7-36r^6s^2-36r^6s+42r^5s^2-4rs^8+18rs^7-24rs^6+2s^9-8s^8+9s^7)-g_{n+s}h^4r^3s(r-1)^3(s-1)(2r^9-4r^8s-8r^8+18r^7s+9r^7-24r^6s+9r^2s^7-36r^2s^6+42r^2s^5-10rs^8+36rs^7-36rs^6+3s^9-10s^8+9s^7)-g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(2r^8-6r^7s-8r^7+3r^6s^2+28r^6s+9r^6+3r^5s^3-20r^5s^2-39r^5s+3r^4s^4-20r^4s^3+45r^4s^2+3r^3s^5-20r^3s^4+45r^3s^3+3r^2s^6-20r^2s^5+45r^2s^4-6rs^7+28rs^6-39rs^5+2s^8-8s^7+9s^6)-g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(2r^8-6r^7s-4r^7+3r^6s^2+14r^6s+3r^5s^3-10r^5s^2+3r^4s^4-10r^4s^3+3r^3s^5-10r^3s^4+3r^2s^6-10r^2s^5-6rs^7+14rs^6+2s^8-4s^7)) \tag{4.14}$$

$$
\begin{aligned}
y''_{n+r} &= \tfrac{1}{2520h^2r^3s^3(r-s)^3(r-1)^3(s-1)^3}(2f_{n+s}h^3r^3(r-1)^3(13r^9-26r^9s+93r^8s^2+22r^8s-\\
&40r^8-82r^7s^3-243r^7s^2+121r^7s+33r^7+2r^6s^4+282r^6s^3+67r^6s^2-165r^6s+\\
&2r^5s^5-12r^5s^4-269r^5s^3+195r^5s^2+2r^4s^6-12r^4s^5+25r^4s^4-15r^4s^3+2r^3s^7-\\
&12r^3s^6+25r^3s^5-15r^3s^4+32r^2s^8-147r^2s^7+229r^2s^6-99r^2s^5-42rs^9+178rs^8-\\
&251rs^7+105rs^6+14s^10-56s^9+74s^8-30s^7)+2f_{n+r}h^3s^3(s-1)^3(301r^{10}-\\
&1008r^9s-994r^9+1132r^8s^2+3383r^8s+1090r^8-422r^7s^3-3879r^7s^2-3775r^7s-\\
&390r^7-2r^6s^4+1482r^6s^3+4433r^6s^2+1365r^6s-2r^5s^5+12r^5s^4-1747r^5s^3-\\
&1623r^5s^2-2r^4s^6+12r^4s^5-25r^4s^4+645r^4s^3-2r^3s^7+12r^3s^6-25r^3s^5+15r^3s^4+\\
&12r^2s^8-51r^2s^7+59r^2s^6+15r^2s^5-4rs^9+8rs^8+17rs^7-45rs^6+2s^9-8s^8+9s^7)-\\
&2f_{n+1}h^3r^3s^3(r-s)^3(13r^9s-26r^9-42r^8s^2+28r^8s+89r^8+39r^7s^3+111r^7s^2-\\
&251r^7s-70r^7-3r^6s^4-183r^6s^3+127r^6s^2+245r^6s-3r^5s^5+27r^5s^4+151r^5s^3-\\
&259r^5s^2-3r^4s^6+27r^4s^5-59r^4s^4+35r^4s^3-3r^3s^7+27r^3s^6-59r^3s^5+35r^3s^4+\\
&6r^2s^8-27r^2s^7+r^2s^6+35r^2s^5-2rs^9-2rs^8+43rs^7-49rs^6+4s^9-16s^8+14s^7)-\\
&2f_nh^3(r-s)^3(r-1)^3(s-1)^3(42r^8s^2-13r^9-13r^9s+59r^8s+40r^8-39r^7s^3-\\
&102r^7s^2-100r^7s-33r^7+3r^6s^4+75r^6s^3+8r^6s^2+66r^6s+3r^5s^5-9r^5s^4+128r^5s^3+\\
&102r^5s^2+3r^4s^6-9r^4s^5-40r^4s^4-360r^4s^3+3r^3s^7-9r^3s^6-40r^3s^5+270r^3s^4-\\
&6r^2s^8+18r^2s^7+8r^2s^6-66r^2s^5+2rs^9-10rs^8+20rs^7-18rs^6+2s^9-8s^8+\\
&9s^7)+5040r^2s^2y_n(r-s)^3(r-1)^3(s-1)^3+5040r^2s^3y_{n+r}(r-s)^2(r-1)^3(s-1)^3-\\
&5040r^3s^2y_{n+s}(r-s)^2(r-1)^3(s-1)^3-g_{n+s}h^4r^3s(r-1)^3(s-1)(35r^8s-13r^9+\\
&40r^8-24r^7s^2-114r^7s-33r^7+84r^6s^2+102r^6s-84r^5s^2+9r^2s^7-36r^2s^6+42r^2s^5-\\
&10rs^8+36rs^7-36rs^6+3s^9-10s^8+9s^7)+g_{n+r}h^4rs^3(r-1)(s-1)^3(155r^8s-42r^9+\\
&110r^8-195r^7s^2-420r^7s-75r^7+84r^6s^3+552r^6s^2+300r^6s-252r^5s^3-420r^5s^2+\\
&210r^4s^3-4rs^8+18rs^7-24rs^6+2s^9-8s^8+9s^7)-g_nh^4rs(r-s)^3(r-1)^3(s-\\
&1)^3(42r^7s-13r^8+40r^7-39r^6s^2-140r^6s-33r^6+3r^5s^3+148r^5s^2+129r^5s+\\
&3r^4s^4-20r^4s^3-165r^4s^2+3r^3s^5-20r^3s^4+45r^3s^3+3r^2s^6-20r^2s^5+45r^2s^4-6rs^7+\\
&28rs^6-39rs^5+2s^8-8s^7+9s^6)-g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(42r^7s-13r^8+\\
&20r^7-39r^6s^2-70r^6s+3r^5s^3+74r^5s^2+3r^4s^4-10r^4s^3+3r^3s^5-10r^3s^4+3r^2s^6-\\
&10r^2s^5-6rs^7+14rs^6+2s^8-4s^7))
\end{aligned}
\tag{4.15}
$$

$$
\begin{aligned}
y''_{n+s} = {} & \frac{1}{2520h^2r^3s^3(r-s)^3(r-1)^3(s-1)^3}(5040r^2s^2y_n(r-s)^3(r-1)^3(s-1)^3 - 2f_{n+r}h^3s^3(s- \\
& 1)^3(14r^{10} - 42r^9s - 56r^9 + 32r^8s^2 + 178r^8s + 74r^8 + 2r^7s^3 - 147r^7s^2 - 251r^7s - \\
& 30r^7 + 2r^6s^4 - 12r^6s^3 + 229r^6s^2 + 105r^6s + 2r^5s^5 - 12r^5s^4 + 25r^5s^3 - 99r^5s^2 + \\
& 2r^4s^6 - 12r^4s^5 + 25r^4s^4 - 15r^4s^3 - 82r^3s^7 + 282r^3s^6 - 269r^3s^5 - 15r^3s^4 + 93r^2s^8 - \\
& 243r^2s^7 + 67r^2s^6 + 195r^2s^5 - 26rs^9 + 22rs^8 + 121rs^7 - 165rs^6 + 13s^9 - 40s^8 + \\
& 33s^7) - 2f_{n+1}h^3r^3s^3(r-s)^3(4r^9 - 2r^9s + 6r^8s^2 - 2r^8s - 16r^8 - 3r^7s^3 - 27r^7s^2 + \\
& 43r^7s + 14r^7 - 3r^6s^4 + 27r^6s^3 + r^6s^2 - 49r^6s - 3r^5s^5 + 27r^5s^4 - 59r^5s^3 + 35r^5s^2 - \\
& 3r^4s^6 + 27r^4s^5 - 59r^4s^4 + 35r^4s^3 + 39r^3s^7 - 183r^3s^6 + 151r^3s^5 + 35r^3s^4 - 42r^2s^8 + \\
& 111r^2s^7 + 127r^2s^6 - 259r^2s^5 + 13rs^9 + 28rs^8 - 251rs^7 + 245rs^6 - 26s^9 + 89s^8 - \\
& 70s^7) - 2f_nh^3(r-s)^3(r-1)^3(s-1)^3(2r^9s + 2r^9 - 6r^8s^2 - 10r^8s - 8r^8 + 3r^7s^3 + \\
& 18r^7s^2 + 20r^7s + 9r^7 + 3r^6s^4 - 9r^6s^3 + 8r^6s^2 - 18r^6s + 3r^5s^5 - 9r^5s^4 - 40r^5s^3 - \\
& 66r^5s^2 + 3r^4s^6 - 9r^4s^5 - 40r^4s^4 + 270r^4s^3 - 39r^3s^7 + 75r^3s^6 + 128r^3s^5 - 360r^3s^4 + \\
& 42r^2s^8 - 102r^2s^7 + 8r^2s^6 + 102r^2s^5 - 13rs^9 + 59rs^8 - 100rs^7 + 66rs^6 - 13s^9 + \\
& 40s^8 - 33s^7) - 2f_{n+s}h^3r^3(r-1)^3(2r^9 - 4r^9s + 12r^8s^2 + 8r^8s - 8r^8 - 2r^7s^3 - 51r^7s^2 + \\
& 17r^7s + 9r^7 - 2r^6s^4 + 12r^6s^3 + 59r^6s^2 - 45r^6s - 2r^5s^5 + 12r^5s^4 - 25r^5s^3 + 15r^5s^2 - \\
& 2r^4s^6 + 12r^4s^5 - 25r^4s^4 + 15r^4s^3 - 422r^3s^7 + 1482r^3s^6 - 1747r^3s^5 + 645r^3s^4 + \\
& 1132r^2s^8 - 3879r^2s^7 + 4433r^2s^6 - 1623r^2s^5 - 1008rs^9 + 3383rs^8 - 3775rs^7 + \\
& 1365rs^6 + 301s^{1}0 - 994s^9 + 1090s^8 - 390s^7) + 5040r^2s^3y_{n+r}(r-s)^2(r-1)^3(s- \\
& 1)^3 - 5040r^3s^2y_{n+s}(r-s)^2(r-1)^3(s-1)^3 - g_{n+r}h^4rs^3(r-1)(s-1)^3(10r^8s - 3r^9 + \\
& 10r^8 - 9r^7s^2 - 36r^7s - 9r^7 + 36r^6s^2 + 36r^6s - 42r^5s^2 + 24r^2s^7 - 84r^2s^6 + 84r^2s^5 - \\
& 35rs^8 + 114rs^7 - 102rs^6 + 13s^9 - 40s^8 + 33s^7) + g_{n+s}h^4r^3s(r-1)^3(s-1)(4r^8s - \\
& 2r^9 + 8r^8 - 18r^7s - 9r^7 + 24r^6s - 84r^3s^6 + 252r^3s^5 - 210r^3s^4 + 195r^2s^7 - 552r^2s^6 + \\
& 420r^2s^5 - 155rs^8 + 420rs^7 - 300rs^6 + 42s^9 - 110s^8 + 75s^7) - g_nh^4rs(r-s)^3(r- \\
& 1)^3(s-1)^3(2r^8 - 6r^7s - 8r^7 + 3r^6s^2 + 28r^6s + 9r^6 + 3r^5s^3 - 20r^5s^2 - 39r^5s + 3r^4s^4 - \\
& 20r^4s^3 + 45r^4s^2 + 3r^3s^5 - 20r^3s^4 + 45r^3s^3 - 39r^2s^6 + 148r^2s^5 - 165r^2s^4 + 42rs^7 - \\
& 140rs^6 + 129rs^5 - 13s^8 + 40s^7 - 33s^6) - g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(2r^8 - 6r^7s - \\
& 4r^7 + 3r^6s^2 + 14r^6s + 3r^5s^3 - 10r^5s^2 + 3r^4s^4 - 10r^4s^3 + 3r^3s^5 - 10r^3s^4 - 39r^2s^6 + \\
& 74r^2s^5 + 42rs^7 - 70rs^6 - 13s^8 + 20s^7))
\end{aligned}
\tag{4.16}
$$

$$y''_{n+1} = \frac{1}{2520h^2r^3s^3(r-s)^3(r-1)^3(s-1)^3}(2f_{n+s}h^3r^3(r-1)^3(4r^9s - 2r^9 - 12r^8s^2 - 8r^8s +$$

$$8r^8 + 2r^7s^3 + 51r^7s^2 - 17r^7s - 9r^7 + 2r^6s^4 - 12r^6s^3 - 59r^6s^2 + 45r^6s + 2r^5s^5 -$$

$$12r^5s^4 + 25r^5s^3 - 15r^5s^2 + 2r^4s^6 - 12r^4s^5 + 25r^4s^4 - 15r^4s^3 + 2r^3s^7 - 12r^3s^6 +$$

$$25r^3s^5 - 15r^3s^4 + 210r^3s^2 - 210r^3s + 42r^3 + 32r^2s^8 - 147r^2s^7 + 229r^2s^6 - 99r^2s^5 -$$

$$294r^2s^3 + 126r^2s^2 + 138r^2s - 48r^2 - 42rs^9 + 178rs^8 - 251rs^7 + 105rs^6 + 294rs^3 -$$

$$294rs^2 + 30rs + 15r + 14s^{10} - 56s^9 + 74s^8 - 30s^7 - 84s^3 + 105s^2 - 30s) -$$

$$2f_{n+r}h^3s^3(s-1)^3(14r^{10} - 42r^9s - 56r^9 + 32r^8s^2 + 178r^8s + 74r^8 + 2r^7s^3 - 147r^7s^2 -$$

$$251r^7s - 30r^7 + 2r^6s^4 - 12r^6s^3 + 229r^6s^2 + 105r^6s + 2r^5s^5 - 12r^5s^4 + 25r^5s^3 -$$

$$99r^5s^2 + 2r^4s^6 - 12r^4s^5 + 25r^4s^4 - 15r^4s^3 + 2r^3s^7 - 12r^3s^6 + 25r^3s^5 - 15r^3s^4 -$$

$$294r^3s^2 + 294r^3s - 84r^3 - 12r^2s^8 + 51r^2s^7 - 59r^2s^6 - 15r^2s^5 + 210r^2s^3 + 126r^2s^2 -$$

$$294r^2s + 105r^2 + 4rs^9 - 8rs^8 - 17rs^7 + 45rs^6 - 210rs^3 + 138rs^2 + 30rs - 30r - 2s^9 +$$

$$8s^8 - 9s^7 + 42s^3 - 48s^2 + 15s) - 2f_nh^3(r-s)^3(r-1)^3(s-1)^3(2r^9s + 2r^9 - 6r^8s^2 -$$

$$10r^8s - 8r^8 + 3r^7s^3 + 18r^7s^2 + 20r^7s + 9r^7 + 3r^6s^4 - 9r^6s^3 + 8r^6s^2 - 18r^6s + 3r^5s^5 -$$

$$9r^5s^4 - 40r^5s^3 - 66r^5s^2 + 3r^4s^6 - 9r^4s^5 - 40r^4s^4 + 270r^4s^3 + 3r^3s^7 - 9r^3s^6 - 40r^3s^5 +$$

$$270r^3s^4 - 630r^3s^3 + 168r^3s^2 + 84r^3s - 42r^3 - 6r^2s^8 + 18r^2s^7 + 8r^2s^6 - 66r^2s^5 +$$

$$168r^2s^3 - 120r^2s + 48r^2 + 2rs^9 - 10rs^8 + 20rs^7 - 18rs^6 + 84rs^3 - 120rs^2 + 69rs -$$

$$15r + 2s^9 - 8s^8 + 9s^7 - 42s^3 + 48s^2 - 15s) - 2f_{n+1}h^3r^3s^3(r-s)^3(4r^9 - 2r^9s + 6r^8s^2 -$$

$$2r^8s - 16r^8 - 3r^7s^3 - 27r^7s^2 + 43r^7s + 14r^7 - 3r^6s^4 + 27r^6s^3 + r^6s^2 - 49r^6s - 3r^5s^5 +$$

$$27r^5s^4 - 59r^5s^3 + 35r^5s^2 - 3r^4s^6 + 27r^4s^5 - 59r^4s^4 + 35r^4s^3 - 3r^3s^7 + 27r^3s^6 -$$

$$59r^3s^5 + 35r^3s^4 - 630r^3s^3 + 1722r^3s^2 - 1470r^3s + 420r^3 + 6r^2s^8 - 27r^2s^7 + r^2s^6 +$$

$$35r^2s^5 + 1722r^2s^3 - 4662r^2s^2 + 4026r^2s - 1164r^2 - 2rs^9 - 2rs^8 + 43rs^7 - 49rs^6 -$$

$$1470rs^3 + 4026rs^2 - 3561rs + 1050r + 4s^9 - 16s^8 + 14s^7 + 420s^3 - 1164s^2 + 1050s -$$

$$315) + 5040r^2s^2y_n(r-s)^3(r-1)^3(s-1)^3 + 5040r^2s^3y_{n+r}(r-s)^2(r-1)^3(s-1)^3 -$$

$$5040r^3s^2y_{n+s}(r-s)^2(r-1)^3(s-1)^3 + g_{n+s}h^4r^3s(r-1)^3(s-1)(4r^8s - 2r^9 + 8r^8 -$$

$$18r^7s - 9r^7 + 24r^6s - 84r^3s + 42r^3 - 9r^2s^7 + 36r^2s^6 - 42r^2s^5 + 84r^2s^2 + 42r^2s -$$

$$48r^2 + 10rs^8 - 36rs^7 + 36rs^6 - 84rs^2 + 24rs + 15r - 3s^9 + 10s^8 - 9s^7 + 24s^2 - 15s) -$$

$$g_{n+r}h^4rs^3(r-1)(s-1)^3(10r^8s - 3r^9 + 10r^8 - 9r^7s^2 - 36r^7s - 9r^7 + 36r^6s^2 + 36r^6s -$$

$$42r^5s^2 + 84r^2s^2 - 84r^2s + 24r^2 + 4rs^8 - 18rs^7 + 24rs^6 - 84rs^3 + 42rs^2 + 24rs - 15r -$$

$$2s^9 + 8s^8 - 9s^7 + 42s^3 - 48s^2 + 15s) - g_{n+1}h^4r^3s^3(r-s)^3(r-1)(s-1)(2r^8 - 6r^7s -$$

$$4r^7+3r^6s^2+14r^6s+3r^5s^3-10r^5s^2+3r^4s^4-10r^4s^3+3r^3s^5-10r^3s^4+3r^2s^6-$$
$$10r^2s^5+210r^2s^2-252r^2s+84r^2-6rs^7+14rs^6-252rs^2+336rs-120r+2s^8-$$
$$4s^7+84s^2-120s+45)-g_nh^4rs(r-s)^3(r-1)^3(s-1)^3(2r^8-6r^7s-8r^7+3r^6s^2+$$
$$28r^6s+9r^6+3r^5s^3-20r^5s^2-39r^5s+3r^4s^4-20r^4s^3+45r^4s^2+3r^3s^5-20r^3s^4+$$
$$45r^3s^3+3r^2s^6-20r^2s^5+45r^2s^4-210r^2s^2+168r^2s-42r^2-6rs^7+28rs^6-39rs^5+$$
$$168rs^2-168rs+48r+2s^8-8s^7+9s^6-42s^2+48s-15)) \tag{4.17}$$

Following the same approach as mentioned in Section 3.3, combining (4.9), (4.10) and (4.14) produces a block in the form

$$\begin{aligned} H^{[3]_2}Y_{n+1}^{[3]_2} &= M_1^{[3]_2}Y_n^{[3]_2}+M_2^{[3]_2}Y_{n-1}^{[3]_2}+M_3^{[3]_2}Y_{n-2}^{[3]_2}+E_1^{[3]_2}F_n^{[3]_2}+E_2^{[3]_2}F_{n+1}^{[3]_2} \\ &\quad +K_1^{[3]_2}G_n^{[3]_2}+K_2^{[3]_2}G_{n+1}^{[3]_2} \end{aligned} \tag{4.18}$$

where

$$H^{[3]_2}=\begin{pmatrix} \frac{(s-1)}{r(r-s)} & \frac{-(r-1)}{s(r-s)} & 1 \\ \frac{s}{hr(r-s)} & \frac{-r}{hs(r-s)} & 0 \\ \frac{-2}{h^2r(r-s)} & \frac{2}{h^2s(r-s)} & 0 \end{pmatrix},\ Y_{n+1}^{[3]_2}=\begin{pmatrix} y_{n+r} \\ y_{n+s} \\ y_{n+1} \end{pmatrix},\ M_1^{[3]_2}=\begin{pmatrix} 0 & 0 & \frac{(r-1)(s-1)}{rs} \\ 0 & 0 & \frac{-(r+s)}{hrs} \\ 0 & 0 & \frac{2}{h^2rs} \end{pmatrix},$$

$$Y_n^{[3]_2}=\begin{pmatrix} y_{n-s} \\ y_{n-r} \\ y_n \end{pmatrix},\ M_2^{[3]_2}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ Y_{n-1}^{[3]_2}=\begin{pmatrix} y'_{n-s} \\ y'_{n-r} \\ y'_n \end{pmatrix},\ Y_{n-2}^{[3]_2}=\begin{pmatrix} y''_{n-s} \\ y''_{n-r} \\ y''_n \end{pmatrix},$$

$$M_3^{[3]_2}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},\quad F_n^{[3]_2}=\begin{pmatrix} f_{n-s} \\ f_{n-r} \\ f_n \end{pmatrix},\quad F_{n+1}^{[3]_2}=\begin{pmatrix} f_{n+r} \\ f_{n+s} \\ f_{n+1} \end{pmatrix},\quad G_n^{[3]_2}=\begin{pmatrix} g_{n-s} \\ g_{n-r} \\ g_n \end{pmatrix},$$

$$G_{n+1}^{[3]_2}=\begin{pmatrix} g_{n+r} \\ g_{n+s} \\ g_{n+1} \end{pmatrix}.$$

Multiplying both sides of (4.18) by the inverse of $H^{[3]_2}$ yields

$$I_3 Y_{n+1}^{[3]_2} = \hat{M}_1^{[3]_2} Y_n^{[3]_2} + h\hat{M}_2^{[3]_2} Y_{n-1}^{[3]_2} + h^2 \hat{M}_3^{[3]_2} Y_{n-2}^{[3]_2} + h^3 \left[\hat{E}_1^{[3]_2} F_n^{[3]_2} + \hat{E}_2^{[3]_2} F_{n+1}^{[3]_2}\right] + h^4 \left[\hat{K}_1^{[3]_2} G_n^{[3]_2} + \hat{K}_2^{[3]_2} G_{n+1}^{[3]_2}\right] \quad (4.19)$$

where

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ \hat{M}_1^{[3]_2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix},\ \hat{M}_2^{[3]_2} = \begin{pmatrix} 0 & 0 & r \\ 0 & 0 & s \\ 0 & 0 & 1 \end{pmatrix},$$

$$\hat{M}_3^{[3]_2} = \begin{pmatrix} 0 & 0 & \frac{r^2}{2} \\ 0 & 0 & \frac{s^2}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix},\ \hat{E}_1^{[3]_2} = \begin{pmatrix} 0 & 0 & \hat{E}_{1_{13}}^{[3]_2} \\ 0 & 0 & \hat{E}_{1_{23}}^{[3]_2} \\ 0 & 0 & \hat{E}_{1_{33}}^{[3]_2} \end{pmatrix},\ \hat{E}_2^{[3]_2} = \begin{pmatrix} \hat{E}_{2_{11}}^{[3]_2} & \hat{E}_{2_{12}}^{[3]_2} & \hat{E}_{2_{13}}^{[3]_2} \\ \hat{E}_{2_{21}}^{[3]_2} & \hat{E}_{2_{22}}^{[3]_2} & \hat{E}_{2_{23}}^{[3]_2} \\ \hat{E}_{2_{31}}^{[3]_2} & \hat{E}_{2_{32}}^{[3]_2} & \hat{E}_{2_{33}}^{[3]_2} \end{pmatrix},$$

$$\hat{K}_1^{[3]_2} = \begin{pmatrix} 0 & 0 & \hat{K}_{1_{13}}^{[3]_2} \\ 0 & 0 & \hat{K}_{1_{23}}^{[3]_2} \\ 0 & 0 & \hat{K}_{1_{33}}^{[3]_2} \end{pmatrix},\ \hat{K}_2^{[3]_2} = \begin{pmatrix} \hat{K}_{2_{11}}^{[3]_2} & \hat{K}_{2_{12}}^{[3]_2} & \hat{K}_{2_{13}}^{[3]_2} \\ \hat{K}_{2_{21}}^{[3]_2} & \hat{K}_{2_{22}}^{[3]_2} & \hat{K}_{2_{23}}^{[3]_2} \\ \hat{K}_{2_{31}}^{[3]_2} & \hat{K}_{2_{32}}^{[3]_2} & \hat{K}_{2_{33}}^{[3]_2} \end{pmatrix}.$$

The elements of $\hat{E}_1^{[3]_2}, \hat{E}_2^{[3]_2}, \hat{K}_1^{[3]_2}$ and $\hat{K}_2^{[3]_2}$ are given below:

$\hat{E}_{1_{13}}^{[3]_2} = \frac{-r^3}{2520s^3}(-2r^5s - 2r^5 + 8r^4s^2 + 12r^4s + 8r^4 - 9r^3s^3 - 28r^3s^2 - 28r^3s$
$-9r^3 + 27r^2s^3 + 12r^2s^2 + 27r^2s + 48rs^3 + 48rs^2 - 336s^3)$,

$\hat{E}_{1_{23}}^{[3]_2} = \frac{-s^3}{2520r^3}(-9r^3s^3 + 27r^3s^2 + 48r^3s - 336r^3 + 8r^2s^4 - 28r^2s^3 + 12r^2s^2 +$
$48r^2s - 2rs^5 + 12rs^4 - 28rs^3 + 27rs^2 - 2s^5 + 8s^4 - 9s^3)$,

$\hat{E}_{1_{33}}^{[3]_2} = \frac{-1}{2520r^3s^3}(-336r^3s^3 + 48r^3s^2 + 27r^3s - 9r^3 + 48r^2s^3 + 12r^2s^2 - 28r^2s +$
$8r^2 + 27rs^3 - 28rs^2 + 12rs - 2r - 9s^3 + 8s^2 - 2s)$,

$\hat{E}_{2_{11}}^{[3]_2} = \frac{r^3}{2520(r-s)^3(r-1)^3}(14r^6 - 56r^5s - 56r^5 + 74r^4s^2 + 234r^4s + 74r^4 -$
$30r^3s^3 - 325r^3s^2 - 325r^3s - 30r^3 + 135r^2s^3 + 480r^2s^2 + 135r^2s - 204$
$rs^3 - 204rs^2 + 84s^3)$,

$\hat{E}_{2_{21}}^{[3]_2} = \frac{-s^7}{2520r^3(r-s)^3(r-1)^3}(-14r^3s^2 + 63r^3s - 84r^3 + 16r^2s^3 - 59r^2s^2 +$
$42r^2s + 60r^2 - 4rs^4 + 6rs^3 + 25rs^2 - 54rs + 2s^4 - 8s^3 + 9s^2)$,

$\hat{E}^{[3]_2}_{2_{31}} = \frac{-1}{2520r^3(r-s)^3(r-1)^3}(-84r^3s^2+63r^3s-14r^3+60r^2s^3+42r^2s^2-$
$59r^2s+16r^2-54rs^3+25rs^2+6rs-4r+9s^3-8s^2+2s),$

$\hat{E}^{[3]_2}_{2_{12}} = \frac{r^7}{2520s^3(r-s)^3(s-1)^3}(-4r^4s+2r^4+16r^3s^2+6r^3s-8r^3-14r^2s^3$
$-59r^2s^2+25r^2s+9r^2+63rs^3+42rs^2-54rs-84s^3+60s^2),$

$\hat{E}^{[3]_2}_{2_{22}} = \frac{-s^3}{2520(r-s)^3(s-1)^3}(-30r^3s^3+135r^3s^2-204r^3s+84r^3+74r^2s^4-$
$325r^2s^3+480r^2s^2-204r^2s-56rs^5+234rs^4-325rs^3+135rs^2+14$
$s^6-56s^5+74s^4-30s^3),$

$\hat{E}^{[3]_2}_{2_{32}} = \frac{1}{2520s^3(r-s)^3(s-1)^3}(60r^3s^2-54r^3s+9r^3-84r^2s^3+42r^2s^2+25r^2s$
$-8r^2+63rs^3-59rs^2+6rs+2r-14s^3+16s^2-4s),$

$\hat{E}^{[3]_2}_{2_{13}} = \frac{-r^7}{2520(r-1)^3(s-1)^3}(2r^4s-4r^4-8r^3s^2+6r^3s+16r^3+9r^2s^3+25r^2$
$s^2-59r^2s-14r^2-54rs^3+42rs^2+63rs+60s^3-84s^2),$

$\hat{E}^{[3]_2}_{2_{23}} = \frac{-s^7}{2520(r-1)^3(s-1)^3}(9r^3s^2-54r^3s+60r^3-8r^2s^3+25r^2s^2+42r^2s-$
$84r^2+2rs^4+6rs^3-59rs^2+63rs-4s^4+16s^3-14s^2),$

$\hat{E}^{[3]_2}_{2_{33}} = \frac{-1}{2520(r-1)^3(s-1)^3}(-84r^3s^3+204r^3s^2-135r^3s+30r^3+204r^2s^3-$
$480r^2s^2+325r^2s-74r^2-135rs^3+325rs^2-234rs+56r+30s^3-74$
$s^2+56s-14),$

$\hat{K}^{[3]_2}_{1_{13}} = \frac{r^4}{5040s^2}(2r^4-8r^3s-8r^3+9r^2s^2+36r^2s+9r^2-48rs^2-48rs+84s^2),$

$\hat{K}^{[3]_2}_{1_{23}} = \frac{s^4}{5040r^2}(9r^2s^2-48r^2s+84r^2-8rs^3+36rs^2-48rs+2s^4-8s^3+9s^2),$

$\hat{K}^{[3]_2}_{1_{33}} = \frac{1}{5040r^2s^2}(84r^2s^2-48r^2s+9r^2-48rs^2+36rs-8r+9s^2-8s+2),$

$\hat{K}^{[3]_2}_{2_{11}} = \frac{-r^4}{5040(r-s)^2(r-1)^2}(3r^4-10r^3s-10r^3+9r^2s^2+36r^2s+9r^2-36rs^2$
$-36rs+42s^2),$

$\hat{K}^{[3]_2}_{2_{21}} = \frac{-s^7}{5040r^2(r-s)^2(r-1)^2}(24r-9s-18rs+4rs^2+8s^2-2s^3),$

$\hat{K}^{[3]_2}_{2_{31}} = \frac{-1}{5040r^2(r-s)^2(r-1)^2}(4r+8s-18rs+24rs^2-9s^2-2),$

$\hat{K}_{2_{12}}^{[3]_2} = \frac{r^7}{5040s^2(r-s)^2(s-1)^2}(9r-24s+18rs-4r^2s-8r^2+2r^3)$,

$\hat{K}_{2_{22}}^{[3]_2} = \frac{-s^4}{5040(r-s)^2(s-1)^2}(9r^2s^2-36r^2s+42r^2-10rs^3+36rs^2-36rs+3s^4-10s^3+9s^2)$,

$\hat{K}_{2_{32}}^{[3]_2} = \frac{-1}{5040s^2(r-s)^2(s-1)^2}(8r+4s-18rs+24r^2s-9r^2-2)$,

$\hat{K}_{2_{13}}^{[3]_2} = \frac{r^7}{5040(r-1)^2(s-1)^2}(2r^3-8r^2s-4r^2+9rs^2+18rs-24s^2)$,

$\hat{K}_{2_{23}}^{[3]_2} = \frac{s^7}{5040(r-1)^2(s-1)^2}(9r^2s-24r^2-8rs^2+18rs+2s^3-4s^2)$,

$\hat{K}_{2_{33}}^{[3]_2} = \frac{-1}{5040(r-1)^2(s-1)^2}(42r^2s^2-36r^2s+9r^2-36rs^2+36rs-10r+9s^2-10s+3)$.

From the main block (4.19), the following equations are obtained

$$y_{n+r} = y_n + hry'_n + \frac{h^2r^2y''_n}{2} - \frac{f_nh^3r^3}{2520s^3}(-2r^5s-2r^5+8r^4s^2+12r^4s+8r^4-9r^3s^3-28r^3s^2 -28r^3s-9r^3+27r^2s^3+12r^2s^2+27r^2s+48rs^3+48rs^2-336s^3) + \frac{g_nh^4r^4}{5040s^2}(2r^4-8r^3s-8r^3+9r^2s^2+36r^2s+9r^2-48rs^2-48rs+84s^2) + \frac{g_{n+1}h^4r^7}{5040(r-1)^2(s-1)^2}(2r^3-8r^2s-4r^2+9rs^2+18rs-24s^2) - \frac{f_{n+1}h^3r^7}{2520(r-1)^3(s-1)^3}(2r^4s-4r^4-8r^3s^2+6r^3s+16r^3+9r^2s^3+25r^2s^2-59r^2s-14r^2-54rs^3+42rs^2+63rs+60s^3-84s^2) + \frac{f_{n+r}h^3r^3}{2520(r-s)^3(r-1)^3}(14r^6-56r^5s-56r^5+74r^4s^2+234r^4s+74r^4-30r^3s^3-325r^3s^2-325r^3s-30r^3+135r^2s^3+480r^2s^2+135r^2s-204rs^3-204rs^2+84s^3) - \frac{g_{n+r}h^4r^4}{5040(r-s)^2(r-1)^2}(3r^4-10r^3s-10r^3+9r^2s^2+36r^2s+9r^2-36rs^2-36rs+42s^2) + \frac{f_{n+s}h^3r^7}{2520s^3(r-s)^3(s-1)^3}(-4r^4s+2r^4+16r^3s^2+6r^3s-8r^3-14r^2s^3-59r^2s^2+25r^2s+9r^2+63rs^3+42rs^2-54rs-84s^3+60s^2) + \frac{g_{n+s}h^4r^7}{5040s^2(r-s)^2(s-1)^2}(9r-24s+18rs-4r^2s-8r^2+2r^3), \tag{4.20}$$

$$y_{n+s} = y_n + hsy'_n + \frac{h^2s^2y''_n}{2} - \frac{f_nh^3s^3}{2520r^3}(-9r^3s^3+27r^3s^2+48r^3s-336r^3+8r^2s^4-28r^2s^3 +12r^2s^2+48r^2s-2rs^5+12rs^4-28rs^3+27rs^2-2s^5+8s^4-9s^3) + \frac{g_nh^4s^4}{5040r^2}(9r^2s^2-48r^2s+84r^2-8rs^3+36rs^2-48rs+2s^4-8s^3+9s^2) + \frac{g_{n+1}h^4s^7}{5040(r-1)^2(s-1)^2}(9r^2s-24r^2-8rs^2+18rs+2s^3-4s^2) - \frac{f_{n+1}h^3s^7}{2520(r-1)^3(s-1)^3}(9r^3s^2-54r^3s+60r^3-8r^2s^3+25r^2s^2+42r^2s-84r^2+2rs^4+6rs^3-59rs^2+63rs-4s^4+16s^3-14s^2) - \frac{f_{n+s}h^3s^3}{2520(r-s)^3(s-1)^3}(-30r^3s^3+135r^3s^2-204r^3s+84r^3+74r^2s^4-325r^2s^3+480r^2s^2-204r^2s-56rs^5+$$

$234rs^4 - 325rs^3 + 135rs^2 + 14s^6 - 56s^5 + 74s^4 - 30s^3) - \frac{g_{n+s}h^4s^4}{5040(r-s)^2(s-1)^2}(9r^2s^2 - 36r^2s + 42r^2 - 10rs^3 + 36rs^2 - 36rs + 3s^4 - 10s^3 + 9s^2) - \frac{f_{n+r}h^3s^7}{2520r^3(r-s)^3(r-1)^3}(-14r^3s^2 + 63r^3s - 84r^3 + 16r^2s^3 - 59r^2s^2 + 42r^2s + 60r^2 - 4rs^4 + 6rs^3 + 25rs^2 - 54rs + 2s^4 - 8s^3 + 9s^2) - \frac{g_{n+r}h^4s^7}{5040r^2(r-s)^2(r-1)^2}(24r - 9s - 18rs + 4rs^2 + 8s^2 - 2s^3),$ (4.21)

$y_{n+1} = y_n + hy'_n + \frac{h^2y''_n}{2} + \frac{g_nh^4}{5040r^2s^2}(84r^2s^2 - 48r^2s + 9r^2 - 48rs^2 + 36rs - 8r + 9s^2 - 8s + 2) - \frac{f_{n+1}h^3}{2520(r-1)^3(s-1)^3}(-84r^3s^3 + 204r^3s^2 - 135r^3s + 30r^3 + 204r^2s^3 - 480r^2s^2 + 325r^2s - 74r^2 - 135rs^3 + 325rs^2 - 234rs + 56r + 30s^3 - 74s^2 + 56s - 14) - \frac{f_nh^3}{2520r^3s^3}(-336r^3s^3 + 48r^3s^2 + 27r^3s - 9r^3 + 48r^2s^3 + 12r^2s^2 - 28r^2s + 8r^2 + 27rs^3 - 28rs^2 + 12rs - 2r - 9s^3 + 8s^2 - 2s) - \frac{g_{n+1}h^4}{5040(r-1)^2(s-1)^2}(42r^2s^2 - 36r^2s + 9r^2 - 36rs^2 + 36rs - 10r + 9s^2 - 10s + 3) - \frac{f_{n+r}h^3}{2520r^3(r-s)^3(r-1)^3}(-84r^3s^2 + 63r^3s - 14r^3 + 60r^2s^3 + 42r^2s^2 - 59r^2s + 16r^2 - 54rs^3 + 25rs^2 + 6rs - 4r + 9s^3 - 8s^2 + 2s) + \frac{f_{n+s}h^3}{2520s^3(r-s)^3(s-1)^3}(60r^3s^2 - 54r^3s + 9r^3 - 84r^2s^3 + 42r^2s^2 + 25r^2s - 8r^2 + 63rs^3 - 59rs^2 + 6rs + 2r - 14s^3 + 16s^2 - 4s) - \frac{g_{n+r}h^4}{5040r^2(r-s)^2(r-1)^2}(4r + 8s - 18rs + 24rs^2 - 9s^2 - 2) - \frac{g_{n+s}h^4}{5040s^2(r-s)^2(s-1)^2}(8r + 4s - 18rs + 24r^2s - 9r^2 - 2).$ (4.22)

Substituting (4.20) and (4.21) into (4.11) – (4.13) gives the equations of first derivative as below

$y'_{n+r} = y'_n + hry''_n - \frac{f_nh^2r^2}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 - 108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3) + \frac{g_nh^3r^3}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2) + \frac{g_{n+1}h^3r^6}{2520(r-1)^2(s-1)^2}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36rs - 42s^2) - \frac{f_{n+1}h^2r^6}{2520(r-1)^3(s-1)^3}(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2) + \frac{f_{n+r}h^2r^2}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2s - 966rs^3 - 966rs^2 + 378s^3) - \frac{g_{n+r}h^3r^3}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2) + \frac{f_{n+s}h^2r^6}{2520s^3(r-s)^3(s-1)^3}(-20r^4s + 10r^4 + 75r^3s^2 +$

$25r^3s-36r^3-63r^2s^3-243r^2s^2+108r^2s+36r^2+252rs^3+138rs^2-198rs-294s^3+210s^2)+\frac{g_{n+s}h^3r^6}{2520s^2(r-s)^2(s-1)^2}(18r-42s+36rs-9r^2s-18r^2+5r^3),$ (4.23)

$y'_{n+s}=y'_n+hsy''_n-\frac{f_nh^2s^2}{2520r^3}(-36r^3s^3+90r^3s^2+168r^3s-882r^3+36r^2s^4-108r^2s^3+24r^2s^2+168r^2s-10rs^5+53rs^4-108rs^3+90rs^2-10s^5+36s^4-36s^3)+\frac{g_nh^3s^3}{2520r^2}(18r^2s^2-84r^2s+126r^2-18rs^3+72rs^2-84rs+5s^4-18s^3+18s^2)+\frac{g_{n+1}h^3s^6}{2520(r-1)^2(s-1)^2}(18r^2s-42r^2-18rs^2+36rs+5s^3-9s^2)-\frac{f_{n+1}h^2s^6}{2520(r-1)^3(s-1)^3}(36r^3s^2-198r^3s+210r^3-36r^2s^3+108r^2s^2+138r^2s-294r^2+10rs^4+25rs^3-243rs^2+252rs-20s^4+75s^3-63s^2)-\frac{f_{n+s}h^2s^2}{2520(r-s)^3(s-1)^3}(-180r^3s^3+720r^3s^2-966r^3s+378r^3+468r^2s^4-1836r^2s^3+2418r^2s^2-966r^2s-385rs^5+1457rs^4-1836rs^3+720rs^2+105s^6-385s^5+468s^4-180s^3)-\frac{g_{n+s}h^3s^3}{1260(r-s)^2(s-1)^2}(12r^2s^2-42r^2s+42r^2-15rs^3+48rs^2-42rs+5s^4-15s^3+12s^2)-\frac{f_{n+r}h^2s^6}{2520r^3(r-s)^3(r-1)^3}(-63r^3s^2+252r^3s-294r^3+75r^2s^3-243r^2s^2+138r^2s+210r^2-20rs^4+25rs^3+108rs^2-198rs+10s^4-36s^3+36s^2)-\frac{g_{n+r}h^3s^6}{2520r^2(r-s)^2(r-1)^2}(42r-18s-36rs+9rs^2+18s^2-5s^3),$ (4.24)

$y'_{n+1}=y'_n+hy''_n+\frac{g_nh^3}{2520r^2s^2}(126r^2s^2-84r^2s+18r^2-84rs^2+72rs-18r+18s^2-18s+5)-\frac{f_{n+1}h^2}{2520(r-1)^3(s-1)^3}(-378r^3s^3+966r^3s^2-720r^3s+180r^3+966r^2s^3-2418r^2s^2+1836r^2s-468r^2-720rs^3+1836rs^2-1457rs+385r+180s^3-468s^2+385s-105)-\frac{f_nh^2}{2520r^3s^3}(-882r^3s^3+168r^3s^2+90r^3s-36r^3+168r^2s^3+24r^2s^2-108r^2s+36r^2+90rs^3-108rs^2+53rs-10r-36s^3+36s^2-10s)-\frac{g_{n+1}h^3}{1260(r-1)^2(s-1)^2}(42r^2s^2-42r^2s+12r^2-42rs^2+48rs-15r+12s^2-15s+5)-\frac{f_{n+r}h^2}{2520r^3(r-s)^3(r-1)^3}(-294r^3s^2+252r^3s-63r^3+210r^2s^3+138r^2s^2-243r^2s+75r^2-198rs^3+108rs^2+25rs-20r+36s^3-36s^2+10s)+\frac{f_{n+s}h^2}{2520s^3(r-s)^3(s-1)^3}(210r^3s^2-198r^3s+36r^3-294r^2s^3+138r^2s^2+108r^2s-36r^2+252rs^3-243rs^2+25rs+10r-63s^3+75s^2-20s)-\frac{g_{n+r}h^3}{2520r^2(r-s)^2(r-1)^2}(9r+18s-36rs+42rs^2-18s^2-5)-\frac{g_{n+s}h^3}{2520s^2(r-s)^2(s-1)^2}(18r+9s-36rs+42r^2s-18r^2-5),$ (4.25)

which can be expressed in block form

$$\begin{aligned} I_3Y'^{[3]_2}_{n+1} &= M'^{[3]_2}_2Y^{[3]_2}_{n-1}+hM'^{[3]_2}_3Y^{[3]_2}_{n-2}+h^2\left[E'^{[3]_2}_1F^{[3]_2}_n+E'^{[3]_2}_2F^{[3]_2}_{n+1}\right] \\ &\quad +h^3\left[K'^{[3]_2}_1G^{[3]_2}_n+K'^{[3]_2}_2G^{[3]_2}_{n+1}\right] \end{aligned} \tag{4.26}$$

where

$$Y_{n+1}^{'[3]_2} = \begin{pmatrix} y'_{n+r} \\ y'_{n+s} \\ y'_{n+1} \end{pmatrix}, \; M_2^{'[3]_2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \; M_3^{'[3]_2} = \begin{pmatrix} 0 & 0 & r \\ 0 & 0 & s \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_1^{'[3]_2} = \begin{pmatrix} 0 & 0 & E_{1_{13}}^{'[3]_2} \\ 0 & 0 & E_{1_{23}}^{'[3]_2} \\ 0 & 0 & E_{1_{33}}^{'[3]_2} \end{pmatrix}, \; E_2^{'[3]_2} = \begin{pmatrix} E_{2_{11}}^{'[3]_2} & E_{2_{12}}^{'[3]_2} & E_{2_{13}}^{'[3]_2} \\ E_{2_{21}}^{'[3]_2} & E_{2_{22}}^{'[3]_2} & E_{2_{23}}^{'[3]_2} \\ E_{2_{31}}^{'[3]_2} & E_{2_{32}}^{'[3]_2} & E_{2_{33}}^{'[3]_2} \end{pmatrix},$$

$$K_1^{'[3]_2} = \begin{pmatrix} 0 & 0 & K_{1_{13}}^{'[3]_2} \\ 0 & 0 & K_{1_{23}}^{'[3]_2} \\ 0 & 0 & K_{1_{33}}^{'[3]_2} \end{pmatrix}, \; K_2^{'[3]_2} = \begin{pmatrix} K_{2_{11}}^{'[3]_2} & K_{2_{12}}^{'[3]_2} & K_{2_{13}}^{'[3]_2} \\ K_{2_{21}}^{'[3]_2} & K_{2_{22}}^{'[3]_2} & K_{2_{23}}^{'[3]_2} \\ K_{2_{31}}^{'[3]_2} & K_{2_{32}}^{'[3]_2} & K_{2_{33}}^{'[3]_2} \end{pmatrix}.$$

The entries of $E_1^{'[3]_2}, E_2^{'[3]_2}, K_1^{'[3]_2}$ and $K_2^{'[3]_2}$ are given by

$E_{1_{13}}^{'[3]_2} = \frac{-r^2}{2520s^3}(-10r^5s - 10r^5 + 36r^4s^2 + 53r^4s + 36r^4 - 36r^3s^3 - 108r^3s^2 -$
$108r^3s - 36r^3 + 90r^2s^3 + 24r^2s^2 + 90r^2s + 168rs^3 + 168rs^2 - 882s^3),$

$E_{1_{23}}^{'[3]_2} = \frac{-s^2}{2520r^3}(-36r^3s^3 + 90r^3s^2 + 168r^3s - 882r^3 + 36r^2s^4 - 108r^2s^3 + 24r^2$
$s^2 + 168r^2s - 10rs^5 + 53rs^4 - 108rs^3 + 90rs^2 - 10s^5 + 36s^4 - 36s^3),$

$E_{1_{33}}^{'[3]_2} = \frac{-1}{2520r^3s^3}(-882r^3s^3 + 168r^3s^2 + 90r^3s - 36r^3 + 168r^2s^3 + 24r^2s^2 - 108$
$r^2s + 36r^2 + 90rs^3 - 108rs^2 + 53rs - 10r - 36s^3 + 36s^2 - 10s),$

$E_{2_{11}}^{'[3]_2} = \frac{r^2}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5s - 385r^5 + 468r^4s^2 + 1457r^4s + 468$
$r^4 - 180r^3s^3 - 1836r^3s^2 - 1836r^3s - 180r^3 + 720r^2s^3 + 2418r^2s^2 + 720r^2$
$s - 966rs^3 - 966rs^2 + 378s^3),$

$E_{2_{21}}^{'[3]_2} = \frac{-s^6}{2520r^3(r-s)^3(r-1)^3}(-63r^3s^2 + 252r^3s - 294r^3 + 75r^2s^3 - 243r^2s^2 +$
$138r^2s + 210r^2 - 20rs^4 + 25rs^3 + 108rs^2 - 198rs + 10s^4 - 36s^3 + 36s^2),$

$E'^{[3]_2}_{2_{31}} = \frac{-1}{2520r^3(r-s)^3(r-1)^3}(-294r^3s^2 + 252r^3s - 63r^3 + 210r^2s^3 + 138r^2s^2 - 243r^2s + 75r^2 - 198rs^3 + 108rs^2 + 25rs - 20r + 36s^3 - 36s^2 + 10s)$,

$E'^{[3]_2}_{2_{12}} = \frac{r^6}{2520s^3(r-s)^3(s-1)^3}(-20r^4s + 10r^4 + 75r^3s^2 + 25r^3s - 36r^3 - 63r^2s^3 - 243r^2s^2 + 108r^2s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2)$,

$E'^{[3]_2}_{2_{22}} = \frac{-s^2}{2520(r-s)^3(s-1)^3}(-180r^3s^3 + 720r^3s^2 - 966r^3s + 378r^3 + 468r^2s^4 - 1836r^2s^3 + 2418r^2s^2 - 966r^2s - 385rs^5 + 1457rs^4 - 1836rs^3 + 720rs^2 + 105s^6 - 385s^5 + 468s^4 - 180s^3)$,

$E'^{[3]_2}_{2_{32}} = \frac{1}{2520s^3(r-s)^3(s-1)^3}(210r^3s^2 - 198r^3s + 36r^3 - 294r^2s^3 + 138r^2s^2 + 108r^2s - 36r^2 + 252rs^3 - 243rs^2 + 25rs + 10r - 63s^3 + 75s^2 - 20s)$,

$E'^{[3]_2}_{2_{13}} = \frac{-r^6}{2520(r-1)^3(s-1)^3}(10r^4s - 20r^4 - 36r^3s^2 + 25r^3s + 75r^3 + 36r^2s^3 + 108r^2s^2 - 243r^2s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2)$,

$E'^{[3]_2}_{2_{23}} = \frac{-s^6}{2520(r-1)^3(s-1)^3}(36r^3s^2 - 198r^3s + 210r^3 - 36r^2s^3 + 108r^2s^2 + 138r^2s - 294r^2 + 10rs^4 + 25rs^3 - 243rs^2 + 252rs - 20s^4 + 75s^3 - 63s^2)$,

$E'^{[3]_2}_{2_{33}} = \frac{-1}{2520(r-1)^3(s-1)^3}(-378r^3s^3 + 966r^3s^2 - 720r^3s + 180r^3 + 966r^2s^3 - 2418r^2s^2 + 1836r^2s - 468r^2 - 720rs^3 + 1836rs^2 - 1457rs + 385r + 180s^3 - 468s^2 + 385s - 105)$,

$K'^{[3]_2}_{1_{13}} = \frac{r^3}{2520s^2}(5r^4 - 18r^3s - 18r^3 + 18r^2s^2 + 72r^2s + 18r^2 - 84rs^2 - 84rs + 126s^2)$,

$K'^{[3]_2}_{1_{23}} = \frac{s^3}{2520r^2}(18r^2s^2 - 84r^2s + 126r^2 - 18rs^3 + 72rs^2 - 84rs + 5s^4 - 18s^3 + 18s^2)$,

$K'^{[3]_2}_{1_{33}} = \frac{1}{2520r^2s^2}(126r^2s^2 - 84r^2s + 18r^2 - 84rs^2 + 72rs - 18r + 18s^2 - 18s + 5)$,

$K'^{[3]_2}_{2_{11}} = \frac{-r^3}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3s - 15r^3 + 12r^2s^2 + 48r^2s + 12r^2 - 42rs^2 - 42rs + 42s^2)$,

$K_{2_{21}}^{'[3]_2} = \frac{-s^6}{2520r^2(r-s)^2(r-1)^2}(42r - 18s - 36rs + 9rs^2 + 18s^2 - 5s^3),$

$K_{2_{31}}^{'[3]_2} = \frac{-1}{2520r^2(r-s)^2(r-1)^2}(9r + 18s - 36rs + 42rs^2 - 18s^2 - 5),$

$K_{2_{12}}^{'[3]_2} = \frac{r^6}{2520s^2(r-s)^2(s-1)^2}(18r - 42s + 36rs - 9r^2s - 18r^2 + 5r^3),$

$K_{2_{22}}^{'[3]_2} = \frac{-s^3}{1260(r-s)^2(s-1)^2}(12r^2s^2 - 42r^2s + 42r^2 - 15rs^3 + 48rs^2 - 42rs + 5$
$s^4 - 15s^3 + 12s^2),$

$K_{2_{32}}^{'[3]_2} = \frac{-1}{2520s^2(r-s)^2(s-1)^2}(18r + 9s - 36rs + 42r^2s - 18r^2 - 5),$

$K_{2_{13}}^{'[3]_2} = \frac{r^6}{2520(r-1)^2(s-1)^2}(5r^3 - 18r^2s - 9r^2 + 18rs^2 + 36rs - 42s^2),$

$K_{2_{23}}^{'[3]_2} = \frac{s^6}{2520(r-1)^2(s-1)^2}(18r^2s - 42r^2 - 18rs^2 + 36rs + 5s^3 - 9s^2),$

$K_{2_{33}}^{'[3]_2} = \frac{-1}{1260(r-1)^2(s-1)^2}(42r^2s^2 - 42r^2s + 12r^2 - 42rs^2 + 48rs - 15r +$
$12s^2 - 15s + 5).$

To obtain the following second derivative equations, we substitute (4.20) and (4.21) into (4.15) – (4.17):

$$y''_{n+r} = y''_n - \frac{f_n hr}{420s^3}(-5r^5s - 5r^5 + 16r^4s^2 + 23r^4s + 16r^4 - 14r^3s^3 - 40r^3s^2 - 40r^3s - 14r^3 + 28r^2s^3 + 28r^2s + 56rs^3 + 56rs^2 - 210s^3) + \frac{g_n h^2 r^2}{840s^2}(5r^4 - 16r^3s - 16r^3 + 14r^2s^2 + 56r^2s + 14r^2 - 56rs^2 - 56rs + 70s^2) + \frac{f_{n+r} hr}{420(r-s)^3(r-1)^3}(105r^6 - 350r^5s - 350r^5 + 388r^4s^2 + 1187r^4s + 388r^4 - 140r^3s^3 - 1342r^3s^2 - 1342r^3s - 140r^3 + 490r^2s^3 + 1554r^2s^2 + 490r^2s - 574rs^3 - 574rs^2 + 210s^3) - \frac{f_{n+1} hr^5}{420(r-1)^3(s-1)^3}(5r^4s - 10r^4 - 16r^3s^2 + 10r^3s + 35r^3 + 14r^2s^3 + 46r^2s^2 - 98r^2s - 28r^2 - 70rs^3 + 42rs^2 + 98rs + 70s^3 - 98s^2) + \frac{g_{n+1} h^2 r^5}{840(r-1)^2(s-1)^2}(5r^3 - 16r^2s - 8r^2 + 14rs^2 + 28rs - 28s^2) - \frac{g_{n+r} h^2 r^2}{840(r-s)^2(r-1)^2}(15r^4 - 40r^3s - 40r^3 + 28r^2s^2 + 112r^2s + 28r^2 - 84rs^2 - 84rs + 70s^2) + \frac{g_{n+s} h^2 r^5}{840s^2(r-s)^2(s-1)^2}(14r - 28s + 28rs - 8r^2s - 16r^2 + 5r^3) + \frac{f_{n+s} hr^5}{420s^3(r-s)^3(s-1)^3}(-10r^4s + 5r^4 + 35r^3s^2 + 10r^3s - 16r^3 - 28r^2s^3 - 98r^2s^2 + 46r^2s + 14r^2 + 98rs^3 + 42rs^2 - 70rs - 98s^3 + 70s^2) \tag{4.27}$$

$y''_{n+s} = y''_n + \frac{f_n hs}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - 56r^2s + 5rs^5 - 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3) + \frac{g_n h^2 s^2}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2) - \frac{f_{n+s}hs}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3) - \frac{f_{n+1}hs^5}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2) + \frac{g_{n+1}h^2s^5}{840(r-1)^2(s-1)^2}(14r^2s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2) - \frac{g_{n+s}h^2s^2}{840(r-s)^2(s-1)^2}(28r^2s^2 - 84r^2s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4 - 40s^3 + 28s^2) - \frac{g_{n+r}h^2s^5}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3) - \frac{f_{n+r}hs^5}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2) \quad (4.28)$

$y''_{n+1} = y''_n + \frac{g_n h^2}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5) - \frac{f_{n+1}h}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105) + \frac{f_n h}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s) - \frac{g_{n+1}h^2}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15) - \frac{g_{n+r}h^2}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5) - \frac{g_{n+s}h^2}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5) - \frac{f_{n+r}h}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s) + \frac{f_{n+s}h}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s) \quad (4.29)$

which can be written as

$$I_3 Y''^{[3]_2}_{n+1} = M''^{[3]_2}_3 Y^{[3]_2}_{n-2} + h[E''^{[3]_2}_1 F^{[3]_2}_n + E''^{[3]_2}_2 F^{[3]_2}_{n+1}] + h^2[K''^{[3]_2}_1 G^{[3]_2}_n + K''^{[3]_2}_2 G^{[3]_2}_{n+1}] \quad (4.30)$$

where

$$Y''^{[3]_2}_{n+1} = \begin{pmatrix} y''_{n+r} \\ y''_{n+s} \\ y''_{n+1} \end{pmatrix}, \; M''^{[3]_2}_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_1^{''[3]_2} = \begin{pmatrix} 0 & 0 & E_{1_{13}}^{''[3]_2} \\ 0 & 0 & E_{1_{23}}^{''[3]_2} \\ 0 & 0 & E_{1_{33}}^{''[3]_2} \end{pmatrix}, E_2^{''[3]_2} = \begin{pmatrix} E_{2_{11}}^{''[3]_2} & E_{2_{12}}^{''[3]_2} & E_{2_{13}}^{''[3]_2} \\ E_{2_{21}}^{''[3]_2} & E_{2_{22}}^{''[3]_2} & E_{2_{23}}^{''[3]_2} \\ E_{2_{31}}^{''[3]_2} & E_{2_{32}}^{''[3]_2} & E_{2_{33}}^{''[3]_2} \end{pmatrix},$$

$$K_1^{''[3]_2} = \begin{pmatrix} 0 & 0 & K_{1_{13}}^{''[3]_2} \\ 0 & 0 & K_{1_{23}}^{''[3]_2} \\ 0 & 0 & K_{1_{33}}^{''[3]_2} \end{pmatrix}, K_2^{''[3]_2} = \begin{pmatrix} K_{2_{11}}^{''[3]_2} & K_{2_{12}}^{''[3]_2} & K_{2_{13}}^{''[3]_2} \\ K_{2_{21}}^{''[3]_2} & K_{2_{22}}^{''[3]_2} & K_{2_{23}}^{''[3]_2} \\ K_{2_{31}}^{''[3]_2} & K_{2_{32}}^{''[3]_2} & K_{2_{33}}^{''[3]_2} \end{pmatrix}$$

whose entries of $E_1^{''[3]_2}, E_2^{''[3]_2}, K_1^{''[3]_2}$ and $K_2^{''[3]_2}$ are defined as below

$E_{1_{13}}^{''[3]_2} = \frac{-r}{420s^3}(-5r^5s - 5r^5 + 16r^4s^2 + 23r^4s + 16r^4 - 14r^3s^3 - 40r^3s^2 - 40r^3s$
$- 14r^3 + 28r^2s^3 + 28r^2s + 56rs^3 + 56rs^2 - 210s^3),$

$E_{1_{23}}^{''[3]_2} = \frac{s}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - 56r^2s + 5rs^5$
$- 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3),$

$E_{1_{33}}^{''[3]_2} = \frac{1}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28r$
$s^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s),$

$E_{1_{11}}^{''[3]_2} = \frac{r}{420(r-s)^3(r-1)^3}(105r^6 - 350r^5s - 350r^5 + 388r^4s^2 + 1187r^4s + 388$
$r^4 - 140r^3s^3 - 1342r^3s^2 - 1342r^3s - 140r^3 + 490r^2s^3 + 1554r^2s^2 + 490$
$r^2s - 574rs^3 - 574rs^2 + 210s^3),$

$E_{2_{21}}^{''[3]_2} = \frac{-s^5}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2$
$s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2),$

$E_{2_{31}}^{''[3]_2} = \frac{-1}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2$
$s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s),$

$E_{2_{12}}^{''[3]_2} = \frac{r^5}{420s^3(r-s)^3(s-1)^3}(-10r^4s + 5r^4 + 35r^3s^2 + 10r^3s - 16r^3 - 28r^2s^3 -$
$98r^2s^2 + 46r^2s + 14r^2 + 98rs^3 + 42rs^2 - 70rs - 98s^3 + 70s^2),$

$E''^{[3]_2}_{2_{22}} = \frac{-s}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3),$

$E''^{[3]_2}_{2_{32}} = \frac{1}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s),$

$E''^{[3]_2}_{2_{13}} = \frac{-r^5}{420(r-1)^3(s-1)^3}(5r^4s - 10r^4 - 16r^3s^2 + 10r^3s + 35r^3 + 14r^2s^3 + 46r^2s^2 - 98r^2s - 28r^2 - 70rs^3 + 42rs^2 + 98rs + 70s^3 - 98s^2),$

$E''^{[3]_2}_{2_{23}} = \frac{-s^5}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2),$

$E''^{[3]_2}_{2_{33}} = \frac{-1}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105),$

$K''^{[3]_2}_{1_{13}} = \frac{r^2}{840s^2}(5r^4 - 16r^3s - 16r^3 + 14r^2s^2 + 56r^2s + 14r^2 - 56rs^2 - 56rs + 70s^2),$

$K''^{[3]_2}_{1_{23}} = \frac{s^2}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2),$

$K''^{[3]_2}_{1_{33}} = \frac{1}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5),$

$K''^{[3]_2}_{2_{11}} = \frac{-r^2}{840(r-s)^2(r-1)^2}(15r^4 - 40r^3s - 40r^3 + 28r^2s^2 + 112r^2s + 28r^2 - 84rs^2 - 84rs + 70s^2),$

$K''^{[3]_2}_{2_{21}} = \frac{-s^5}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3),$

$K''^{[3]_2}_{2_{31}} = \frac{-1}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5),$

$K''^{[3]_2}_{2_{12}} = \frac{r^5}{840s^2(r-s)^2(s-1)^2}(14r - 28s + 28rs - 8r^2s - 16r^2 + 5r^3),$

$K''^{[3]_2}_{2_{22}} = \frac{-s^2}{840(r-s)^2(s-1)^2}(28r^2s^2 - 84r^2s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4$
$-40s^3 + 28s^2)$,

$K''^{[3]_2}_{2_{32}} = \frac{-1}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5)$,

$K''^{[3]_2}_{2_{13}} = \frac{r^5}{840(r-1)^2(s-1)^2}(5r^3 - 16r^2s - 8r^2 + 14rs^2 + 28rs - 28s^2)$,

$K''^{[3]_2}_{2_{23}} = \frac{s^5}{840(r-1)^2(s-1)^2}(14r^2s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2)$,

$K''^{[3]_2}_{2_{33}} = \frac{-1}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2$
$-40s + 15)$.

4.2.1 Properties of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

This section investigates the basic properties of HBM with generalised two off-step points for solving third order ODEs such as order and error constants, zero-stability, consistency, convergence and region of absolute stability.

4.2.1.1 Order of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

The linear difference operator ∇ associated with Equation (4.19) is

$$\begin{aligned}\nabla[y(x),h] &= Y^{[3]_2}_{n+1} - \hat{M}^{[3]_2}_1 Y^{[3]_2}_n - h\hat{M}^{[3]_2}_2 Y^{[3]_2}_{n-1} - h^2\hat{M}^{[3]_2}_3 Y^{[3]_2}_{n-2} - h^3\left[\hat{E}^{[3]_2}_1 F^{[3]_2}_n + \hat{E}^{[3]_2}_2 F^{[3]_2}_{n+1}\right] \\ &\quad - h^4\left[\hat{K}^{[3]_2}_1 G^{[3]_2}_n + \hat{K}^{[3]_2}_2 G^{[3]_2}_{n+1}\right] \qquad (4.31)\end{aligned}$$

Now, expanding each function in $Y^{[3]_2}_{n+1}, F^{[3]_2}_{n+1}$ and $G^{[3]_2}_{n+1}$ about x_n and equating to $\mathbf{0}$, we have

$$\left[Q^{[3]_2}_{11},\ Q^{[3]_2}_{21},\ Q^{[3]_2}_{31}\right]^T = \left[0,\ 0,\ 0\right]^T$$

where

$$Q_{11}^{[3]_2} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n - rhy'_n - \frac{h^2r^2y''_n}{2} + \frac{y'''_n h^3 r^3}{2520s^3}(-2r^5s - 2r^5 + 8r^4s^2 + 12r^4s + 8r^4 - 9r^3s^3 - 28r^3s^2 - 28r^3s - 9r^3 + 27r^2s^3 + 12r^2s^2 + 27r^2s + 48rs^3 + 48rs^2 - 336s^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+3} r^3 y_n^{(i+3)}}{2520(r-s)^3(r-1)^3}(14r^6 - 56r^5s - 56r^5 + 74r^4s^2 + 234r^4s + 74r^4 - 30r^3s^3 - 325r^3s^2 - 325r^3s - 30r^3 + 135r^2s^3 + 480r^2s^2 + 135r^2s - 204rs^3 - 204rs^2 + 84s^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+3)} r^7}{2520s^3(r-s)^3(s-1)^3}(-4r^4s + 2r^4 + 16r^3s^2 + 6r^3s - 8r^3 - 14r^2s^3 - 59r^2s^2 + 25r^2s + 9r^2 + 63rs^3 + 42rs^2 - 54rs - 84s^3 + 60s^2) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)} r^7}{2520(r-1)^3(s-1)^3}(2r^4s - 4r^4 - 8r^3s^2 + 6r^3s + 16r^3 + 9r^2s^3 + 25r^2s^2 - 59r^2s - 14r^2 - 54rs^3 + 42rs^2 + 63rs + 60s^3 - 84s^2) - \frac{y_n^{iv} h^4 r^4}{5040s^2}(2r^4 - 8r^3s - 8r^3 + 9r^2s^2 + 36r^2s + 9r^2 - 48rs^2 - 48rs + 84s^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} r^4}{5040(r-s)^2(r-1)^2)}(3r^4 - 10r^3s - 10r^3 + 9r^2s^2 + 36r^2s + 9r^2 - 36rs^2 - 36rs + 42s^2) - \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)} r^7}{5040s^2(r-s)^2(s-1)^2}(9r - 24s + 18rs - 4r^2s - 8r^2 + 2r^3) - \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+4)} r^7}{5040(r-1)^2(s-1)^2}(2r^3 - 8r^2s - 4r^2 + 9rs^2 + 18rs - 24s^2),$$

$$Q_{21}^{[3]_2} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i)} - y_n - shy'_n - \frac{h^2s^2y''_n}{2} + \frac{y'''_n h^3 s^3}{2520r^3}(-9r^3s^3 + 27r^3s^2 + 48r^3s - 336r^3 + 8r^2s^4 - 28r^2s^3 + 12r^2s^2 + 48r^2s - 2rs^5 + 12rs^4 - 28rs^3 + 27rs^2 - 2s^5 + 8s^4 - 9s^3) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+3)} s^7}{2520r^3(r-s)^3(r-1)^3}(-14r^3s^2 + 63r^3s - 84r^3 + 16r^2s^3 - 59r^2s^2 + 42r^2s + 60r^2 - 4rs^4 + 6rs^3 + 25rs^2 - 54rs + 2s^4 - 8s^3 + 9s^2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+3)} s^3}{2520(r-s)^3(s-1)^3}(-30r^3s^3 + 135r^3s^2 - 204r^3s + 84r^3 + 74r^2s^4 - 325r^2s^3 + 480r^2s^2 - 204r^2s - 56rs^5 + 234rs^4 - 325rs^3 + 135rs^2 + 14s^6 - 56s^5 + 74s^4 - 30s^3) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)} s^7}{2520(r-1)^3(s-1)^3}(9r^3s^2 - 54r^3s + 60r^3 - 8r^2s^3 + 25r^2s^2 + 42r^2s - 84r^2 + 2rs^4 + 6rs^3 - 59rs^2 + 63rs - 4s^4 + 16s^3 - 14s^2) - \frac{y_n^{iv} h^4 s^4}{5040r^2}(9r^2s^2 - 48r^2s + 84r^2 - 8rs^3 + 36rs^2 - 48rs + 2s^4 - 8s^3 + 9s^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} s^7}{5040r^2(r-s)^2(r-1)^2}(24r - 9s - 18rs + 4rs^2 + 8s^2 - 2s^3) + \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)} s^4}{5040(r-s)^2(s-1)^2}(9r^2s^2 - 36r^2s + 42r^2 - 10rs^3 + 36rs^2 - 36rs + 3s^4 - 10s^3 + 9s^2) - \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+4)} s^7}{5040(r-1)^2(s-1)^2}(9r^2s - 24r^2 - 8rs^2 + 18rs + 2s^3 - 4s^2),$$

$$Q_{31}^{[3]_2} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i)} - y_n - hy'_n - \frac{h^2y''_n}{2} + + \frac{y'''_n h^3}{2520r^3s^3}(-336r^3s^3 + 48r^3s^2 + 27r^3s - 9r^3 + 48r^2s^3 + 12r^2s^2 - 28r^2s + 8r^2 + 27rs^3 - 28rs^2 + 12rs - 2r - 9s^3 + 8s^2 - 2s) +$$

$\sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+3)}}{2520r^3(r-s)^3(r-1)^3}(-84r^3s^2 + 63r^3s - 14r^3 + 60r^2s^3 + 42r^2s^2 - 59r^2s + 16r^2 - 54rs^3 + 25rs^2 + 6rs - 4r + 9s^3 - 8s^2 + 2s) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+3)}}{2520s^3(r-s)^3(s-1)^3}(60r^3s^2 - 54r^3s + 9r^3 - 84r^2s^3 + 42r^2s^2 + 25r^2s - 8r^2 + 63rs^3 - 59rs^2 + 6rs + 2r - 14s^3 + 16s^2 - 4s) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)}}{2520(r-1)^3(s-1)^3}(-84r^3s^3 + 204r^3s^2 - 135r^3s + 30r^3 + 204r^2s^3 - 480r^2s^2 + 325r^2s - 74r^2 - 135rs^3 + 325rs^2 - 234rs + 56r + 30s^3 - 74s^2 + 56s - 14) - \frac{y_n^{iv} h^4}{5040r^2s^2}(84r^2s^2 - 48r^2s + 9r^2 - 48rs^2 + 36rs - 8r + 9s^2 - 8s + 2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)}}{5040r^2(r-s)^2(r-1)^2}(4r + 8s - 18rs + 24rs^2 - 9s^2 - 2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)}}{5040s^2(r-s)^2(s-1)^2}(8r + 4s - 18rs + 24r^2s - 9r^2 - 2) + \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+4)}}{5040(r-1)^2(s-1)^2}(42r^2s^2 - 36r^2s + 9r^2 - 36rs^2 + 36rs - 10r + 9s^2 - 10s + 3)$.

Collecting the same terms and comparing the coefficients of h^j and $y^{(j)}$ leads to $\bar{D}_0 = \bar{D}_1 = ... = \bar{D}_{10} = 0$ and $\bar{D}_{11} \neq 0$. Therefore, the order of the main block is $[8,8,8]^T$ for all $0 < r < s < 1$ with vector of error constants

$$\bar{D}_{11} = \begin{bmatrix} \frac{r^7(6r^4-22r^3s-22r^3+22r^2s^2+88r^2s+22r^2-99rs^2-99rs+132s^2)}{1117670400} \\ \frac{s^7(22r^2s^2-99r^2s+132r^2-22rs^3+88rs^2-99rs+6s^4-22s^3+22s^2)}{1117670400} \\ \frac{(132r^2s^2-99r^2s+22r^2-99rs^2+88rs-22r+22s^2-22s+6)}{1117670400} \end{bmatrix}.$$

A similar process is also employed in finding the order of first derivative block (4.26). The linear difference operator ∇ associated with Equation (4.26) is

$$\begin{aligned} \nabla[y(x),h] \;=\; & Y_{n+1}^{'[3]_2} - M_2^{'[3]_2} Y_{n-1}^{[3]_2} - hM_3^{'[3]_2} Y_{n-2}^{[3]_2} - h^2\left[E_1^{'[3]_2} F_n^{[3]_2} + E_2^{'[3]_2} F_{n+1}^{[3]_2}\right] \\ & -h^3\left[K_1^{'[3]_2} G_n^{[3]_2} + K_2^{'[3]_2} G_{n+1}^{[3]_2}\right]. \end{aligned} \tag{4.32}$$

Using Taylor series expansion, we expand every function in $Y_{n+1}^{'[3]_2}, F_{n+1}^{[3]_2}$ and $G_{n+1}^{[3]_2}$ about x_n and this leads to

$$\left[Q_{11}^{'[3]_2}\ Q_{21}^{'[3]_2}\ Q_{31}^{'[3]_2}\right]^T = \left[0\ 0\ 0\right]^T$$

where

$Q_{11}^{'[3]_2} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n' - hry_n'' + \frac{y_n''' h^2 r^2}{2520 s^3}(-10r^5 s - 10r^5 + 36r^4 s^2 + 53r^4 s + 36r^4 - 36r^3 s^3 - 108r^3 s^2 - 108r^3 s - 36r^3 + 90r^2 s^3 + 24r^2 s^2 + 90r^2 s + 168rs^3 + 168rs^2 - 882s^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+2} r^2 y_n^{(i+3)}}{2520(r-s)^3(r-1)^3}(105r^6 - 385r^5 s - 385r^5 + 468r^4 s^2 + 1457r^4 s + 468r^4 - 180r^3 s^3 - 1836r^3 s^2 - 1836r^3 s - 180r^3 + 720r^2 s^3 + 2418r^2 s^2 + 720r^2 s - 966rs^3 - 966rs^2 + 378s^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} r^6 y_n^{(i+3)}}{2520 s^3 (r-s)^3 (s-1)^3}(-20r^4 s + 10r^4 + 75r^3 s^2 + 25r^3 s - 36r^3 - 63r^2 s^3 - 243r^2 s^2 + 108r^2 s + 36r^2 + 252rs^3 + 138rs^2 - 198rs - 294s^3 + 210s^2) + \sum_{i=0}^{\infty} \frac{h^{i+3} r^6 y_n^{(i+2)}}{2520(r-1)^3(s-1)^3}(10r^4 s - 20r^4 - 36r^3 s^2 + 25r^3 s + 75r^3 + 36r^2 s^3 + 108r^2 s^2 - 243r^2 s - 63r^2 - 198rs^3 + 138rs^2 + 252rs + 210s^3 - 294s^2) - \frac{y_n^{iv} h^3 r^3}{5040 s^2}(5r^4 - 18r^3 s - 18r^3 + 18r^2 s^2 + 72r^2 s + 18r^2 - 84rs^2 - 84rs + 126s^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} r^3}{1260(r-s)^2(r-1)^2}(5r^4 - 15r^3 s - 15r^3 + 12r^2 s^2 + 48r^2 s + 12r^2 - 42rs^2 - 42rs + 42s^2) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} r^6}{2520 s^2 (r-s)^2 (s-1)^2}(18r - 42s + 36rs - 9r^2 s - 18r^2 + 5r^3) - \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)} r^6}{2520 s^2 (r-s)^2 (s-1)^2}(5r^3 - 18r^2 s - 9r^2 + 18rs^2 + 36rs - 42s^2),$

$Q_{21}^{'[3]_2} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i)} - y_n' - hsy_n'' + \frac{y_n''' h^2 s^2}{2520 r^3}(-36r^3 s^3 + 90r^3 s^2 + 168r^3 s - 882r^3 + 36r^2 s^4 - 108r^2 s^3 + 24r^2 s^2 + 168r^2 s - 10rs^5 + 53rs^4 - 108rs^3 + 90rs^2 - 10s^5 + 36s^4 - 36s^3) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} s^6 y_n^{(i+3)}}{2520 r^3 (r-s)^3 (r-1)^3}(-63r^3 s^2 + 252r^3 s - 294r^3 + 75r^2 s^3 - 243r^2 s^2 + 138r^2 s + 210r^2 - 20rs^4 + 25rs^3 + 108rs^2 - 198rs + 10s^4 - 36s^3 + 36s^2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+2} s^2 y_n^{(i+3)}}{2520 s^3 (r-s)^3 (s-1)^3}(-180r^3 s^3 + 720r^3 s^2 - 966r^3 s + 378r^3 + 468r^2 s^4 - 1836r^2 s^3 + 2418r^2 s^2 - 966r^2 s - 385rs^5 + 1457rs^4 - 1836rs^3 + 720rs^2 + 105s^6 - 385s^5 + 468s^4 - 180s^3) + \sum_{i=0}^{\infty} \frac{h^{i+2} s^6 y_n^{(i+3)}}{2520(r-1)^3(s-1)^3}(36r^3 s^2 - 198r^3 s + 210r^3 - 36r^2 s^3 + 108r^2 s^2 + 138r^2 s - 294r^2 + 10rs^4 + 25rs^3 - 243rs^2 + 252rs - 20s^4 + 75s^3 - 63s^2) - \frac{y_n^{iv} h^3 s^3}{2520 r^2}(18r^2 s^2 - 84r^2 s + 126r^2 - 18rs^3 + 72rs^2 - 84rs + 5s^4 - 18s^3 + 18s^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} s^6}{2520 r^2 (r-s)^2 (r-1)^2}(42r - 18s - 36rs + 9rs^2 + 18s^2 - 5s^3) + \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} s^3}{1260(r-s)^2(s-1)^2}(12r^2 s^2 - 42r^2 s + 42r^2 - 15rs^3 + 48rs^2 - 42rs + 5s^4 - 15s^3 + 12s^2) - \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)} s^6}{2520(r-1)^2(s-1)^2}(18r^2 s - 42r^2 - 18rs^2 + 36rs + 5s^3 - 9s^2),$

$Q_{31}^{'[3]_2} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i)} - y_n' - hy_n'' + \frac{y_n''' h^2}{2520 r^3 s^3}(-882r^3 s^3 + 168r^3 s^2 + 90r^3 s - 36r^3 + 168r^2 s^3 + 24r^2 s^2 - 108r^2 s + 36r^2 + 90rs^3 - 108rs^2 + 53rs - 10r - 36s^3 + 36s^2 - 10s) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)}}{2520 r^3 (r-s)^3 (r-1)^3}(-294r^3 s^2 + 252r^3 s - 63r^3 + 210r^2 s^3 + 138r^2 s^2 - 243$

$r^2s + 75r^2 - 198rs^3 + 108rs^2 + 25rs - 20r + 36s^3 - 36s^2 + 10s) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)}}{2520s^3(r-s)^3(s-1)^3}(210r^3s^2 - 198r^3s + 36r^3 - 294r^2s^3 + 138r^2s^2 + 108r^2s - 36r^2 + 252rs^3 - 243rs^2 + 25rs + 10r - 63s^3 + 75s^2 - 20s) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)}}{2520(r-1)^3(s-1)^3}(-378r^3s^3 + 966r^3s^2 - 720r^3s + 180r^3 + 966r^2s^3 - 2418r^2s^2 + 1836r^2s - 468r^2 - 720rs^3 + 1836rs^2 - 1457rs + 385r + 180s^3 - 468s^2 + 385s - 105) - \frac{y_n^{iv} h^3}{2520r^2s^2}(126r^2s^2 - 84r^2s + 18r^2 - 84rs^2 + 72rs - 18r + 18s^2 - 18s + 5) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)}}{2520r^2(r-s)^2(r-1)^2}(9r + 18s - 36rs + 42rs^2 - 18s^2 - 5) + \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)}}{2520s^2(r-s)^2(s-1)^2}(18r + 9s - 36rs + 42r^2s - 18r^2 - 5) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)}}{1260(r-1)^2(s-1)^2}(42r^2s^2 - 42r^2s + 12r^2 - 42rs^2 + 48rs - 15r + 12s^2 - 15s + 5).$

Again, comparing the coefficients of h^j and $y^{(j)}$ gives $\bar{D}'_0 = \bar{D}'_1 = ... = \bar{D}'_{10} = 0$ and $\bar{D}'_{11} \neq 0$. As a result, we have found that the first derivative of the block has order $[8,8,8]^T$ with vector of error constants

$$\bar{D}'_{11} = \begin{bmatrix} \frac{(9r^8-36r^7+42r^6)s^2+(-10r^9+36r^8-36r^7)s+3r^{10}-10r^9+9r^8}{101606400} \\ \frac{3s^{10}+(-10r-10)s^9+(9r^2+36r+9)s^8+(-36r^2-36r)s^7+(42r^2)s^6}{101606400} \\ \frac{(42r^2-36r+9)s^2+(-36r^2+36r-10)s+9r^2-10r+3}{101606400} \end{bmatrix}.$$

In finding the order of second derivative (4.30), we set the linear difference operator ∇ below

$$\nabla[y(x),h] = Y''^{[3]_2}_{n+1} - M''^{[3]_2}_3 Y^{[3]_2}_{n-2} - h[E''^{[3]_2}_1 F^{[3]_2}_n + E''^{[3]_2}_2 F^{[3]_2}_{n+1}] - h^2[K''^{[3]_2}_1 G^{[3]_2}_n + K''^{[3]_2}_2 G^{[3]_2}_{n+1}] \quad (4.33)$$

Then, by expanding every function in $Y''^{[3]_2}_{n+1}, F^{[3]_2}_{n+1}\ G^{[3]_2}_{n+1}$ about x_n we get

$$\begin{bmatrix} Q''^{[3]_2}_{11}\ Q''^{[3]_2}_{21}\ Q''^{[3]_2}_{31} \end{bmatrix}^T = \begin{bmatrix} 0\ 0\ 0 \end{bmatrix}^T$$

where

$$Q_{11}^{''[3]_2} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i+2)} - y_n'' + \frac{y_n''' hr}{420s^3}(-5r^5s - 5r^5 + 16r^4s^2 + 23r^4s + 16r^4 - 14r^3s^3 - 40r^3s^2 - 40r^3s - 14r^3 + 28r^2s^3 + 28r^2s + 56rs^3 + 56rs^2 - 210s^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+1} r y_n^{(i+3)}}{420(r-s)^3(r-1)^3}(105r^6 - 350r^5s - 350r^5 + 388r^4s^2 + 1187r^4s + 388r^4 - 140r^3s^3 - 1342r^3s^2 - 1342r^3s - 140r^3 + 490r^2s^3 + 1554r^2s^2 + 490r^2s - 574rs^3 - 574rs^2 + 210s^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+1} r^5 y_n^{(i+3)}}{420s^3(r-s)^3(s-1)^3}(-10r^4s + 5r^4 + 35r^3s^2 + 10r^3s - 16r^3 - 28r^2s^3 - 98r^2s^2 + 46r^2s + 14r^2 + 98rs^3 + 42rs^2 - 70rs - 98s^3 + 70s^2) + \sum_{i=0}^{\infty} \frac{h^{i+1} r^5 y_n^{(i+3)}}{420(r-1)^3(s-1)^3}(5r^4s - 10r^4 - 16r^3s^2 + 10r^3s + 35r^3 + 14r^2s^3 + 46r^2s^2 - 98r^2s - 28r^2 - 70rs^3 + 42rs^2 + 98rs + 70s^3 - 98s^2) - \frac{y_n^{iv} h^2 r^2}{840s^2}(5r^4 - 16r^3s - 16r^3 + 14r^2s^2 + 56r^2s + 14r^2 - 56rs^2 - 56rs + 70s^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+4)} r^2}{840(r-s)^2(r-1)^2}(15r^4 - 40r^3s - 40r^3 + 28r^2s^2 + 112r^2s + 28r^2 - 84rs^2 - 84rs + 70s^2) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+4)} r^5}{840s^2(r-s)^2(s-1)^2}(14r - 28s + 28rs - 8r^2s - 16r^2 + 5r^3) - \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)} r^5}{840(r-1)^2(s-1)^2}(5r^3 - 16r^2s - 8r^2 + 14rs^2 + 28rs - 28s^2),$$

$$Q_{21}^{''[3]_2} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i+2)} - y_n'' - \frac{y_n''' hs}{420r^3}(14r^3s^3 - 28r^3s^2 - 56r^3s + 210r^3 - 16r^2s^4 + 40r^2s^3 - \frac{y_n''' h}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s) + \sum_{i=0}^{\infty} \frac{r^i h^{i+1} y_n^{(i+3)}}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s) - \sum_{i=0}^{\infty} \frac{s^i h^{i+1} y_n^{(i+3)}}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+3)}}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105) - \frac{y_n^{iv} h^2}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+4)}}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5) + \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+4)}}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)}}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15) - 56r^2s + 5rs^5 - 23rs^4 + 40rs^3 - 28rs^2 + 5s^5 - 16s^4 + 14s^3) + \sum_{i=0}^{\infty} \frac{r^i h^{i+1} s^5 y_n^{(i+3)}}{420r^3(r-s)^3(r-1)^3}(-28r^3s^2 + 98r^3s - 98r^3 + 35r^2s^3 - 98r^2s^2 + 42r^2s + 70r^2 - 10rs^4 + 10rs^3 + 46rs^2 - 70rs + 5s^4 - 16s^3 + 14s^2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+1} s y_n^{(i+3)}}{420(r-s)^3(s-1)^3}(-140r^3s^3 + 2490r^3s^2 - 574r^3s + 210r^3 + 388r^2s^4 - 1342r^2s^3 + 1554r^2s^2 - 574r^2s - 350rs^5 + 1187rs^4 - 1342rs^3 + 490rs^2 + 105s^6 - 350s^5 + 388s^4 - 140s^3) +$$

$\sum_{i=0}^{\infty} \frac{h^{i+1}s^5 y_n^{(i+3)}}{420(r-1)^3(s-1)^3}(14r^3s^2 - 70r^3s + 70r^3 - 16r^2s^3 + 46r^2s^2 + 42r^2s - 98r^2 + 5rs^4 + 10rs^3 - 98rs^2 + 98rs - 10s^4 + 35s^3 - 28s^2) - \frac{y_n^{iv}h^2s^2}{840r^2}(14r^2s^2 - 56r^2s + 70r^2 - 16rs^3 + 56rs^2 - 56rs + 5s^4 - 16s^3 + 14s^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+2}s^5 y_n^{(i+4)}}{840r^2(r-s)^2(r-1)^2}(28r - 14s - 28rs + 8rs^2 + 16s^2 - 5s^3) + \sum_{i=0}^{\infty} \frac{s^ih^{i+2}s^2 y_n^{(i+4)}}{840(r-s)^2(s-1)^2}(28r^2s^2 - 84r^2s + 70r^2 - 40rs^3 + 112rs^2 - 84rs + 15s^4 - 40s^3 + 28s^2) - \sum_{i=0}^{\infty} \frac{h^{i+2}s^5 y_n^{(i+4)}}{840(r-1)^2(s-1)^2}(14r^2s - 28r^2 - 16rs^2 + 28rs + 5s^3 - 8s^2)$,

$Q_{31}^{''[3]_2} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i+2)} - y_n'' - \frac{y_n'''h}{420r^3s^3}(210r^3s^3 - 56r^3s^2 - 28r^3s + 14r^3 - 56r^2s^3 + 40r^2s - 16r^2 - 28rs^3 + 40rs^2 - 23rs + 5r + 14s^3 - 16s^2 + 5s) + \sum_{i=0}^{\infty} \frac{r^ih^{i+1} y_n^{(i+3)}}{420r^3(r-s)^3(r-1)^3}(-98r^3s^2 + 98r^3s - 28r^3 + 70r^2s^3 + 42r^2s^2 - 98r^2s + 35r^2 - 70rs^3 + 46rs^2 + 10rs - 10r + 14s^3 - 16s^2 + 5s) - \sum_{i=0}^{\infty} \frac{s^ih^{i+1} y_n^{(i+3)}}{420s^3(r-s)^3(s-1)^3}(70r^3s^2 - 70r^3s + 14r^3 - 98r^2s^3 + 42r^2s^2 + 46r^2s - 16r^2 + 98rs^3 - 98rs^2 + 10rs + 5r - 28s^3 + 35s^2 - 10s) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+3)}}{420(r-1)^3(s-1)^3}(-210r^3s^3 + 574r^3s^2 - 490r^3s + 140r^3 + 574r^2s^3 - 1554r^2s^2 + 1342r^2s - 388r^2 - 490rs^3 + 1342rs^2 - 1187rs + 350r + 140s^3 - 388s^2 + 350s - 105) - \frac{y_n^{iv}h^2}{840r^2s^2}(70r^2s^2 - 56r^2s + 14r^2 - 56rs^2 + 56rs - 16r + 14s^2 - 16s + 5) + \sum_{i=0}^{\infty} \frac{r^ih^{i+2} y_n^{(i+4)}}{840r^2(r-s)^2(r-1)^2}(8r + 16s - 28rs + 28rs^2 - 14s^2 - 5) + \sum_{i=0}^{\infty} \frac{s^ih^{i+2} y_n^{(i+4)}}{840s^2(r-s)^2(s-1)^2}(16r + 8s - 28rs + 28r^2s - 14r^2 - 5) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)}}{840(r-1)^2(s-1)^2}(70r^2s^2 - 84r^2s + 28r^2 - 84rs^2 + 112rs - 40r + 28s^2 - 40s + 15)$.

Again, collecting the like terms in the provided equations and comparing coefficients of h^j and $y^{(j)}$ yields $\bar{D}''_0 = \bar{D}''_1 = ... = \bar{D}''_{10} = 0$ and $\bar{D}''_{11} \neq 0$. As a result, the second derivative of the main block has order $[8,8,8]^T$ with vector of error constants

$$\bar{D}''_{11} = \begin{bmatrix} \frac{(12r^7-42r^6+42r^5)s^2+(-15r^8+48r^7-42r^6)s+5r^9-15r^8+12r^7}{50803200} \\ \frac{5s^9+(-15r-15)s^8+(12r^2+48r+12)s^7+(-42r^2-42r)s^6+(42r^2)s^5}{50803200} \\ \frac{(42r^2-42r+12)s^2+(-42r^2+48r-15)s+12r^2-15r+5}{50803200} \end{bmatrix}.$$

4.2.1.2 zero-stability of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

For the main block, first and second derivatives block, the following first characteristic polynomial are obtained by substituting $m = 3$ and $z = 2$ in (4.19), (4.26) and (4.30).

$$\begin{aligned}
\psi^{[3]_2}(q) &= |qI_3 - \hat{M}_1^{[3]_2}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^2(q-1) \\
\psi'^{[3]_2}(q) &= |qI_3 - M_2'^{[3]_2}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^2(q-1) \\
\psi''^{[3]_2}(q) &= |qI_3 - M_3''^{[3]_2}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^2(q-1).
\end{aligned}$$

Equating each characteristic polynomial to 0 we get $q = \{0,0,1\}$. This implies that one-step HBM with two off-step points is zero stable.

4.2.1.3 Consistency of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

According to Definition 2.4.4, the main block method (4.19) and its derivatives (4.26) and (4.30) are consistent.

4.2.1.4 Convergence of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

By Theorem (2.1), the main block method (4.19) and its derivatives (4.26) and (4.30) are convergent .

4.2.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised Two Off-Step Points for Solving Third Order ODEs

Now, substituting $m = 3$ and $z = 2$ in (3.27) gives

$$M^{[3]_2}(q) = (I_3 - q^3\hat{E}_2^{[3]_2} - q^4\hat{K}_2^{[3]_2})^{-1}(\hat{M}_1^{[3]_2} + q\hat{M}_2^{[3]_2} + q^2\hat{M}_3^{[3]_2} + q^3\hat{E}_1^{[3]_2} + q^4\hat{K}_1^{[3]_2})$$

where the eigenvalues of $M^{[3]_2}$ are $\{0, 0, \eta_3^{[3]_2}\}$. Then the non-zero eigenvalue $\eta_3^{[3]_2}$ is in terms of q given below

$$\eta_3^{[3]_2} = eig(M^{[3]_2}(q)). \tag{4.34}$$

To sketch the graph of region of absolute stability, we consider two specific values , i.e $r = \frac{1}{3}$ and $s = \frac{2}{3}$. Substituting these values in (4.34), we have

$$\eta_3^{[3]_2} = \frac{\sum_{i=0}^{12} c_i q^i}{K\sum_{j=0}^{13} d_j q^j}$$

where $K = 4096$ and the values c_i and d_j are listed in the two tables below:

Table 4.1

Coefficients of the Eigenvalue $(\eta_3^{[3]_2})$ *for the Matrix* $M^{[3]_2}$

c-value	q^i Coefficients
c_0	4652109034955913531058025085646064340906383769600 0
c_1	46521090349559135310580250856460643409063837696000
c_2	23260545174779567655290125428230321704531918848000
c_3	7199692554098553125404947443419105612802713190400
c_4	1480848444769152633906799660274101213507407052800
c_5	207056685197836613550788168564910582166192128000
c_6	19240298493726626320669041948058112000923975680
c_7	1063079864337172164585771170117057316900963840
c_8	23649677670190006258843478410343478240215040
c_9	1088079452789205407442305756503687272026400
c_{10}	52436050591689905588005595404975532192494 5
c_{11}	7252932558998954433219522084769172546347 9
c_{12}	343605436910402548631906509882168180736 0
d-value	q^j Coefficients
d_0	0
d_1	11357688073622835769184631556753086769790976000
d_2	0
d_3	0
d_4	- 1352105723050240513472560402907616742735872 00
d_5	235088340736118367258500236759170054881280 00
d_6	0
d_7	- 29654992211143401706620344051594763067392 0
d_8	1821619997251512687969728278810664632320 0
d_9	1363932009913148027065997765120769392640
d_{10}	- 19463324774226300612355724106439483205 1
d_{11}	736008079697217799922105243085203531 1
d_{12}	45432597512556736020807597019889664
d_{13}	3245185536584267267831560205762560 0

By plotting the function $(\eta_3^{[3]_2})$ we obtain the region of absolute stability as indicated by the dark area Figure 4.2

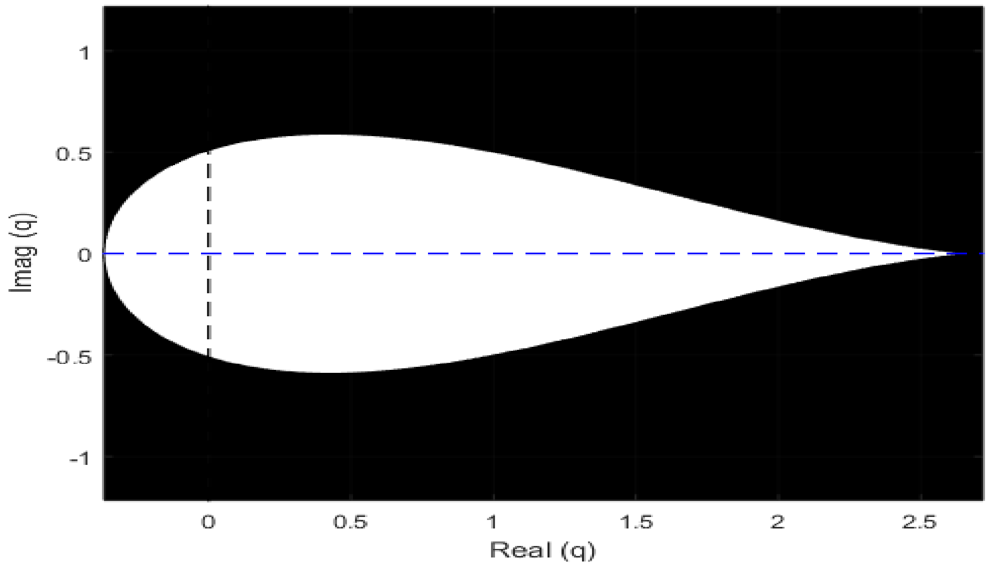


Figure 4.2. Region of Absolute Stability of One-Step HBM with Two Off-Step Points $r = \frac{1}{3}$ and $s = \frac{2}{3}$ for Third Order ODEs

In the same fashion, implementing the same procedure as before we arrive at the graph of the region of absolute stability for the function $(\eta_3^{[3]_2})$ corresponding to the specific $r = \frac{1}{4}$ and $s = \frac{3}{4}$ as illustrated in dark area in Figure 4.3 below

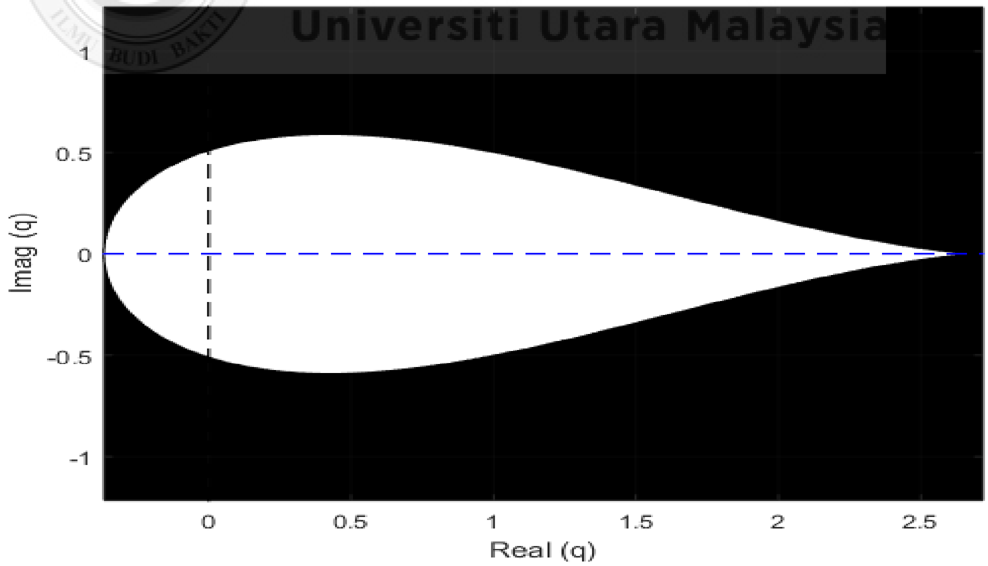


Figure 4.3. Region of Absolute Stability of One-Step HBM with Two Off-Step Points $r = \frac{1}{4}$ and $s = \frac{3}{4}$ for Third Order ODEs

Henceforth, the function $(\eta_3^{[3]_2})$ obtains a sketch for all values r and s as shown above.

In the coming section, the procedure for deriving the HBM with generalised three off-step points is proposed.

4.3 Derivation of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

Following Section 4.2 in deriving the continuous scheme for third order ODEs with three off-step points, the approximate solution Equation (3.2) is interpolated at three points, i.e x_n, x_{n+r} and x_{n+s} while its third and fourth derivatives (4.4) and (4.5) are collocated at all points as depicted in Figures 4.4 below

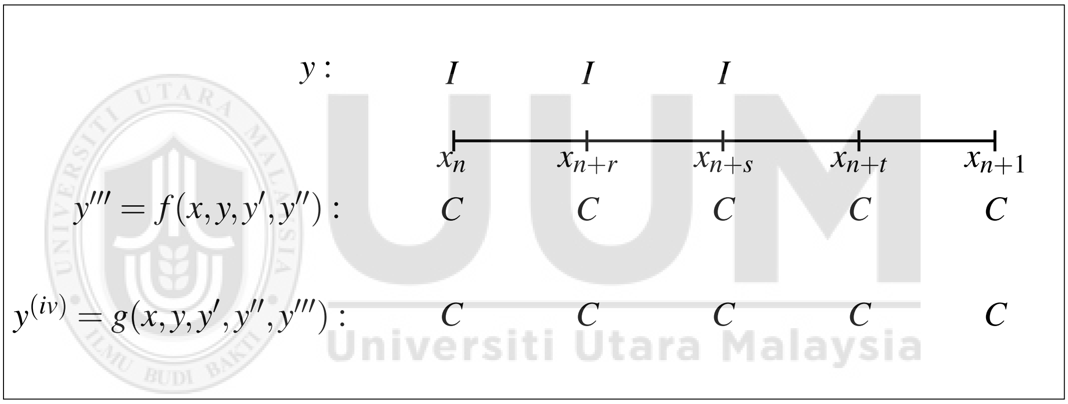


Figure 4.4. Interpolation and Collocation Strategy for One-Step HBM with Three Off-Step Points for Solving Third Order ODEs

Now, substituting $u = 3$ and $v = 5$ in (3.2), (4.2) and (4.3) produces the following equations

$$
\begin{aligned}
y_n &= a_0, \\
y_{n+r} &= a_0 + ra_1 + r^2a_2 + r^3a_3 + r^4a_4 + r^5a_5 + r^6a_6 + r^7a_7 + r^8a_8 + r^9a_9 + \\
&\quad r^{10}a_{10} + r^{11}a_{11} + r^{12}a_{12}, \\
y_{n+s} &= a_0 + sa_1 + s^2a_2 + s^3a_3 + s^4a_4 + s^5a_5 + s^6a_6 + s^7a_7 + s^8a_8 + s^9a_9 + \\
&\quad s^{10}a_{10} + s^{11}a_{11} + s^{12}a_{12},
\end{aligned}
$$

$$
\begin{aligned}
f_n &= \frac{6}{h^3}a_3, \\
f_{n+r} &= \frac{6}{h^3}a_3 + \frac{24r}{h^3}a_4 + \frac{60r^2}{h^3}a_5 + \frac{120r^3}{h^3}a_6 + \frac{210r^4}{h^3}a_7 + \frac{336r^5}{h^3}a_8 + \frac{504r^6}{h^3}a_9 \\
&\quad + \frac{720r^7}{h^3}a_{10} + \frac{990r^8}{h^3}a_{11} + \frac{1320r^9}{h^3}a_{12}, \\
f_{n+s} &= \frac{6}{h^3}a_3 + \frac{24s}{h^3}a_4 + \frac{60s^2}{h^3}a_5 + \frac{120s^3}{h^3}a_6 + \frac{210s^4}{h^3}a_7 + \frac{336s^5}{h^3}a_8 + \frac{504s^6}{h^3}a_9 \\
&\quad + \frac{720s^7}{h^3}a_{10} + \frac{990s^8}{h^3}a_{11} + \frac{1320s^9}{h^3}a_{12}, \\
f_{n+t} &= \frac{6}{h^3}a_3 + \frac{24t}{h^3}a_4 + \frac{60t^2}{h^3}a_5 + \frac{120t^3}{h^3}a_6 + \frac{210t^4}{h^3}a_7 + \frac{336t^5}{h^3}a_8 + \frac{504t^6}{h^3}a_9 \\
&\quad + \frac{720t^7}{h^3}a_{10} + \frac{990t^8}{h^3}a_{11} + \frac{1320t^9}{h^3}a_{12}, \\
f_{n+1} &= \frac{6}{h^3}a_3 + \frac{24}{h^3}a_4 + \frac{60}{h^3}a_5 + \frac{120}{h^3}a_6 + \frac{210}{h^3}a_7 + \frac{336}{h^3}a_8 + \frac{504}{h^3}a_9 \\
&\quad + \frac{720}{h^3}a_{10} + \frac{990}{h^3}a_{11} + \frac{1320}{h^3}a_{12}, \\
g_n &= \frac{24}{h^4}a_4, \\
g_{n+r} &= \frac{24}{h^4}a_4 + \frac{120r}{h^4}a_5 + \frac{360r^2}{h^4}a_6 + \frac{840r^3}{h^4}a_7 + \frac{1680r^4}{h^4}a_8 + \frac{3024r^5}{h^4}a_9 + \\
&\quad \frac{5040r^6}{h^4}a_{10} + \frac{7920r^7}{h^4}a_{11} + \frac{11880r^8}{h^4}a_{12}, \\
g_{n+s} &= \frac{24}{h^4}a_4 + \frac{120s}{h^4}a_5 + \frac{360s^2}{h^4}a_6 + \frac{840s^3}{h^4}a_7 + \frac{1680s^4}{h^4}a_8 + \frac{3024s^5}{h^4}a_9 + \\
&\quad \frac{5040s^6}{h^4}a_{10} + \frac{7920s^7}{h^4}a_{11} + \frac{11880s^8}{h^4}a_{12}, \\
g_{n+t} &= \frac{24}{h^4}a_4 + \frac{120t}{h^4}a_5 + \frac{360t^2}{h^4}a_6 + \frac{840t^3}{h^4}a_7 + \frac{1680t^4}{h^4}a_8 + \frac{3024t^5}{h^4}a_9 + \\
&\quad \frac{5040t^6}{h^4}a_{10} + \frac{7920t^7}{h^4}a_{11} + \frac{11880t^8}{h^4}a_{12}, \\
g_{n+1} &= \frac{24}{h^4}a_4 + \frac{120}{h^4}a_5 + \frac{360}{h^4}a_6 + \frac{840}{h^4}a_7 + \frac{1680}{h^4}a_8 + \frac{3024}{h^4}a_9 + \frac{5040}{h^4}a_{10} \\
&\quad + \frac{7920}{h^4}a_{11} + \frac{11880}{h^4}a_{12}
\end{aligned}
$$

which can be transformed into matrix form $AX = B$ where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & r^{12} \\ 1 & s & s^2 & s^3 & s^4 & s^5 & s^6 & s^7 & s^8 & s^9 & s^{10} & s^{11} & s^{12} \\ 0 & 0 & 0 & \frac{6}{h^3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24r}{h^3} & \frac{60r^2}{h^3} & \frac{120r^3}{h^3} & \frac{210r^4}{h^3} & \frac{336r^5}{h^3} & \frac{504r^6}{h^3} & \frac{720r^7}{h^3} & \frac{990r^8}{h^3} & \frac{1320r^9}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24s}{h^3} & \frac{60s^2}{h^3} & \frac{120s^3}{h^3} & \frac{210s^4}{h^3} & \frac{336s^5}{h^3} & \frac{504s^6}{h^3} & \frac{720s^7}{h^3} & \frac{990s^8}{h^3} & \frac{1320s^9}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24t}{h^3} & \frac{60t^2}{h^3} & \frac{120t^3}{h^3} & \frac{210t^4}{h^3} & \frac{336t^5}{h^3} & \frac{504t^6}{h^3} & \frac{720t^7}{h^3} & \frac{990t^8}{h^3} & \frac{1320t^9}{h^3} \\ 0 & 0 & 0 & \frac{6}{h^3} & \frac{24}{h^3} & \frac{60}{h^3} & \frac{120}{h^3} & \frac{210}{h^3} & \frac{336}{h^3} & \frac{504}{h^3} & \frac{720}{h^3} & \frac{990}{h^3} & \frac{1320}{h^3} \\ 0 & 0 & 0 & 0 & \frac{24}{h^4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120r}{h^4} & \frac{360r^2}{h^4} & \frac{840r^3}{h^4} & \frac{1680r^4}{h^4} & \frac{3024r^5}{h^4} & \frac{5040r^6}{h^4} & \frac{7920r^7}{h^4} & \frac{11880r^8}{h^4} \\ 0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120s}{h^4} & \frac{360s^2}{h^4} & \frac{840s^3}{h^4} & \frac{1680s^4}{h^4} & \frac{3024s^5}{h^4} & \frac{5040s^6}{h^4} & \frac{7920s^7}{h^4} & \frac{11880s^8}{h^4} \\ 0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120t}{h^4} & \frac{360t^2}{h^4} & \frac{840t^3}{h^4} & \frac{1680t^4}{h^4} & \frac{3024t^5}{h^4} & \frac{5040t^6}{h^4} & \frac{7920t^7}{h^4} & \frac{11880t^8}{h^4} \\ 0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120}{h^4} & \frac{360}{h^4} & \frac{840}{h^4} & \frac{1680}{h^4} & \frac{3024}{h^4} & \frac{5040}{h^4} & \frac{7920}{h^4} & \frac{11880}{h^4} \end{pmatrix},$$

$X = [a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8,\ a_9,\ a_{10},\ a_{11},\ a_{12}]^T$,

$B = [y_n,\ y_{n+r},\ ,y_{n+s},\ f_n,\ f_{n+r},\ f_{n+s},\ f_{n+t},\ f_{n+1},\ g_n,\ g_{n+r},\ g_{n+s},\ g_{n+t},\ g_{n+1}]^T$.

The unknown values a_i's; $i = 0(1)(12)$ are obtained by solving the above system of equations. Then, substituting these values into (3.2) yields a continuous implicit scheme of the form

$$\begin{aligned} y(x) \;=\; & \sum_{i=0,r,s} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \sum_{i=r,s,t} \beta_i(x) f_{n+i} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} \\ & + \sum_{i=r,s,t} \gamma_i(x) g_{n+i} \end{aligned} \tag{4.35}$$

where

$\alpha_0 = \frac{(x_n - x + hr)(x_n - x + hs)}{h^2 rs}$,

$\alpha_r = -\frac{(x - x_n)(xn - x + hs)}{h^2 r(r-s)}$,

$\alpha_s = \frac{(x - x_n)(x_n - x + hr)}{h^2 s(r-s)}$,

$$\beta_0 = \frac{(x-x_n)^3}{6} + \frac{(x-x_n)^6}{60h^3r^3s^3t^3}(r^3s^3t^3 + 4r^3s^3t^2 + 4r^3s^3t + r^3s^3 + 4r^3s^2t^3 + 8r^3s^2t^2 + 4r^3s^2t +$$
$$4r^3st^3 + 4r^3st^2 + r^3t^3 + 4r^2s^3t^3 + 8r^2s^3t^2 + 4r^2s^3t + 8r^2s^2t^3 + 8r^2s^2t^2 + 4r^2st^3 +$$
$$4rs^3t^3 + 4rs^3t^2 + 4rs^2t^3 + s^3t^3) - \frac{(x-x_n)^9}{504h^6r^3s^3t^3}(4r^3s^2t + 4r^3s^2 + 4r^3st^2 + 11r^3st + 4r^3s +$$
$$4r^3t^2 + 4r^3t + 4r^2s^3t + 4r^2s^3 + 16r^2s^2t^2 + 36r^2s^2t + 16r^2s^2 + 4r^2st^3 + 36r^2st^2 +$$
$$36r^2st + 4r^2s + 4r^2t^3 + 16r^2t^2 + 4r^2t + 4rs^3t^2 + 11rs^3t + 4rs^3 + 4rs^2t^3 + 36rs^2t^2 +$$
$$36rs^2t + 4rs^2 + 11rst^3 + 36rst^2 + 11rst + 4rt^3 + 4rt^2 + 4s^3t^2 + 4s^3t + 4s^2t^3 + 16s^2t^2 +$$
$$4s^2t + 4st^3 + 4st^2) + \frac{(x-x_n)^{10}}{360h^7r^3s^3t^3}(r^3st + r^3s + r^3t + 4r^2s^2t + 4r^2s^2 + 4r^2st^2 + 12r^2st +$$
$$4r^2s + 4r^2t^2 + 4r^2t + rs^3t + rs^3 + 4rs^2t^2 + 12rs^2t + 4rs^2 + rst^3 + 12rst^2 + 12rst +$$
$$rs + rt^3 + 4rt^2 + rt + s^3t + 4s^2t^2 + 4s^2t + st^3 + 4st^2 + st) - \frac{h(x-x_n)^2}{27720r^3s^3t^3}(7r^{11}st + 7r^{11}s +$$
$$7r^{11}t - 17r^{10}s^2t - 17r^{10}s^2 - 24r^{10}st^2 - 52r^{10}st - 24r^{10}s - 24r^{10}t^2 - 24r^{10}t + 5r^9s^3t +$$
$$5r^9s^3 + 64r^9s^2t^2 + 100r^9s^2t + 64r^9s^2 + 22r^9st^3 + 128r^9st^2 + 128r^9st + 22r^9s +$$
$$22r^9t^3 + 88r^9t^2 + 22r^9t + 5r^8s^4t + 5r^8s^4 - 24r^8s^3t^2 - 32r^8s^3t - 24r^8s^3 - 66r^8s^2t^3 -$$
$$224r^8s^2t^2 - 224r^8s^2t - 66r^8s^2 - 110r^8st^3 - 264r^8st^2 - 110r^8st - 88r^8t^3 - 88r^8t^2 +$$
$$5r^7s^5t + 5r^7s^5 - 24r^7s^4t^2 - 32r^7s^4t - 24r^7s^4 + 33r^7s^3t^3 + 84r^7s^3t^2 + 84r^7s^3t +$$
$$33r^7s^3 + 198r^7s^2t^3 + 176r^7s^2t^2 + 198r^7s^2t + 220r^7st^3 + 220r^7st^2 + 99r^7t^3 + 5r^6s^6t +$$
$$5r^6s^6 - 24r^6s^5t^2 - 32r^6s^5t - 24r^6s^5 + 33r^6s^4t^3 + 84r^6s^4t^2 + 84r^6s^4t + 33r^6s^4 -$$
$$99r^6s^3t^3 + 44r^6s^3t^2 - 99r^6s^3t + 88r^6s^2t^3 + 88r^6s^2t^2 - 198r^6st^3 + 5r^5s^7t + 5r^5s^7 -$$
$$24r^5s^6t^2 - 32r^5s^6t - 24r^5s^6 + 33r^5s^5t^3 + 84r^5s^5t^2 + 84r^5s^5t + 33r^5s^5 - 99r^5s^4t^3 +$$
$$44r^5s^4t^2 - 99r^5s^4t - 440r^5s^3t^3 - 440r^5s^3t^2 - 726r^5s^2t^3 + 5r^4s^8t + 5r^4s^8 - 24r^4s^7t^2 -$$
$$32r^4s^7t - 24r^4s^7 + 33r^4s^6t^3 + 84r^4s^6t^2 + 84r^4s^6t + 33r^4s^6 - 99r^4s^5t^3 + 44r^4s^5t^2 -$$
$$99r^4s^5t - 440r^4s^4t^3 - 440r^4s^4t^2 + 2970r^4s^3t^3 + 5r^3s^9t + 5r^3s^9 - 24r^3s^8t^2 - 32r^3s^8t -$$
$$24r^3s^8 + 33r^3s^7t^3 + 84r^3s^7t^2 + 84r^3s^7t + 33r^3s^7 - 99r^3s^6t^3 + 44r^3s^6t^2 - 99r^3s^6t -$$
$$440r^3s^5t^3 - 440r^3s^5t^2 + 2970r^3s^4t^3 - 17r^2s^{10}t - 17r^2s^{10} + 64r^2s^9t^2 + 100r^2s^9t +$$
$$64r^2s^9 - 66r^2s^8t^3 - 224r^2s^8t^2 - 224r^2s^8t - 66r^2s^8 + 198r^2s^7t^3 + 176r^2s^7t^2 +$$
$$198r^2s^7t + 88r^2s^6t^3 + 88r^2s^6t^2 - 726r^2s^5t^3 + 7rs^{11}t + 7rs^{11} - 24rs^{10}t^2 - 52rs^{10}t -$$
$$24rs^{10} + 22rs^9t^3 + 128rs^9t^2 + 128rs^9t + 22rs^9 - 110rs^8t^3 - 264rs^8t^2 - 110rs^8t +$$
$$220rs^7t^3 + 220rs^7t^2 - 198rs^6t^3 + 7s^{11}t - 24s^{10}t^2 - 24s^{10}t + 22s^9t^3 + 88s^9t^2 + 22s^9t -$$
$$88s^8t^3 - 88s^8t^2 + 99s^7t^3) + \frac{(x-x_n)^{12}}{660h^9r^3s^3t^3}(rs + rt + st + rst) - \frac{(x-x_n)^5}{60h^2r^2s^2t^2}(3r^2s^2t^2 + 4r^2s^2t +$$
$$3r^2s^2 + 4r^2st^2 + 4r^2st + 3r^2t^2 + 4rs^2t^2 + 4rs^2t + 4rst^2 + 3s^2t^2) + \frac{(x-x_n)^8}{168h^5r^3s^3t^3}(r^3s^3t +$$

$$r^3s^3 + 4r^3s^2t^2 + 8r^3s^2t + 4r^3s^2 + r^3st^3 + 8r^3st^2 + 8r^3st + r^3s + r^3t^3 + 4r^3t^2 + r^3t + 4r^2s^3t^2 + 8r^2s^3t + 4r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 + 24r^2s^2t + 4r^2s^2 + 8r^2st^3 + 24r^2st^2 + 8r^2st + 4r^2t^3 + 4r^2t^2 + rs^3t^3 + 8rs^3t^2 + 8rs^3t + rs^3 + 8rs^2t^3 + 24rs^2t^2 + 8rs^2t + 8rst^3 + 8rst^2 + rt^3 + s^3t^3 + 4s^3t^2 + s^3t + 4s^2t^3 + 4s^2t^2 + st^3) - \frac{(x-x_n)^{11}}{990h^8r^3s^3t^3}(4r^2st + 4r^2s + 4r^2t + 4rs^2t + 4rs^2 + 4rst^2 + 15rst + 4rs + 4rt^2 + 4rt + 4s^2t + 4st^2 + 4st) + \frac{h^2(x-x_n)}{27720r^2s^2t^3}(7r^{10}st + 7r^{10}s + 7r^{10}t - 17r^9s^2t - 17r^9s^2 - 24r^9st^2 - 52r^9st - 24r^9s - 24r^9t^2 - 24r^9t + 5r^8s^3t + 5r^8s^3 + 64r^8s^2t^2 + 100r^8s^2t + 64r^8s^2 + 22r^8st^3 + 128r^8st^2 + 128r^8st + 22r^8s + 22r^8t^3 + 88r^8t^2 + 22r^8t + 5r^7s^4t + 5r^7s^4 - 24r^7s^3t^2 - 32r^7s^3t - 24r^7s^3 - 66r^7s^2t^3 - 224r^7s^2t^2 - 224r^7s^2t - 66r^7s^2 - 110r^7st^3 - 264r^7st^2 - 110r^7st - 88r^7t^3 - 88r^7t^2 + 5r^6s^5t + 5r^6s^5 - 24r^6s^4t^2 - 32r^6s^4t - 24r^6s^4 + 33r^6s^3t^3 + 84r^6s^3t^2 + 84r^6s^3t + 33r^6s^3 + 198r^6s^2t^3 + 176r^6s^2t^2 + 198r^6s^2t + 220r^6st^3 + 220r^6st^2 + 99r^6t^3 + 5r^5s^6t + 5r^5s^6 - 24r^5s^5t^2 - 32r^5s^5t - 24r^5s^5 + 33r^5s^4t^3 + 84r^5s^4t^2 + 84r^5s^4t + 33r^5s^4 - 99r^5s^3t^3 + 44r^5s^3t^2 - 99r^5s^3t + 88r^5s^2t^3 + 88r^5s^2t^2 - 198r^5st^3 + 5r^4s^7t + 5r^4s^7 - 24r^4s^6t^2 - 32r^4s^6t - 24r^4s^6 + 33r^4s^5t^3 + 84r^4s^5t^2 + 84r^4s^5t + 33r^4s^5 - 99r^4s^4t^3 + 44r^4s^4t^2 - 99r^4s^4t - 440r^4s^3t^3 - 440r^4s^3t^2 - 726r^4s^2t^3 + 5r^3s^8t + 5r^3s^8 - 24r^3s^7t^2 - 32r^3s^7t - 24r^3s^7 + 33r^3s^6t^3 + 84r^3s^6t^2 + 84r^3s^6t + 33r^3s^6 - 99r^3s^5t^3 + 44r^3s^5t^2 - 99r^3s^5t - 440r^3s^4t^3 - 440r^3s^4t^2 + 2970r^3s^3t^3 - 17r^2s^9t - 17r^2s^9 + 64r^2s^8t^2 + 100r^2s^8t + 64r^2s^8 - 66r^2s^7t^3 - 224r^2s^7t^2 - 224r^2s^7t - 66r^2s^7 + 198r^2s^6t^3 + 176r^2s^6t^2 + 198r^2s^6t + 88r^2s^5t^3 + 88r^2s^5t^2 - 726r^2s^4t^3 + 7rs^{10}t + 7rs^{10} - 24rs^9t^2 - 52rs^9t - 24rs^9 + 22rs^8t^3 + 128rs^8t^2 + 128rs^8t + 22rs^8 - 110rs^7t^3 - 264rs^7t^2 - 110rs^7t + 220rs^6t^3 + 220rs^6t^2 - 198rs^5t^3 + 7s^{10}t - 24s^9t^2 - 24s^9t + 22s^8t^3 + 88s^8t^2 + 22s^8t - 88s^7t^3 - 88s^7t^2 + 99s^6t^3) - \frac{(x-x_n)^7}{210h^4r^3s^3t^3}(4r^3s^3t^2 + 7r^3s^3t + 4r^3s^3 + 4r^3s^2t^3 + 20r^3s^2t^2 + 20r^3s^2t + 4r^3s^2 + 7r^3st^3 + 20r^3st^2 + 7r^3st + 4r^3t^3 + 4r^3t^2 + 4r^2s^3t^3 + 20r^2s^3t^2 + 20r^2s^3t + 4r^2s^3 + 20r^2s^2t^3 + 48r^2s^2t^2 + 20r^2s^2t + 20r^2st^3 + 20r^2st^2 + 4r^2t^3 + 7rs^3t^3 + 20rs^3t^2 + 7rs^3t + 20rs^2t^3 + 20rs^2t^2 + 7rst^3 + 4s^3t^3 + 4s^3t^2 + 4s^2t^3),$$

$$\beta_r = \frac{(x-x_n)^{11}}{990h^8r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4 + 9r^3s + 9r^3t + 9r^3 - 12r^2s^2 - 19r^2st - 19r^2s - 12r^2t^2 - 19r^2t - 12r^2 + 8rs^2t + 8rs^2 + 8rst^2 + 21rst + 8rs + 8rt^2 + 8rt - 4s^2t - 4st^2 - 4st) + \frac{h^2(x-x_n)}{27720r^2(r-s)^3(r-t)^3(r-1)^3}(84r^{12}s - 231r^{11}s^2 - 315r^{11}st - 315r^{11}s + 159r^{10}s^3 +$$

$$
895r^{10}s^2t + 895r^{10}s^2 + 390r^{10}st^2 + 1210r^{10}st + 390r^{10}s + 5r^9s^4 - 641r^9s^3t -
641r^9s^3 - 1146r^9s^2t^2 - 3583r^9s^2t - 1146r^9s^2 - 154r^9st^3 - 1536r^9st^2 - 1536r^9st -
154r^9s + 5r^8s^5 - 25r^8s^4t - 25r^8s^4 + 856r^8s^3t^2 + 2707r^8s^3t + 856r^8s^3 + 462r^8s^2t^3 +
4754r^8s^2t^2 + 4754r^8s^2t + 462r^8s^2 + 616r^8st^3 + 2002r^8st^2 + 616r^8st + 5r^7s^6 -
25r^7s^5t - 25r^7s^5 + 42r^7s^4t^2 + 133r^7s^4t + 42r^7s^4 - 352r^7s^3t^3 - 3815r^7s^3t^2 -
3815r^7s^3t - 352r^7s^3 - 1958r^7s^2t^3 - 6567r^7s^2t^2 - 1958r^7s^2t - 814r^7st^3 - 814r^7st^2 +
5r^6s^7 - 25r^6s^6t - 25r^6s^6 + 42r^6s^5t^2 + 133r^6s^5t + 42r^6s^5 - 22r^6s^4t^3 - 240r^6s^4t^2 -
240r^6s^4t - 22r^6s^4 + 1617r^6s^3t^3 + 5753r^6s^3t^2 + 1617r^6s^3t + 2761r^6s^2t^3 +
2761r^6s^2t^2 + 330r^6st^3 + 5r^5s^8 - 25r^5s^7t - 25r^5s^7 + 42r^5s^6t^2 + 133r^5s^6t +
42r^5s^6 - 22r^5s^5t^3 - 240r^5s^5t^2 - 240r^5s^5t - 22r^5s^5 + 132r^5s^4t^3 + 473r^5s^4t^2 +
132r^5s^4t - 2519r^5s^3t^3 - 2519r^5s^3t^2 - 1155r^5s^2t^3 + 5r^4s^9 - 25r^4s^8t - 25r^4s^8 +
42r^4s^7t^2 + 133r^4s^7t + 42r^4s^7 - 22r^4s^6t^3 - 240r^4s^6t^2 - 240r^4s^6t - 22r^4s^6 +
132r^4s^5t^3 + 473r^4s^5t^2 + 132r^4s^5t - 275r^4s^4t^3 - 275r^4s^4t^2 + 1089r^4s^3t^3 - 49r^3s^{10} +
173r^3s^9t + 173r^3s^9 - 156r^3s^8t^2 - 659r^3s^8t - 156r^3s^8 - 22r^3s^7t^3 + 651r^3s^7t^2 +
651r^3s^7t - 22r^3s^7 + 132r^3s^6t^3 - 715r^3s^6t^2 + 132r^3s^6t - 275r^3s^5t^3 - 275r^3s^5t^2 +
165r^3s^4t^3 + 21r^2s^{11} - 35r^2s^{10}t - 35r^2s^{10} - 68r^2s^9t^2 + r^2s^9t - 68r^2s^9 + 132r^2s^8t^3 +
376r^2s^8t^2 + 376r^2s^8t + 132r^2s^8 - 561r^2s^7t^3 - 781r^2s^7t^2 - 561r^2s^7t + 649r^2s^6t^3 +
649r^2s^6t^2 + 165r^2s^5t^3 - 14rs^{11}t - 14rs^{11} + 48rs^{10}t^2 + 71rs^{10}t + 48rs^{10} - 44rs^9t^3 -
130rs^9t^2 - 130rs^9t - 44rs^9 + 88rs^8t^3 + 88rs^8t + 187rs^7t^3 + 187rs^7t^2 - 495rs^6t^3 +
7s^{11}t - 24s^{10}t^2 - 24s^{10}t + 22s^9t^3 + 88s^9t^2 + 22s^9t - 88s^8t^3 - 88s^8t^2 + 99s^7t^3) +
\frac{(x-x_n)^7}{210h^4r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4s^2t^2 + 36r^4s^2t + 9r^4s^2 + 36r^4st^2 + 36r^4st + 9r^4t^2 -
7r^3s^3t^2 - 28r^3s^3t - 7r^3s^3 - 7r^3s^2t^3 - 47r^3s^2t^2 - 47r^3s^2t - 7r^3s^2 - 28r^3st^3 -
47r^3st^2 - 28r^3st - 7r^3t^3 - 7r^3t^2 + 5r^2s^3t^3 + 13r^2s^3t^2 + 13r^2s^3t + 5r^2s^3 + 13r^2s^2t^3 +
24r^2s^2t^2 + 13r^2s^2t + 13r^2st^3 + 13r^2st^2 + 5r^2t^3 + 5rs^3t^3 + 4rs^3t^2 + 5rs^3t + 4rs^2t^3 +
4rs^2t^2 + 5rst^3 - 4s^3t^3 - 4s^3t^2 - 4s^2t^3) - \frac{(x-x_n)^8}{168h^5r^3(r-s)^3(r-t)^3(r-1)^3}(9r^4s^2t + 9r^4s^2 +
9r^4st^2 + 36r^4st + 9r^4s + 9r^4t^2 + 9r^4t - 7r^3s^3t - 7r^3s^3 - 10r^3s^2t^2 - 26r^3s^2t - 10r^3s^2 -
7r^3st^3 - 26r^3st^2 - 26r^3st - 7r^3s - 7r^3t^3 - 10r^3t^2 - 7r^3t + 2r^2s^3t^2 - 2r^2s^3t + 2r^2s^3 +
2r^2s^2t^3 + 3r^2s^2t^2 + 3r^2s^2t + 2r^2s^2 - 2r^2st^3 + 3r^2st^2 - 2r^2st + 2r^2t^3 + 2r^2t^2 + 2rs^3t^3 +
7rs^3t^2 + 7rs^3t + 2rs^3 + 7rs^2t^3 + 12rs^2t^2 + 7rs^2t + 7rst^3 + 7rst^2 + 2rt^3 - s^3t^3 - 4s^3t^2
$$

$$
-s^3t-4s^2t^3-4s^2t^2-st^3)-\frac{(x-x_n)^9}{504h^6r^3(r-s)^3(r-t)^3(r-1)^3}(7r^3s^3-36r^4st-36r^4s-9r^4t^2-
36r^4t-9r^4-9r^4s^2+19r^3s^2t+19r^3s^2+19r^3st^2+20r^3st+19r^3s+7r^3t^3+19r^3t^2+
19r^3t+7r^3+7r^2s^3t+7r^2s^3+4r^2s^2t^2+27r^2s^2t+4r^2s^2+7r^2st^3+27r^2st^2+27r^2st+
7r^2s+7r^2t^3+4r^2t^2+7r^2t-8rs^3t^2-13rs^3t-8rs^3-8rs^2t^3-36rs^2t^2-36rs^2t-
8rs^2-13rst^3-36rst^2-13rst-8rt^3-8rt^2+4s^3t^2+4s^3t+4s^2t^3+16s^2t^2+
4s^2t+4st^3+4st^2)-\frac{h(x-x_n)^2}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(84r^{13}-231r^{12}s-315r^{12}t-315r^{12}+
159r^{11}s^2+895r^{11}st+895r^{11}s+390r^{11}t^2+1210r^{11}t+390r^{11}+5r^{10}s^3-641r^{10}s^2t-
641r^{10}s^2-1146r^{10}st^2-3583r^{10}st-1146r^{10}s-154r^{10}t^3-1536r^{10}t^2-1536r^{10}t-
154r^{10}+5r^9s^4-25r^9s^3t-25r^9s^3+856r^9s^2t^2+2707r^9s^2t+856r^9s^2+462r^9st^3+
4754r^9st^2+4754r^9st+462r^9s+616r^9t^3+2002r^9t^2+616r^9t+5r^8s^5-25r^8s^4t-
25r^8s^4+42r^8s^3t^2+133r^8s^3t+42r^8s^3-352r^8s^2t^3-3815r^8s^2t^2-3815r^8s^2t-
352r^8s^2-1958r^8st^3-6567r^8st^2-1958r^8st-814r^8t^3-814r^8t^2+5r^7s^6-25r^7s^5t-
25r^7s^5+42r^7s^4t^2+133r^7s^4t+42r^7s^4-22r^7s^3t^3-240r^7s^3t^2-240r^7s^3t-22r^7s^3+
1617r^7s^2t^3+5753r^7s^2t^2+1617r^7s^2t+2761r^7st^3+2761r^7st^2+330r^7t^3+5r^6s^7-
25r^6s^6t-25r^6s^6+42r^6s^5t^2+133r^6s^5t+42r^6s^5-22r^6s^4t^3-240r^6s^4t^2-240r^6s^4t-
22r^6s^4+132r^6s^3t^3+473r^6s^3t^2+132r^6s^3t-2519r^6s^2t^3-2519r^6s^2t^2-1155r^6st^3+
5r^5s^8-25r^5s^7t-25r^5s^7+42r^5s^6t^2+133r^5s^6t+42r^5s^6-22r^5s^5t^3-240r^5s^5t^2-
240r^5s^5t-22r^5s^5+132r^5s^4t^3+473r^5s^4t^2+132r^5s^4t-275r^5s^3t^3-275r^5s^3t^2+
1089r^5s^2t^3+5r^4s^9-25r^4s^8t-25r^4s^8+42r^4s^7t^2+133r^4s^7t+42r^4s^7-22r^4s^6t^3-
240r^4s^6t^2-240r^4s^6t-22r^4s^6+132r^4s^5t^3+473r^4s^5t^2+132r^4s^5t-275r^4s^4t^3-
275r^4s^4t^2+165r^4s^3t^3-49r^3s^{10}+173r^3s^9t+173r^3s^9-156r^3s^8t^2-659r^3s^8t-
156r^3s^8-22r^3s^7t^3+651r^3s^7t^2+651r^3s^7t-22r^3s^7+132r^3s^6t^3-715r^3s^6t^2+
132r^3s^6t-275r^3s^5t^3-275r^3s^5t^2+165r^3s^4t^3+21r^2s^{11}-35r^2s^{10}t-35r^2s^{10}-
68r^2s^9t^2+r^2s^9t-68r^2s^9+132r^2s^8t^3+376r^2s^8t^2+376r^2s^8t+132r^2s^8-561r^2s^7t^3-
781r^2s^7t^2-561r^2s^7t+649r^2s^6t^3+649r^2s^6t^2+165r^2s^5t^3-14rs^{11}t-14rs^{11}+
48rs^{10}t^2+71rs^{10}t+48rs^{10}-44rs^9t^3-130rs^9t^2-130rs^9t-44rs^9+88rs^8t^3+
88rs^8t+187rs^7t^3+187rs^7t^2-495rs^6t^3+7s^{11}t-24s^{10}t^2-24s^{10}t+22s^9t^3+88s^9t^2+
22s^9t-88s^8t^3-88s^8t^2+99s^7t^3)+\frac{(x-x_n)^{10}}{360h^7r^3(r-s)^3(r-t)^3(r-1)^3}(3r^3s^2-9r^4t-9r^4-
9r^4s-2r^3st-2r^3s+3r^3t^2-2r^3t+3r^3+3r^2s^3+10r^2s^2t+10r^2s^2+10r^2st^2+21r^2
$$

$st+10r^2s+3r^2t^3+10r^2t^2+10r^2t+3r^2-2rs^3t-2rs^3-8rs^2t^2-15rs^2t-8rs^2-2rst^3-15rst^2-15rst-2rs-2rt^3-8rt^2-2rt+s^3t+4s^2t^2+4s^2t+st^3+4st^2+st)-\frac{(x-x_n)^{12}}{660h^9r^3(r-s)^3(r-t)^3(r-1)^3}(2rs+2rt-st-3r^2s-3r^2t-3r^2+4r^3+2rst)+\frac{s^2t^2(x-x_n)^5}{60h^2r^2(r-s)^3(r-t)^3(r-1)^3}(5rs+5rt-3st-7r^2s-7r^2t-7r^2+9r^3+5rst)+\frac{st(x-x_n)^6}{60h^3r^3(r-s)^3(r-t)^3(r-1)^3}(7r^3s^2t-9r^4s-9r^4t-9r^4st+7r^3s^2+7r^3st^2+17r^3st+7r^3s+7r^3t^2+7r^3t-5r^2s^2t^2-7r^2s^2t-5r^2s^2-7r^2st^2-7r^2st-5r^2t^2+rs^2t^2+rs^2t+rst^2+s^2t^2),$

$\beta_s=\frac{(x-x_n)^8}{168h^5s^3(r-s)^3(s-t)^3(s-1)^3}(2r^3s^2t^2-7r^3s^3-7r^3s^3t-2r^3s^2t+2r^3s^2+2r^3st^3+7r^3st^2+7r^3st+2r^3s-r^3t^3-4r^3t^2-r^3t+9r^2s^4t+9r^2s^4-10r^2s^3t^2-26r^2s^3t-10r^2s^3+2r^2s^2t^3+3r^2s^2t^2+3r^2s^2t+2r^2s^2+7r^2st^3+12r^2st^2+7r^2st-4r^2t^3-4r^2t^2+9rs^4t^2+36rs^4t+9rs^4-7rs^3t^3-26rs^3t^2-26rs^3t-7rs^3-2rs^2t^3+3rs^2t^2-2rs^2t+7rst^3+7rst^2-rt^3+9s^4t^2+9s^4t-7s^3t^3-10s^3t^2-7s^3t+2s^2t^3+2s^2t^2+2st^3)-\frac{(x-x_n)^{11}}{990h^8s^3(r-s)^3(s-t)^3(s-1)^3}(8r^2st-12r^2s^2+8r^2s-4r^2t+9rs^3-19rs^2t-19rs^2+8rst^2+21rst+8rs-4rt^2-4rt+9s^4+9s^3t+9s^3-12s^2t^2-19s^2t-12s^2+8st^2+8st)-\frac{(x-x_n)^7}{210h^4s^3(r-s)^3(s-t)^3(s-1)^3}(5r^3s^2t^3-28r^3s^3t-7r^3s^3-7r^3s^3t^2+13r^3s^2t^2+13r^3s^2t+5r^3s^2+5r^3st^3+4r^3st^2+5r^3st-4r^3t^3-4r^3t^2+9r^2s^4t^2+36r^2s^4t+9r^2s^4-7r^2s^3t^3-47r^2s^3t^2-47r^2s^3t-7r^2s^3+13r^2s^2t^3+24r^2s^2t^2+13r^2s^2t+4r^2st^3+4r^2st^2-4r^2t^3+36rs^4t^2+36rs^4t-28rs^3t^3-47rs^3t^2-28rs^3t+13rs^2t^3+13rs^2t^2+5rst^3+9s^4t^2-7s^3t^3-7s^3t^2+5s^2t^3)-\frac{h^2(x-x_n)}{27720s^2(r-s)^3(s-t)^3(s-1)^3}(21r^{11}s^2-14r^{11}st-14r^{11}s+7r^{11}t-49r^{10}s^3-35r^{10}s^2t-35r^{10}s^2+48r^{10}st^2+71r^{10}st+48r^{10}s-24r^{10}t^2-24r^{10}t+5r^9s^4+173r^9s^3t+173r^9s^3-68r^9s^2t^2+r^9s^2t-68r^9s^2-44r^9st^3-130r^9st^2-130r^9st-44r^9s+22r^9t^3+88r^9t^2+22r^9t+5r^8s^5-25r^8s^4t-25r^8s^4-156r^8s^3t^2-659r^8s^3t-156r^8s^3+132r^8s^2t^3+376r^8s^2t^2+376r^8s^2t+132r^8s^2+88r^8st^3+88r^8st-88r^8t^3-88r^8t^2+5r^7s^6-25r^7s^5t-25r^7s^5+42r^7s^4t^2+133r^7s^4t+42r^7s^4-22r^7s^3t^3+651r^7s^3t^2+651r^7s^3t-22r^7s^3-561r^7s^2t^3-781r^7s^2t^2-561r^7s^2t+187r^7st^3+187r^7st^2+99r^7t^3+5r^6s^7-25r^6s^6t-25r^6s^6+42r^6s^5t^2+133r^6s^5t+42r^6s^5-22r^6s^4t^3-240r^6s^4t^2-240r^6s^4t-22r^6s^4+132r^6s^3t^3-715r^6s^3t^2+132r^6s^3t+649r^6s^2t^3+649r^6s^2t^2-495r^6st^3+5r^5s^8-25r^5$

$$s^7t - 25r^5s^7 + 42r^5s^6t^2 + 133r^5s^6t + 42r^5s^6 - 22r^5s^5t^3 - 240r^5s^5t^2 - 240r^5s^5t -$$
$$22r^5s^5 + 132r^5s^4t^3 + 473r^5s^4t^2 + 132r^5s^4t - 275r^5s^3t^3 - 275r^5s^3t^2 + 165r^5s^2t^3 +$$
$$5r^4s^9 - 25r^4s^8t - 25r^4s^8 + 42r^4s^7t^2 + 133r^4s^7t + 42r^4s^7 - 22r^4s^6t^3 - 240r^4s^6t^2 -$$
$$240r^4s^6t - 22r^4s^6 + 132r^4s^5t^3 + 473r^4s^5t^2 + 132r^4s^5t - 275r^4s^4t^3 - 275r^4s^4t^2 +$$
$$165r^4s^3t^3 + 159r^3s^{10} - 641r^3s^9t - 641r^3s^9 + 856r^3s^8t^2 + 2707r^3s^8t + 856r^3s^8 -$$
$$352r^3s^7t^3 - 3815r^3s^7t^2 - 3815r^3s^7t - 352r^3s^7 + 1617r^3s^6t^3 + 5753r^3s^6t^2 +$$
$$1617r^3s^6t - 2519r^3s^5t^3 - 2519r^3s^5t^2 + 1089r^3s^4t^3 - 231r^2s^{11} + 895r^2s^{10}t +$$
$$895r^2s^{10} - 1146r^2s^9t^2 - 3583r^2s^9t - 1146r^2s^9 + 462r^2s^8t^3 + 4754r^2s^8t^2 +$$
$$4754r^2s^8t + 462r^2s^8 - 1958r^2s^7t^3 - 6567r^2s^7t^2 - 1958r^2s^7t + 2761r^2s^6t^3 +$$
$$2761r^2s^6t^2 - 1155r^2s^5t^3 + 84rs^{12} - 315rs^{11}t - 315rs^{11} + 390rs^{10}t^2 + 1210rs^{10}t +$$
$$390rs^{10} - 154rs^9t^3 - 1536rs^9t^2 - 1536rs^9t - 154rs^9 + 616rs^8t^3 + 2002rs^8t^2 +$$
$$616rs^8t - 814rs^7t^3 - 814rs^7t^2 + 330rs^6t^3) + \frac{(x-x_n)^9}{504h^6s^3(r-s)^3(s-t)^3(s-1)^3}(7r^3s^3 + 7r^3s^2t +$$
$$7r^3s^2 - 8r^3st^2 - 13r^3st - 8r^3s + 4r^3t^2 + 4r^3t - 9r^2s^4 + 19r^2s^3t + 19r^2s^3 + 4r^2s^2t^2 +$$
$$27r^2s^2t + 4r^2s^2 - 8r^2st^3 - 36r^2st^2 - 36r^2st - 8r^2s + 4r^2t^3 + 16r^2t^2 + 4r^2t -$$
$$36rs^4t - 36rs^4 + 19rs^3t^2 + 20rs^3t + 19rs^3 + 7rs^2t^3 + 27rs^2t^2 + 27rs^2t + 7rs^2 -$$
$$13rst^3 - 36rst^2 - 13rst + 4rt^3 + 4rt^2 - 9s^4t^2 - 36s^4t - 9s^4 + 7s^3t^3 + 19s^3t^2 +$$
$$19s^3t + 7s^3 + 7s^2t^3 + 4s^2t^2 + 7s^2t - 8st^3 - 8st^2) + \frac{h(x-x_n)^2}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(21r^{11}s^2 -$$
$$14r^{11}st - 14r^{11}s + 7r^{11}t - 49r^{10}s^3 - 35r^{10}s^2t - 35r^{10}s^2 + 48r^{10}st^2 + 71r^{10}st +$$
$$48r^{10}s - 24r^{10}t^2 - 24r^{10}t + 5r^9s^4 + 173r^9s^3t + 173r^9s^3 - 68r^9s^2t^2 + r^9s^2t -$$
$$68r^9s^2 - 44r^9st^3 - 130r^9st^2 - 130r^9st - 44r^9s + 22r^9t^3 + 88r^9t^2 + 22r^9t + 5r^8s^5 -$$
$$25r^8s^4t - 25r^8s^4 - 156r^8s^3t^2 - 659r^8s^3t - 156r^8s^3 + 132r^8s^2t^3 + 376r^8s^2t^2 +$$
$$376r^8s^2t + 132r^8s^2 + 88r^8st^3 + 88r^8st - 88r^8t^3 - 88r^8t^2 + 5r^7s^6 - 25r^7s^5t -$$
$$25r^7s^5 + 42r^7s^4t^2 + 133r^7s^4t + 42r^7s^4 - 22r^7s^3t^3 + 651r^7s^3t^2 + 651r^7s^3t - 22r^7s^3 -$$
$$561r^7s^2t^3 - 781r^7s^2t^2 - 561r^7s^2t + 187r^7st^3 + 187r^7st^2 + 99r^7t^3 + 5r^6s^7 - 25r^6s^6t -$$
$$25r^6s^6 + 42r^6s^5t^2 + 133r^6s^5t + 42r^6s^5 - 22r^6s^4t^3 - 240r^6s^4t^2 - 240r^6s^4t - 22r^6s^4 +$$
$$132r^6s^3t^3 - 715r^6s^3t^2 + 132r^6s^3t + 649r^6s^2t^3 + 649r^6s^2t^2 - 495r^6st^3 + 5r^5s^8 -$$
$$25r^5s^7t - 25r^5s^7 + 42r^5s^6t^2 + 133r^5s^6t + 42r^5s^6 - 22r^5s^5t^3 - 240r^5s^5t^2 - 240r^5s^5t -$$
$$22r^5s^5 + 132r^5s^4t^3 + 473r^5s^4t^2 + 132r^5s^4t - 275r^5s^3t^3 - 275r^5s^3t^2 + 165r^5s^2t^3 +$$
$$5r^4s^9 - 25r^4s^8t - 25r^4s^8 + 42r^4s^7t^2 + 133r^4s^7t + 42r^4s^7 - 22r^4s^6t^3 - 240r^4s^6t^2 -$$

$240r^4s^6t - 22r^4s^6 + 132r^4s^5t^3 + 473r^4s^5t^2 + 132r^4s^5t - 275r^4s^4t^3 - 275r^4s^4t^2 + 165r^4s^3t^3 + 5r^3s^{10} - 25r^3s^9t - 25r^3s^9 + 42r^3s^8t^2 + 133r^3s^8t + 42r^3s^8 - 22r^3s^7t^3 - 240r^3s^7t^2 - 240r^3s^7t - 22r^3s^7 + 132r^3s^6t^3 + 473r^3s^6t^2 + 132r^3s^6t - 275r^3s^5t^3 - 275r^3s^5t^2 + 165r^3s^4t^3 + 159r^2s^{11} - 641r^2s^{10}t - 641r^2s^{10} + 856r^2s^9t^2 + 2707r^2s^9t + 856r^2s^9 - 352r^2s^8t^3 - 3815r^2s^8t^2 - 3815r^2s^8t - 352r^2s^8 + 1617r^2s^7t^3 + 5753r^2s^7t^2 + 1617r^2s^7t - 2519r^2s^6t^3 - 2519r^2s^6t^2 + 1089r^2s^5t^3 - 231rs^{12} + 895rs^{11}t + 895rs^{11} - 1146rs^{10}t^2 - 3583rs^{10}t - 1146rs^{10} + 462rs^9t^3 + 4754rs^9t^2 + 4754rs^9t + 462rs^9 - 1958rs^8t^3 - 6567rs^8t^2 - 1958rs^8t + 2761rs^7t^3 + 2761rs^7t^2 - 1155rs^6t^3 + 84s^13 - 315s^{12}t - 315s^{12} + 390s^{11}t^2 + 1210s^{11}t + 390s^{11} - 154s^{10}t^3 - 1536s^{10}t^2 - 1536s^{10}t - 154s^{10} + 616s^9t^3 + 2002s^9t^2 + 616s^9t - 814s^8t^3 - 814s^8t^2 + 330s^7t^3) - \frac{(x-x_n)^{10}}{360h^7s^3(r-s)^3(s-t)^3(s-1)^3}(3r^3s^2 - 2r^3st - 2r^3s + r^3t + 3r^2s^3 + 10r^2s^2t + 10r^2s^2 - 8r^2st^2 - 15r^2st - 8r^2s + 4r^2t^2 + 4r^2t - 9rs^4 - 2rs^3t - 2rs^3 + 10rs^2t^2 + 21rs^2t + 10rs^2 - 2rst^3 - 15rst^2 - 15rst - 2rs + rt^3 + 4rt^2 + rt - 9s^4t - 9s^4 + 3s^3t^2 - 2s^3t + 3s^3 + 3s^2t^3 + 10s^2t^2 + 10s^2t + 3s^2 - 2st^3 - 8st^2 - 2st) + \frac{(x-x_n)^{12}}{660h^9s^3(r-s)^3(s-t)^3(s-1)^3}(2rs - rt + 2st - 3rs^2 - 3s^2t - 3s^2 + 4s^3 + 2rst) - \frac{r^2t^2(x-x_n)^5}{60h^2s^2(r-s)^3(s-t)^3(s-1)^3}(5rs - 3rt + 5st - 7rs^2 - 7s^2t - 7s^2 + 9s^3 + 5rst) - \frac{rt(x-x_n)^6}{60h^3s^3(r-s)^3(s-t)^3(s-1)^3}(7r^2s^3t + 7r^2s^3 - 5r^2s^2t^2 - 7r^2s^2t - 5r^2s^2 + r^2st^2 + r^2st + r^2t^2 - 9rs^4t - 9rs^4 + 7rs^3t^2 + 17rs^3t + 7rs^3 - 7rs^2t^2 - 7rs^2t + rst^2 - 9s^4t + 7s^3t^2 + 7s^3t - 5s^2t^2),$

$\beta_t = \frac{(x-x_n)^{11}}{990h^8t^3(r-t)^3(s-t)^3(t-1)^3}(8r^2st - 4r^2s - 12r^2t^2 + 8r^2t + 8rs^2t - 4rs^2 - 19rst^2 + 21rst - 4rs + 9rt^3 - 19rt^2 + 8rt - 12s^2t^2 + 8s^2t + 9st^3 - 19st^2 + 8st + 9t^4 + 9t^3 - 12t^2) + \frac{(x-x_n)^7}{210h^4t^3(r-t)^3(s-t)^3(t-1)^3}(5r^3s^3t^2 + 5r^3s^3t - 4r^3s^3 - 7r^3s^2t^3 + 13r^3s^2t^2 + 4r^3s^2t - 4r^3s^2 - 28r^3st^3 + 13r^3st^2 + 5r^3st - 7r^3t^3 + 5r^3t^2 - 7r^2s^3t^3 + 13r^2s^3t^2 + 4r^2s^3t - 4r^2s^3 + 9r^2s^2t^4 - 47r^2s^2t^3 + 24r^2s^2t^2 + 4r^2s^2t + 36r^2st^4 - 47r^2st^3 + 13r^2st^2 + 9r^2t^4 - 7r^2t^3 - 28rs^3t^3 + 13rs^3t^2 + 5rs^3t + 36rs^2t^4 - 47rs^2t^3 + 13rs^2t^2 + 36rst^4 - 28rst^3 - 7s^3t^3 + 5s^3t^2 + 9s^2t^4 - 7s^2t^3) - \frac{(x-x_n)^8}{168h^5t^3(r-t)^3(s-t)^3(t-1)^3}(2r^3s^3t - r^3s^3 + 2r^3s^2t^2 + 7r^3s^2t - 4r^3s^2 - 7r^3st^3 - 2r^3st^2 + 7r^3st - r^3s - 7r^3t^3 + 2r^3t^2 + 2r^3t + 2r^2s^3t^2 + 7r^2s^3t - 4r^2s^3 - 10r^2s^2t^3 + 3r^2s^2t^2 + 12r^2s^2t - 4r^2s^2 + 9r^2st^4 - 26r^2st^3 + 3r^2st^2 + 7r^2st + 9r^2t^4 - 10r^2t^3 + 2r^2t^2 - 7rs^3t^3 - 2rs^3t^2 + 7rs^3t - rs^3 + 9rs^2t^4 -$

$$26rs^2t^3 + 3rs^2t^2 + 7rs^2t + 36rst^4 - 26rst^3 - 2rst^2 + 9rt^4 - 7rt^3 - 7s^3t^3 + 2s^3t^2 +$$
$$2s^3t + 9s^2t^4 - 10s^2t^3 + 2s^2t^2 + 9st^4 - 7st^3) - \frac{h(x-x_n)^2}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(7r^{11}s - 14r^{11}st +$$
$$21r^{11}t^2 - 14r^{11}t + 34r^{10}s^2t - 17r^{10}s^2 + 50r^{10}st - 24r^{10}s - 70r^{10}t^3 - 21r^{10}t^2 +$$
$$48r^{10}t - 10r^9s^3t + 5r^9s^3 - 116r^9s^2t^2 - 56r^9s^2t + 64r^9s^2 + 138r^9st^3 - 91r^9st^2 -$$
$$58r^9st + 22r^9s + 54r^9t^4 + 208r^9t^3 - 116r^9t^2 - 44r^9t - 10r^8s^4t + 5r^8s^4 + 60r^8s^3t^2 +$$
$$10r^8s^3t - 24r^8s^3 + 50r^8s^2t^3 + 415r^8s^2t^2 - 146r^8s^2t - 66r^8s^2 - 144r^8st^4 - 452r^8st^3 +$$
$$390r^8st^2 + 22r^8st - 198r^8t^4 - 88r^8t^3 + 176r^8t^2 - 10r^7s^5t + 5r^7s^5 + 60r^7s^4t^2 +$$
$$10r^7s^4t - 24r^7s^4 - 104r^7s^3t^3 - 234r^7s^3t^2 + 129r^7s^3t + 33r^7s^3 + 54r^7s^2t^4 -$$
$$177r^7s^2t^3 - 391r^7s^2t^2 + 297r^7s^2t + 594r^7st^4 + 187r^7st^3 - 473r^7st^2 + 198r^7t^4 -$$
$$154r^7t^3 - 10r^6s^6t + 5r^6s^6 + 60r^6s^5t^2 + 10r^6s^5t - 24r^6s^5 - 104r^6s^4t^3 - 234r^6s^4t^2 +$$
$$129r^6s^4t + 33r^6s^4 + 54r^6s^3t^4 + 516r^6s^3t^3 + 71r^6s^3t^2 - 297r^6s^3t - 297r^6s^2t^4 +$$
$$253r^6s^2t^3 - 11r^6s^2t^2 - 693r^6st^4 + 539r^6st^3 - 10r^5s^7t + 5r^5s^7 + 60r^5s^6t^2 + 10r^5s^6t -$$
$$24r^5s^6 - 104r^5s^5t^3 - 234r^5s^5t^2 + 129r^5s^5t + 33r^5s^5 + 54r^5s^4t^4 + 516r^5s^4t^3 +$$
$$71r^5s^4t^2 - 297r^5s^4t - 297r^5s^3t^4 - 671r^5s^3t^3 + 649r^5s^3t^2 + 495r^5s^2t^4 - 385r^5s^2t^3 -$$
$$10r^4s^8t + 5r^4s^8 + 60r^4s^7t^2 + 10r^4s^7t - 24r^4s^7 - 104r^4s^6t^3 - 234r^4s^6t^2 + 129r^4s^6t +$$
$$33r^4s^6 + 54r^4s^5t^4 + 516r^4s^5t^3 + 71r^4s^5t^2 - 297r^4s^5t - 297r^4s^4t^4 - 671r^4s^4t^3 +$$
$$649r^4s^4t^2 + 495r^4s^3t^4 - 385r^4s^3t^3 - 10r^3s^9t + 5r^3s^9 + 60r^3s^8t^2 + 10r^3s^8t - 24r^3s^8 -$$
$$104r^3s^7t^3 - 234r^3s^7t^2 + 129r^3s^7t + 33r^3s^7 + 54r^3s^6t^4 + 516r^3s^6t^3 + 71r^3s^6t^2 -$$
$$297r^3s^6t - 297r^3s^5t^4 - 671r^3s^5t^3 + 649r^3s^5t^2 + 495r^3s^4t^4 - 385r^3s^4t^3 + 34r^2s^{10}t -$$
$$17r^2s^{10} - 116r^2s^9t^2 - 56r^2s^9t + 64r^2s^9 + 50r^2s^8t^3 + 415r^2s^8t^2 - 146r^2s^8t -$$
$$66r^2s^8 + 54r^2s^7t^4 - 177r^2s^7t^3 - 391r^2s^7t^2 + 297r^2s^7t - 297r^2s^6t^4 + 253r^2s^6t^3 -$$
$$11r^2s^6t^2 + 495r^2s^5t^4 - 385r^2s^5t^3 - 14rs^{11}t + 7rs^{11} + 50rs^{10}t - 24rs^{10} + 138rs^9t^3 -$$
$$91rs^9t^2 - 58rs^9t + 22rs^9 - 144rs^8t^4 - 452rs^8t^3 + 390rs^8t^2 + 22rs^8t + 594rs^7t^4 +$$
$$187rs^7t^3 - 473rs^7t^2 - 693rs^6t^4 + 539rs^6t^3 + 21s^{11}t^2 - 14s^{11}t - 70s^{10}t^3 - 21s^{10}t^2 +$$
$$48s^{10}t + 54s^9t^4 + 208s^9t^3 - 116s^9t^2 - 44s^9t - 198s^8t^4 - 88s^8t^3 + 176s^8t^2 + 198s^7t^4 -$$
$$154s^7t^3) + \frac{h^2(x-x_n)}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(7r^{11}s^2 - 14r^{11}s^2t + 21r^{11}st^2 - 14r^{11}st + 34r^{10}s^3t -$$
$$17r^{10}s^3 + 50r^{10}s^2t - 24r^{10}s^2 - 70r^{10}st^3 - 21r^{10}st^2 + 48r^{10}st - 10r^9s^4t + 5r^9s^4 -$$
$$116r^9s^3t^2 - 56r^9s^3t + 64r^9s^3 + 138r^9s^2t^3 - 91r^9s^2t^2 - 58r^9s^2t + 22r^9s^2 + 54r^9st^4 +$$
$$208r^9st^3 - 116r^9st^2 - 44r^9st - 10r^8s^5t + 5r^8s^5 + 60r^8s^4t^2 + 10r^8s^4t - 24r^8s^4 +$$

$50r^8s^3t^3 + 415r^8s^3t^2 - 146r^8s^3t - 66r^8s^3 - 144r^8s^2t^4 - 452r^8s^2t^3 + 390r^8s^2t^2 +$
$22r^8s^2t - 198r^8st^4 - 88r^8st^3 + 176r^8st^2 - 10r^7s^6t + 5r^7s^6 + 60r^7s^5t^2 + 10r^7s^5t -$
$24r^7s^5 - 104r^7s^4t^3 - 234r^7s^4t^2 + 129r^7s^4t + 33r^7s^4 + 54r^7s^3t^4 - 177r^7s^3t^3 -$
$391r^7s^3t^2 + 297r^7s^3t + 594r^7s^2t^4 + 187r^7s^2t^3 - 473r^7s^2t^2 + 198r^7st^4 - 154r^7st^3 -$
$10r^6s^7t + 5r^6s^7 + 60r^6s^6t^2 + 10r^6s^6t - 24r^6s^6 - 104r^6s^5t^3 - 234r^6s^5t^2 + 129r^6s^5t +$
$33r^6s^5 + 54r^6s^4t^4 + 516r^6s^4t^3 + 71r^6s^4t^2 - 297r^6s^4t - 297r^6s^3t^4 + 253r^6s^3t^3 -$
$11r^6s^3t^2 - 693r^6s^2t^4 + 539r^6s^2t^3 - 10r^5s^8t + 5r^5s^8 + 60r^5s^7t^2 + 10r^5s^7t -$
$24r^5s^7 - 104r^5s^6t^3 - 234r^5s^6t^2 + 129r^5s^6t + 33r^5s^6 + 54r^5s^5t^4 + 516r^5s^5t^3 +$
$71r^5s^5t^2 - 297r^5s^5t - 297r^5s^4t^4 - 671r^5s^4t^3 + 649r^5s^4t^2 + 495r^5s^3t^4 - 385r^5s^3t^3 -$
$10r^4s^9t + 5r^4s^9 + 60r^4s^8t^2 + 10r^4s^8t - 24r^4s^8 - 104r^4s^7t^3 - 234r^4s^7t^2 + 129r^4s^7t +$
$33r^4s^7 + 54r^4s^6t^4 + 516r^4s^6t^3 + 71r^4s^6t^2 - 297r^4s^6t - 297r^4s^5t^4 - 671r^4s^5t^3 +$
$649r^4s^5t^2 + 495r^4s^4t^4 - 385r^4s^4t^3 + 34r^3s^{10}t - 17r^3s^{10} - 116r^3s^9t^2 - 56r^3s^9t +$
$64r^3s^9 + 50r^3s^8t^3 + 415r^3s^8t^2 - 146r^3s^8t - 66r^3s^8 + 54r^3s^7t^4 - 177r^3s^7t^3 -$
$391r^3s^7t^2 + 297r^3s^7t - 297r^3s^6t^4 + 253r^3s^6t^3 - 11r^3s^6t^2 + 495r^3s^5t^4 - 385r^3s^5t^3 -$
$14r^2s^{11}t + 7r^2s^{11} + 50r^2s^{10}t - 24r^2s^{10} + 138r^2s^9t^3 - 91r^2s^9t^2 - 58r^2s^9t + 22r^2s^9 -$
$144r^2s^8t^4 - 452r^2s^8t^3 + 390r^2s^8t^2 + 22r^2s^8t + 594r^2s^7t^4 + 187r^2s^7t^3 - 473r^2s^7t^2 -$
$693r^2s^6t^4 + 539r^2s^6t^3 + 21rs^{11}t^2 - 14rs^{11}t - 70rs^{10}t^3 - 21rs^{10}t^2 + 48rs^{10}t +$
$54rs^9t^4 + 208rs^9t^3 - 116rs^9t^2 - 44rs^9t - 198rs^8t^4 - 88rs^8t^3 + 176rs^8t^2 + 198rs^7t^4 -$
$154rs^7t^3) - \frac{(x-x_n)^9}{504h^6t^3(r-t)^3(s-t)^3(t-1)^3}(4r^3s^2 - 8r^3s^2t + 7r^3st^2 - 13r^3st + 4r^3s +$
$7r^3t^3 + 7r^3t^2 - 8r^3t - 8r^2s^3t + 4r^2s^3 + 4r^2s^2t^2 - 36r^2s^2t + 16r^2s^2 + 19r^2st^3 +$
$27r^2st^2 - 36r^2st + 4r^2s - 9r^2t^4 + 19r^2t^3 + 4r^2t^2 - 8r^2t + 7rs^3t^2 - 13rs^3t + 4rs^3 +$
$19rs^2t^3 + 27rs^2t^2 - 36rs^2t + 4rs^2 - 36rst^4 + 20rst^3 + 27rst^2 - 13rst - 36rt^4 +$
$19rt^3 + 7rt^2 + 7s^3t^3 + 7s^3t^2 - 8s^3t - 9s^2t^4 + 19s^2t^3 + 4s^2t^2 - 8s^2t - 36st^4 + 19st^3 +$
$7st^2 - 9t^4 + 7t^3) + \frac{(x-x_n)^{12}}{660h^9t^3(r-t)^3(s-t)^3(t-1)^3}(rs - 2rt - 2st + 3rt^2 + 3st^2 + 3t^2 - 4t^3 -$
$2rst) + \frac{(x-x_n)^{10}}{360h^7t^3(r-t)^3(s-t)^3(t-1)^3}(r^3s - 2r^3st + 3r^3t^2 - 2r^3t - 8r^2s^2t + 4r^2s^2 + 10r^2st^2 -$
$15r^2st + 4r^2s + 3r^2t^3 + 10r^2t^2 - 8r^2t - 2rs^3t + rs^3 + 10rs^2t^2 - 15rs^2t + 4rs^2 - 2rst^3 +$
$21rst^2 - 15rst + rs - 9rt^4 - 2rt^3 + 10rt^2 - 2rt + 3s^3t^2 - 2s^3t + 3s^2t^3 + 10s^2t^2 - 8s^2t -$
$9st^4 - 2st^3 + 10st^2 - 2st - 9t^4 + 3t^3 + 3t^2) - \frac{(r^2s^2(x-x_n)^5}{60h^2t^2(r-t)^3(s-t)^3(t-1)^3}(3rs - 5rt - 5st +$
$7rt^2 + 7st^2 + 7t^2 - 9t^3 - 5rst) + \frac{rs(x-x_n)^6}{60h^3t^3(r-t)^3(s-t)^3(t-1)^3}(r^2s^2t - 5r^2s^2t^2 + r^2s^2 + 7r^2st^3$

$-7r^2st^2 + r^2st + 7r^2t^3 - 5r^2t^2 + 7rs^2t^3 - 7rs^2t^2 + rs^2t - 9rst^4 + 17rst^3 - 7rst^2 - 9rt^4 + 7rt^3 + 7s^2t^3 - 5s^2t^2 - 9st^4 + 7st^3)$

$\beta_1 = \frac{(x-x_n)^8}{168h^5(r-1)^3(s-1)^3(t-1)^3}(2r^3s^3 - r^3s^3t - 4r^3s^2t^2 + 7r^3s^2t + 2r^3s^2 - r^3st^3 + 7r^3st^2 - 2r^3st - 7r^3s + 2r^3t^3 + 2r^3t^2 - 7r^3t - 4r^2s^3t^2 + 7r^2s^3t + 2r^2s^3 - 4r^2s^2t^3 + 12r^2s^2t^2 + 3r^2s^2t - 10r^2s^2 + 7r^2st^3 + 3r^2st^2 - 26r^2st + 9r^2s + 2r^2t^3 - 10r^2t^2 + 9r^2t - rs^3t^3 + 7rs^3t^2 - 2rs^3t - 7rs^3 + 7rs^2t^3 + 3rs^2t^2 - 26rs^2t + 9rs^2 - 2rst^3 - 26rst^2 + 36rst - 7rt^3 + 9rt^2 + 2s^3t^3 + 2s^3t^2 - 7s^3t + 2s^2t^3 - 10s^2t^2 + 9s^2t - 7st^3 + 9st^2) - \frac{h^2(x-x_n)}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^{11}s^2t - 14r^{11}s^2 - 14r^{11}st + 21r^{11}s - 17r^{10}s^3t + 34r^{10}s^3 - 24r^{10}s^2t^2 + 50r^{10}s^2t + 48r^{10}st^2 - 21r^{10}st - 70r^{10}s + 5r^9s^4t - 10r^9s^4 + 64r^9s^3t^2 - 56r^9s^3t - 116r^9s^3 + 22r^9s^2t^3 - 58r^9s^2t^2 - 91r^9s^2t + 138r^9s^2 - 44r^9st^3 - 116r^9st^2 + 208r^9st + 54r^9s + 5r^8s^5t - 10r^8s^5 - 24r^8s^4t^2 + 10r^8s^4t + 60r^8s^4 - 66r^8s^3t^3 - 146r^8s^3t^2 + 415r^8s^3t + 50r^8s^3 + 22r^8s^2t^3 + 390r^8s^2t^2 - 452r^8s^2t - 144r^8s^2 + 176r^8st^3 - 88r^8st^2 - 198r^8st + 5r^7s^6t - 10r^7s^6 - 24r^7s^5t^2 + 10r^7s^5t + 60r^7s^5 + 33r^7s^4t^3 + 129r^7s^4t^2 - 234r^7s^4t - 104r^7s^4 + 297r^7s^3t^3 - 391r^7s^3t^2 - 177r^7s^3t + 54r^7s^3 - 473r^7s^2t^3 + 187r^7s^2t^2 + 594r^7s^2t - 154r^7st^3 + 198r^7st^2 + 5r^6s^7t - 10r^6s^7 - 24r^6s^6t^2 + 10r^6s^6t + 60r^6s^6 + 33r^6s^5t^3 + 129r^6s^5t^2 - 234r^6s^5t - 104r^6s^5 - 297r^6s^4t^3 + 71r^6s^4t^2 + 516r^6s^4t + 54r^6s^4 - 11r^6s^3t^3 + 253r^6s^3t^2 - 297r^6s^3t + 539r^6s^2t^3 - 693r^6s^2t^2 + 5r^5s^8t - 10r^5s^8 - 24r^5s^7t^2 + 10r^5s^7t + 60r^5s^7 + 33r^5s^6t^3 + 129r^5s^6t^2 - 234r^5s^6t - 104r^5s^6 - 297r^5s^5t^3 + 71r^5s^5t^2 + 516r^5s^5t + 54r^5s^5 + 649r^5s^4t^3 - 671r^5s^4t^2 - 297r^5s^4t - 385r^5s^3t^3 + 495r^5s^3t^2 + 5r^4s^9t - 10r^4s^9 - 24r^4s^8t^2 + 10r^4s^8t + 60r^4s^8 + 33r^4s^7t^3 + 129r^4s^7t^2 - 234r^4s^7t - 104r^4s^7 - 297r^4s^6t^3 + 71r^4s^6t^2 + 516r^4s^6t + 54r^4s^6 + 649r^4s^5t^3 - 671r^4s^5t^2 - 297r^4s^5t - 385r^4s^4t^3 + 495r^4s^4t^2 - 17r^3s^{10}t + 34r^3s^{10} + 64r^3s^9t^2 - 56r^3s^9t - 116r^3s^9 - 66r^3s^8t^3 - 146r^3s^8t^2 + 415r^3s^8t + 50r^3s^8 + 297r^3s^7t^3 - 391r^3s^7t^2 - 177r^3s^7t + 54r^3s^7 - 11r^3s^6t^3 + 253r^3s^6t^2 - 297r^3s^6t - 385r^3s^5t^3 + 495r^3s^5t^2 + 7r^2s^{11}t - 14r^2s^{11} - 24r^2s^{10}t^2 + 50r^2s^{10}t + 22r^2s^9t^3 - 58r^2s^9t^2 - 91r^2s^9t + 138r^2s^9 + 22r^2s^8t^3 + 390r^2s^8t^2 - 452r^2s^8t - 144r^2s^8 - 473r^2s^7t^3 + 187r^2s^7t^2 + 594r^2s^7t + 539r^2s^6t^3 - 693r^2s^6t^2 - 14rs^{11}t + 21rs^{11} + 48rs^{10}t^2 - 21rs^{10}t - 70rs^{10} - 44rs^9t^3 - 116rs^9t^2 +$

$208rs^9t + 54rs^9 + 176rs^8t^3 - 88rs^8t^2 - 198rs^8t - 154rs^7t^3 + 198rs^7t^2) - \frac{(x-x_n)^{12}}{660h^9(r-1)^3(s-1)^3(t-1)^3}(3r + 3s + 3t - 2rs - 2rt - 2st + rst - 4) - \frac{(x-x_n)^{10}}{360h^7(r-1)^3(s-1)^3(t-1)^3}(r^3st - 2r^3s - 2r^3t + 3r^3 + 4r^2s^2t - 8r^2s^2 + 4r^2st^2 - 15r^2st + 10r^2s - 8r^2t^2 + 10r^2t + 3r^2 + rs^3t - 2rs^3 + 4rs^2t^2 - 15rs^2t + 10rs^2 + rst^3 - 15rst^2 + 21rst - 2rs - 2rt^3 + 10rt^2 - 2rt - 9r - 2s^3t + 3s^3 - 8s^2t^2 + 10s^2t + 3s^2 - 2st^3 + 10st^2 - 2st - 9s + 3t^3 + 3t^2 - 9t) + \frac{(x-x_n)^9}{504h^6(r-1)^3(s-1)^3(t-1)^3}(4r^3s^2t - 8r^3s^2 + 4r^3st^2 - 13r^3st + 7r^3s - 8r^3t^2 + 7r^3t + 7r^3 + 4r^2s^3t - 8r^2s^3 + 16r^2s^2t^2 - 36r^2s^2t + 4r^2s^2 + 4r^2st^3 - 36r^2st^2 + 27r^2st + 19r^2s - 8r^2t^3 + 4r^2t^2 + 19r^2t - 9r^2 + 4rs^3t^2 - 13rs^3t + 7rs^3 + 4rs^2t^3 - 36rs^2t^2 + 27rs^2t + 19rs^2 - 13rst^3 + 27rst^2 + 20rst - 36rs + 7rt^3 + 19rt^2 - 36rt - 8s^3t^2 + 7s^3t + 7s^3 - 8s^2t^3 + 4s^2t^2 + 19s^2t - 9s^2 + 7st^3 + 19st^2 - 36st + 7t^3 - 9t^2) - \frac{(x-x_n)^7}{210h^4(r-1)^3(s-1)^3(t-1)^3}(5r^3s^3t - 4r^3s^3t^2 + 5r^3s^3 - 4r^3s^2t^3 + 4r^3s^2t^2 + 13r^3s^2t - 7r^3s^2 + 5r^3st^3 + 13r^3st^2 - 28r^3st + 5r^3t^3 - 7r^3t^2 - 4r^2s^3t^3 + 4r^2s^3t^2 + 13r^2s^3t - 7r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 - 47r^2s^2t + 9r^2s^2 + 13r^2st^3 - 47r^2st^2 + 36r^2st - 7r^2t^3 + 9r^2t^2 + 5rs^3t^3 + 13rs^3t^2 - 28rs^3t + 13rs^2t^3 - 47rs^2t^2 + 36rs^2t - 28rst^3 + 36rst^2 + 5s^3t^3 - 7s^3t^2 - 7s^2t^3 + 9s^2t^2) + \frac{h(x-x_n)^2}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^{11}st - 14r^{11}s - 14r^{11}t + 21r^{11} - 17r^{10}s^2t + 34r^{10}s^2 - 24r^{10}st^2 + 50r^{10}st + 48r^{10}t^2 - 21r^{10}t - 70r^{10} + 5r^9s^3t - 10r^9s^3 + 64r^9s^2t^2 - 56r^9s^2t - 116r^9s^2 + 22r^9st^3 - 58r^9st^2 - 91r^9st + 138r^9s - 44r^9t^3 - 116r^9t^2 + 208r^9t + 54r^9 + 5r^8s^4t - 10r^8s^4 - 24r^8s^3t^2 + 10r^8s^3t + 60r^8s^3 - 66r^8s^2t^3 - 146r^8s^2t^2 + 415r^8s^2t + 50r^8s^2 + 22r^8st^3 + 390r^8st^2 - 452r^8st - 144r^8s + 176r^8t^3 - 88r^8t^2 - 198r^8t + 5r^7s^5t - 10r^7s^5 - 24r^7s^4t^2 + 10r^7s^4t + 60r^7s^4 + 33r^7s^3t^3 + 129r^7s^3t^2 - 234r^7s^3t - 104r^7s^3 + 297r^7s^2t^3 - 391r^7s^2t^2 - 177r^7s^2t + 54r^7s^2 - 473r^7st^3 + 187r^7st^2 + 594r^7st - 154r^7t^3 + 198r^7t^2 + 5r^6s^6t - 10r^6s^6 - 24r^6s^5t^2 + 10r^6s^5t + 60r^6s^5 + 33r^6s^4t^3 + 129r^6s^4t^2 - 234r^6s^4t - 104r^6s^4 - 297r^6s^3t^3 + 71r^6s^3t^2 + 516r^6s^3t + 54r^6s^3 - 11r^6s^2t^3 + 253r^6s^2t^2 - 297r^6s^2t + 539r^6st^3 - 693r^6st^2 + 5r^5s^7t - 10r^5s^7 - 24r^5s^6t^2 + 10r^5s^6t + 60r^5s^6 + 33r^5s^5t^3 + 129r^5s^5t^2 - 234r^5s^5t - 104r^5s^5 - 297r^5s^4t^3 + 71r^5s^4t^2 + 516r^5s^4t + 54r^5s^4 + 649r^5s^3t^3 - 671r^5s^3t^2 - 297r^5s^3t - 385r^5s^2t^3 + 495r^5s^2t^2 + 5r^4s^8t - 10r^4s^8 - 24r^4s^7t^2 + 10r^4s^7t + 60r^4s^7 + 33r^4s^6t^3 + 129r^4s^6t^2 - 234r^4s^6t - 104r^4s^6 - 297r^4s^5t^3 + 71r^4s^5t^2 + 516r^4s^5t + 54r^4s^5 + 649r^4s^4t^3 - 671r^4s^4t^2 - 297r^4s^4t -$

$385r^4s^3t^3+495r^4s^3t^2+5r^3s^9t-10r^3s^9-24r^3s^8t^2+10r^3s^8t+60r^3s^8+33r^3s^7t^3+129r^3s^7t^2-234r^3s^7t-104r^3s^7-297r^3s^6t^3+71r^3s^6t^2+516r^3s^6t+54r^3s^6+649r^3s^5t^3-671r^3s^5t^2-297r^3s^5t-385r^3s^4t^3+495r^3s^4t^2-17r^2s^{10}t+34r^2s^{10}+64r^2s^9t^2-56r^2s^9t-116r^2s^9-66r^2s^8t^3-146r^2s^8t^2+415r^2s^8t+50r^2s^8+297r^2s^7t^3-391r^2s^7t^2-177r^2s^7t+54r^2s^7-11r^2s^6t^3+253r^2s^6t^2-297r^2s^6t-385r^2s^5t^3+495r^2s^5t^2+7rs^{11}t-14rs^{11}-24rs^{10}t^2+50rs^{10}t+22rs^9t^3-58rs^9t^2-91rs^9t+138rs^9+22rs^8t^3+390rs^8t^2-452rs^8t-144rs^8-473rs^7t^3+187rs^7t^2+594rs^7t+539rs^6t^3-693rs^6t^2-14s^{11}t+21s^{11}+48s^{10}t^2-21s^{10}t-70s^{10}-44s^9t^3-116s^9t^2+208s^9t+54s^9+176s^8t^3-88s^8t^2-198s^8t-154s^7t^3+198s^7t^2)-\frac{(x-x_n)^{11}}{990h^8(r-1)^3(s-1)^3(t-1)^3}(8r^2s-4r^2st+8r^2t-12r^2-4rs^2t+8rs^2-4rst^2+21rst-19rs+8rt^2-19rt+9r+8s^2t-12s^2+8st^2-19st+9s-12t^2+9t+9)-\frac{rst(x-x_n)^6}{60h^3(r-1)^3(s-1)^3(t-1)^3}(r^2s^2t^2+r^2s^2t-5r^2s^2+r^2st^2-7r^2st+7r^2s-5r^2t^2+7r^2t+rs^2t^2-7rs^2t+7rs^2-7rst^2+17rst-9rs+7rt^2-9rt-5s^2t^2+7s^2t+7st^2-9st)+\frac{r^2s^2t^2(x-x_n)^5}{60h^2(r-1)^3(s-1)^3(t-1)^3}(7r+7s+7t-5rs-5rt-5st+3rst-9),$

$\gamma_0=\frac{(x-x_n)^4}{24}+\frac{(x-x_n)^{12}}{1320h^8r^2s^2t^2}+\frac{(x-x_n)^8}{336h^4r^2s^2t^2}(r^2s^2+4r^2st+4r^2s+r^2t^2+4r^2t+r^2+4rs^2t+4rs^2+4rst^2+16rst+4rs+4rt^2+4rt+s^2t^2+4s^2t+s^2+4st^2+4st+t^2)-\frac{(x-x_n)^9}{252h^5r^2s^2t^2}(r^2s+r^2t+r^2+rs^2+4rst+4rs+rt^2+4rt+r+s^2t+s^2+st^2+4st+s+t^2+t)+\frac{h^3(x-x_n)}{55440rst^2}(7r^9-17r^8s-24r^8t-24r^8+5r^7s^2+64r^7st+64r^7s+22r^7t^2+88r^7t+22r^7+5r^6s^3-24r^6s^2t-24r^6s^2-66r^6st^2-264r^6st-66r^6s-88r^6t^2-88r^6t+5r^5s^4-24r^5s^3t-24r^5s^3+33r^5s^2t^2+132r^5s^2t+33r^5s^2+308r^5st^2+308r^5st+99r^5t^2+5r^4s^5-24r^4s^4t-24r^4s^4+33r^4s^3t^2+132r^4s^3t+33r^4s^3-220r^4s^2t^2-220r^4s^2t-429r^4st^2+5r^3s^6-24r^3s^5t-24r^3s^5+33r^3s^4t^2+132r^3s^4t+33r^3s^4-220r^3s^3t^2-220r^3s^3t+495r^3s^2t^2+5r^2s^7-24r^2s^6t-24r^2s^6+33r^2s^5t^2+132r^2s^5t+33r^2s^5-220r^2s^4t^2-220r^2s^4t+495r^2s^3t^2-17rs^8+64rs^7t+64rs^7-66rs^6t^2-264rs^6t-66rs^6+308rs^5t^2+308rs^5t-429rs^4t^2+7s^9-24s^8t-24s^8+22s^7t^2+88s^7t+22s^7-88s^6t^2-88s^6t+99s^5t^2)-\frac{(x-x_n)^7}{105h^3r^2s^2t^2}(r^2s^2t+r^2s^2+r^2st^2+4r^2st+r^2s+r^2t^2+r^2t+rs^2t^2+4rs^2t+rs^2+4rst^2+4rst+rt^2+s^2t^2+s^2t+st^2)-\frac{(x-x_n)^{11}}{495h^7r^2s^2t^2}(r+s+t+1)-\frac{(x-x_n)^5}{30hrst}(rs+rt+st+rst)+\frac{(x-x_n)^6}{120h^2r^2s^2t^2}(r^2s^2t^2+4r^2$

$s^2t + r^2s^2 + 4r^2st^2 + 4r^2st + r^2t^2 + 4rs^2t^2 + 4rs^2t + 4rst^2 + s^2t^2) - \frac{h^2(x-x_n)^2}{55440r^2s^2t^2}(7r^{10} -$
$17r^9s - 24r^9t - 24r^9 + 5r^8s^2 + 64r^8st + 64r^8s + 22r^8t^2 + 88r^8t + 22r^8 + 5r^7s^3 -$
$24r^7s^2t - 24r^7s^2 - 66r^7st^2 - 264r^7st - 66r^7s - 88r^7t^2 - 88r^7t + 5r^6s^4 - 24r^6s^3t -$
$24r^6s^3 + 33r^6s^2t^2 + 132r^6s^2t + 33r^6s^2 + 308r^6st^2 + 308r^6st + 99r^6t^2 + 5r^5s^5 -$
$24r^5s^4t - 24r^5s^4 + 33r^5s^3t^2 + 132r^5s^3t + 33r^5s^3 - 220r^5s^2t^2 - 220r^5s^2t - 429r^5st^2 +$
$5r^4s^6 - 24r^4s^5t - 24r^4s^5 + 33r^4s^4t^2 + 132r^4s^4t + 33r^4s^4 - 220r^4s^3t^2 - 220r^4s^3t +$
$495r^4s^2t^2 + 5r^3s^7 - 24r^3s^6t - 24r^3s^6 + 33r^3s^5t^2 + 132r^3s^5t + 33r^3s^5 - 220r^3s^4t^2 -$
$220r^3s^4t + 495r^3s^3t^2 + 5r^2s^8 - 24r^2s^7t - 24r^2s^7 + 33r^2s^6t^2 + 132r^2s^6t + 33r^2s^6 -$
$220r^2s^5t^2 - 220r^2s^5t + 495r^2s^4t^2 - 17rs^9 + 64rs^8t + 64rs^8 - 66rs^7t^2 - 264rs^7t -$
$66rs^7 + 308rs^6t^2 + 308rs^6t - 429rs^5t^2 + 7s^{10} - 24s^9t - 24s^9 + 22s^8t^2 + 88s^8t + 22s^8 -$
$88s^7t^2 - 88s^7t + 99s^6t^2) + \frac{(x-x_n)^{10}}{720h^6r^2s^2t^2}(r^2 + 4rs + 4rt + 4r + s^2 + 4st + 4s + t^2 + 4t + 1),$

$\gamma_r = \frac{(x-x_n)^{12}}{1320h^8r^2(r-s)^2(r-t)^2(r-1)^2} - \frac{(x-x_n)^9}{504h^5r^2(r-s)^2(r-t)^2(r-1)^2}(r + 2s + 2t + 4rs + 4rt +$
$8st + rs^2 + rt^2 + 2st^2 + 2s^2t + 2s^2 + 2t^2 + 4rst) + \frac{h^2(x-x_n)^2}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(14r^{10} -$
$28r^9s - 42r^9t - 42r^9 + 5r^8s^2 + 90r^8st + 90r^8s + 33r^8t^2 + 132r^8t + 33r^8 + 5r^7s^3 -$
$20r^7s^2t - 20r^7s^2 - 77r^7st^2 - 308r^7st - 77r^7s - 110r^7t^2 - 110r^7t + 5r^6s^4 - 20r^6s^3t -$
$20r^6s^3 + 22r^6s^2t^2 + 88r^6s^2t + 22r^6s^2 + 286r^6st^2 + 286r^6st + 99r^6t^2 + 5r^5s^5 -$
$20r^5s^4t - 20r^5s^4 + 22r^5s^3t^2 + 88r^5s^3t + 22r^5s^3 - 110r^5s^2t^2 - 110r^5s^2t - 297r^5st^2 +$
$5r^4s^6 - 20r^4s^5t - 20r^4s^5 + 22r^4s^4t^2 + 88r^4s^4t + 22r^4s^4 - 110r^4s^3t^2 - 110r^4s^3t +$
$165r^4s^2t^2 + 5r^3s^7 - 20r^3s^6t - 20r^3s^6 + 22r^3s^5t^2 + 88r^3s^5t + 22r^3s^5 - 110r^3s^4t^2 -$
$110r^3s^4t + 165r^3s^3t^2 + 5r^2s^8 - 20r^2s^7t - 20r^2s^7 + 22r^2s^6t^2 + 88r^2s^6t + 22r^2s^6 -$
$110r^2s^5t^2 - 110r^2s^5t + 165r^2s^4t^2 + 5rs^9 - 20rs^8t - 20rs^8 + 22rs^7t^2 + 88rs^7t + 22rs^7 -$
$110rs^6t^2 - 110rs^6t + 165rs^5t^2 - 7s^{10} + 24s^9t + 24s^9 - 22s^8t^2 - 88s^8t - 22s^8 +$
$88s^7t^2 + 88s^7t - 99s^6t^2) - \frac{h^3(x-x_n)}{55440r(r-s)^2(r-t)^2(r-1)^2}(14r^9s - 28r^8s^2 - 42r^8st - 42r^8s +$
$5r^7s^3 + 90r^7s^2t + 90r^7s^2 + 33r^7st^2 + 132r^7st + 33r^7s + 5r^6s^4 - 20r^6s^3t - 20r^6s^3 -$
$77r^6s^2t^2 - 308r^6s^2t - 77r^6s^2 - 110r^6st^2 - 110r^6st + 5r^5s^5 - 20r^5s^4t - 20r^5s^4 +$
$22r^5s^3t^2 + 88r^5s^3t + 22r^5s^3 + 286r^5s^2t^2 + 286r^5s^2t + 99r^5st^2 + 5r^4s^6 - 20r^4s^5t -$
$20r^4s^5 + 22r^4s^4t^2 + 88r^4s^4t + 22r^4s^4 - 110r^4s^3t^2 - 110r^4s^3t - 297r^4s^2t^2 + 5r^3s^7 -$
$20r^3s^6t - 20r^3s^6 + 22r^3s^5t^2 + 88r^3s^5t + 22r^3s^5 - 110r^3s^4t^2 - 110r^3s^4t + 165r^3s^3t^2$

$+5r^2s^8-20r^2s^7t-20r^2s^7+22r^2s^6t^2+88r^2s^6t+22r^2s^6-110r^2s^5t^2-110r^2s^5t+165r^2s^4t^2+5rs^9-20rs^8t-20rs^8+22rs^7t^2+88rs^7t+22rs^7-110rs^6t^2-110rs^6t+165rs^5t^2-7s^{10}+24s^9t+24s^9-22s^8t^2-88s^8t-22s^8+88s^7t^2+88s^7t-99s^6t^2)+\frac{(x-x_n)^{10}}{720h^6r^2(r-s)^2(r-t)^2(r-1)^2}(2r+4s+4t+2rs+2rt+4st+s^2+t^2+1)+\frac{(x-x_n)^8}{336h^4r^2(r-s)^2(r-t)^2(r-1)^2}(s^2t^2+2rs+2rt+4st+2rs^2+2rt^2+4st^2+4s^2t+s^2+t^2+2rst^2+2rs^2t+8rst)-\frac{(x-x_n)^7}{210h^3r^2(r-s)^2(r-t)^2(r-1)^2}(2s^2t^2+rs^2+rt^2+2st^2+2s^2t+4rst^2+4rs^2t+rs^2t^2+4rst)-\frac{(x-x_n)^{11}}{990h^7r^2(r-s)^2(r-t)^2(r-1)^2}(r+2s+2t+2)-\frac{s^2t^2(x-x_n)^5}{60hr(r-s)^2(r-t)^2(r-1)^2}+\frac{st(x-x_n)^6}{120h^2r^2(r-s)^2(r-t)^2(r-1)^2}(2rs+2rt+st+2rst),$

$\gamma_s=\frac{(x-x_n)^{12}}{1320h^8s^2(r-s)^2(s-t)^2(s-1)^2}-\frac{(x-x_n)^9}{504h^5s^2(r-s)^2(s-t)^2(s-1)^2}(2r+s+2t+4rs+8rt+4st+r^2s+2rt^2+2r^2t+st^2+2r^2+2t^2+4rst)+\frac{h^2(x-x_n)^2}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(5r^9s-7r^{10}+24r^9t+24r^9+5r^8s^2-20r^8st-20r^8s-22r^8t^2-88r^8t-22r^8+5r^7s^3-20r^7s^2t-20r^7s^2+22r^7st^2+88r^7st+22r^7s+88r^7t^2+88r^7t+5r^6s^4-20r^6s^3t-20r^6s^3+22r^6s^2t^2+88r^6s^2t+22r^6s^2-110r^6st^2-110r^6st-99r^6t^2+5r^5s^5-20r^5s^4t-20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3-110r^5s^2t^2-110r^5s^2t+165r^5st^2+5r^4s^6-20r^4s^5t-20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-110r^4s^3t^2-110r^4s^3t+165r^4s^2t^2+5r^3s^7-20r^3s^6t-20r^3s^6+22r^3s^5t^2+88r^3s^5t+22r^3s^5-110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2+5r^2s^8-20r^2s^7t-20r^2s^7+22r^2s^6t^2+88r^2s^6t+22r^2s^6-110r^2s^5t^2-110r^2s^5t+165r^2s^4t^2-28rs^9+90rs^8t+90rs^8-77rs^7t^2-308rs^7t-77rs^7+286rs^6t^2+286rs^6t-297rs^5t^2+14s^{10}-42s^9t-42s^9+33s^8t^2+132s^8t+33s^8-110s^7t^2-110s^7t+99s^6t^2)-\frac{h^3(x-x_n)}{55440s(r-s)^2(s-t)^2(s-1)^2}(5r^9s-7r^{10}+24r^9t+24r^9+5r^8s^2-20r^8st-20r^8s-22r^8t^2-88r^8t-22r^8+5r^7s^3-20r^7s^2t-20r^7s^2+22r^7st^2+88r^7st+22r^7s+88r^7t^2+88r^7t+5r^6s^4-20r^6s^3t-20r^6s^3+22r^6s^2t^2+88r^6s^2t+22r^6s^2-110r^6st^2-110r^6st-99r^6t^2+5r^5s^5-20r^5s^4t-20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3-110r^5s^2t^2-110r^5s^2t+165r^5st^2+5r^4s^6-20r^4s^5t-20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-110r^4s^3t^2-110r^4s^3t+165r^4s^2t^2+5r^3s^7-20r^3s^6t-20r^3s^6+22r^3s^5t^2+88r^3s^5t+22r^3s^5-110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2-28r^2s^8+90r^2s^7t+90r^2s^7-77r^2s^6t^2-308r^2s^6t-77r^2s^6+286r^2s^5t^2+286r^2s^5t-297r^2s^4t^2+14rs^9-42rs^8t-42rs^8+33rs^7t^2+132rs^7t+33rs^7-110rs^6t^2-110rs^6t+99rs^5t^2)+$

$$\frac{(x-x_n)^{10}}{720h^6s^2(r-s)^2(s-t)^2(s-1)^2}(4r+2s+4t+2rs+4rt+2st+r^2+t^2+1)+$$
$$\frac{(x-x_n)^8}{336h^4s^2(r-s)^2(s-t)^2(s-1)^2}(r^2t^2+2rs+4rt+2st+2r^2s+4rt^2+4r^2t+2st^2+r^2+$$
$$t^2+2rst^2+2r^2st+8rst)-\frac{(x-x_n)^7}{210h^3s^2(r-s)^2(s-t)^2(s-1)^2}(2r^2t^2+r^2s+2rt^2+2r^2t+$$
$$st^2+4rst^2+4r^2st+r^2st^2+4rst)-\frac{(x-x_n)^{11}}{990h^7s^2(r-s)^2(s-t)^2(s-1)^2}(2r+s+2t+2)-$$
$$\frac{r^2t^2(x-x_n)^5}{60hs(r-s)^2(s-t)^2(s-1)^2}+\frac{rt(x-x_n)^6}{120h^2s^2(r-s)^2(s-t)^2(s-1)^2}(2rs+rt+2st+2rst),$$

$$\gamma_t=\frac{(x-x_n)^{12}}{1320h^8t^2(r-t)^2(s-t)^2(t-1)^2}-\frac{(x-x_n)^9}{504h^5t^2(r-t)^2(s-t)^2(t-1)^2}(2r+2s+t+8rs+4rt+$$
$$4st+2rs^2+2r^2s+r^2t+s^2t+2r^2+2s^2+4rst)-\frac{h^2(x-x_n)^2}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(33r^2s^6+$$
$$33r^3s^5+33r^4s^4+33r^5s^3+33r^6s^2-24r^2s^7-24r^3s^6-24r^4s^5-24r^5s^4-24r^6s^3-$$
$$24r^7s^2+5r^2s^8+5r^3s^7+5r^4s^6+5r^5s^5+5r^6s^4+5r^7s^3+5r^8s^2-66rs^7-66r^7s+$$
$$64rs^8+64r^8s-17rs^9-17r^9s-44r^7t+44r^8t-12r^9t-44s^7t+44s^8t-12s^9t+$$
$$22r^8-24r^9+7r^{10}+22s^8-24s^9+7s^{10}+154rs^6t+154r^6st-132rs^7t-132r^7st+$$
$$32rs^8t+32r^8st-110r^2s^5t-110r^3s^4t-110r^4s^3t-110r^5s^2t+66r^2s^6t+66r^3s^5t+$$
$$66r^4s^4t+66r^5s^3t+66r^6s^2t-12r^2s^7t-12r^3s^6t-12r^4s^5t-12r^5s^4t-12r^6s^3t-$$
$$12r^7s^2t)+\frac{(x-x_n)^{10}}{720h^6t^2(r-t)^2(s-t)^2(t-1)^2}(4r+4s+2t+4rs+2rt+2st+r^2+s^2+1)+$$
$$\frac{(x-x_n)^8}{336h^4t^2(r-t)^2(s-t)^2(t-1)^2}(r^2s^2+4rs+2rt+2st+4rs^2+4r^2s+2r^2t+2s^2t+r^2+s^2+$$
$$2rs^2t+2r^2st+8rst)-\frac{(x-x_n)^7}{210h^3t^2(r-t)^2(s-t)^2(t-1)^2}(2r^2s^2+2rs^2+2r^2s+r^2t+s^2t+4rs^2t+$$
$$4r^2st+r^2s^2t+4rst)-\frac{(x-x_n)^{11}}{990h^7t^2(r-t)^2(s-t)^2(t-1)^2}(2r+2s+t+2)-\frac{r^2s^2(x-x_n)^5}{60ht(r-t)^2(s-t)^2(t-1)^2}+$$
$$\frac{rs(x-x_n)^6}{120h^2t^2(r-t)^2(s-t)^2(t-1)^2}(rs+2rt+2st+2rst)+\frac{h^3rs(x-x_n)}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(33r^2s^5+$$
$$33r^3s^4+33r^4s^3+33r^5s^2-24r^2s^6-24r^3s^5-24r^4s^4-24r^5s^3-24r^6s^2+5r^2s^7+$$
$$5r^3s^6+5r^4s^5+5r^5s^4+5r^6s^3+5r^7s^2-66rs^6-66r^6s+64rs^7+64r^7s-17rs^8-$$
$$17r^8s-44r^6t+44r^7t-12r^8t-44s^6t+44s^7t-12s^8t+22r^7-24r^8+7r^9+22s^7-$$
$$24s^8+7s^9+154rs^5t+154r^5st-132rs^6t-132r^6st+32rs^7t+32r^7st-110r^2s^4t-$$
$$110r^3s^3t-110r^4s^2t+66r^2s^5t+66r^3s^4t+66r^4s^3t+66r^5s^2t-12r^2s^6t-12r^3s^5t-$$
$$12r^4s^4t-12r^5s^3t-12r^6s^2t),$$

$$\gamma_1=\frac{(x-x_n)^{12}}{1320h^8(r-1)^2(s-1)^2(t-1)^2}-\frac{(x-x_n)^9}{504h^5(r-1)^2(s-1)^2(t-1)^2}(2r^2s+2r^2t+r^2+2rs^2+8rst+$$
$$4rs+2rt^2+4rt+2s^2t+s^2+2st^2+4st+t^2)+\frac{(x-x_n)^8}{336h^4(r-1)^2(s-1)^2(t-1)^2}(r^2s^2+4r^2st+$$
$$2r^2s+r^2t^2+2r^2t+4rs^2t+2rs^2+4rst^2+8rst+2rt^2+s^2t^2+2s^2t+2st^2)-$$
$$\frac{h^2(x-x_n)^2}{55440(r-1)^2(s-1)^2(t-1)^2}(7r^{10}-17r^9s-24r^9t-12r^9+5r^8s^2+64r^8st+32r^8s+22r^8t^2$$

$+44r^8t+5r^7s^3-24r^7s^2t-12r^7s^2-66r^7st^2-132r^7st-44r^7t^2+5r^6s^4-24r^6s^3t-12r^6s^3+33r^6s^2t^2+66r^6s^2t+154r^6st^2+5r^5s^5-24r^5s^4t-12r^5s^4+33r^5s^3t^2+66r^5s^3t-110r^5s^2t^2+5r^4s^6-24r^4s^5t-12r^4s^5+33r^4s^4t^2+66r^4s^4t-110r^4s^3t^2+5r^3s^7-24r^3s^6t-12r^3s^6+33r^3s^5t^2+66r^3s^5t-110r^3s^4t^2+5r^2s^8-24r^2s^7t-12r^2s^7+33r^2s^6t^2+66r^2s^6t-110r^2s^5t^2-17rs^9+64rs^8t+32rs^8-66rs^7t^2-132rs^7t+154rs^6t^2+7s^{10}-24s^9t-12s^9+22s^8t^2+44s^8t-44s^7t^2)-\frac{(x-x_n)^{11}}{990h^7(r-1)^2(s-1)^2(t-1)^2}(2r+2s+2t+1)-\frac{(x-x_n)^7}{210h^3(r-1)^2(s-1)^2(t-1)^2}(2r^2s^2t+r^2s^2+2r^2st^2+4r^2st+r^2t^2+2rs^2t^2+4rs^2t+4rst^2+s^2t^2)+\frac{(x-x_n)^{10}}{720h^6(r-1)^2(s-1)^2(t-1)^2}(r^2+4rs+4rt+2r+s^2+4st+2s+t^2+2t)+\frac{h^3rs(x-x_n)}{55440(r-1)^2(s-1)^2(t-1)^2}(7r^9-17r^8s-24r^8t-12r^8+5r^7s^2+64r^7st+32r^7s+22r^7t^2+44r^7t+5r^6s^3-24r^6s^2t-12r^6s^2-66r^6st^2-132r^6st-44r^6t^2+5r^5s^4-24r^5s^3t-12r^5s^3+33r^5s^2t^2+66r^5s^2t+154r^5st^2+5r^4s^5-24r^4s^4t-12r^4s^4+33r^4s^3t^2+66r^4s^3t-110r^4s^2t^2+5r^3s^6-24r^3s^5t-12r^3s^5+33r^3s^4t^2+66r^3s^4t-110r^3s^3t^2+5r^2s^7-24r^2s^6t-12r^2s^6+33r^2s^5t^2+66r^2s^5t-110r^2s^4t^2-17rs^8+64rs^7t+32rs^7-66rs^6t^2-132rs^6t+154rs^5t^2+7s^9-24s^8t-12s^8+22s^7t^2+44s^7t-44s^6t^2)-\frac{r^2s^2t^2(x-x_n)^5}{60h(r-1)^2(s-1)^2(t-1)^2}+\frac{rst(x-x_n)^6}{120h^2(r-1)^2(s-1)^2(t-1)^2}(2rs+2rt+2st+rst).$

Differentiating Equation (4.35) once and twice gives

$$y'(x) = \frac{d}{dx}[\sum_{i=0,r,s}\alpha_i(x)y_{n+i}+\sum_{i=0}^{1}\beta_i(x)f_{n+i}+\sum_{i=r,s,t}\beta_i(x)f_{n+i}+\sum_{i=0}^{1}\gamma_i(x)g_{n+i} + \sum_{i=r,s,t}\gamma_i(x)g_{n+i}], \tag{4.36}$$

$$y''(x) = \frac{d^2}{dx^2}[\sum_{i=0,r,s}\alpha_i(x)y_{n+i}+\sum_{i=0}^{1}\beta_i(x)f_{n+i}+\sum_{i=r,s,t}\beta_i(x)f_{n+i}+\sum_{i=0}^{1}\gamma_i(x)g_{n+i} + \sum_{i=r,s,t}\gamma_i(x)g_{n+i}]. \tag{4.37}$$

Following the same argument as discussed in Section (4.2), Equation (4.35) is evaluated at the non-interpolating points, i.e at x_{n+t} and x_{n+1}, while evaluating (4.36) - (4.37) at all points, i.e $x_n, x_{n+r}, x_{n+s}, x_{n+t}$ and x_{n+1} to produce the block in the form

$$
\begin{aligned}
H^{[3]_3}Y_{n+1}^{[3]_3} &= M_1^{[3]_3}Y_n^{[3]_3}+M_2^{[3]_3}Y_{n-1}^{[3]_3}+M_3^{[3]_3}Y_{n-2}^{[3]_3}+E_1^{[3]_3}F_n^{[3]_3}+E_2^{[3]_3}F_{n+1}^{[3]_3} \\
&\quad +K_1^{[3]_3}G_n^{[3]_3}+K_2^{[3]_3}G_{n+1}^{[3]_3} \qquad (4.38)
\end{aligned}
$$

where

$$
H^{[3]_3}=\begin{pmatrix} \frac{t(s-t)}{r(r-s)} & -\frac{t(r-t)}{s(r-s)} & 1 & 0 \\ \frac{s-1}{r(r-s)} & -\frac{r-1}{s(r-s)} & 0 & 1 \\ \frac{s}{hr(r-s)} & \frac{-r}{hs(r-s)} & 0 & 0 \\ \frac{-2}{h^2r(r-s)} & \frac{2}{h^2s(r-s)} & 0 & 0 \end{pmatrix},\ Y_{n+1}^{[3]_3}=\begin{pmatrix} y_{n+r} \\ y_{n+s} \\ y_{n+t} \\ y_{n+1} \end{pmatrix},\ Y_n^{[3]_3}=\begin{pmatrix} y_{n-t} \\ y_{n-s} \\ y_{n-r} \\ y_n \end{pmatrix},
$$

$$
F_n^{[3]_3}=\begin{pmatrix} f_{n-t} \\ f_{n-s} \\ f_{n-r} \\ f_n \end{pmatrix},M_1^{[3]_3}=\begin{pmatrix} 0 & 0 & 0 & \frac{(r-t)(s-t)}{rs} \\ 0 & 0 & 0 & \frac{(r-1)(s-1)}{rs} \\ 0 & 0 & 0 & \frac{-(r+s)}{hrs} \\ 0 & 0 & 0 & \frac{2}{h^2rs} \end{pmatrix},\ M_2^{[3]_3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},
$$

$$
Y_{n-1}^{[3]_3}=\begin{pmatrix} y'_{n-t} \\ y'_{n-s} \\ y'_{n-r} \\ y'_n \end{pmatrix},Y_{n-2}^{[3]_3}=\begin{pmatrix} y''_{n-t} \\ y''_{n-s} \\ y''_{n-r} \\ y''_n \end{pmatrix},\ M_3^{[3]_3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},
$$

$$
F_{n+1}^{[3]_3}=\begin{pmatrix} f_{n+r} \\ f_{n+s} \\ f_{n+t} \\ f_{n+1} \end{pmatrix},\quad G_n^{[3]_3}=\begin{pmatrix} g_{n-t} \\ g_{n-s} \\ g_{n-r} \\ g_n \end{pmatrix},\quad G_{n+1}^{[3]_3}=\begin{pmatrix} g_{n+r} \\ g_{n+s} \\ g_{n+t} \\ g_{n+1} \end{pmatrix}.
$$

Now, both sides of Equation (4.38) are multiplied by the inverse of $H^{[3]_3}$ to get

$$
\begin{aligned}
I_4Y_{n+1}^{[3]_3} &= \hat{M}_1^{[3]_3}Y_n^{[3]_3}+h\hat{M}_2^{[3]_3}Y_{n-1}^{[3]_3}+h^2\hat{M}_3^{[3]_3}Y_{n-2}^{[3]_3}+h^3\left[\hat{E}_1^{[3]_3}F_n^{[3]_3}+\hat{E}_2^{[3]_3}F_{n+1}^{[3]_3}\right] \\
&\quad +h^4\left[\hat{K}_1^{[3]_3}G_n^{[3]_3}+\hat{K}_2^{[3]_3}G_{n+1}^{[3]_3}\right] \qquad (4.39)
\end{aligned}
$$

where

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{M}_1^{[3]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{M}_2^{[3]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\hat{M}_3^{[3]_3} = \begin{pmatrix} 0 & 0 & 0 & \frac{r^2}{2} \\ 0 & 0 & 0 & \frac{s^2}{2} \\ 0 & 0 & 0 & \frac{t^2}{2} \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \hat{E}_1^{[3]_3} = \begin{pmatrix} 0 & 0 & 0 & \hat{E}_{1_{14}}^{[3]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{24}}^{[3]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{34}}^{[3]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{44}}^{[3]_3} \end{pmatrix},$$

$$\hat{E}_2^{[3]_3} = \begin{pmatrix} \hat{E}_{2_{11}}^{[3]_3} & \hat{E}_{2_{12}}^{[3]_3} & \hat{E}_{2_{13}}^{[3]_3} & \hat{E}_{2_{14}}^{[3]_3} \\ \hat{E}_{2_{21}}^{[3]_3} & \hat{E}_{2_{22}}^{[3]_3} & \hat{E}_{2_{23}}^{[3]_3} & \hat{E}_{2_{24}}^{[3]_3} \\ \hat{E}_{2_{31}}^{[3]_3} & \hat{E}_{2_{32}}^{[3]_3} & \hat{E}_{2_{33}}^{[3]_3} & \hat{E}_{2_{34}}^{[3]_3} \\ \hat{E}_{2_{41}}^{[3]_3} & \hat{E}_{2_{42}}^{[3]_3} & \hat{E}_{2_{43}}^{[3]_3} & \hat{E}_{2_{44}}^{[3]_3} \end{pmatrix}, \hat{K}_1^{[3]_3} = \begin{pmatrix} 0 & 0 & 0 & \hat{K}_{1_{14}}^{[3]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{24}}^{[3]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{34}}^{[3]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{44}}^{[3]_3} \end{pmatrix},$$

$$\hat{K}_2^{[3]_3} = \begin{pmatrix} \hat{K}_{2_{11}}^{[3]_3} & \hat{K}_{2_{12}}^{[3]_3} & \hat{K}_{2_{13}}^{[3]_3} & \hat{K}_{2_{14}}^{[3]_3} \\ \hat{K}_{2_{21}}^{[3]_3} & \hat{K}_{2_{22}}^{[3]_3} & \hat{K}_{2_{23}}^{[3]_3} & \hat{K}_{2_{24}}^{[3]_3} \\ \hat{K}_{2_{31}}^{[3]_3} & \hat{K}_{2_{32}}^{[3]_3} & \hat{K}_{2_{33}}^{[3]_3} & \hat{K}_{2_{34}}^{[3]_3} \\ \hat{K}_{2_{41}}^{[3]_3} & \hat{K}_{2_{42}}^{[3]_3} & \hat{K}_{2_{43}}^{[3]_3} & \hat{K}_{2_{44}}^{[3]_3} \end{pmatrix}.$$

The entries of $\hat{E}_1^{[3]_3}, \hat{E}_2^{[3]_3}, \hat{K}_1^{[3]_3}$ and $\hat{K}_2^{[3]_3}$ are given below:

$\hat{E}_{1_{14}}^{[3]_3} = -\frac{r^3}{27720 s^3 t^3}(-7r^7 st - 7r^7 s - 7r^7 t + 24r^6 s^2 t + 24r^6 s^2 + 24r^6 st^2 + 59r^6 st + 24r^6 s + 24r^6 t^2 + 24r^6 t - 22r^5 s^3 t - 22r^5 s^3 - 88r^5 s^2 t^2 - 152r^5 s^2 t - 88r^5 s^2 - 22r^5 st^3 - 152r^5 st^2 - 152r^5 st - 22r^5 s - 22r^5 t^3 - 88r^5 t^2 - 22r^5 t + 88r^4 s^3 t^2 + 132r^4 s^3 t + 88r^4 s^3 + 88r^4 s^2 t^3 + 352r^4 s^2 t^2 + 352r^4 s^2 t + 88r^4 s^2 + 132r^4 st^3 + 352r^4 st^2 + 132r^4 st + 88r^4 t^3 + 88r^4 t^2 - 99r^3 s^3 t^3 - 308r^3 s^3 t^2 - 308r^3 s^3 t - 99r^3 s^3 - 308r^3 s^2 t^3 - 440r^3 s^2 t^2 - 308r^3 s^2 t - 308r^3 st^3 - 308r^3 st^2 - 99r^3 t^3 + 297r^2 s^3 t^3 + 132r^2 s^3 t^2 + 297r^2 s^3 t + 132r^2 s^2 t^3 + 132r^2 s^2 t^2 + 297r^2 st^3 + 528rs^3 t^3 + 528rs^3 t^2 + 528rs^2 t^3 - 3696s^3 t^3)$,

$\hat{E}^{[3]_3}_{1_{24}} = -\frac{s^3}{27720r^3t^3}(-22r^3s^5t - 22r^3s^5 + 88r^3s^4t^2 + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 + 297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t + 24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t - 7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t - 22rs^5 + 132rs^4t^3 + 352rs^4t^2 + 132rs^4t - 308rs^3t^3 - 308rs^3t^2 + 297rs^2t^3 - 7s^7t + 24s^6t^2 + 24s^6t - 22s^5t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3)$,

$\hat{E}^{[3]_3}_{1_{34}} = -\frac{t^3}{27720r^3s^3}(-99r^3s^3t^3 + 297r^3s^3t^2 + 528r^3s^3t - 3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 + 132r^3st^4 - 308r^3st^3 + 297r^3st^2 - 22r^3t^5 + 88r^3t^4 - 99r^3t^3 + 88r^2s^3t^4 - 308r^2s^3t^3 + 132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 - 152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 - 22rs^3t^5 + 132rs^3t^4 - 308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 - 152rst^5 + 132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 - 88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5)$,

$\hat{E}^{[3]_3}_{1_{44}} = -\frac{1}{27720r^3s^3t^3}(-3696r^3s^3t^3 + 528r^3s^3t^2 + 297r^3s^3t - 99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2 - 308r^3s^2t + 88r^3s^2 + 297r^3st^3 - 308r^3st^2 + 132r^3st - 22r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t + 528r^2s^3t^3 + 132r^2s^3t^2 - 308r^2s^3t + 88r^2s^3 + 132r^2s^2t^3 - 440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 - 152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3t^2 + 132rs^3t - 22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs - 22rt^3 + 24rt^2 - 7rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st)$,

$\hat{E}^{[3]_3}_{2_{11}} = \frac{r^3}{27720(r-s)^3(r-t)^3(r-1)^3}(84r^9 - 315r^8s - 315r^8t - 315r^8 + 390r^7s^2 + 1210r^7st + 1210r^7s + 390r^7t^2 + 1210r^7t + 390r^7 - 154r^6s^3 - 1536r^6s^2t - 1536r^6s^2 - 1536r^6st^2 - 4793r^6st - 1536r^6s - 154r^6t^3 - 1536r^6t^2 - 1536r^6t - 154r^6 + 616r^5s^3t + 616r^5s^3 + 2002r^5s^2t^2 + 6290r^5s^2t + 2002r^5s^2 + 616r^5st^3 + 6290r^5st^2 + 6290r^5st + 616r^5s + 616r^5t^3 + 2002r^5t^2 + 616r^5t - 814r^4s^3t^2 - 2574r^4s^3t - 814r^4s^3 - 814r^4s^2t^3 - 8569r^4s^2t^2 - 8569r^4s^2t - 814r^4s^2 - 2574r^4st^3 - 8569r^4st^2 - 2574r^4st - 814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 +$

$3575r^3s^2t^3 + 12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 - 5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 + 2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3)$,

$\hat{E}^{[3]_3}_{2_{21}} = \frac{s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3)$,

$\hat{E}^{[3]_3}_{2_{31}} = \frac{t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(198r^4s^2t^2 - 891r^4s^2t + 1188r^4s^2 - 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 - 198r^4t^3 + 198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t - 924r^3s^2 + 208r^3st^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 - 154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 + 462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 - 781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 - 649r^2st^2 + 21r^2t^6 - 21r^2t^5 - 116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594rs^3t + 48rs^2t^5 - 106rs^2t^4 - 88rs^2t^3 + 275rs^2t^2 - 14rst^6 + 64rst^5 - 106rst^4 + 66rst^3 - 14rt^6 + 48rt^5 - 44rt^4 + 22s^3t^4 - 88s^3t^3 + 99s^3t^2 - 24s^2t^5 + 88s^2t^4 - 88s^2t^3 + 7st^6 - 24st^5 + 22st^4)$,

$\hat{E}^{[3]_3}_{2_{41}} = -\frac{1}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-1188r^4s^2t^2 + 891r^4s^2t - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3$

$$+88rs^2t^2+106rs^2t-48rs^2-66rst^3+106rst^2-64rst+14rs+44rt^3-48rt^2+$$
$$14rt-99s^3t^3+88s^3t^2-22s^3t+88s^2t^3-88s^2t^2+24s^2t-22st^3+24st^2-7st),$$

$$\hat{E}_{2_{12}}^{[3]_3}=\frac{r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2+14r^6st+14r^6s-7r^6t+70r^5s^3+21r^5s^2t+$$
$$21r^5s^2-48r^5st^2-64r^5st-48r^5s+24r^5t^2+24r^5t-54r^4s^4-208r^4s^3t-208r^4s^3+$$
$$116r^4s^2t^2+70r^4s^2t+116r^4s^2+44r^4st^3+106r^4st^2+106r^4st+44r^4s-22r^4t^3-$$
$$88r^4t^2-22r^4t+198r^3s^4t+198r^3s^4+88r^3s^3t^2+660r^3s^3t+88r^3s^3-176r^3s^2t^3-$$
$$506r^3s^2t^2-506r^3s^2t-176r^3s^2-66r^3st^3+88r^3st^2-66r^3st+88r^3t^3+88r^3t^2-$$
$$198r^2s^4t^2-792r^2s^4t-198r^2s^4+154r^2s^3t^3-275r^2s^3t^2-275r^2s^3t+154r^2s^3+$$
$$649r^2s^2t^3+781r^2s^2t^2+649r^2s^2t-275r^2st^3-275r^2st^2-99r^2t^3+891rs^4t^2+$$
$$891rs^4t-693rs^3t^3-66rs^3t^2-693rs^3t-462rs^2t^3-462rs^2t^2+594rst^3-1188s^4t^2+$$
$$924s^3t^3+924s^3t^2-660s^2t^3),$$

$$\hat{E}_{2_{22}}^{[3]_3}=\frac{s^3}{27720(r-s)^3(s-t)^3(s-1)^3}(154r^3s^6-616r^3s^5t-616r^3s^5+814r^3s^4t^2+$$
$$2574r^3s^4t+814r^3s^4-330r^3s^3t^3-3575r^3s^3t^2-3575r^3s^3t-330r^3s^3+1485r^3s^2t^3+$$
$$5280r^3s^2t^2+1485r^3s^2t-2244r^3st^3-2244r^3st^2+924r^3t^3-390r^2s^7+1536r^2s^6t+$$
$$1536r^2s^6-2002r^2s^5t^2-6290r^2s^5t-2002r^2s^5+814r^2s^4t^3+8569r^2s^4t^2+$$
$$8569r^2s^4t+814r^2s^4-3575r^2s^3t^3-12320r^2s^3t^2-3575r^2s^3t+5280r^2s^2t^3+$$
$$5280r^2s^2t^2-2244r^2st^3+315rs^8-1210rs^7t-1210rs^7+1536rs^6t^2+4793rs^6t+$$
$$1536rs^6-616rs^5t^3-6290rs^5t^2-6290rs^5t-616rs^5+2574rs^4t^3+8569rs^4t^2+$$
$$2574rs^4t-3575rs^3t^3-3575rs^3t^2+1485rs^2t^3-84s^9+315s^8t+315s^8-390s^7t^2-$$
$$1210s^7t-390s^7+154s^6t^3+1536s^6t^2+1536s^6t+154s^6-616s^5t^3-2002s^5t^2-$$
$$616s^5t+814s^4t^3+814s^4t^2-330s^3t^3),$$

$$\hat{E}_{2_{32}}^{[3]_3}=\frac{t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(154r^3s^3t^2-693r^3s^3t+924r^3s^3-176r^3s^2t^3+$$
$$649r^3s^2t^2-462r^3s^2t-660r^3s^2+44r^3st^4-66r^3st^3-275r^3st^2+594r^3st-22r^3t^4+$$
$$88r^3t^3-99r^3t^2-198r^2s^4t^2+891r^2s^4t-1188r^2s^4+88r^2s^3t^3-275r^2s^3t^2-66r^2s^3t+$$
$$924r^2s^3+116r^2s^2t^4-506r^2s^2t^3+781r^2s^2t^2-462r^2s^2t-48r^2st^5+106r^2st^4+$$
$$88r^2st^3-275r^2st^2+24r^2t^5-88r^2t^4+88r^2t^3+198rs^4t^3-792rs^4t^2+891rs^4t-$$
$$208rs^3t^4+660rs^3t^3-275rs^3t^2-693rs^3t+21rs^2t^5+70rs^2t^4-506rs^2t^3+649rs^2t^2+$$
$$14rst^6-64rst^5+106rst^4-66rst^3-7rt^6+24rt^5-22rt^4-54s^4t^4+198s^4t^3$$

$-198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-176s^2t^3+14st^6-48st^5+44st^4)$,

$\hat{E}^{[3]_3}_{2_{42}}=\frac{1}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(924r^3s^3t^2-693r^3s^3t+154r^3s^3-660r^3s^2t^3-462r^3s^2t^2+649r^3s^2t-176r^3s^2+594r^3st^3-275r^3st^2-66r^3st+44r^3s-99r^3t^3+88r^3t^2-22r^3t-1188r^2s^4t^2+891r^2s^4t-198r^2s^4+924r^2s^3t^3-66r^2s^3t^2-275r^2s^3t+88r^2s^3-462r^2s^2t^3+781r^2s^2t^2-506r^2s^2t+116r^2s^2-275r^2st^3+88r^2st^2+106r^2st-48r^2s+88r^2t^3-88r^2t^2+24r^2t+891rs^4t^2-792rs^4t+198rs^4-693rs^3t^3-275rs^3t^2+660rs^3t-208rs^3+649rs^2t^3-506rs^2t^2+70rs^2t+21rs^2-66rst^3+106rst^2-64rst+14rs-22rt^3+24rt^2-7rt-198s^4t^2+198s^4t-54s^4+154s^3t^3+88s^3t^2-208s^3t+70s^3-176s^2t^3+116s^2t^2+21s^2t-21s^2+44st^3-48st^2+14st)$,

$\hat{E}^{[3]_3}_{2_{13}}=\frac{r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st+7r^6s+21r^6t^2-14r^6t+48r^5s^2t-24r^5s^2-21r^5st^2+64r^5st-24r^5s-70r^5t^3-21r^5t^2+48r^5t-44r^4s^3t+22r^4s^3-116r^4s^2t^2-106r^4s^2t+88r^4s^2+208r^4st^3-70r^4st^2-106r^4st+22r^4s+54r^4t^4+208r^4t^3-116r^4t^2-44r^4t+176r^3s^3t^2+66r^3s^3t-88r^3s^3-88r^3s^2t^3+506r^3s^2t^2-88r^3s^2t-88r^3s^2-198r^3st^4-660r^3st^3+506r^3st^2+66r^3st-198r^3t^4-88r^3t^3+176r^3t^2-154r^2s^3t^3-649r^2s^3t^2+275r^2s^3t+99r^2s^3+198r^2s^2t^4+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2t+792r^2st^4+275r^2st^3-649r^2st^2+198r^2t^4-154r^2t^3+693rs^3t^3+462rs^3t^2-594rs^3t-891rs^2t^4+66rs^2t^3+462rs^2t^2-891rst^4+693rst^3-924s^3t^3+660s^3t^2+1188s^2t^4-924s^2t^3)$,

$\hat{E}^{[3]_3}_{2_{23}}=-\frac{s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(44r^3s^4t-22r^3s^4-176r^3s^3t^2-66r^3s^3t+88r^3s^3+154r^3s^2t^3+649r^3s^2t^2-275r^3s^2t-99r^3s^2-693r^3st^3-462r^3st^2+594r^3st+924r^3t^3-660r^3t^2-48r^2s^5t+24r^2s^5+116r^2s^4t^2+106r^2s^4t-88r^2s^4+88r^2s^3t^3-506r^2s^3t^2+88r^2s^3t+88r^2s^3-198r^2s^2t^4-275r^2s^2t^3+781r^2s^2t^2-275r^2s^2t+891r^2st^4-66r^2st^3-462r^2st^2-1188r^2t^4+924r^2t^3+14rs^6t-7rs^6+21rs^5t^2-64rs^5t+24rs^5-208rs^4t^3+70rs^4t^2+106rs^4t-22rs^4+198rs^3t^4+660rs^3t^3-506rs^3t^2-66rs^3t-792rs^2t^4-275rs^2t^3+649rs^2t^2+891rst^4-693rst^3-21s^6t^2+14s^6t+70s^5t^3+21s^5t^2-48s^5t-54s^4t^4-208s^4t^3+116s^4t^2+44s^4t+198s^3t^4$

$+88s^3t^3-176s^3t^2-198s^2t^4+154s^2t^3),$

$\hat{E}_{2_{33}}^{[3]_3} = -\frac{t^3}{27720(r-t)^3(s-t)^3(t-1)^3}(-330r^3s^3t^3+1485r^3s^3t^2-2244r^3s^3t+924r^3s^3+814r^3s^2t^4-3575r^3s^2t^3+5280r^3s^2t^2-2244r^3s^2t-616r^3st^5+2574r^3st^4-3575r^3st^3+1485r^3st^2+154r^3t^6-616r^3t^5+814r^3t^4-330r^3t^3+814r^2s^3t^4-3575r^2s^3t^3+5280r^2s^3t^2-2244r^2s^3t-2002r^2s^2t^5+8569r^2s^2t^4-12320r^2s^2t^3+5280r^2s^2t^2+1536r^2st^6-6290r^2st^5+8569r^2st^4-3575r^2st^3-390r^2t^7+1536r^2t^6-2002r^2t^5+814r^2t^4-616rs^3t^5+2574rs^3t^4-3575rs^3t^3+1485rs^3t^2+1536rs^2t^6-6290rs^2t^5+8569rs^2t^4-3575rs^2t^3-1210rst^7+4793rst^6-6290rst^5+2574rst^4+315rt^8-1210rt^7+1536rt^6-616rt^5+154s^3t^6-616s^3t^5+814s^3t^4-330s^3t^3-390s^2t^7+1536s^2t^6-2002s^2t^5+814s^2t^4+315st^8-1210st^7+1536st^6-616st^5-84t^9+315t^8-390t^7+154t^6),$

$\hat{E}_{2_{43}}^{[3]_3} = \frac{1}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(660r^3s^3t^2-594r^3s^3t+99r^3s^3-924r^3s^2t^3+462r^3s^2t^2+275r^3s^2t-88r^3s^2+693r^3st^3-649r^3st^2+66r^3st+22r^3s-154r^3t^3+176r^3t^2-44r^3t-924r^2s^3t^3+462r^2s^3t^2+275r^2s^3t-88r^2s^3+1188r^2s^2t^4+66r^2s^2t^3-781r^2s^2t^2-88r^2s^2t+88r^2s^2-891r^2st^4+275r^2st^3+506r^2st^2-106r^2st-24r^2s+198r^2t^4-88r^2t^3-116r^2t^2+48r^2t+693rs^3t^3-649rs^3t^2+66rs^3t+22rs^3-891rs^2t^4+275rs^2t^3+506rs^2t^2-106rs^2t-24rs^2+792rst^4-660rst^3-70rst^2+64rst+7rs-198rt^4+208rt^3-21rt^2-14rt-154s^3t^3+176s^3t^2-44s^3t+198s^2t^4-88s^2t^3-116s^2t^2+48s^2t-198st^4+208st^3-21st^2-14st+54t^4-70t^3+21t^2),$

$\hat{E}_{2_{14}}^{[3]_3} = -\frac{r^7}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^6st-14r^6s-14r^6t+21r^6-24r^5s^2t+48r^5s^2-24r^5st^2+64r^5st-21r^5s+48r^5t^2-21r^5t-70r^5+22r^4s^3t-44r^4s^3+88r^4s^2t^2-106r^4s^2t-116r^4s^2+22r^4st^3-106r^4st^2-70r^4st+208r^4s-44r^4t^3-116r^4t^2+208r^4t+54r^4-88r^3s^3t^2+66r^3s^3t+176r^3s^3-88r^3s^2t^3-88r^3s^2t^2+506r^3s^2t-88r^3s^2+66r^3st^3+506r^3st^2-660r^3st-198r^3s+176r^3t^3-88r^3t^2-198r^3t+99r^2s^3t^3+275r^2s^3t^2-649r^2s^3t-154r^2s^3+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2t+198r^2s^2-649r^2st^3+275r^2st^2+792r^2st-154r^2t^3+198r^2t^2-594rs^3t^3+462rs^3t^2+693rs^3t+462rs^2t^3+66rs^2t^2-891rs^2t+693rst^3-891rst^2+660s^3t^3-924s^3t^2-924s^2t^3+1188s^2t^2),$

$\hat{E}^{[3]_3}_{2_{24}} = -\frac{s^7}{27720(r-1)^3(s-1)^3(t-1)^3}(22r^3s^4t - 44r^3s^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 - 594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 - 106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 + 1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 - 70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 + 792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 - 116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2)$,

$\hat{E}^{[3]_3}_{2_{34}} = -\frac{t^7}{27720(r-1)^3(s-1)^3(t-1)^3}(99r^3s^3t^2 - 594r^3s^3t + 660r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 462r^3s^2t - 924r^3s^2 + 22r^3st^4 + 66r^3st^3 - 649r^3st^2 + 693r^3st - 44r^3t^4 + 176r^3t^3 - 154r^3t^2 - 88r^2s^3t^3 + 275r^2s^3t^2 + 462r^2s^3t - 924r^2s^3 + 88r^2s^2t^4 - 88r^2s^2t^3 - 781r^2s^2t^2 + 66r^2s^2t + 1188r^2s^2 - 24r^2st^5 - 106r^2st^4 + 506r^2st^3 + 275r^2st^2 - 891r^2st + 48r^2t^5 - 116r^2t^4 - 88r^2t^3 + 198r^2t^2 + 22rs^3t^4 + 66rs^3t^3 - 649rs^3t^2 + 693rs^3t - 24rs^2t^5 - 106rs^2t^4 + 506rs^2t^3 + 275rs^2t^2 - 891rs^2t + 7rst^6 + 64rst^5 - 70rst^4 - 660rst^3 + 792rst^2 - 14rt^6 - 21rt^5 + 208rt^4 - 198rt^3 - 44s^3t^4 + 176s^3t^3 - 154s^3t^2 + 48s^2t^5 - 116s^2t^4 - 88s^2t^3 + 198s^2t^2 - 14st^6 - 21st^5 + 208st^4 - 198st^3 + 21t^6 - 70t^5 + 54t^4)$,

$\hat{E}^{[3]_3}_{2_{44}} = \frac{1}{27720(r-1)^3(s-1)^3(t-1)^3}(924r^3s^3t^3 - 2244r^3s^3t^2 + 1485r^3s^3t - 330r^3s^3 - 2244r^3s^2t^3 + 5280r^3s^2t^2 - 3575r^3s^2t + 814r^3s^2 + 1485r^3st^3 - 3575r^3st^2 + 2574r^3st - 616r^3s - 330r^3t^3 + 814r^3t^2 - 616r^3t + 154r^3 - 2244r^2s^3t^3 + 5280r^2s^3t^2 - 3575r^2s^3t + 814r^2s^3 + 5280r^2s^2t^3 - 12320r^2s^2t^2 + 8569r^2s^2t - 2002r^2s^2 - 3575r^2st^3 + 8569r^2st^2 - 6290r^2st + 1536r^2s + 814r^2t^3 - 2002r^2t^2 + 1536r^2t - 390r^2 + 1485rs^3t^3 - 3575rs^3t^2 + 2574rs^3t - 616rs^3 - 3575rs^2t^3 + 8569rs^2t^2 - 6290rs^2t + 1536rs^2 + 2574rst^3 - 6290rst^2 + 4793rst - 1210rs - 616rt^3 + 1536rt^2 - 1210rt + 315r - 330s^3t^3 + 814s^3t^2 - 616s^3t + 154s^3 + 814s^2t^3 - 2002s^2t^2 + 1536s^2t - 390s^2 - 616st^3 + 1536st^2 - 1210st + 315s + 154t^3 - 390t^2 + 315t - 84)$,

$\hat{K}^{[3]_3}_{1_{14}} = \frac{r^4}{55440s^2t^2}(7r^6 - 24r^5s - 24r^5t - 24r^5 + 22r^4s^2 + 88r^4st + 88r^4s + 22r^4t^2 + 88r^4t + 22r^4 - 88r^3s^2t - 88r^3s^2 - 88r^3st^2 - 352r^3st - 88r^3s - 88r^3t^2 - 88r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 528rs^2t^2 - 528rs^2t - 528rst^2 + 924s^2t^2)$,

$\hat{K}^{[3]_3}_{1_{24}} = \frac{s^4}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 +$

$396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2)$,

$\hat{K}^{[3]_3}_{1_{34}} = \frac{t^4}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 - 528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 - 88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 - 352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 + 99s^2t^2 - 24st^5 + 88st^4 - 88st^3 + 7t^6 - 24t^5 + 22t^4)$,

$\hat{K}^{[3]_3}_{1_{44}} = \frac{1}{55440r^2s^2t^2}(924r^2s^2t^2 - 528r^2s^2t + 99r^2s^2 - 528r^2st^2 + 396r^2st - 88r^2s + 99r^2t^2 - 88r^2t + 22r^2 - 528rs^2t^2 + 396rs^2t - 88rs^2 + 396rst^2 - 352rst + 88rs - 88rt^2 + 88rt - 24r + 99s^2t^2 - 88s^2t + 22s^2 - 88st^2 + 88st - 24s + 22t^2 - 24t + 7)$,

$\hat{K}^{[3]_3}_{2_{11}} = -\frac{r^4}{55440(r-s)^2(r-t)^2(r-1)^2}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$,

$\hat{K}^{[3]_3}_{2_{21}} = -\frac{s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst)$,

$\hat{K}^{[3]_3}_{2_{31}} = -\frac{t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^2t^3 + 264rs^2 + 44rt^2 - 44rt^3 + 12rt^4 + 88st^2 - 99s^2t - 88st^3 + 24st^4 - 22t^3 + 24t^4 - 7t^5 + 176rst^2 - 198rs^2t - 44rst^3 + 44rs^2t^2 - 198rst)$,

$\hat{K}^{[3]_3}_{2_{41}} = -\frac{1}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(12r+24s+24t-99s^2t^2-44rs-44rt-88st+$
$44rs^2+44rt^2+88st^2+88s^2t-22s^2-22t^2-198rst^2-198rs^2t+264rs^2t^2+176rst-$
$7)$,

$\hat{K}^{[3]_3}_{2_{12}} = -\frac{r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2-22r^3t^2+44r^2s-44r^3s+12r^4s-99rt^2+$
$88r^2t-88r^3t+24r^4t+264st^2-22r^3+24r^4-7r^5-198rst^2+176r^2st-44r^3st+$
$44r^2st^2-198rst)$,

$\hat{K}^{[3]_3}_{2_{22}} = -\frac{s^4}{55440(r-s)^2(s-t)^2(s-1)^2}(33r^2s^4-110r^2s^3t-110r^2s^3+99r^2s^2t^2+396r^2s^2t+$
$99r^2s^2-396r^2st^2-396r^2st+462r^2t^2-42rs^5+132rs^4t+132rs^4-110rs^3t^2-$
$440rs^3t-110rs^3+396rs^2t^2+396rs^2t-396rst^2+14s^6-42s^5t-42s^5+33s^4t^2+$
$132s^4t+33s^4-110s^3t^2-110s^3t+99s^2t^2)$,

$\hat{K}^{[3]_3}_{2_{32}} = -\frac{t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2-22r^2t^3+264r^2s+88rt^2-99r^2t-88rt^3$
$+24rt^4+44st^2-44st^3+12st^4-22t^3+24t^4-7t^5+176rst^2-198r^2st-44rst^3+$
$44r^2st^2-198rst)$,

$\hat{K}^{[3]_3}_{2_{42}} = -\frac{1}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(24r+12s+24t-99r^2t^2-44rs-88rt-44st+$
$44r^2s+88rt^2+88r^2t+44st^2-22r^2-22t^2-198rst^2-198r^2st+264r^2st^2+$
$176rst-7)$,

$\hat{K}^{[3]_3}_{2_{13}} = -\frac{r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2-22r^3s^2-99rs^2+88r^2s-88r^3s+24r^4s+$
$44r^2t-44r^3t+12r^4t+264s^2t-22r^3+24r^4-7r^5-198rs^2t+176r^2st-44r^3st+$
$44r^2s^2t-198rst)$,

$\hat{K}^{[3]_3}_{2_{23}} = -\frac{s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2-22r^2s^3+88rs^2-99r^2s-88rs^3+24rs^4+$
$264r^2t+44s^2t-44s^3t+12s^4t-22s^3+24s^4-7s^5+176rs^2t-198r^2st-44rs^3t+$
$44r^2s^2t-198rst)$,

$\hat{K}^{[3]_3}_{2_{33}} = -\frac{t^4}{55440(r-t)^2(s-t)^2(t-1)^2}(99r^2s^2t^2-396r^2s^2t+462r^2s^2-110r^2st^3+$
$396r^2st^2-396r^2st+33r^2t^4-110r^2t^3+99r^2t^2-110rs^2t^3+396rs^2t^2-396rs^2t+$
$132rst^4-440rst^3+396rst^2-42rt^5+132rt^4-110rt^3+33s^2t^4-110s^2t^3+99s^2t^2-$
$42st^5+132st^4-110st^3+14t^6-42t^5+33t^4)$,

$$\hat{K}^{[3]_3}_{2_{43}} = -\frac{1}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(24r+24s+12t-99r^2s^2-88rs-44rt-44st+88rs^2+88r^2s+44r^2t+44s^2t-22r^2-22s^2-198rs^2t-198r^2st+264r^2s^2t+176rst-7),$$

$$\hat{K}^{[3]_3}_{2_{14}} = -\frac{r^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-7r^5+24r^4s+24r^4t+12r^4-22r^3s^2-88r^3st-44r^3s-22r^3t^2-44r^3t+88r^2s^2t+44r^2s^2+88r^2st^2+176r^2st+44r^2t^2-99rs^2t^2-198rs^2t-198rst^2+264s^2t^2),$$

$$\hat{K}^{[3]_3}_{2_{24}} = -\frac{s^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-22r^2s^3+88r^2s^2t+44r^2s^2-99r^2st^2-198r^2st+264r^2t^2+24rs^4-88rs^3t-44rs^3+88rs^2t^2+176rs^2t-198rst^2-7s^5+24s^4t+12s^4-22s^3t^2-44s^3t+44s^2t^2),$$

$$\hat{K}^{[3]_3}_{2_{34}} = -\frac{t^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-99r^2s^2t+264r^2s^2+88r^2st^2-198r^2st-22r^2t^3+44r^2t^2+88rs^2t^2-198rs^2t-88rst^3+176rst^2+24rt^4-44rt^3-22s^2t^3+44s^2t^2+24st^4-44st^3-7t^5+12t^4),$$

$$\hat{K}^{[3]_3}_{2_{44}} = -\frac{1}{55440(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2-396r^2s^2t+99r^2s^2-396r^2st^2+396r^2st-110r^2s+99r^2t^2-110r^2t+33r^2-396rs^2t^2+396rs^2t-110rs^2+396rst^2-440rst+132rs-110rt^2+132rt-42r+99s^2t^2-110s^2t+33s^2-110st^2+132st-42s+33t^2-42t+14).$$

From (4.39), we obtain the following schemes :

$$y_{n+r} = y_n + \frac{h^2r^2y''_n}{2} + hry'_n + \frac{g_nh^4r^4}{55440s^2t^2}(7r^6-24r^5s-24r^5t-24r^5+22r^4s^2+88r^4st+88r^4s+22r^4t^2+88r^4t+22r^4-88r^3s^2t-88r^3s^2-88r^3st^2-352r^3st-88r^3s-88r^3t^2-88r^3t+99r^2s^2t^2+396r^2s^2t+99r^2s^2+396r^2st^2+396r^2st+99r^2t^2-528rs^2t^2-528rs^2t-528rst^2+924s^2t^2) - \frac{f_nh^3r^3}{27720s^3t^3}(24r^6s^2t-7r^7s-7r^7t-7r^7st+24r^6s^2+24r^6st^2+59r^6st+24r^6s+24r^6t^2+24r^6t-22r^5s^3t-22r^5s^3-88r^5s^2t^2-152r^5s^2t-88r^5s^2-22r^5st^3-152r^5st^2-152r^5st-22r^5s-22r^5t^3-88r^5t^2-22r^5t+88r^4s^3t^2+132r^4s^3t+88r^4s^3+88r^4s^2t^3+352r^4s^2t^2+352r^4s^2t+88r^4s^2+132r^4st^3+352r^4st^2+132r^4st+88r^4t^3+88r^4t^2-99r^3s^3t^3-308r^3s^3t^2-308r^3s^3t-99r^3s^3-$$

$$308r^3s^2t^3 - 440r^3s^2t^2 - 308r^3s^2t - 308r^3st^3 - 308r^3st^2 - 99r^3t^3 + 297r^2s^3t^3 +$$
$$132r^2s^3t^2 + 297r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 297r^2st^3 + 528rs^3t^3 + 528rs^3t^2 +$$
$$528rs^2t^3 - 3696s^3t^3) - \frac{g_{n+1}h^4r^7}{55440(r-1)^2(s-1)^2(t-1)^2}(24r^4s - 7r^5 + 24r^4t + 12r^4 - 22r^3s^2 -$$
$$88r^3st - 44r^3s - 22r^3t^2 - 44r^3t + 88r^2s^2t + 44r^2s^2 + 88r^2st^2 + 176r^2st + 44r^2t^2 -$$
$$99rs^2t^2 - 198rs^2t - 198rst^2 + 264s^2t^2) - \frac{g_{n+r}h^4r^4}{55440(r-s)^2(r-t)^2(r-1)^2}(14r^6 - 42r^5s -$$
$$42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t -$$
$$110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t +$$
$$99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2) -$$
$$\frac{f_{n+1}h^3r^7}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 24r^5s^2t + 48r^5s^2 - 24r^5st^2 +$$
$$64r^5st - 21r^5s + 48r^5t^2 - 21r^5t - 70r^5 + 22r^4s^3t - 44r^4s^3 + 88r^4s^2t^2 - 106r^4s^2t -$$
$$116r^4s^2 + 22r^4st^3 - 106r^4st^2 - 70r^4st + 208r^4s - 44r^4t^3 - 116r^4t^2 + 208r^4t +$$
$$54r^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 - 88r^3s^2t^3 - 88r^3s^2t^2 + 506r^3s^2t - 88r^3s^2 +$$
$$66r^3st^3 + 506r^3st^2 - 660r^3st - 198r^3s + 176r^3t^3 - 88r^3t^2 - 198r^3t + 99r^2s^3t^3 +$$
$$275r^2s^3t^2 - 649r^2s^3t - 154r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 -$$
$$649r^2st^3 + 275r^2st^2 + 792r^2st - 154r^2t^3 + 198r^2t^2 - 594rs^3t^3 + 462rs^3t^2 + 693rs^3t +$$
$$462rs^2t^3 + 66rs^2t^2 - 891rs^2t + 693rst^3 - 891rst^2 + 660s^3t^3 - 924s^3t^2 - 924s^2t^3 +$$
$$1188s^2t^2) + \frac{f_{n+r}h^3r^3}{27720(r-s)^3(r-t)^3(r-1)^3}(84r^9 - 315r^8s - 315r^8t - 315r^8 + 390r^7s^2 +$$
$$1210r^7st + 1210r^7s + 390r^7t^2 + 1210r^7t + 390r^7 - 154r^6s^3 - 1536r^6s^2t - 1536r^6s^2 -$$
$$1536r^6st^2 - 4793r^6st - 1536r^6s - 154r^6t^3 - 1536r^6t^2 - 1536r^6t - 154r^6 + 616r^5s^3t +$$
$$616r^5s^3 + 2002r^5s^2t^2 + 6290r^5s^2t + 2002r^5s^2 + 616r^5st^3 + 6290r^5st^2 + 6290r^5st +$$
$$616r^5s + 616r^5t^3 + 2002r^5t^2 + 616r^5t - 814r^4s^3t^2 - 2574r^4s^3t - 814r^4s^3 -$$
$$814r^4s^2t^3 - 8569r^4s^2t^2 - 8569r^4s^2t - 814r^4s^2 - 2574r^4st^3 - 8569r^4st^2 - 2574r^4st -$$
$$814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 + 3575r^3s^2t^3 +$$
$$12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 -$$
$$5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 +$$
$$2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3) - \frac{g_{n+s}h^4r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2 - 22r^3t^2 +$$
$$44r^2s - 44r^3s + 12r^4s - 99rt^2 + 88r^2t - 88r^3t + 24r^4t + 264st^2 - 22r^3 + 24r^4 - 7r^5 -$$
$$198rst^2 + 176r^2st - 44r^3st + 44r^2st^2 - 198rst) - \frac{g_{n+t}h^4r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 -$$
$$22r^3s^2 - 99rs^2 + 88r^2s - 88r^3s + 24r^4s + 44r^2t - 44r^3t + 12r^4t + 264s^2t - 22r^3 +$$

$24r^4 - 7r^5 - 198rs^2t + 176r^2st - 44r^3st + 44r^2s^2t - 198rst) + \frac{f_{n+s}h^3r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t + 70r^5s^3 + 21r^5s^2t + 21r^5s^2 - 48r^5st^2 - 64r^5st - 48r^5s + 24r^5t^2 + 24r^5t - 54r^4s^4 - 208r^4s^3t - 208r^4s^3 + 116r^4s^2t^2 + 70r^4s^2t + 116r^4s^2 + 44r^4st^3 + 106r^4st^2 + 106r^4st + 44r^4s - 22r^4t^3 - 88r^4t^2 - 22r^4t + 198r^3s^4t + 198r^3s^4 + 88r^3s^3t^2 + 660r^3s^3t + 88r^3s^3 - 176r^3s^2t^3 - 506r^3s^2t^2 - 506r^3s^2t - 176r^3s^2 - 66r^3st^3 + 88r^3st^2 - 66r^3st + 88r^3t^3 + 88r^3t^2 - 198r^2s^4t^2 - 792r^2s^4t - 198r^2s^4 + 154r^2s^3t^3 - 275r^2s^3t^2 - 275r^2s^3t + 154r^2s^3 + 649r^2s^2t^3 + 781r^2s^2t^2 + 649r^2s^2t - 275r^2st^3 - 275r^2st^2 - 99r^2t^3 + 891rs^4t^2 + 891rs^4t - 693rs^3t^3 - 66rs^3t^2 - 693rs^3t - 462rs^2t^3 - 462rs^2t^2 + 594rst^3 - 1188s^4t^2 + 924s^3t^3 + 924s^3t^2 - 660s^2t^3) + \frac{f_{n+t}h^3r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(7r^6s - 14r^6st + 21r^6t^2 - 14r^6t + 48r^5s^2t - 24r^5s^2 - 21r^5st^2 + 64r^5st - 24r^5s - 70r^5t^3 - 21r^5t^2 + 48r^5t - 44r^4s^3t + 22r^4s^3 - 116r^4s^2t^2 - 106r^4s^2t + 88r^4s^2 + 208r^4st^3 - 70r^4st^2 - 106r^4st + 22r^4s + 54r^4t^4 + 208r^4t^3 - 116r^4t^2 - 44r^4t + 176r^3s^3t^2 + 66r^3s^3t - 88r^3s^3 - 88r^3s^2t^3 + 506r^3s^2t^2 - 88r^3s^2t - 88r^3s^2 - 198r^3st^4 - 660r^3st^3 + 506r^3st^2 + 66r^3st - 198r^3t^4 - 88r^3t^3 + 176r^3t^2 - 154r^2s^3t^3 - 649r^2s^3t^2 + 275r^2s^3t + 99r^2s^3 + 198r^2s^2t^4 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 792r^2st^4 + 275r^2st^3 - 649r^2st^2 + 198r^2t^4 - 154r^2t^3 + 693rs^3t^3 + 462rs^3t^2 - 594rs^3t - 891rs^2t^4 + 66rs^2t^3 + 462rs^2t^2 - 891rst^4 + 693rst^3 - 924s^3t^3 + 660s^3t^2 + 1188s^2t^4 - 924s^2t^3), \qquad (4.40)$

$y_{n+s} = y_n + \frac{h^2s^2y''_n}{2} + hsy'_n + \frac{g_nh^4s^4}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2) - \frac{f_nh^3s^3}{27720r^3t^3}(88r^3s^4t^2 - 22r^3s^5 - 22r^3s^5t + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 + 297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t + 24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t - 7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t - 22rs^5 + 132rs^4t^3 + 352rs^4t^2 + 132rs^4t - 308rs^3t^3 - 308rs^3t^2 + 297rs^2t^3 - 7s^7t + 24s^6t^2 + 24s^6t - 22s^5$

$t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3) - \frac{g_{n+1}h^4s^7}{55440(r-1)^2(s-1)^2(t-1)^2}(88r^2s^2t - 22r^2s^3 + 44r^2s^2 - 99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 + 88rs^2t^2 + 176rs^2t - 198rst^2 - 7s^5 + 24s^4t + 12s^4 - 22s^3t^2 - 44s^3t + 44s^2t^2) - \frac{g_{n+s}h^4s^4}{55440(r-s)^2(s-t)^2(s-1)^2}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2) - \frac{f_{n+1}h^3s^7}{27720(r-1)^3(s-1)^3(t-1)^3}(22r^3s^4t - 44r^3s^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 - 594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 - 106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 + 1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 - 70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 + 792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 - 116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2) + \frac{f_{n+s}h^3s^3}{27720(r-s)^3(s-t)^3(s-1)^3}(154r^3s^6 - 616r^3s^5t - 616r^3s^5 + 814r^3s^4t^2 + 2574r^3s^4t + 814r^3s^4 - 330r^3s^3t^3 - 3575r^3s^3t^2 - 3575r^3s^3t - 330r^3s^3 + 1485r^3s^2t^3 + 5280r^3s^2t^2 + 1485r^3s^2t - 2244r^3st^3 - 2244r^3st^2 + 924r^3t^3 - 390r^2s^7 + 1536r^2s^6t + 1536r^2s^6 - 2002r^2s^5t^2 - 6290r^2s^5t - 2002r^2s^5 + 814r^2s^4t^3 + 8569r^2s^4t^2 + 8569r^2s^4t + 814r^2s^4 - 3575r^2s^3t^3 - 12320r^2s^3t^2 - 3575r^2s^3t + 5280r^2s^2t^3 + 5280r^2s^2t^2 - 2244r^2st^3 + 315rs^8 - 1210rs^7t - 1210rs^7 + 1536rs^6t^2 + 4793rs^6t + 1536rs^6 - 616rs^5t^3 - 6290rs^5t^2 - 6290rs^5t - 616rs^5 + 2574rs^4t^3 + 8569rs^4t^2 + 2574rs^4t - 3575rs^3t^3 - 3575rs^3t^2 + 1485rs^2t^3 - 84s^9 + 315s^8t + 315s^8 - 390s^7t^2 - 1210s^7t - 390s^7 + 154s^6t^3 + 1536s^6t^2 + 1536s^6t + 154s^6 - 616s^5t^3 - 2002s^5t^2 - 616s^5t + 814s^4t^3 + 814s^4t^2 - 330s^3t^3) - \frac{f_{n+t}h^3s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(44r^3s^4t - 22r^3s^4 - 176r^3s^3t^2 - 66r^3s^3t + 88r^3s^3 + 154r^3s^2t^3 + 649r^3s^2t^2 - 275r^3s^2t - 99r^3s^2 - 693r^3st^3 - 462r^3st^2 + 594r^3st + 924r^3t^3 - 660r^3t^2 - 48r^2s^5t + 24r^2s^5 + 116r^2s^4t^2 + 106r^2s^4t - 88r^2s^4 + 88r^2s^3t^3 - 506r^2s^3t^2 + 88r^2s^3t + 88r^2s^3 - 198r^2s^2t^4 - 275r^2s^2t^3 + 781r^2s^2t^2 - 275r^2s^2t + 891r^2st^4 - 66r^2st^3 - 462r^2st^2 - 1188r^2t^4 + 924r^2t^3 + 14rs^6t -$

$7rs^6 + 21rs^5t^2 - 64rs^5t + 24rs^5 - 208rs^4t^3 + 70rs^4t^2 + 106rs^4t - 22rs^4 + 198rs^3t^4 + 660rs^3t^3 - 506rs^3t^2 - 66rs^3t - 792rs^2t^4 - 275rs^2t^3 + 649rs^2t^2 + 891rst^4 - 693rst^3 - 21s^6t^2 + 14s^6t + 70s^5t^3 + 21s^5t^2 - 48s^5t - 54s^4t^4 - 208s^4t^3 + 116s^4t^2 + 44s^4t + 198s^3t^4 + 88s^3t^3 - 176s^3t^2 - 198s^2t^4 + 154s^2t^3) - \frac{g_{n+r}h^4s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst) - \frac{g_{n+t}h^4s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 - 22r^2s^3 + 88rs^2 - 99r^2s - 88rs^3 + 24rs^4 + 264r^2t + 44s^2t - 44s^3t + 12s^4t - 22s^3 + 24s^4 - 7s^5 + 176rs^2t - 198r^2st - 44rs^3t + 44r^2s^2t - 198rst) + \frac{f_{n+r}h^3s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3),$ (4.41)

$y_{n+t} = y_n + \frac{h^2t^2y''_n}{2} + hty'_n + \frac{g_nh^4t^4}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 - 528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 - 88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 - 352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 + 99s^2t^2 - 24st^5 + 88st^4 - 88st^3 + 7t^6 - 24t^5 + 22t^4) - \frac{f_nh^3t^3}{27720r^3s^3}(297r^3s^3t^2 - 99r^3s^3t^3 + 528r^3s^3t - 3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 + 132r^3st^4 - 308r^3st^3 + 297r^3st^2 - 22r^3t^5 + 88r^3t^4 - 99r^3t^3 + 88r^2s^3t^4 - 308r^2s^3t^3 + 132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 - 152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 - 22rs^3t^5 + 132rs^3t^4 - 308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 - 152rst^5 + 132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 - 88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5) - \frac{g_{n+1}h^4t^7}{55440(r-1)^2(s-1)^2(t-1)^2}(264r^2s^2 - 99r^2s^2t$

$+88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4) - \frac{g_{n+t}h^4t^4}{55440(r-t)^2(s-t)^2(t-1)^2}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4) - \frac{f_{n+1}h^3t^7}{27720(r-1)^3(s-1)^3(t-1)^3}(99r^3s^3t^2 - 594r^3s^3t + 660r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 462r^3s^2t - 924r^3s^2 + 22r^3st^4 + 66r^3st^3 - 649r^3st^2 + 693r^3st - 44r^3t^4 + 176r^3t^3 - 154r^3t^2 - 88r^2s^3t^3 + 275r^2s^3t^2 + 462r^2s^3t - 924r^2s^3 + 88r^2s^2t^4 - 88r^2s^2t^3 - 781r^2s^2t^2 + 66r^2s^2t + 1188r^2s^2 - 24r^2st^5 - 106r^2st^4 + 506r^2st^3 + 275r^2st^2 - 891r^2st + 48r^2t^5 - 116r^2t^4 - 88r^2t^3 + 198r^2t^2 + 22rs^3t^4 + 66rs^3t^3 - 649rs^3t^2 + 693rs^3t - 24rs^2t^5 - 106rs^2t^4 + 506rs^2t^3 + 275rs^2t^2 - 891rs^2t + 7rst^6 + 64rst^5 - 70rst^4 - 660rst^3 + 792rst^2 - 14rt^6 - 21rt^5 + 208rt^4 - 198rt^3 - 44s^3t^4 + 176s^3t^3 - 154s^3t^2 + 48s^2t^5 - 116s^2t^4 - 88s^2t^3 + 198s^2t^2 - 14st^6 - 21st^5 + 208st^4 - 198st^3 + 21t^6 - 70t^5 + 54t^4) - \frac{f_{n+t}h^3t^3}{27720(r-t)^3(s-t)^3(t-1)^3}(1485r^3s^3t^2 - 330r^3s^3t^3 - 2244r^3s^3t + 924r^3s^3 + 814r^3s^2t^4 - 3575r^3s^2t^3 + 5280r^3s^2t^2 - 2244r^3s^2t - 616r^3st^5 + 2574r^3st^4 - 3575r^3st^3 + 1485r^3st^2 + 154r^3t^6 - 616r^3t^5 + 814r^3t^4 - 330r^3t^3 + 814r^2s^3t^4 - 3575r^2s^3t^3 + 5280r^2s^3t^2 - 2244r^2s^3t - 2002r^2s^2t^5 + 8569r^2s^2t^4 - 12320r^2s^2t^3 + 5280r^2s^2t^2 + 1536r^2st^6 - 6290r^2st^5 + 8569r^2st^4 - 3575r^2st^3 - 390r^2t^7 + 1536r^2t^6 - 2002r^2t^5 + 814r^2t^4 - 616rs^3t^5 + 2574rs^3t^4 - 3575rs^3t^3 + 1485rs^3t^2 + 1536rs^2t^6 - 6290rs^2t^5 + 8569rs^2t^4 - 3575rs^2t^3 - 1210rst^7 + 4793rst^6 - 6290rst^5 + 2574rst^4 + 315rt^8 - 1210rt^7 + 1536rt^6 - 616rt^5 + 154s^3t^6 - 616s^3t^5 + 814s^3t^4 - 330s^3t^3 - 390s^2t^7 + 1536s^2t^6 - 2002s^2t^5 + 814s^2t^4 + 315st^8 - 1210st^7 + 1536st^6 - 616st^5 - 84t^9 + 315t^8 - 390t^7 + 154t^6) + \frac{f_{n+r}h^3t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(198r^4s^2t^2 - 891r^4s^2t + 1188r^4s^2 - 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 - 198r^4t^3 + 198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t - 924r^3s^2 + 208r^3st^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 - 154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 + 462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 - 781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 - 649r^2st^2 + 21r^2t^6 - 21r^2t^5 - 116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594$

$rs^3t+48rs^2t^5-106rs^2t^4-88rs^2t^3+275rs^2t^2-14rst^6+64rst^5-106rst^4+66rst^3-14rt^6+48rt^5-44rt^4+22s^3t^4-88s^3t^3+99s^3t^2-24s^2t^5+88s^2t^4-88s^2t^3+7st^6-24st^5+22st^4)+\frac{f_{n+s}h^3t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(154r^3s^3t^2-693r^3s^3t+924r^3s^3-176r^3s^2t^3+649r^3s^2t^2-462r^3s^2t-660r^3s^2+44r^3st^4-66r^3st^3-275r^3st^2+594r^3st-22r^3t^4+88r^3t^3-99r^3t^2-198r^2s^4t^2+891r^2s^4t-1188r^2s^4+88r^2s^3t^3-275r^2s^3t^2-66r^2s^3t+924r^2s^3+116r^2s^2t^4-506r^2s^2t^3+781r^2s^2t^2-462r^2s^2t-48r^2st^5+106r^2st^4+88r^2st^3-275r^2st^2+24r^2t^5-88r^2t^4+88r^2t^3+198rs^4t^3-792rs^4t^2+891rs^4t-208rs^3t^4+660rs^3t^3-275rs^3t^2-693rs^3t+21rs^2t^5+70rs^2t^4-506rs^2t^3+649rs^2t^2+14rst^6-64rst^5+106rst^4-66rst^3-7rt^6+24rt^5-22rt^4-54s^4t^4+198s^4t^3-198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-176s^2t^3+14st^6-48st^5+44st^4)-\frac{g_{n+r}h^4t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2-22s^2t^3+264rs^2+44rt^2-44rt^3+12rt^4+88st^2-99s^2t-88st^3+24st^4-22t^3+24t^4-7t^5+176rst^2-198rs^2t-44rst^3+44rs^2t^2-198rst)-\frac{g_{n+s}h^4t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2-22r^2t^3+264r^2s+88rt^2-99r^2t-88rt^3+24rt^4+44st^2-44st^3+12st^4-22t^3+24t^4-7t^5+176rst^2-198r^2st-44rst^3+44r^2st^2-198rst) \quad (4.42)$

$y_{n+1}=y_n+hy'_n+\frac{h^2y''_n}{2}+\frac{g_nh^4}{55440r^2s^2t^2}(924r^2s^2t^2-528r^2s^2t+99r^2s^2-528r^2st^2+396r^2st-88r^2s+99r^2t^2-88r^2t+22r^2-528rs^2t^2+396rs^2t-88rs^2+396rst^2-352rst+88rs-88rt^2+88rt-24r+99s^2t^2-88s^2t+22s^2-88st^2+88st-24s+22t^2-24t+7)+\frac{f_{n+1}h^3}{27720(r-1)^3(s-1)^3(t-1)^3}(924r^3s^3t^3-2244r^3s^3t^2+1485r^3s^3t-330r^3s^3-2244r^3s^2t^3+5280r^3s^2t^2-3575r^3s^2t+814r^3s^2+1485r^3st^3-3575r^3st^2+2574r^3st-616r^3s-330r^3t^3+814r^3t^2-616r^3t+154r^3-2244r^2s^3t^3+5280r^2s^3t^2-3575r^2s^3t+814r^2s^3+5280r^2s^2t^3-12320r^2s^2t^2+8569r^2s^2t-2002r^2s^2-3575r^2st^3+8569r^2st^2-6290r^2st+1536r^2s+814r^2t^3-2002r^2t^2+1536r^2t-390r^2+1485rs^3t^3-3575rs^3t^2+2574rs^3t-616rs^3-3575rs^2t^3+8569rs^2t^2-6290rs^2t+1536rs^2+2574rst^3-6290rst^2+4793rst-1210rs-616rt^3+1536rt^2-1210rt+315r-330s^3t^3+814s^3t^2-616s^3t+154s^3+814s^2t^3-2002s^2t^2+1536s^2t-390s^2-616st^3+1536st^2-1210st+315s+154t^3-390t^2+315t-84)$

$-\frac{g_{n+1}h^4}{55440(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st - 110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 - 440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14) - \frac{f_nh^3}{27720r^3s^3t^3}(528r^3s^3t^2 - 3696r^3s^3t^3 + 297r^3s^3t - 99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2 - 308r^3s^2t + 88r^3s^2 + 297r^3st^3 - 308r^3st^2 + 132r^3st - 22r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t + 528r^2s^3t^3 + 132r^2s^3t^2 - 308r^2s^3t + 88r^2s^3 + 132r^2s^2t^3 - 440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 - 152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3t^2 + 132rs^3t - 22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs - 22rt^3 + 24rt^2 - 7rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st)\frac{f_{n+r}h^3}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t - 1188r^4s^2t^2 - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + -88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs + 44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st) + \frac{f_{n+s}h^3}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 - 660r^3s^2t^3 - 462r^3s^2t^2 + 649r^3s^2t - 176r^3s^2 + 594r^3st^3 - 275r^3st^2 - 66r^3st + 44r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t - 1188r^2s^4t^2 + 891r^2s^4t - 198r^2s^4 + 924r^2s^3t^3 - 66r^2s^3t^2 - 275r^2s^3t + 88r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 - 275r^2st^3 + 88r^2st^2 + 106r^2st - 48r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 891rs^4t^2 - 792rs^4t + 198rs^4 - 693rs^3t^3 - 275rs^3t^2 + 660rs^3t - 208rs^3 + 649rs^2t^3 - 506rs^2t^2 + 70rs^2t + 21rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs - 22rt^3 + 24rt^2 - 7rt - 198s^4t^2 + 198s^4t - 54s^4 + 154s^3t^3 + 88s^3t^2 - 208s^3t + 70s^3 - 176s^2t^3 + 116s^2t^2 + 21s^2t - 21s^2 + 44st^3 - 48st^2 + 14st) + \frac{f_{n+t}h^3}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(660r^3s^3t^2 - 594r^3s^3t + 99r^3s^3 - 924r^3s^2t^3 + 462r^3s^2t^2 + 275r^3s^2t - 88r^3s^2 + 693r^3st^3 - 649r^3st^2 + 66r^3st + 22r^3s - 154r^3t^3 + 176r^3t^2 - 44r^3t - 924r^2s^3t^3 + 462r^2s^3t^2 + 275r^2s^3t - 88r^2s^3 + 1188r^2s^2t^4 + 66r^2s^2$

$t^3 - 781r^2s^2t^2 - 88r^2s^2t + 88r^2s^2 - 891r^2st^4 + 275r^2st^3 + 506r^2st^2 - 106r^2st - 24r^2s + 198r^2t^4 - 88r^2t^3 - 116r^2t^2 + 48r^2t + 693rs^3t^3 - 649rs^3t^2 + 66rs^3t + 22rs^3 - 891rs^2t^4 + 275rs^2t^3 + 506rs^2t^2 - 106rs^2t - 24rs^2 + 792rst^4 - 660rst^3 - 70rst^2 + 64rst + 7rs - 198rt^4 + 208rt^3 - 21rt^2 - 14rt - 154s^3t^3 + 176s^3t^2 - 44s^3t + 198s^2t^4 - 88s^2t^3 - 116s^2t^2 + 48s^2t - 198st^4 + 208st^3 - 21st^2 - 14st + 54t^4 - 70t^3 + 21t^2) - \frac{g_{n+r}h^4}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(12r + 24s + 24t - 99s^2t^2 - 44rs - 44rt - 88st + 44rs^2 + 44rt^2 + 88st^2 + 88s^2t - 22s^2 - 22t^2 - 198rst^2 - 198rs^2t + 264rs^2t^2 + 176rst - 7) - \frac{g_{n+s}h^4}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(24r + 12s + 24t - 99r^2t^2 - 44rs - 88rt - 44st + 44r^2s + 88rt^2 + 88r^2t + 44st^2 - 22r^2 - 22t^2 - 198rst^2 - 198r^2st + 264r^2st^2 + 176rst - 7) - \frac{g_{n+t}h^4}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(24r + 24s + 12t - 99r^2s^2 - 88rs - 44rt - 44st + 88rs^2 + 88r^2 s + 44r^2t + 44s^2t - 22r^2 - 22s^2 - 198rs^2t - 198r^2st + 264r^2s^2t + 176rst - 7). \quad (4.43)$

The first derivative corresponding to the main block (4.39) can be expressed in a block form

$$\begin{aligned} I_4 Y_{n+1}^{'[3]_3} &= M_2^{'[3]_3} Y_{n-1}^{[3]_3} + hM_3^{'[3]_3} Y_{n-2}^{[3]_3} + h^2[E_1^{'[3]_3} F_n^{[3]_3} + E_2^{'[3]_3} F_{n+1}^{[3]_3}] + h^3[K_1^{'[3]_3} \\ & G_n^{[3]_3} + K_2^{'[3]_3} G_{n+1}^{[3]_3}] \end{aligned} \tag{4.44}$$

where

$$Y_{n+1}^{'[3]_3} = \begin{pmatrix} y'_{n+r} \\ y'_{n+s} \\ y'_{n+t} \\ y'_{n+1} \end{pmatrix}, \; M_2^{'[3]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; M_3^{'[3]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_1^{'[3]_3} = \begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{'[3]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{'[3]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{'[3]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{'[3]_3} \end{pmatrix}, \; E_2^{'[3]_3} = \begin{pmatrix} E_{2_{11}}^{'[3]_3} & E_{2_{12}}^{'[3]_3} & E_{2_{13}}^{'[3]_3} & E_{2_{14}}^{'[3]_3} \\ E_{2_{21}}^{'[3]_3} & E_{2_{22}}^{'[3]_3} & E_{2_{23}}^{'[3]_3} & E_{2_{24}}^{'[3]_3} \\ E_{2_{31}}^{'[3]_3} & E_{2_{32}}^{'[3]_3} & E_{2_{33}}^{'[3]_3} & E_{2_{34}}^{'[3]_3} \\ E_{2_{41}}^{'[3]_3} & E_{2_{42}}^{'[3]_3} & E_{2_{43}}^{'[3]_3} & E_{2_{44}}^{'[3]_3} \end{pmatrix},$$

$$K_1^{'[3]_3} = \begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{'[3]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{'[3]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{'[3]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{'[3]_3} \end{pmatrix}, \; K_2^{'[3]_3} = \begin{pmatrix} K_{2_{11}}^{'[3]_3} & K_{2_{12}}^{'[3]_3} & K_{2_{13}}^{'[3]_3} & K_{2_{14}}^{'[3]_3} \\ K_{2_{21}}^{'[3]_3} & K_{2_{22}}^{'[3]_3} & K_{2_{23}}^{'[3]_3} & K_{2_{24}}^{'[3]_3} \\ K_{2_{31}}^{'[3]_3} & K_{2_{32}}^{'[3]_3} & K_{2_{33}}^{'[3]_3} & K_{2_{34}}^{'[3]_3} \\ K_{2_{41}}^{'[3]_3} & K_{2_{42}}^{'[3]_3} & K_{2_{43}}^{'[3]_3} & K_{2_{44}}^{'[3]_3} \end{pmatrix}.$$

The entries of $E_1^{'[3]_3}, E_2^{'[3]_3}, K_1^{'[3]_3}$ and $K_2^{'[3]_3}$ are given by

$E_{1_{14}}^{'[3]_3} = -\frac{r^2}{27720s^3t^3}(-42r^7st - 42r^7s - 42r^7t + 132r^6s^2t + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 - 1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188 r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 - 9702s^3t^3)$,

$E_{1_{24}}^{'[3]_3} = -\frac{s^2}{27720r^3t^3}(-110r^3s^5t - 110r^3s^5 + 396r^3s^4t^2 + 583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 + 264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t + 132r^2s^6 - 440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 - 1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t - 110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t + 132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3)$,

$E_{1_{34}}^{'[3]_3} = -\frac{t^2}{27720r^3s^3}(-396r^3s^3t^3 + 990r^3s^3t^2 + 1848r^3s^3t - 9702r^3s^3 + 396r^3s^2t^4 - 1188r^3s^2t^3 + 264r^3s^2t^2 + 1848r^3s^2t - 110r^3st^5 + 583r^3st^4 - 1188r^3st^3 + 990r^3st^2 - 110r^3t^5 + 396r^3t^4 - 396r^3t^3 + 396r^2s^3t^4 - 1188r^2s^3t^3 + 264r^2s^3t^2 + 1848r^2s^3t - 440r^2s^2t^5 + 1540r^2s^2t^4 - 1584r^2s^2t^3 + 264r^2s^2t^2 + 132r^2st^6 - 748r^2st^5 + 1540r^2st^4 - 1188r^2st^3 + 132r^2t^6 - 440r^2t^5 + 396r^2t^4 - 110rs^3t^5 + 583rs^3t^4 - 1188rs^3t^3 + 990rs^3t^2 + 132rs^2t^6 - 748rs^2t^5 + 1540rs^2t^4 - 1188rs^2t^3 - 42rst^7 +$

$321rst^6-748rst^5+583rst^4-42rt^7+132rt^6-110rt^5-110s^3t^5+396s^3t^4-396s^3t^3+132s^2t^6-440s^2t^5+396s^2t^4-42st^7+132st^6-110st^5)$,

$E_{1_{44}}^{'[3]_3}=-\frac{1}{27720r^3s^3t^3}(-9702r^3s^3t^3+1848r^3s^3t^2+990r^3s^3t-396r^3s^3+1848r^3s^2t^3+264r^3s^2t^2-1188r^3s^2t+396r^3s^2+990r^3st^3-1188r^3st^2+583r^3st-110r^3s-396r^3t^3+396r^3t^2-110r^3t+1848r^2s^3t^3+264r^2s^3t^2-1188r^2s^3t+396r^2s^3+264r^2s^2t^3-1584r^2s^2t^2+1540r^2s^2t-440r^2s^2-1188r^2st^3+1540r^2st^2-748r^2st+132r^2s+396r^2t^3-440r^2t^2+132r^2t+990rs^3t^3-1188rs^3t^2+583rs^3t-110rs^3-1188rs^2t^3+1540rs^2t^2-748rs^2t+132rs^2+583rst^3-748rst^2+321rst-42rs-110rt^3+132rt^2-42rt-396s^3t^3+396s^3t^2-110s^3t+396s^2t^3-440s^2t^2+132s^2t-110st^3+132st^2-42st)$,

$E_{2_{11}}^{'[3]_3}=\frac{r^2}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^9-2646r^8s-2646r^8t-2646r^8+3069r^7s^2+9420r^7st+9420r^7s+3069r^7t^2+9420r^7t+3069r^7-1155r^6s^3-11121r^6s^2t-11121r^6s^2-11121r^6st^2-34278r^6st-11121r^6s-1155r^6t^3-11121r^6t^2-11121r^6t-1155r^6+4235r^5s^3t+4235r^5s^3+13376r^5s^2t^2+41437r^5s^2t+13376r^5s^2+4235r^5st^3+41437r^5st^2+41437r^5st+4235r^5s+4235r^5t^3+13376r^5t^2+4235r^5t-5148r^4s^3t^2-16027r^4s^3t-5148r^4s^3-5148r^4s^2t^3-51436r^4s^2t^2-51436r^4s^2t-5148r^4s^2-16027r^4st^3-51436r^4st^2-16027r^4st-5148r^4t^3-5148r^4t^2+1980r^3s^3t^3+20196r^3s^3t^2+20196r^3s^3t+1980r^3s^3+20196r^3s^2t^3+66330r^3s^2t^2+20196r^3s^2t+20196r^3st^3+20196r^3st^2+1980r^3t^3-7920r^2s^3t^3-26598r^2s^3t^2-7920r^2s^3t-26598r^2s^2t^3-26598r^2s^2t^2-7920r^2st^3+10626rs^3t^3+10626rs^3t^2+10626rs^2t^3-4158s^3t^3)$,

$E_{2_{21}}^{'[3]_3}=\frac{s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(297r^4s^4-990r^4s^3t-990r^4s^3+891r^4s^2t^2+3564r^4s^2t+891r^4s^2-3564r^4st^2-3564r^4st+4158r^4t^2-399r^3s^5+1067r^3s^4t+1067r^3s^4-363r^3s^3t^2-2992r^3s^3t-363r^3s^3-693r^3s^2t^3+891r^3s^2t^2+891r^3s^2t-693r^3s^2+2772r^3st^3+726r^3st^2+2772r^3st-3234r^3t^3-3234r^3t^2+126r^2s^6-105r^2s^5t-105r^2s^5-616r^2s^4t^2-407r^2s^4t-616r^2s^4+825r^2s^3t^3+2387r^2s^3t^2+2387r^2s^3t+825r^2s^3-2673r^2s^2t^3-3168r^2s^2t^2-2673r^2s^2t+1518r^2st^3+1518r^2st^2$

$+2310r^2t^3 - 84rs^6t - 84rs^6 + 264rs^5t^2 + 345rs^5t + 264rs^5 - 220rs^4t^3 - 506rs^4t^2 - 506rs^4t - 220rs^4 + 275rs^3t^3 - 484rs^3t^2 + 275rs^3t + 1188rs^2t^3 + 1188rs^2t^2 - 2178rst^3 + 42s^6t - 132s^5t^2 - 132s^5t + 110s^4t^3 + 440s^4t^2 + 110s^4t - 396s^3t^3 - 396s^3t^2 + 396s^2t^3)$,

$E_{2_{31}}^{'[3]_3} = \frac{t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2 - 3564r^4s^2t + 4158r^4s^2 - 990r^4st^3 + 3564r^4st^2 - 3564r^4st + 297r^4t^4 - 990r^4t^3 + 891r^4t^2 - 693r^3s^3t^2 + 2772r^3s^3t - 3234r^3s^3 - 363r^3s^2t^3 + 891r^3s^2t^2 + 726r^3s^2t - 3234r^3s^2 + 1067r^3st^4 - 2992r^3st^3 + 891r^3st^2 + 2772r^3st - 399r^3t^5 + 1067r^3t^4 - 363r^3t^3 - 693r^3t^2 + 825r^2s^3t^3 - 2673r^2s^3t^2 + 1518r^2s^3t + 2310r^2s^3 - 616r^2s^2t^4 + 2387r^2s^2t^3 - 3168r^2s^2t^2 + 1518r^2s^2t - 105r^2st^5 - 407r^2st^4 + 2387r^2st^3 - 2673r^2st^2 + 126r^2t^6 - 105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t + 264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 - 84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 - 396s^2t^3 + 42st^6 - 132st^5 + 110st^4)$,

$E_{2_{41}}^{'[3]_3} = \frac{-1}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-4158r^4s^2t^2 + 3564r^4s^2t - 891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 + 3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - 891r^3st^2 + 2992r^3st - 1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - 1518r^2s^3t^2 + 2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 + 2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t - 126r^2 + 2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + 506rs^2t - 264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st)$,

$E_{2_{12}}^{'[3]_3} = \frac{r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-126r^6s^2 + 84r^6st + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 - 345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 + 616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s - 110r^4t^3 - 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 + 2992r^3s^3t + 363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3+$

$484r^3st^2 - 275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 + 693r^2s^3t^3 - 891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t - 1188r^2st^3 - 1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 - 2772rs^3t - 1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 - 2310s^2t^3)$,

$E_{2_{22}}^{'[3]_3} = \frac{s^2}{27720(r-s)^3(s-t)^3(s-1)^3}(1155r^3s^6 - 4235r^3s^5t - 4235r^3s^5 + 5148r^3s^4t^2 + 16027r^3s^4t + 5148r^3s^4 - 1980r^3s^3t^3 - 20196r^3s^3t^2 - 20196r^3s^3t - 1980r^3s^3 + 7920r^3s^2t^3 + 26598r^3s^2t^2 + 7920r^3s^2t - 10626r^3st^3 - 10626r^3st^2 + 4158r^3t^3 - 3069r^2s^7 + 11121r^2s^6t + 11121r^2s^6 - 13376r^2s^5t^2 - 41437r^2s^5t - 13376r^2s^5 + 5148r^2s^4t^3 + 51436r^2s^4t^2 + 51436r^2s^4t + 5148r^2s^4 - 20196r^2s^3t^3 - 66330r^2s^3t^2 - 20196r^2s^3t + 26598r^2s^2t^3 + 26598r^2s^2t^2 - 10626r^2st^3 + 2646rs^8 - 9420rs^7t - 9420rs^7 + 11121rs^6t^2 + 34278rs^6t + 11121rs^6 - 4235rs^5t^3 - 41437rs^5t^2 - 41437rs^5t - 4235rs^5 + 16027rs^4t^3 + 51436rs^4t^2 + 16027rs^4t - 20196rs^3t^3 - 20196rs^3t^2 + 7920rs^2t^3 - 756s^9 + 2646s^8t + 2646s^8 - 3069s^7t^2 - 9420s^7t - 3069s^7 + 1155s^6t^3 + 11121s^6t^2 + 11121s^6t + 1155s^6 - 4235s^5t^3 - 13376s^5t^2 - 4235s^5t + 5148s^4t^3 + 5148s^4t^2 - 1980s^3t^3)$,

$E_{2_{32}}^{'[3]_3} = \frac{t^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^3s^3t^2 - 2772r^3s^3t + 3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 - 275r^3st^3 - 1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 + 3564r^2s^4t - 4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 - 2387r^2s^2t^3 + 3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 + 132r^2t^5 - 440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 + 2992rs^3t^3 - 891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 + 84rst^6 - 345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 - 891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + 616s^2t^4 - 825s^2t^3 + 84st^6 - 264st^5 + 220st^4)$,

$E_{2_{42}}^{'[3]_3} = \frac{1}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s -$

$396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 -$
$726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 -$
$1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 -$
$3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 -$
$2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 +$
$132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 -$
$825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st)$,

$E_{2_{13}}^{'[3]_3} = \frac{r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-84r^6st + 42r^6s + 126r^6t^2 - 84r^6t + 264r^5s^2t -$
$132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t +$
$110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st +$
$110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 -$
$363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 +$
$275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t +$
$396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 + 891r^2st^3 -$
$2673r^2st^2 + 891r^2t^4 - 693r^2t^3 + 2772rs^3t^3 + 1518rs^3t^2 - 2178rs^3t - 3564rs^2t^4 +$
$726rs^2t^3 + 1518rs^2t^2 - 3564rst^4 + 2772rst^3 - 3234s^3t^3 + 2310s^3t^2 + 4158s^2t^4 -$
$3234s^2t^3)$,

$E_{2_{23}}^{'[3]_3} = -\frac{s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t - 110r^3s^4 - 825r^3s^3t^2 - 275r^3s^3t +$
$396r^3s^3 + 693r^3s^2t^3 + 2673r^3s^2t^2 - 1188r^3s^2t - 396r^3s^2 - 2772r^3st^3 - 1518r^3st^2 +$
$2178r^3st + 3234r^3t^3 - 2310r^3t^2 - 264r^2s^5t + 132r^2s^5 + 616r^2s^4t^2 + 506r^2s^4t -$
$440r^2s^4 + 363r^2s^3t^3 - 2387r^2s^3t^2 + 484r^2s^3t + 396r^2s^3 - 891r^2s^2t^4 - 891r^2s^2t^3 +$
$3168r^2s^2t^2 - 1188r^2s^2t + 3564r^2st^4 - 726r^2st^3 - 1518r^2st^2 - 4158r^2t^4 + 3234r^2t^3 +$
$84rs^6t - 42rs^6 + 105rs^5t^2 - 345rs^5t + 132rs^5 - 1067rs^4t^3 + 407rs^4t^2 + 506rs^4t -$
$110rs^4 + 990rs^3t^4 + 2992rs^3t^3 - 2387rs^3t^2 - 275rs^3t - 3564rs^2t^4 - 891rs^2t^3 +$
$2673rs^2t^2 + 3564rst^4 - 2772rst^3 - 126s^6t^2 + 84s^6t + 399s^5t^3 + 105s^5t^2 - 264s^5t -$
$297s^4t^4 - 1067s^4t^3 + 616s^4t^2 + 220s^4t + 990s^3t^4 + 363s^3t^3 - 825s^3t^2 - 891s^2t^4 +$
$693s^2t^3)$,

$E^{'[3]_3}_{2_{33}} = -\frac{t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(-1980r^3s^3t^3 + 7920r^3s^3t^2 - 10626r^3s^3t + 4158r^3s^3 + 5148r^3s^2t^4 - 20196r^3s^2t^3 + 26598r^3s^2t^2 - 10626r^3s^2t - 4235r^3st^5 + 16027r^3st^4 - 20196r^3st^3 + 7920r^3st^2 + 1155r^3t^6 - 4235r^3t^5 + 5148r^3t^4 - 1980r^3t^3 + 5148r^2s^3t^4 - 20196r^2s^3t^3 + 26598r^2s^3t^2 - 10626r^2s^3t - 13376r^2s^2t^5 + 51436r^2s^2t^4 - 66330r^2s^2t^3 + 26598r^2s^2t^2 + 11121r^2st^6 - 41437r^2st^5 + 51436r^2st^4 - 20196r^2st^3 - 3069r^2t^7 + 11121r^2t^6 - 13376r^2t^5 + 5148r^2t^4 - 4235rs^3t^5 + 16027rs^3t^4 - 20196rs^3t^3 + 7920rs^3t^2 + 11121rs^2t^6 - 41437rs^2t^5 + 51436rs^2t^4 - 20196rs^2t^3 - 9420rst^7 + 34278rst^6 - 41437rst^5 + 16027rst^4 + 2646rt^8 - 9420rt^7 + 11121rt^6 - 4235rt^5 + 1155s^3t^6 - 4235s^3t^5 + 5148s^3t^4 - 1980s^3t^3 - 3069s^2t^7 + 11121s^2t^6 - 13376s^2t^5 + 5148s^2t^4 + 2646st^8 - 9420st^7 + 11121st^6 - 4235st^5 - 756t^9 + 2646t^8 - 3069t^7 + 1155t^6)$,

$E^{'[3]_3}_{2_{43}} = \frac{1}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2 - 2178r^3s^3t + 396r^3s^3 - 3234r^3s^2t^3 + 1518r^3s^2t^2 + 1188r^3s^2t - 396r^3s^2 + 2772r^3st^3 - 2673r^3st^2 + 275r^3st + 110r^3s - 693r^3t^3 + 825r^3t^2 - 220r^3t - 3234r^2s^3t^3 + 1518r^2s^3t^2 + 1188r^2s^3t - 396r^2s^3 + 4158r^2s^2t^4 + 726r^2s^2t^3 - 3168r^2s^2t^2 - 484r^2s^2t + 440r^2s^2 - 3564r^2st^4 + 891r^2st^3 + 2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t + 2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 - 506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 + 1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 - 616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2)$,

$E^{'[3]_3}_{2_{14}} = -\frac{r^6}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^6st - 84r^6s - 84r^6t + 126r^6 - 132r^5s^2t + 264r^5s^2 - 132r^5st^2 + 345r^5st - 105r^5s + 264r^5t^2 - 105r^5t - 399r^5 + 110r^4s^3t - 220r^4s^3 + 440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 - 506r^4st^2 - 407r^4st + 1067r^4s - 220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 - 396r^3s^2t^3 - 484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 + 2387r^3st^2 - 2992r^3st - 990r^3s + 825r^3t^3 - 363r^3t^2 - 990r^3t + 396r^2s^3t^3 + 1188r^2s^3t^2 - 2673r^2s^3t - 693r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 + 726$

$rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2)$,

$E'^{[3]_3}_{2_{24}} = -\frac{s^6}{27720(r-1)^3(s-1)^3(t-1)^3}(110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + 440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 + 2387rs^3t^2 - 2992rs^3t - 990rs^3 - 2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - 616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2)$,

$E'^{[3]_3}_{2_{34}} = -\frac{t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 + 1188r^3s^2t^2 + 1518r^3s^2t - 3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st - 220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + 1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 + 440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - 506r^2st^4 + 2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 + 275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 - 3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4)$,

$E'^{[3]_3}_{2_{44}} = \frac{1}{27720(r-1)^3(s-1)^3(t-1)^3}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - 10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 -$

$20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - 41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756)$,

$K_{1_{14}}^{'[3]_3} = \frac{r^3}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2)$,

$K_{1_{24}}^{'[3]_3} = \frac{s^3}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st + 1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 + 21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2)$,

$K_{1_{34}}^{'[3]_3} = \frac{t^3}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4)$,

$K_{1_{44}}^{'[3]_3} = \frac{1}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21)$,

$K_{2_{11}}^{'[3]_3} = \frac{-r^3}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2)$,

$K_{2_{21}}^{'[3]_3} = \frac{-s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^3t^2 + 99rs^2 - 110rs^3 + 33rs^4 + 462rt^2 - 198st^2 + 198s^2t - 220s^3t + 66s^4t - 55s^3 + 66s^4 - 21s^5 - 396rst^2 + 396rs^2t - 110rs^3t + 99rs^2t^2 - 396rst)$,

$K_{2_{31}}^{'[3]_3} = \frac{-t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst)$,

$K_{2_{41}}^{'[3]_3} = \frac{-1}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21)$,

$K_{2_{12}}^{'[3]_3} = \frac{-r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 + 396r^2st - 110r^3st + 99r^2st^2 - 396rst)$,

$K_{2_{22}}^{'[3]_3} = \frac{-s^3}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2)$,

$K_{2_{32}}^{'[3]_3} = \frac{-t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + 66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + 99r^2st^2 - 396rst)$,

$K_{2_{42}}^{'[3]_3} = \frac{-1}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + 99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + 396rst - 21)$,

$K_{2_{13}}^{'[3]_3} = \frac{-r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst)$,

$K_{2_{23}}^{'[3]_3} = \frac{-s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^2s^3 + 198rs^2 - 198r^2s - 220rs^3 + 66rs^4 + 462r^2t + 99s^2t - 110s^3t + 33s^4t - 55s^3 + 66s^4 - 21s^5 + 396rs^2t - 396r^2st - 110rs^3t + 99r^2s^2t - 396rst)$,

$K_{2_{33}}^{'[3]_3} = \frac{-t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)$,

$K_{2_{43}}^{'[3]_3} = \frac{-1}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t + 396rst - 21)$,

$K_{2_{14}}^{'[3]_3} = \frac{-r^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-21r^5 + 66r^4s + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st - 110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 - 198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$,

$K_{2_{24}}^{'[3]_3} = \frac{-s^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-55r^2s^3 + 198r^2s^2t + 99r^2s^2 - 198r^2st^2 - 396r^2st + 462r^2t^2 + 66rs^4 - 220rs^3t - 110rs^3 + 198rs^2t^2 + 396rs^2t - 396rst^2 - 21s^5 + 66s^4t + 33s^4 - 55s^3t^2 - 110s^3t + 99s^2t^2)$,

$K_{2_{34}}^{'[3]_3} = \frac{-t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-198r^2s^2t + 462r^2s^2 + 198r^2st^2 - 396r^2st - 55r^2t^3 + 99r^2t^2 + 198rs^2t^2 - 396rs^2t - 220rst^3 + 396rst^2 + 66rt^4 - 110rt^3 - 55s^2t^3 + 99s^2t^2 + 66st^4 - 110st^3 - 21t^5 + 33t^4)$,

$K_{2_{44}}^{'[3]_3} = \frac{-1}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$.

Similarly, the second derivative corresponding to (4.39) is given by

$$\begin{aligned} I_4Y_{n+1}^{''[3]_3} &= M_3^{''[3]_3}Y_{n-2}^{[3]_3} + h[E_1^{''[3]_3}F_n^{[3]_3} + E_2^{''[3]_3}F_{n+1}^{[3]_3}] + h^2[K_1^{''[3]_3}G_n^{[3]_3} \\ &\quad + K_2^{''[3]_3}G_{n+1}^{[3]_3}] \end{aligned} \quad (4.45)$$

where

$$Y_{n+1}^{''[3]_3} = \begin{pmatrix} y''_{n+r} \\ y''_{n+s} \\ y''_{n+t} \\ y''_{n+1} \end{pmatrix}, \ M_3^{''[3]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$E_1^{''[3]_3} = \begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{''[3]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{''[3]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{''[3]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{''[3]_3} \end{pmatrix}, \ E_2^{''[3]_3} = \begin{pmatrix} E_{2_{11}}^{''[3]_3} & E_{2_{12}}^{''[3]_3} & E_{2_{13}}^{''[3]_3} & E_{2_{14}}^{''[3]_3} \\ E_{2_{21}}^{''[3]_3} & E_{2_{22}}^{''[3]_3} & E_{2_{23}}^{''[3]_3} & E_{2_{24}}^{''[3]_3} \\ E_{2_{31}}^{''[3]_3} & E_{2_{32}}^{''[3]_3} & E_{2_{33}}^{''[3]_3} & E_{2_{34}}^{''[3]_3} \\ E_{2_{41}}^{''[3]_3} & E_{2_{42}}^{''[3]_3} & E_{2_{43}}^{''[3]_3} & E_{2_{44}}^{''[3]_3} \end{pmatrix},$$

$$K_1^{''[3]_3} = \begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{''[3]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{''[3]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{''[3]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{''[3]_3} \end{pmatrix}, \ K_2^{''[3]_3} = \begin{pmatrix} K_{2_{11}}^{''[3]_3} & K_{2_{12}}^{''[3]_3} & K_{2_{13}}^{''[3]_3} & K_{2_{14}}^{''[3]_3} \\ K_{2_{21}}^{''[3]_3} & K_{2_{22}}^{''[3]_3} & K_{2_{23}}^{''[3]_3} & K_{2_{24}}^{''[3]_3} \\ K_{2_{31}}^{''[3]_3} & K_{2_{32}}^{''[3]_3} & K_{2_{33}}^{''[3]_3} & K_{2_{34}}^{''[3]_3} \\ K_{2_{41}}^{''[3]_3} & K_{2_{42}}^{''[3]_3} & K_{2_{43}}^{''[3]_3} & K_{2_{44}}^{''[3]_3} \end{pmatrix}$$

whose entries of $E_1^{''[3]_3}, E_2^{''[3]_3}, K_1^{''[3]_3}$ and $K_2^{''[3]_3}$ as below:

$E_{1_{14}}^{''[3]_3} = \frac{r}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 - 20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t + 60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 - 69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 - 69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 + 144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 - 168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3)$,

$E_{1_{24}}^{''[3]_3} = \frac{s}{1260r^3t^3}(15r^3s^5t + 15r^3s^5 - 48r^3s^4t^2 - 69r^3s^4t - 48r^3s^4 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 - 84r^3s^2t^3 - 84r^3s^2t - 168r^3st^3 - 168r^3st^2 + 630r^3t^3 - 20r^2s^6t - 20r^2s^6 + 60r^2s^5t^2 + 100r^2s^5t + 60r^2s^5 - 48r^2s^4t^3 - 180r^2s^4t^2 - 180r^2s^4t - 48r^2s^4 + 120r^2s^3t^3 + 144r^2s^3t^2 + 120r^2s^3t - 168r^2st^3 + 7rs^7t + 7rs^7 - 20rs^6t^2 - 48$

$rs^6t - 20rs^6 + 15rs^5t^3 + 100rs^5t^2 + 100rs^5t + 15rs^5 - 69rs^4t^3 - 180rs^4t^2 - 69rs^4t + 120rs^3t^3 + 120rs^3t^2 - 84rs^2t^3 + 7s^7t - 20s^6t^2 - 20s^6t + 15s^5t^3 + 60s^5t^2 + 15s^5t - 48s^4t^3 - 48s^4t^2 + 42s^3t^3)$,

$E''^{[3]_3}_{1_{34}} = \frac{t}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 20rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5)$,

$E''^{[3]_3}_{1_{44}} = \frac{1}{1260r^3s^3t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - 168r^3s^2t^3 + 120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - 48r^3t^2 + 15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + 120r^2st^3 - 180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t - 84rs^3t^3 + 120rs^3t^2 - 69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + 100rst^2 - 48rst + 7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + 60s^2t^2 - 20s^2t + 15st^3 - 20st^2 + 7st)$,

$E''^{[3]_3}_{2_{11}} = \frac{r}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9 - 819r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t + 885r^7 - 315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s - 315r^6t^3 - 2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 + 9847r^5s^2t + 3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 + 3228r^5t^2 + 1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 - 11034r^4s^2t^2 - 11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st - 1164r^4t^3 - 1164r^4t^2 + 420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 + 12618r^3s^2t^2 + 4026r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 - 4662r^2s^3t^2 - 1470r^2s^3t - 4662r^2s^2t^3 - 4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 + 1722rs^3t^2 + 1722rs^2t^3 - 630s^3t^3)$,

$$E_{2_{21}}^{''[3]_3} = \frac{s^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4 - 135r^4s^3t - 135r^4s^3 + 108r^4s^2t^2 + 432r^4s^2t + 108r^4s^2 - 378r^4st^2 - 378r^4st + 378r^4t^2 - 63r^3s^5 + 150r^3s^4t + 150r^3s^4 - 39r^3s^3t^2 - 366r^3s^3t - 39r^3s^3 - 84r^3s^2t^3 + 66r^3s^2t^2 + 66r^3s^2t - 84r^3s^2 + 294r^3st^3 + 126r^3st^2 + 294r^3st - 294r^3t^3 - 294r^3t^2 + 21r^2s^6 - 14r^2s^5t - 14r^2s^5 - 90r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 + 105r^2s^3t^3 + 306r^2s^3t^2 + 306r^2s^3t + 105r^2s^3 - 294r^2s^2t^3 - 342r^2s^2t^2 - 294r^2s^2t + 126r^2st^3 + 126r^2st^2 + 210r^2t^3 - 14rs^6t - 14rs^6 + 40rs^5t^2 + 51rs^5t + 40rs^5 - 30rs^4t^3 - 65rs^4t^2 - 65rs^4t - 30rs^4 + 30rs^3t^3 - 72rs^3t^2 + 30rs^3t + 138rs^2t^3 + 138rs^2t^2 - 210rst^3 + 7s^6t - 20s^5t^2 - 20s^5t + 15s^4t^3 + 60s^4t^2 + 15s^4t - 48s^3t^3 - 48s^3t^2 + 42s^2t^3),$$

$$E_{2_{31}}^{''[3]_3} = \frac{t^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 - 135r^4st^3 + 432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t - 294r^3s^3 - 39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 + 66r^3st^2 + 294r^3st - 63r^3t^5 + 150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 + 126r^2s^3t + 210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 - 65r^2st^4 + 306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 + 30rs^3t^3 + 138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 + 51rst^5 - 65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 - 20s^2t^5 + 60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4),$$

$$E_{2_{41}}^{''[3]_3} = -\frac{1}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(-378r^4s^2t^2 + 378r^4s^2t - 108r^4s^2 + 378r^4st^2 - 432r^4st + 135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + 294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 + 294r^3s^2t^3 - 126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - 294r^3st^3 - 66r^3st^2 + 366r^3st - 150r^3s + 84r^3t^3 + 39r^3t^2 - 150r^3t + 63r^3 - 210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - 105r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 + 294r^2st^3 - 306r^2st^2 + 65r^2st + 14r^2s - 105r^2t^3 + 90r^2t^2 + 14r^2t - 21r^2 + 210rs^3t^3 - 138rs^3t^2 - 30rs^3t + 30rs^3 - 138rs^2t^3 + 72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs + 30rt^3 - 40rt^2 + 14rt - 42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - 60s^2t^2 + 20s^2t - 15st^3 + 20st^2 - 7st),$$

$$E_{2_{12}}^{''[3]_3} = \frac{r^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2 + 14r^6st + 14r^6s - 7r^6t + 63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 + 20r^5t - 45r^4s^4 - 150r^4s^3t - 150r^4s^3 + 90$$

$$r^4s^2t^2+65r^4s^2t+90r^4s^2+30r^4st^3+65r^4st^2+65r^4st+30r^4s-15r^4t^3-60r^4t^2-15r^4t+135r^3s^4t+135r^3s^4+39r^3s^3t^2+366r^3s^3t+39r^3s^3-105r^3s^2t^3-306r^3s^2t^2-306r^3s^2t-105r^3s^2-30r^3st^3+72r^3st^2-30r^3st+48r^3t^3+48r^3t^2-108r^2s^4t^2-432r^2s^4t-108r^2s^4+84r^2s^3t^3-66r^2s^3t^2-66r^2s^3t+84r^2s^3+294r^2s^2t^3+342r^2s^2t^2+294r^2s^2t-138r^2st^3-138r^2st^2-42r^2t^3+378rs^4t^2+378rs^4t-294rs^3t^3-126rs^3t^2-294rs^3t-126rs^2t^3-126rs^2t^2+210rst^3-378s^4t^2+294s^3t^3+294s^3t^2-210s^2t^3),$$

$$E_{2_{22}}^{''[3]_3}=\frac{s}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6-1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-9847r^2s^5t-3228r^2s^5+1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-4026r^2s^3t^3-12618r^2s^3t^2-4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+819rs^8-2686rs^7t-2686rs^7+2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-9847rs^5t^2-9847rs^5t-1050rs^5+3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-4026rs^3t^2+1470rs^2t^3-252s^9+819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+315s^6t^3+2930s^6t^2+2930s^6t+315s^6-1050s^5t^3-3228s^5t^2-1050s^5t+1164s^4t^3+1164s^4t^2-420s^3t^3),$$

$$E_{2_{32}}^{''[3]_3}=\frac{t^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2-294r^3s^3t+294r^3s^3-105r^3s^2t^3+294r^3s^2t^2-126r^3s^2t-210r^3s^2+30r^3st^4-30r^3st^3-138r^3st^2+210r^3st-15r^3t^4+48r^3t^3-42r^3t^2-108r^2s^4t^2+378r^2s^4t-378r^2s^4+39r^2s^3t^3-66r^2s^3t^2-126r^2s^3t+294r^2s^3+90r^2s^2t^4-306r^2s^2t^3+342r^2s^2t^2-126r^2s^2t-40r^2st^5+65r^2st^4+72r^2st^3-138r^2st^2+20r^2t^5-60r^2t^4+48r^2t^3+135rs^4t^3-432rs^4t^2+378rs^4t-150rs^3t^4+366rs^3t^3-66rs^3t^2-294rs^3t+14rs^2t^5+65rs^2t^4-306rs^2t^3+294rs^2t^2+14rst^6-51rst^5+65rst^4-30rst^3-7rt^6+20rt^5-15rt^4-45s^4t^4+135s^4t^3-108s^4t^2+63s^3t^5-150s^3t^4+39s^3t^3+84s^3t^2-21s^2t^6+14s^2t^5+90s^2t^4-105s^2t^3+14st^6-40st^5+30st^4),$$

$$E_{2_{42}}^{''[3]_3}=\frac{1}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2-294r^3s^3t+84r^3s^3-210r^3s^2t^3-126r^3s^2t^2+294r^3s^2t-105r^3s^2+210r^3st^3-138r^3st^2-30r^3st+30r^3s-42r^3t^3+$$

$48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - 108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + 39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - 40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + 378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - 66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - 306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - 108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + 39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + 14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st),$

$E''^{[3]_3}_{2_{13}} = \frac{r^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st + 7r^6s + 21r^6t^2 - 14r^6t + 40r^5s^2t - 20r^5s^2 - 14r^5st^2 + 51r^5st - 20r^5s - 63r^5t^3 - 14r^5t^2 + 40r^5t - 30r^4s^3t + 15r^4s^3 - 90r^4s^2t^2 - 65r^4s^2t + 60r^4s^2 + 150r^4st^3 - 65r^4st^2 - 65r^4st + 15r^4s + 45r^4t^4 + 150r^4t^3 - 90r^4t^2 - 30r^4t + 105r^3s^3t^2 + 30r^3s^3t - 48r^3s^3 - 39r^3s^2t^3 + 306r^3s^2t^2 - 72r^3s^2t - 48r^3s^2 - 135r^3st^4 - 366r^3st^3 + 306r^3st^2 + 30r^3st - 135r^3t^4 - 39r^3t^3 + 105r^3t^2 - 84r^2s^3t^3 - 294r^2s^3t^2 + 138r^2s^3t + 42r^2s^3 + 108r^2s^2t^4 + 66r^2s^2t^3 - 342r^2s^2t^2 + 138r^2s^2t + 432r^2st^4 + 66r^2st^3 - 294r^2st^2 + 108r^2t^4 - 84r^2t^3 + 294rs^3t^3 + 126rs^3t^2 - 210rs^3t - 378rs^2t^4 + 126rs^2t^3 + 126rs^2t^2 - 378rst^4 + 294rst^3 - 294s^3t^3 + 210s^3t^2 + 378s^2t^4 - 294s^2t^3),$

$E''^{[3]_3}_{2_{23}} = -\frac{s^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t - 15r^3s^4 - 105r^3s^3t^2 - 30r^3s^3t + 48r^3s^3 + 84r^3s^2t^3 + 294r^3s^2t^2 - 138r^3s^2t - 42r^3s^2 - 294r^3st^3 - 126r^3st^2 + 210r^3st + 294r^3t^3 - 210r^3t^2 - 40r^2s^5t + 20r^2s^5 + 90r^2s^4t^2 + 65r^2s^4t - 60r^2s^4 + 39r^2s^3t^3 - 306r^2s^3t^2 + 72r^2s^3t + 48r^2s^3 - 108r^2s^2t^4 - 66r^2s^2t^3 + 342r^2s^2t^2 - 138r^2s^2t + 378r^2st^4 - 126r^2st^3 - 126r^2st^2 - 378r^2t^4 + 294r^2t^3 + 14rs^6t - 7rs^6 + 14rs^5t^2 - 51rs^5t + 20rs^5 - 150rs^4t^3 + 65rs^4t^2 + 65rs^4t - 15rs^4 + 135rs^3t^4 + 366rs^3t^3 - 306rs^3t^2 - 30rs^3t - 432rs^2t^4 - 66rs^2t^3 + 294rs^2t^2 + 378rst^4 - 294rst^3 - 21s^6t^2 + 14s^6t + 63s^5t^3 + 14s^5t^2 - 40s^5t - 45s^4t^4 - 150s^4t^3 + 90s^4t^2 + 30s^4t + 135s^3t^4 + 39s^3t^3 - 105s^3t^2 - 108s^2t^4 + 84s^2t^3),$

$E''^{[3]_3}_{2_{33}} = -\frac{t}{1260(r-t)^3(s-t)^3(t-1)^3}(-420r^3s^3t^3 + 1470r^3s^3t^2 - 1722r^3s^3t + 630r^3s^3 + 1164r^3s^2t^4 - 4026r^3s^2t^3 + 4662r^3s^2t^2 - 1722r^3s^2t - 1050r^3st^5 + 3561r^3st^4 - 4026r^3st^3 + 1470r^3st^2 + 315r^3t^6 - 1050r^3t^5 + 1164r^3t^4 - 420r^3t^3 + 1164r^2s^3t^4 -$

$4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686rst^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 + 315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 + 1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6),$

$E''^{[3]_3}_{2_{43}} = \frac{1}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2 - 210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + 138r^3s^2t - 48r^3s^2 + 294r^3st^3 - 294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - 30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + 138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - 342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - 378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + 108r^2t^4 - 39r^2t^3 - 90r^2t^2 + 40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - 378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - 65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + 7rs - 135rt^4 + 150rt^3 - 14rt^2 - 14rt - 84s^3t^3 + 105s^3t^2 - 30s^3t + 108s^2t^4 - 39s^2t^3 - 90s^2t^2 + 40s^2t - 135st^4 + 150st^3 - 14st^2 - 14st + 45t^4 - 63t^3 + 21t^2),$

$E''^{[3]_3}_{2_{14}} = -\frac{r^5}{1260(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 20r^5s^2t + 40r^5s^2 - 20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t - 63r^5 + 15r^4s^3t - 30r^4s^3 + 60r^4s^2t^2 - 65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st + 150r^4s - 30r^4t^3 - 90r^4t^2 + 150r^4t + 45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3 - 72r^3s^2t^2 + 306r^3s^2t - 39r^3s^2 + 30r^3st^3 + 306r^3st^2 - 366r^3st - 135r^3s + 105r^3t^3 - 39r^3t^2 - 135r^3t + 42r^2s^3t^3 + 138r^2s^3t^2 - 294r^2s^3t - 84r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 - 294r^2st^3 + 66r^2st^2 + 432r^2st - 84r^2t^3 + 108r^2t^2 - 210rs^3t^3 + 126rs^3t^2 + 294rs^3t + 126rs^2t^3 + 126rs^2t^2 - 378rs^2t + 294rst^3 - 378rst^2 + 210s^3t^3 - 294s^3t^2 - 294s^2t^3 + 378s^2t^2),$

$E''^{[3]_3}_{2_{24}} = -\frac{s^5}{1260(r-1)^3(s-1)^3(t-1)^3}(15r^3s^4t - 30r^3s^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - 84r^3s^2 - 210r^3st^3 + 126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 - 48r^2s^3t^3 -$

$72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 + 126r^2st^3 + 126r^2st^2 - 378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + 40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2),$

$E''^{[3]_3}_{2_{34}} = -\frac{t^5}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 - 366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 + 40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 + 45t^4),$

$E''^{[3]_3}_{2_{44}} = \frac{1}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3 - 1722r^3s^3t^2 + 1470r^3s^3t - 420r^3s^3 - 1722r^3s^2t^3 + 4662r^3s^2t^2 - 4026r^3s^2t + 1164r^3s^2 + 1470r^3st^3 - 4026r^3st^2 + 3561r^3st - 1050r^3s - 420r^3t^3 + 1164r^3t^2 - 1050r^3t + 315r^3 - 1722r^2s^3t^3 + 4662r^2s^3t^2 - 4026r^2s^3t + 1164r^2s^3 + 4662r^2s^2t^3 - 12618r^2s^2t^2 + 11034r^2s^2t - 3228r^2s^2 - 4026r^2st^3 + 11034r^2st^2 - 9847r^2st + 2930r^2s + 1164r^2t^3 - 3228r^2t^2 + 2930r^2t - 885r^2 + 1470rs^3t^3 - 4026rs^3t^2 + 3561rs^3t - 1050rs^3 - 4026rs^2t^3 + 11034rs^2t^2 - 9847rs^2t + 2930rs^2 + 3561rst^3 - 9847rst^2 + 8913rst - 2686rs - 1050rt^3 + 2930rt^2 - 2686rt + 819r - 420s^3t^3 + 1164s^3t^2 - 1050s^3t + 315s^3 + 1164s^2t^3 - 3228s^2t^2 + 2930s^2t - 885s^2 - 1050st^3 + 2930st^2 - 2686st + 819s + 315t^3 - 885t^2 + 819t - 252),$

$K''^{[3]_3}_{1_{14}} = \frac{r^2}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2$

$s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2)$,

$K''^{[3]_3}_{1_{24}} = \frac{s^2}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t - 48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 - 48s^3t^2 - 48s^3t + 42s^2t^2)$,

$K''^{[3]_3}_{1_{34}} = \frac{t^2}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4)$,

$K''^{[3]_3}_{1_{44}} = \frac{1}{2520r^2s^2t^2}(210r^2s^2t^2 - 168r^2s^2t + 42r^2s^2 - 168r^2st^2 + 168r^2st - 48r^2s + 42r^2t^2 - 48r^2t + 15r^2 - 168rs^2t^2 + 168rs^2t - 48rs^2 + 168rst^2 - 192rst + 60rs - 48rt^2 + 60rt - 20r + 42s^2t^2 - 48s^2t + 15s^2 - 48st^2 + 60st - 20s + 15t^2 - 20t + 7)$,

$K''^{[3]_3}_{2_{11}} = \frac{-r^2}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 70r^5s - 70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t - 120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2)$,

$K''^{[3]_3}_{2_{21}} = \frac{-s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^3t^2 + 24rs^2 - 30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 + 96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst)$,

$K''^{[3]_3}_{2_{31}} = \frac{-t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 + 48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 + 24rs^2t^2 - 84rst)$,

$K''^{[3]_3}_{2_{41}} = \frac{-1}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r + 20s + 20t - 42s^2t^2 - 30rs - 30rt - 60st + 24rs^2 + 24rt^2 + 48st^2 + 48s^2t - 15s^2 - 15t^2 - 84rst^2 - 84rs^2t + 84rs^2t^2 + 96rst - 7)$,

$$K_{2_{12}}^{''[3]_3} = \frac{-r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^3t^2 + 24r^2s - 30r^3s + 10r^4s - 42rt^2 + 48r^2t - 60r^3t + 20r^4t + 84st^2 - 15r^3 + 20r^4 - 7r^5 - 84rst^2 + 96r^2st - 30r^3st + 24r^2st^2 - 84rst),$$

$$K_{2_{22}}^{''[3]_3} = \frac{-s^2}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4 - 120r^2s^3t - 120r^2s^3 + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 - 252r^2st^2 - 252r^2st + 210r^2t^2 - 70rs^5 + 180rs^4t + 180rs^4 - 120rs^3t^2 - 480rs^3t - 120rs^3 + 336rs^2t^2 + 336rs^2t - 252rst^2 + 28s^6 - 70s^5t - 70s^5 + 45s^4t^2 + 180s^4t + 45s^4 - 120s^3t^2 - 120s^3t + 84s^2t^2),$$

$$K_{2_{32}}^{''[3]_3} = \frac{-t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^2t^3 + 84r^2s + 48rt^2 - 42r^2t - 60rt^3 + 20rt^4 + 24st^2 - 30st^3 + 10st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84r^2st - 30rst^3 + 24r^2st^2 - 84rst),$$

$$K_{2_{42}}^{''[3]_3} = \frac{-1}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r + 10s + 20t - 42r^2t^2 - 30rs - 60rt - 30st + 24r^2s + 48rt^2 + 48r^2t + 24st^2 - 15r^2 - 15t^2 - 84rst^2 - 84r^2st + 84r^2st^2 + 96rst - 7),$$

$$K_{2_{13}}^{''[3]_3} = \frac{-r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^3s^2 - 42rs^2 + 48r^2s - 60r^3s + 20r^4s + 24r^2t - 30r^3t + 10r^4t + 84s^2t - 15r^3 + 20r^4 - 7r^5 - 84rs^2t + 96r^2st - 30r^3st + 24r^2s^2t - 84rst),$$

$$K_{2_{23}}^{''[3]_3} = \frac{-s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 + 84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t + 24r^2s^2t - 84rst),$$

$$K_{2_{33}}^{''[3]_3} = \frac{-t^2}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4),$$

$$K_{2_{43}}^{''[3]_3} = \frac{-1}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r + 20s + 10t - 42r^2s^2 - 60rs - 30rt - 30st + 48rs^2 + 48r^2s + 24r^2t + 24s^2t - 15r^2 - 15s^2 - 84rs^2t - 84r^2st + 84r^2s^2t + 96rst - 7),$$

$K''^{[3]_3}_{2_{14}} = \frac{-r^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-7r^5 + 20r^4s + 20r^4t + 10r^4 - 15r^3s^2 - 60r^3st - 30r^3s - 15r^3t^2 - 30r^3t + 48r^2s^2t + 24r^2s^2 + 48r^2st^2 + 96r^2st + 24r^2t^2 - 42rs^2t^2 - 84rs^2t - 84rst^2 + 84s^2t^2)$,

$K''^{[3]_3}_{2_{24}} = \frac{-s^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-15r^2s^3 + 48r^2s^2t + 24r^2s^2 - 42r^2st^2 - 84r^2st + 84r^2t^2 + 20rs^4 - 60rs^3t - 30rs^3 + 48rs^2t^2 + 96rs^2t - 84rst^2 - 7s^5 + 20s^4t + 10s^4 - 15s^3t^2 - 30s^3t + 24s^2t^2)$,

$K''^{[3]_3}_{2_{34}} = \frac{-t^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-42r^2s^2t + 84r^2s^2 + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4)$,

$K''^{[3]_3}_{2_{44}} = \frac{-1}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2 - 252r^2s^2t + 84r^2s^2 - 252r^2st^2 + 336r^2st - 120r^2s + 84r^2t^2 - 120r^2t + 45r^2 - 252rs^2t^2 + 336rs^2t - 120rs^2 + 336rst^2 - 480rst + 180rs - 120rt^2 + 180rt - 70r + 84s^2t^2 - 120s^2t + 45s^2 - 120st^2 + 180st - 70s + 45t^2 - 70t + 28)$.

4.3.1 Properties of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

In this section, the properties of the developed method (4.39) such as order and error constant, zero-stability, consistency, convergence and region of absolute stability are established.

4.3.1.1 Order of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

The linear difference operator ∇ corresponding to (4.39) is given as

$$\begin{aligned}\nabla[y(x),h] \quad = \quad & Y^{[3]_3}_{n+1} - \hat{M}^{[3]_3}_1 Y^{[3]_3}_n - h\hat{M}^{[3]_3}_2 Y^{[3]_3}_{n-1} - h^2\hat{M}^{[3]_3}_3 Y^{[3]_3}_{n-2} - h^3\left[\hat{E}^{[3]_3}_1 F^{[3]_3}_n + \hat{E}^{[3]_3}_2 F^{[3]_3}_{n+1}\right] \\ & -h^4\left[\hat{K}^{[3]_3}_1 G^{[3]_3}_n + \hat{K}^{[3]_3}_2 G^{[3]_3}_{n+1}\right]. \end{aligned} \quad (4.46)$$

Following the same procedure as described earlier, we expand each function in $Y_{n+1}^{[3]_3}$, $F_{n+1}^{[3]_3}$ and $G_{n+1}^{[3]_3}$ about x_n to get

$$\left[Q_{11}^{[3]_3}\ Q_{21}^{[3]_3}\ Q_{31}^{[3]_3}\ Q_{41}^{[3]_3}\right]^T = \left[0\ 0\ 0\ 0\right]^T$$

where

$Q_{11}^{[3]_3} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n - rhy'_n - \frac{h^2r^2y''_n}{2} + \frac{h^3r^3y'''_n}{27720s^3t^3}(24r^6s^2t - 7r^7s - 7r^7t - 7r^7st + 24r^6s^2 + 24r^6st^2 + 59r^6st + 24r^6s + 24r^6t^2 + 24r^6t - 22r^5s^3t - 22r^5s^3 - 88r^5s^2t^2 - 152r^5s^2t - 88r^5s^2 - 22r^5st^3 - 152r^5st^2 - 152r^5st - 22r^5s - 22r^5t^3 - 88r^5t^2 - 22r^5t + 88r^4s^3t^2 + 132r^4s^3t + 88r^4s^3 + 88r^4s^2t^3 + 352r^4s^2t^2 + 352r^4s^2t + 88r^4s^2 + 132r^4st^3 + 352r^4st^2 + 132r^4st + 88r^4t^3 + 88r^4t^2 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 - 308r^3s^2t^3 - 440r^3s^2t^2 - 308r^3s^2t - 308r^3st^3 - 308r^3st^2 - 99r^3t^3 + 297r^2s^3t^3 + 132r^2s^3t^2 + 297r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 297r^2st^3 + 528rs^3t^3 + 528rs^3t^2 - 528rs^2t^3 - 3696s^3t^3) - \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+3)}r^3}{27720(r-s)^3(r-t)^3(r-1)^3i!}(84r^9 - 315r^8s - 315r^8t - 315r^8 + 390r^7s^2 + 1210r^7st + 1210r^7s + 390r^7t^2 + 1210r^7t + 390r^7 - 154r^6s^3 - 1536r^6s^2t - 1536r^6s^2 - 1536r^6st^2 - 4793r^6st - 1536r^6s - 154r^6t^3 - 1536r^6t^2 - 1536r^6t - 154r^6 + 616r^5s^3t + 616r^5s^3 + 2002r^5s^2t^2 + 6290r^5s^2t + 2002r^5s^2 + 616r^5st^3 + 6290r^5st^2 + 6290r^5st + 616r^5s + 616r^5t^3 + 2002r^5t^2 + 616r^5t - 814r^4s^3t^2 - 2574r^4s^3t - 814r^4s^3 - 814r^4s^2t^3 - 8569r^4s^2t^2 - 8569r^4s^2t - 814r^4s^2 - 2574r^4st^3 - 8569r^4st^2 - 2574r^4st - 814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 + 3575r^3s^2t^3 + 12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 - 5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 + 2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3) - \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+3)}r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3i!}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t + 70r^5s^3 + 21r^5s^2t + 21r^5s^2 - 48r^5st^2 - 64r^5st - 48r^5s + 24r^5t^2 + 24r^5t - 54r^4s^4 - 208r^4s^3t - 208r^4s^3 + 116r^4s^2t^2 + 70r^4s^2t + 116r^4s^2 + 44r^4st^3 + 106r^4st^2 + 106r^4st + 44r^4s - 22r^4t^3 - 88r^4t^2 - 22r^4t + 198r^3s^4t + 198r^3s^4 + 88r^3s^3t^2 + 660r^3s^3t + 88r^3s^3 - 176r^3s^2t^3 - 506r^3s^2t^2 - 506r^3s^2t - 176r^3s^2 - 66r^3st^3 + 88r^3st^2 - 66r^3st + 88r^3t^3 + 88r^3t^2 - 198r^2s^4t^2 - 792r^2s^4t - 198r^2s^4 + 154r^2s^3t^3 - 275r^2s^3t^2 - 275r^2s^3t + 154r^2s^3 + 649$

$r^2s^2t^3 + 781r^2s^2t^2 + 649r^2s^2t - 275r^2st^3 - 275r^2st^2 - 99r^2t^3 + 891rs^4t^2 + 891rs^4t - 693rs^3t^3 - 66rs^3t^2 - 693rs^3t - 462rs^2t^3 - 462rs^2t^2 + 594rst^3 - 1188s^4t^2 + 924s^3t^3 + 924s^3t^2 - 660s^2t^3) - \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+3)}r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3i!}(7r^6s - 14r^6st + 21r^6t^2 - 14r^6t + 48r^5s^2t - 24r^5s^2 - 21r^5st^2 + 64r^5st - 24r^5s - 70r^5t^3 - 21r^5t^2 + 48r^5t - 44r^4s^3t + 22r^4s^3 - 116r^4s^2t^2 - 106r^4s^2t + 88r^4s^2 + 208r^4st^3 - 70r^4st^2 - 106r^4st + 22r^4s + 54r^4t^4 + 208r^4t^3 - 116r^4t^2 - 44r^4t + 176r^3s^3t^2 + 66r^3s^3t - 88r^3s^3 - 88r^3s^2t^3 + 506r^3s^2t^2 - 88r^3s^2t - 88r^3s^2 - 198r^3st^4 - 660r^3st^3 + 506r^3st^2 + 66r^3st - 198r^3t^4 - 88r^3t^3 + 176r^3t^2 - 154r^2s^3t^3 - 649r^2s^3t^2 + 275r^2s^3t + 99r^2s^3 + 198r^2s^2t^4 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 792r^2st^4 + 275r^2st^3 - 649r^2st^2 + 198r^2t^4 - 154r^2t^3 + 693rs^3t^3 + 462rs^3t^2 - 594rs^3t - 891rs^2t^4 + 66rs^2t^3 + 462rs^2t^2 - 891rst^4 + 693rst^3 - 924s^3t^3 + 660s^3t^2 + 1188s^2t^4 - 924s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+3)}r^7}{27720(r-1)^3(s-1)^3(t-1)^3i!}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 24r^5s^2t + 48r^5s^2 - 24r^5st^2 + 64r^5st - 21r^5s + 48r^5t^2 - 21r^5t - 70r^5 + 22r^4s^3t - 44r^4s^3 + 88r^4s^2t^2 - 106r^4s^2t - 116r^4s^2 + 22r^4st^3 - 106r^4st^2 - 70r^4st + 208r^4s - 44r^4t^3 - 116r^4t^2 + 208r^4t + 54r^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 - 88r^3s^2t^3 - 88r^3s^2t^2 + 506r^3s^2t - 88r^3s^2 + 66r^3st^3 + 506r^3st^2 - 660r^3st - 198r^3s + 176r^3t^3 - 88r^3t^2 - 198r^3t + 99r^2s^3t^3 + 275r^2s^3t^2 - 649r^2s^3t - 154r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 - 649r^2st^3 + 275r^2st^2 + 792r^2st - 154r^2t^3 + 198r^2t^2 - 594rs^3t^3 + 462rs^3t^2 + 693rs^3t + 462rs^2t^3 + 66rs^2t^2 - 891rs^2t + 693rst^3 - 891rst^2 + 660s^3t^3 - 924s^3t^2 - 924s^2t^3 + 1188s^2t^2) + \frac{h^4r^4y_n^{iv}}{55440s^2t^2}(7r^6 - 24r^5s - 24r^5t - 24r^5 + 22r^4s^2 + 88r^4st + 88r^4s + 22r^4t^2 + 88r^4t + 22r^4 - 88r^3s^2t - 88r^3s^2 - 88r^3st^2 - 352r^3st - 88r^3s - 88r^3t^2 - 88r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 528rs^2t^2 - 528rs^2t - 528rst^2 + 924s^2t^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+4}y_n^{(i+4)}r^4}{55440(r-s)^2(r-t)^2(r-1)^2i!}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2) + \sum_{i=0}^{\infty} \frac{s^ih^{i+4}y_n^{(i+4)}r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2i!}(88r^2t^2 - 22r^3t^2 + 44r^2s - 44r^3s + 12r^4s - 99rt^2 + 88r^2t - 88r^3t + 24r^4t + 264st^2 - 22r^3 + 24r^4 - 7r^5 - 198rst^2 + 176r^2st - 44r^3st + 44r^2st^2 - 198rst) + \sum_{i=0}^{\infty} \frac{t^ih^{i+4}y_n^{(i+4)}r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2i!}(88r^2s^2 - 22r^3s^2 - 99rs^2 +$

$88r^2s - 88r^3s + 24r^4s + 44r^2t - 44r^3t + 12r^4t + 264s^2t - 22r^3 + 24r^4 - 7r^5 - 198rs^2t + 176r^2st - 44r^3st + 44r^2s^2t - 198rst) + \sum_{i=0}^{\infty} \frac{h^{i+4}y_n^{(i+4)}r^7}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(24r^4s - 7r^5 + 24r^4t + 12r^4 - 22r^3s^2 - 88r^3st - 44r^3s - 22r^3t^2 - 44r^3t + 88r^2s^2t + 44r^2s^2 + 88r^2st^2 + 176r^2st + 44r^2t^2 - 99rs^2t^2 - 198rs^2t - 198rst^2 + 264s^2t^2),$

$Q_{21}^{[3]_3} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!}y_n^{(i)} - y_n - shy_n' - \frac{h^2s^2y_n''}{2} + \frac{h^3s^3y_n'''}{27720r^3t^3}(88r^3s^4t^2 - 22r^3s^5 - 22r^3s^5t + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 + 297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t + 24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t - 7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t - 22rs^5 + 132rs^4t^3 + 352rs^4t^2 + 132rs^4t - 308rs^3t^3 - 308rs^3t^2 + 297rs^2t^3 - 7s^7t + 24s^6t^2 + 24s^6t - 22s^5t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3) - \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+3)}s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3) - \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+3)}s^3}{27720(r-s)^3(s-t)^3(s-1)^3 i!}(154r^3s^6 - 616r^3s^5t - 616r^3s^5 + 814r^3s^4t^2 + 2574r^3s^4t + 814r^3s^4 - 330r^3s^3t^3 - 3575r^3s^3t^2 - 3575r^3s^3t - 330r^3s^3 + 1485r^3s^2t^3 + 5280r^3s^2t^2 + 1485r^3s^2t - 2244r^3st^3 - 2244r^3st^2 + 924r^3t^3 - 390r^2s^7 + 1536r^2s^6t + 1536r^2s^6 - 2002r^2s^5t^2 - 6290r^2s^5t - 2002r^2s^5 + 814r^2s^4t^3 + 8569r^2s^4t^2 + 8569r^2s^4t + 814r^2s^4 - 3575r^2s^3t^3 - 12320r^2s^3t^2 - 3575r^2s^3t + 5280r^2s^2t^3 + 5280r^2s^2t^2 - 2244r^2st^3 + 315rs^8 - 1210rs^7t - 1210rs^7 + 1536rs^6t^2 + 4793rs^6t + 1536rs^6 - 616rs^5t^3 - 6290rs^5t^2 - 6290rs^5t - 616rs^5 + 2574rs^4t^3 + 8569rs^4t^2 + 2574rs^4t - 3575rs^3t^3 - 3575rs^3t^2 + 1485rs^2t^3 - 84s^9 + 315s^8t + 315s^8$

$$-390s^7t^2 - 1210s^7t - 390s^7 + 154s^6t^3 + 1536s^6t^2 + 1536s^6t + 154s^6 - 616s^5t^3 - 2002s^5t^2 - 616s^5t + 814s^4t^3 + 814s^4t^2 - 330s^3t^3) + \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+3)} s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(44r^3s^4t - 22r^3s^4 - 176r^3s^3t^2 - 66r^3s^3t + 88r^3s^3 + 154r^3s^2t^3 + 649r^3s^2t^2 - 275r^3s^2t - 99r^3s^2 - 693r^3st^3 - 462r^3st^2 + 594r^3st + 924r^3t^3 - 660r^3t^2 - 48r^2s^5t + 24r^2s^5 + 116r^2s^4t^2 + 106r^2s^4t - 88r^2s^4 + 88r^2s^3t^3 - 506r^2s^3t^2 + 88r^2s^3t + 88r^2s^3 - 198r^2s^2t^4 - 275r^2s^2t^3 + 781r^2s^2t^2 - 275r^2s^2t + 891r^2st^4 - 66r^2st^3 - 462r^2st^2 - 1188r^2t^4 + 924r^2t^3 + 14rs^6t - 7rs^6 + 21rs^5t^2 - 64rs^5t + 24rs^5 - 208rs^4t^3 + 70rs^4t^2 + 106rs^4t - 22rs^4 + 198rs^3t^4 + 660rs^3t^3 - 506rs^3t^2 - 66rs^3t - 792rs^2t^4 - 275rs^2t^3 + 649rs^2t^2 + 891rst^4 - 693rst^3 - 21s^6t^2 + 14s^6t + 70s^5t^3 + 21s^5t^2 - 48s^5t - 54s^4t^4 - 208s^4t^3 + 116s^4t^2 + 44s^4t + 198s^3t^4 + 88s^3t^3 - 176s^3t^2 - 198s^2t^4 + 154s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)} s^7}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(22r^3s^4t - 44r^3s^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 - 594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 - 106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 + 1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 - 70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 + 792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 - 116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2) - \frac{h^4s^4y_n^{iv}}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2 i!}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst) + \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)} s^4}{55440(r-s)^2(s-t)^2(s-1)^2 i!}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+4)} s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2 i!}(88r^2s^2 - 22r^2s^3 + 88rs^2 - 99r^2s - 88rs^3 + 24rs^4 +$$

$264r^2t+44s^2t-44s^3t+12s^4t-22s^3+24s^4-7s^5+176rs^2t-198r^2st-44rs^3t+$
$44r^2s^2t - 198rst) + \sum_{i=0}^{\infty}\frac{h^{i+4}y_n^{(i+4)}s^7}{55440(r-1)^2(s-1)^2(t-1)^2i!}(88r^2s^2t - 22r^2s^3 + 44r^2s^2 -$
$99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 + 88rs^2t^2 + 176rs^2t -$
$198rst^2-7s^5+24s^4t+12s^4-22s^3t^2-44s^3t+44s^2t^2),$

$Q_{31}^{[3]_3} = \sum_{i=0}^{\infty}\frac{(th)^i}{i!}y_n^{(i)} - y_n - thy'_n - \frac{h^2t^2y''_n}{2} - \frac{h^3t^3y'''_n}{27720r^3s^3}(297r^3s^3t^2 - 99r^3s^3t^3 +$
$528r^3s^3t - 3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 +$
$132r^3st^4-308r^3st^3+297r^3st^2-22r^3t^5+88r^3t^4-99r^3t^3+88r^2s^3t^4-308r^2s^3t^3+$
$132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 -$
$152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 - 22rs^3t^5 + 132rs^3t^4 -$
$308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 -$
$152rst^5 + 132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 -$
$88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5) + \sum_{i=0}^{\infty}\frac{r^ih^{i+3}y_n^{(i+3)}t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3i!}(198r^4s^2t^2 -$
$891r^4s^2t + 1188r^4s^2 - 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 - 198r^4t^3 +$
$198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t -$
$924r^3s^2 + 208r^3st^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 -$
$154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 + 462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 -$
$781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 - 649r^2st^2 + 21r^2t^6 - 21r^2t^5 -$
$116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594rs^3t + 48rs^2t^5 - 106rs^2t^4 -$
$88rs^2t^3 + 275rs^2t^2 - 14rst^6 + 64rst^5 - 106rst^4 + 66rst^3 - 14rt^6 + 48rt^5 - 44rt^4 +$
$22s^3t^4 - 88s^3t^3 + 99s^3t^2 - 24s^2t^5 + 88s^2t^4 - 88s^2t^3 + 7st^6 - 24st^5 + 22st^4) +$
$\sum_{i=0}^{\infty}\frac{s^ih^{i+3}y_n^{(i+3)}t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3i!}(154r^3s^3t^2 - 693r^3s^3t + 924r^3s^3 - 176r^3s^2t^3 +$
$649r^3s^2t^2 - 462r^3s^2t - 660r^3s^2 + 44r^3st^4 - 66r^3st^3 - 275r^3st^2 + 594r^3st -$
$22r^3t^4 + 88r^3t^3 - 99r^3t^2 - 198r^2s^4t^2 + 891r^2s^4t - 1188r^2s^4 + 88r^2s^3t^3 - 275r^2s^3t^2 -$
$66r^2s^3t + 924r^2s^3 + 116r^2s^2t^4 - 506r^2s^2t^3 + 781r^2s^2t^2 - 462r^2s^2t - 48r^2st^5 +$
$106r^2st^4 + 88r^2st^3 - 275r^2st^2 + 24r^2t^5 - 88r^2t^4 + 88r^2t^3 + 198rs^4t^3 - 792rs^4t^2 +$
$891rs^4t - 208rs^3t^4 + 660rs^3t^3 - 275rs^3t^2 - 693rs^3t + 21rs^2t^5 + 70rs^2t^4 - 506rs^2t^3 +$
$649rs^2t^2 + 14rst^6 - 64rst^5 + 106rst^4 - 66rst^3 - 7rt^6 + 24rt^5 - 22rt^4 - 54s^4t^4 +$
$198s^4t^3 - 198s^4t^2 + 70s^3t^5 - 208s^3t^4 + 88s^3t^3 + 154s^3t^2 - 21s^2t^6 + 21s^2t^5 + 116s^2t^4 -$

$$176s^2t^3 + 14st^6 - 48st^5 + 44st^4) - \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+3)} t^3}{27720(r-t)^3(s-t)^3(t-1)^3 i!}(1485r^3s^3t^2 -$$

$$330r^3s^3t^3 - 2244r^3s^3t + 924r^3s^3 + 814r^3s^2t^4 - 3575r^3s^2t^3 + 5280r^3s^2t^2 -$$

$$2244r^3s^2t - 616r^3st^5 + 2574r^3st^4 - 3575r^3st^3 + 1485r^3st^2 + 154r^3t^6 - 616r^3t^5 +$$

$$814r^3t^4 - 330r^3t^3 + 814r^2s^3t^4 - 3575r^2s^3t^3 + 5280r^2s^3t^2 - 2244r^2s^3t - 2002r^2s^2t^5 +$$

$$8569r^2s^2t^4 - 12320r^2s^2t^3 + 5280r^2s^2t^2 + 1536r^2st^6 - 6290r^2st^5 + 8569r^2st^4 -$$

$$3575r^2st^3 - 390r^2t^7 + 1536r^2t^6 - 2002r^2t^5 + 814r^2t^4 - 616rs^3t^5 + 2574rs^3t^4 -$$

$$3575rs^3t^3 + 1485rs^3t^2 + 1536rs^2t^6 - 6290rs^2t^5 + 8569rs^2t^4 - 3575rs^2t^3 -$$

$$1210rst^7 + 4793rst^6 - 6290rst^5 + 2574rst^4 + 315rt^8 - 1210rt^7 + 1536rt^6 -$$

$$616rt^5 + 154s^3t^6 - 616s^3t^5 + 814s^3t^4 - 330s^3t^3 - 390s^2t^7 + 1536s^2t^6 - 2002s^2t^5 +$$

$$814s^2t^4 + 315st^8 - 1210st^7 + 1536st^6 - 616st^5 - 84t^9 + 315t^8 - 390t^7 + 154t^6) -$$

$$\sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+3)} t^7}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(99r^3s^3t^2 - 594r^3s^3t + 660r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 +$$

$$462r^3s^2t - 924r^3s^2 + 22r^3st^4 + 66r^3st^3 - 649r^3st^2 + 693r^3st - 44r^3t^4 + 176r^3t^3 -$$

$$154r^3t^2 - 88r^2s^3t^3 + 275r^2s^3t^2 + 462r^2s^3t - 924r^2s^3 + 88r^2s^2t^4 - 88r^2s^2t^3 -$$

$$781r^2s^2t^2 + 66r^2s^2t + 1188r^2s^2 - 24r^2st^5 - 106r^2st^4 + 506r^2st^3 + 275r^2st^2 -$$

$$891r^2st + 48r^2t^5 - 116r^2t^4 - 88r^2t^3 + 198r^2t^2 + 22rs^3t^4 + 66rs^3t^3 - 649rs^3t^2 +$$

$$693rs^3t - 24rs^2t^5 - 106rs^2t^4 + 506rs^2t^3 + 275rs^2t^2 - 891rs^2t + 7rst^6 + 64rst^5 -$$

$$70rst^4 - 660rst^3 + 792rst^2 - 14rt^6 - 21rt^5 + 208rt^4 - 198rt^3 - 44s^3t^4 + 176s^3t^3 -$$

$$154s^3t^2 + 48s^2t^5 - 116s^2t^4 - 88s^2t^3 + 198s^2t^2 - 14st^6 - 21st^5 + 208st^4 - 198st^3 +$$

$$21t^6 - 70t^5 + 54t^4) - \frac{h^4 t^4 y_n^{iv}}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 -$$

$$528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 - 88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 -$$

$$352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 + 99s^2t^2 - 24st^5 + 88st^4 -$$

$$88st^3 + 7t^6 - 24t^5 + 22t^4) - \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2 i!}(88s^2t^2 - 22s^2t^3 + 264rs^2 +$$

$$44rt^2 - 44rt^3 + 12rt^4 + 88st^2 - 99s^2t - 88st^3 + 24st^4 - 22t^3 + 24t^4 - 7t^5 + 176rst^2 -$$

$$198rs^2t - 44rst^3 + 44rs^2t^2 - 198rst) - \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)} t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2 i!}(88r^2t^2 -$$

$$22r^2t^3 + 264r^2s + 88rt^2 - 99r^2t - 88rt^3 + 24rt^4 + 44st^2 - 44st^3 + 12st^4 -$$

$$22t^3 + 24t^4 - 7t^5 + 176rst^2 - 198r^2st - 44rst^3 + 44r^2st^2 - 198rst) -$$

$$\sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+4)} t^4}{55440(r-t)^2(s-t)^2(t-1)^2 i!}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 -$$

$$396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 -$$

$$440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 +$$

$132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4) - \sum_{i=0}^{\infty} \frac{h^{i+4}y_n^{(i+4)}t^7}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(264r^2s^2 - 99r^2s^2t + 88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4),$

$Q_{41}^{[3]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i)} - y_n - hy_n' - \frac{h^2y_n''}{2} - \frac{h^3y_n'''}{27720r^3s^3t^3}(528r^3s^3t^2 - 3696r^3s^3t^3 + 297r^3s^3t - 99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2 - 308r^3s^2t + 88r^3s^2 + 297r^3st^3 - 308r^3st^2 + 132r^3st - 22r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t + 528r^2s^3t^3 + 132r^2s^3t^2 - 308r^2s^3t + 88r^2s^3 + 132r^2s^2t^3 - 440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 - 152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3t^2 + 132rs^3t - 22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs - 22rt^3 + 24rt^2 - 7rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st) - \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+3)}}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(891r^4s^2t - 1188r^4s^2t^2 - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs + 44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st) + \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+3)}}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}(924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 - 660r^3s^2t^3 - 462r^3s^2t^2 + 649r^3s^2t - 176r^3s^2 + 594r^3st^3 - 275r^3st^2 - 66r^3st + 44r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t - 1188r^2s^4t^2 + 891r^2s^4t - 198r^2s^4 + 924r^2s^3t^3 - 66r^2s^3t^2 - 275r^2s^3t + 88r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 - 275r^2st^3 + 88r^2st^2 + 106r^2st - 48r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 891rs^4t^2 - 792rs^4t + 198rs^4 - 693rs^3t^3 - 275rs^3t^2 + 660rs^3t - 208rs^3 + 649rs^2t^3 - 506rs^2t^2 + 70rs^2t + 21rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs - 22rt^3 + 24rt^2 - 7rt - 198s^4t^2 + 198s^4t - 54s^4 + 154s^3t^3 + 88s^3t^2 - 208s^3t + 70s^3 - 176s^2t^3 + 116s^2t^2 + 21s^2t - 21s^2 + 44st^3 - 48st^2 + 14st) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+3)}}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(660r^3s^3t^2 - 594r^3s^3t + 99r^3s^3 - 924r^3s^2t^3 + 462r^3s^2t^2 + 275r^3s^2t - 88r^3s^2 + 693r^3st^3 -$

$649r^3st^2+66r^3st+22r^3s-154r^3t^3+176r^3t^2-44r^3t-924r^2s^3t^3+462r^2s^3t^2+275r^2s^3t-88r^2s^3+1188r^2s^2t^4+66r^2s^2t^3-781r^2s^2t^2-88r^2s^2t+88r^2s^2-891r^2st^4+275r^2st^3+506r^2st^2-106r^2st-24r^2s+198r^2t^4-88r^2t^3-116r^2t^2+48r^2t+693rs^3t^3-649rs^3t^2+66rs^3t+22rs^3-891rs^2t^4+275rs^2t^3+506rs^2t^2-106rs^2t-24rs^2+792rst^4-660rst^3-70rst^2+64rst+7rs-198rt^4+208rt^3-21rt^2-14rt-154s^3t^3+176s^3t^2-44s^3t+198s^2t^4-88s^2t^3-116s^2t^2+48s^2t-198st^4+208st^3-21st^2-14st+54t^4-70t^3+21t^2)+\sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+3)}}{27720(r-1)^3(s-1)^3(t-1)^3i!}(924r^3s^3t^3-2244r^3s^3t^2+1485r^3s^3t-330r^3s^3-2244r^3s^2t^3+5280r^3s^2t^2-3575r^3s^2t+814r^3s^2+1485r^3st^3-3575r^3st^2+2574r^3st-616r^3s-330r^3t^3+814r^3t^2-616r^3t+154r^3-2244r^2s^3t^3+5280r^2s^3t^2-3575r^2s^3t+814r^2s^3+5280r^2s^2t^3-12320r^2s^2t^2+8569r^2s^2t-2002r^2s^2-3575r^2st^3+8569r^2st^2-6290r^2st+1536r^2s+814r^2t^3-2002r^2t^2+1536r^2t-390r^2+1485rs^3t^3-3575rs^3t^2+2574rs^3t-616rs^3-3575rs^2t^3+8569rs^2t^2-6290rs^2t+1536rs^2+2574rst^3-6290rst^2+4793rst-1210rs-616rt^3+1536rt^2-1210rt+315r-330s^3t^3+814s^3t^2-616s^3t+154s^3+814s^2t^3-2002s^2t^2+1536s^2t-390s^2-616st^3+1536st^2-1210st+315s+154t^3-390t^2+315t-84)-\frac{h^4y_n^{iv}}{55440r^2s^2t^2}(924r^2s^2t^2-528r^2s^2t+99r^2s^2-528r^2st^2+396r^2st-88r^2s+99r^2t^2-88r^2t+22r^2-528rs^2t^2+396rs^2t-88rs^2+396rst^2-352rst+88rs-88rt^2+88rt-24r+99s^2t^2-88s^2t+22s^2-88st^2+88st-24s+22t^2-24t+7)-\sum_{i=0}^{\infty}\frac{r^ih^{i+4}y_n^{(i+4)}}{55440r^2(r-s)^2(r-t)^2(r-1)^2i!}(12r+24s+24t-99s^2t^2-44rs-44rt-88st+44rs^2+44rt^2+88st^2+88s^2t-22s^2-22t^2-198rst^2-198rs^2t+264rs^2t^2+176rst-7)-\sum_{i=0}^{\infty}\frac{s^ih^{i+4}y_n^{(i+4)}}{55440s^2(r-s)^2(s-t)^2(s-1)^2i!}(24r+12s+24t-99r^2t^2-44rs-88rt-44st+44r^2s+88rt^2+88r^2t+44st^2-22r^2-22t^2-198rst^2-198r^2st+264r^2st^2+176rst-7)-\sum_{i=0}^{\infty}\frac{t^ih^{i+4}y_n^{(i+4)}}{55440t^2(r-t)^2(s-t)^2(t-1)^2i!}(24r+24s+12t-99r^2s^2-88rs-44rt-44st+88rs^2+88r^2s+44r^2t+44s^2t-22r^2-22s^2-198rs^2t-198r^2st+264r^2s^2t+176rst-7)-\sum_{i=0}^{\infty}\frac{h^{i+4}y_n^{(i+4)}}{55440(r-1)^2(s-1)^2(t-1)^2i!}(462r^2s^2t^2-396r^2s^2t+99r^2s^2-396r^2st^2+396r^2st-110r^2s+99r^2t^2-110r^2t+33r^2-396rs^2t^2+396rs^2t-110rs^2+396rst^2-440rst+132rs-110rt^2+132rt-42r+99s^2t^2-110s^2t+33s^2-110st^2+132st-42s+33t^2-42t+14).$

By collecting the like terms and comparing the coefficients of h^j and $y^{(j)}$ as mentioned earlier, we obtain $\bar{D}_0 = \bar{D}_1 = ... = \bar{D}_{12} = 0$ and $\bar{D}_{13} \neq 0$ which implies that , the order of the main block is $[10,10,10,10]^T$ and the vector of error constants is

$\bar{D}_{13} = \left[\bar{D}_{13_1}\ \bar{D}_{13_2}\ \bar{D}_{13_3}\ \bar{D}_{13_4}\right]^T$

where

$$\bar{D}_{13_1} = \tfrac{r^7}{1307674368000}(28r^6 - 91r^5s - 91r^5t - 91r^5 + 78r^4s^2 + 312r^4st + 312r^4s + 78r^4t^2 + 312r^4t + 78r^4 - 286r^3s^2t - 286r^3s^2 - 286r^3st^2 - 1144r^3st - 286r^3s - 286r^3t^2 - 286r^3t + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 + 1144r^2st^2 + 1144r^2st + 286r^2t^2 - 1287rs^2t^2 - 1287rs^2t - 1287rst^2 + 1716s^2t^2),$$

$$\bar{D}_{13_2} = \tfrac{s^7}{1307674368000}(78r^2s^4 - 286r^2s^3t - 286r^2s^3 + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 - 1287r^2st^2 - 1287r^2st + 1716r^2t^2 - 91rs^5 + 312rs^4t + 312rs^4 - 286rs^3t^2 - 1144rs^3t - 286rs^3 + 1144rs^2t^2 + 1144rs^2t - 1287rst^2 + 28s^6 - 91s^5t - 91s^5 + 78s^4t^2 + 312s^4t + 78s^4 - 286s^3t^2 - 286s^3t + 286s^2t^2),$$

$$\bar{D}_{13_3} = \tfrac{t^7}{1307674368000}(286r^2s^2t^2 - 1287r^2s^2t + 1716r^2s^2 - 286r^2st^3 + 1144r^2st^2 - 1287r^2st + 78r^2t^4 - 286r^2t^3 + 286r^2t^2 - 286rs^2t^3 + 1144rs^2t^2 - 1287rs^2t + 312rst^4 - 1144rst^3 + 1144rst^2 - 91rt^5 + 312rt^4 - 286rt^3 + 78s^2t^4 - 286s^2t^3 + 286s^2t^2 - 91st^5 + 312st^4 - 286st^3 + 28t^6 - 91t^5 + 78t^4),$$

$$\bar{D}_{13_4} = \tfrac{1}{1307674368000}(1716r^2s^2t^2 - 1287r^2s^2t + 286r^2s^2 - 1287r^2st^2 + 1144r^2st - 286r^2s + 286r^2t^2 - 286r^2t + 78r^2 - 1287rs^2t^2 + 1144rs^2t - 286rs^2 + 1144rst^2 - 1144rst + 312rs - 286rt^2 + 312rt - 91r + 286s^2t^2 - 286s^2t + 78s^2 - 286st^2 + 312st - 91s + 78t^2 - 91t + 28).$$

Now, define the linear difference operator ∇ corresponding to (4.44) as follows

$$\begin{aligned}\nabla[y(x),h] \quad = \quad & Y_{n+1}^{'[3]_3} - M_2^{'[3]_3} Y_{n-1}^{[3]_3} - hM_3^{'[3]_3} Y_{n-2}^{[3]_3} - h^2[E_1^{'[3]_3} F_n^{[3]_3} + E_2^{'[3]_3} F_{n+1}^{[3]_3}] \\ & -h^3[K_1^{'[3]_3} G_n^{[3]_3} + K_2^{'[3]_3} G_{n+1}^{[3]_3}]. \qquad (4.47)\end{aligned}$$

After expanding every functions in $Y_{n+1}^{'[3]_3}$, $F_{n+1}^{[3]_3}$ and $G_{n+1}^{[3]_3}$ about x_n, the following matrix is obtained

$$\begin{bmatrix} Q_{11}^{'[3]_3} \; Q_{21}^{'[3]_3} \; Q_{31}^{'[3]_3} \; Q_{41}^{'[3]_3} \end{bmatrix}^T = \begin{bmatrix} 0 \; 0 \; 0 \; 0 \end{bmatrix}^T$$

where

$Q_{11}^{'[3]_3} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n' - hry_n'' + \frac{h^2 r^2 y_n'''}{27720 s^3 t^3}(132r^6s^2t - 42r^7s - 42r^7t - 42r^7st + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 - 1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 - 9702s^3t^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)} r^2}{27720(r-s)^3(r-t)^3(r-1)^3 i!}(756r^9 - 2646r^8s - 2646r^8t - 2646r^8 + 3069r^7s^2 + 9420r^7st + 9420r^7s + 3069r^7t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 - 11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 - 11121r^6t^2 - 11121r^6t - 1155r^6 + 4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 + 41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t - 5148r^4s^3t^2 - 16027r^4s^3t - 5148r^4s^3 - 5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 - 16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 - 5148r^4t^2 + 1980r^3s^3t^3 + 20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t + 20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t - 26598r^2s^2t^3 - 26598r^2s^2t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 - 4158s^3t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}$

$(84r^6st - 126r^6s^2 + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 -$
$345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 +$
$616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s -$
$110r^4t^3 - 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 + 2992r^3s^3t +$
$363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3 + 484r^3st^2 -$
$275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 + 693r^2s^3t^3 -$
$891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t - 1188r^2st^3 -$
$1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 - 2772rs^3t -$
$1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 - 2310s^2t^3) -$
$\sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(42r^6s - 84r^6st + 126r^6t^2 - 84r^6t + 264r^5s^2t -$
$132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t +$
$110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st +$
$110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 -$
$363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 +$
$275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t +$
$396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 + 891r^2st^3 -$
$2673r^2st^2 + 891r^2t^4 - 693r^2t^3 + 2772rs^3t^3 + 1518rs^3t^2 - 2178rs^3t - 3564rs^2t^4 +$
$726rs^2t^3 + 1518rs^2t^2 - 3564rst^4 + 2772rst^3 - 3234s^3t^3 + 2310s^3t^2 + 4158s^2t^4 -$
$3234s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)} r^6}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(42r^6st - 84r^6s - 84r^6t + 126r^6 - 132r^5s^2t +$
$264r^5s^2 - 132r^5st^2 + 345r^5st - 105r^5s + 264r^5t^2 - 105r^5t - 399r^5 + 110r^4s^3t -$
$220r^4s^3 + 440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 - 506r^4st^2 - 407r^4st +$
$1067r^4s - 220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 -$
$396r^3s^2t^3 - 484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 + 2387r^3st^2 - 2992r^3st -$
$990r^3s + 825r^3t^3 - 363r^3t^2 - 990r^3t + 396r^2s^3t^3 + 1188r^2s^3t^2 - 2673r^2s^3t -$
$693r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 +$
$3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 +$
$726rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 +$
$4158s^2t^2) - \frac{h^3 r^3 y_n^{iv}}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55$

$r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s -$
$198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 -$
$924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} r^3}{13860(r-s)^2(r-t)^2(r-1)^2 i!}(28r^6 -$
$77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 -$
$165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 +$
$528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 +$
$462s^2t^2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2 i!}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s +$
$33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 +$
$396r^2st - 110r^3st + 99r^2st^2 - 396rst) + \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+4)} r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2 i!}(198r^2s^2 -$
$55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t -$
$55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst) +$
$\sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)} r^6}{27720(r-1)^2(s-1)^2(t-1)^2 i!}(66r^4s - 21r^5 + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st -$
$110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 -$
$198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2),$

$Q_{21}^{'[3]_3} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i+1)} - y'_n - hsy''_n + \frac{h^2 s^2 y'''_n}{27720r^3t^3}(396r^3s^4t^2 - 110r^3s^5 - 110r^3s^5t +$
$583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 +$
$264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t + 132r^2s^6 -$
$440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 -$
$1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 -$
$42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t -$
$110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 -$
$42s^7t + 132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 -$
$396s^3t^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)} s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(297r^4s^4 - 990r^4s^3t - 990r^4s^3 + 891r^4s^2t^2 +$
$3564r^4s^2t + 891r^4s^2 - 3564r^4st^2 - 3564r^4st + 4158r^4t^2 - 399r^3s^5 + 1067r^3s^4t +$
$1067r^3s^4 - 363r^3s^3t^2 - 2992r^3s^3t - 363r^3s^3 - 693r^3s^2t^3 + 891r^3s^2t^2 + 891r^3s^2t -$
$693r^3s^2 + 2772r^3st^3 + 726r^3st^2 + 2772r^3st - 3234r^3t^3 - 3234r^3t^2 + 126r^2s^6 -$
$105r^2s^5t - 105r^2s^5 - 616r^2s^4t^2 - 407r^2s^4t - 616r^2s^4 + 825r^2s^3t^3 + 2387r^2s^3t^2 +$
$2387r^2s^3t + 825r^2s^3 - 2673r^2s^2t^3 - 3168r^2s^2t^2 - 2673r^2s^2t + 1518r^2st^3 + 1518r^2$

$st^2 + 2310r^2t^3 - 84rs^6t - 84rs^6 + 264rs^5t^2 + 345rs^5t + 264rs^5 - 220rs^4t^3 - 506rs^4t^2 - 506rs^4t - 220rs^4 + 275rs^3t^3 - 484rs^3t^2 + 275rs^3t + 1188rs^2t^3 + 1188rs^2t^2 - 2178rst^3 + 42s^6t - 132s^5t^2 - 132s^5t + 110s^4t^3 + 440s^4t^2 + 110s^4t - 396s^3t^3 - 396s^3t^2 + 396s^2t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} s^2}{27720(r-s)^3(s-t)^3(s-1)^3 i!}(1155r^3s^6 - 4235r^3s^5t - 4235r^3s^5 + 5148r^3s^4t^2 + 16027r^3s^4t + 5148r^3s^4 - 1980r^3s^3t^3 - 20196r^3s^3t^2 - 20196r^3s^3t - 1980r^3s^3 + 7920r^3s^2t^3 + 26598r^3s^2t^2 + 7920r^3s^2t - 10626r^3st^3 - 10626r^3st^2 + 4158r^3t^3 - 3069r^2s^7 + 11121r^2s^6t + 11121r^2s^6 - 13376r^2s^5t^2 - 41437r^2s^5t - 13376r^2s^5 + 5148r^2s^4t^3 + 51436r^2s^4t^2 + 51436r^2s^4t + 5148r^2s^4 - 20196r^2s^3t^3 - 66330r^2s^3t^2 - 20196r^2s^3t + 26598r^2s^2t^3 + 26598r^2s^2t^2 - 10626r^2st^3 + 2646rs^8 - 9420rs^7t - 9420rs^7 + 11121rs^6t^2 + 34278rs^6t + 11121rs^6 - 4235rs^5t^3 - 41437rs^5t^2 - 41437rs^5t - 4235rs^5 + 16027rs^4t^3 + 51436rs^4t^2 + 16027rs^4t - 20196rs^3t^3 - 20196rs^3t^2 + 7920rs^2t^3 - 756s^9 + 2646s^8t + 2646s^8 - 3069s^7t^2 - 9420s^7t - 3069s^7 + 1155s^6t^3 + 11121s^6t^2 + 11121s^6t + 1155s^6 - 4235s^5t^3 - 13376s^5t^2 - 4235s^5t + 5148s^4t^3 + 5148s^4t^2 - 1980s^3t^3) + \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(220r^3s^4t - 110r^3s^4 - 825r^3s^3t^2 - 275r^3s^3t + 396r^3s^3 + 693r^3s^2t^3 + 2673r^3s^2t^2 - 1188r^3s^2t - 396r^3s^2 - 2772r^3st^3 - 1518r^3st^2 + 2178r^3st + 3234r^3t^3 - 2310r^3t^2 - 264r^2s^5t + 132r^2s^5 + 616r^2s^4t^2 + 506r^2s^4t - 440r^2s^4 + 363r^2s^3t^3 - 2387r^2s^3t^2 + 484r^2s^3t + 396r^2s^3 - 891r^2s^2t^4 - 891r^2s^2t^3 + 3168r^2s^2t^2 - 1188r^2s^2t + 3564r^2st^4 - 726r^2st^3 - 1518r^2st^2 - 4158r^2t^4 + 3234r^2t^3 + 84rs^6t - 42rs^6 + 105rs^5t^2 - 345rs^5t + 132rs^5 - 1067rs^4t^3 + 407rs^4t^2 + 506rs^4t - 110rs^4 + 990rs^3t^4 + 2992rs^3t^3 - 2387rs^3t^2 - 275rs^3t - 3564rs^2t^4 - 891rs^2t^3 + 2673rs^2t^2 + 3564rst^4 - 2772rst^3 - 126s^6t^2 + 84s^6t + 399s^5t^3 + 105s^5t^2 - 264s^5t - 297s^4t^4 - 1067s^4t^3 + 616s^4t^2 + 220s^4t + 990s^3t^4 + 363s^3t^3 - 825s^3t^2 - 891s^2t^4 + 693s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)} s^6}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + 440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 +$

$2387rs^3t^2 - 2992rs^3t - 990rs^3 - 2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - 616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2) - \frac{h^3s^3y_n^{iv}}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st + 1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 + 21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+4)}s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2i!}(198s^2t^2 - 55s^3t^2 + 99rs^2 - 110rs^3 + 33rs^4 + 462rt^2 - 198st^2 + 198s^2t - 220s^3t + 66s^4t - 55s^3 + 66s^4 - 21s^5 - 396rst^2 + 396rs^2t - 110rs^3t + 99rs^2t^2 - 396rst) + \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+4)}s^3}{13860(r-s)^2(s-t)^2(s-1)^2i!}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+4)}s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2i!}(198r^2s^2 - 55r^2s^3 + 198rs^2 - 198r^2s - 220rs^3 + 66rs^4 + 462r^2t + 99s^2t - 110s^3t + 33s^4t - 55s^3 + 66s^4 - 21s^5 + 396rs^2t - 396r^2st - 110rs^3t + 99r^2s^2t - 396rst) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+4)}s^6}{27720(r-1)^2(s-1)^2(t-1)^2i!}(198r^2s^2t - 55r^2s^3 + 99r^2s^2 - 198r^2st^2 - 396r^2st + 462r^2t^2 + 66rs^4 - 220rs^3t - 110rs^3 + 198rs^2t^2 + 396rs^2t - 396rst^2 - 21s^5 + 66s^4t + 33s^4 - 55s^3t^2 - 110s^3t + 99s^2t^2),$

$Q_{31}^{'[3]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!}y_n^{(i+1)} - y'_n - hty''_n + \frac{h^2t^2y'''_n}{27720r^3s^3}(990r^3s^3t^2 - 396r^3s^3t^3 + 1848r^3s^3t - 9702r^3s^3 + 396r^3s^2t^4 - 1188r^3s^2t^3 + 264r^3s^2t^2 + 1848r^3s^2t - 110r^3st^5 + 583r^3st^4 - 1188r^3st^3 + 990r^3st^2 - 110r^3t^5 + 396r^3t^4 - 396r^3t^3 + 396r^2s^3t^4 - 1188r^2s^3t^3 + 264r^2s^3t^2 + 1848r^2s^3t - 440r^2s^2t^5 + 1540r^2s^2t^4 - 1584r^2s^2t^3 + 264r^2s^2t^2 + 132r^2st^6 - 748r^2st^5 + 1540r^2st^4 - 1188r^2st^3 + 132r^2t^6 - 440r^2t^5 + 396r^2t^4 - 110rs^3t^5 + 583rs^3t^4 - 1188rs^3t^3 + 990rs^3t^2 + 132rs^2t^6 - 748rs^2t^5 + 1540rs^2t^4 - 1188rs^2t^3 - 42rst^7 + 321rst^6 - 748rst^5 + 583rst^4 - 42rt^7 + 132rt^6 - 110rt^5 - 110s^3t^5 + 396s^3t^4 - 396s^3t^3 + 132s^2t^6 - 440s^2t^5 + 396s^2t^4 - 42st^7 + 132st^6 - 110st^5) - \sum_{i=0}^{\infty} \frac{r^ih^{i+2}y_n^{(i+3)}t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3i!}(891r^4s^2t^2 - 3564r^4s^2t + 4158r^4s^2 - 990r^4st^3 + 3564r^4st^2 - 3564r^4st + 297r^4t^4 - 990r^4t^3 + 891r^4t^2 - 693r^3s^3t^2 + 2772$

$$r^3s^3t - 3234r^3s^3 - 363r^3s^2t^3 + 891r^3s^2t^2 + 726r^3s^2t - 3234r^3s^2 + 1067r^3st^4 - 2992r^3st^3 + 891r^3st^2 + 2772r^3st - 399r^3t^5 + 1067r^3t^4 - 363r^3t^3 - 693r^3t^2 + 825r^2s^3t^3 - 2673r^2s^3t^2 + 1518r^2s^3t + 2310r^2s^3 - 616r^2s^2t^4 + 2387r^2s^2t^3 - 3168r^2s^2t^2 + 1518r^2s^2t - 105r^2st^5 - 407r^2st^4 + 2387r^2st^3 - 2673r^2st^2 + 126r^2t^6 - 105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t + 264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 - 84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 - 396s^2t^3 + 42st^6 - 132st^5 + 110st^4) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} t^6}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3 i!} (693r^3s^3t^2 - 2772r^3s^3t + 3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 - 275r^3st^3 - 1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 + 3564r^2s^4t - 4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 - 2387r^2s^2t^3 + 3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 + 132r^2t^5 - 440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 + 2992rs^3t^3 - 891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 + 84rst^6 - 345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 - 891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + 616s^2t^4 - 825s^2t^3 + 84st^6 - 264st^5 + 220st^4) + \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} t^2}{27720 (r-t)^3 (s-t)^3 (t-1)^3 i!} (7920r^3s^3t^2 - 1980r^3s^3t^3 - 10626r^3s^3t + 4158r^3s^3 + 5148r^3s^2t^4 - 20196r^3s^2t^3 + 26598r^3s^2t^2 - 10626r^3s^2t - 4235r^3st^5 + 16027r^3st^4 - 20196r^3st^3 + 7920r^3st^2 + 1155r^3t^6 - 4235r^3t^5 + 5148r^3t^4 - 1980r^3t^3 + 5148r^2s^3t^4 - 20196r^2s^3t^3 + 26598r^2s^3t^2 - 10626r^2s^3t - 13376r^2s^2t^5 + 51436r^2s^2t^4 - 66330r^2s^2t^3 + 26598r^2s^2t^2 + 11121r^2st^6 - 41437r^2st^5 + 51436r^2st^4 - 20196r^2st^3 - 3069r^2t^7 + 11121r^2t^6 - 13376r^2t^5 + 5148r^2t^4 - 4235rs^3t^5 + 16027rs^3t^4 - 20196rs^3t^3 + 7920rs^3t^2 + 11121rs^2t^6 - 41437rs^2t^5 + 51436rs^2t^4 - 20196rs^2t^3 - 9420rst^7 + 34278rst^6 - 41437rst^5 + 16027rst^4 + 2646rt^8 - 9420rt^7 + 11121rt^6 - 4235rt^5 + 1155s^3t^6 - 4235s^3t^5 + 5148s^3t^4 - 1980s^3t^3 - 3069s^2t^7 + 11121s^2t^6 - 13376s^2t^5 + 5148s^2t^4 + 2646st^8 - 9420st^7 + 11121st^6 - 4235st^5 - 756t^9 + 2646t^8 - 3069t^7 + 1155t^6) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)} t^6}{27720 (r-1)^3 (s-1)^3 (t-1)^3 i!} (396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 + 1188r^3s^2t^2 + 1518r^3s^2t - 3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st -$$

$220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + 1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 + 440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - 506r^2st^4 + 2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 + 275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 - 3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4) - \frac{h^3t^3y_n^{iv}}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4) + \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+4)}t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2i!}(198s^2t^2 - 55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst) + \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+4)}t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2i!}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + 66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + 99r^2st^2 - 396rst) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+4)}t^3}{13860(r-t)^2(s-t)^2(t-1)^2i!}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+4)}t^6}{27720(r-1)^2(s-1)^2(t-1)^2i!}(462r^2s^2 - 198r^2s^2t + 198r^2st^2 - 396r^2st - 55r^2t^3 + 99r^2t^2 + 198rs^2t^2 - 396rs^2t - 220rst^3 + 396rst^2 + 66rt^4 - 110rt^3 - 55s^2t^3 + 99s^2t^2 + 66st^4 - 110st^3 - 21t^5 + 33t^4),$

$Q_{41}^{'[3]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!}y_n^{(i+1)} - y_n' - hy_n'' + \frac{h^2y_n'''}{27720r^3s^3t^3}(1848r^3s^3t^2 - 9702r^3s^3t^3 + 990r^3s^3t - 396r^3s^3 + 1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 + 990r^3st^3 - 1188r^3st^2 + 583r^3st - 110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t + 396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - 748r^2st + 132r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - 110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 + 583rst^3 - 748rst^2 +$

$321rst - 42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) +$
$\sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)} r}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(3564r^4s^2t - 4158r^4s^2t^2 - 891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 + 3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - 891r^3st^2 + 2992r^3st - 1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - 1518r^2s^3t^2 + 2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 + 2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t - 126r^2 + 2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + 506rs^2t - 264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) -$
$\sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} s}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - 726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st) -$
$\sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} t}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(2310r^3s^3t^2 - 2178r^3s^3t + 396r^3s^3 - 3234r^3s^2t^3 + 1518r^3s^2t^2 + 1188r^3s^2t - 396r^3s^2 + 2772r^3st^3 - 2673r^3st^2 + 275r^3st + 110r^3s - 693r^3t^3 + 825r^3t^2 - 220r^3t - 3234r^2s^3t^3 + 1518r^2s^3t^2 + 1188r^2s^3t - 396r^2s^3 + 4158r^2s^2t^4 + 726r^2s^2t^3 - 3168r^2s^2t^2 - 484r^2s^2t + 440r^2s^2 - 3564r^2st^4 + 891r^2st^3 + 2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t + 2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 - 506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 + 1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 - 616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2) -$

$\sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)}}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - 10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - 41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756) - \frac{h^3 y_n^{iv}}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)}}{27720r^2(r-s)^2(r-t)^2(r-1)^2 i!}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21) + \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)}}{27720s^2(r-s)^2(s-t)^2(s-1)^2 i!}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + 99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + 396rst - 21) + \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+4)}}{27720t^2(r-t)^2(s-t)^2(t-1)^2 i!}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t + 396rst - 21) + \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)}}{13860(r-1)^2(s-1)^2(t-1)^2 i!}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$

which leads to $\bar{D}'_0 = \bar{D}'_1 = ... = \bar{D}'_{12} = 0$ and $\bar{D}'_{13} \neq 0$. Thus, the first derivative block has order $[10, 10, 10, 10]^T$ together with the following vector of error constants

$$\bar{D}'_{13} = \begin{bmatrix} \bar{D}'_{13_1} & \bar{D}'_{13_2} & \bar{D}'_{13_3} & \bar{D}'_{13_4} \end{bmatrix}^T$$

where

$\bar{D}'_{13_1} = \frac{r^6}{100590336000}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$,

$\bar{D}'_{13_2} = \frac{s^6}{100590336000}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2)$,

$\bar{D}'_{13_3} = \frac{t^6}{100590336000}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4)$,

$\bar{D}'_{13_4} = \frac{1}{100590336000}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st - 110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 - 440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14)$.

The same procedure as used earlier is employed to find the order of second derivative block (4.45), the linear difference operator ∇ related with Equation (4.45) is evaluated as below

$$\nabla[y(x),h] = Y''^{[3]_3}_{n+1} - M''^{[3]_3}_3 Y^{[3]_3}_{n-2} - h[E''^{[3]_3}_1 F^{[3]_3}_n + E''^{[3]_3}_2 F^{[3]_3}_{n+1}] - h^2[K''^{[3]_3}_1 G^{[3]_3}_n + K''^{[3]_3}_2 G^{[3]_3}_{n+1}]. \tag{4.48}$$

Expanding $Y''^{[3]_3}_{n+1}$, $F^{[3]_3}_{n+1}$ and $G^{[3]_3}_{n+1}$-functions in Taylor series about x_n produces

$$\left[Q''^{[3]_3}_{11}\; Q''^{[3]_3}_{21}\; Q''^{[3]_3}_{31}\; Q''^{[3]_3}_{41}\right]^T = \left[0\;0\;0\;\;0\right]^T$$

where

$Q''^{[3]_3}_{11} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i+2)} - y''_n - \frac{hry'''_n}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 -$
$20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t +$
$60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 -$
$69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 -$
$69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 +$
$144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 -$
$168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3) - \sum_{i=0}^{\infty} \frac{r^ih^{i+1}y_n^{(i+3)}r}{1260(r-s)^3(r-t)^3(r-1)^3i!}(252r^9 -$
$819r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t +$
$885r^7 - 315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s -$
$315r^6t^3 - 2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 +$
$9847r^5s^2t + 3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 +$
$3228r^5t^2 + 1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 -$
$11034r^4s^2t^2 - 11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st -$
$1164r^4t^3 - 1164r^4t^2 + 420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 +$
$12618r^3s^2t^2 + 4026r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 -$
$4662r^2s^3t^2 - 1470r^2s^3t - 4662r^2s^2t^3 - 4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 +$
$1722rs^3t^2 + 1722rs^2t^3 - 630s^3t^3) - \sum_{i=0}^{\infty} \frac{s^ih^{i+1}y_n^{(i+3)}r^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3i!}(14r^6st - 21r^6s^2 +$
$14r^6s - 7r^6t + 63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 +$
$20r^5t - 45r^4s^4 - 150r^4s^3t - 150r^4s^3 + 90r^4s^2t^2 + 65r^4s^2t + 90r^4s^2 + 30r^4st^3 +$
$65r^4st^2 + 65r^4st + 30r^4s - 15r^4t^3 - 60r^4t^2 - 15r^4t + 135r^3s^4t + 135r^3s^4 +$
$39r^3s^3t^2 + 366r^3s^3t + 39r^3s^3 - 105r^3s^2t^3 - 306r^3s^2t^2 - 306r^3s^2t - 105r^3s^2 -$
$30r^3st^3 + 72r^3st^2 - 30r^3st + 48r^3t^3 + 48r^3t^2 - 108r^2s^4t^2 - 432r^2s^4t - 108r^2s^4 +$
$84r^2s^3t^3 - 66r^2s^3t^2 - 66r^2s^3t + 84r^2s^3 + 294r^2s^2t^3 + 342r^2s^2t^2 + 294r^2s^2t - 138r^2s$

$t^3 - 138r^2st^2 - 42r^2t^3 + 378rs^4t^2 + 378rs^4t - 294rs^3t^3 - 126rs^3t^2 - 294rs^3t -$
$126rs^2t^3 - 126rs^2t^2 + 210rst^3 - 378s^4t^2 + 294s^3t^3 + 294s^3t^2 - 210s^2t^3) -$
$\sum_{i=0}^{\infty} \frac{t^i h^{i+1} y_n^{(i+3)} r^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3 i!}(7r^6s - 14r^6st + 21r^6t^2 - 14r^6t + 40r^5s^2t - 20r^5s^2 -$
$14r^5st^2 + 51r^5st - 20r^5s - 63r^5t^3 - 14r^5t^2 + 40r^5t - 30r^4s^3t + 15r^4s^3 - 90r^4s^2t^2 -$
$65r^4s^2t + 60r^4s^2 + 150r^4st^3 - 65r^4st^2 - 65r^4st + 15r^4s + 45r^4t^4 + 150r^4t^3 - 90r^4t^2 -$
$30r^4t + 105r^3s^3t^2 + 30r^3s^3t - 48r^3s^3 - 39r^3s^2t^3 + 306r^3s^2t^2 - 72r^3s^2t - 48r^3s^2 -$
$135r^3st^4 - 366r^3st^3 + 306r^3st^2 + 30r^3st - 135r^3t^4 - 39r^3t^3 + 105r^3t^2 - 84r^2s^3t^3 -$
$294r^2s^3t^2 + 138r^2s^3t + 42r^2s^3 + 108r^2s^2t^4 + 66r^2s^2t^3 - 342r^2s^2t^2 + 138r^2s^2t +$
$432r^2st^4 + 66r^2st^3 - 294r^2st^2 + 108r^2t^4 - 84r^2t^3 + 294rs^3t^3 + 126rs^3t^2 - 210rs^3t -$
$378rs^2t^4 + 126rs^2t^3 + 126rs^2t^2 - 378rst^4 + 294rst^3 - 294s^3t^3 + 210s^3t^2 + 378s^2t^4 -$
$294s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+3)} r^5}{1260(r-1)^3(s-1)^3(t-1)^3 i!}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 20r^5s^2t +$
$40r^5s^2 - 20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t - 63r^5 + 15r^4s^3t - 30r^4s^3 +$
$60r^4s^2t^2 - 65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st + 150r^4s - 30r^4t^3 - 90r^4t^2 +$
$150r^4t + 45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3 - 72r^3s^2t^2 + 306r^3s^2t -$
$39r^3s^2 + 30r^3st^3 + 306r^3st^2 - 366r^3st - 135r^3s + 105r^3t^3 - 39r^3t^2 - 135r^3t +$
$42r^2s^3t^3 + 138r^2s^3t^2 - 294r^2s^3t - 84r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t +$
$108r^2s^2 - 294r^2st^3 + 66r^2st^2 + 432r^2st - 84r^2t^3 + 108r^2t^2 - 210rs^3t^3 + 126rs^3t^2 +$
$294rs^3t + 126rs^2t^3 + 126rs^2t^2 - 378rs^2t + 294rst^3 - 378rst^2 + 210s^3t^3 - 294s^3t^2 -$
$294s^2t^3 + 378s^2t^2) - \frac{h^2 r^2 y_n^{iv}}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st +$
$60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s -$
$48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 -$
$168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)} r^2}{2520(r-s)^2(r-t)^2(r-1)^2 i!}(28r^6 - 70r^5s -$
$70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t -$
$120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t +$
$84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2) +$
$\sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2 i!}(48r^2t^2 - 15r^3t^2 + 24r^2s - 30r^3s + 10r^4s - 42rt^2 +$
$48r^2t - 60r^3t + 20r^4t + 84st^2 - 15r^3 + 20r^4 - 7r^5 - 84rst^2 + 96r^2st - 30r^3st +$
$24r^2st^2 - 84rst) + \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2 i!}(48r^2s^2 - 15r^3s^2 - 42rs^2 + 48r^2s - 60$

$r^3s+20r^4s+24r^2t-30r^3t+10r^4t+84s^2t-15r^3+20r^4-7r^5-84rs^2t+96r^2st-30r^3st+24r^2s^2t-84rst)+\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+3)}r^5}{2520(r-1)^2(s-1)^2(t-1)^2i!}(20r^4s-7r^5+20r^4t+10r^4-15r^3s^2-60r^3st-30r^3s-15r^3t^2-30r^3t+48r^2s^2t+24r^2s^2+48r^2st^2+96r^2st+24r^2t^2-42rs^2t^2-84rs^2t-84rst^2+84s^2t^2),$

$Q_{21}^{''[3]_3}=\sum_{i=0}^{\infty}\frac{(sh)^i}{i!}y_n^{(i+2)}-y_n''-\frac{hsy_n'''}{1260r^3t^3}(15r^3s^5t+15r^3s^5-48r^3s^4t^2-69r^3s^4t-48r^3s^4+42r^3s^3t^3+120r^3s^3t^2+120r^3s^3t+42r^3s^3-84r^3s^2t^3-84r^3s^2t-168r^3st^3-168r^3st^2+630r^3t^3-20r^2s^6t-20r^2s^6+60r^2s^5t^2+100r^2s^5t+60r^2s^5-48r^2s^4t^3-180r^2s^4t^2-180r^2s^4t-48r^2s^4+120r^2s^3t^3+144r^2s^3t^2+120r^2s^3t-168r^2st^3+7rs^7t+7rs^7-20rs^6t^2-48rs^6t-20rs^6+15rs^5t^3+100rs^5t^2+100rs^5t+15rs^5-69rs^4t^3-180rs^4t^2-69rs^4t+120rs^3t^3+120rs^3t^2-84rs^2t^3+7s^7t-20s^6t^2-20s^6t+15s^5t^3+60s^5t^2+15s^5t-48s^4t^3-48s^4t^2+42s^3t^3)-\sum_{i=0}^{\infty}\frac{r^ih^{i+1}y_n^{(i+3)}s^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3i!}(45r^4s^4-135r^4s^3t-135r^4s^3+108r^4s^2t^2+432r^4s^2t+108r^4s^2-378r^4st^2-378r^4st+378r^4t^2-63r^3s^5+150r^3s^4t+150r^3s^4-39r^3s^3t^2-366r^3s^3t-39r^3s^3-84r^3s^2t^3+66r^3s^2t^2+66r^3s^2t-84r^3s^2+294r^3st^3+126r^3st^2+294r^3st-294r^3t^3-294r^3t^2+21r^2s^6-14r^2s^5t-14r^2s^5-90r^2s^4t^2-65r^2s^4t-90r^2s^4+105r^2s^3t^3+306r^2s^3t^2+306r^2s^3t+105r^2s^3-294r^2s^2t^3-342r^2s^2t^2-294r^2s^2t+126r^2st^3+126r^2st^2+210r^2t^3-14rs^6t-14rs^6+40rs^5t^2+51rs^5t+40rs^5-30rs^4t^3-65rs^4t^2-65rs^4t-30rs^4+30rs^3t^3-72rs^3t^2+30rs^3t+138rs^2t^3+138rs^2t^2-210rst^3+7s^6t-20s^5t^2-20s^5t+15s^4t^3+60s^4t^2+15s^4t-48s^3t^3-48s^3t^2+42s^2t^3)-\sum_{i=0}^{\infty}\frac{s^ih^{i+1}y_n^{(i+3)}s}{1260(r-s)^3(s-t)^3(s-1)^3i!}(315r^3s^6-1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-9847r^2s^5t-3228r^2s^5+1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-4026r^2s^3t^3-12618r^2s^3t^2-4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+819rs^8-2686rs^7t-2686rs^7+2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-9847rs^5t^2-9847rs^5t-1050rs^5+3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-4026rs^3t^2+1470rs^2t^3-252s^9+819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+315s^6t^3+2930s^6t^2+2930s^6t+315s^6-1050s^5$

$$t^3 - 3228s^5t^2 - 1050s^5t + 1164s^4t^3 + 1164s^4t^2 - 420s^3t^3) +$$

$$\sum_{i=0}^{\infty} \frac{t^i h^{i+1} y_n^{(i+3)} s^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3 i!}(30r^3s^4t - 15r^3s^4 - 105r^3s^3t^2 - 30r^3s^3t + 48r^3s^3 +$$

$$84r^3s^2t^3 + 294r^3s^2t^2 - 138r^3s^2t - 42r^3s^2 - 294r^3st^3 - 126r^3st^2 + 210r^3st +$$

$$294r^3t^3 - 210r^3t^2 - 40r^2s^5t + 20r^2s^5 + 90r^2s^4t^2 + 65r^2s^4t - 60r^2s^4 + 39r^2s^3t^3 -$$

$$306r^2s^3t^2 + 72r^2s^3t + 48r^2s^3 - 108r^2s^2t^4 - 66r^2s^2t^3 + 342r^2s^2t^2 - 138r^2s^2t +$$

$$378r^2st^4 - 126r^2st^3 - 126r^2st^2 - 378r^2t^4 + 294r^2t^3 + 14rs^6t - 7rs^6 + 14rs^5t^2 -$$

$$51rs^5t + 20rs^5 - 150rs^4t^3 + 65rs^4t^2 + 65rs^4t - 15rs^4 + 135rs^3t^4 + 366rs^3t^3 -$$

$$306rs^3t^2 - 30rs^3t - 432rs^2t^4 - 66rs^2t^3 + 294rs^2t^2 + 378rst^4 - 294rst^3 - 21s^6t^2 +$$

$$14s^6t + 63s^5t^3 + 14s^5t^2 - 40s^5t - 45s^4t^4 - 150s^4t^3 + 90s^4t^2 + 30s^4t + 135s^3t^4 +$$

$$39s^3t^3 - 105s^3t^2 - 108s^2t^4 + 84s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+3)} s^5}{1260(r-1)^3(s-1)^3(t-1)^3 i!}(15r^3s^4t - 30r^3s^4 -$$

$$48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - 84r^3s^2 - 210r^3st^3 +$$

$$126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t -$$

$$90r^2s^4 - 48r^2s^3t^3 - 72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t +$$

$$108r^2s^2 + 126r^2st^3 + 126r^2st^2 - 378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 -$$

$$20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 -$$

$$366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 +$$

$$40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t -$$

$$84s^2t^3 + 108s^2t^2) - \frac{h^2 s^2 y_n^{iv}}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t +$$

$$42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t -$$

$$48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 -$$

$$48s^3t^2 - 48s^3t + 42s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+3)} s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2 i!}(48s^2t^2 - 15s^3t^2 + 24rs^2 -$$

$$30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 +$$

$$96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst) + \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+3)} s^2}{2520(r-s)^2(s-t)^2(s-1)^2 i!}(45r^2s^4 - 120r^2s^3t -$$

$$120r^2s^3 + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 - 252r^2st^2 - 252r^2st + 210r^2t^2 - 70rs^5 +$$

$$180rs^4t + 180rs^4 - 120rs^3t^2 - 480rs^3t - 120rs^3 + 336rs^2t^2 + 336rs^2t - 252rst^2 +$$

$$28s^6 - 70s^5t - 70s^5 + 45s^4t^2 + 180s^4t + 45s^4 - 120s^3t^2 - 120s^3t + 84s^2t^2) +$$

$$\sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+3)} s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2 i!}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 +$$

$$84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t +$$

$$24r^2s^2t - 84rst) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+3)} s^5}{2520(r-1)^2(s-1)^2(t-1)^2 i!}(48r^2s^2t - 15r^2s^3 + 24r^2s^2 - 42r^2st^2$$

$$-84r^2st+84r^2t^2+20rs^4-60rs^3t-30rs^3+48rs^2t^2+96rs^2t-84rst^2-7s^5+20s^4t+$$
$$10s^4-15s^3t^2-30s^3t+24s^2t^2),$$

$$Q_{31}^{''[3]_3}=\sum_{i=0}^{\infty}\frac{(th)^i}{i!}y_n^{(i+2)}-y_n''-\frac{hty_n'''}{1260r^3s^3}(42r^3s^3t^3-84r^3s^3t^2-168r^3s^3t+630r^3s^3-$$
$$48r^3s^2t^4+120r^3s^2t^3-168r^3s^2t+15r^3st^5-69r^3st^4+120r^3st^3-84r^3st^2+$$
$$15r^3t^5-48r^3t^4+42r^3t^3-48r^2s^3t^4+120r^2s^3t^3-168r^2s^3t+60r^2s^2t^5-$$
$$180r^2s^2t^4+144r^2s^2t^3-20r^2st^6+100r^2st^5-180r^2st^4+120r^2st^3-20r^2t^6+$$
$$60r^2t^5-48r^2t^4+15rs^3t^5-69rs^3t^4+120rs^3t^3-84rs^3t^2-20rs^2t^6+100rs^2t^5-$$
$$180rs^2t^4+120rs^2t^3+7rst^7-48rst^6+100rst^5-69rst^4+7rt^7-20rt^6+15rt^5+$$
$$15s^3t^5-48s^3t^4+42s^3t^3-20s^2t^6+60s^2t^5-48s^2t^4+7st^7-20st^6+15st^5)-$$
$$\sum_{i=0}^{\infty}\frac{r^ih^{i+1}y_n^{(i+3)}t^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3i!}(108r^4s^2t^2-378r^4s^2t+378r^4s^2-135r^4st^3+432r^4st^2-$$
$$378r^4st+45r^4t^4-135r^4t^3+108r^4t^2-84r^3s^3t^2+294r^3s^3t-294r^3s^3-39r^3s^2t^3+$$
$$66r^3s^2t^2+126r^3s^2t-294r^3s^2+150r^3st^4-366r^3st^3+66r^3st^2+294r^3st-63r^3t^5+$$
$$150r^3t^4-39r^3t^3-84r^3t^2+105r^2s^3t^3-294r^2s^3t^2+126r^2s^3t+210r^2s^3-90r^2s^2t^4+$$
$$306r^2s^2t^3-342r^2s^2t^2+126r^2s^2t-14r^2st^5-65r^2st^4+306r^2st^3-294r^2st^2+$$
$$21r^2t^6-14r^2t^5-90r^2t^4+105r^2t^3-30rs^3t^4+30rs^3t^3+138rs^3t^2-210rs^3t+$$
$$40rs^2t^5-65rs^2t^4-72rs^2t^3+138rs^2t^2-14rst^6+51rst^5-65rst^4+30rst^3-14rt^6+$$
$$40rt^5-30rt^4+15s^3t^4-48s^3t^3+42s^3t^2-20s^2t^5+60s^2t^4-48s^2t^3+7st^6-20st^5+$$
$$15st^4)-\sum_{i=0}^{\infty}\frac{s^ih^{i+1}y_n^{(i+3)}t^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3i!}(84r^3s^3t^2-294r^3s^3t+294r^3s^3-105r^3s^2t^3+$$
$$294r^3s^2t^2-126r^3s^2t-210r^3s^2+30r^3st^4-30r^3st^3-138r^3st^2+210r^3st-15r^3t^4+$$
$$48r^3t^3-42r^3t^2-108r^2s^4t^2+378r^2s^4t-378r^2s^4+39r^2s^3t^3-66r^2s^3t^2-126r^2s^3t+$$
$$294r^2s^3+90r^2s^2t^4-306r^2s^2t^3+342r^2s^2t^2-126r^2s^2t-40r^2st^5+65r^2st^4+$$
$$72r^2st^3-138r^2st^2+20r^2t^5-60r^2t^4+48r^2t^3+135rs^4t^3-432rs^4t^2+378rs^4t-$$
$$150rs^3t^4+366rs^3t^3-66rs^3t^2-294rs^3t+14rs^2t^5+65rs^2t^4-306rs^2t^3+294rs^2t^2+$$
$$14rst^6-51rst^5+65rst^4-30rst^3-7rt^6+20rt^5-15rt^4-45s^4t^4+135s^4t^3-$$
$$108s^4t^2+63s^3t^5-150s^3t^4+39s^3t^3+84s^3t^2-21s^2t^6+14s^2t^5+90s^2t^4-105s^2t^3+$$
$$14st^6-40st^5+30st^4)+\sum_{i=0}^{\infty}\frac{t^ih^{i+1}y_n^{(i+3)}t}{1260(r-t)^3(s-t)^3(t-1)^3i!}(1470r^3s^3t^2-420r^3s^3t^3-$$
$$1722r^3s^3t+630r^3s^3+1164r^3s^2t^4-4026r^3s^2t^3+4662r^3s^2t^2-1722r^3s^2t-$$
$$1050r^3st^5+3561r^3st^4-4026r^3st^3+1470r^3st^2+315r^3t^6-1050r^3t^5+1164r^3t^4$$

$-420r^3t^3 + 1164r^2s^3t^4 - 4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686rst^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 + 315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 + 1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6) + \sum_{i=0}^{\infty} \frac{h^{i+1}y_n^{(i+3)}t^5}{1260(r-1)^3(s-1)^3(t-1)^3 i!}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 - 366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 + 40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 + 45t^4) - \frac{h^2t^2y_n^{iv}}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4) + \sum_{i=0}^{\infty} \frac{r^i h^{i+2}y_n^{(i+3)}t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2 i!}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 + 48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 + 24rs^2t^2 - 84rst) + \sum_{i=0}^{\infty} \frac{s^i h^{i+2}y_n^{(i+3)}t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2 i!}(48r^2t^2 - 15r^2t^3 + 84r^2s + 48rt^2 - 42r^2t - 60rt^3 + 20rt^4 + 24st^2 - 30st^3 + 10st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84r^2st - 30rst^3 + 24r^2st^2 - 84rst) + \sum_{i=0}^{\infty} \frac{t^i h^{i+2}y_n^{(i+3)}t^2}{2520(r-t)^2(s-t)^2(t-1)^2 i!}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4) + \sum_{i=0}^{\infty} \frac{h^{i+2}y_n^{(i+3)}t^5}{2520(r-1)^2(s-1)^2(t-1)^2 i!}(84r^2s^2 - 42r^2s^2t + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4),$

$$
\begin{aligned}
&Q_{41}^{''[3]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i+2)} - y_n'' - \frac{h y_n'''}{1260 r^3 s^3 t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - \\
&168r^3s^2t^3 + 120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - \\
&48r^3t^2 + 15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + \\
&120r^2st^3 - 180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t - 84rs^3t^3 + \\
&120rs^3t^2 - 69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + \\
&100rst^2 - 48rst + 7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + \\
&60s^2t^2 - 20s^2t + 15st^3 - 20st^2 + 7st) + \sum_{i=0}^{\infty} \frac{r^i h^{i+1} y_n^{(i+3)}}{1260r^3(r-s)^3(r-t)^3(r-1)^3 i!}(378r^4s^2t - \\
&378r^4s^2t^2 - 108r^4s^2 + 378r^4st^2 - 432r^4st + 135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + \\
&294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 + 294r^3s^2t^3 - 126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - \\
&294r^3st^3 - 66r^3st^2 + 366r^3st - 150r^3s + 84r^3t^3 + 39r^3t^2 - 150r^3t + 63r^3 - \\
&210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - 105r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + \\
&90r^2s^2 + 294r^2st^3 - 306r^2st^2 + 65r^2st + 14r^2s - 105r^2t^3 + 90r^2t^2 + 14r^2t - 21r^2 + \\
&210rs^3t^3 - 138rs^3t^2 - 30rs^3t + 30rs^3 - 138rs^2t^3 + 72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + \\
&65rst^2 - 51rst + 14rs + 30rt^3 - 40rt^2 + 14rt - 42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - \\
&60s^2t^2 + 20s^2t - 15st^3 + 20st^2 - 7st) - \sum_{i=0}^{\infty} \frac{s^i h^{i+1} y_n^{(i+3)}}{1260s^3(r-s)^3(s-t)^3(s-1)^3 i!}(294r^3s^3t^2 - \\
&294r^3s^3t + 84r^3s^3 - 210r^3s^2t^3 - 126r^3s^2t^2 + 294r^3s^2t - 105r^3s^2 + 210r^3st^3 - \\
&138r^3st^2 - 30r^3st + 30r^3s - 42r^3t^3 + 48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - \\
&108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + 39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - \\
&306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - 40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + \\
&378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - 66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - \\
&306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - \\
&108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + 39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + \\
&14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st) - \sum_{i=0}^{\infty} \frac{t^i h^{i+1} y_n^{(i+3)}}{1260t^3(r-t)^3(s-t)^3(t-1)^3 i!}(210r^3s^3t^2 - \\
&210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + 138r^3s^2t - 48r^3s^2 + 294r^3st^3 - \\
&294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - 30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + \\
&138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - 342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - \\
&378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + 108r^2t^4 - 39r^2t^3 - 90r^2t^2 + \\
&40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - 378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - \\
&65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + 7rs - 135rt^4 + 150rt^3 - 14rt^2
\end{aligned}
$$

$$
\begin{aligned}
&-14rt-84s^3t^3+105s^3t^2-30s^3t+108s^2t^4-39s^2t^3-90s^2t^2+40s^2t-135st^4+\\
&150st^3-14st^2-14st+45t^4-63t^3+21t^2)-\textstyle\sum_{i=0}^{\infty}\frac{h^{i+1}y_n^{(i+3)}}{1260(r-1)^3(s-1)^3(t-1)^3i!}(630r^3s^3t^3-\\
&1722r^3s^3t^2+1470r^3s^3t-420r^3s^3-1722r^3s^2t^3+4662r^3s^2t^2-4026r^3s^2t+\\
&1164r^3s^2+1470r^3st^3-4026r^3st^2+3561r^3st-1050r^3s-420r^3t^3+1164r^3t^2-\\
&1050r^3t+315r^3-1722r^2s^3t^3+4662r^2s^3t^2-4026r^2s^3t+1164r^2s^3+4662r^2s^2t^3-\\
&12618r^2s^2t^2+11034r^2s^2t-3228r^2s^2-4026r^2st^3+11034r^2st^2-9847r^2st+\\
&2930r^2s+1164r^2t^3-3228r^2t^2+2930r^2t-885r^2+1470rs^3t^3-4026rs^3t^2+\\
&3561rs^3t-1050rs^3-4026rs^2t^3+11034rs^2t^2-9847rs^2t+2930rs^2+3561rst^3-\\
&9847rst^2+8913rst-2686rs-1050rt^3+2930rt^2-2686rt+819r-420s^3t^3+\\
&1164s^3t^2-1050s^3t+315s^3+1164s^2t^3-3228s^2t^2+2930s^2t-885s^2-1050st^3+\\
&2930st^2-2686st+819s+315t^3-885t^2+819t-252)-\tfrac{h^2y_n^{iv}}{2520r^2s^2t^2}(210r^2s^2t^2-\\
&168r^2s^2t+42r^2s^2-168r^2st^2+168r^2st-48r^2s+42r^2t^2-48r^2t+15r^2-168rs^2t^2+\\
&168rs^2t-48rs^2+168rst^2-192rst+60rs-48rt^2+60rt-20r+42s^2t^2-48s^2t+\\
&15s^2-48st^2+60st-20s+15t^2-20t+7)+\textstyle\sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+3)}}{2520r^2(r-s)^2(r-t)^2(r-1)^2i!}(10r+20s+\\
&20t-42s^2t^2-30rs-30rt-60st+24rs^2+24rt^2+48st^2+48s^2t-15s^2-15t^2-\\
&84rst^2-84rs^2t+84rs^2t^2+96rst-7)+\textstyle\sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+3)}}{2520s^2(r-s)^2(s-t)^2(s-1)^2i!}(20r+10s+\\
&20t-42r^2t^2-30rs-60rt-30st+24r^2s+48rt^2+48r^2t+24st^2-15r^2-15t^2-\\
&84rst^2-84r^2st+84r^2st^2+96rst-7)+\textstyle\sum_{i=0}^{\infty}\frac{t^ih^{i+2}y_n^{(i+3)}}{2520t^2(r-t)^2(s-t)^2(t-1)^2i!}(20r+20s+\\
&10t-42r^2s^2-60rs-30rt-30st+48rs^2+48r^2s+24r^2t+24s^2t-15r^2-15s^2-\\
&84rs^2t-84r^2st+84r^2s^2t+96rst-7)+\textstyle\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+3)}}{2520(r-1)^2(s-1)^2(t-1)^2i!}(210r^2s^2t^2-\\
&252r^2s^2t+84r^2s^2-252r^2st^2+336r^2st-120r^2s+84r^2t^2-120r^2t+45r^2-\\
&252rs^2t^2+336rs^2t-120rs^2+336rst^2-480rst+180rs-120rt^2+180rt-70r+\\
&84s^2t^2-120s^2t+45s^2-120st^2+180st-70s+45t^2-70t+28).
\end{aligned}
$$

Now, comparing the coefficients of h^j and $y^{(j)}$ this leads to $\bar{D''}_0=\bar{D''}_1=\ldots=\bar{D''}_{12}=0$ and $\bar{D''}_{13}\neq 0$. Consequently, the second derivative of the main block has order $[10,10,10,10]^T$ with vector of error constants as follows

$$
\bar{D''}_{13}=\begin{bmatrix}\bar{D''}_{13_1} & \bar{D''}_{13_2} & \bar{D''}_{13_3} & \bar{D''}_{13_4}\end{bmatrix}^T
$$

where

$\bar{D}''_{13_1} = \frac{r^5}{50295168000}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2)$,

$\bar{D}''_{13_2} = \frac{s^5}{50295168000}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2)$,

$\bar{D}''_{13_3} = \frac{t^5}{50295168000}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)$,

$\bar{D}''_{13_4} = \frac{1}{50295168000}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$.

4.3.1.2 Zero-Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

Regarding the main block, first and second derivatives block, we Substitute $m = 3$ and $z = 3$ in (4.39), (4.44) and (4.45) which gives the first characteristic polynomials for the main block, first and second derivatives block respectively as below :

$$
\begin{aligned}
\psi^{[3]_3}(q) &= |qI_4 - \hat{M}_1^{[3]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1) \\
\psi'^{[3]_3}(q) &= |qI_4 - M_2'^{[3]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1) \\
\psi''^{[3]_3}(q) &= |qI_4 - M_3''^{[3]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1).
\end{aligned}
$$

Then, equating each characteristic polynomial to 0 we obtain $q = \{0,0,0,1\}$. This shows that one-step HBM with three off-step points is zero stable.

4.3.1.3 Consistency of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

By Definition 2.4.4 the main block method (4.39) and its derivatives (4.44) and (4.45) are consistent.

4.3.1.4 Convergence of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

The main block method (4.39) and its derivatives (4.44) and (4.45) are convergent by Theorem (2.1).

4.3.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Third Order ODEs

On substituting $m = 3$ and $z = 3$ in (3.27) yields

$$M^{[3]_3}(q) = (I_4 - q^3\hat{E}_2^{[3]_3} - q^4\hat{K}_2^{[3]_3})^{-1}(\hat{M}_1^{[3]_3} + q\hat{M}_2^{[3]_3} + q^2\hat{M}_3^{[3]_3} + q^3\hat{E}_1^{[3]_3} + q^4\hat{K}_1^{[3]_3}).$$

Calculating the eigenvalues of matrix $M^{[3]_2}(q)$, yields $\{0, 0, 0, \eta_4^{[3]_3}\}$, where the eigenvalue $\eta_4^{[3]_3}$ is in terms of q obtained below

$$\eta_4^{[3]_3} = eig(M^{[3]_3}(q)). \tag{4.49}$$

To inspect the absolute stability region, we shall substitute $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ into Equation (4.49) which produce

$$\eta_4^{[3]_3} = \frac{\sum_{i=0}^{16} c_i q^i}{K\sum_{j=0}^{17} d_j q^j}$$

where $K = 140444754379548917760$ and the values c_i and d_j are listed as in Table 4.3

Table 4.2
Coefficients of the Eigenvalue $(\eta_4^{[3]_3})$ *for the Matrix* $M^{[3]_3}$

c-value	q^i Coefficients	d-value	q^j Coefficients
c_0	4692960166059432678168249813851701248000	d_0	0
c_1	4692960166059432678168249813851701248000	d_1	334149907327746048000
c_2	2346480083029716339084124906925850624000	d_2	0
c_3	716321641508510487352280555200708608000	d_3	0
c_4	141961540213350322238508761691979776000	d_4	- 4687849429401600000
c_5	18448727762066712470780974784210534400	d_5	872935377076224000
c_6	1481808084806293306781441077668741120	d_6	0
c_7	55718034730419879505525258699407360	d_7	- 13748289601536000
c_8	- 1963795129952310749203949577830400	d_8	1253791236096000
c_9	3269711545371987789722103704453120	d_9	12027312138240
c_{10}	1175463559361586016299305344696320	d_{10}	- 8647264174080
c_{11}	165141739276162386828369461084160	d_{11}	519234263040
c_{12}	13823828404383136440180637569024	d_{12}	-3043814400
c_{13}	706404692014914119131940603904	d_{13}	-808602952
c_{14}	14611664949190805539641888000	d_{14}	26680320
c_{15}	- 584565898026768841531835784	d_{15}	1029024
c_{16}	- 33210409289114409515101539	d_{16}	-20700
		d_{17}	3213

Graphing the function $(\eta_4^{[3]_3})$ using Matlab software, we get the region of absolute stability as presented by dark area as given in Figure 4.5

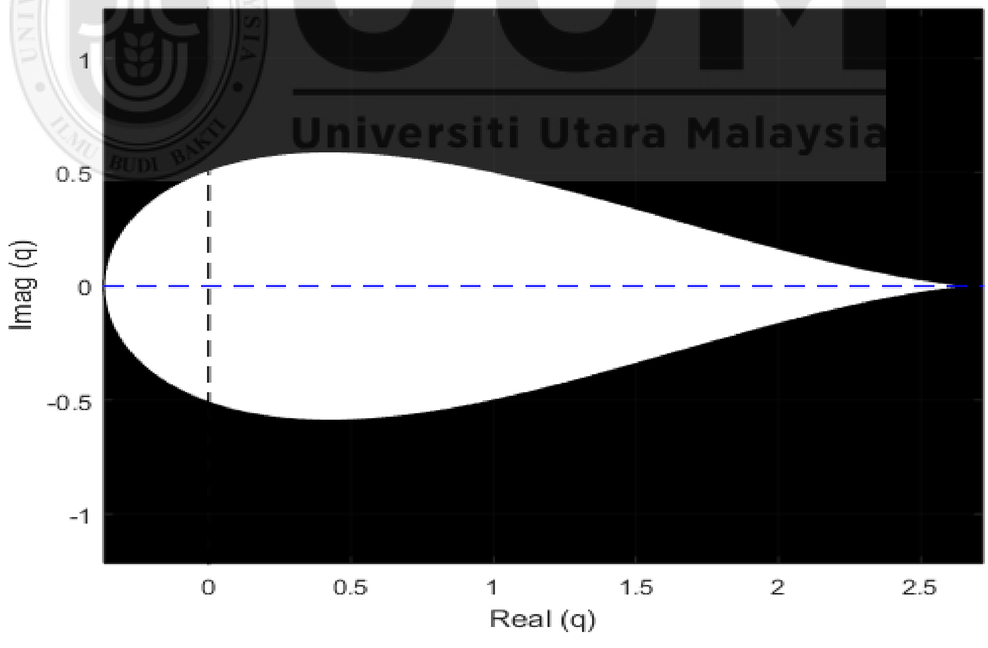


Figure 4.5. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ for Third Order ODEs

Employing the same strategy as before to plot the function $(\eta_4^{[3]_3})$ using the same software with the values $r = \frac{1}{5}$, $s = \frac{2}{5}$ and $t = \frac{3}{5}$ leads to the region of absolute stability as seen in Figure 4.6

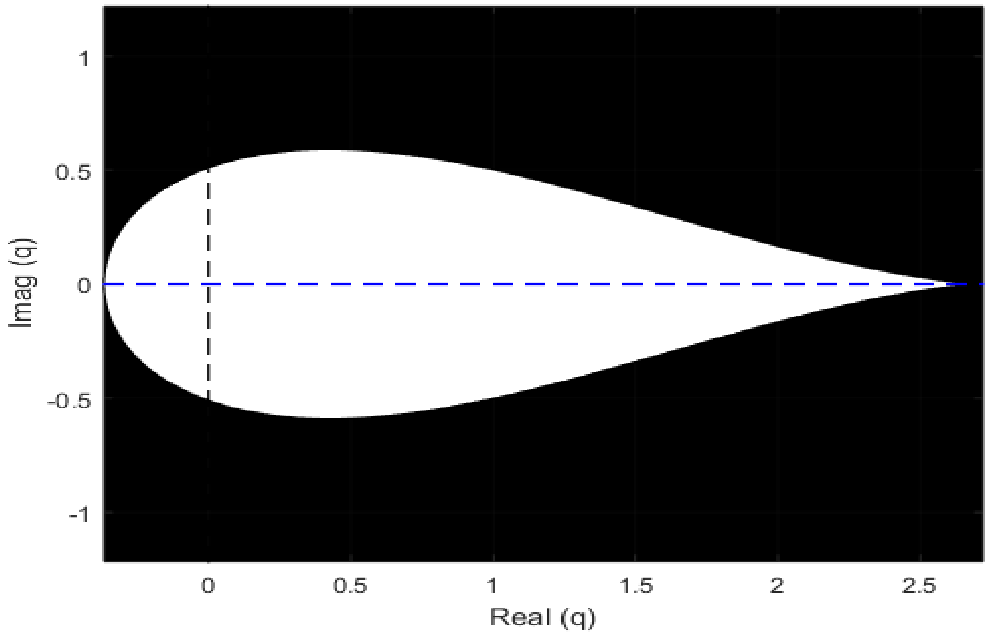


Figure 4.6. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{5}$, $s = \frac{2}{5}$ and $t = \frac{3}{5}$ for Third Order ODEs

In conclusion, the function $(\eta_4^{[2]_3})$ can be sketched for different values r, s and t and still have similar graph.

Next section, a discussion on how to transform a BVPs of third order ODEs to the equivalent IVPs is presented.

4.4 Transforming Boundary Value Problems of Third Order ODEs to the Equivalent Initial Value Problems Using Shooting Method

For the case of third order BVPs, a similar algorithm to Section 3.5 is employed, with slight modifications will be made in Step 1 and 2. It is also worth noting that not all boundary conditions for third order cases were considered. Thus, if any new boundary conditions encountered we shall deal with it in a similar manner.

Now, a general third order BVP is given by

$$y''' = f(x, y, y', y''), \quad a \leq x \leq b \tag{4.50}$$

subject to the boundary conditions below

$$y(a) = \alpha,\ y'(a) = \alpha_1,\ y(b) = \beta. \tag{4.51}$$

Equation (4.50) can be transformed to a third order IVP with initial conditions

$$y(a) = \alpha,\ y'(a) = \alpha_1,\ y''(a) = t. \tag{4.52}$$

The derivative of (4.50) with respect to t, i.e

$$\frac{\partial y'''(x,t)}{\partial t} = v'''(x,t) = f_y(x,y,y',y'')v(x,t) + f_{y'}(x,y,y',y'')v'(x,t) + f_{y''}(x,y,y',y'')v''(x,t)$$

with initial conditions

$$v(a,t) = 0,\ \ v'(a,t) = 0,\ v''(a,t) = 1\ .$$

4.5 Numerical Results for Solving Third Order ODEs

In order to compare the performance of the newly developed HBMs with existing methods in terms of error, we first need to specify the off-step points which was done randomly. Substituting $r = \frac{2}{5}$ and $s = \frac{3}{5}$ into Equations (4.20)-(4.22), (4.23)-(4.25) and (4.27)-(4.29) gives the block and derivatives for HBM with specific two off-step points as below

$$\begin{aligned}
y_{n+\frac{2}{5}} &= y_n + \frac{2h}{5}y'_n + \frac{2h^2}{25}y''_n + h^3[\frac{347686898432257}{49873847435919360}f_n + \frac{333950919568777019392}{1196972338462064697265625}f_{n+1} \\
&\quad - \frac{9004}{1063125}f_{n+\frac{2}{5}} + \frac{1456831613221211838873 6}{1225699674585153564453125}f_{n+\frac{3}{5}}] + h^4[\frac{3428}{14765625}g_n - \frac{1076}{44296875} \\
&\quad g_{n+1} - \frac{3602}{1771875}g_{n+\frac{2}{5}} - \frac{668}{590625}g_{n+\frac{3}{5}}],
\end{aligned}$$

$$\begin{aligned}
y_{n+\frac{3}{5}} &= y_n + \frac{3h}{5}y'_n + \frac{9h^2}{50}y''_n + h^3[\frac{231368647832 47233}{1261007895663738880}f_n + \frac{108404654813058642739 2}{1196972338462064697265625} \\
&\quad f_{n+1} - \frac{7062000000087499997184}{3064249186462883300781 25}f_{n+\frac{2}{5}} + \frac{10035821409632175}{252201579132747776}f_{n+\frac{3}{5}}] + h^4[\frac{23031}{35000000} \\
&\quad g_n - \frac{1377}{17500000}g_{n+1} - \frac{9801}{1400000}g_{n+\frac{2}{5}} - \frac{2619}{700000}g_{n+\frac{3}{5}}],
\end{aligned} \tag{4.53}$$

$$\begin{aligned}
y_{n+1} &= y_n + hy'_n + \frac{h^2}{2}y''_n + h^3[\frac{4440408495098953728}{7660622966157214 0625}f_n + \frac{322851798287122432}{76606229661572140625}f_{n+1} \\
&\quad - \frac{67734138395655}{1255116466775197}f_{n+\frac{2}{5}} + \frac{2485986994308513792}{15688955834689965625}f_{n+\frac{3}{5}}] + h^4[\frac{19}{8640}g_n - \frac{31}{90720}g_{n+1} \\
&\quad - \frac{925}{36288}g_{n+\frac{2}{5}} - \frac{25}{2016}g_{n+\frac{3}{5}}],
\end{aligned}$$

$$y'_{n+\frac{2}{5}} = y'_n + \frac{2h}{5}y''_n + h^2[\frac{47631028758991}{1108307720798208}f_n + \frac{52371459346766023884 8}{23939446769241293945312 5}f_{n+1}$$
$$-\frac{693179042646125}{11399736556781568}f_{n+\frac{2}{5}} + \frac{2344581171768484010393 6}{24513993491703071289062 5}f_{n+\frac{3}{5}}] + h^3[\frac{13886}{8859375}g_n -$$
$$-\frac{1684}{8859375}g_{n+1}\frac{6004}{354375}g_{n+\frac{2}{5}} - \frac{3196}{354375}g_{n+\frac{3}{5}}],$$

$$y'_{n+\frac{3}{5}} = y'_n + \frac{3h}{5}y''_n + h^2[\frac{8928152074081389}{126100789566373888}f_n + \frac{97939696653933163315 2}{23939446769241293945312 5}f_{n+1}$$
$$-\frac{500348117161011157401 6}{612849837292576660156 25}f_{n+\frac{2}{5}} + \frac{747}{4000}f_{n+\frac{3}{5}}] + h^3[\frac{9423}{3500000}g_n - \frac{621}{1750000}g_{n+1}$$
$$-\frac{4563}{140000}g_{n+\frac{2}{5}} - \frac{603}{35000}g_{n+\frac{3}{5}}],$$

$$y'_{n+1} = y'_n + hy''_n + h^2[\frac{9811091788226625536h^2}{76606229661572140625}f_n + \frac{100862617254584}{4902798698340617}f_{n+1}$$
$$-\frac{46116860184268}{1255116466775197}f_{n+\frac{2}{5}} + \frac{278348191826528}{717209409585827}f_{n+\frac{3}{5}}] + h^3[\frac{461}{90720}g_n - \frac{31}{22680}g_{n+1}$$
$$-\frac{1025}{18144}g_{n+\frac{2}{5}} - \frac{25}{1296}g_{n+\frac{3}{5}}],$$

$$y''_{n+\frac{2}{5}} = y''_n + h[\frac{16417}{118125}f_n + \frac{1108}{118125}f_{n+1} - \frac{3088718744438125}{17732923532771328}f_{n+\frac{2}{5}}$$
$$+\frac{463935613453795223142 4}{108951082185346972656 25}f_{n+\frac{3}{5}}] + h^2[\frac{221}{39375}g_n - \frac{32}{39375}g_{n+1} - \frac{211}{2625}g_{n+\frac{2}{5}}$$
$$-\frac{104g_{n+\frac{3}{5}}}{2625}],$$

$$y''_{n+\frac{3}{5}} = y''_n + h[\frac{19497}{140000}f_n + \frac{1353}{140000}f_{n+1} - \frac{486777870763818378854 4}{653706493112081933593 75}f_{n+\frac{2}{5}}$$
$$+\frac{4141904282296875}{7881299347898368}f_{n+\frac{3}{5}}] + h^2[\frac{789}{140000}g_n - \frac{117}{140000}g_{n+1}$$
$$-\frac{2151}{28000}g_{n+\frac{2}{5}} - \frac{1209}{28000}g_{n+\frac{3}{5}}],$$

$$y''_{n+1} = y''_n + h[\frac{899}{6048}f_n + \frac{189784503047153125}{1276770494359535616}f_{n+1} + \frac{2351959869397632}{6693954489467719}$$
$$+f_{n+\frac{2}{5}}\frac{1224979098644774912}{3486434629931103125}f_{n+\frac{3}{5}}] + h^2[\frac{13}{2016}g_n - \frac{13}{2016}g_{n+1}$$
$$-\frac{25}{672}g_{n+\frac{2}{5}} + \frac{25}{672}g_{n+\frac{3}{5}}].$$

Replacing the values $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ into Equations (4.40) -(4.45) produces the main block and derivatives for HBM with specific three off-step points as shown below

$$y_{n+\frac{1}{4}} = y_n + \frac{h}{4}y'_n + \frac{h^2}{32}y''_n + h^3[\frac{21033953}{12262440960}f_n + \frac{487679}{12262440960}f_{n+1}$$
$$+\frac{1447}{76640256}f_{n+\frac{1}{4}} + \frac{135}{315392}f_{n+\frac{1}{2}} + \frac{154109}{383201280}f_{n+\frac{3}{4}} + h^4[\frac{26587}{743178240}g_n$$
$$-\frac{18311}{8174960640}g_{n+1} - \frac{20869}{102187008}g_{n+\frac{1}{4}} - \frac{40291}{227082240}g_{n+\frac{1}{2}} - \frac{719}{14598144}g_{n+\frac{3}{4}}]$$

$$
\begin{aligned}
y_{n+\frac{1}{2}} &= y_n+\frac{h}{2}y'_n+\frac{h^2}{8}y''_n+h^3[\frac{12683}{1451520}f_n+\frac{1}{3584}f_{n+1}\\
&+\frac{37}{6930}f_{n+\frac{1}{4}}+\frac{5}{1386}f_{n+\frac{1}{2}}+\frac{179}{62370}f_{n+\frac{3}{4}}]+h^4[\frac{241}{1182720}g_n\\
&-\frac{167g_{n+1}}{10644480}-\frac{25}{16632}g_{n+\frac{1}{4}}-\frac{1}{768}g_{n+\frac{1}{2}}-\frac{29}{83160}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{3}{4}} &= y_n+\frac{3h}{4}y'_n+\frac{9h^2}{32}y''_n+h^3[\frac{1078587}{50462720}f_n+\frac{39717}{50462720}f_{n+1}\\
&+\frac{7101}{315392}f_{n+\frac{1}{4}}+\frac{26973}{1576960}f_{n+\frac{1}{2}}+\frac{2691}{315392}f_{n+\frac{3}{4}}]+h^4[\frac{52137}{100925440}g_n\\
&-\frac{81}{1835008}g_{n+1}-\frac{3483}{901120}g_{n+\frac{1}{4}}-\frac{95499}{25231360}g_{n+\frac{1}{2}}-\frac{1269}{1261568}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} &= y_n+hy'_n+\frac{h^2}{2}y''_n+h^3[\frac{1061}{26730}f_n+\frac{5}{2673}f_{n+1}\\
&+\frac{64}{1215}f_{n+\frac{1}{4}}+\frac{8}{165}f_{n+\frac{1}{2}}+\frac{64}{2673}f_{n+\frac{3}{4}}]+h^4[\frac{61}{62370}g_n\\
&-\frac{5}{49896}g_{n+1}-\frac{32}{4455}g_{n+\frac{1}{4}}-\frac{5}{693}g_{n+\frac{1}{2}}-\frac{64}{31185}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{1}{4}} &= y'_n+\frac{h}{4}y''_n+h^2[\frac{2602339}{153280512}f_n+\frac{382169}{766402560}f_{n+1}\\
&+\frac{148231}{47900160}f_{n+\frac{1}{4}}+\frac{1807}{322560}f_{n+\frac{1}{2}}+\frac{243193}{47900160}f_{n+\frac{3}{4}}]+h^3[\frac{28343}{72990720}g_n\\
&-\frac{14339}{510935040}g_{n+1}-\frac{551}{199584}g_{n+\frac{1}{4}}-\frac{32027}{14192640}g_{n+\frac{1}{2}}-\frac{3959}{6386688}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{1}{2}} &= y'_n+\frac{hy''_n}{2}+h^2[\frac{35}{891}f_n+\frac{13}{8910}f_{n+1}\\
&+\frac{196}{4455}f_{n+\frac{1}{4}}+\frac{1}{40}f_{n+\frac{1}{2}}+\frac{68}{4455}f_{n+\frac{3}{4}}]+h^3[\frac{1277}{1330560}g_n\\
&-\frac{109}{1330560}g_{n+1}-\frac{41}{5544}g_{n+\frac{1}{4}}-\frac{5}{693}g_{n+\frac{1}{2}}-\frac{17}{9240}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+\frac{3}{4}} &= y'_n+\frac{3h}{4}y''_n+h^2[\frac{39015}{630784}f_n+\frac{8469}{3153920}f_{n+1}\\
&+\frac{18531}{197120}f_{n+\frac{1}{4}}+\frac{3159}{35840}f_{n+\frac{1}{2}}+\frac{6813}{197120}f_{n+\frac{3}{4}}]+h^3[\frac{9747}{6307840}g_n\\
&-\frac{27}{180224}g_{n+1}-\frac{4509}{394240}g_{n+\frac{1}{4}}-\frac{19197}{1576960}g_{n+\frac{1}{2}}-\frac{9}{2464}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y'_{n+1} &= y'_n+hy''_n+h^2[\frac{6353}{74844}f_n+\frac{3457}{374220}f_{n+1}\\
&+\frac{13952}{93555}f_{n+\frac{1}{4}}+\frac{52}{315}f_{n+\frac{1}{2}}+\frac{8576}{93555}f_{n+\frac{3}{4}}]+h^3[\frac{269}{124740}g_n\\
&-\frac{5}{12474}g_{n+1}-\frac{464}{31185}g_{n+\frac{1}{4}}-\frac{10}{693}g_{n+\frac{1}{2}}-\frac{16}{4455}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y''_{n+\frac{1}{4}} &= y''_n + h[\frac{1539551}{17418240}f_n + \frac{59681}{17418240}f_{n+1} \\
&+ \frac{89371}{1088640}f_{n+\frac{1}{4}} + \frac{103}{2520}f_{n+\frac{1}{2}} + \frac{38341}{1088640}f_{n+\frac{3}{4}}] + h^2[\frac{26051}{11612160}g_n \\
&- \frac{2237}{11612160}g_{n+1} - \frac{31207}{1451520}g_{n+\frac{1}{4}} - \frac{81}{5120}g_{n+\frac{1}{2}} - \frac{1243}{290304}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y''_{n+\frac{1}{2}} &= y''_n + h[\frac{24463}{272160}f_n + \frac{1153}{272160}f_{n+1} \\
&+ \frac{1654}{8505}f_{n+\frac{1}{4}} + \frac{52}{315}f_{n+\frac{1}{2}} + \frac{394}{8505}f_{n+\frac{3}{4}}] + h^2[\frac{421}{181440}g_n \\
&- \frac{43}{181440}g_{n+1} - \frac{19}{1134}g_{n+\frac{1}{4}} - \frac{1}{40}g_{n+\frac{1}{2}} - \frac{31}{5670}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y''_{n+\frac{3}{4}} &= y''_n + h[\frac{6501}{71680}f_n + \frac{411}{71680}f_{n+1} \\
&+ \frac{921}{4480}f_{n+\frac{1}{4}} + \frac{81}{280}f_{n+\frac{1}{2}} + \frac{711}{4480}f_{n+\frac{3}{4}}] + h^2[\frac{339}{143360}g_n \\
&- \frac{9}{28672}g_{n+1} - \frac{279}{17920}g_{n+\frac{1}{4}} - \frac{81}{5120}g_{n+\frac{1}{2}} - \frac{183}{17920}g_{n+\frac{3}{4}}]
\end{aligned}
$$

$$
\begin{aligned}
y''_{n+1} &= y''_n + h[\frac{1601}{17010}f_n + \frac{1601}{17010}f_{n+1} \\
&+ \frac{2048}{8505}f_{n+\frac{1}{4}} + \frac{104}{315}f_{n+\frac{1}{2}} + \frac{2048}{8505}f_{n+\frac{3}{4}}] + h^2[\frac{29}{11340}g_n \\
&- \frac{29}{11340}g_{n+1} - \frac{32}{2835}g_{n+\frac{1}{4}} + \frac{32}{2835}g_{n+\frac{3}{4}}].
\end{aligned}
$$

HBMs with different two and three specific off-step points can be obtained by replacing different values of r, s and t in the corresponding equations.

4.6 Test Problems and Numerical Results

The same notations in Section 3.7 are also adopted in this section. The following test problems are considered to compare the performance of the proposed HBMs with the existing methods:

Problem 8: $y''' = 3\sin(x), \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = -2,\ x \in [0,1].$

Exact solution: $y(x) = 3\cos(x) + \dfrac{x^2}{2} - 2$

Source: (Olabode, 2014)

Problem 9: $y''' + 4y' - x = 0,\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 1,\ x \in [0,1].$

Exact solution: $y(x) = \frac{3}{16}(1 - \cos(2x)) + \frac{x^2}{8}$

Source: (Mohammed & Adeniyi, 2014)

Problem 10: $y''' - 2xy'y'' - (y')^2 = 0,\ y(0) = 1,\ y'(0) = \frac{1}{2},$

$y''(0) = 0,\ x \in [0,1].$

Exact solution: $y(x) = 1 + \frac{1}{2}\log\frac{(2+x)}{(2-x)}$

Source: (Gbenga et al., 2015)

Problem 11: $y''' - xy - (x^3 - 2x^2 - 5x - 3)e^x = 0,\ y(0) = 0,\ y'(0) = 1,$

$y'(1) = -e,\ x \in [0,1].$

Exact solution: $y(x) = x(1-x)e^x$

Source: (Jator, 2008b)

Problem 12: $y''' + y - (7 - x^2)\cos(x) - (x^2 - 6x - 1)\sin(x) = 0,$

$y(0) = 0,\ y'(0) = -1,\ y'(1) = 2\sin(1),\ x \in [0,1].$

Exact solution: $y(x) = (x^2 - 1)\sin(x)$

Source: (Abdullah et al., 2013)

Problem 13: $y''' + 2e^{-3y} - 4(1+x)^{-3} = 0,$

$y(0) = 0,\ y'(0) = 1,\ y(1) = \ln(2),\ x \in [0,1].$

Exact solution: $y(x) = \ln(1+x)$

Source: (Sahi et al., 2013)

Problem 14: $y''' + y - (x-4)\sin(x) - (1-x)\cos(x) = 0,$

$y(0) = 0,\ y'(0) = -1,\ y'(1) = \sin(1),\ x \in [0,1].$

Exact solution: $y(x) = (x-1)\sin(x)$

Source: (Abdullah et al., 2013)

Table 4.3

Comparison of One-Step HBMs with FSBM of order 7 and TSBM of order 5 for Solving Problem 8

x		HBM with $r=\frac{3}{8}, s=\frac{4}{9}$	HBM with $r=\frac{1}{10}, s=\frac{3}{10}, t=\frac{7}{10}$	FSBM	TSBM
	Exact solution	0.990012495	0.990012495	0.990012495	0.990012495
0.1	Computed solution	0.990012495	0.990012495	0.990012495	0.990012496
	Error	3.330E(-16)	2.220E(-16)	3.407E(-11)	1.659E(-10)
	Exact solution	0.960199733	0.960199733	0.960199733	0.960199733
0.2	Computed Solution	0.960199733	0.960199733	0.960199733	0.960199734
	Error	3.330E(-16)	2.220E(-16)	1.237E(-10)	4.762E(-10)
	Exact solution	0.911009467	0.911009467	0.911009467	0.911009467
0.3	Computed solution	0.911009467	0.911009467	0.911009467	0.911009468
	Error	5.551E(-16)	1.110E(-16)	1.7681E(-10)	6.231E(-10)
	Exact solution	0.843182982	0.843182982	0.843182982	0.843182982
0.4	Computed solution	0.843182982	0.843182982	0.843182981	0.843182984
	Error	1.776E(-15)	3.330E(-16)	4.086E(-10)	2.913E(-10)
	Exact Solution	0.757747685	0.757747685	0.757747685	0.757747685
0.5	Computed solution	0.757747685	0.757747685	0.757747685	0.757747686
	Error	4.218E(-15)	6.661E(-16)	3.711E(-10)	8.7118E(-10)
	Exact Solution	0.656006844	0.656006844	0.656006844	0.656006844
0.6	Computed solution	0.656006844	0.656006844	0.656006844	0.656006846
	Error	8.992E(-15)	1.776E(-15)	7.096E(-10)	3.929E(-9)
	Exact solution	0.539526561	0.539526561	0.539526561	0.539526561
0.7	Computed solution	0.539526561	0.539526561	0.539526562	0.539526567
	Error	1.620E(-14)	3.108E(-15)	7.465E(-10)	9.553E(-9)
	Exact solution	0.410120128	0.410120128	0.410120128	0.410120128
0.8	Computed solution	0.410120128	0.410120128	0.410120130	0.410120139
	Error	2.714E(-14)	5.162E(-15)	1.958E(-9)	1.804E(-8)
	Exact solution	0.269829904	0.269829904	0.269829904	0.269829904
0.9	Computed Solution	0.269829904	0.269829904	0.269829908	0.269829925
	Error	4.363E(-14)	9.0483E(-15)	3.888E(-9)	3.031E(-8)
	Exact solution	0.120906917	0.120906917	0.120906917	0.120906917
1.0	Computed solution	0.120906917	0.120906917	0.120906924	0.120906953
	Error	6.616E(-14)	1.453E(-14)	6.395E(-9)	4.730E(-8)

Table 4.4

Comparison of One-Step HBMs with TSHBM of order 5 for Solving Problem 9

x		HBM with $r=\frac{2}{5}, s=\frac{4}{5}$	HBM with $r=\frac{1}{7}, s=\frac{3}{7}, t=\frac{5}{7}$	TSHBM
0.1	Exact solution	0.00498751665	0.00498751665	0.00498751665
	Computed solution	0.00498751665	0.00498751665	0.00498751766
	Error	0.0000 E(+00)	0.0000 E(+00)	9.6100E(-10)
0.2	Exact solution	0.01980106362	0.01980106362	0.01980106362
	Computed Solution	0.01980106362	0.01980106362	0.01980107010
	Error	2.0816E(-17)	1.0408E(-17)	6.5000E(-9)
0.3	Exact solution	0.04399957220	0.04399957220	0.04399957220
	Computed solution	0.04399957220	0.04399957220	0.04399958817
	Error	7.6327E(-17)	8.3266E(-17)	1.5970E(-8)
0.4	Exact solution	0.07686749199	0.07686749199	0.07686749199
	Computed solution	0.07686749199	0.07686749199	0.07686750864
	Error	2.7755E(-16)	1.9428E(-16)	1.6640E(-8)
0.5	Exact Solution	0.1174433176	0.1174433176	0.1174433176
	Computed solution	0.1174433176	0.1174433176	0.1174433379
	Error	7.2164E(-16)	3.3306E(-16)	2.0300E(-8)
0.6	Exact Solution	0.1645579210	0.1645579210	0.1645579210
	Computed solution	0.1645579210	0.1645579210	0.1645579476
	Error	1.4710E(-15)	5.2735E(-16)	2.6600E(-8)
0.7	Exact solution	0.2168811607	0.2168811607	0.2168811607
	Computed solution	0.2168811607	0.2168811607	0.2168811874
	Error	2.6367E(-15)	7.2164E(-16)	2.6700E(-8)
0.8	Exact solution	0.2729749104	0.2729749104	0.2729749104
	Computed solution	0.2729749104	0.2729749104	0.2729749375
	Error	4.3298E(-15)	8.3266E(-16)	2.7100E(-8)
0.9	Exact solution	0.3313503927	0.3313503927	0.3313503927
	Computed Solution	0.3313503927	0.3313503927	0.3313504205
	Error	6.6613E(-15)	8.8817E(-16)	2.7700E(-8)
1.0	Exact solution	0.3905275318	0.3905275318	0.3905275318
	Computed solution	0.3905275318	0.3905275318	0.3905275591
	Error	9.6034E(-15)	7.7715E(-16)	2.7200E(-8)

Table 4.5

Comparison of One-Step HBMs with THOSHBM of order 5 and HNHIS of order 6 for Solving Problem 10

x		HBM with $r=\frac{2}{7}, s=\frac{5}{7}$	HBM with $r=\frac{1}{10}, s=\frac{4}{10}, t=\frac{7}{10}$	THOSHBM	HNHIS
0.21	Exact solution	1.1053884478384988	1.1053884478384988	1.1053884478384988	1.1053884478384988
	Computed solution	1.1053884478384988	1.1053884478384988	1.1053884478384957	1.1053884478384823
	Error	0.000000E(+00)	0.000000E(+00)	3.108624E(-15)	1.643130E(-14)
0.31	Exact solution	1.1562594977993601	1.1562594977993601	1.1562594977993601	1.1562594977993601
	Computed Solution	1.1562594977993597	1.1562594977993601	1.1562594977993363	1.1562594977992764
	Error	4.440892E(-16)	0.000000E(+00)	2.375877E(-14)	8.371082E(-14)
0.41	Exact solution	1.2079463656352118	1.2079463656352118	1.2079463656352118	1.2079463656352118
	Computed solution	1.2079463656352114	1.2079463656352118	1.2079463656351106	1.2079463656349305
	Error	4.440892E(-16)	0.000000E(+00)	1.012523E(-13)	2.813305E(-13)
0.51	Exact solution	1.2607533165931626	1.2607533165931626	1.2607533165931626	1.2607533165931626
	Computed solution	1.2607533165931619	1.2607533165931628	1.2607533165928335	1.2607533165923959
	Error	6.661338E(-16)	2.220446E(-16)	3.290701E(-13)	7.667200E(-13)
0.61	Exact Solution	1.3150232370960011	1.3150232370960011	1.3150232370960011	1.3150232370960011
	Computed solution	1.3150232370960004	1.3150232370960016	1.3150232370951040	1.3150232370941486
	Error	6.661338E(-16)	4.440892E(-16)	8.970602E(-13)	1.852518E(-12)
0.71	Exact Solution	1.3711532082590145	1.3711532082590145	1.3711532082590145	1.3711532082590145
	Computed solution	1.3711532082590141	1.3711532082590161	1.3711532082568312	1.3711532082548517
	Error	4.440892E(-16)	1.554312E(-15)	2.183365E(-12)	4.162892E(-12)
0.81	Exact solution	1.4296155881111083	1.4296155881111083	1.4296155881111083	1.4296155881111083
	Computed solution	1.4296155881111079	1.4296155881111108	1.4296155881061783	1.4296155881021435
	Error	4.440892E(-16)	2.442491E(-15)	4.930056E(-12)	8.964829E(-12)

Table 4.6

Comparison of One-Step HBMs with NITOM of order 4 for Solving Problem 11

h		HBM with $r=\frac{2}{7}, s=\frac{3}{7}$	HBM with $r=\frac{1}{8}, s=\frac{3}{8}, t=\frac{5}{8}$	NITOM
$\frac{1}{6}$	Exact solution	0.000000000000000302	0.000000000000000302	N/A
	Computed solution	-0.000000000000119983	-0.000000000000000674	N/A
	Max Error	1.20E(-13)	9.75E(-16)	1.52E(-05)
$\frac{1}{9}$	Exact solution	-0.000000000000000604	0.419517165797803850	N/A
	Computed Solution	-0.000000000000071992	0.419517165797803570	N/A
	Max Error	7.13E(-14)	2.77E(-16)	2.93E(-06)
$\frac{1}{12}$	Exact solution	0.000000000000000000	0.306527738415440790	N/A
	Computed solution	-0.000000000000069897	0.306527738415440450	N/A
	Max Error	6.98E(-14)	3.33E(-16)	9.26E(-07)
$\frac{1}{15}$	Exact solution	0.000000000000000302	0.064622200767701365	N/A
	Computed solution	-0.000000000000071166	0.064622200767701102	N/A
	Max Error	7.14E(-14)	2.63E(-16)	3.85E(-07)

Table 4.7

Comparison of One-Step HBMs with FOBM of order 4 for Solving Problem 12

h		HBM with $r=\frac{2}{5}, s=\frac{4}{5}$	HBM with $r=\frac{1}{7}, s=\frac{3}{7}, t=\frac{5}{7}$	FOBM
$\frac{1}{8}$	Exact solution	0.000000000000000000	0.000000000000000000	N/A
	Computed solution	0.000000000000035468	0.000000000000023888	N/A
	Max Error	3.54E(-14)	2.38E(-14)	3.27E(-05)
$\frac{1}{16}$	Exact solution	0.000000000000000000	-0.029437974542430287	N/A
	Computed Solution	0.000000000000034553	-0.029437974542410181	N/A
	Max Error	3.45E(-14)	2.01E(-14)	2.39E(-06)
$\frac{1}{32}$	Exact solution	0.000000000000000000	-0.029437974542430287	N/A
	Computed solution	0.000000000000034844	-0.029437974542413165	N/A
	Max Error	3.48E(-14)	1.71E(-14)	1.59E(-07)
$\frac{1}{64}$	Exact solution	-0.005240414251907476	0.000000000000000000	N/A
	Computed solution	-0.005240414251873007	0.000000000000015676	N/A
	Max Error	3.44E(-14)	1.56E(-14)	1.01E(-08)
$\frac{1}{128}$	Exact solution	0.000000000000000000	-0.016710324398911097	N/A
	Computed solution	0.000000000000034946	-0.016710324398896841	N/A
	Max Error	3.49E(-14)	1.42E(-14)	6.43E(-10)

Table 4.8

Comparison of One-Step HBMs with FDM of order 8 for Solving Problem 13

h		HBM with $r=\frac{2}{5}, s=\frac{3}{5}$	HBM with $r=\frac{1}{4}, s=\frac{1}{2}, t=\frac{3}{4}$	FDM
$\frac{1}{7}$	Exact solution	0.451985123743057220	0.428995605518358410	N/A
	Computed solution	0.451985123745303470	0.428995605518360750	N/A
	Max Error	2.24E(-12)	2.33E(-15)	5.24E(-09)
$\frac{1}{14}$	Exact solution	0.451985123743057100	0.609377297494486330	N/A
	Computed Solution	0.451985123743066760	0.609377297494486550	N/A
	Max Error	9.65E(-15)	2.22E(-16)	2.39E(-11)
$\frac{1}{28}$	Exact solution	0.461034959262974940	0.693147180559945060	N/A
	Computed solution	0.461034959262976160	0.693147180559945180	N/A
	Max Error	1.22E(-15)	1.11E(-16)	9.50E(-14)

Table 4.9

Comparison of One-Step HBMs with FOBM of order 4 for Solving Problem 14

h		HBM with $r=\frac{5}{8}, s=\frac{5}{7}$	HBM with $r=\frac{3}{8}, s=\frac{5}{8}, t=\frac{7}{8}$	FOBM
$\frac{1}{16}$	Exact solution	-0.014851590760145026	-0.006540814347317457	N/A
	Computed solution	-0.014851590760140178	-0.006540814347317499	N/A
	Max Error	4.84E(-15)	4.25E(-17)	4.18E(-06)
$\frac{1}{32}$	Exact solution	-0.009786113478684858	-0.129026502334850940	N/A
	Computed Solution	-0.009786113478676296	-0.129026502334851110	N/A
	Max Error	8.56E(-15)	1.66E(-16)	5.46E(-07)
$\frac{1}{64}$	Exact solution	0.000000000000000000	-0.132602302084352840	N/A
	Computed solution	0.000000000000010219	-0.132602302084352950	N/A
	Max Error	1.02E(-14)	1.11E(-16)	6.96E(-08)
$\frac{1}{128}$	Exact solution	-0.001875586735667530	-0.002460599000863589	N/A
	Computed solution	-0.001875586735656803	-0.002460599000863905	N/A
	Max Error	1.07E(-14)	3.16E(-16)	8.79E(-09)

4.7 Comments on the Results

Both Problem 8 and Problem 9 are linear IVPs of third order ODEs. It can be observed from Tables 4.3-4.4 that both new HBMs perform better than the existing methods in terms of error. The numerical results also suggest that among the two HBMs, HBM with three off-step points gives better accuracy than HBM with two-off step points. This may be due to the fact that the more points used, the more accurate, normally, the method will be.

The numerical results for solving a nonlinear IVP of third order ODE in Problem 10 also indicate that the proposed HBMs are more accurate if compared to the existing ones. Surprisingly, for this test problem, HBM with two-off step points produces better accuracy than HBM with three-off step points as the values of x increase. It is worth mentioning that only the numerical results at the x values considered in the previous existing methods are reported.

Problems 11-12 are both linear BVPs of third order ODEs. This time, both problems were solved using the newly developed HBMs with different step sizes. Again, the numerical results in Tables 4.6-4.7 demonstrate that HBMs outperform the existing methods in terms of maximum error. In Problem 11, HBM with three off-step points really show its superiority by producing the least maximum error. Although HBM with three off-step points has the same order of accuracy as its two-off step points counterpart in solving Problem 12, the former method still produces better results.

Meanwhile, a nonlinear BVP in Problem 13 was solved using the proposed HBMs with three different step sizes. As expected, as the step sizes decrease the accuracy of the methods increase. The numerical results suggest that HBM with three off-step points has the advantages over HBM with two off-step points in terms of maximum error. Both HBMs show their superiority over the existing FDM method as depicted in Table 4.8.

Another linear BVP of third order ODE in Problem 14 was solved directly using the developed HBMs. As anticipated, HBM with three off-step points performs the best, followed by HBM with two off-step points. The existing FOBM method fails to compete with HBMs in terms of maximum error.

4.8 Summary

HBMs with two and three off-steps points in the presence of fourth derivative for solving third order ODEs directly using interpolation and collocation approach have been successfully developed in this chapter. Based on the numerical results obtained, these two methods outperform the previous existing methods in terms of error. The results also indicate that, in general, HBM with three off-step points performs the best among all methods considered.

CHAPTER FIVE

ONE-STEP HYBRID BLOCK METHODS FOR DIRECTLY SOLVING FOURTH ORDER ODES IN THE PRESENCE OF FIFTH DERIVATIVE

5.1 Introduction

In this chapter, the development of one-step HBMs involving three generalised off-step points using collocation and interpolation method for solving fourth order IVPs and BVPs of ODEs are considered.

5.2 Derivation of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

The power series in Equation (3.2) is also employed to approximate the solution of the general fourth order IVP of ODE of the form

$$y^{(iv)}(x) = f(x,y,y^{'},y^{''},y^{'''}) \;\; y^{(i)}(a) = \omega_i, \;\; i = 0,1,2,3. \tag{5.1}$$

Now, differentiating (3.2) four and five times gives

$$y^{(iv)}(x) = \sum_{j=4}^{2v+u-1} a_j \frac{j!}{h^4(j-4)!}(\frac{x-x_n}{h})^{j-4}, \tag{5.2}$$

$$y^{(v)}(x) = \sum_{j=5}^{2v+u-1} a_j \frac{j!}{h^5(j-5)!}(\frac{x-x_n}{h})^{j-5}, \tag{5.3}$$

which leads to

$$\sum_{j=4}^{2v+u-1} a_j \frac{j!}{h^4(j-4)!}(\frac{x-x_n}{h})^{j-4} = f(x,y,y^{'},y^{''},y^{'''}), \tag{5.4}$$

$$\sum_{j=5}^{2v+u-1} a_j \frac{j!}{h^5(j-5)!}(\frac{x-x_n}{h})^{j-5} = g(x,y,y^{'},y^{''},y^{'''},y^{(iv)}). \tag{5.5}$$

The approximate solution (3.2) is then interpolated at four points, i.e x_n, x_{n+r}, x_{n+s} and x_{n+t} where $0 < r < s < t < 1$, while (5.4) and (5.5) are collocated at all points as shown in Figure 5.1 below :

$$
\begin{array}{rccccc}
y: & I & I & I & I & \\
 & x_n & x_{n+r} & x_{n+s} & x_{n+t} & x_{n+1} \\
y^{(iv)} = f(x,y,y',y'',y'''): & C & C & C & C & C \\
y^{(v)} = g(x,y,y',y'',y''',y^{(iv)}): & C & C & C & C & C
\end{array}
$$

Figure 5.1. Interpolation and Collocation Strategy for One-Step HBM with Three Off-Step Points for Solving Fourth Order ODEs

Substituting $u = 4$ and $v = 5$ in (3.2), (5.4) and (5.5) yields

$$
\begin{aligned}
y_n &= a_0, \\
y_{n+r} &= a_0 + ra_1 + r^2a_2 + r^3a_3 + r^4a_4 + r^5a_5 + r^6a_6 + r^7a_7 + r^8a_8 + r^9a_9 + \\
&\quad r^{10}a_{10} + r^{11}a_{11} + r^{12}a_{12} + r^{13}a_{13}, \\
y_{n+s} &= a_0 + sa_1 + s^2a_2 + s^3a_3 + s^4a_4 + s^5a_5 + s^6a_6 + s^7a_7 + s^8a_8 + s^9a_9 + \\
&\quad s^{10}a_{10} + s^{11}a_{11} + s^{12}a_{12} + s^{13}a_{13}, \\
y_{n+t} &= a_0 + ta_1 + t^2a_2 + t^3a_3 + t^4a_4 + t^5a_5 + t^6a_6 + t^7a_7 + t^8a_8 + t^9a_9 + \\
&\quad t^{10}a_{10} + t^{11}a_{11} + t^{12}a_{12} + t^{13}a_{13}, \\
f_n &= \frac{24}{h^4}a_4, \\
f_{n+r} &= \frac{24}{h^4}a_4 + \frac{120r}{h^4}a_5 + \frac{360r^2}{h^4}a_6 + \frac{840r^3}{h^4}a_7 + \frac{1680r^4}{h^4}a_8 + \frac{3024r^5}{h^4}a_9 + \\
&\quad \frac{5040r^6}{h^4}a_{10} + \frac{7920r^7}{h^4}a_{11} + \frac{11880r^8}{h^4}a_{12} + \frac{17160r^9}{h^4}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
f_{n+s} &= \frac{24}{h^4}a_4+\frac{120s}{h^4}a_5+\frac{360s^2}{h^4}a_6+\frac{840s^3}{h^4}a_7+\frac{1680s^4}{h^4}a_8+\frac{3024s^5}{h^4}a_9+\\
&\quad \frac{5040s^6}{h^4}a_{10}+\frac{7920s^7}{h^4}a_{11}+\frac{11880s^8}{h^4}a_{12}+\frac{17160s^9}{h^4}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
f_{n+t} &= \frac{24}{h^4}a_4+\frac{120t}{h^4}a_5+\frac{360t^2}{h^4}a_6+\frac{840t^3}{h^4}a_7+\frac{1680t^4}{h^4}a_8+\frac{3024t^5}{h^4}a_9+\\
&\quad \frac{5040t^6}{h^4}a_{10}+\frac{7920t^7}{h^4}a_{11}+\frac{11880t^8}{h^4}a_{12}+\frac{17160t^9}{h^4}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
f_{n+1} &= \frac{24}{h^4}a_4+\frac{120}{h^4}a_5+\frac{360}{h^4}a_6+\frac{840}{h^4}a_7+\frac{1680}{h^4}a_8+\frac{3024}{h^4}a_9+\\
&\quad \frac{5040}{h^4}a_{10}+\frac{7920}{h^4}a_{11}+\frac{11880}{h^4}a_{12}+\frac{17160}{h^4}a_{13},
\end{aligned}
$$

$$
g_n = \frac{120}{h^5}a_5,
$$

$$
\begin{aligned}
g_{n+r} &= \frac{120}{h^5}a_5+\frac{720r}{h^5}a_6+\frac{2520r^2}{h^5}a_7+\frac{6720r^3}{h^5}a_8+\frac{15120r^4}{h^5}a_9+\frac{30240r^5}{h^5}\\
&\quad a_{10}+\frac{55440r^6}{h^5}a_{11}+\frac{95040r^7}{h^5}a_{12}+\frac{154440r^8}{h^5}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
g_{n+s} &= \frac{120}{h^5}a_5+\frac{720s}{h^5}a_6+\frac{2520s^2}{h^5}a_7+\frac{6720s^3}{h^5}a_8+\frac{15120s^4}{h^5}a_9+\frac{30240s^5}{h^5}\\
&\quad a_{10}+\frac{55440s^6}{h^5}a_{11}+\frac{95040s^7}{h^5}a_{12}+\frac{154440s^8}{h^5}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
g_{n+t} &= \frac{120}{h^5}a_5+\frac{720t}{h^5}a_6+\frac{2520t^2}{h^5}a_7+\frac{6720t^3}{h^5}a_8+\frac{15120t^4}{h^5}a_9+\frac{30240t^5}{h^5}\\
&\quad a_{10}+\frac{55440t^6}{h^5}a_{11}+\frac{95040t^7}{h^5}a_{12}+\frac{154440t^8}{h^5}a_{13},
\end{aligned}
$$

$$
\begin{aligned}
g_{n+1} &= \frac{120}{h^5}a_5+\frac{720}{h^5}a_6+\frac{2520}{h^5}a_7+\frac{6720}{h^5}a_8+\frac{15120}{h^5}a_9+\frac{30240}{h^5}a_{10}+\\
&\quad \frac{55440}{h^5}a_{11}+\frac{95040}{h^5}a_{12}+\frac{154440}{h^5}a_{13},
\end{aligned}
$$

which can be represented in a matrix form $AX = B$ where

$$A = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & r^{12} & r^{13} \\
1 & s & s^2 & s^3 & s^4 & s^5 & s^6 & s^7 & s^8 & s^9 & s^{10} & s^{11} & s^{12} & s^{13} \\
1 & t & t^2 & t^3 & t^4 & t^5 & t^6 & t^7 & t^8 & t^9 & t^{10} & t^{11} & t^{12} & t^{13} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120r}{h^4} & \frac{360r^2}{h^4} & \frac{840r^3}{h^4} & \frac{1680r^4}{h^4} & \frac{3024r^5}{h^4} & \frac{5040r^6}{h^4} & \frac{7920r^7}{h^4} & \frac{11880r^8}{h^4} & \frac{17160r^9}{h^4} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120s}{h^4} & \frac{360s^2}{h^4} & \frac{840s^3}{h^4} & \frac{1680s^4}{h^4} & \frac{3024s^5}{h^4} & \frac{5040s^6}{h^4} & \frac{7920s^7}{h^4} & \frac{11880s^8}{h^4} & \frac{17160s^9}{h^4} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120t}{h^4} & \frac{360t^2}{h^4} & \frac{840t^3}{h^4} & \frac{1680t^4}{h^4} & \frac{3024t^5}{h^4} & \frac{5040t^6}{h^4} & \frac{7920t^7}{h^4} & \frac{11880t^8}{h^4} & \frac{17160t^9}{h^4} \\
0 & 0 & 0 & 0 & \frac{24}{h^4} & \frac{120}{h^4} & \frac{360}{h^4} & \frac{840}{h^4} & \frac{1680}{h^4} & \frac{3024}{h^4} & \frac{5040}{h^4} & \frac{7920}{h^4} & \frac{11880}{h^4} & \frac{17160}{h^4} \\
0 & 0 & 0 & 0 & 0 & \frac{120}{h^5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{120}{h^5} & \frac{720r}{h^5} & \frac{2520r^2}{h^5} & \frac{6720r^3}{h^5} & \frac{15120r^4}{h^5} & \frac{30240r^5}{h^5} & \frac{55440r^6}{h^5} & \frac{95040r^7}{h^5} & \frac{154440r^8}{h^5} \\
0 & 0 & 0 & 0 & 0 & \frac{120}{h^5} & \frac{720s}{h^5} & \frac{2520s^2}{h^5} & \frac{6720s^3}{h^5} & \frac{15120s^4}{h^5} & \frac{30240s^5}{h^5} & \frac{55440s^6}{h^5} & \frac{95040s^7}{h^5} & \frac{154440s^8}{h^5} \\
0 & 0 & 0 & 0 & 0 & \frac{120}{h^5} & \frac{720t}{h^5} & \frac{2520t^2}{h^5} & \frac{6720t^3}{h^5} & \frac{15120t^4}{h^5} & \frac{30240t^5}{h^5} & \frac{55440t^6}{h^5} & \frac{95040t^7}{h^5} & \frac{154440t^8}{h^5} \\
0 & 0 & 0 & 0 & 0 & \frac{120}{h^5} & \frac{720}{h^5} & \frac{2520}{h^5} & \frac{6720}{h^5} & \frac{15120}{h^5} & \frac{30240}{h^5} & \frac{55440}{h^5} & \frac{95040}{h^5} & \frac{154440}{h^5}
\end{pmatrix},$$

$X = [a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8,\ a_9,\ a_{10},\ a_{11},\ a_{12},\ a_{13}]^T,$

$B = [y_n,\ y_{n+r},\ y_{n+s},\ y_{n+t},\ f_n,\ f_{n+r},\ f_{n+s},\ f_{n+t},\ f_{n+1},\ g_n,\ g_{n+r},\ g_{n+s},\ g_{n+t},\ g_{n+1}]^T.$

Now, we used matrix manipulation for solving the previous system to find the unknown values a_j's, $j = 0(1)13$. Then, these values are substituted into Equation (3.2) to produce a continuous implicit scheme of the form

$$\begin{aligned} y(x) &= \sum_{i=0,r,s,t} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \sum_{i=r,s,t} \beta_i(x) f_{n+i} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} \\ &\quad + \sum_{i=r,s,t} \gamma_i(x) g_{n+i}. \end{aligned} \tag{5.6}$$

The corresponding first, second and third derivatives to Equation (5.6) are

$$\begin{aligned} y'(x) &= \frac{d}{dx}[\sum_{i=0,r,s,t} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \sum_{i=r,s,t} \beta_i(x) f_{n+i} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} \\ &\quad + \sum_{i=r,s,t} \gamma_i(x) g_{n+i}] \end{aligned} \tag{5.7}$$

$$y''(x) = \frac{d^2}{dx^2}[\sum_{i=0,r,s,t} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \sum_{i=r,s,t} \beta_i(x) f_{n+i} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} + \sum_{i=r,s,t} \gamma_i(x) g_{n+i}] \quad (5.8)$$

$$y'''(x) = \frac{d^3}{dx^3}[\sum_{i=0,r,s,t} \alpha_i(x) y_{n+i} + \sum_{i=0}^{1} \beta_i(x) f_{n+i} + \sum_{i=r,s,t} \beta_i(x) f_{n+i} + \sum_{i=0}^{1} \gamma_i(x) g_{n+i} + \sum_{i=r,s,t} \gamma_i(x) g_{n+i}] \quad (5.9)$$

where

$\alpha_0 = \frac{(x_n-x+hr)(x_n-x+hs)(x_n-x+ht)}{h^3 rst}$,

$\alpha_r = \frac{(x-x_n)(x_n-x+hs)(x_n-x+ht)}{h^3 r(r-s)(r-t)}$,

$\alpha_s = -\frac{(x-x_n)(x_n-x+hr)(x_n-x+ht)}{h^3 s(r-s)(s-t)}$,

$\alpha_t = \frac{(x-x_n)(x_n-x+hr)(x_n-x+hs)}{h^3 t(r-t)(s-t)}$,

$\beta_0 = \frac{(x-x_n)^4}{24} + \frac{(x-x_n)^7}{420h^3r^3s^3t^3}(r^3s^3 + r^3t^3 + s^3t^3 + 4rs^2t^3 + 4rs^3t^2 + 4r^2st^3 + 4r^2s^3t + 4r^3st^2 + 4r^3s^2t + 4rs^3t^3 + 4r^3st^3 + 4r^3s^3t + 8r^2s^2t^2 + 8r^2s^2t^3 + 8r^2s^3t^2 + 8r^3s^2t^2 + 4r^2s^3t^3 + 4r^3s^2t^3 + 4r^3s^3t^2 + r^3s^3t^3) - \frac{(x-x_n)^{10}}{5040h^6r^3s^3t^3}(16r^2s^2 + 4r^2s^3 + 4r^3s^2 + 16r^2t^2 + 4r^2t^3 + 4r^3t^2 + 16s^2t^2 + 4s^2t^3 + 4s^3t^2 + 4rs^2 + 4r^2s + 4rs^3 + 4r^3s + 4rt^2 + 4r^2t + 4rt^3 + 4r^3t + 4st^2 + 4s^2t + 4st^3 + 4s^3t + 36rst^2 + 36rs^2t + 36r^2st + 11rst^3 + 11rs^3t + 11r^3st + 36rs^2t^2 + 36r^2st^2 + 36r^2s^2t + 4rs^2t^3 + 4rs^3t^2 + 4r^2st^3 + 4r^2s^3t + 4r^3st^2 + 4r^3s^2t + 11rst + 16r^2s^2t^2) + \frac{(x-x_n)^{11}}{3960h^7r^3s^3t^3}(4r^2s^2 + 4r^2t^2 + 4s^2t^2 + rs + rt + st + 4rs^2 + 4r^2s + rs^3 + r^3s + 4rt^2 + 4r^2t + rt^3 + r^3t + 4st^2 + 4s^2t + st^3 + s^3t + 12rst^2 + 12rs^2t + 12r^2st + rst^3 + rs^3t + r^3st + 4rs^2t^2 + 4r^2st^2 + 4r^2s^2t + 12rst) - \frac{h(x-x_n)^3}{2162160r^3s^3t^3}(546r^3s^7 - 884r^2s^8 + 546r^4s^6 + 546r^5s^5 + 546r^6s^4 + 546r^7s^3 - 884r^8s^2 + 780r^2s^9 - 364r^3s^8 - 364r^4s^7 - 364r^5s^6 - 364r^6s^5 - 364r^7s^4 - 364r^8s^3 + 780r^9s^2 - 190r^2s^{10} + 70r^3s^9 + 70r^4s^8 + 70r^5s^7 + 70r^6s^6 + 70r^7s^5 + 70r^8s^4 + 70r^9s^3 - 190r^{10}s^2 - 884r^2t^8 + 546r^3t^7 + 546r^4t^6 + 546r^5t^5 + 546r^6t^4 + 546r^7t^3 - 884r^8t^2 + 780r^2t^9 - 364r^3t^8 - 364r^4t^7 - 364r^5t^6 - 364r^6t^5 - 364r^7t^4 - 364r^8t^3 + 780r^9t^2 - 190r^2t^{10} + 70r^3t^9 + 70r^4t^8 + 70r^5t^7 + 70r^6t^6 + 70r^7t^5 + 70r^8t^4 + 70r^9t^3 - 190r^{10}t^2 - 884s^2t^8 + 546s^3t^7 +$

$546s^4t^6 + 546s^5t^5 + 546s^6t^4 + 546s^7t^3 - 884s^8t^2 + 780s^2t^9 - 364s^3t^8 - 364s^4t^7 - 364s^5t^6 - 364s^6t^5 - 364s^7t^4 - 364s^8t^3 + 780s^9t^2 - 190s^2t^{10} + 70s^3t^9 + 70s^4t^8 + 70s^5t^7 + 70s^6t^6 + 70s^7t^5 + 70s^8t^4 + 70s^9t^3 - 190s^{10}t^2 + 260rs^9 + 260r^9s - 260rs^{10} - 260r^{10}s + 70rs^{11} + 70r^{11}s + 260rt^9 + 260r^9t - 260rt^{10} - 260r^{10}t + 70rt^{11} + 70r^{11}t + 260st^9 + 260s^9t - 260st^{10} - 260s^{10}t + 70st^{11} + 70s^{11}t - 1222rst^8 - 1222rs^8t - 1222r^8st + 1300rst^9 + 1300rs^9t + 1300r^9st - 505rst^{10} - 505rs^{10}t - 505r^{10}st + 70rst^{11} + 70rs^{11}t + 70r^{11}st + 2210rs^2t^7 - 1365rs^3t^6 - 1365rs^4t^5 - 1365rs^5t^4 - 1365rs^6t^3 + 2210rs^7t^2 + 2210r^2st^7 + 2210r^2s^7t - 1365r^3st^6 - 1365r^3s^6t - 1365r^4st^5 - 1365r^4s^5t - 1365r^5st^4 - 1365r^5s^4t - 1365r^6st^3 - 1365r^6s^3t + 2210r^7st^2 + 2210r^7s^2t - 2340rs^2t^8 + 1092rs^3t^7 + 1092rs^4t^6 + 1092rs^5t^5 + 1092rs^6t^4 + 1092rs^7t^3 - 2340rs^8t^2 - 2340r^2st^8 - 2340r^2s^8t + 1092r^3st^7 + 1092r^3s^7t + 1092r^4st^6 + 1092r^4s^6t + 1092r^5st^5 + 1092r^5s^5t + 1092r^6st^4 + 1092r^6s^4t + 1092r^7st^3 + 1092r^7s^3t - 2340r^8st^2 - 2340r^8s^2t + 1055rs^2t^9 - 427rs^3t^8 - 427rs^4t^7 - 427rs^5t^6 - 427rs^6t^5 - 427rs^7t^4 - 427rs^8t^3 + 1055rs^9t^2 + 1055r^2st^9 + 1055r^2s^9t - 427r^3st^8 - 427r^3s^8t - 427r^4st^7 - 427r^4s^7t - 427r^5st^6 - 427r^5s^6t - 427r^6st^5 - 427r^6s^5t - 427r^7st^4 - 427r^7s^4t - 427r^8st^3 - 427r^8s^3t + 1055r^9st^2 + 1055r^9s^2t - 190rs^2t^{10} + 70rs^3t^9 + 70rs^4t^8 + 70rs^5t^7 + 70rs^6t^6 + 70rs^7t^5 + 70rs^8t^4 + 70rs^9t^3 - 190rs^{10}t^2 - 190r^2st^{10} - 190r^2s^{10}t + 70r^3st^9 + 70r^3s^9t + 70r^4st^8 + 70r^4s^8t + 70r^5st^7 + 70r^5s^7t + 70r^6st^6 + 70r^6s^6t + 70r^7st^5 + 70r^7s^5t + 70r^8st^4 + 70r^8s^4t + 70r^9st^3 + 70r^9s^3t - 190r^{10}st^2 - 190r^{10}s^2t + 2782r^2s^2t^6 - 9373r^2s^3t^5 - 9373r^2s^4t^4 - 9373r^2s^5t^3 + 2782r^2s^6t^2 - 9373r^3s^2t^5 + 55692r^3s^3t^4 + 55692r^3s^4t^3 - 9373r^3s^5t^2 - 9373r^4s^2t^4 + 55692r^4s^3t^3 - 9373r^4s^4t^2 - 9373r^5s^2t^3 - 9373r^5s^3t^2 + 2782r^6s^2t^2 + 884r^2s^2t^7 + 1456r^2s^3t^6 + 1456r^2s^4t^5 + 1456r^2s^5t^4 + 1456r^2s^6t^3 + 884r^2s^7t^2 + 1456r^3s^2t^6 - 6552r^3s^3t^5 - 6552r^3s^4t^4 - 6552r^3s^5t^3 + 1456r^3s^6t^2 + 1456r^4s^2t^5 - 6552r^4s^3t^4 - 6552r^4s^4t^3 + 1456r^4s^5t^2 + 1456r^5s^2t^4 - 6552r^5s^3t^3 + 1456r^5s^4t^2 + 1456r^6s^2t^3 + 1456r^6s^3t^2 + 884r^7s^2t^2 - 2065r^2s^2t^8 + 1029r^2s^3t^7 + 1029r^2s^4t^6 + 1029r^2s^5t^5 + 1029r^2s^6t^4 + 1029r^2s^7t^3 - 2065r^2s^8t^2 + 1029r^3s^2t^7 - 882r^3s^3t^6 - 882r^3s^4t^5 - 882r^3s^5t^4 - 882r^3s^6t^3 + 1029r^3s^7t^2 + 1029r^4s^2t^6 - 882r^4$

$$
s^3t^5 - 882r^4s^4t^4 - 882r^4s^5t^3 + 1029r^4s^6t^2 + 1029r^5s^2t^5 - 882r^5s^3t^4 - 882r^5s^4t^3 + 1029r^5s^5t^2 + 1029r^6s^2t^4 - 882r^6s^3t^3 + 1029r^6s^4t^2 + 1029r^7s^2t^3 + 1029r^7s^3t^2 - 2065r^8s^2t^2 + 590r^2s^2t^9 - 294r^2s^3t^8 - 294r^2s^4t^7 - 294r^2s^5t^6 - 294r^2s^6t^5 - 294r^2s^7t^4 - 294r^2s^8t^3 + 590r^2s^9t^2 - 294r^3s^2t^8 + 252r^3s^3t^7 + 252r^3s^4t^6 + 252r^3s^5t^5 + 252r^3s^6t^4 + 252r^3s^7t^3 - 294r^3s^8t^2 - 294r^4s^2t^7 + 252r^4s^3t^6 + 252r^4s^4t^5 + 252r^4s^5t^4 + 252r^4s^6t^3 - 294r^4s^7t^2 - 294r^5s^2t^6 + 252r^5s^3t^5 + 252r^5s^4t^4 + 252r^5s^5t^3 - 294r^5s^6t^2 - 294r^6s^2t^5 + 252r^6s^3t^4 + 252r^6s^4t^3 - 294r^6s^5t^2 - 294r^7s^2t^4 + 252r^7s^3t^3 - 294r^7s^4t^2 - 294r^8s^2t^3 - 294r^8s^3t^2 + 590r^9s^2t^2) + \frac{(x-x_n)^{13}}{8580h^9r^3s^3t^3}(rs + rt + st + rst) - \frac{(x-x_n)^6}{360h^2r^2s^2t^2}(3r^2s^2 + 3r^2t^2 + 3s^2t^2 + 4rst^2 + 4rs^2t + 4r^2st + 4rs^2t^2 + 4r^2st^2 + 4r^2s^2t + 3r^2s^2t^2) + \frac{(x-x_n)^9}{1512h^5r^3s^3t^3}(4r^2s^2 + 4r^2s^3 + 4r^3s^2 + r^3s^3 + 4r^2t^2 + 4r^2t^3 + 4r^3t^2 + r^3t^3 + 4s^2t^2 + 4s^2t^3 + 4s^3t^2 + s^3t^3 + rs^3 + r^3s + rt^3 + r^3t + st^3 + s^3t + 8rst^2 + 8rs^2t + 8r^2st + 8rst^3 + 8rs^3t + 8r^3st + 24rs^2t^2 + 24r^2st^2 + 24r^2s^2t + 8rs^2t^3 + 8rs^3t^2 + 8r^2st^3 + 8r^2s^3t + 8r^3st^2 + 8r^3s^2t + rs^3t^3 + r^3st^3 + r^3s^3t + 24r^2s^2t^2 + 4r^2s^2t^3 + 4r^2s^3t^2 + 4r^3s^2t^2) - \frac{(x-x_n)^{12}}{11880h^8r^3s^3t^3}(4rs + 4rt + 4st + 4rs^2 + 4r^2s + 4rt^2 + 4r^2t + 4st^2 + 4s^2t + 4rst^2 + 4rs^2t + 4r^2st + 15rst) + \frac{h^2(x-x_n)^2}{2162160r^3s^3t^3}(260r^2s^9 - 884r^3s^8 + 546r^4s^7 + 546r^5s^6 + 546r^6s^5 + 546r^7s^4 - 884r^8s^3 + 260r^9s^2 - 260r^2s^{10} + 780r^3s^9 - 364r^4s^8 - 364r^5s^7 - 364r^6s^6 - 364r^7s^5 - 364r^8s^4 + 780r^9s^3 - 260r^{10}s^2 + 70r^2s^{11} - 190r^3s^{10} + 70r^4s^9 + 70r^5s^8 + 70r^6s^7 + 70r^7s^6 + 70r^8s^5 + 70r^9s^4 - 190r^{10}s^3 + 70r^{11}s^2 + 260r^2t^9 - 884r^3t^8 + 546r^4t^7 + 546r^5t^6 + 546r^6t^5 + 546r^7t^4 - 884r^8t^3 + 260r^9t^2 - 260r^2t^{10} + 780r^3t^9 - 364r^4t^8 - 364r^5t^7 - 364r^6t^6 - 364r^7t^5 - 364r^8t^4 + 780r^9t^3 - 260r^{10}t^2 + 70r^2t^{11} - 190r^3t^{10} + 70r^4t^9 + 70r^5t^8 + 70r^6t^7 + 70r^7t^6 + 70r^8t^5 + 70r^9t^4 - 190r^{10}t^3 + 70r^{11}t^2 + 260s^2t^9 - 884s^3t^8 + 546s^4t^7 + 546s^5t^6 + 546s^6t^5 + 546s^7t^4 - 884s^8t^3 + 260s^9t^2 - 260s^2t^{10} + 780s^3t^9 - 364s^4t^8 - 364s^5t^7 - 364s^6t^6 - 364s^7t^5 - 364s^8t^4 + 780s^9t^3 - 260s^{10}t^2 + 70s^2t^{11} - 190s^3t^{10} + 70s^4t^9 + 70s^5t^8 + 70s^6t^7 + 70s^7t^6 + 70s^8t^5 + 70s^9t^4 - 190s^{10}t^3 + 70s^{11}t^2 + 520rst^9 + 520rs^9t + 520r^9st - 520rst^{10} - 520rs^{10}t - 520r^{10}st + 140rst^{11} + 140rs^{11}t + 140r^{11}st - 2106rs^2t^8 + 2756rs^3t^7 - 819rs^4t^6 - 819rs^5t^5 - 819rs^6t^4 + 2756rs^7t^3 - 2106rs^8t^2 - 2106r^2st^8 - 2106r^2s^8t + 2756r^3st^7 + 2756r^3s^7t - 819r^4st^6 - 819r^4s^6t - 819r^5st^5 - 819r^5s^5t - 819r^6st^4 - 819r^6s^4t + 2756r^7st^3 + 2756r^7s^3t - 2106r^8st^2 - 2106r^8s^2t + 2080rs^2t^9 - 2704rs^3t^8 +
$$

$728rs^4t^7 + 728rs^5t^6 + 728rs^6t^5 + 728rs^7t^4 - 2704rs^8t^3 + 2080rs^9t^2 + 2080r^2st^9 +$
$2080r^2s^9t - 2704r^3st^8 - 2704r^3s^8t + 728r^4st^7 + 728r^4s^7t + 728r^5st^6 + 728r^5s^6t +$
$728r^6st^5 + 728r^6s^5t + 728r^7st^4 + 728r^7s^4t - 2704r^8st^3 - 2704r^8s^3t + 2080r^9st^2 +$
$2080r^9s^2t - 695rs^2t^{10} + 1125rs^3t^9 - 357rs^4t^8 - 357rs^5t^7 - 357rs^6t^6 - 357rs^7t^5 -$
$357rs^8t^4 + 1125rs^9t^3 - 695rs^{10}t^2 - 695r^2st^{10} - 695r^2s^{10}t + 1125r^3st^9 + 1125r^3s^9t -$
$357r^4st^8 - 357r^4s^8t - 357r^5st^7 - 357r^5s^7t - 357r^6st^6 - 357r^6s^6t - 357r^7st^5 -$
$357r^7s^5t - 357r^8st^4 - 357r^8s^4t + 1125r^9st^3 + 1125r^9s^3t - 695r^{10}st^2 - 695r^{10}s^2t +$
$70rs^2t^{11} - 190rs^3t^{10} + 70rs^4t^9 + 70rs^5t^8 + 70rs^6t^7 + 70rs^7t^6 + 70rs^8t^5 +$
$70rs^9t^4 - 190rs^{10}t^3 + 70rs^{11}t^2 + 70r^2st^{11} + 70r^2s^{11}t - 190r^3st^{10} - 190r^3s^{10}t +$
$70r^4st^9 + 70r^4s^9t + 70r^5st^8 + 70r^5s^8t + 70r^6st^7 + 70r^6s^7t + 70r^7st^6 + 70r^7s^6t +$
$70r^8st^5 + 70r^8s^5t + 70r^9st^4 + 70r^9s^4t - 190r^{10}st^3 - 190r^{10}s^3t + 70r^{11}st^2 +$
$70r^{11}s^2t + 4420r^2s^2t^7 + 1417r^2s^3t^6 - 10738r^2s^4t^5 - 10738r^2s^5t^4 + 1417r^2s^6t^3 +$
$4420r^2s^7t^2 + 1417r^3s^2t^6 - 18746r^3s^3t^5 + 46319r^3s^4t^4 - 18746r^3s^5t^3 + 1417r^3s^6t^2 -$
$10738r^4s^2t^5 + 46319r^4s^3t^4 + 46319r^4s^4t^3 - 10738r^4s^5t^2 - 10738r^5s^2t^4 -$
$18746r^5s^3t^3 - 10738r^5s^4t^2 + 1417r^6s^2t^3 + 1417r^6s^3t^2 + 4420r^7s^2t^2 - 4680r^2s^2t^8 +$
$1976r^2s^3t^7 + 2548r^2s^4t^6 + 2548r^2s^5t^5 + 2548r^2s^6t^4 + 1976r^2s^7t^3 - 4680r^2s^8t^2 +$
$1976r^3s^2t^7 + 2912r^3s^3t^6 - 5096r^3s^4t^5 - 5096r^3s^5t^4 + 2912r^3s^6t^3 + 1976r^3s^7t^2 +$
$2548r^4s^2t^6 - 5096r^4s^3t^5 - 13104r^4s^4t^4 - 5096r^4s^5t^3 + 2548r^4s^6t^2 + 2548r^5s^2t^5 -$
$5096r^5s^3t^4 - 5096r^5s^4t^3 + 2548r^5s^5t^2 + 2548r^6s^2t^4 + 2912r^6s^3t^3 + 2548r^6s^4t^2 +$
$1976r^7s^2t^3 + 1976r^7s^3t^2 - 4680r^8s^2t^2 + 2110r^2s^2t^9 - 2492r^2s^3t^8 + 602r^2s^4t^7 +$
$602r^2s^5t^6 + 602r^2s^6t^5 + 602r^2s^7t^4 - 2492r^2s^8t^3 + 2110r^2s^9t^2 - 2492r^3s^2t^8 +$
$2058r^3s^3t^7 + 147r^3s^4t^6 + 147r^3s^5t^5 + 147r^3s^6t^4 + 2058r^3s^7t^3 - 2492r^3s^8t^2 +$
$602r^4s^2t^7 + 147r^4s^3t^6 - 1764r^4s^4t^5 - 1764r^4s^5t^4 + 147r^4s^6t^3 + 602r^4s^7t^2 +$
$602r^5s^2t^6 + 147r^5s^3t^5 - 1764r^5s^4t^4 + 147r^5s^5t^3 + 602r^5s^6t^2 + 602r^6s^2t^5 +$
$147r^6s^3t^4 + 147r^6s^4t^3 + 602r^6s^5t^2 + 602r^7s^2t^4 + 2058r^7s^3t^3 + 602r^7s^4t^2 -$
$2492r^8s^2t^3 - 2492r^8s^3t^2 + 2110r^9s^2t^2 - 380r^2s^2t^{10} + 660r^2s^3t^9 - 224r^2s^4t^8 -$
$224r^2s^5t^7 - 224r^2s^6t^6 - 224r^2s^7t^5 - 224r^2s^8t^4 + 660r^2s^9t^3 - 380r^2s^{10}t^2 +$
$660r^3s^2t^9 - 588r^3s^3t^8 - 42r^3s^4t^7 - 42r^3s^5t^6 - 42r^3s^6t^5 - 42r^3s^7t^4 - 588r^3s^8t^3 +$
$660r^3s^9t^2 - 224r^4s^2t^8 - 42r^4s^3t^7 + 504r^4s^4t^6 + 504r^4s^5t^5 + 504r^4s^6t^4 - 42r^4s^7t^3 -$

$$224r^4s^8t^2 - 224r^5s^2t^7 - 42r^5s^3t^6 + 504r^5s^4t^5 + 504r^5s^5t^4 - 42r^5s^6t^3 - 224r^5s^7t^2 -$$

$$224r^6s^2t^6 - 42r^6s^3t^5 + 504r^6s^4t^4 - 42r^6s^5t^3 - 224r^6s^6t^2 - 224r^7s^2t^5 - 42r^7s^3t^4 -$$

$$42r^7s^4t^3 - 224r^7s^5t^2 - 224r^8s^2t^4 - 588r^8s^3t^3 - 224r^8s^4t^2 + 660r^9s^2t^3 + 660r^9s^3t^2 -$$

$$380r^{10}s^2t^2) - \frac{h^3(x-x_n)}{2162160r^2s^2t^2}(546r^3s^6 - 884r^2s^7 + 546r^4s^5 + 546r^5s^4 + 546r^6s^3 -$$

$$884r^7s^2 + 780r^2s^8 - 364r^3s^7 - 364r^4s^6 - 364r^5s^5 - 364r^6s^4 - 364r^7s^3 + 780r^8s^2 -$$

$$190r^2s^9 + 70r^3s^8 + 70r^4s^7 + 70r^5s^6 + 70r^6s^5 + 70r^7s^4 + 70r^8s^3 - 190r^9s^2 - 884r^2t^7 +$$

$$546r^3t^6 + 546r^4t^5 + 546r^5t^4 + 546r^6t^3 - 884r^7t^2 + 780r^2t^8 - 364r^3t^7 - 364r^4t^6 -$$

$$364r^5t^5 - 364r^6t^4 - 364r^7t^3 + 780r^8t^2 - 190r^2t^9 + 70r^3t^8 + 70r^4t^7 + 70r^5t^6 +$$

$$70r^6t^5 + 70r^7t^4 + 70r^8t^3 - 190r^9t^2 - 884s^2t^7 + 546s^3t^6 + 546s^4t^5 + 546s^5t^4 +$$

$$546s^6t^3 - 884s^7t^2 + 780s^2t^8 - 364s^3t^7 - 364s^4t^6 - 364s^5t^5 - 364s^6t^4 - 364s^7t^3 +$$

$$780s^8t^2 - 190s^2t^9 + 70s^3t^8 + 70s^4t^7 + 70s^5t^6 + 70s^6t^5 + 70s^7t^4 + 70s^8t^3 - 190s^9t^2 +$$

$$260rs^8 + 260r^8s - 260rs^9 - 260r^9s + 70rs^{10} + 70r^{10}s + 260rt^8 + 260r^8t - 260rt^9 -$$

$$260r^9t + 70rt^{10} + 70r^{10}t + 260st^8 + 260s^8t - 260st^9 - 260s^9t + 70st^{10} + 70s^{10}t -$$

$$1222rst^7 - 1222rs^7t - 1222r^7st + 1300rst^8 + 1300rs^8t + 1300r^8st - 505rst^9 -$$

$$505rs^9t - 505r^9st + 70rst^{10} + 70rs^{10}t + 70r^{10}st + 2210rs^2t^6 - 1365rs^3t^5 -$$

$$1365rs^4t^4 - 1365rs^5t^3 + 2210rs^6t^2 + 2210r^2st^6 + 2210r^2s^6t - 1365r^3st^5 -$$

$$1365r^3s^5t - 1365r^4st^4 - 1365r^4s^4t - 1365r^5st^3 - 1365r^5s^3t + 2210r^6st^2 +$$

$$2210r^6s^2t - 2340rs^2t^7 + 1092rs^3t^6 + 1092rs^4t^5 + 1092rs^5t^4 + 1092rs^6t^3 -$$

$$2340rs^7t^2 - 2340r^2st^7 - 2340r^2s^7t + 1092r^3st^6 + 1092r^3s^6t + 1092r^4st^5 +$$

$$1092r^4s^5t + 1092r^5st^4 + 1092r^5s^4t + 1092r^6st^3 + 1092r^6s^3t - 2340r^7st^2 -$$

$$2340r^7s^2t + 1055rs^2t^8 - 427rs^3t^7 - 427rs^4t^6 - 427rs^5t^5 - 427rs^6t^4 - 427rs^7t^3 +$$

$$1055rs^8t^2 + 1055r^2st^8 + 1055r^2s^8t - 427r^3st^7 - 427r^3s^7t - 427r^4st^6 - 427r^4s^6t -$$

$$427r^5st^5 - 427r^5s^5t - 427r^6st^4 - 427r^6s^4t - 427r^7st^3 - 427r^7s^3t + 1055r^8st^2 +$$

$$1055r^8s^2t - 190rs^2t^9 + 70rs^3t^8 + 70rs^4t^7 + 70rs^5t^6 + 70rs^6t^5 + 70rs^7t^4 + 70rs^8t^3 -$$

$$190rs^9t^2 - 190r^2st^9 - 190r^2s^9t + 70r^3st^8 + 70r^3s^8t + 70r^4st^7 + 70r^4s^7t + 70r^5st^6 +$$

$$70r^5s^6t + 70r^6st^5 + 70r^6s^5t + 70r^7st^4 + 70r^7s^4t + 70r^8st^3 + 70r^8s^3t - 190r^9st^2 -$$

$$190r^9s^2t + 2782r^2s^2t^5 - 9373r^2s^3t^4 - 9373r^2s^4t^3 + 2782r^2s^5t^2 - 9373r^3s^2t^4 +$$

$$55692r^3s^3t^3 - 9373r^3s^4t^2 - 9373r^4s^2t^3 - 9373r^4s^3t^2 + 2782r^5s^2t^2 + 884r^2s^2t^6 +$$

$$1456r^2s^3t^5 + 1456r^2s^4t^4 + 1456r^2s^5t^3 + 884r^2s^6t^2 + 1456r^3s^2t^5 - 6552r^3s^3t^4 - 6552$$

$r^3s^4t^3 + 1456r^3s^5t^2 + 1456r^4s^2t^4 - 6552r^4s^3t^3 + 1456r^4s^4t^2 + 1456r^5s^2t^3 + 1456r^5s^3t^2 + 884r^6s^2t^2 - 2065r^2s^2t^7 + 1029r^2s^3t^6 + 1029r^2s^4t^5 + 1029r^2s^5t^4 + 1029r^2s^6t^3 - 2065r^2s^7t^2 + 1029r^3s^2t^6 - 882r^3s^3t^5 - 882r^3s^4t^4 - 882r^3s^5t^3 + 1029r^3s^6t^2 + 1029r^4s^2t^5 - 882r^4s^3t^4 - 882r^4s^4t^3 + 1029r^4s^5t^2 + 1029r^5s^2t^4 - 882r^5s^3t^3 + 1029r^5s^4t^2 + 1029r^6s^2t^3 + 1029r^6s^3t^2 - 2065r^7s^2t^2 + 590r^2s^2t^8 - 294r^2s^3t^7 - 294r^2s^4t^6 - 294r^2s^5t^5 - 294r^2s^6t^4 - 294r^2s^7t^3 + 590r^2s^8t^2 - 294r^3s^2t^7 + 252r^3s^3t^6 + 252r^3s^4t^5 + 252r^3s^5t^4 + 252r^3s^6t^3 - 294r^3s^7t^2 - 294r^4s^2t^6 + 252r^4s^3t^5 + 252r^4s^4t^4 + 252r^4s^5t^3 - 294r^4s^6t^2 - 294r^5s^2t^5 + 252r^5s^3t^4 + 252r^5s^4t^3 - 294r^5s^5t^2 - 294r^6s^2t^4 + 252r^6s^3t^3 - 294r^6s^4t^2 - 294r^7s^2t^3 - 294r^7s^3t^2 + 590r^8s^2t^2) - \frac{(x-x_n)^8}{1680h^4r^3s^3t^3}(4r^2s^3 + 4r^3s^2 + 4r^3s^3 + 4r^2t^3 + 4r^3t^2 + 4r^3t^3 + 4s^2t^3 + 4s^3t^2 + 4s^3t^3 + 7rst^3 + 7rs^3t + 7r^3st + 20rs^2t^2 + 20r^2st^2 + 20r^2s^2t + 20rs^2t^3 + 20rs^3t^2 + 20r^2st^3 + 20r^2s^3t + 20r^3st^2 + 20r^3s^2t + 7rs^3t^3 + 7r^3st^3 + 7r^3s^3t + 48r^2s^2t^2 + 20r^2s^2t^3 + 20r^2s^3t^2 + 20r^3s^2t^2 + 4r^2s^3t^3 + 4r^3s^2t^3 + 4r^3s^3t^2)$,

$\beta_r = \frac{(x-x_n)^{12}}{11880h^8r^3(r-s)^3(r-t)^3(r-1)^3}(8rs - 12r^2t^2 - 12r^2s^2 + 8rt - 4st + 8rs^2 - 19r^2s + 9r^3s + 8rt^2 - 19r^2t + 9r^3t - 4st^2 - 4s^2t - 12r^2 + 9r^3 + 9r^4 + 8rst^2 + 8rs^2t - 19r^2st + 21rst) + \frac{(x-x_n)^8}{1680h^4r^3(r-s)^3(r-t)^3(r-1)^3}(5r^2s^3 - 7r^3s^2 - 7r^3s^3 + 9r^4s^2 + 5r^2t^3 - 7r^3t^2 - 7r^3t^3 + 9r^4t^2 - 4s^2t^3 - 4s^3t^2 - 4s^3t^3 + 5rst^3 + 5rs^3t - 28r^3st + 36r^4st + 4rs^2t^2 + 13r^2st^2 + 13r^2s^2t + 4rs^2t^3 + 4rs^3t^2 + 13r^2st^3 + 13r^2s^3t - 47r^3st^2 - 47r^3s^2t + 5rs^3t^3 - 28r^3st^3 - 28r^3s^3t + 36r^4st^2 + 36r^4s^2t + 24r^2s^2t^2 + 13r^2s^2t^3 + 13r^2s^3t^2 - 47r^3s^2t^2 + 5r^2s^3t^3 - 7r^3s^2t^3 - 7r^3s^3t^2 + 9r^4s^2t^2) - \frac{(x-x_n)^9}{1512h^5r^3(r-s)^3(r-t)^3(r-1)^3}(2r^2s^2 + 2r^2s^3 - 10r^3s^2 - 7r^3s^3 + 9r^4s^2 + 2r^2t^2 + 2r^2t^3 - 10r^3t^2 - 7r^3t^3 + 9r^4t^2 - 4s^2t^2 - 4s^2t^3 - 4s^3t^2 - s^3t^3 + 2rs^3 - 7r^3s + 9r^4s + 2rt^3 - 7r^3t + 9r^4t - st^3 - s^3t + 7rst^2 + 7rs^2t - 2r^2st + 7rst^3 + 7rs^3t - 26r^3st + 36r^4st + 12rs^2t^2 + 3r^2st^2 + 3r^2s^2t + 7rs^2t^3 + 7rs^3t^2 - 2r^2st^3 - 2r^2s^3t - 26r^3st^2 - 26r^3s^2t + 2rs^3t^3 - 7r^3st^3 - 7r^3s^3t + 9r^4st^2 + 9r^4s^2t + 3r^2s^2t^2 + 2r^2s^2t^3 + 2r^2s^3t^2 - 10r^3s^2t^2) + \frac{h^2(x-x_n)^2}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(1690r^3s^8 - 520r^2s^9 - 312r^4s^7 - 312r^5s^6 - 312r^6s^5 - 312r^7s^4 - 3887r^8s^3 + 4550r^9s^2 + 520r^2s^{10} - 780r^3s^9 - 2002r^4s^8 + 572r^5s^7 + 572r^6s^6 + 572r^7s^5 + 572r^8s^4 + 9009r^9s^3 - 10790r^{10}s^2 - 140r^2s^{11} - 385r^3s^{10} + 2020r^4s^9 - 320r^5s^8 - 320r^6s^7 -$

$320r^7s^6 - 320r^8s^5 - 320r^9s^4 - 6235r^{10}s^3 + 7870r^{11}s^2 + 210r^3s^{11} - 525r^4s^{10} + 60r^5s^9 + 60r^6s^8 + 60r^7s^7 + 60r^8s^6 + 60r^9s^5 + 60r^{10}s^4 + 1425r^{11}s^3 - 1890r^{12}s^2 - 520r^2t^9 + 1690r^3t^8 - 312r^4t^7 - 312r^5t^6 - 312r^6t^5 - 312r^7t^4 - 3887r^8t^3 + 4550r^9t^2 + 520r^2t^{10} - 780r^3t^9 - 2002r^4t^8 + 572r^5t^7 + 572r^6t^6 + 572r^7t^5 + 572r^8t^4 + 9009r^9t^3 - 10790r^{10}t^2 - 140r^2t^{11} - 385r^3t^{10} + 2020r^4t^9 - 320r^5t^8 - 320r^6t^7 - 320r^7t^6 - 320r^8t^5 - 320r^9t^4 - 6235r^{10}t^3 + 7870r^{11}t^2 + 210r^3t^{11} - 525r^4t^{10} + 60r^5t^9 + 60r^6t^8 + 60r^7t^7 + 60r^8t^6 + 60r^9t^5 + 60r^{10}t^4 + 1425r^{11}t^3 - 1890r^{12}t^2 + 260s^2t^9 - 884s^3t^8 + 546s^4t^7 + 546s^5t^6 + 546s^6t^5 + 546s^7t^4 - 884s^8t^3 + 260s^9t^2 - 260s^2t^{10} + 780s^3t^9 - 364s^4t^8 - 364s^5t^7 - 364s^6t^6 - 364s^7t^5 - 364s^8t^4 + 780s^9t^3 - 260s^{10}t^2 + 70s^2t^{11} - 190s^3t^{10} + 70s^4t^9 + 70s^5t^8 + 70s^6t^7 + 70s^7t^6 + 70s^8t^5 + 70s^9t^4 - 190s^{10}t^3 + 70s^{11}t^2 - 1365r^{10}s + 3315r^{11}s - 2520r^{12}s + 630r^{13}s - 1365r^{10}t + 3315r^{11}t - 2520r^{12}t + 630r^{13}t - 260rst^9 - 260rs^9t + 9100r^9st + 260rst^{10} + 260rs^{10}t - 21580r^{10}st - 70rst^{11} - 70rs^{11}t + 15740r^{11}st - 3780r^{12}st - 234rs^2t^8 + 3770rs^3t^7 - 4095rs^4t^6 - 4095rs^5t^5 - 4095rs^6t^4 + 3770rs^7t^3 - 234rs^8t^2 + 2340r^2st^8 + 2340r^2s^8t - 6604r^3st^7 - 6604r^3s^7t + 1404r^4st^6 + 1404r^4s^6t + 1404r^5st^5 + 1404r^5s^5t + 1404r^6st^4 + 1404r^6s^4t + 15704r^7st^3 + 15704r^7s^3t - 20475r^8st^2 - 20475r^8s^2t - 260rs^2t^9 - 1300rs^3t^8 + 1274rs^4t^7 + 1274rs^5t^6 + 1274rs^6t^5 + 1274rs^7t^4 - 1300rs^8t^3 - 260rs^9t^2 - 1820r^2st^9 - 1820r^2s^9t + 1976r^3st^8 + 1976r^3s^8t + 7904r^4st^7 + 7904r^4s^7t - 2392r^5st^6 - 2392r^5s^6t - 2392r^6st^5 - 2392r^6s^5t - 2392r^7st^4 - 2392r^7s^4t - 34996r^8st^3 - 34996r^8s^3t + 47125r^9st^2 + 47125r^9s^2t + 445rs^2t^{10} - 855rs^3t^9 + 315rs^4t^8 + 315rs^5t^7 + 315rs^6t^6 + 315rs^7t^5 + 315rs^8t^4 - 855rs^9t^3 + 445rs^{10}t^2 + 250r^2st^{10} + 250r^2s^{10}t + 1695r^3st^9 + 1695r^3s^9t - 6768r^4st^8 - 6768r^4s^8t + 1188r^5st^7 + 1188r^5s^7t + 1188r^6st^6 + 1188r^6s^6t + 1188r^7st^5 + 1188r^7s^5t + 1188r^8st^4 + 1188r^8s^4t + 22326r^9st^3 + 22326r^9s^3t - 32495r^{10}st^2 - 32495r^{10}s^2t - 140rs^2t^{11} + 380rs^3t^{10} - 140rs^4t^9 - 140rs^5t^8 - 140rs^6t^7 - 140rs^7t^6 - 140rs^8t^5 - 140rs^9t^4 + 380rs^{10}t^3 - 140rs^{11}t^2 + 70r^2st^{11} + 70r^2s^{11}t - 700r^3st^{10} - 700r^3s^{10}t + 1555r^4st^9 + 1555r^4s^9t - 200r^5st^8 - 200r^5s^8t - 200r^6st^7 - 200r^6s^7t - 200r^7st^6 - 200r^7s^6t - 200r^8st^5 - 200r^8s^5t - 200r^9st^4 - 200r^9s^4t - 4750r^{10}st^3 - 4750r^{10}s^3t + 7405r^{11}st^2 + 7405r^{11}s^2t - 3068r^2s^2t^7 - 494r^2s^3t^6 + 2366r^2s^4t^5 + 2366r^2s^5t^4 -$

$494r^2s^6t^3 - 3068r^2s^7t^2 + 5863r^3s^2t^6 + 4147r^3s^3t^5 + 7007r^3s^4t^4 + 4147r^3s^5t^3 + 5863r^3s^6t^2 - 1144r^4s^2t^5 - 2860r^4s^3t^4 - 2860r^4s^4t^3 - 1144r^4s^5t^2 - 1144r^5s^2t^4 - 5720r^5s^3t^3 - 1144r^5s^4t^2 - 16874r^6s^2t^3 - 16874r^6s^3t^2 + 32032r^7s^2t^2 + 3042r^2s^2t^8 - 5434r^2s^3t^7 + 5005r^2s^4t^6 + 5005r^2s^5t^5 + 5005r^2s^6t^4 - 5434r^2s^7t^3 + 3042r^2s^8t^2 + 260r^3s^2t^7 - 1066r^3s^3t^6 - 5642r^3s^4t^5 - 5642r^3s^5t^4 - 1066r^3s^6t^3 + 260r^3s^7t^2 - 7397r^4s^2t^6 - 4433r^4s^3t^5 - 9009r^4s^4t^4 - 4433r^4s^5t^3 - 7397r^4s^6t^2 + 1612r^5s^2t^5 + 4576r^5s^3t^4 + 4576r^5s^4t^3 + 1612r^5s^5t^2 + 1612r^6s^2t^4 + 9152r^6s^3t^3 + 1612r^6s^4t^2 + 36218r^7s^2t^3 + 36218r^7s^3t^2 - 71136r^8s^2t^2 - 1250r^2s^2t^9 + 4678r^2s^3t^8 - 3304r^2s^4t^7 - 3304r^2s^5t^6 - 3304r^2s^6t^5 - 3304r^2s^7t^4 + 4678r^2s^8t^3 - 1250r^2s^9t^2 - 2015r^3s^2t^8 - 663r^3s^3t^7 + 1365r^3s^4t^6 + 1365r^3s^5t^5 + 1365r^3s^6t^4 - 663r^3s^7t^3 - 2015r^3s^8t^2 + 4394r^4s^2t^7 + 2886r^4s^3t^6 + 4914r^4s^4t^5 + 4914r^4s^5t^4 + 2886r^4s^6t^3 + 4394r^4s^7t^2 - 520r^5s^2t^6 - 2028r^5s^3t^5 - 2028r^5s^5t^3 - 520r^5s^6t^2 - 520r^6s^2t^5 - 2028r^6s^3t^4 - 2028r^6s^4t^3 - 520r^6s^5t^2 - 520r^7s^2t^4 - 4056r^7s^3t^3 - 520r^7s^4t^2 - 20423r^8s^2t^3 - 20423r^8s^3t^2 + 45292r^9s^2t^2 + 205r^2s^2t^{10} - 1095r^2s^3t^9 + 595r^2s^4t^8 + 595r^2s^5t^7 + 595r^2s^6t^6 + 595r^2s^7t^5 + 595r^2s^8t^4 - 1095r^2s^9t^3 + 205r^2s^{10}t^2 + 540r^3s^2t^9 + 228r^3s^3t^8 - 84r^3s^4t^7 - 84r^3s^5t^6 - 84r^3s^6t^5 - 84r^3s^7t^4 + 228r^3s^8t^3 + 540r^3s^9t^2 - 767r^4s^2t^8 - 507r^4s^3t^7 - 819r^4s^4t^6 - 819r^4s^5t^5 - 819r^4s^6t^4 - 507r^4s^7t^3 - 767r^4s^8t^2 + 52r^5s^2t^7 + 312r^5s^3t^6 + 312r^5s^6t^3 + 52r^5s^7t^2 + 52r^6s^2t^6 + 312r^6s^3t^5 + 312r^6s^5t^3 + 52r^6s^6t^2 + 52r^7s^2t^5 + 312r^7s^3t^4 + 312r^7s^4t^3 + 52r^7s^5t^2 + 52r^8s^2t^4 + 624r^8s^3t^3 + 52r^8s^4t^2 + 3939r^9s^2t^3 + 3939r^9s^3t^2 - 9620r^{10}s^2t^2) - \frac{(x-x_n)^{10}}{5040h^6r^3(r-s)^3(r-t)^3(r-1)^3}(4r^2s^2 + 7r^2s^3 + 19r^3s^2 + 7r^3s^3 - 9r^4s^2 + 4r^2t^2 + 7r^2t^3 + 19r^3t^2 + 7r^3t^3 - 9r^4t^2 + 16s^2t^2 + 4s^2t^3 + 4s^3t^2 - 8rs^2 + 7r^2s - 8rs^3 + 19r^3s - 36r^4s - 8rt^2 + 7r^2t - 8rt^3 + 19r^3t - 36r^4t + 4st^2 + 4s^2t + 4st^3 + 4s^3t + 7r^3 - 9r^4 - 36rst^2 - 36rs^2t + 27r^2st - 13rst^3 - 13rs^3t + 20r^3st - 36r^4st - 36rs^2t^2 + 27r^2st^2 + 27r^2s^2t - 8rs^2t^3 - 8rs^3t^2 + 7r^2st^3 + 7r^2s^3t + 19r^3st^2 + 19r^3s^2t - 13rst + 4r^2s^2t^2) + \frac{(x-x_n)^{11}}{3960h^7r^3(r-s)^3(r-t)^3(r-1)^3}(10r^2s^2 + 3r^2s^3 + 3r^3s^2 + 10r^2t^2 + 3r^2t^3 + 3r^3t^2 + 4s^2t^2 - 2rs - 2rt + st - 8rs^2 + 10r^2s - 2rs^3 - 2r^3s - 9r^4s - 8rt^2 + 10r^2t - 2rt^3 - 2r^3t - 9r^4t + 4st^2 + 4s^2t + st^3 + s^3t + 3r^2 + 3r^3 - 9r^4 - 15rst^2 - 15rs^2t + 21r^2st - 2rst^3 - 2rs^3t - 2r^3st - 8rs^2t^2 + 10r^2st^2 + 10r^2s^2t - 15rst) - \frac{(x-x_n)^{13}}{8580h^9r^3(r-s)^3(r-t)^3(r-1)^3}(2rs + 2rt - st - 3r^2s - 3r^2t - 3r^2 + 4r^3 + 2rst) -$

$\frac{h(x-x_n)^3}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(1690r^2s^8 - 312r^3s^7 - 312r^4s^6 - 312r^5s^5 - 312r^6s^4 - 312r^7s^3 - 3887r^8s^2 - 780r^2s^9 - 2002r^3s^8 + 572r^4s^7 + 572r^5s^6 + 572r^6s^5 + 572r^7s^4 + 572r^8s^3 + 9009r^9s^2 - 385r^2s^{10} + 2020r^3s^9 - 320r^4s^8 - 320r^5s^7 - 320r^6s^6 - 320r^7s^5 - 320r^8s^4 - 320r^9s^3 - 6235r^{10}s^2 + 210r^2s^{11} - 525r^3s^{10} + 60r^4s^9 + 60r^5s^8 + 60r^6s^7 + 60r^7s^6 + 60r^8s^5 + 60r^9s^4 + 60r^{10}s^3 + 1425r^{11}s^2 + 1690r^2t^8 - 312r^3t^7 - 312r^4t^6 - 312r^5t^5 - 312r^6t^4 - 312r^7t^3 - 3887r^8t^2 - 780r^2t^9 - 2002r^3t^8 + 572r^4t^7 + 572r^5t^6 + 572r^6t^5 + 572r^7t^4 + 572r^8t^3 + 9009r^9t^2 - 385r^2t^{10} + 2020r^3t^9 - 320r^4t^8 - 320r^5t^7 - 320r^6t^6 - 320r^7t^5 - 320r^8t^4 - 320r^9t^3 - 6235r^{10}t^2 + 210r^2t^{11} - 525r^3t^{10} + 60r^4t^9 + 60r^5t^8 + 60r^6t^7 + 60r^7t^6 + 60r^8t^5 + 60r^9t^4 + 60r^{10}t^3 + 1425r^{11}t^2 - 884s^2t^8 + 546s^3t^7 + 546s^4t^6 + 546s^5t^5 + 546s^6t^4 + 546s^7t^3 - 884s^8t^2 + 780s^2t^9 - 364s^3t^8 - 364s^4t^7 - 364s^5t^6 - 364s^6t^5 - 364s^7t^4 - 364s^8t^3 + 780s^9t^2 - 190s^2t^{10} + 70s^3t^9 + 70s^4t^8 + 70s^5t^7 + 70s^6t^6 + 70s^7t^5 + 70s^8t^4 + 70s^9t^3 - 190s^{10}t^2 - 520rs^9 + 4550r^9s + 520rs^{10} - 10790r^{10}s - 140rs^{11} + 7870r^{11}s - 1890r^{12}s - 520rt^9 + 4550r^9t + 520rt^{10} - 10790r^{10}t - 140rt^{11} + 7870r^{11}t - 1890r^{12}t + 260st^9 + 260s^9t - 260st^{10} - 260s^{10}t + 70st^{11} + 70s^{11}t - 1365r^{10} + 3315r^{11} - 2520r^{12} + 630r^{13} + 650rst^8 + 650rs^8t - 16588r^8st - 1040rst^9 - 1040rs^9t + 38116r^9st + 635rst^{10} + 635rs^{10}t - 26260r^{10}st - 140rst^{11} - 140rs^{11}t + 5980r^{11}st + 3224rs^2t^7 - 4641rs^3t^6 - 4641rs^4t^5 - 4641rs^5t^4 - 4641rs^6t^3 + 3224rs^7t^2 - 6292r^2st^7 - 6292r^2s^7t + 1716r^3st^6 + 1716r^3s^6t + 1716r^4st^5 + 1716r^4s^5t + 1716r^5st^4 + 1716r^5s^4t + 1716r^6st^3 + 1716r^6s^3t + 16016r^7st^2 + 16016r^7s^2t - 936rs^2t^8 + 1638rs^3t^7 + 1638rs^4t^6 + 1638rs^5t^5 + 1638rs^6t^4 + 1638rs^7t^3 - 936rs^8t^2 + 3978r^2st^8 + 3978r^2s^8t + 7332r^3st^7 + 7332r^3s^7t - 2964r^4st^6 - 2964r^4s^6t - 2964r^5st^5 - 2964r^5s^5t - 2964r^6st^4 - 2964r^6s^4t - 2964r^7st^3 - 2964r^7s^3t - 35568r^8st^2 - 35568r^8s^2t - 925rs^2t^9 + 245rs^3t^8 + 245rs^4t^7 + 245rs^5t^6 + 245rs^6t^5 + 245rs^7t^4 + 245rs^8t^3 - 925rs^9t^2 - 325r^2st^9 - 325r^2s^9t - 6448r^3st^8 - 6448r^3s^8t + 1508r^4st^7 + 1508r^4s^7t + 1508r^5st^6 + 1508r^5s^6t + 1508r^6st^5 + 1508r^6s^5t + 1508r^7st^4 + 1508r^7s^4t + 1508r^8st^3 + 1508r^8s^3t + 22646r^9st^2 + 22646r^9s^2t + 380rs^2t^{10} - 140rs^3t^9 - 140rs^4t^8 - 140rs^5t^7 - 140rs^6t^6 - 140rs^7t^5 - 140rs^8t^4 - 140rs^9t^3 + 380rs^{10}t^2 - 175r^2st^{10} - 175r^2s^{10}t + 1495r^3st^9 + 1495r^3s^9t - 260r^4st^8 - 260r^4s^8t - 260r^5st^7 - 260r^5s^7t - 260r^6st^6 -$

$$260r^6s^6t - 260r^7st^5 - 260r^7s^5t - 260r^8st^4 - 260r^8s^4t - 260r^9st^3 - 260r^9s^3t -$$
$$4810r^{10}st^2 - 4810r^{10}s^2t + 4147r^2s^2t^6 + 7007r^2s^3t^5 + 7007r^2s^4t^4 + 7007r^2s^5t^3 +$$
$$4147r^2s^6t^2 - 2860r^3s^2t^5 - 2860r^3s^5t^2 - 2860r^4s^2t^4 - 2860r^4s^4t^2 - 2860r^5s^2t^3 -$$
$$2860r^5s^3t^2 - 18590r^6s^2t^2 - 7072r^2s^2t^7 + 3367r^2s^3t^6 + 3367r^2s^4t^5 + 3367r^2s^5t^4 +$$
$$3367r^2s^6t^3 - 7072r^2s^7t^2 - 4433r^3s^2t^6 - 9009r^3s^3t^5 - 9009r^3s^4t^4 - 9009r^3s^5t^3 -$$
$$4433r^3s^6t^2 + 4576r^4s^2t^5 + 4576r^4s^5t^2 + 4576r^5s^2t^4 + 4576r^5s^4t^2 + 4576r^6s^2t^3 +$$
$$4576r^6s^3t^2 + 39182r^7s^2t^2 + 4433r^2s^2t^8 - 3549r^2s^3t^7 - 3549r^2s^4t^6 - 3549r^2s^5t^5 -$$
$$3549r^2s^6t^4 - 3549r^2s^7t^3 + 4433r^2s^8t^2 + 2886r^3s^2t^7 + 4914r^3s^3t^6 + 4914r^3s^4t^5 +$$
$$4914r^3s^5t^4 + 4914r^3s^6t^3 + 2886r^3s^7t^2 - 2028r^4s^2t^6 - 2028r^4s^6t^2 - 2028r^5s^2t^5 -$$
$$2028r^5s^5t^2 - 2028r^6s^2t^4 - 2028r^6s^4t^2 - 2028r^7s^2t^3 - 2028r^7s^3t^2 - 21931r^8s^2t^2 -$$
$$955r^2s^2t^9 + 735r^2s^3t^8 + 735r^2s^4t^7 + 735r^2s^5t^6 + 735r^2s^6t^5 + 735r^2s^7t^4 + 735r^2s^8t^3 -$$
$$955r^2s^9t^2 - 507r^3s^2t^8 - 819r^3s^3t^7 - 819r^3s^4t^6 - 819r^3s^5t^5 - 819r^3s^6t^4 - 819r^3s^7t^3 -$$
$$507r^3s^8t^2 + 312r^4s^2t^7 + 312r^4s^7t^2 + 312r^5s^2t^6 + 312r^5s^6t^2 + 312r^6s^2t^5 +$$
$$312r^6s^5t^2 + 312r^7s^2t^4 + 312r^7s^4t^2 + 312r^8s^2t^3 + 312r^8s^3t^2 + 4199r^9s^2t^2) -$$
$$\frac{h^3st(x-x_n)}{2162160r^2(r-s)^3(r-t)^3(r-1)^3}(1690r^2s^7 - 312r^3s^6 - 312r^4s^5 - 312r^5s^4 - 312r^6s^3 -$$
$$3887r^7s^2 - 780r^2s^8 - 2002r^3s^7 + 572r^4s^6 + 572r^5s^5 + 572r^6s^4 + 572r^7s^3 +$$
$$9009r^8s^2 - 385r^2s^9 + 2020r^3s^8 - 320r^4s^7 - 320r^5s^6 - 320r^6s^5 - 320r^7s^4 -$$
$$320r^8s^3 - 6235r^9s^2 + 210r^2s^{10} - 525r^3s^9 + 60r^4s^8 + 60r^5s^7 + 60r^6s^6 + 60r^7s^5 +$$
$$60r^8s^4 + 60r^9s^3 + 1425r^{10}s^2 + 1690r^2t^7 - 312r^3t^6 - 312r^4t^5 - 312r^5t^4 - 312r^6t^3 -$$
$$3887r^7t^2 - 780r^2t^8 - 2002r^3t^7 + 572r^4t^6 + 572r^5t^5 + 572r^6t^4 + 572r^7t^3 + 9009r^8t^2 -$$
$$385r^2t^9 + 2020r^3t^8 - 320r^4t^7 - 320r^5t^6 - 320r^6t^5 - 320r^7t^4 - 320r^8t^3 - 6235r^9t^2 +$$
$$210r^2t^{10} - 525r^3t^9 + 60r^4t^8 + 60r^5t^7 + 60r^6t^6 + 60r^7t^5 + 60r^8t^4 + 60r^9t^3 +$$
$$1425r^{10}t^2 - 884s^2t^7 + 546s^3t^6 + 546s^4t^5 + 546s^5t^4 + 546s^6t^3 - 884s^7t^2 + 780s^2t^8 -$$
$$364s^3t^7 - 364s^4t^6 - 364s^5t^5 - 364s^6t^4 - 364s^7t^3 + 780s^8t^2 - 190s^2t^9 + 70s^3t^8 +$$
$$70s^4t^7 + 70s^5t^6 + 70s^6t^5 + 70s^7t^4 + 70s^8t^3 - 190s^9t^2 - 520rs^8 + 4550r^8s + 520rs^9 -$$
$$10790r^9s - 140rs^{10} + 7870r^{10}s - 1890r^{11}s - 520rt^8 + 4550r^8t + 520rt^9 - 10790r^9t -$$
$$140rt^{10} + 7870r^{10}t - 1890r^{11}t + 260st^8 + 260s^8t - 260st^9 - 260s^9t + 70st^{10} +$$
$$70s^{10}t - 1365r^9 + 3315r^{10} - 2520r^{11} + 630r^{12} + 650rst^7 + 650rs^7t - 16588r^7st -$$
$$1040rst^8 - 1040rs^8t + 38116r^8st + 635rst^9 + 635rs^9t - 26260r^9st - 140rst^{10} -$$

$140rs^{10}t+5980r^{10}st+3224rs^2t^6-4641rs^3t^5-4641rs^4t^4-4641rs^5t^3+3224rs^6t^2-$

$6292r^2st^6-6292r^2s^6t+1716r^3st^5+1716r^3s^5t+1716r^4st^4+1716r^4s^4t+$

$1716r^5st^3+1716r^5s^3t+16016r^6st^2+16016r^6s^2t-936rs^2t^7+1638rs^3t^6+$

$1638rs^4t^5+1638rs^5t^4+1638rs^6t^3-936rs^7t^2+3978r^2st^7+3978r^2s^7t+7332r^3st^6+$

$7332r^3s^6t-2964r^4st^5-2964r^4s^5t-2964r^5st^4-2964r^5s^4t-2964r^6st^3-$

$2964r^6s^3t-35568r^7st^2-35568r^7s^2t-925rs^2t^8+245rs^3t^7+245rs^4t^6+245rs^5t^5+$

$245rs^6t^4+245rs^7t^3-925rs^8t^2-325r^2st^8-325r^2s^8t-6448r^3st^7-6448r^3s^7t+$

$1508r^4st^6+1508r^4s^6t+1508r^5st^5+1508r^5s^5t+1508r^6st^4+1508r^6s^4t+$

$1508r^7st^3+1508r^7s^3t+22646r^8st^2+22646r^8s^2t+380rs^2t^9-140rs^3t^8-$

$140rs^4t^7-140rs^5t^6-140rs^6t^5-140rs^7t^4-140rs^8t^3+380rs^9t^2-175r^2st^9-$

$175r^2s^9t+1495r^3st^8+1495r^3s^8t-260r^4st^7-260r^4s^7t-260r^5st^6-260r^5s^6t-$

$260r^6st^5-260r^6s^5t-260r^7st^4-260r^7s^4t-260r^8st^3-260r^8s^3t-4810r^9st^2-$

$4810r^9s^2t+4147r^2s^2t^5+7007r^2s^3t^4+7007r^2s^4t^3+4147r^2s^5t^2-2860r^3s^2t^4-$

$2860r^3s^4t^2-2860r^4s^2t^3-2860r^4s^3t^2-18590r^5s^2t^2-7072r^2s^2t^6+3367r^2s^3t^5+$

$3367r^2s^4t^4+3367r^2s^5t^3-7072r^2s^6t^2-4433r^3s^2t^5-9009r^3s^3t^4-9009r^3s^4t^3-$

$4433r^3s^5t^2+4576r^4s^2t^4+4576r^4s^4t^2+4576r^5s^2t^3+4576r^5s^3t^2+39182r^6s^2t^2+$

$4433r^2s^2t^7-3549r^2s^3t^6-3549r^2s^4t^5-3549r^2s^5t^4-3549r^2s^6t^3+4433r^2s^7t^2+$

$2886r^3s^2t^6+4914r^3s^3t^5+4914r^3s^4t^4+4914r^3s^5t^3+2886r^3s^6t^2-2028r^4s^2t^5-$

$2028r^4s^5t^2-2028r^5s^2t^4-2028r^5s^4t^2-2028r^6s^2t^3-2028r^6s^3t^2-21931r^7s^2t^2-$

$955r^2s^2t^8+735r^2s^3t^7+735r^2s^4t^6+735r^2s^5t^5+735r^2s^6t^4+735r^2s^7t^3-955r^2s^8t^2-$

$507r^3s^2t^7-819r^3s^3t^6-819r^3s^4t^5-819r^3s^5t^4-819r^3s^6t^3-507r^3s^7t^2+312r^4s^2t^6+$

$312r^4s^6t^2+312r^5s^2t^5+312r^5s^5t^2+312r^6s^2t^4+312r^6s^4t^2+312r^7s^2t^3+312r^7s^3t^2+$

$4199r^8s^2t^2)+\frac{s^2t^2(x-x_n)^6}{360h^2r^2(r-s)^3(r-t)^3(r-1)^3}(5rs+5rt-3st-7r^2s-7r^2t-7r^2+9r^3+$

$5rst)+\frac{st(x-x_n)^7}{420h^3r^3(r-s)^3(r-t)^3(r-1)^3}(7r^3s^2-5r^2s^2-5r^2t^2+7r^3t^2+s^2t^2+7r^3s-9r^4s+$

$7r^3t-9r^4t+rst^2+rs^2t-7r^2st+17r^3st-9r^4st+rs^2t^2-7r^2st^2-7r^2s^2t+7r^3st^2+$

$7r^3s^2t-5r^2s^2t^2),$

$\beta_s=\frac{(x-x_n)^9}{1512h^5s^3(r-s)^3(s-t)^3(s-1)^3}(2r^2s^2-10r^2s^3+2r^3s^2+9r^2s^4-7r^3s^3-4r^2t^2-4r^2t^3-$

$4r^3t^2-r^3t^3+2s^2t^2+2s^2t^3-10s^3t^2-7s^3t^3+9s^4t^2-7rs^3+2r^3s+9rs^4-rt^3-$

$r^3t + 2st^3 - 7s^3t + 9s^4t + 7rst^2 - 2rs^2t + 7r^2st + 7rst^3 - 26rs^3t + 7r^3st + 36rs^4t + 3rs^2t^2 + 12r^2st^2 + 3r^2s^2t - 2rs^2t^3 - 26rs^3t^2 + 7r^2st^3 - 26r^2s^3t + 7r^3st^2 - 2r^3s^2t - 7rs^3t^3 + 9rs^4t^2 + 9r^2s^4t + 2r^3st^3 - 7r^3s^3t + 3r^2s^2t^2 + 2r^2s^2t^3 - 10r^2s^3t^2 + 2r^3s^2t^2) - \frac{(x-x_n)^8}{1680h^4s^3(r-s)^3(s-t)^3(s-1)^3}(5r^3s^2 - 7r^2s^3 + 9r^2s^4 - 7r^3s^3 - 4r^2t^3 - 4r^3t^2 - 4r^3t^3 + 5s^2t^3 - 7s^3t^2 - 7s^3t^3 + 9s^4t^2 + 5rst^3 - 28rs^3t + 5r^3st + 36rs^4t + 13rs^2t^2 + 4r^2st^2 + 13r^2s^2t + 13rs^2t^3 - 47rs^3t^2 + 4r^2st^3 - 47r^2s^3t + 4r^3st^2 + 13r^3s^2t - 28rs^3t^3 + 36rs^4t^2 + 36r^2s^4t + 5r^3st^3 - 28r^3s^3t + 24r^2s^2t^2 + 13r^2s^2t^3 - 47r^2s^3t^2 + 13r^3s^2t^2 - 7r^2s^3t^3 + 9r^2s^4t^2 + 5r^3s^2t^3 - 7r^3s^3t^2) - \frac{(x-x_n)^{12}}{11880h^8s^3(r-s)^3(s-t)^3(s-1)^3}(8rs - 12s^2t^2 - 12r^2s^2 - 4rt + 8st - 19rs^2 + 8r^2s + 9rs^3 - 4rt^2 - 4r^2t + 8st^2 - 19s^2t + 9s^3t - 12s^2 + 9s^3 + 9s^4 + 8rst^2 - 19rs^2t + 8r^2st + 21rst) - \frac{h^2(x-x_n)^2}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(4550r^2s^9 - 3887r^3s^8 - 312r^4s^7 - 312r^5s^6 - 312r^6s^5 - 312r^7s^4 + 1690r^8s^3 - 520r^9s^2 - 10790r^2s^{10} + 9009r^3s^9 + 572r^4s^8 + 572r^5s^7 + 572r^6s^6 + 572r^7s^5 - 2002r^8s^4 - 780r^9s^3 + 520r^{10}s^2 + 7870r^2s^{11} - 6235r^3s^{10} - 320r^4s^9 - 320r^5s^8 - 320r^6s^7 - 320r^7s^6 - 320r^8s^5 + 2020r^9s^4 - 385r^{10}s^3 - 140r^{11}s^2 - 1890r^2s^{12} + 1425r^3s^{11} + 60r^4s^{10} + 60r^5s^9 + 60r^6s^8 + 60r^7s^7 + 60r^8s^6 + 60r^9s^5 - 525r^{10}s^4 + 210r^{11}s^3 + 260r^2t^9 - 884r^3t^8 + 546r^4t^7 + 546r^5t^6 + 546r^6t^5 + 546r^7t^4 - 884r^8t^3 + 260r^9t^2 - 260r^2t^{10} + 780r^3t^9 - 364r^4t^8 - 364r^5t^7 - 364r^6t^6 - 364r^7t^5 - 364r^8t^4 + 780r^9t^3 - 260r^{10}t^2 + 70r^2t^{11} - 190r^3t^{10} + 70r^4t^9 + 70r^5t^8 + 70r^6t^7 + 70r^7t^6 + 70r^8t^5 + 70r^9t^4 - 190r^{10}t^3 + 70r^{11}t^2 - 520s^2t^9 + 1690s^3t^8 - 312s^4t^7 - 312s^5t^6 - 312s^6t^5 - 312s^7t^4 - 3887s^8t^3 + 4550s^9t^2 + 520s^2t^{10} - 780s^3t^9 - 2002s^4t^8 + 572s^5t^7 + 572s^6t^6 + 572s^7t^5 + 572s^8t^4 + 9009s^9t^3 - 10790s^{10}t^2 - 140s^2t^{11} - 385s^3t^{10} + 2020s^4t^9 - 320s^5t^8 - 320s^6t^7 - 320s^7t^6 - 320s^8t^5 - 320s^9t^4 - 6235s^{10}t^3 + 7870s^{11}t^2 + 210s^3t^{11} - 525s^4t^{10} + 60s^5t^9 + 60s^6t^8 + 60s^7t^7 + 60s^8t^6 + 60s^9t^5 + 60s^{10}t^4 + 1425s^{11}t^3 - 1890s^{12}t^2 - 1365rs^{10} + 3315rs^{11} - 2520rs^{12} + 630rs^{13} - 1365s^{10}t + 3315s^{11}t - 2520s^{12}t + 630s^{13}t - 260rst^9 + 9100rs^9t - 260r^9st + 260rst^{10} - 21580rs^{10}t + 260r^{10}st - 70rst^{11} + 15740rs^{11}t - 70r^{11}st - 3780rs^{12}t + 2340rs^2t^8 - 6604rs^3t^7 + 1404rs^4t^6 + 1404rs^5t^5 + 1404rs^6t^4 + 15704rs^7t^3 - 20475rs^8t^2 - 234r^2st^8 - 20475r^2s^8t + 3770r^3st^7 + 15704r^3s^7t - 4095r^4st^6 + 1404r^4s^6t - 4095r^5$

$st^5 + 1404r^5s^5t - 4095r^6st^4 + 1404r^6s^4t + 3770r^7st^3 - 6604r^7s^3t - 234r^8st^2 +$
$2340r^8s^2t - 1820rs^2t^9 + 1976rs^3t^8 + 7904rs^4t^7 - 2392rs^5t^6 - 2392rs^6t^5 -$
$2392rs^7t^4 - 34996rs^8t^3 + 47125rs^9t^2 - 260r^2st^9 + 47125r^2s^9t - 1300r^3st^8 -$
$34996r^3s^8t + 1274r^4st^7 - 2392r^4s^7t + 1274r^5st^6 - 2392r^5s^6t + 1274r^6st^5 -$
$2392r^6s^5t + 1274r^7st^4 + 7904r^7s^4t - 1300r^8st^3 + 1976r^8s^3t - 260r^9st^2 -$
$1820r^9s^2t + 250rs^2t^{10} + 1695rs^3t^9 - 6768rs^4t^8 + 1188rs^5t^7 + 1188rs^6t^6 +$
$1188rs^7t^5 + 1188rs^8t^4 + 22326rs^9t^3 - 32495rs^{10}t^2 + 445r^2st^{10} - 32495r^2s^{10}t -$
$855r^3st^9 + 22326r^3s^9t + 315r^4st^8 + 1188r^4s^8t + 315r^5st^7 + 1188r^5s^7t + 315r^6st^6 +$
$1188r^6s^6t + 315r^7st^5 + 1188r^7s^5t + 315r^8st^4 - 6768r^8s^4t - 855r^9st^3 + 1695r^9s^3t +$
$445r^{10}st^2 + 250r^{10}s^2t + 70rs^2t^{11} - 700rs^3t^{10} + 1555rs^4t^9 - 200rs^5t^8 - 200rs^6t^7 -$
$200rs^7t^6 - 200rs^8t^5 - 200rs^9t^4 - 4750rs^{10}t^3 + 7405rs^{11}t^2 - 140r^2st^{11} + 7405r^2s^{11}t +$
$380r^3st^{10} - 4750r^3s^{10}t - 140r^4st^9 - 200r^4s^9t - 140r^5st^8 - 200r^5s^8t - 140r^6st^7 -$
$200r^6s^7t - 140r^7st^6 - 200r^7s^6t - 140r^8st^5 - 200r^8s^5t - 140r^9st^4 + 1555r^9s^4t +$
$380r^{10}st^3 - 700r^{10}s^3t - 140r^{11}st^2 + 70r^{11}s^2t - 3068r^2s^2t^7 + 5863r^2s^3t^6 -$
$1144r^2s^4t^5 - 1144r^2s^5t^4 - 16874r^2s^6t^3 + 32032r^2s^7t^2 - 494r^3s^2t^6 + 4147r^3s^3t^5 -$
$2860r^3s^4t^4 - 5720r^3s^5t^3 - 16874r^3s^6t^2 + 2366r^4s^2t^5 + 7007r^4s^3t^4 - 2860r^4s^4t^3 -$
$1144r^4s^5t^2 + 2366r^5s^2t^4 + 4147r^5s^3t^3 - 1144r^5s^4t^2 - 494r^6s^2t^3 + 5863r^6s^3t^2 -$
$3068r^7s^2t^2 + 3042r^2s^2t^8 + 260r^2s^3t^7 - 7397r^2s^4t^6 + 1612r^2s^5t^5 + 1612r^2s^6t^4 +$
$36218r^2s^7t^3 - 71136r^2s^8t^2 - 5434r^3s^2t^7 - 1066r^3s^3t^6 - 4433r^3s^4t^5 + 4576r^3s^5t^4 +$
$9152r^3s^6t^3 + 36218r^3s^7t^2 + 5005r^4s^2t^6 - 5642r^4s^3t^5 - 9009r^4s^4t^4 + 4576r^4s^5t^3 +$
$1612r^4s^6t^2 + 5005r^5s^2t^5 - 5642r^5s^3t^4 - 4433r^5s^4t^3 + 1612r^5s^5t^2 + 5005r^6s^2t^4 -$
$1066r^6s^3t^3 - 7397r^6s^4t^2 - 5434r^7s^2t^3 + 260r^7s^3t^2 + 3042r^8s^2t^2 - 1250r^2s^2t^9 -$
$2015r^2s^3t^8 + 4394r^2s^4t^7 - 520r^2s^5t^6 - 520r^2s^6t^5 - 520r^2s^7t^4 - 20423r^2s^8t^3 +$
$45292r^2s^9t^2 + 4678r^3s^2t^8 - 663r^3s^3t^7 + 2886r^3s^4t^6 - 2028r^3s^5t^5 - 2028r^3s^6t^4 -$
$4056r^3s^7t^3 - 20423r^3s^8t^2 - 3304r^4s^2t^7 + 1365r^4s^3t^6 + 4914r^4s^4t^5 - 2028r^4s^6t^3 -$
$520r^4s^7t^2 - 3304r^5s^2t^6 + 1365r^5s^3t^5 + 4914r^5s^4t^4 - 2028r^5s^5t^3 - 520r^5s^6t^2 -$
$3304r^6s^2t^5 + 1365r^6s^3t^4 + 2886r^6s^4t^3 - 520r^6s^5t^2 - 3304r^7s^2t^4 - 663r^7s^3t^3 +$
$4394r^7s^4t^2 + 4678r^8s^2t^3 - 2015r^8s^3t^2 - 1250r^9s^2t^2 + 205r^2s^2t^{10} + 540r^2s^3t^9 -$
$767r^2s^4t^8 + 52r^2s^5t^7 + 52r^2s^6t^6 + 52r^2s^7t^5 + 52r^2s^8t^4 + 3939r^2s^9t^3 - 9620r^2s^{10}t^2 -$

$$1095r^3s^2t^9 + 228r^3s^3t^8 - 507r^3s^4t^7 + 312r^3s^5t^6 + 312r^3s^6t^5 + 312r^3s^7t^4 +$$
$$624r^3s^8t^3 + 3939r^3s^9t^2 + 595r^4s^2t^8 - 84r^4s^3t^7 - 819r^4s^4t^6 + 312r^4s^7t^3 + 52r^4s^8t^2 +$$
$$595r^5s^2t^7 - 84r^5s^3t^6 - 819r^5s^4t^5 + 312r^5s^6t^3 + 52r^5s^7t^2 + 595r^6s^2t^6 - 84r^6s^3t^5 -$$
$$819r^6s^4t^4 + 312r^6s^5t^3 + 52r^6s^6t^2 + 595r^7s^2t^5 - 84r^7s^3t^4 - 507r^7s^4t^3 + 52r^7s^5t^2 +$$
$$595r^8s^2t^4 + 228r^8s^3t^3 - 767r^8s^4t^2 - 1095r^9s^2t^3 + 540r^9s^3t^2 + 205r^{10}s^2t^2) +$$
$$\frac{(x-x_n)^{10}}{5040h^6s^3(r-s)^3(s-t)^3(s-1)^3}(4r^2s^2 + 19r^2s^3 + 7r^3s^2 - 9r^2s^4 + 7r^3s^3 + 16r^2t^2 + 4r^2t^3 +$$
$$4r^3t^2 + 4s^2t^2 + 7s^2t^3 + 19s^3t^2 + 7s^3t^3 - 9s^4t^2 + 7rs^2 - 8r^2s + 19rs^3 - 8r^3s -$$
$$36rs^4 + 4rt^2 + 4r^2t + 4rt^3 + 4r^3t - 8st^2 + 7s^2t - 8st^3 + 19s^3t - 36s^4t + 7s^3 - 9s^4 -$$
$$36rst^2 + 27rs^2t - 36r^2st - 13rst^3 + 20rs^3t - 13r^3st - 36rs^4t + 27rs^2t^2 - 36r^2st^2 +$$
$$27r^2s^2t + 7rs^2t^3 + 19rs^3t^2 - 8r^2st^3 + 19r^2s^3t - 8r^3st^2 + 7r^3s^2t - 13rst + 4r^2s^2t^2) -$$
$$\frac{(x-x_n)^{11}}{3960h^7s^3(r-s)^3(s-t)^3(s-1)^3}(10r^2s^2 + 3r^2s^3 + 3r^3s^2 + 4r^2t^2 + 10s^2t^2 + 3s^2t^3 + 3s^3t^2 -$$
$$2rs + rt - 2st + 10rs^2 - 8r^2s - 2rs^3 - 2r^3s - 9rs^4 + 4rt^2 + 4r^2t + rt^3 + r^3t - 8st^2 +$$
$$10s^2t - 2st^3 - 2s^3t - 9s^4t + 3s^2 + 3s^3 - 9s^4 - 15rst^2 + 21rs^2t - 15r^2st - 2rst^3 -$$
$$2rs^3t - 2r^3st + 10rs^2t^2 - 8r^2st^2 + 10r^2s^2t - 15rst) + \frac{(x-x_n)^{13}}{8580h^9s^3(r-s)^3(s-t)^3(s-1)^3}(2rs -$$
$$rt + 2st - 3rs^2 - 3s^2t - 3s^2 + 4s^3 + 2rst) + \frac{h(x-x_n)^3}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(1690r^8s^2 -$$
$$312r^3s^7 - 312r^4s^6 - 312r^5s^5 - 312r^6s^4 - 312r^7s^3 - 3887r^2s^8 + 9009r^2s^9 + 572r^3s^8 +$$
$$572r^4s^7 + 572r^5s^6 + 572r^6s^5 + 572r^7s^4 - 2002r^8s^3 - 780r^9s^2 - 6235r^2s^{10} -$$
$$320r^3s^9 - 320r^4s^8 - 320r^5s^7 - 320r^6s^6 - 320r^7s^5 - 320r^8s^4 + 2020r^9s^3 - 385r^{10}s^2 +$$
$$1425r^2s^{11} + 60r^3s^{10} + 60r^4s^9 + 60r^5s^8 + 60r^6s^7 + 60r^7s^6 + 60r^8s^5 + 60r^9s^4 -$$
$$525r^{10}s^3 + 210r^{11}s^2 - 884r^2t^8 + 546r^3t^7 + 546r^4t^6 + 546r^5t^5 + 546r^6t^4 + 546r^7t^3 -$$
$$884r^8t^2 + 780r^2t^9 - 364r^3t^8 - 364r^4t^7 - 364r^5t^6 - 364r^6t^5 - 364r^7t^4 - 364r^8t^3 +$$
$$780r^9t^2 - 190r^2t^{10} + 70r^3t^9 + 70r^4t^8 + 70r^5t^7 + 70r^6t^6 + 70r^7t^5 + 70r^8t^4 +$$
$$70r^9t^3 - 190r^{10}t^2 + 1690s^2t^8 - 312s^3t^7 - 312s^4t^6 - 312s^5t^5 - 312s^6t^4 - 312s^7t^3 -$$
$$3887s^8t^2 - 780s^2t^9 - 2002s^3t^8 + 572s^4t^7 + 572s^5t^6 + 572s^6t^5 + 572s^7t^4 + 572s^8t^3 +$$
$$9009s^9t^2 - 385s^2t^{10} + 2020s^3t^9 - 320s^4t^8 - 320s^5t^7 - 320s^6t^6 - 320s^7t^5 - 320s^8t^4 -$$
$$320s^9t^3 - 6235s^{10}t^2 + 210s^2t^{11} - 525s^3t^{10} + 60s^4t^9 + 60s^5t^8 + 60s^6t^7 + 60s^7t^6 +$$
$$60s^8t^5 + 60s^9t^4 + 60s^{10}t^3 + 1425s^{11}t^2 + 4550rs^9 - 520r^9s - 10790rs^{10} + 520r^{10}s +$$
$$7870rs^{11} - 140r^{11}s - 1890rs^{12} + 260rt^9 + 260r^9t - 260rt^{10} - 260r^{10}t + 70rt^{11} +$$
$$70r^{11}t - 520st^9 + 4550s^9t + 520st^{10} - 10790s^{10}t - 140st^{11} + 7870s^{11}t - 1890s^{12}t -$$

$$1365s^{10}+3315s^{11}-2520s^{12}+630s^{13}+650rst^8-16588rs^8t+650r^8st-1040rst^9+$$
$$38116rs^9t-1040r^9st+635rst^{10}-26260rs^{10}t+635r^{10}st-140rst^{11}+5980rs^{11}t-$$
$$140r^{11}st-6292rs^2t^7+1716rs^3t^6+1716rs^4t^5+1716rs^5t^4+1716rs^6t^3+$$
$$16016rs^7t^2+3224r^2st^7+16016r^2s^7t-4641r^3st^6+1716r^3s^6t-4641r^4st^5+$$
$$1716r^4s^5t-4641r^5st^4+1716r^5s^4t-4641r^6st^3+1716r^6s^3t+3224r^7st^2-$$
$$6292r^7s^2t+3978rs^2t^8+7332rs^3t^7-2964rs^4t^6-2964rs^5t^5-2964rs^6t^4-$$
$$2964rs^7t^3-35568rs^8t^2-936r^2st^8-35568r^2s^8t+1638r^3st^7-2964r^3s^7t+$$
$$1638r^4st^6-2964r^4s^6t+1638r^5st^5-2964r^5s^5t+1638r^6st^4-2964r^6s^4t+$$
$$1638r^7st^3+7332r^7s^3t-936r^8st^2+3978r^8s^2t-325rs^2t^9-6448rs^3t^8+1508rs^4t^7+$$
$$1508rs^5t^6+1508rs^6t^5+1508rs^7t^4+1508rs^8t^3+22646rs^9t^2-925r^2st^9+$$
$$22646r^2s^9t+245r^3st^8+1508r^3s^8t+245r^4st^7+1508r^4s^7t+245r^5st^6+1508r^5s^6t+$$
$$245r^6st^5+1508r^6s^5t+245r^7st^4+1508r^7s^4t+245r^8st^3-6448r^8s^3t-925r^9st^2-$$
$$325r^9s^2t-175rs^2t^{10}+1495rs^3t^9-260rs^4t^8-260rs^5t^7-260rs^6t^6-260rs^7t^5-$$
$$260rs^8t^4-260rs^9t^3-4810rs^{10}t^2+380r^2st^{10}-4810r^2s^{10}t-140r^3st^9-260r^3s^9t-$$
$$140r^4st^8-260r^4s^8t-140r^5st^7-260r^5s^7t-140r^6st^6-260r^6s^6t-140r^7st^5-$$
$$260r^7s^5t-140r^8st^4-260r^8s^4t-140r^9st^3+1495r^9s^3t+380r^{10}st^2-175r^{10}s^2t+$$
$$4147r^2s^2t^6-2860r^2s^3t^5-2860r^2s^4t^4-2860r^2s^5t^3-18590r^2s^6t^2+7007r^3s^2t^5-$$
$$2860r^3s^5t^2+7007r^4s^2t^4-2860r^4s^4t^2+7007r^5s^2t^3-2860r^5s^3t^2+4147r^6s^2t^2-$$
$$7072r^2s^2t^7-4433r^2s^3t^6+4576r^2s^4t^5+4576r^2s^5t^4+4576r^2s^6t^3+39182r^2s^7t^2+$$
$$3367r^3s^2t^6-9009r^3s^3t^5+4576r^3s^6t^2+3367r^4s^2t^5-9009r^4s^3t^4+4576r^4s^5t^2+$$
$$3367r^5s^2t^4-9009r^5s^3t^3+4576r^5s^4t^2+3367r^6s^2t^3-4433r^6s^3t^2-7072r^7s^2t^2+$$
$$4433r^2s^2t^8+2886r^2s^3t^7-2028r^2s^4t^6-2028r^2s^5t^5-2028r^2s^6t^4-2028r^2s^7t^3-$$
$$21931r^2s^8t^2-3549r^3s^2t^7+4914r^3s^3t^6-2028r^3s^7t^2-3549r^4s^2t^6+4914r^4s^3t^5-$$
$$2028r^4s^6t^2-3549r^5s^2t^5+4914r^5s^3t^4-2028r^5s^5t^2-3549r^6s^2t^4+4914r^6s^3t^3-$$
$$2028r^6s^4t^2-3549r^7s^2t^3+2886r^7s^3t^2+4433r^8s^2t^2-955r^2s^2t^9-507r^2s^3t^8+$$
$$312r^2s^4t^7+312r^2s^5t^6+312r^2s^6t^5+312r^2s^7t^4+312r^2s^8t^3+4199r^2s^9t^2+$$
$$735r^3s^2t^8-819r^3s^3t^7+312r^3s^8t^2+735r^4s^2t^7-819r^4s^3t^6+312r^4s^7t^2+735r^5s^2t^6-$$
$$819r^5s^3t^5+312r^5s^6t^2+735r^6s^2t^5-819r^6s^3t^4+312r^6s^5t^2+735r^7s^2t^4-819r^7s^3t^3+$$
$$312r^7s^4t^2+735r^8s^2t^3-507r^8s^3t^2-955r^9s^2t^2)-\frac{r^2t^2(x-x_n)^6}{360h^2s^2(r-s)^3(s-t)^3(s-1)^3}(5rs-3rt+$$

$5st-7rs^2-7s^2t-7s^2+9s^3+5rst)+\frac{h^3rt(x-x_n)}{2162160s^2(r-s)^3(s-t)^3(s-1)^3}(1690r^7s^2-312r^3s^6-$
$312r^4s^5-312r^5s^4-312r^6s^3-3887r^2s^7+9009r^2s^8+572r^3s^7+572r^4s^6+572r^5s^5+$
$572r^6s^4-2002r^7s^3-780r^8s^2-6235r^2s^9-320r^3s^8-320r^4s^7-320r^5s^6-$
$320r^6s^5-320r^7s^4+2020r^8s^3-385r^9s^2+1425r^2s^{10}+60r^3s^9+60r^4s^8+60r^5s^7+$
$60r^6s^6+60r^7s^5+60r^8s^4-525r^9s^3+210r^{10}s^2-884r^2t^7+546r^3t^6+546r^4t^5+$
$546r^5t^4+546r^6t^3-884r^7t^2+780r^2t^8-364r^3t^7-364r^4t^6-364r^5t^5-364r^6t^4-$
$364r^7t^3+780r^8t^2-190r^2t^9+70r^3t^8+70r^4t^7+70r^5t^6+70r^6t^5+70r^7t^4+70r^8t^3-$
$190r^9t^2+1690s^2t^7-312s^3t^6-312s^4t^5-312s^5t^4-312s^6t^3-3887s^7t^2-780s^2t^8-$
$2002s^3t^7+572s^4t^6+572s^5t^5+572s^6t^4+572s^7t^3+9009s^8t^2-385s^2t^9+2020s^3t^8-$
$320s^4t^7-320s^5t^6-320s^6t^5-320s^7t^4-320s^8t^3-6235s^9t^2+210s^2t^{10}-525s^3t^9+$
$60s^4t^8+60s^5t^7+60s^6t^6+60s^7t^5+60s^8t^4+60s^9t^3+1425s^{10}t^2+4550rs^8-$
$520r^8s-10790rs^9+520r^9s+7870rs^{10}-140r^{10}s-1890rs^{11}+260rt^8+260r^8t-$
$260rt^9-260r^9t+70rt^{10}+70r^{10}t-520st^8+4550s^8t+520st^9-10790s^9t-$
$140st^{10}+7870s^{10}t-1890s^{11}t-1365s^9+3315s^{10}-2520s^{11}+630s^{12}+650rst^7-$
$16588rs^7t+650r^7st-1040rst^8+38116rs^8t-1040r^8st+635rst^9-26260rs^9t+$
$635r^9st-140rst^{10}+5980rs^{10}t-140r^{10}st-6292rs^2t^6+1716rs^3t^5+1716rs^4t^4+$
$1716rs^5t^3+16016rs^6t^2+3224r^2st^6+16016r^2s^6t-4641r^3st^5+1716r^3s^5t-$
$4641r^4st^4+1716r^4s^4t-4641r^5st^3+1716r^5s^3t+3224r^6st^2-6292r^6s^2t+$
$3978rs^2t^7+7332rs^3t^6-2964rs^4t^5-2964rs^5t^4-2964rs^6t^3-35568rs^7t^2-$
$936r^2st^7-35568r^2s^7t+1638r^3st^6-2964r^3s^6t+1638r^4st^5-2964r^4s^5t+$
$1638r^5st^4-2964r^5s^4t+1638r^6st^3+7332r^6s^3t-936r^7st^2+3978r^7s^2t-325rs^2t^8-$
$6448rs^3t^7+1508rs^4t^6+1508rs^5t^5+1508rs^6t^4+1508rs^7t^3+22646rs^8t^2-$
$925r^2st^8+22646r^2s^8t+245r^3st^7+1508r^3s^7t+245r^4st^6+1508r^4s^6t+245r^5st^5+$
$1508r^5s^5t+245r^6st^4+1508r^6s^4t+245r^7st^3-6448r^7s^3t-925r^8st^2-325r^8s^2t-$
$175rs^2t^9+1495rs^3t^8-260rs^4t^7-260rs^5t^6-260rs^6t^5-260rs^7t^4-260rs^8t^3-$
$4810rs^9t^2+380r^2st^9-4810r^2s^9t-140r^3st^8-260r^3s^8t-140r^4st^7-260r^4s^7t-$
$140r^5st^6-260r^5s^6t-140r^6st^5-260r^6s^5t-140r^7st^4-260r^7s^4t-140r^8st^3+$
$1495r^8s^3t+380r^9st^2-175r^9s^2t+4147r^2s^2t^5-2860r^2s^3t^4-2860r^2s^4t^3-$
$18590r^2s^5t^2+7007r^3s^2t^4-2860r^3s^4t^2+7007r^4s^2t^3-2860r^4s^3t^2+4147r^5s^2t^2-$

$7072r^2s^2t^6 - 4433r^2s^3t^5 + 4576r^2s^4t^4 + 4576r^2s^5t^3 + 39182r^2s^6t^2 + 3367r^3s^2t^5 - 9009r^3s^3t^4 + 4576r^3s^5t^2 + 3367r^4s^2t^4 - 9009r^4s^3t^3 + 4576r^4s^4t^2 + 3367r^5s^2t^3 - 4433r^5s^3t^2 - 7072r^6s^2t^2 + 4433r^2s^2t^7 + 2886r^2s^3t^6 - 2028r^2s^4t^5 - 2028r^2s^5t^4 - 2028r^2s^6t^3 - 21931r^2s^7t^2 - 3549r^3s^2t^6 + 4914r^3s^3t^5 - 2028r^3s^6t^2 - 3549r^4s^2t^5 + 4914r^4s^3t^4 - 2028r^4s^5t^2 - 3549r^5s^2t^4 + 4914r^5s^3t^3 - 2028r^5s^4t^2 - 3549r^6s^2t^3 + 2886r^6s^3t^2 + 4433r^7s^2t^2 - 955r^2s^2t^8 - 507r^2s^3t^7 + 312r^2s^4t^6 + 312r^2s^5t^5 + 312r^2s^6t^4 + 312r^2s^7t^3 + 4199r^2s^8t^2 + 735r^3s^2t^7 - 819r^3s^3t^6 + 312r^3s^7t^2 + 735r^4s^2t^6 - 819r^4s^3t^5 + 312r^4s^6t^2 + 735r^5s^2t^5 - 819r^5s^3t^4 + 312r^5s^5t^2 + 735r^6s^2t^4 - 819r^6s^3t^3 + 312r^6s^4t^2 + 735r^7s^2t^3 - 507r^7s^3t^2 - 955r^8s^2t^2) - \frac{rt(x-x_n)^7}{420h^3s^3(r-s)^3(s-t)^3(s-1)^3}(7r^2s^3 - 5r^2s^2 + r^2t^2 - 5s^2t^2 + 7s^3t^2 + 7rs^3 - 9rs^4 + 7s^3t - 9s^4t + rst^2 - 7rs^2t + r^2st + 17rs^3t - 9rs^4t - 7rs^2t^2 + r^2st^2 - 7r^2s^2t + 7rs^3t^2 + 7r^2s^3t - 5r^2s^2t^2)$,

$\beta_t = \frac{(x-x_n)^{12}}{11880h^8t^3(r-t)^3(s-t)^3(t-1)^3}(8rt - 12s^2t^2 - 4rs - 12r^2t^2 + 8st - 4rs^2 - 4r^2s - 19rt^2 + 8r^2t + 9rt^3 - 19st^2 + 8s^2t + 9st^3 - 12t^2 + 9t^3 + 9t^4 - 19rst^2 + 8rs^2t + 8r^2st + 21rst) - \frac{h(x-x_n)^3}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(546r^3s^7 - 884r^2s^8 + 546r^4s^6 + 546r^5s^5 + 546r^6s^4 + 546r^7s^3 - 884r^8s^2 + 780r^2s^9 - 364r^3s^8 - 364r^4s^7 - 364r^5s^6 - 364r^6s^5 - 364r^7s^4 - 364r^8s^3 + 780r^9s^2 - 190r^2s^{10} + 70r^3s^9 + 70r^4s^8 + 70r^5s^7 + 70r^6s^6 + 70r^7s^5 + 70r^8s^4 + 70r^9s^3 - 190r^{10}s^2 - 3887r^2t^8 - 312r^3t^7 - 312r^4t^6 - 312r^5t^5 - 312r^6t^4 - 312r^7t^3 + 1690r^8t^2 + 9009r^2t^9 + 572r^3t^8 + 572r^4t^7 + 572r^5t^6 + 572r^6t^5 + 572r^7t^4 - 2002r^8t^3 - 780r^9t^2 - 6235r^2t^{10} - 320r^3t^9 - 320r^4t^8 - 320r^5t^7 - 320r^6t^6 - 320r^7t^5 - 320r^8t^4 + 2020r^9t^3 - 385r^{10}t^2 + 1425r^2t^{11} + 60r^3t^{10} + 60r^4t^9 + 60r^5t^8 + 60r^6t^7 + 60r^7t^6 + 60r^8t^5 + 60r^9t^4 - 525r^{10}t^3 + 210r^{11}t^2 - 3887s^2t^8 - 312s^3t^7 - 312s^4t^6 - 312s^5t^5 - 312s^6t^4 - 312s^7t^3 + 1690s^8t^2 + 9009s^2t^9 + 572s^3t^8 + 572s^4t^7 + 572s^5t^6 + 572s^6t^5 + 572s^7t^4 - 2002s^8t^3 - 780s^9t^2 - 6235s^2t^{10} - 320s^3t^9 - 320s^4t^8 - 320s^5t^7 - 320s^6t^6 - 320s^7t^5 - 320s^8t^4 + 2020s^9t^3 - 385s^{10}t^2 + 1425s^2t^{11} + 60s^3t^{10} + 60s^4t^9 + 60s^5t^8 + 60s^6t^7 + 60s^7t^6 + 60s^8t^5 + 60s^9t^4 - 525s^{10}t^3 + 210s^{11}t^2 + 260rs^9 + 260r^9s - 260rs^{10} - 260r^{10}s + 70rs^{11} + 70r^{11}s + 4550rt^9 - 520r^9t - 10790rt^{10} + 520r^{10}t + 7870rt^{11} - 140r^{11}t - 1890rt^{12} + 4550st^9 - 520s^9t - 10790st^{10} + 520s^{10}t$

$+7870st^{11} - 140s^{11}t - 1890st^{12} - 1365t^{10} + 3315t^{11} - 2520t^{12} + 630t^{13} - 16588rst^{8} + 650rs^{8}t + 650r^{8}st + 38116rst^{9} - 1040rs^{9}t - 1040r^{9}st - 26260rst^{10} + 635rs^{10}t + 635r^{10}st + 5980rst^{11} - 140rs^{11}t - 140r^{11}st + 16016rs^{2}t^{7} + 1716rs^{3}t^{6} + 1716rs^{4}t^{5} + 1716rs^{5}t^{4} + 1716rs^{6}t^{3} - 6292rs^{7}t^{2} + 16016r^{2}st^{7} + 3224r^{2}s^{7}t + 1716r^{3}st^{6} - 4641r^{3}s^{6}t + 1716r^{4}st^{5} - 4641r^{4}s^{5}t + 1716r^{5}st^{4} - 4641r^{5}s^{4}t + 1716r^{6}st^{3} - 4641r^{6}s^{3}t - 6292r^{7}st^{2} + 3224r^{7}s^{2}t - 35568rs^{2}t^{8} - 2964rs^{3}t^{7} - 2964rs^{4}t^{6} - 2964rs^{5}t^{5} - 2964rs^{6}t^{4} + 7332rs^{7}t^{3} + 3978rs^{8}t^{2} - 35568r^{2}st^{8} - 936r^{2}s^{8}t - 2964r^{3}st^{7} + 1638r^{3}s^{7}t - 2964r^{4}st^{6} + 1638r^{4}s^{6}t - 2964r^{5}st^{5} + 1638r^{5}s^{5}t - 2964r^{6}st^{4} + 1638r^{6}s^{4}t + 7332r^{7}st^{3} + 1638r^{7}s^{3}t + 3978r^{8}st^{2} - 936r^{8}s^{2}t + 22646rs^{2}t^{9} + 1508rs^{3}t^{8} + 1508rs^{4}t^{7} + 1508rs^{5}t^{6} + 1508rs^{6}t^{5} + 1508rs^{7}t^{4} - 6448rs^{8}t^{3} - 325rs^{9}t^{2} + 22646r^{2}st^{9} - 925r^{2}s^{9}t + 1508r^{3}st^{8} + 245r^{3}s^{8}t + 1508r^{4}st^{7} + 245r^{4}s^{7}t + 1508r^{5}st^{6} + 245r^{5}s^{6}t + 1508r^{6}st^{5} + 245r^{6}s^{5}t + 1508r^{7}st^{4} + 245r^{7}s^{4}t - 6448r^{8}st^{3} + 245r^{8}s^{3}t - 325r^{9}st^{2} - 925r^{9}s^{2}t - 4810rs^{2}t^{10} - 260rs^{3}t^{9} - 260rs^{4}t^{8} - 260rs^{5}t^{7} - 260rs^{6}t^{6} - 260rs^{7}t^{5} - 260rs^{8}t^{4} + 1495rs^{9}t^{3} - 175rs^{10}t^{2} - 4810r^{2}st^{10} + 380r^{2}s^{10}t - 260r^{3}st^{9} - 140r^{3}s^{9}t - 260r^{4}st^{8} - 140r^{4}s^{8}t - 260r^{5}st^{7} - 140r^{5}s^{7}t - 260r^{6}st^{6} - 140r^{6}s^{6}t - 260r^{7}st^{5} - 140r^{7}s^{5}t - 260r^{8}st^{4} - 140r^{8}s^{4}t + 1495r^{9}st^{3} - 140r^{9}s^{3}t - 175r^{10}st^{2} + 380r^{10}s^{2}t - 18590r^{2}s^{2}t^{6} - 2860r^{2}s^{3}t^{5} - 2860r^{2}s^{4}t^{4} - 2860r^{2}s^{5}t^{3} + 4147r^{2}s^{6}t^{2} - 2860r^{3}s^{2}t^{5} + 7007r^{3}s^{5}t^{2} - 2860r^{4}s^{2}t^{4} + 7007r^{4}s^{4}t^{2} - 2860r^{5}s^{2}t^{3} + 7007r^{5}s^{3}t^{2} + 4147r^{6}s^{2}t^{2} + 39182r^{2}s^{2}t^{7} + 4576r^{2}s^{3}t^{6} + 4576r^{2}s^{4}t^{5} + 4576r^{2}s^{5}t^{4} - 4433r^{2}s^{6}t^{3} - 7072r^{2}s^{7}t^{2} + 4576r^{3}s^{2}t^{6} - 9009r^{3}s^{5}t^{3} + 3367r^{3}s^{6}t^{2} + 4576r^{4}s^{2}t^{5} - 9009r^{4}s^{4}t^{3} + 3367r^{4}s^{5}t^{2} + 4576r^{5}s^{2}t^{4} - 9009r^{5}s^{3}t^{3} + 3367r^{5}s^{4}t^{2} - 4433r^{6}s^{2}t^{3} + 3367r^{6}s^{3}t^{2} - 7072r^{7}s^{2}t^{2} - 21931r^{2}s^{2}t^{8} - 2028r^{2}s^{3}t^{7} - 2028r^{2}s^{4}t^{6} - 2028r^{2}s^{5}t^{5} - 2028r^{2}s^{6}t^{4} + 2886r^{2}s^{7}t^{3} + 4433r^{2}s^{8}t^{2} - 2028r^{3}s^{2}t^{7} + 4914r^{3}s^{6}t^{3} - 3549r^{3}s^{7}t^{2} - 2028r^{4}s^{2}t^{6} + 4914r^{4}s^{5}t^{3} - 3549r^{4}s^{6}t^{2} - 2028r^{5}s^{2}t^{5} + 4914r^{5}s^{4}t^{3} - 3549r^{5}s^{5}t^{2} - 2028r^{6}s^{2}t^{4} + 4914r^{6}s^{3}t^{3} - 3549r^{6}s^{4}t^{2} + 2886r^{7}s^{2}t^{3} - 3549r^{7}s^{3}t^{2} + 4433r^{8}s^{2}t^{2} + 4199r^{2}s^{2}t^{9} + 312r^{2}s^{3}t^{8} + 312r^{2}s^{4}t^{7} + 312r^{2}s^{5}t^{6} + 312r^{2}s^{6}t^{5} + 312r^{2}s^{7}t^{4} - 507r^{2}s^{8}t^{3} - 955r^{2}s^{9}t^{2} + 312r^{3}s^{2}t^{8} - 819r^{3}s^{7}t^{3} + 735r^{3}s^{8}t^{2} + 312r^{4}s^{2}t^{7} - 819r^{4}s^{6}t^{3} + 735r^{4}s^{7}t^{2} + 312r^{5}s^{2}t^{6} - 819r^{5}s^{5}t^{3} + 735r^{5}s^{6}t^{2} + 312r^{6}s^{2}t^{5} - 819r^{6}s^{4}t^{3} + 735r^{6}s^{5}t^{2} + 312r^{7}s^{2}t^{4} - 819r^{7}s^{3}t^{3} + 735r^{7}s^{4}t^{2} - 507r^{8}s^{2}t^{3} + 735r^{8}s^{3}t^{2} - 955r^{9}s^{2}t^{2})$

$+\frac{(x-x_n)^8}{1680h^4t^3(r-t)^3(s-t)^3(t-1)^3}(5r^3t^2-4r^3s^2-4r^3s^3-7r^2t^3-4r^2s^3+9r^2t^4-7r^3t^3-$
$7s^2t^3+5s^3t^2+9s^2t^4-7s^3t^3-28rst^3+5rs^3t+5r^3st+36rst^4+13rs^2t^2+13r^2st^2+$
$4r^2s^2t-47rs^2t^3+13rs^3t^2-47r^2st^3+4r^2s^3t+13r^3st^2+4r^3s^2t+36rs^2t^4-28rs^3t^3+$
$36r^2st^4-28r^3st^3+5r^3s^3t+24r^2s^2t^2-47r^2s^2t^3+13r^2s^3t^2+13r^3s^2t^2+9r^2s^2t^4-$
$7r^2s^3t^3-7r^3s^2t^3+5r^3s^3t^2)-\frac{(x-x_n)^9}{1512h^5t^3(r-t)^3(s-t)^3(t-1)^3}(2r^2t^2-4r^2s^3-4r^3s^2-r^3s^3-$
$4r^2s^2-10r^2t^3+2r^3t^2+9r^2t^4-7r^3t^3+2s^2t^2-10s^2t^3+2s^3t^2+9s^2t^4-7s^3t^3-$
$rs^3-r^3s-7rt^3+2r^3t+9rt^4-7st^3+2s^3t+9st^4-2rst^2+7rs^2t+7r^2st-26rst^3+$
$7rs^3t+7r^3st+36rst^4+3rs^2t^2+3r^2st^2+12r^2s^2t-26rs^2t^3-2rs^3t^2-26r^2st^3+$
$7r^2s^3t-2r^3st^2+7r^3s^2t+9rs^2t^4-7rs^3t^3+9r^2st^4-7r^3st^3+2r^3s^3t+3r^2s^2t^2-$
$10r^2s^2t^3+2r^2s^3t^2+2r^3s^2t^2)-\frac{(x-x_n)^{10}}{5040h^6t^3(r-t)^3(s-t)^3(t-1)^3}(16r^2s^2+4r^2s^3+4r^3s^2+$
$4r^2t^2+19r^2t^3+7r^3t^2-9r^2t^4+7r^3t^3+4s^2t^2+19s^2t^3+7s^3t^2-9s^2t^4+7s^3t^3+$
$4rs^2+4r^2s+4rs^3+4r^3s+7rt^2-8r^2t+19rt^3-8r^3t-36rt^4+7st^2-8s^2t+19st^3-$
$8s^3t-36st^4+7t^3-9t^4+27rst^2-36rs^2t-36r^2st+20rst^3-13rs^3t-13r^3st-$
$36rst^4+27rs^2t^2+27r^2st^2-36r^2s^2t+19rs^2t^3+7rs^3t^2+19r^2st^3-8r^2s^3t+7r^3st^2-$
$8r^3s^2t-13rst+4r^2s^2t^2)+\frac{h^2(x-x_n)^2}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(260r^2s^9-884r^3s^8+546r^4s^7+$
$546r^5s^6+546r^6s^5+546r^7s^4-884r^8s^3+260r^9s^2-260r^2s^{10}+780r^3s^9-364r^4s^8-$
$364r^5s^7-364r^6s^6-364r^7s^5-364r^8s^4+780r^9s^3-260r^{10}s^2+70r^2s^{11}-190r^3s^{10}+$
$70r^4s^9+70r^5s^8+70r^6s^7+70r^7s^6+70r^8s^5+70r^9s^4-190r^{10}s^3+70r^{11}s^2+$
$4550r^2t^9-3887r^3t^8-312r^4t^7-312r^5t^6-312r^6t^5-312r^7t^4+1690r^8t^3-520r^9t^2-$
$10790r^2t^{10}+9009r^3t^9+572r^4t^8+572r^5t^7+572r^6t^6+572r^7t^5-2002r^8t^4-$
$780r^9t^3+520r^{10}t^2+7870r^2t^{11}-6235r^3t^{10}-320r^4t^9-320r^5t^8-320r^6t^7-$
$320r^7t^6-320r^8t^5+2020r^9t^4-385r^{10}t^3-140r^{11}t^2-1890r^2t^{12}+1425r^3t^{11}+$
$60r^4t^{10}+60r^5t^9+60r^6t^8+60r^7t^7+60r^8t^6+60r^9t^5-525r^{10}t^4+210r^{11}t^3+$
$4550s^2t^9-3887s^3t^8-312s^4t^7-312s^5t^6-312s^6t^5-312s^7t^4+1690s^8t^3-520s^9t^2-$
$10790s^2t^{10}+9009s^3t^9+572s^4t^8+572s^5t^7+572s^6t^6+572s^7t^5-2002s^8t^4-$
$780s^9t^3+520s^{10}t^2+7870s^2t^{11}-6235s^3t^{10}-320s^4t^9-320s^5t^8-320s^6t^7-$
$320s^7t^6-320s^8t^5+2020s^9t^4-385s^{10}t^3-140s^{11}t^2-1890s^2t^{12}+1425s^3t^{11}+$
$60s^4t^{10}+60s^5t^9+60s^6t^8+60s^7t^7+60s^8t^6+60s^9t^5-525s^{10}t^4+210s^{11}t^3-$
$1365rt^{10}+3315rt^{11}-2520rt^{12}+630rt^{13}-1365st^{10}+3315st^{11}-2520st^{12}+630$

$st^{13} + 9100rst^9 - 260rs^9t - 260r^9st - 21580rst^{10} + 260rs^{10}t + 260r^{10}st +$
$15740rst^{11} - 70rs^{11}t - 70r^{11}st - 3780rst^{12} - 20475rs^2t^8 + 15704rs^3t^7 + 1404rs^4t^6 +$
$1404rs^5t^5 + 1404rs^6t^4 - 6604rs^7t^3 + 2340rs^8t^2 - 20475r^2st^8 - 234r^2s^8t +$
$15704r^3st^7 + 3770r^3s^7t + 1404r^4st^6 - 4095r^4s^6t + 1404r^5st^5 - 4095r^5s^5t +$
$1404r^6st^4 - 4095r^6s^4t - 6604r^7st^3 + 3770r^7s^3t + 2340r^8st^2 - 234r^8s^2t +$
$47125rs^2t^9 - 34996rs^3t^8 - 2392rs^4t^7 - 2392rs^5t^6 - 2392rs^6t^5 + 7904rs^7t^4 +$
$1976rs^8t^3 - 1820rs^9t^2 + 47125r^2st^9 - 260r^2s^9t - 34996r^3st^8 - 1300r^3s^8t -$
$2392r^4st^7 + 1274r^4s^7t - 2392r^5st^6 + 1274r^5s^6t - 2392r^6st^5 + 1274r^6s^5t +$
$7904r^7st^4 + 1274r^7s^4t + 1976r^8st^3 - 1300r^8s^3t - 1820r^9st^2 - 260r^9s^2t -$
$32495rs^2t^{10} + 22326rs^3t^9 + 1188rs^4t^8 + 1188rs^5t^7 + 1188rs^6t^6 + 1188rs^7t^5 -$
$6768rs^8t^4 + 1695rs^9t^3 + 250rs^{10}t^2 - 32495r^2st^{10} + 445r^2s^{10}t + 22326r^3st^9 -$
$855r^3s^9t + 1188r^4st^8 + 315r^4s^8t + 1188r^5st^7 + 315r^5s^7t + 1188r^6st^6 + 315r^6s^6t +$
$1188r^7st^5 + 315r^7s^5t - 6768r^8st^4 + 315r^8s^4t + 1695r^9st^3 - 855r^9s^3t + 250r^{10}st^2 +$
$445r^{10}s^2t + 7405rs^2t^{11} - 4750rs^3t^{10} - 200rs^4t^9 - 200rs^5t^8 - 200rs^6t^7 - 200rs^7t^6 -$
$200rs^8t^5 + 1555rs^9t^4 - 700rs^{10}t^3 + 70rs^{11}t^2 + 7405r^2st^{11} - 140r^2s^{11}t - 4750r^3st^{10} +$
$380r^3s^{10}t - 200r^4st^9 - 140r^4s^9t - 200r^5st^8 - 140r^5s^8t - 200r^6st^7 - 140r^6s^7t -$
$200r^7st^6 - 140r^7s^6t - 200r^8st^5 - 140r^8s^5t + 1555r^9st^4 - 140r^9s^4t - 700r^{10}st^3 +$
$380r^{10}s^3t + 70r^{11}st^2 - 140r^{11}s^2t + 32032r^2s^2t^7 - 16874r^2s^3t^6 - 1144r^2s^4t^5 -$
$1144r^2s^5t^4 + 5863r^2s^6t^3 - 3068r^2s^7t^2 - 16874r^3s^2t^6 - 5720r^3s^3t^5 - 2860r^3s^4t^4 +$
$4147r^3s^5t^3 - 494r^3s^6t^2 - 1144r^4s^2t^5 - 2860r^4s^3t^4 + 7007r^4s^4t^3 + 2366r^4s^5t^2 -$
$1144r^5s^2t^4 + 4147r^5s^3t^3 + 2366r^5s^4t^2 + 5863r^6s^2t^3 - 494r^6s^3t^2 - 3068r^7s^2t^2 -$
$71136r^2s^2t^8 + 36218r^2s^3t^7 + 1612r^2s^4t^6 + 1612r^2s^5t^5 - 7397r^2s^6t^4 + 260r^2s^7t^3 +$
$3042r^2s^8t^2 + 36218r^3s^2t^7 + 9152r^3s^3t^6 + 4576r^3s^4t^5 - 4433r^3s^5t^4 - 1066r^3s^6t^3 -$
$5434r^3s^7t^2 + 1612r^4s^2t^6 + 4576r^4s^3t^5 - 9009r^4s^4t^4 - 5642r^4s^5t^3 + 5005r^4s^6t^2 +$
$1612r^5s^2t^5 - 4433r^5s^3t^4 - 5642r^5s^4t^3 + 5005r^5s^5t^2 - 7397r^6s^2t^4 - 1066r^6s^3t^3 +$
$5005r^6s^4t^2 + 260r^7s^2t^3 - 5434r^7s^3t^2 + 3042r^8s^2t^2 + 45292r^2s^2t^9 - 20423r^2s^3t^8 -$
$520r^2s^4t^7 - 520r^2s^5t^6 - 520r^2s^6t^5 + 4394r^2s^7t^4 - 2015r^2s^8t^3 - 1250r^2s^9t^2 -$
$20423r^3s^2t^8 - 4056r^3s^3t^7 - 2028r^3s^4t^6 - 2028r^3s^5t^5 + 2886r^3s^6t^4 - 663r^3s^7t^3 +$
$4678r^3s^8t^2 - 520r^4s^2t^7 - 2028r^4s^3t^6 + 4914r^4s^5t^4 + 1365r^4s^6t^3 - 3304r^4s^7t^2 - 520$

$$r^5s^2t^6 - 2028r^5s^3t^5 + 4914r^5s^4t^4 + 1365r^5s^5t^3 - 3304r^5s^6t^2 - 520r^6s^2t^5 + 2886r^6s^3t^4 + 1365r^6s^4t^3 - 3304r^6s^5t^2 + 4394r^7s^2t^4 - 663r^7s^3t^3 - 3304r^7s^4t^2 - 2015r^8s^2t^3 + 4678r^8s^3t^2 - 1250r^9s^2t^2 - 9620r^2s^2t^{10} + 3939r^2s^3t^9 + 52r^2s^4t^8 + 52r^2s^5t^7 + 52r^2s^6t^6 + 52r^2s^7t^5 - 767r^2s^8t^4 + 540r^2s^9t^3 + 205r^2s^{10}t^2 + 3939r^3s^2t^9 + 624r^3s^3t^8 + 312r^3s^4t^7 + 312r^3s^5t^6 + 312r^3s^6t^5 - 507r^3s^7t^4 + 228r^3s^8t^3 - 1095r^3s^9t^2 + 52r^4s^2t^8 + 312r^4s^3t^7 - 819r^4s^6t^4 - 84r^4s^7t^3 + 595r^4s^8t^2 + 52r^5s^2t^7 + 312r^5s^3t^6 - 819r^5s^5t^4 - 84r^5s^6t^3 + 595r^5s^7t^2 + 52r^6s^2t^6 + 312r^6s^3t^5 - 819r^6s^4t^4 - 84r^6s^5t^3 + 595r^6s^6t^2 + 52r^7s^2t^5 - 507r^7s^3t^4 - 84r^7s^4t^3 + 595r^7s^5t^2 - 767r^8s^2t^4 + 228r^8s^3t^3 + 595r^8s^4t^2 + 540r^9s^2t^3 - 1095r^9s^3t^2 + 205r^{10}s^2t^2) + \frac{(x-x_n)^{13}}{8580h^9t^3(r-t)^3(s-t)^3(t-1)^3}(rs - 2rt - 2st + 3rt^2 + 3st^2 + 3t^2 - 4t^3 - 2rst) + \frac{(x-x_n)^{11}}{3960h^7t^3(r-t)^3(s-t)^3(t-1)^3}(4r^2s^2 + 10r^2t^2 + 3r^2t^3 + 3r^3t^2 + 10s^2t^2 + 3s^2t^3 + 3s^3t^2 + rs - 2rt - 2st + 4rs^2 + 4r^2s + rs^3 + r^3s + 10rt^2 - 8r^2t - 2rt^3 - 2r^3t - 9rt^4 + 10st^2 - 8s^2t - 2st^3 - 2s^3t - 9st^4 + 3t^2 + 3t^3 - 9t^4 + 21rst^2 - 15rs^2t - 15r^2st - 2rst^3 - 2rs^3t - 2r^3st + 10rs^2t^2 + 10r^2st^2 - 8r^2s^2t - 15rst) - \frac{r^2s^2(x-x_n)^6}{360h^2t^2(r-t)^3(s-t)^3(t-1)^3}(3rs - 5rt - 5st + 7rt^2 + 7st^2 + 7t^2 - 9t^3 - 5rst) - \frac{h^3rs(x-x_n)}{2162160t^2(r-t)^3(s-t)^3(t-1)^3}(546r^3s^6 - 884r^2s^7 + 546r^4s^5 + 546r^5s^4 + 546r^6s^3 - 884r^7s^2 + 780r^2s^8 - 364r^3s^7 - 364r^4s^6 - 364r^5s^5 - 364r^6s^4 - 364r^7s^3 + 780r^8s^2 - 190r^2s^9 + 70r^3s^8 + 70r^4s^7 + 70r^5s^6 + 70r^6s^5 + 70r^7s^4 + 70r^8s^3 - 190r^9s^2 - 3887r^2t^7 - 312r^3t^6 - 312r^4t^5 - 312r^5t^4 - 312r^6t^3 + 1690r^7t^2 + 9009r^2t^8 + 572r^3t^7 + 572r^4t^6 + 572r^5t^5 + 572r^6t^4 - 2002r^7t^3 - 780r^8t^2 - 6235r^2t^9 - 320r^3t^8 - 320r^4t^7 - 320r^5t^6 - 320r^6t^5 - 320r^7t^4 + 2020r^8t^3 - 385r^9t^2 + 1425r^2t^{10} + 60r^3t^9 + 60r^4t^8 + 60r^5t^7 + 60r^6t^6 + 60r^7t^5 + 60r^8t^4 - 525r^9t^3 + 210r^{10}t^2 - 3887s^2t^7 - 312s^3t^6 - 312s^4t^5 - 312s^5t^4 - 312s^6t^3 + 1690s^7t^2 + 9009s^2t^8 + 572s^3t^7 + 572s^4t^6 + 572s^5t^5 + 572s^6t^4 - 2002s^7t^3 - 780s^8t^2 - 6235s^2t^9 - 320s^3t^8 - 320s^4t^7 - 320s^5t^6 - 320s^6t^5 - 320s^7t^4 + 2020s^8t^3 - 385s^9t^2 + 1425s^2t^{10} + 60s^3t^9 + 60s^4t^8 + 60s^5t^7 + 60s^6t^6 + 60s^7t^5 + 60s^8t^4 - 525s^9t^3 + 210s^{10}t^2 + 260rs^8 + 260r^8s - 260rs^9 - 260r^9s + 70rs^{10} + 70r^{10}s + 4550rt^8 - 520r^8t - 10790rt^9 + 520r^9t + 7870rt^{10} - 140r^{10}t - 1890rt^{11} + 4550st^8 - 520s^8t - 10790st^9 + 520s^9t + 7870st^{10} - 140s^{10}t - 1890st^{11} - 1365t^9 + 3315t^{10} - 2520t^{11} + 630t^{12} - 16588rst^7 + 650rs^7t + 650r^7st + 38116rst^8 - 1040rs^8t - 1040r^8st - 26260rst^9 + 635rs^9t + 635r^9st + 5980rst^{10} - 140rs^{10}t - 140$$

$r^{10}st + 16016rs^2t^6 + 1716rs^3t^5 + 1716rs^4t^4 + 1716rs^5t^3 - 6292rs^6t^2 + 16016r^2st^6 +$
$3224r^2s^6t + 1716r^3st^5 - 4641r^3s^5t + 1716r^4st^4 - 4641r^4s^4t + 1716r^5st^3 -$
$4641r^5s^3t - 6292r^6st^2 + 3224r^6s^2t - 35568rs^2t^7 - 2964rs^3t^6 - 2964rs^4t^5 -$
$2964rs^5t^4 + 7332rs^6t^3 + 3978rs^7t^2 - 35568r^2st^7 - 936r^2s^7t - 2964r^3st^6 +$
$1638r^3s^6t - 2964r^4st^5 + 1638r^4s^5t - 2964r^5st^4 + 1638r^5s^4t + 7332r^6st^3 +$
$1638r^6s^3t + 3978r^7st^2 - 936r^7s^2t + 22646rs^2t^8 + 1508rs^3t^7 + 1508rs^4t^6 +$
$1508rs^5t^5 + 1508rs^6t^4 - 6448rs^7t^3 - 325rs^8t^2 + 22646r^2st^8 - 925r^2s^8t + 1508r^3st^7 +$
$245r^3s^7t + 1508r^4st^6 + 245r^4s^6t + 1508r^5st^5 + 245r^5s^5t + 1508r^6st^4 + 245r^6s^4t -$
$6448r^7st^3 + 245r^7s^3t - 325r^8st^2 - 925r^8s^2t - 4810rs^2t^9 - 260rs^3t^8 - 260rs^4t^7 -$
$260rs^5t^6 - 260rs^6t^5 - 260rs^7t^4 + 1495rs^8t^3 - 175rs^9t^2 - 4810r^2st^9 + 380r^2s^9t -$
$260r^3st^8 - 140r^3s^8t - 260r^4st^7 - 140r^4s^7t - 260r^5st^6 - 140r^5s^6t - 260r^6st^5 -$
$140r^6s^5t - 260r^7st^4 - 140r^7s^4t + 1495r^8st^3 - 140r^8s^3t - 175r^9st^2 + 380r^9s^2t -$
$18590r^2s^2t^5 - 2860r^2s^3t^4 - 2860r^2s^4t^3 + 4147r^2s^5t^2 - 2860r^3s^2t^4 + 7007r^3s^4t^2 -$
$2860r^4s^2t^3 + 7007r^4s^3t^2 + 4147r^5s^2t^2 + 39182r^2s^2t^6 + 4576r^2s^3t^5 + 4576r^2s^4t^4 -$
$4433r^2s^5t^3 - 7072r^2s^6t^2 + 4576r^3s^2t^5 - 9009r^3s^4t^3 + 3367r^3s^5t^2 + 4576r^4s^2t^4 -$
$9009r^4s^3t^3 + 3367r^4s^4t^2 - 4433r^5s^2t^3 + 3367r^5s^3t^2 - 7072r^6s^2t^2 - 21931r^2s^2t^7 -$
$2028r^2s^3t^6 - 2028r^2s^4t^5 - 2028r^2s^5t^4 + 2886r^2s^6t^3 + 4433r^2s^7t^2 - 2028r^3s^2t^6 +$
$4914r^3s^5t^3 - 3549r^3s^6t^2 - 2028r^4s^2t^5 + 4914r^4s^4t^3 - 3549r^4s^5t^2 - 2028r^5s^2t^4 +$
$4914r^5s^3t^3 - 3549r^5s^4t^2 + 2886r^6s^2t^3 - 3549r^6s^3t^2 + 4433r^7s^2t^2 + 4199r^2s^2t^8 +$
$312r^2s^3t^7 + 312r^2s^4t^6 + 312r^2s^5t^5 + 312r^2s^6t^4 - 507r^2s^7t^3 - 955r^2s^8t^2 +$
$312r^3s^2t^7 - 819r^3s^6t^3 + 735r^3s^7t^2 + 312r^4s^2t^6 - 819r^4s^5t^3 + 735r^4s^6t^2 + 312r^5s^2t^5 -$
$819r^5s^4t^3 + 735r^5s^5t^2 + 312r^6s^2t^4 - 819r^6s^3t^3 + 735r^6s^4t^2 - 507r^7s^2t^3 + 735r^7s^3t^2 -$
$955r^8s^2t^2) + \frac{rs(x-x_n)^7}{420h^3t^3(r-t)^3(s-t)^3(t-1)^3}(r^2s^2 - 5r^2t^2 + 7r^2t^3 - 5s^2t^2 + 7s^2t^3 + 7rt^3 -$
$9rt^4 + 7st^3 - 9st^4 - 7rst^2 + rs^2t + r^2st + 17rst^3 - 9rst^4 - 7rs^2t^2 - 7r^2st^2 + r^2s^2t +$
$7rs^2t^3 + 7r^2st^3 - 5r^2s^2t^2),$

$\beta_1 = \frac{h(x-x_n)^3}{2162160(r-1)^3(s-1)^3(t-1)^3}(819r^2s^7 + 819r^3s^6 + 819r^4s^5 + 819r^5s^4 + 819r^6s^3 +$
$819r^7s^2 + 448r^2s^8 - 1554r^3s^7 - 1554r^4s^6 - 1554r^5s^5 - 1554r^6s^4 - 1554r^7s^3 +$
$448r^8s^2 - 1335r^2s^9 + 875r^3s^8 + 875r^4s^7 + 875r^5s^6 + 875r^6s^5 + 875r^7s^4 + 875r^8s^3 -$

$1335r^9s^2+380r^2s^{10}-140r^3s^9-140r^4s^8-140r^5s^7-140r^6s^6-140r^7s^5-140r^8s^4-140r^9s^3+380r^{10}s^2+819r^2t^7+819r^3t^6+819r^4t^5+819r^5t^4+819r^6t^3+819r^7t^2+448r^2t^8-1554r^3t^7-1554r^4t^6-1554r^5t^5-1554r^6t^4-1554r^7t^3+448r^8t^2-1335r^2t^9+875r^3t^8+875r^4t^7+875r^5t^6+875r^6t^5+875r^7t^4+875r^8t^3-1335r^9t^2+380r^2t^{10}-140r^3t^9-140r^4t^8-140r^5t^7-140r^6t^6-140r^7t^5-140r^8t^4-140r^9t^3+380r^{10}t^2+819s^2t^7+819s^3t^6+819s^4t^5+819s^5t^4+819s^6t^3+819s^7t^2+448s^2t^8-1554s^3t^7-1554s^4t^6-1554s^5t^5-1554s^6t^4-1554s^7t^3+448s^8t^2-1335s^2t^9+875s^3t^8+875s^4t^7+875s^5t^6+875s^6t^5+875s^7t^4+875s^8t^3-1335s^9t^2+380s^2t^{10}-140s^3t^9-140s^4t^8-140s^5t^7-140s^6t^6-140s^7t^5-140s^8t^4-140s^9t^3+380s^{10}t^2-1755rs^8-1755r^8s+1670rs^9+1670r^9s-35rs^{10}-35r^{10}s-140rs^{11}-140r^{11}s-1755rt^8-1755r^8t+1670rt^9+1670r^9t-35rt^{10}-35r^{10}t-140rt^{11}-140r^{11}t-1755st^8-1755s^8t+1670st^9+1670s^9t-35st^{10}-35s^{10}t-140st^{11}-140s^{11}t+585r^9-735r^{10}+210r^{11}+585s^9-735s^{10}+210s^{11}+585t^9-735t^{10}+210t^{11}+6201rst^7+6201rs^7t+6201r^7st-4453rst^8-4453rs^8t-4453r^8st-995rst^9-995rs^9t-995r^9st+425rst^{10}+425rs^{10}t+425r^{10}st+70rst^{11}+70rs^{11}t+70r^{11}st-4095rs^2t^6-4095rs^3t^5-4095rs^4t^4-4095rs^5t^3-4095rs^6t^2-4095r^2st^6-4095r^2s^6t-4095r^3st^5-4095r^3s^5t-4095r^4st^4-4095r^4s^4t-4095r^5st^3-4095r^5s^3t-4095r^6st^2-4095r^6s^2t-1099rs^2t^7+6909rs^3t^6+6909rs^4t^5+6909rs^5t^4+6909rs^6t^3-1099rs^7t^2-1099r^2st^7-1099r^2s^7t+6909r^3st^6+6909r^3s^6t+6909r^4st^5+6909r^4s^5t+6909r^5st^4+6909r^5s^4t+6909r^6st^3+6909r^6s^3t-1099r^7st^2-1099r^7s^2t+4023rs^2t^8-2919rs^3t^7-2919rs^4t^6-2919rs^5t^5-2919rs^6t^4-2919rs^7t^3+4023rs^8t^2+4023r^2st^8+4023r^2s^8t-2919r^3st^7-2919r^3s^7t-2919r^4st^6-2919r^4s^6t-2919r^5st^5-2919r^5s^5t-2919r^6st^4-2919r^6s^4t-2919r^7st^3-2919r^7s^3t+4023r^8st^2+4023r^8s^2t-355rs^2t^9+35rs^3t^8+35rs^4t^7+35rs^5t^6+35rs^6t^5+35rs^7t^4+35rs^8t^3-355rs^9t^2-355r^2st^9-355r^2s^9t+35r^3st^8+35r^3s^8t+35r^4st^7+35r^4s^7t+35r^5st^6+35r^5s^6t+35r^6st^5+35r^6s^5t+35r^7st^4+35r^7s^4t+35r^8st^3+35r^8s^3t-355r^9st^2-355r^9s^2t-190rs^2t^{10}+70rs^3t^9+70rs^4t^8+70rs^5t^7+70rs^6t^6+70rs^7t^5+70rs^8t^4+70rs^9t^3-190rs^{10}t^2-190r^2st^{10}-190r^2s^{10}t+70r^3st^9+70r^3s^9t+70r^4st^8+70r^4s^8t+70r^5st^7+70r^5s^7t+$

$$70r^6st^6 + 70r^6s^6t + 70r^7st^5 + 70r^7s^5t + 70r^8st^4 + 70r^8s^4t + 70r^9st^3 + 70r^9s^3t -$$
$$190r^{10}st^2 - 190r^{10}s^2t + 4914r^2s^2t^5 + 4914r^2s^3t^4 + 4914r^2s^4t^3 + 4914r^2s^5t^2 +$$
$$4914r^3s^2t^4 + 4914r^3s^3t^3 + 4914r^3s^4t^2 + 4914r^4s^2t^3 + 4914r^4s^3t^2 + 4914r^5s^2t^2 +$$
$$1540r^2s^2t^6 - 5467r^2s^3t^5 - 5467r^2s^4t^4 - 5467r^2s^5t^3 + 1540r^2s^6t^2 - 5467r^3s^2t^5 -$$
$$12474r^3s^3t^4 - 12474r^3s^4t^3 - 5467r^3s^5t^2 - 5467r^4s^2t^4 - 12474r^4s^3t^3 - 5467r^4s^4t^2 -$$
$$5467r^5s^2t^3 - 5467r^5s^3t^2 + 1540r^6s^2t^2 - 2113r^2s^2t^7 - 1190r^2s^3t^6 - 1190r^2s^4t^5 -$$
$$1190r^2s^5t^4 - 1190r^2s^6t^3 - 2113r^2s^7t^2 - 1190r^3s^2t^6 + 10458r^3s^3t^5 + 10458r^3s^4t^4 +$$
$$10458r^3s^5t^3 - 1190r^3s^6t^2 - 1190r^4s^2t^5 + 10458r^4s^3t^4 + 10458r^4s^4t^3 - 1190r^4s^5t^2 -$$
$$1190r^5s^2t^4 + 10458r^5s^3t^3 - 1190r^5s^4t^2 - 1190r^6s^2t^3 - 1190r^6s^3t^2 - 2113r^7s^2t^2 -$$
$$2071r^2s^2t^8 + 2037r^2s^3t^7 + 2037r^2s^4t^6 + 2037r^2s^5t^5 + 2037r^2s^6t^4 + 2037r^2s^7t^3 -$$
$$2071r^2s^8t^2 + 2037r^3s^2t^7 - 3150r^3s^3t^6 - 3150r^3s^4t^5 - 3150r^3s^5t^4 - 3150r^3s^6t^3 +$$
$$2037r^3s^7t^2 + 2037r^4s^2t^6 - 3150r^4s^3t^5 - 3150r^4s^4t^4 - 3150r^4s^5t^3 + 2037r^4s^6t^2 +$$
$$2037r^5s^2t^5 - 3150r^5s^3t^4 - 3150r^5s^4t^3 + 2037r^5s^5t^2 + 2037r^6s^2t^4 - 3150r^6s^3t^3 +$$
$$2037r^6s^4t^2 + 2037r^7s^2t^3 + 2037r^7s^3t^2 - 2071r^8s^2t^2 + 590r^2s^2t^9 - 294r^2s^3t^8 -$$
$$294r^2s^4t^7 - 294r^2s^5t^6 - 294r^2s^6t^5 - 294r^2s^7t^4 - 294r^2s^8t^3 + 590r^2s^9t^2 - 294r^3s^2t^8 +$$
$$252r^3s^3t^7 + 252r^3s^4t^6 + 252r^3s^5t^5 + 252r^3s^6t^4 + 252r^3s^7t^3 - 294r^3s^8t^2 - 294r^4s^2t^7 +$$
$$252r^4s^3t^6 + 252r^4s^4t^5 + 252r^4s^5t^4 + 252r^4s^6t^3 - 294r^4s^7t^2 - 294r^5s^2t^6 + 252r^5s^3t^5 +$$
$$252r^5s^4t^4 + 252r^5s^5t^3 - 294r^5s^6t^2 - 294r^6s^2t^5 + 252r^6s^3t^4 + 252r^6s^4t^3 - 294r^6s^5t^2 -$$
$$294r^7s^2t^4 + 252r^7s^3t^3 - 294r^7s^4t^2 - 294r^8s^2t^3 - 294r^8s^3t^2 + 590r^9s^2t^2) -$$
$$\frac{(x-x_n)^{11}}{3960h^7(r-1)^3(s-1)^3(t-1)^3}(10rs^2 - 9s - 9t - 8r^2s^2 - 8r^2t^2 - 8s^2t^2 - 2rs - 2rt - 2st - 9r +$$
$$10r^2s - 2rs^3 - 2r^3s + 10rt^2 + 10r^2t - 2rt^3 - 2r^3t + 10st^2 + 10s^2t - 2st^3 - 2s^3t + 3r^2 +$$
$$3r^3 + 3s^2 + 3s^3 + 3t^2 + 3t^3 - 15rst^2 - 15rs^2t - 15r^2st + rst^3 + rs^3t + r^3st + 4rs^2t^2 +$$
$$4r^2st^2 + 4r^2s^2t + 21rst) + \frac{h^2(x-x_n)^2}{2162160(r-1)^3(s-1)^3(t-1)^3}(1755r^2s^8 - 819r^3s^7 - 819r^4s^6 -$$
$$819r^5s^5 - 819r^6s^4 - 819r^7s^3 + 1755r^8s^2 - 1670r^2s^9 - 448r^3s^8 + 1554r^4s^7 +$$
$$1554r^5s^6 + 1554r^6s^5 + 1554r^7s^4 - 448r^8s^3 - 1670r^9s^2 + 35r^2s^{10} + 1335r^3s^9 -$$
$$875r^4s^8 - 875r^5s^7 - 875r^6s^6 - 875r^7s^5 - 875r^8s^4 + 1335r^9s^3 + 35r^{10}s^2 + 140r^2s^{11} -$$
$$380r^3s^{10} + 140r^4s^9 + 140r^5s^8 + 140r^6s^7 + 140r^7s^6 + 140r^8s^5 + 140r^9s^4 - 380r^{10}s^3 +$$
$$140r^{11}s^2 + 1755r^2t^8 - 819r^3t^7 - 819r^4t^6 - 819r^5t^5 - 819r^6t^4 - 819r^7t^3 + 1755r^8t^2 -$$
$$1670r^2t^9 - 448r^3t^8 + 1554r^4t^7 + 1554r^5t^6 + 1554r^6t^5 + 1554r^7t^4 - 448r^8t^3 - 1670r^9$$

$$
\begin{aligned}
&t^2 + 35r^2t^{10} + 1335r^3t^9 - 875r^4t^8 - 875r^5t^7 - 875r^6t^6 - 875r^7t^5 - 875r^8t^4 + \\
&1335r^9t^3 + 35r^{10}t^2 + 140r^2t^{11} - 380r^3t^{10} + 140r^4t^9 + 140r^5t^8 + 140r^6t^7 + 140r^7t^6 + \\
&140r^8t^5 + 140r^9t^4 - 380r^{10}t^3 + 140r^{11}t^2 + 1755s^2t^8 - 819s^3t^7 - 819s^4t^6 - 819s^5t^5 - \\
&819s^6t^4 - 819s^7t^3 + 1755s^8t^2 - 1670s^2t^9 - 448s^3t^8 + 1554s^4t^7 + 1554s^5t^6 + \\
&1554s^6t^5 + 1554s^7t^4 - 448s^8t^3 - 1670s^9t^2 + 35s^2t^{10} + 1335s^3t^9 - 875s^4t^8 - 875s^5t^7 - \\
&875s^6t^6 - 875s^7t^5 - 875s^8t^4 + 1335s^9t^3 + 35s^{10}t^2 + 140s^2t^{11} - 380s^3t^{10} + 140s^4t^9 + \\
&140s^5t^8 + 140s^6t^7 + 140s^7t^6 + 140s^8t^5 + 140s^9t^4 - 380s^{10}t^3 + 140s^{11}t^2 - 585rs^9 - \\
&585r^9s + 735rs^{10} + 735r^{10}s - 210rs^{11} - 210r^{11}s - 585rt^9 - 585r^9t + 735rt^{10} + \\
&735r^{10}t - 210rt^{11} - 210r^{11}t - 585st^9 - 585s^9t + 735st^{10} + 735s^{10}t - 210st^{11} - \\
&210s^{11}t + 3510rst^8 + 3510rs^8t + 3510r^8st - 3340rst^9 - 3340rs^9t - 3340r^9st + \\
&70rst^{10} + 70rs^{10}t + 70r^{10}st + 280rst^{11} + 280rs^{11}t + 280r^{11}st - 7020rs^2t^7 + \\
&3276rs^3t^6 + 3276rs^4t^5 + 3276rs^5t^4 + 3276rs^6t^3 - 7020rs^7t^2 - 7020r^2st^7 - \\
&7020r^2s^7t + 3276r^3st^6 + 3276r^3s^6t + 3276r^4st^5 + 3276r^4s^5t + 3276r^5st^4 + \\
&3276r^5s^4t + 3276r^6st^3 + 3276r^6s^3t - 7020r^7st^2 - 7020r^7s^2t + 4005rs^2t^8 + \\
&2653rs^3t^7 - 5355rs^4t^6 - 5355rs^5t^5 - 5355rs^6t^4 + 2653rs^7t^3 + 4005rs^8t^2 + \\
&4005r^2st^8 + 4005r^2s^8t + 2653r^3st^7 + 2653r^3s^7t - 5355r^4st^6 - 5355r^4s^6t - \\
&5355r^5st^5 - 5355r^5s^5t - 5355r^6st^4 - 5355r^6s^4t + 2653r^7st^3 + 2653r^7s^3t + \\
&4005r^8st^2 + 4005r^8s^2t + 2330rs^2t^9 - 4898rs^3t^8 + 2044rs^4t^7 + 2044rs^5t^6 + \\
&2044rs^6t^5 + 2044rs^7t^4 - 4898rs^8t^3 + 2330rs^9t^2 + 2330r^2st^9 + 2330r^2s^9t - \\
&4898r^3st^8 - 4898r^3s^8t + 2044r^4st^7 + 2044r^4s^7t + 2044r^5st^6 + 2044r^5s^6t + \\
&2044r^6st^5 + 2044r^6s^5t + 2044r^7st^4 + 2044r^7s^4t - 4898r^8st^3 - 4898r^8s^3t + \\
&2330r^9st^2 + 2330r^9s^2t - 805rs^2t^{10} + 495rs^3t^9 + 105rs^4t^8 + 105rs^5t^7 + 105rs^6t^6 + \\
&105rs^7t^5 + 105rs^8t^4 + 495rs^9t^3 - 805rs^{10}t^2 - 805r^2st^{10} - 805r^2s^{10}t + 495r^3st^9 + \\
&495r^3s^9t + 105r^4st^8 + 105r^4s^8t + 105r^5st^7 + 105r^5s^7t + 105r^6st^6 + 105r^6s^6t + \\
&105r^7st^5 + 105r^7s^5t + 105r^8st^4 + 105r^8s^4t + 495r^9st^3 + 495r^9s^3t - 805r^{10}st^2 - \\
&805r^{10}s^2t - 70rs^2t^{11} + 190rs^3t^{10} - 70rs^4t^9 - 70rs^5t^8 - 70rs^6t^7 - 70rs^7t^6 - 70rs^8t^5 - \\
&70rs^9t^4 + 190rs^{10}t^3 - 70rs^{11}t^2 - 70r^2st^{11} - 70r^2s^{11}t + 190r^3st^{10} + 190r^3s^{10}t - \\
&70r^4st^9 - 70r^4s^9t - 70r^5st^8 - 70r^5s^8t - 70r^6st^7 - 70r^6s^7t - 70r^7st^6 - 70r^7s^6t - \\
&70r^8st^5 - 70r^8s^5t - 70r^9st^4 - 70r^9s^4t + 190r^{10}st^3 + 190r^{10}s^3t - 70r^{11}st^2 - 70r^{11}s^2t +
\end{aligned}
$$

$8190r^2s^2t^6 - 819r^2s^3t^5 - 819r^2s^4t^4 - 819r^2s^5t^3 + 8190r^2s^6t^2 - 819r^3s^2t^5 -$
$9828r^3s^3t^4 - 9828r^3s^4t^3 - 819r^3s^5t^2 - 819r^4s^2t^4 - 9828r^4s^3t^3 - 819r^4s^4t^2 -$
$819r^5s^2t^3 - 819r^5s^3t^2 + 8190r^6s^2t^2 + 2198r^2s^2t^7 - 8449r^2s^3t^6 - 1442r^2s^4t^5 -$
$1442r^2s^5t^4 - 8449r^2s^6t^3 + 2198r^2s^7t^2 - 8449r^3s^2t^6 + 10934r^3s^3t^5 + 17941r^3s^4t^4 +$
$10934r^3s^5t^3 - 8449r^3s^6t^2 - 1442r^4s^2t^5 + 17941r^4s^3t^4 + 17941r^4s^4t^3 - 1442r^4s^5t^2 -$
$1442r^5s^2t^4 + 10934r^5s^3t^3 - 1442r^5s^4t^2 - 8449r^6s^2t^3 - 8449r^6s^3t^2 + 2198r^7s^2t^2 -$
$8046r^2s^2t^8 + 5032r^2s^3t^7 + 4109r^2s^4t^6 + 4109r^2s^5t^5 + 4109r^2s^6t^4 + 5032r^2s^7t^3 -$
$8046r^2s^8t^2 + 5032r^3s^2t^7 + 2380r^3s^3t^6 - 9268r^3s^4t^5 - 9268r^3s^5t^4 + 2380r^3s^6t^3 +$
$5032r^3s^7t^2 + 4109r^4s^2t^6 - 9268r^4s^3t^5 - 20916r^4s^4t^4 - 9268r^4s^5t^3 + 4109r^4s^6t^2 +$
$4109r^5s^2t^5 - 9268r^5s^3t^4 - 9268r^5s^4t^3 + 4109r^5s^5t^2 + 4109r^6s^2t^4 + 2380r^6s^3t^3 +$
$4109r^6s^4t^2 + 5032r^7s^2t^3 + 5032r^7s^3t^2 - 8046r^8s^2t^2 + 710r^2s^2t^9 + 2036r^2s^3t^8 -$
$2072r^2s^4t^7 - 2072r^2s^5t^6 - 2072r^2s^6t^5 - 2072r^2s^7t^4 + 2036r^2s^8t^3 + 710r^2s^9t^2 +$
$2036r^3s^2t^8 - 4074r^3s^3t^7 + 1113r^3s^4t^6 + 1113r^3s^5t^5 + 1113r^3s^6t^4 - 4074r^3s^7t^3 +$
$2036r^3s^8t^2 - 2072r^4s^2t^7 + 1113r^4s^3t^6 + 6300r^4s^4t^5 + 6300r^4s^5t^4 + 1113r^4s^6t^3 -$
$2072r^4s^7t^2 - 2072r^5s^2t^6 + 1113r^5s^3t^5 + 6300r^5s^4t^4 + 1113r^5s^5t^3 - 2072r^5s^6t^2 -$
$2072r^6s^2t^5 + 1113r^6s^3t^4 + 1113r^6s^4t^3 - 2072r^6s^5t^2 - 2072r^7s^2t^4 - 4074r^7s^3t^3 -$
$2072r^7s^4t^2 + 2036r^8s^2t^3 + 2036r^8s^3t^2 + 710r^9s^2t^2 + 380r^2s^2t^{10} - 660r^2s^3t^9 +$
$224r^2s^4t^8 + 224r^2s^5t^7 + 224r^2s^6t^6 + 224r^2s^7t^5 + 224r^2s^8t^4 - 660r^2s^9t^3 +$
$380r^2s^{10}t^2 - 660r^3s^2t^9 + 588r^3s^3t^8 + 42r^3s^4t^7 + 42r^3s^5t^6 + 42r^3s^6t^5 + 42r^3s^7t^4 +$
$588r^3s^8t^3 - 660r^3s^9t^2 + 224r^4s^2t^8 + 42r^4s^3t^7 - 504r^4s^4t^6 - 504r^4s^5t^5 - 504r^4s^6t^4 +$
$42r^4s^7t^3 + 224r^4s^8t^2 + 224r^5s^2t^7 + 42r^5s^3t^6 - 504r^5s^4t^5 - 504r^5s^5t^4 + 42r^5s^6t^3 +$
$224r^5s^7t^2 + 224r^6s^2t^6 + 42r^6s^3t^5 - 504r^6s^4t^4 + 42r^6s^5t^3 + 224r^6s^6t^2 + 224r^7s^2t^5 +$
$42r^7s^3t^4 + 42r^7s^4t^3 + 224r^7s^5t^2 + 224r^8s^2t^4 + 588r^8s^3t^3 + 224r^8s^4t^2 - 660r^9s^2t^3 -$
$660r^9s^3t^2 + 380r^{10}s^2t^2) - \frac{(x-x_n)^{13}}{8580h^9(r-1)^3(s-1)^3(t-1)^3}(3r + 3s + 3t - 2rs - 2rt - 2st +$
$rst - 4) + \frac{(x-x_n)^9}{1512h^5(r-1)^3(s-1)^3(t-1)^3}(2r^2s^3 - 10r^2s^2 + 2r^3s^2 + 2r^3s^3 - 10r^2t^2 + 2r^2t^3 +$
$2r^3t^2 + 2r^3t^3 - 10s^2t^2 + 2s^2t^3 + 2s^3t^2 + 2s^3t^3 + 9rs^2 + 9r^2s - 7rs^3 - 7r^3s + 9rt^2 +$
$9r^2t - 7rt^3 - 7r^3t + 9st^2 + 9s^2t - 7st^3 - 7s^3t - 26rst^2 - 26rs^2t - 26r^2st - 2rst^3 -$
$2rs^3t - 2r^3st + 3rs^2t^2 + 3r^2st^2 + 3r^2s^2t + 7rs^2t^3 + 7rs^3t^2 + 7r^2st^3 + 7r^2s^3t + 7r^3st^2 +$
$7r^3s^2t - rs^3t^3 - r^3st^3 - r^3s^3t + 36rst + 12r^2s^2t^2 - 4r^2s^2t^3 - 4r^2s^3t^2 - 4r^3s^2t^2)+$

$$\frac{(x-x_n)^{10}}{5040h^6(r-1)^3(s-1)^3(t-1)^3}(4r^2s^2 - 8r^2s^3 - 8r^3s^2 + 4r^2t^2 - 8r^2t^3 - 8r^3t^2 + 4s^2t^2 -$$
$$8s^2t^3 - 8s^3t^2 - 36rs - 36rt - 36st + 19rs^2 + 19r^2s + 7rs^3 + 7r^3s + 19rt^2 +$$
$$19r^2t + 7rt^3 + 7r^3t + 19st^2 + 19s^2t + 7st^3 + 7s^3t - 9r^2 + 7r^3 - 9s^2 + 7s^3 - 9t^2 +$$
$$7t^3 + 27rst^2 + 27rs^2t + 27r^2st - 13rst^3 - 13rs^3t - 13r^3st - 36rs^2t^2 - 36r^2st^2 -$$
$$36r^2s^2t + 4rs^2t^3 + 4rs^3t^2 + 4r^2st^3 + 4r^2s^3t + 4r^3st^2 + 4r^3s^2t + 20rst + 16r^2s^2t^2) -$$
$$\frac{(x-x_n)^8}{1680h^4(r-1)^3(s-1)^3(t-1)^3}(9r^2s^2 - 7r^2s^3 - 7r^3s^2 + 5r^3s^3 + 9r^2t^2 - 7r^2t^3 - 7r^3t^2 +$$
$$5r^3t^3 + 9s^2t^2 - 7s^2t^3 - 7s^3t^2 + 5s^3t^3 + 36rst^2 + 36rs^2t + 36r^2st - 28rst^3 - 28rs^3t -$$
$$28r^3st - 47rs^2t^2 - 47r^2st^2 - 47r^2s^2t + 13rs^2t^3 + 13rs^3t^2 + 13r^2st^3 + 13r^2s^3t +$$
$$13r^3st^2 + 13r^3s^2t + 5rs^3t^3 + 5r^3st^3 + 5r^3s^3t + 24r^2s^2t^2 + 4r^2s^2t^3 + 4r^2s^3t^2 + 4r^3s^2t^2 -$$
$$4r^2s^3t^3 - 4r^3s^2t^3 - 4r^3s^3t^2) - \frac{(x-x_n)^{12}}{11880h^8(r-1)^3(s-1)^3(t-1)^3}(9r + 9s + 9t - 19rs - 19rt -$$
$$19st + 8rs^2 + 8r^2s + 8rt^2 + 8r^2t + 8st^2 + 8s^2t - 12r^2 - 12s^2 - 12t^2 - 4rst^2 -$$
$$4rs^2t - 4r^2st + 21rst + 9) + \frac{h^3rst(x-x_n)}{2162160(r-1)^3(s-1)^3(t-1)^3}(819r^2s^6 + 819r^3s^5 + 819r^4s^4 +$$
$$819r^5s^3 + 819r^6s^2 + 448r^2s^7 - 1554r^3s^6 - 1554r^4s^5 - 1554r^5s^4 - 1554r^6s^3 +$$
$$448r^7s^2 - 1335r^2s^8 + 875r^3s^7 + 875r^4s^6 + 875r^5s^5 + 875r^6s^4 + 875r^7s^3 - 1335r^8s^2 +$$
$$380r^2s^9 - 140r^3s^8 - 140r^4s^7 - 140r^5s^6 - 140r^6s^5 - 140r^7s^4 - 140r^8s^3 + 380r^9s^2 +$$
$$819r^2t^6 + 819r^3t^5 + 819r^4t^4 + 819r^5t^3 + 819r^6t^2 + 448r^2t^7 - 1554r^3t^6 - 1554r^4t^5 -$$
$$1554r^5t^4 - 1554r^6t^3 + 448r^7t^2 - 1335r^2t^8 + 875r^3t^7 + 875r^4t^6 + 875r^5t^5 + 875r^6t^4 +$$
$$875r^7t^3 - 1335r^8t^2 + 380r^2t^9 - 140r^3t^8 - 140r^4t^7 - 140r^5t^6 - 140r^6t^5 - 140r^7t^4 -$$
$$140r^8t^3 + 380r^9t^2 + 819s^2t^6 + 819s^3t^5 + 819s^4t^4 + 819s^5t^3 + 819s^6t^2 + 448s^2t^7 -$$
$$1554s^3t^6 - 1554s^4t^5 - 1554s^5t^4 - 1554s^6t^3 + 448s^7t^2 - 1335s^2t^8 + 875s^3t^7 +$$
$$875s^4t^6 + 875s^5t^5 + 875s^6t^4 + 875s^7t^3 - 1335s^8t^2 + 380s^2t^9 - 140s^3t^8 - 140s^4t^7 -$$
$$140s^5t^6 - 140s^6t^5 - 140s^7t^4 - 140s^8t^3 + 380s^9t^2 - 1755rs^7 - 1755r^7s + 1670rs^8 +$$
$$1670r^8s - 35rs^9 - 35r^9s - 140rs^{10} - 140r^{10}s - 1755rt^7 - 1755r^7t + 1670rt^8 +$$
$$1670r^8t - 35rt^9 - 35r^9t - 140rt^{10} - 140r^{10}t - 1755st^7 - 1755s^7t + 1670st^8 +$$
$$1670s^8t - 35st^9 - 35s^9t - 140st^{10} - 140s^{10}t + 585r^8 - 735r^9 + 210r^{10} + 585s^8 -$$
$$735s^9 + 210s^{10} + 585t^8 - 735t^9 + 210t^{10} + 6201rst^6 + 6201rs^6t + 6201r^6st -$$
$$4453rst^7 - 4453rs^7t - 4453r^7st - 995rst^8 - 995rs^8t - 995r^8st + 425rst^9 + 425rs^9t +$$
$$425r^9st + 70rst^{10} + 70rs^{10}t + 70r^{10}st - 4095rs^2t^5 - 4095rs^3t^4 - 4095rs^4t^3 -$$
$$4095rs^5t^2 - 4095r^2st^5 - 4095r^2s^5t - 4095r^3st^4 - 4095r^3s^4t - 4095r^4st^3 - 4095r^4s^3t$$

$$-4095r^5st^2 - 4095r^5s^2t - 1099rs^2t^6 + 6909rs^3t^5 + 6909rs^4t^4 + 6909rs^5t^3 -$$
$$1099rs^6t^2 - 1099r^2st^6 - 1099r^2s^6t + 6909r^3st^5 + 6909r^3s^5t + 6909r^4st^4 +$$
$$6909r^4s^4t + 6909r^5st^3 + 6909r^5s^3t - 1099r^6st^2 - 1099r^6s^2t + 4023rs^2t^7 -$$
$$2919rs^3t^6 - 2919rs^4t^5 - 2919rs^5t^4 - 2919rs^6t^3 + 4023rs^7t^2 + 4023r^2st^7 +$$
$$4023r^2s^7t - 2919r^3st^6 - 2919r^3s^6t - 2919r^4st^5 - 2919r^4s^5t - 2919r^5st^4 -$$
$$2919r^5s^4t - 2919r^6st^3 - 2919r^6s^3t + 4023r^7st^2 + 4023r^7s^2t - 355rs^2t^8 +$$
$$35rs^3t^7 + 35rs^4t^6 + 35rs^5t^5 + 35rs^6t^4 + 35rs^7t^3 - 355rs^8t^2 - 355r^2st^8 - 355r^2s^8t +$$
$$35r^3st^7 + 35r^3s^7t + 35r^4st^6 + 35r^4s^6t + 35r^5st^5 + 35r^5s^5t + 35r^6st^4 + 35r^6s^4t +$$
$$35r^7st^3 + 35r^7s^3t - 355r^8st^2 - 355r^8s^2t - 190rs^2t^9 + 70rs^3t^8 + 70rs^4t^7 + 70rs^5t^6 +$$
$$70rs^6t^5 + 70rs^7t^4 + 70rs^8t^3 - 190rs^9t^2 - 190r^2st^9 - 190r^2s^9t + 70r^3st^8 + 70r^3s^8t +$$
$$70r^4st^7 + 70r^4s^7t + 70r^5st^6 + 70r^5s^6t + 70r^6st^5 + 70r^6s^5t + 70r^7st^4 + 70r^7s^4t +$$
$$70r^8st^3 + 70r^8s^3t - 190r^9st^2 - 190r^9s^2t + 4914r^2s^2t^4 + 4914r^2s^3t^3 + 4914r^2s^4t^2 +$$
$$4914r^3s^2t^3 + 4914r^3s^3t^2 + 4914r^4s^2t^2 + 1540r^2s^2t^5 - 5467r^2s^3t^4 - 5467r^2s^4t^3 +$$
$$1540r^2s^5t^2 - 5467r^3s^2t^4 - 12474r^3s^3t^3 - 5467r^3s^4t^2 - 5467r^4s^2t^3 - 5467r^4s^3t^2 +$$
$$1540r^5s^2t^2 - 2113r^2s^2t^6 - 1190r^2s^3t^5 - 1190r^2s^4t^4 - 1190r^2s^5t^3 - 2113r^2s^6t^2 -$$
$$1190r^3s^2t^5 + 10458r^3s^3t^4 + 10458r^3s^4t^3 - 1190r^3s^5t^2 - 1190r^4s^2t^4 + 10458r^4s^3t^3 -$$
$$1190r^4s^4t^2 - 1190r^5s^2t^3 - 1190r^5s^3t^2 - 2113r^6s^2t^2 - 2071r^2s^2t^7 + 2037r^2s^3t^6 +$$
$$2037r^2s^4t^5 + 2037r^2s^5t^4 + 2037r^2s^6t^3 - 2071r^2s^7t^2 + 2037r^3s^2t^6 - 3150r^3s^3t^5 -$$
$$3150r^3s^4t^4 - 3150r^3s^5t^3 + 2037r^3s^6t^2 + 2037r^4s^2t^5 - 3150r^4s^3t^4 - 3150r^4s^4t^3 +$$
$$2037r^4s^5t^2 + 2037r^5s^2t^4 - 3150r^5s^3t^3 + 2037r^5s^4t^2 + 2037r^6s^2t^3 + 2037r^6s^3t^2 -$$
$$2071r^7s^2t^2 + 590r^2s^2t^8 - 294r^2s^3t^7 - 294r^2s^4t^6 - 294r^2s^5t^5 - 294r^2s^6t^4 -$$
$$294r^2s^7t^3 + 590r^2s^8t^2 - 294r^3s^2t^7 + 252r^3s^3t^6 + 252r^3s^4t^5 + 252r^3s^5t^4 + 252r^3s^6t^3 -$$
$$294r^3s^7t^2 - 294r^4s^2t^6 + 252r^4s^3t^5 + 252r^4s^4t^4 + 252r^4s^5t^3 - 294r^4s^6t^2 - 294r^5s^2t^5 +$$
$$252r^5s^3t^4 + 252r^5s^4t^3 - 294r^5s^5t^2 - 294r^6s^2t^4 + 252r^6s^3t^3 - 294r^6s^4t^2 - 294r^7s^2t^3 -$$
$$294r^7s^3t^2 + 590r^8s^2t^2) - \frac{rst(x-x_n)^7}{420h^3(r-1)^3(s-1)^3(t-1)^3}(7rs^2 - 5r^2t^2 - 5s^2t^2 - 9rs - 9rt -$$
$$9st - 5r^2s^2 + 7r^2s + 7rt^2 + 7r^2t + 7st^2 + 7s^2t - 7rst^2 - 7rs^2t - 7r^2st + rs^2t^2 + r^2st^2 +$$
$$r^2s^2t + 17rst + r^2s^2t^2) + \frac{r^2s^2t^2(x-x_n)^6}{360h^2(r-1)^3(s-1)^3(t-1)^3}(7r + 7s + 7t - 5rs - 5rt - 5st + 3rst -$$
$$9),$$

$$\gamma_0 = \frac{(x-x_n)^5}{120} + \frac{(x-x_n)^{13}}{17160h^8r^2s^2t^2} - \frac{h^2(x-x_n)^3}{2162160r^2s^2t^2}(35r^{10} - 95r^9s - 95r^9t - 130r^9 + 35r^8s^2 + 295r^8st + 390r^8s + 35r^8t^2 + 390r^8t + 130r^8 + 35r^7s^3 - 147r^7s^2t - 182r^7s^2 - 147r^7st^2 - 1378r^7st - 442r^7s + 35r^7t^3 - 182r^7t^2 - 442r^7t + 35r^6s^4 - 147r^6s^3t - 182r^6s^3 + 126r^6s^2t^2 + 910r^6s^2t + 273r^6s^2 - 147r^6st^3 + 910r^6st^2 + 1846r^6st + 35r^6t^4 - 182r^6t^3 + 273r^6t^2 + 35r^5s^5 - 147r^5s^4t - 182r^5s^4 + 126r^5s^3t^2 + 910r^5s^3t + 273r^5s^3 + 126r^5s^2t^3 - 1092r^5s^2t^2 - 1729r^5s^2t - 147r^5st^4 + 910r^5st^3 - 1729r^5st^2 + 35r^5t^5 - 182r^5t^4 + 273r^5t^3 + 35r^4s^6 - 147r^4s^5t - 182r^4s^5 + 126r^4s^4t^2 + 910r^4s^4t + 273r^4s^4 + 126r^4s^3t^3 - 1092r^4s^3t^2 - 1729r^4s^3t + 126r^4s^2t^4 - 1092r^4s^2t^3 + 3276r^4s^2t^2 - 147r^4st^5 + 910r^4st^4 - 1729r^4st^3 + 35r^4t^6 - 182r^4t^5 + 273r^4t^4 + 35r^3s^7 - 147r^3s^6t - 182r^3s^6 + 126r^3s^5t^2 + 910r^3s^5t + 273r^3s^5 + 126r^3s^4t^3 - 1092r^3s^4t^2 - 1729r^3s^4t + 126r^3s^3t^4 - 1092r^3s^3t^3 + 3276r^3s^3t^2 + 126r^3s^2t^5 - 1092r^3s^2t^4 + 3276r^3s^2t^3 - 147r^3st^6 + 910r^3st^5 - 1729r^3st^4 + 35r^3t^7 - 182r^3t^6 + 273r^3t^5 + 35r^2s^8 - 147r^2s^7t - 182r^2s^7 + 126r^2s^6t^2 + 910r^2s^6t + 273r^2s^6 + 126r^2s^5t^3 - 1092r^2s^5t^2 - 1729r^2s^5t + 126r^2s^4t^4 - 1092r^2s^4t^3 + 3276r^2s^4t^2 + 126r^2s^3t^5 - 1092r^2s^3t^4 + 3276r^2s^3t^3 + 126r^2s^2t^6 - 1092r^2s^2t^5 + 3276r^2s^2t^4 - 147r^2st^7 + 910r^2st^6 - 1729r^2st^5 + 35r^2t^8 - 182r^2t^7 + 273r^2t^6 - 95rs^9 + 295rs^8t + 390rs^8 - 147rs^7t^2 - 1378rs^7t - 442rs^7 - 147rs^6t^3 + 910rs^6t^2 + 1846rs^6t - 147rs^5t^4 + 910rs^5t^3 - 1729rs^5t^2 - 147rs^4t^5 + 910rs^4t^4 - 1729rs^4t^3 - 147rs^3t^6 + 910rs^3t^5 - 1729rs^3t^4 - 147rs^2t^7 + 910rs^2t^6 - 1729rs^2t^5 + 295rst^8 - 1378rst^7 + 1846rst^6 - 95rt^9 + 390rt^8 - 442rt^7 + 35s^{10} - 95s^9t - 130s^9 + 35s^8t^2 + 390s^8t + 130s^8 + 35s^7t^3 - 182s^7t^2 - 442s^7t + 35s^6t^4 - 182s^6t^3 + 273s^6t^2 + 35s^5t^5 - 182s^5t^4 + 273s^5t^3 + 35s^4t^6 - 182s^4t^5 + 273s^4t^4 + 35s^3t^7 - 182s^3t^6 + 273s^3t^5 + 35s^2t^8 - 182s^2t^7 + 273s^2t^6 - 95st^9 + 390st^8 - 442st^7 + 35t^{10} - 130t^9 + 130t^8) + \frac{(x-x_n)^9}{3024h^4r^2s^2t^2}(r^2s^2 + 4r^2st + 4r^2s + r^2t^2 + 4r^2t + r^2 + 4rs^2t + 4rs^2 + 4rst^2 + 16rst + 4rs + 4rt^2 + 4rt + s^2t^2 + 4s^2t + s^2 + 4st^2 + 4st + t^2) - \frac{(x-x_n)^{10}}{2520h^5r^2s^2t^2}(r^2s + r^2t + r^2 + rs^2 + 4rst + 4rs + rt^2 + 4rt + r + s^2t + s^2 + st^2 + 4st + s + t^2 + t) - \frac{h^3(x-x_n)^2}{2162160r^2s^2t^2}(95r^9s^2 - 35r^{10}t - 35r^{10}s + 190r^9st + 130r^9s + 95r^9t^2 + 130r^9t - 35r^8s^3 - 330r^8s^2t - 390r^8s^2 - 330r^8st^2 - 780r^8st - 130r^8s - 35r^8t^3 - 390r^8t^2 - 130r^8t - 35r^7s^4 + 112r^7s^3t + 182r^7s^3 + 294r^7s^2t^2 + 1560r^7s^2t + 442r^7s^2 + 112r^7st^3 + 1560r^7st^2 + 884r^7st - 35r^7t^4 + 182r^7t^3 + 442r^7$$

$$t^2 - 35r^6s^5 + 112r^6s^4t + 182r^6s^4 + 21r^6s^3t^2 - 728r^6s^3t - 273r^6s^3 + 21r^6s^2t^3 -$$
$$1820r^6s^2t^2 - 2119r^6s^2t + 112r^6st^4 - 728r^6st^3 - 2119r^6st^2 - 35r^6t^5 + 182r^6t^4 -$$
$$273r^6t^3 - 35r^5s^6 + 112r^5s^5t + 182r^5s^5 + 21r^5s^4t^2 - 728r^5s^4t - 273r^5s^4 -$$
$$252r^5s^3t^3 + 182r^5s^3t^2 + 1456r^5s^3t + 21r^5s^2t^4 + 182r^5s^2t^3 + 3458r^5s^2t^2 +$$
$$112r^5st^5 - 728r^5st^4 + 1456r^5st^3 - 35r^5t^6 + 182r^5t^5 - 273r^5t^4 - 35r^4s^7 + 112r^4s^6t +$$
$$182r^4s^6 + 21r^4s^5t^2 - 728r^4s^5t - 273r^4s^5 - 252r^4s^4t^3 + 182r^4s^4t^2 + 1456r^4s^4t -$$
$$252r^4s^3t^4 + 2184r^4s^3t^3 - 1547r^4s^3t^2 + 21r^4s^2t^5 + 182r^4s^2t^4 - 1547r^4s^2t^3 +$$
$$112r^4st^6 - 728r^4st^5 + 1456r^4st^4 - 35r^4t^7 + 182r^4t^6 - 273r^4t^5 - 35r^3s^8 + 112r^3s^7t +$$
$$182r^3s^7 + 21r^3s^6t^2 - 728r^3s^6t - 273r^3s^6 - 252r^3s^5t^3 + 182r^3s^5t^2 + 1456r^3s^5t -$$
$$252r^3s^4t^4 + 2184r^3s^4t^3 - 1547r^3s^4t^2 - 252r^3s^3t^5 + 2184r^3s^3t^4 - 6552r^3s^3t^3 +$$
$$21r^3s^2t^6 + 182r^3s^2t^5 - 1547r^3s^2t^4 + 112r^3st^7 - 728r^3st^6 + 1456r^3st^5 - 35r^3t^8 +$$
$$182r^3t^7 - 273r^3t^6 + 95r^2s^9 - 330r^2s^8t - 390r^2s^8 + 294r^2s^7t^2 + 1560r^2s^7t +$$
$$442r^2s^7 + 21r^2s^6t^3 - 1820r^2s^6t^2 - 2119r^2s^6t + 21r^2s^5t^4 + 182r^2s^5t^3 + 3458r^2s^5t^2 +$$
$$21r^2s^4t^5 + 182r^2s^4t^4 - 1547r^2s^4t^3 + 21r^2s^3t^6 + 182r^2s^3t^5 - 1547r^2s^3t^4 + 294r^2s^2t^7 -$$
$$1820r^2s^2t^6 + 3458r^2s^2t^5 - 330r^2st^8 + 1560r^2st^7 - 2119r^2st^6 + 95r^2t^9 - 390r^2t^8 +$$
$$442r^2t^7 - 35rs^{10} + 190rs^9t + 130rs^9 - 330rs^8t^2 - 780rs^8t - 130rs^8 + 112rs^7t^3 +$$
$$1560rs^7t^2 + 884rs^7t + 112rs^6t^4 - 728rs^6t^3 - 2119rs^6t^2 + 112rs^5t^5 - 728rs^5t^4 +$$
$$1456rs^5t^3 + 112rs^4t^6 - 728rs^4t^5 + 1456rs^4t^4 + 112rs^3t^7 - 728rs^3t^6 + 1456rs^3t^5 -$$
$$330rs^2t^8 + 1560rs^2t^7 - 2119rs^2t^6 + 190rst^9 - 780rst^8 + 884rst^7 - 35rt^{10} + 130rt^9 -$$
$$130rt^8 - 35s^{10}t + 95s^9t^2 + 130s^9t - 35s^8t^3 - 390s^8t^2 - 130s^8t - 35s^7t^4 + 182s^7t^3 +$$
$$442s^7t^2 - 35s^6t^5 + 182s^6t^4 - 273s^6t^3 - 35s^5t^6 + 182s^5t^5 - 273s^5t^4 - 35s^4t^7 +$$
$$182s^4t^6 - 273s^4t^5 - 35s^3t^8 + 182s^3t^7 - 273s^3t^6 + 95s^2t^9 - 390s^2t^8 + 442s^2t^7 -$$
$$35st^{10} + 130st^9 - 130st^8) - \frac{h^4(x-x_n)}{2162160rst}(35r^9 - 95r^8s - 95r^8t - 130r^8 + 35r^7s^2 +$$
$$295r^7st + 390r^7s + 35r^7t^2 + 390r^7t + 130r^7 + 35r^6s^3 - 147r^6s^2t - 182r^6s^2 -$$
$$147r^6st^2 - 1378r^6st - 442r^6s + 35r^6t^3 - 182r^6t^2 - 442r^6t + 35r^5s^4 - 147r^5s^3t -$$
$$182r^5s^3 + 126r^5s^2t^2 + 910r^5s^2t + 273r^5s^2 - 147r^5st^3 + 910r^5st^2 + 1846r^5st +$$
$$35r^5t^4 - 182r^5t^3 + 273r^5t^2 + 35r^4s^5 - 147r^4s^4t - 182r^4s^4 + 126r^4s^3t^2 + 910r^4s^3t +$$
$$273r^4s^3 + 126r^4s^2t^3 - 1092r^4s^2t^2 - 1729r^4s^2t - 147r^4st^4 + 910r^4st^3 - 1729r^4st^2 +$$
$$35r^4t^5 - 182r^4t^4 + 273r^4t^3 + 35r^3s^6 - 147r^3s^5t - 182r^3s^5 + 126r^3s^4t^2 + 910r^3s^4t +$$

$273r^3s^4 + 126r^3s^3t^3 - 1092r^3s^3t^2 - 1729r^3s^3t + 126r^3s^2t^4 - 1092r^3s^2t^3 +$
$3276r^3s^2t^2 - 147r^3st^5 + 910r^3st^4 - 1729r^3st^3 + 35r^3t^6 - 182r^3t^5 + 273r^3t^4 +$
$35r^2s^7 - 147r^2s^6t - 182r^2s^6 + 126r^2s^5t^2 + 910r^2s^5t + 273r^2s^5 + 126r^2s^4t^3 -$
$1092r^2s^4t^2 - 1729r^2s^4t + 126r^2s^3t^4 - 1092r^2s^3t^3 + 3276r^2s^3t^2 + 126r^2s^2t^5 -$
$1092r^2s^2t^4 + 3276r^2s^2t^3 - 147r^2st^6 + 910r^2st^5 - 1729r^2st^4 + 35r^2t^7 - 182r^2t^6 +$
$273r^2t^5 - 95rs^8 + 295rs^7t + 390rs^7 - 147rs^6t^2 - 1378rs^6t - 442rs^6 - 147rs^5t^3 +$
$910rs^5t^2 + 1846rs^5t - 147rs^4t^4 + 910rs^4t^3 - 1729rs^4t^2 - 147rs^3t^5 + 910rs^3t^4 -$
$1729rs^3t^3 - 147rs^2t^6 + 910rs^2t^5 - 1729rs^2t^4 + 295rst^7 - 1378rst^6 + 1846rst^5 -$
$95rt^8 + 390rt^7 - 442rt^6 + 35s^9 - 95s^8t - 130s^8 + 35s^7t^2 + 390s^7t + 130s^7 + 35s^6t^3 -$
$182s^6t^2 - 442s^6t + 35s^5t^4 - 182s^5t^3 + 273s^5t^2 + 35s^4t^5 - 182s^4t^4 + 273s^4t^3 +$
$35s^3t^6 - 182s^3t^5 + 273s^3t^4 + 35s^2t^7 - 182s^2t^6 + 273s^2t^5 - 95st^8 + 390st^7 - 442st^6 +$
$35t^9 - 130t^8 + 130t^7) - \frac{(x-x_n)^8}{840h^3r^2s^2t^2}(r^2s^2t + r^2s^2 + r^2st^2 + 4r^2st + r^2s + r^2t^2 + r^2t +$
$rs^2t^2 + 4rs^2t + rs^2 + 4rst^2 + 4rst + rt^2 + s^2t^2 + s^2t + st^2) - \frac{(x-x_n)^{12}}{5940h^7r^2s^2t^2}(r + s + t +$
$1) - \frac{(x-x_n)^6}{180hrst}(rs + rt + st + rst) + \frac{(x-x_n)^7}{840h^2r^2s^2t^2}(r^2s^2t^2 + 4r^2s^2t + r^2s^2 + 4r^2st^2 + 4r^2st +$
$r^2t^2 + 4rs^2t^2 + 4rs^2t + 4rst^2 + s^2t^2) + \frac{(x-x_n)^{11}}{7920h^6r^2s^2t^2}(r^2 + 4rs + 4rt + 4r + s^2 + 4st +$
$4s + t^2 + 4t + 1),$

$\gamma_r = \frac{(x-x_n)^{13}}{17160h^8r^2(r-s)^2(r-t)^2(r-1)^2} - \frac{(x-x_n)^{10}}{5040h^5r^2(r-s)^2(r-t)^2(r-1)^2}(r + 2s + 2t + 4rs + 4rt +$
$8st + rs^2 + rt^2 + 2st^2 + 2s^2t + 2s^2 + 2t^2 + 4rst) + \frac{(x-x_n)^{11}}{7920h^6r^2(r-s)^2(r-t)^2(r-1)^2}(2r +$
$4s + 4t + 2rs + 2rt + 4st + s^2 + t^2 + 1) + \frac{(x-x_n)^9}{3024h^4r^2(r-s)^2(r-t)^2(r-1)^2}(s^2t^2 +$
$2rs + 2rt + 4st + 2rs^2 + 2rt^2 + 4st^2 + 4s^2t + s^2 + t^2 + 2rst^2 + 2rs^2t + 8rst) -$
$\frac{(x-x_n)^8}{1680h^3r^2(r-s)^2(r-t)^2(r-1)^2}(2s^2t^2 + rs^2 + rt^2 + 2st^2 + 2s^2t + 4rst^2 + 4rs^2t + rs^2t^2 +$
$4rst) + \frac{h^2(x-x_n)^3}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(56r^{10} - 126r^9s - 126r^9t - 182r^9 + 30r^8s^2 +$
$316r^8st + 442r^8s + 30r^8t^2 + 442r^8t + 156r^8 + 30r^7s^3 - 100r^7s^2t - 130r^7s^2 -$
$100r^7st^2 - 1222r^7st - 416r^7s + 30r^7t^3 - 130r^7t^2 - 416r^7t + 30r^6s^4 - 100r^6s^3t -$
$130r^6s^3 + 56r^6s^2t^2 + 494r^6s^2t + 156r^6s^2 - 100r^6st^3 + 494r^6st^2 + 1300r^6st +$
$30r^6t^4 - 130r^6t^3 + 156r^6t^2 + 30r^5s^5 - 100r^5s^4t - 130r^5s^4 + 56r^5s^3t^2 + 494r^5s^3t +$
$156r^5s^3 + 56r^5s^2t^3 - 364r^5s^2t^2 - 702r^5s^2t - 100r^5st^4 + 494r^5st^3 - 702r^5st^2 +$
$30r^5t^5 - 130r^5t^4 + 156r^5t^3 + 30r^4s^6 - 100r^4s^5t - 130r^4s^5 + 56r^4s^4t^2 + 494r^4s^4t +$

$156r^4s^4 + 56r^4s^3t^3 - 364r^4s^3t^2 - 702r^4s^3t + 56r^4s^2t^4 - 364r^4s^2t^3 + 728r^4s^2t^2 - 100r^4st^5 + 494r^4st^4 - 702r^4st^3 + 30r^4t^6 - 130r^4t^5 + 156r^4t^4 + 30r^3s^7 - 100r^3s^6t - 130r^3s^6 + 56r^3s^5t^2 + 494r^3s^5t + 156r^3s^5 + 56r^3s^4t^3 - 364r^3s^4t^2 - 702r^3s^4t + 56r^3s^3t^4 - 364r^3s^3t^3 + 728r^3s^3t^2 + 56r^3s^2t^5 - 364r^3s^2t^4 + 728r^3s^2t^3 - 100r^3st^6 + 494r^3st^5 - 702r^3st^4 + 30r^3t^7 - 130r^3t^6 + 156r^3t^5 + 30r^2s^8 - 100r^2s^7t - 130r^2s^7 + 56r^2s^6t^2 + 494r^2s^6t + 156r^2s^6 + 56r^2s^5t^3 - 364r^2s^5t^2 - 702r^2s^5t + 56r^2s^4t^4 - 364r^2s^4t^3 + 728r^2s^4t^2 + 56r^2s^3t^5 - 364r^2s^3t^4 + 728r^2s^3t^3 + 56r^2s^2t^6 - 364r^2s^2t^5 + 728r^2s^2t^4 - 100r^2st^7 + 494r^2st^6 - 702r^2st^5 + 30r^2t^8 - 130r^2t^7 + 156r^2t^6 + 30rs^9 - 100rs^8t - 130rs^8 + 56rs^7t^2 + 494rs^7t + 156rs^7 + 56rs^6t^3 - 364rs^6t^2 - 702rs^6t + 56rs^5t^4 - 364rs^5t^3 + 728rs^5t^2 + 56rs^4t^5 - 364rs^4t^4 + 728rs^4t^3 + 56rs^3t^6 - 364rs^3t^5 + 728rs^3t^4 + 56rs^2t^7 - 364rs^2t^6 + 728rs^2t^5 - 100rst^8 + 494rst^7 - 702rst^6 + 30rt^9 - 130rt^8 + 156rt^7 - 35s^{10} + 95s^9t + 130s^9 - 35s^8t^2 - 390s^8t - 130s^8 - 35s^7t^3 + 182s^7t^2 + 442s^7t - 35s^6t^4 + 182s^6t^3 - 273s^6t^2 - 35s^5t^5 + 182s^5t^4 - 273s^5t^3 - 35s^4t^6 + 182s^4t^5 - 273s^4t^4 - 35s^3t^7 + 182s^3t^6 - 273s^3t^5 - 35s^2t^8 + 182s^2t^7 - 273s^2t^6 + 95st^9 - 390st^8 + 442st^7 - 35t^{10} + 130t^9 - 130t^8) - \frac{(x-x_n)^{12}}{11880h^7r^2(r-s)^2(r-t)^2(r-1)^2}(r + 2s + 2t + 2) - \frac{h^3(x-x_n)^2}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(56r^{10}s + 56r^{10}t - 126r^9s^2 - 252r^9st - 182r^9s - 126r^9t^2 - 182r^9t + 30r^8s^3 + 346r^8s^2t + 442r^8s^2 + 346r^8st^2 + 884r^8st + 156r^8s + 30r^8t^3 + 442r^8t^2 + 156r^8t + 30r^7s^4 - 70r^7s^3t - 130r^7s^3 - 200r^7s^2t^2 - 1352r^7s^2t - 416r^7s^2 - 70r^7st^3 - 1352r^7st^2 - 832r^7st + 30r^7t^4 - 130r^7t^3 - 416r^7t^2 + 30r^6s^5 - 70r^6s^4t - 130r^6s^4 - 44r^6s^3t^2 + 364r^6s^3t + 156r^6s^3 - 44r^6s^2t^3 + 988r^6s^2t^2 + 1456r^6s^2t - 70r^6st^4 + 364r^6st^3 + 1456r^6st^2 + 30r^6t^5 - 130r^6t^4 + 156r^6t^3 + 30r^5s^6 - 70r^5s^5t - 130r^5s^5 - 44r^5s^4t^2 + 364r^5s^4t + 156r^5s^4 + 112r^5s^3t^3 + 130r^5s^3t^2 - 546r^5s^3t - 44r^5s^2t^4 + 130r^5s^2t^3 - 1404r^5s^2t^2 - 70r^5st^5 + 364r^5st^4 - 546r^5st^3 + 30r^5t^6 - 130r^5t^5 + 156r^5t^4 + 30r^4s^7 - 70r^4s^6t - 130r^4s^6 - 44r^4s^5t^2 + 364r^4s^5t + 156r^4s^5 + 112r^4s^4t^3 + 130r^4s^4t^2 - 546r^4s^4t + 112r^4s^3t^4 - 728r^4s^3t^3 + 26r^4s^3t^2 - 44r^4s^2t^5 + 130r^4s^2t^4 + 26r^4s^2t^3 - 70r^4st^6 + 364r^4st^5 - 546r^4st^4 + 30r^4t^7 - 130r^4t^6 + 156r^4t^5 + 30r^3s^8 - 70r^3s^7t - 130r^3s^7 - 44r^3s^6t^2 + 364r^3s^6t + 156r^3s^6 + 112r^3s^5t^3 + 130r^3s^5t^2 - 546r^3s^5t + 112r^3s^4t^4 - 728r^3s^4t^3 + 26r^3s^4t^2 + 112r^3s^3t^5 - 728r^3s^3t^4 + 1456r^3s^3t^3 - 44r^3s^2t^6 +$

$130r^3s^2t^5 + 26r^3s^2t^4 - 70r^3st^7 + 364r^3st^6 - 546r^3st^5 + 30r^3t^8 - 130r^3t^7 +$
$156r^3t^6 + 30r^2s^9 - 70r^2s^8t - 130r^2s^8 - 44r^2s^7t^2 + 364r^2s^7t + 156r^2s^7 + 112r^2s^6t^3 +$
$130r^2s^6t^2 - 546r^2s^6t + 112r^2s^5t^4 - 728r^2s^5t^3 + 26r^2s^5t^2 + 112r^2s^4t^5 - 728r^2s^4t^4 +$
$1456r^2s^4t^3 + 112r^2s^3t^6 - 728r^2s^3t^5 + 1456r^2s^3t^4 - 44r^2s^2t^7 + 130r^2s^2t^6 + 26r^2s^2t^5 -$
$70r^2st^8 + 364r^2st^7 - 546r^2st^6 + 30r^2t^9 - 130r^2t^8 + 156r^2t^7 - 35rs^{10} + 125rs^9t +$
$130rs^9 - 135rs^8t^2 - 520rs^8t - 130rs^8 + 21rs^7t^3 + 676rs^7t^2 + 598rs^7t + 21rs^6t^4 -$
$182rs^6t^3 - 975rs^6t^2 + 21rs^5t^5 - 182rs^5t^4 + 455rs^5t^3 + 21rs^4t^6 - 182rs^4t^5 +$
$455rs^4t^4 + 21rs^3t^7 - 182rs^3t^6 + 455rs^3t^5 - 135rs^2t^8 + 676rs^2t^7 - 975rs^2t^6 +$
$125rst^9 - 520rst^8 + 598rst^7 - 35rt^{10} + 130rt^9 - 130rt^8 - 35s^{10}t + 95s^9t^2 +$
$130s^9t - 35s^8t^3 - 390s^8t^2 - 130s^8t - 35s^7t^4 + 182s^7t^3 + 442s^7t^2 - 35s^6t^5 +$
$182s^6t^4 - 273s^6t^3 - 35s^5t^6 + 182s^5t^5 - 273s^5t^4 - 35s^4t^7 + 182s^4t^6 - 273s^4t^5 -$
$35s^3t^8 + 182s^3t^7 - 273s^3t^6 + 95s^2t^9 - 390s^2t^8 + 442s^2t^7 - 35st^{10} + 130st^9 -$
$130st^8) - \frac{s^2t^2(x-x_n)^6}{360hr(r-s)^2(r-t)^2(r-1)^2} + \frac{st(x-x_n)^7}{840h^2r^2(r-s)^2(r-t)^2(r-1)^2}(2rs + 2rt + st + 2rst) +$
$\frac{h^4st(x-x_n)}{2162160r(r-s)^2(r-t)^2(r-1)^2}(56r^9 - 126r^8s - 126r^8t - 182r^8 + 30r^7s^2 + 316r^7st +$
$442r^7s + 30r^7t^2 + 442r^7t + 156r^7 + 30r^6s^3 - 100r^6s^2t - 130r^6s^2 - 100r^6st^2 -$
$1222r^6st - 416r^6s + 30r^6t^3 - 130r^6t^2 - 416r^6t + 30r^5s^4 - 100r^5s^3t - 130r^5s^3 +$
$56r^5s^2t^2 + 494r^5s^2t + 156r^5s^2 - 100r^5st^3 + 494r^5st^2 + 1300r^5st + 30r^5t^4 - 130r^5t^3 +$
$156r^5t^2 + 30r^4s^5 - 100r^4s^4t - 130r^4s^4 + 56r^4s^3t^2 + 494r^4s^3t + 156r^4s^3 + 56r^4s^2t^3 -$
$364r^4s^2t^2 - 702r^4s^2t - 100r^4st^4 + 494r^4st^3 - 702r^4st^2 + 30r^4t^5 - 130r^4t^4 +$
$156r^4t^3 + 30r^3s^6 - 100r^3s^5t - 130r^3s^5 + 56r^3s^4t^2 + 494r^3s^4t + 156r^3s^4 + 56r^3s^3t^3 -$
$364r^3s^3t^2 - 702r^3s^3t + 56r^3s^2t^4 - 364r^3s^2t^3 + 728r^3s^2t^2 - 100r^3st^5 + 494r^3st^4 -$
$702r^3st^3 + 30r^3t^6 - 130r^3t^5 + 156r^3t^4 + 30r^2s^7 - 100r^2s^6t - 130r^2s^6 + 56r^2s^5t^2 +$
$494r^2s^5t + 156r^2s^5 + 56r^2s^4t^3 - 364r^2s^4t^2 - 702r^2s^4t + 56r^2s^3t^4 - 364r^2s^3t^3 +$
$728r^2s^3t^2 + 56r^2s^2t^5 - 364r^2s^2t^4 + 728r^2s^2t^3 - 100r^2st^6 + 494r^2st^5 - 702r^2st^4 +$
$30r^2t^7 - 130r^2t^6 + 156r^2t^5 + 30rs^8 - 100rs^7t - 130rs^7 + 56rs^6t^2 + 494rs^6t +$
$156rs^6 + 56rs^5t^3 - 364rs^5t^2 - 702rs^5t + 56rs^4t^4 - 364rs^4t^3 + 728rs^4t^2 + 56rs^3t^5 -$
$364rs^3t^4 + 728rs^3t^3 + 56rs^2t^6 - 364rs^2t^5 + 728rs^2t^4 - 100rst^7 + 494rst^6 - 702rst^5 +$
$30rt^8 - 130rt^7 + 156rt^6 - 35s^9 + 95s^8t + 130s^8 - 35s^7t^2 - 390s^7t - 130s^7 - 35s^6t^3 +$
$182s^6t^2 + 442s^6t - 35s^5t^4 + 182s^5t^3 - 273s^5t^2 - 35s^4t^5 + 182s^4t^4 - 273s^4t^3 -$

$35s^3t^6+182s^3t^5-273s^3t^4-35s^2t^7+182s^2t^6-273s^2t^5+95st^8-390st^7+442st^6-35t^9+130t^8-130t^7),$

$\gamma_s = \frac{(x-x_n)^{13}}{17160h^8s^2(r-s)^2(s-t)^2(s-1)^2} - \frac{(x-x_n)^{10}}{5040h^5s^2(r-s)^2(s-t)^2(s-1)^2}(2r+s+2t+4rs+8rt+4st+r^2s+2rt^2+2r^2t+st^2+2r^2+2t^2+4rst) + \frac{(x-x_n)^{11}}{7920h^6s^2(r-s)^2(s-t)^2(s-1)^2}(4r+2s+4t+2rs+4rt+2st+r^2+t^2+1) + \frac{(x-x_n)^9}{3024h^4s^2(r-s)^2(s-t)^2(s-1)^2}(r^2t^2+2rs+4rt+2st+2r^2s+4rt^2+4r^2t+2st^2+r^2+t^2+2rst^2+2r^2st+8rst) - \frac{(x-x_n)^8}{1680h^3s^2(r-s)^2(s-t)^2(s-1)^2}(2r^2t^2+r^2s+2rt^2+2r^2t+st^2+4rst^2+4r^2st+r^2st^2+4rst) + \frac{h^2(x-x_n)^3}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(30r^9s-35r^{10}+95r^9t+130r^9+30r^8s^2-100r^8st-130r^8s-35r^8t^2-390r^8t-130r^8+30r^7s^3-100r^7s^2t-130r^7s^2+56r^7st^2+494r^7st+156r^7s-35r^7t^3+182r^7t^2+442r^7t+30r^6s^4-100r^6s^3t-130r^6s^3+56r^6s^2t^2+494r^6s^2t+156r^6s^2+56r^6st^3-364r^6st^2-702r^6st-35r^6t^4+182r^6t^3-273r^6t^2+30r^5s^5-100r^5s^4t-130r^5s^4+56r^5s^3t^2+494r^5s^3t+156r^5s^3+56r^5s^2t^3-364r^5s^2t^2-702r^5s^2t+56r^5st^4-364r^5st^3+728r^5st^2-35r^5t^5+182r^5t^4-273r^5t^3+30r^4s^6-100r^4s^5t-130r^4s^5+56r^4s^4t^2+494r^4s^4t+156r^4s^4+56r^4s^3t^3-364r^4s^3t^2-702r^4s^3t+56r^4s^2t^4-364r^4s^2t^3+728r^4s^2t^2+56r^4st^5-364r^4st^4+728r^4st^3-35r^4t^6+182r^4t^5-273r^4t^4+30r^3s^7-100r^3s^6t-130r^3s^6+56r^3s^5t^2+494r^3s^5t+156r^3s^5+56r^3s^4t^3-364r^3s^4t^2-702r^3s^4t+56r^3s^3t^4-364r^3s^3t^3+728r^3s^3t^2+56r^3s^2t^5-364r^3s^2t^4+728r^3s^2t^3+56r^3st^6-364r^3st^5+728r^3st^4-35r^3t^7+182r^3t^6-273r^3t^5+30r^2s^8-100r^2s^7t-130r^2s^7+56r^2s^6t^2+494r^2s^6t+156r^2s^6+56r^2s^5t^3-364r^2s^5t^2-702r^2s^5t+56r^2s^4t^4-364r^2s^4t^3+728r^2s^4t^2+56r^2s^3t^5-364r^2s^3t^4+728r^2s^3t^3+56r^2s^2t^6-364r^2s^2t^5+728r^2s^2t^4+56r^2st^7-364r^2st^6+728r^2st^5-35r^2t^8+182r^2t^7-273r^2t^6-126rs^9+316rs^8t+442rs^8-100rs^7t^2-1222rs^7t-416rs^7-100rs^6t^3+494rs^6t^2+1300rs^6t-100rs^5t^4+494rs^5t^3-702rs^5t^2-100rs^4t^5+494rs^4t^4-702rs^4t^3-100rs^3t^6+494rs^3t^5-702rs^3t^4-100rs^2t^7+494rs^2t^6-702rs^2t^5-100rst^8+494rst^7-702rst^6+95rt^9-390rt^8+442rt^7+56s^{10}-126s^9t-182s^9+30s^8t^2+442s^8t+156s^8+30s^7t^3-130s^7t^2-416s^7t+30s^6t^4-130s^6t^3+156s^6t^2+30s^5t^5-130s^5t^4+156s^5t^3+30s^4t^6-130s^4t^5+156s^4t^4+30s^3t^7-130s^3t^6+156s^3t^5+30s^2t^8-130s^2t^7+156$

$s^2t^6 + 30st^9 - 130st^8 + 156st^7 - 35t^{10} + 130t^9 - 130t^8) -$
$\frac{(x-x_n)^{12}}{11880h^7s^2(r-s)^2(s-t)^2(s-1)^2}(2r + s + 2t + 2) - \frac{h^3(x-x_n)^2}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(30r^9s^2 -$
$35r^{10}t - 35r^{10}s + 125r^9st + 130r^9s + 95r^9t^2 + 130r^9t + 30r^8s^3 - 70r^8s^2t -$
$130r^8s^2 - 135r^8st^2 - 520r^8st - 130r^8s - 35r^8t^3 - 390r^8t^2 - 130r^8t + 30r^7s^4 -$
$70r^7s^3t - 130r^7s^3 - 44r^7s^2t^2 + 364r^7s^2t + 156r^7s^2 + 21r^7st^3 + 676r^7st^2 + 598r^7st -$
$35r^7t^4 + 182r^7t^3 + 442r^7t^2 + 30r^6s^5 - 70r^6s^4t - 130r^6s^4 - 44r^6s^3t^2 + 364r^6s^3t +$
$156r^6s^3 + 112r^6s^2t^3 + 130r^6s^2t^2 - 546r^6s^2t + 21r^6st^4 - 182r^6st^3 - 975r^6st^2 -$
$35r^6t^5 + 182r^6t^4 - 273r^6t^3 + 30r^5s^6 - 70r^5s^5t - 130r^5s^5 - 44r^5s^4t^2 + 364r^5s^4t +$
$156r^5s^4 + 112r^5s^3t^3 + 130r^5s^3t^2 - 546r^5s^3t + 112r^5s^2t^4 - 728r^5s^2t^3 + 26r^5s^2t^2 +$
$21r^5st^5 - 182r^5st^4 + 455r^5st^3 - 35r^5t^6 + 182r^5t^5 - 273r^5t^4 + 30r^4s^7 - 70r^4s^6t -$
$130r^4s^6 - 44r^4s^5t^2 + 364r^4s^5t + 156r^4s^5 + 112r^4s^4t^3 + 130r^4s^4t^2 - 546r^4s^4t +$
$112r^4s^3t^4 - 728r^4s^3t^3 + 26r^4s^3t^2 + 112r^4s^2t^5 - 728r^4s^2t^4 + 1456r^4s^2t^3 + 21r^4st^6 -$
$182r^4st^5 + 455r^4st^4 - 35r^4t^7 + 182r^4t^6 - 273r^4t^5 + 30r^3s^8 - 70r^3s^7t - 130r^3s^7 -$
$44r^3s^6t^2 + 364r^3s^6t + 156r^3s^6 + 112r^3s^5t^3 + 130r^3s^5t^2 - 546r^3s^5t + 112r^3s^4t^4 -$
$728r^3s^4t^3 + 26r^3s^4t^2 + 112r^3s^3t^5 - 728r^3s^3t^4 + 1456r^3s^3t^3 + 112r^3s^2t^6 - 728r^3s^2t^5 +$
$1456r^3s^2t^4 + 21r^3st^7 - 182r^3st^6 + 455r^3st^5 - 35r^3t^8 + 182r^3t^7 - 273r^3t^6 - 126r^2s^9 +$
$346r^2s^8t + 442r^2s^8 - 200r^2s^7t^2 - 1352r^2s^7t - 416r^2s^7 - 44r^2s^6t^3 + 988r^2s^6t^2 +$
$1456r^2s^6t - 44r^2s^5t^4 + 130r^2s^5t^3 - 1404r^2s^5t^2 - 44r^2s^4t^5 + 130r^2s^4t^4 + 26r^2s^4t^3 -$
$44r^2s^3t^6 + 130r^2s^3t^5 + 26r^2s^3t^4 - 44r^2s^2t^7 + 130r^2s^2t^6 + 26r^2s^2t^5 - 135r^2st^8 +$
$676r^2st^7 - 975r^2st^6 + 95r^2t^9 - 390r^2t^8 + 442r^2t^7 + 56rs^{10} - 252rs^9t - 182rs^9 +$
$346rs^8t^2 + 884rs^8t + 156rs^8 - 70rs^7t^3 - 1352rs^7t^2 - 832rs^7t - 70rs^6t^4 + 364rs^6t^3 +$
$1456rs^6t^2 - 70rs^5t^5 + 364rs^5t^4 - 546rs^5t^3 - 70rs^4t^6 + 364rs^4t^5 - 546rs^4t^4 -$
$70rs^3t^7 + 364rs^3t^6 - 546rs^3t^5 - 70rs^2t^8 + 364rs^2t^7 - 546rs^2t^6 + 125rst^9 - 520rst^8 +$
$598rst^7 - 35rt^{10} + 130rt^9 - 130rt^8 + 56s^{10}t - 126s^9t^2 - 182s^9t + 30s^8t^3 + 442s^8t^2 +$
$156s^8t + 30s^7t^4 - 130s^7t^3 - 416s^7t^2 + 30s^6t^5 - 130s^6t^4 + 156s^6t^3 + 30s^5t^6 -$
$130s^5t^5 + 156s^5t^4 + 30s^4t^7 - 130s^4t^6 + 156s^4t^5 + 30s^3t^8 - 130s^3t^7 + 156s^3t^6 +$
$30s^2t^9 - 130s^2t^8 + 156s^2t^7 - 35st^{10} + 130st^9 - 130st^8) - \frac{r^2t^2(x-x_n)^6}{360hs(r-s)^2(s-t)^2(s-1)^2} +$
$\frac{rt(x-x_n)^7}{840h^2s^2(r-s)^2(s-t)^2(s-1)^2}(2rs + rt + 2st + 2rst) + \frac{h^4rt(x-x_n)}{2162160s(r-s)^2(s-t)^2(s-1)^2}(30r^8s -$
$35r^9 + 95r^8t + 130r^8 + 30r^7s^2 - 100r^7st - 130r^7s - 35r^7t^2 - 390r^7t - 130r^7 + 30r^6s^3$

$-100r^6s^2t - 130r^6s^2 + 56r^6st^2 + 494r^6st + 156r^6s - 35r^6t^3 + 182r^6t^2 + 442r^6t + 30r^5s^4 - 100r^5s^3t - 130r^5s^3 + 56r^5s^2t^2 + 494r^5s^2t + 156r^5s^2 + 56r^5st^3 - 364r^5st^2 - 702r^5st - 35r^5t^4 + 182r^5t^3 - 273r^5t^2 + 30r^4s^5 - 100r^4s^4t - 130r^4s^4 + 56r^4s^3t^2 + 494r^4s^3t + 156r^4s^3 + 56r^4s^2t^3 - 364r^4s^2t^2 - 702r^4s^2t + 56r^4st^4 - 364r^4st^3 + 728r^4st^2 - 35r^4t^5 + 182r^4t^4 - 273r^4t^3 + 30r^3s^6 - 100r^3s^5t - 130r^3s^5 + 56r^3s^4t^2 + 494r^3s^4t + 156r^3s^4 + 56r^3s^3t^3 - 364r^3s^3t^2 - 702r^3s^3t + 56r^3s^2t^4 - 364r^3s^2t^3 + 728r^3s^2t^2 + 56r^3st^5 - 364r^3st^4 + 728r^3st^3 - 35r^3t^6 + 182r^3t^5 - 273r^3t^4 + 30r^2s^7 - 100r^2s^6t - 130r^2s^6 + 56r^2s^5t^2 + 494r^2s^5t + 156r^2s^5 + 56r^2s^4t^3 - 364r^2s^4t^2 - 702r^2s^4t + 56r^2s^3t^4 - 364r^2s^3t^3 + 728r^2s^3t^2 + 56r^2s^2t^5 - 364r^2s^2t^4 + 728r^2s^2t^3 + 56r^2st^6 - 364r^2st^5 + 728r^2st^4 - 35r^2t^7 + 182r^2t^6 - 273r^2t^5 - 126rs^8 + 316rs^7t + 442rs^7 - 100rs^6t^2 - 1222rs^6t - 416rs^6 - 100rs^5t^3 + 494rs^5t^2 + 1300rs^5t - 100rs^4t^4 + 494rs^4t^3 - 702rs^4t^2 - 100rs^3t^5 + 494rs^3t^4 - 702rs^3t^3 - 100rs^2t^6 + 494rs^2t^5 - 702rs^2t^4 - 100rst^7 + 494rst^6 - 702rst^5 + 95rt^8 - 390rt^7 + 442rt^6 + 56s^9 - 126s^8t - 182s^8 + 30s^7t^2 + 442s^7t + 156s^7 + 30s^6t^3 - 130s^6t^2 - 416s^6t + 30s^5t^4 - 130s^5t^3 + 156s^5t^2 + 30s^4t^5 - 130s^4t^4 + 156s^4t^3 + 30s^3t^6 - 130s^3t^5 + 156s^3t^4 + 30s^2t^7 - 130s^2t^6 + 156s^2t^5 + 30st^8 - 130st^7 + 156st^6 - 35t^9 + 130t^8 - 130t^7),$

$\gamma_t = \frac{(x-x_n)^{13}}{17160h^8t^2(r-t)^2(s-t)^2(t-1)^2} - \frac{(x-x_n)^{10}}{5040h^5t^2(r-t)^2(s-t)^2(t-1)^2}(2r + 2s + t + 8rs + 4rt + 4st + 2rs^2 + 2r^2s + r^2t + s^2t + 2r^2 + 2s^2 + 4rst) + \frac{(x-x_n)^{11}}{7920h^6t^2(r-t)^2(s-t)^2(t-1)^2}(4r + 4s + 2t + 4rs + 2rt + 2st + r^2 + s^2 + 1) + \frac{(x-x_n)^9}{3024h^4t^2(r-t)^2(s-t)^2(t-1)^2}(r^2s^2 + 4rs + 2rt + 2st + 4rs^2 + 4r^2s + 2r^2t + 2s^2t + r^2 + s^2 + 2rs^2t + 2r^2st + 8rst) - \frac{(x-x_n)^8}{1680h^3t^2(r-t)^2(s-t)^2(t-1)^2}(2r^2s^2 + 2rs^2 + 2r^2s + r^2t + s^2t + 4rs^2t + 4r^2st + r^2s^2t + 4rst) + \frac{h^2(x-x_n)^3}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(95r^9s - 35r^{10} + 30r^9t + 130r^9 - 35r^8s^2 - 100r^8st - 390r^8s + 30r^8t^2 - 130r^8t - 130r^8 - 35r^7s^3 + 56r^7s^2t + 182r^7s^2 - 100r^7st^2 + 494r^7st + 442r^7s + 30r^7t^3 - 130r^7t^2 + 156r^7t - 35r^6s^4 + 56r^6s^3t + 182r^6s^3 + 56r^6s^2t^2 - 364r^6s^2t - 273r^6s^2 - 100r^6st^3 + 494r^6st^2 - 702r^6st + 30r^6t^4 - 130r^6t^3 + 156r^6t^2 - 35r^5s^5 + 56r^5s^4t + 182r^5s^4 + 56r^5s^3t^2 - 364r^5s^3t - 273r^5s^3 + 56r^5s^2t^3 - 364r^5s^2t^2 + 728r^5s^2t - 100r^5st^4 + 494r^5st^3 - 702r^5st^2 + 30r^5t^5 - 130r^5t^4 + 156r^5t^3 - 35r^4s^6 + 56r^4s^5t + 182r^4s^5 + 56r^4s^4t^2 - 364r^4s^4t - 273r^4s^4 + 56r^4s^3t^3 -$

$$364r^4s^3t^2+728r^4s^3t+56r^4s^2t^4-364r^4s^2t^3+728r^4s^2t^2-100r^4st^5+494r^4st^4-$$
$$702r^4st^3+30r^4t^6-130r^4t^5+156r^4t^4-35r^3s^7+56r^3s^6t+182r^3s^6+56r^3s^5t^2-$$
$$364r^3s^5t-273r^3s^5+56r^3s^4t^3-364r^3s^4t^2+728r^3s^4t+56r^3s^3t^4-364r^3s^3t^3+$$
$$728r^3s^3t^2+56r^3s^2t^5-364r^3s^2t^4+728r^3s^2t^3-100r^3st^6+494r^3st^5-702r^3st^4+$$
$$30r^3t^7-130r^3t^6+156r^3t^5-35r^2s^8+56r^2s^7t+182r^2s^7+56r^2s^6t^2-364r^2s^6t-$$
$$273r^2s^6+56r^2s^5t^3-364r^2s^5t^2+728r^2s^5t+56r^2s^4t^4-364r^2s^4t^3+728r^2s^4t^2+$$
$$56r^2s^3t^5-364r^2s^3t^4+728r^2s^3t^3+56r^2s^2t^6-364r^2s^2t^5+728r^2s^2t^4-100r^2st^7+$$
$$494r^2st^6-702r^2st^5+30r^2t^8-130r^2t^7+156r^2t^6+95rs^9-100rs^8t-390rs^8-$$
$$100rs^7t^2+494rs^7t+442rs^7-100rs^6t^3+494rs^6t^2-702rs^6t-100rs^5t^4+494rs^5t^3-$$
$$702rs^5t^2-100rs^4t^5+494rs^4t^4-702rs^4t^3-100rs^3t^6+494rs^3t^5-702rs^3t^4-$$
$$100rs^2t^7+494rs^2t^6-702rs^2t^5+316rst^8-1222rst^7+1300rst^6-126rt^9+442rt^8-$$
$$416rt^7-35s^{10}+30s^9t+130s^9+30s^8t^2-130s^8t-130s^8+30s^7t^3-130s^7t^2+$$
$$156s^7t+30s^6t^4-130s^6t^3+156s^6t^2+30s^5t^5-130s^5t^4+156s^5t^3+30s^4t^6-$$
$$130s^4t^5+156s^4t^4+30s^3t^7-130s^3t^6+156s^3t^5+30s^2t^8-130s^2t^7+156s^2t^6-$$
$$126st^9+442st^8-416st^7+56t^{10}-182t^9+156t^8)-\frac{(x-x_n)^{12}}{11880h^7t^2(r-t)^2(s-t)^2(t-1)^2}(2r+$$
$$2s+t+2)-\frac{h^3(x-x_n)^2}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(95r^9s^2-35r^{10}t-35r^{10}s+125r^9st+130r^9s+$$
$$30r^9t^2+130r^9t-35r^8s^3-135r^8s^2t-390r^8s^2-70r^8st^2-520r^8st-130r^8s+$$
$$30r^8t^3-130r^8t^2-130r^8t-35r^7s^4+21r^7s^3t+182r^7s^3-44r^7s^2t^2+676r^7s^2t+$$
$$442r^7s^2-70r^7st^3+364r^7st^2+598r^7st+30r^7t^4-130r^7t^3+156r^7t^2-35r^6s^5+$$
$$21r^6s^4t+182r^6s^4+112r^6s^3t^2-182r^6s^3t-273r^6s^3-44r^6s^2t^3+130r^6s^2t^2-$$
$$975r^6s^2t-70r^6st^4+364r^6st^3-546r^6st^2+30r^6t^5-130r^6t^4+156r^6t^3-35r^5s^6+$$
$$21r^5s^5t+182r^5s^5+112r^5s^4t^2-182r^5s^4t-273r^5s^4+112r^5s^3t^3-728r^5s^3t^2+$$
$$455r^5s^3t-44r^5s^2t^4+130r^5s^2t^3+26r^5s^2t^2-70r^5st^5+364r^5st^4-546r^5st^3+$$
$$30r^5t^6-130r^5t^5+156r^5t^4-35r^4s^7+21r^4s^6t+182r^4s^6+112r^4s^5t^2-182r^4s^5t-$$
$$273r^4s^5+112r^4s^4t^3-728r^4s^4t^2+455r^4s^4t+112r^4s^3t^4-728r^4s^3t^3+1456r^4s^3t^2-$$
$$44r^4s^2t^5+130r^4s^2t^4+26r^4s^2t^3-70r^4st^6+364r^4st^5-546r^4st^4+30r^4t^7-130r^4t^6+$$
$$156r^4t^5-35r^3s^8+21r^3s^7t+182r^3s^7+112r^3s^6t^2-182r^3s^6t-273r^3s^6+112r^3s^5t^3-$$
$$728r^3s^5t^2+455r^3s^5t+112r^3s^4t^4-728r^3s^4t^3+1456r^3s^4t^2+112r^3s^3t^5-728r^3s^3t^4+$$
$$1456r^3s^3t^3-44r^3s^2t^6+130r^3s^2t^5+26r^3s^2t^4-70r^3st^7+364r^3st^6-546r^3st^5+$$

$$30r^3t^8 - 130r^3t^7 + 156r^3t^6 + 95r^2s^9 - 135r^2s^8t - 390r^2s^8 - 44r^2s^7t^2 + 676r^2s^7t +$$
$$442r^2s^7 - 44r^2s^6t^3 + 130r^2s^6t^2 - 975r^2s^6t - 44r^2s^5t^4 + 130r^2s^5t^3 + 26r^2s^5t^2 -$$
$$44r^2s^4t^5 + 130r^2s^4t^4 + 26r^2s^4t^3 - 44r^2s^3t^6 + 130r^2s^3t^5 + 26r^2s^3t^4 - 200r^2s^2t^7 +$$
$$988r^2s^2t^6 - 1404r^2s^2t^5 + 346r^2st^8 - 1352r^2st^7 + 1456r^2st^6 - 126r^2t^9 + 442r^2t^8 -$$
$$416r^2t^7 - 35rs^{10} + 125rs^9t + 130rs^9 - 70rs^8t^2 - 520rs^8t - 130rs^8 - 70rs^7t^3 +$$
$$364rs^7t^2 + 598rs^7t - 70rs^6t^4 + 364rs^6t^3 - 546rs^6t^2 - 70rs^5t^5 + 364rs^5t^4 -$$
$$546rs^5t^3 - 70rs^4t^6 + 364rs^4t^5 - 546rs^4t^4 - 70rs^3t^7 + 364rs^3t^6 - 546rs^3t^5 +$$
$$346rs^2t^8 - 1352rs^2t^7 + 1456rs^2t^6 - 252rst^9 + 884rst^8 - 832rst^7 + 56rt^{10} - 182rt^9 +$$
$$156rt^8 - 35s^{10}t + 30s^9t^2 + 130s^9t + 30s^8t^3 - 130s^8t^2 - 130s^8t + 30s^7t^4 - 130s^7t^3 +$$
$$156s^7t^2 + 30s^6t^5 - 130s^6t^4 + 156s^6t^3 + 30s^5t^6 - 130s^5t^5 + 156s^5t^4 + 30s^4t^7 -$$
$$130s^4t^6 + 156s^4t^5 + 30s^3t^8 - 130s^3t^7 + 156s^3t^6 - 126s^2t^9 + 442s^2t^8 - 416s^2t^7 +$$
$$56st^{10} - 182st^9 + 156st^8) - \frac{r^2s^2(x-x_n)^6}{360ht(r-t)^2(s-t)^2(t-1)^2} + \frac{rs(x-x_n)^7}{840h^2t^2(r-t)^2(s-t)^2(t-1)^2}(rs + 2rt +$$
$$2st + 2rst) + \frac{h^4rs(x-x_n)}{2162160t(r-t)^2(s-t)^2(t-1)^2}(95r^8s - 35r^9 + 30r^8t + 130r^8 - 35r^7s^2 -$$
$$100r^7st - 390r^7s + 30r^7t^2 - 130r^7t - 130r^7 - 35r^6s^3 + 56r^6s^2t + 182r^6s^2 -$$
$$100r^6st^2 + 494r^6st + 442r^6s + 30r^6t^3 - 130r^6t^2 + 156r^6t - 35r^5s^4 + 56r^5s^3t +$$
$$182r^5s^3 + 56r^5s^2t^2 - 364r^5s^2t - 273r^5s^2 - 100r^5st^3 + 494r^5st^2 - 702r^5st + 30r^5t^4 -$$
$$130r^5t^3 + 156r^5t^2 - 35r^4s^5 + 56r^4s^4t + 182r^4s^4 + 56r^4s^3t^2 - 364r^4s^3t - 273r^4s^3 +$$
$$56r^4s^2t^3 - 364r^4s^2t^2 + 728r^4s^2t - 100r^4st^4 + 494r^4st^3 - 702r^4st^2 + 30r^4t^5 -$$
$$130r^4t^4 + 156r^4t^3 - 35r^3s^6 + 56r^3s^5t + 182r^3s^5 + 56r^3s^4t^2 - 364r^3s^4t - 273r^3s^4 +$$
$$56r^3s^3t^3 - 364r^3s^3t^2 + 728r^3s^3t + 56r^3s^2t^4 - 364r^3s^2t^3 + 728r^3s^2t^2 - 100r^3st^5 +$$
$$494r^3st^4 - 702r^3st^3 + 30r^3t^6 - 130r^3t^5 + 156r^3t^4 - 35r^2s^7 + 56r^2s^6t + 182r^2s^6 +$$
$$56r^2s^5t^2 - 364r^2s^5t - 273r^2s^5 + 56r^2s^4t^3 - 364r^2s^4t^2 + 728r^2s^4t + 56r^2s^3t^4 -$$
$$364r^2s^3t^3 + 728r^2s^3t^2 + 56r^2s^2t^5 - 364r^2s^2t^4 + 728r^2s^2t^3 - 100r^2st^6 + 494r^2st^5 -$$
$$702r^2st^4 + 30r^2t^7 - 130r^2t^6 + 156r^2t^5 + 95rs^8 - 100rs^7t - 390rs^7 - 100rs^6t^2 +$$
$$494rs^6t + 442rs^6 - 100rs^5t^3 + 494rs^5t^2 - 702rs^5t - 100rs^4t^4 + 494rs^4t^3 -$$
$$702rs^4t^2 - 100rs^3t^5 + 494rs^3t^4 - 702rs^3t^3 - 100rs^2t^6 + 494rs^2t^5 - 702rs^2t^4 +$$
$$316rst^7 - 1222rst^6 + 1300rst^5 - 126rt^8 + 442rt^7 - 416rt^6 - 35s^9 + 30s^8t + 130s^8 +$$
$$30s^7t^2 - 130s^7t - 130s^7 + 30s^6t^3 - 130s^6t^2 + 156s^6t + 30s^5t^4 - 130s^5t^3 + 156s^5t^2 +$$
$$30s^4t^5 - 130s^4t^4 + 156s^4t^3 + 30s^3t^6 - 130s^3t^5 + 156s^3t^4 + 30s^2t^7 - 130s^2t^6 +$$
$$156s^2t^5 - 126st^8 + 442st^7 - 416st^6 + 56t^9 - 182t^8 + 156t^7),$$

$$\gamma_1 = \frac{(x-x_n)^{13}}{17160h^8(r-1)^2(s-1)^2(t-1)^2} - \frac{(x-x_n)^{10}}{5040h^5(r-1)^2(s-1)^2(t-1)^2}(2r^2s+2r^2t+r^2+2rs^2+8rst+$$
$$4rs+2rt^2+4rt+2s^2t+s^2+2st^2+4st+t^2) + \frac{(x-x_n)^9}{3024h^4(r-1)^2(s-1)^2(t-1)^2}(r^2s^2+$$
$$4r^2st+2r^2s+r^2t^2+2r^2t+4rs^2t+2rs^2+4rst^2+8rst+2rt^2+s^2t^2+2s^2t+$$
$$2st^2) - \frac{h^3(x-x_n)^2}{2162160(r-1)^2(s-1)^2(t-1)^2}(95r^9s^2-35r^{10}t-35r^{10}s+190r^9st+65r^9s+95r^9t^2+$$
$$65r^9t-35r^8s^3-330r^8s^2t-195r^8s^2-330r^8st^2-390r^8st-35r^8t^3-195r^8t^2-$$
$$35r^7s^4+112r^7s^3t+91r^7s^3+294r^7s^2t^2+780r^7s^2t+112r^7st^3+780r^7st^2-35r^7t^4+$$
$$91r^7t^3-35r^6s^5+112r^6s^4t+91r^6s^4+21r^6s^3t^2-364r^6s^3t+21r^6s^2t^3-910r^6s^2t^2+$$
$$112r^6st^4-364r^6st^3-35r^6t^5+91r^6t^4-35r^5s^6+112r^5s^5t+91r^5s^5+21r^5s^4t^2-$$
$$364r^5s^4t-252r^5s^3t^3+91r^5s^3t^2+21r^5s^2t^4+91r^5s^2t^3+112r^5st^5-364r^5st^4-$$
$$35r^5t^6+91r^5t^5-35r^4s^7+112r^4s^6t+91r^4s^6+21r^4s^5t^2-364r^4s^5t-252r^4s^4t^3+$$
$$91r^4s^4t^2-252r^4s^3t^4+1092r^4s^3t^3+21r^4s^2t^5+91r^4s^2t^4+112r^4st^6-364r^4st^5-$$
$$35r^4t^7+91r^4t^6-35r^3s^8+112r^3s^7t+91r^3s^7+21r^3s^6t^2-364r^3s^6t-252r^3s^5t^3+$$
$$91r^3s^5t^2-252r^3s^4t^4+1092r^3s^4t^3-252r^3s^3t^5+1092r^3s^3t^4+21r^3s^2t^6+91r^3s^2t^5+$$
$$112r^3st^7-364r^3st^6-35r^3t^8+91r^3t^7+95r^2s^9-330r^2s^8t-195r^2s^8+294r^2s^7t^2+$$
$$780r^2s^7t+21r^2s^6t^3-910r^2s^6t^2+21r^2s^5t^4+91r^2s^5t^3+21r^2s^4t^5+91r^2s^4t^4+$$
$$21r^2s^3t^6+91r^2s^3t^5+294r^2s^2t^7-910r^2s^2t^6-330r^2st^8+780r^2st^7+95r^2t^9-$$
$$195r^2t^8-35rs^{10}+190rs^9t+65rs^9-330rs^8t^2-390rs^8t+112rs^7t^3+780rs^7t^2+$$
$$112rs^6t^4-364rs^6t^3+112rs^5t^5-364rs^5t^4+112rs^4t^6-364rs^4t^5+112rs^3t^7-$$
$$364rs^3t^6-330rs^2t^8+780rs^2t^7+190rst^9-390rst^8-35rt^{10}+65rt^9-35s^{10}t+$$
$$95s^9t^2+65s^9t-35s^8t^3-195s^8t^2-35s^7t^4+91s^7t^3-35s^6t^5+91s^6t^4-35s^5t^6+$$
$$91s^5t^5-35s^4t^7+91s^4t^6-35s^3t^8+91s^3t^7+95s^2t^9-195s^2t^8-35st^{10}+65st^9)-$$
$$\frac{h^2(x-x_n)^3}{2162160(r-1)^2(s-1)^2(t-1)^2}(35r^{10}-95r^9s-95r^9t-65r^9+35r^8s^2+295r^8st+195r^8s+$$
$$35r^8t^2+195r^8t+35r^7s^3-147r^7s^2t-91r^7s^2-147r^7st^2-689r^7st+35r^7t^3-$$
$$91r^7t^2+35r^6s^4-147r^6s^3t-91r^6s^3+126r^6s^2t^2+455r^6s^2t-147r^6st^3+455r^6st^2+$$
$$35r^6t^4-91r^6t^3+35r^5s^5-147r^5s^4t-91r^5s^4+126r^5s^3t^2+455r^5s^3t+126r^5s^2t^3-$$
$$546r^5s^2t^2-147r^5st^4+455r^5st^3+35r^5t^5-91r^5t^4+35r^4s^6-147r^4s^5t-91r^4s^5+$$
$$126r^4s^4t^2+455r^4s^4t+126r^4s^3t^3-546r^4s^3t^2+126r^4s^2t^4-546r^4s^2t^3-147r^4st^5+$$
$$455r^4st^4+35r^4t^6-91r^4t^5+35r^3s^7-147r^3s^6t-91r^3s^6+126r^3s^5t^2+455r^3s^5t+$$
$$126r^3s^4t^3-546r^3s^4t^2+126r^3s^3t^4-546r^3s^3t^3+126r^3s^2t^5-546r^3s^2t^4-147r^3st^6+$$

$$
\begin{aligned}
&455r^3st^5+35r^3t^7-91r^3t^6+35r^2s^8-147r^2s^7t-91r^2s^7+126r^2s^6t^2+455r^2s^6t+\\
&126r^2s^5t^3-546r^2s^5t^2+126r^2s^4t^4-546r^2s^4t^3+126r^2s^3t^5-546r^2s^3t^4+126r^2s^2t^6-\\
&546r^2s^2t^5-147r^2st^7+455r^2st^6+35r^2t^8-91r^2t^7-95rs^9+295rs^8t+195rs^8-\\
&147rs^7t^2-689rs^7t-147rs^6t^3+455rs^6t^2-147rs^5t^4+455rs^5t^3-147rs^4t^5+\\
&455rs^4t^4-147rs^3t^6+455rs^3t^5-147rs^2t^7+455rs^2t^6+295rst^8-689rst^7-95rt^9+\\
&195rt^8+35s^{10}-95s^9t-65s^9+35s^8t^2+195s^8t+35s^7t^3-91s^7t^2+35s^6t^4-\\
&91s^6t^3+35s^5t^5-91s^5t^4+35s^4t^6-91s^4t^5+35s^3t^7-91s^3t^6+35s^2t^8-91s^2t^7-\\
&95st^9+195st^8+35t^{10}-65t^9)-\tfrac{(x-x_n)^{12}}{11880h^7(r-1)^2(s-1)^2(t-1)^2}(2r+2s+2t+1)-\\
&\tfrac{(x-x_n)^8}{1680h^3(r-1)^2(s-1)^2(t-1)^2}(2r^2s^2t+r^2s^2+2r^2st^2+4r^2st+r^2t^2+2rs^2t^2+4rs^2t+\\
&4rst^2+s^2t^2)+\tfrac{(x-x_n)^{11}}{7920h^6(r-1)^2(s-1)^2(t-1)^2}(r^2+4rs+4rt+2r+s^2+4st+2s+\\
&t^2+2t)-\tfrac{r^2s^2t^2(x-x_n)^6}{360h(r-1)^2(s-1)^2(t-1)^2}+\tfrac{rst(x-x_n)^7}{840h^2(r-1)^2(s-1)^2(t-1)^2}(2rs+2rt+2st+rst)-\\
&\tfrac{h^4rst(x-x_n)}{2162160(r-1)^2(s-1)^2(t-1)^2}(35r^9-95r^8s-95r^8t-65r^8+35r^7s^2+295r^7st+195r^7s+\\
&35r^7t^2+195r^7t+35r^6s^3-147r^6s^2t-91r^6s^2-147r^6st^2-689r^6st+35r^6t^3-\\
&91r^6t^2+35r^5s^4-147r^5s^3t-91r^5s^3+126r^5s^2t^2+455r^5s^2t-147r^5st^3+455r^5st^2+\\
&35r^5t^4-91r^5t^3+35r^4s^5-147r^4s^4t-91r^4s^4+126r^4s^3t^2+455r^4s^3t+126r^4s^2t^3-\\
&546r^4s^2t^2-147r^4st^4+455r^4st^3+35r^4t^5-91r^4t^4+35r^3s^6-147r^3s^5t-91r^3s^5+\\
&126r^3s^4t^2+455r^3s^4t+126r^3s^3t^3-546r^3s^3t^2+126r^3s^2t^4-546r^3s^2t^3-147r^3st^5+\\
&455r^3st^4+35r^3t^6-91r^3t^5+35r^2s^7-147r^2s^6t-91r^2s^6+126r^2s^5t^2+455r^2s^5t+\\
&126r^2s^4t^3-546r^2s^4t^2+126r^2s^3t^4-546r^2s^3t^3+126r^2s^2t^5-546r^2s^2t^4-147r^2st^6+\\
&455r^2st^5+35r^2t^7-91r^2t^6-95rs^8+295rs^7t+195rs^7-147rs^6t^2-689rs^6t-\\
&147rs^5t^3+455rs^5t^2-147rs^4t^4+455rs^4t^3-147rs^3t^5+455rs^3t^4-147rs^2t^6+\\
&455rs^2t^5+295rst^7-689rst^6-95rt^8+195rt^7+35s^9-95s^8t-65s^8+35s^7t^2+\\
&195s^7t+35s^6t^3-91s^6t^2+35s^5t^4-91s^5t^3+35s^4t^5-91s^4t^4+35s^3t^6-91s^3t^5+\\
&35s^2t^7-91s^2t^6-95st^8+195st^7+35t^9-65t^8).
\end{aligned}
$$

Now, (5.6) is evaluated at the non-interpolating point x_{n+1}, and (5.7) - (5.9) are evaluated at all points, i.e $x_n, x_{n+r}, x_{n+s}, x_{n+t}$ and x_{n+1} to produce a main block as shown below

$$\begin{aligned} H^{[4]_3}Y_{n+1}^{[4]_3} &= M_1^{[4]_3}Y_n^{[4]_3}+M_2^{[4]_3}Y_{n-1}^{[4]_3}+M_3^{[4]_3}Y_{n-2}^{[4]_3}+M_4^{[4]_3}Y_{n-3}^{[4]_3}+E_1^{[4]_3}F_n^{[4]_3} \\ &\quad +E_2^{[4]_3}F_{n+1}^{[4]_3}+K_1^{[4]_3}G_n^{[4]_3}+K_2^{[4]_3}G_{n+1}^{[4]_3} \end{aligned} \tag{5.10}$$

where

$$H^{[4]_3}=\begin{pmatrix} \frac{-(s-1)(t-1)}{r(r-s)(r-t)} & \frac{(r-1)(t-1)}{s(r-s)(s-t)} & \frac{-(r-1)(s-1)}{t(r-t)(s-t)} & 1 \\ \frac{-st}{hr(r-s)(r-t)} & \frac{rt}{hs(r-s)(s-t)} & \frac{-rs}{ht(r-t)(s-t)} & 0 \\ \frac{-6}{h^3r(r-s)(r-t)} & -\frac{2r+2t}{h^2s(r-s)(s-t)} & \frac{2r+2s}{h^2t(r-t)(s-t)} & 0 \\ \frac{-6}{h^3r(r-s)(r-t)} & \frac{6}{h^3s(r-s)(s-t)} & \frac{-6}{h^3t(r-t)(s-t)} & 0 \end{pmatrix},\ Y_{n+1}^{[4]_3}=\begin{pmatrix} y_{n+r} \\ y_{n+s} \\ y_{n+t} \\ y_{n+1} \end{pmatrix},$$

$$M_1^{[4]_3}=\begin{pmatrix} 0 & 0 & 0 & \frac{(r-1)(s-1)(t-1)}{rst} \\ 0 & 0 & 0 & \frac{-(rs+rt+st)}{hrst} \\ 0 & 0 & 0 & \frac{2(r+s+t)}{h^2rst} \\ 0 & 0 & 0 & \frac{-6}{h^3rst} \end{pmatrix},\ Y_n^{[4]_3}=\begin{pmatrix} y_{n-t} \\ y_{n-s} \\ y_{n-r} \\ y_n \end{pmatrix},$$

$$M_2^{[4]_3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},\ Y_{n-1}^{[4]_3}=\begin{pmatrix} y'_{n-t} \\ y'_{n-s} \\ y'_{n-r} \\ y'_n \end{pmatrix},\ \ Y_{n-2}^{[4]_3}=\begin{pmatrix} y''_{n-t} \\ y''_{n-s} \\ y''_{n-r} \\ y''_n \end{pmatrix},$$

$$M_3^{[4]_3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},\ Y_{n-3}^{[4]_3}=\begin{pmatrix} y'''_{n-t} \\ y'''_{n-s} \\ y'''_{n-r} \\ y'''_n \end{pmatrix},\ M_4^{[4]_3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$F_n^{[4]_3}=\begin{pmatrix} f_{n-t} \\ f_{n-s} \\ f_{n-r} \\ f_n \end{pmatrix},\ F_{n+1}^{[4]_3}=\begin{pmatrix} f_{n+r} \\ f_{n+s} \\ f_{n+t} \\ f_{n+1} \end{pmatrix},\ G_n^{[4]_3}=\begin{pmatrix} g_{n-t} \\ g_{n-s} \\ g_{n-r} \\ g_n \end{pmatrix},\ G_{n+1}^{[4]_3}=\begin{pmatrix} g_{n+r} \\ g_{n+s} \\ g_{n+t} \\ g_{n+1} \end{pmatrix}.$$

Then, multiplying both sides of Equation (5.10) by the inverse of $H^{[4]_3}$ gives

$$\begin{aligned} I_4 Y_{n+1}^{[4]_3} &= \hat{M}_1^{[4]_3} Y_n^{[4]_3} + h\hat{M}_2^{[4]_3} Y_{n-1}^{[4]_3} + h^2 \hat{M}_3^{[4]_3} Y_{n-2}^{[4]_3} + h^3 \hat{M}_4^{[4]_3} Y_{n-3}^{[4]_3} \\ &\quad + h^4 \left[\hat{E}_1^{[4]_3} F_n^{[4]_3} + \hat{E}_2^{[4]_3} F_{n+1}^{[4]_3} \right] + h^5 \left[\hat{K}_1^{[4]_3} G_n^{[4]_3} + \hat{K}_2^{[4]_3} G_{n+1}^{[4]_3} \right] \quad (5.11) \end{aligned}$$

where

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ \hat{M}_1^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ \hat{M}_2^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\hat{M}_3^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & \frac{r^2}{2} \\ 0 & 0 & 0 & \frac{s^2}{2} \\ 0 & 0 & 0 & \frac{t^2}{2} \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \ \hat{M}_4^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & \frac{r^3}{6} \\ 0 & 0 & 0 & \frac{s^3}{6} \\ 0 & 0 & 0 & \frac{t^3}{6} \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix},$$

$$\hat{E}_1^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & \hat{E}_{1_{14}}^{[4]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{24}}^{[4]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{34}}^{[4]_3} \\ 0 & 0 & 0 & \hat{E}_{1_{44}}^{[4]_3} \end{pmatrix}, \ \hat{E}_2^{[4]_3} = \begin{pmatrix} \hat{E}_{2_{11}}^{[4]_3} & \hat{E}_{2_{12}}^{[4]_3} & \hat{E}_{2_{13}}^{[4]_3} & \hat{E}_{2_{14}}^{[4]_3} \\ \hat{E}_{2_{21}}^{[4]_3} & \hat{E}_{2_{22}}^{[4]_3} & \hat{E}_{2_{23}}^{[4]_3} & \hat{E}_{2_{24}}^{[4]_3} \\ \hat{E}_{2_{31}}^{[4]_3} & \hat{E}_{2_{32}}^{[4]_3} & \hat{E}_{2_{33}}^{[4]_3} & \hat{E}_{2_{34}}^{[4]_3} \\ \hat{E}_{2_{41}}^{[4]_3} & \hat{E}_{2_{42}}^{[4]_3} & \hat{E}_{2_{43}}^{[4]_3} & \hat{E}_{2_{44}}^{[4]_3} \end{pmatrix},$$

$$\hat{K}_1^{[4]_3} = \begin{pmatrix} 0 & 0 & 0 & \hat{K}_{1_{14}}^{[4]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{24}}^{[4]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{34}}^{[4]_3} \\ 0 & 0 & 0 & \hat{K}_{1_{44}}^{[4]_3} \end{pmatrix}, \ \hat{K}_2^{[4]_3} = \begin{pmatrix} \hat{K}_{2_{11}}^{[4]_3} & \hat{K}_{2_{12}}^{[4]_3} & \hat{K}_{2_{13}}^{[4]_3} & \hat{K}_{2_{14}}^{[4]_3} \\ \hat{K}_{2_{21}}^{[4]_3} & \hat{K}_{2_{22}}^{[4]_3} & \hat{K}_{2_{23}}^{[4]_3} & \hat{K}_{2_{24}}^{[4]_3} \\ \hat{K}_{2_{31}}^{[4]_3} & \hat{K}_{2_{32}}^{[4]_3} & \hat{K}_{2_{33}}^{[4]_3} & \hat{K}_{2_{34}}^{[4]_3} \\ \hat{K}_{2_{41}}^{[4]_3} & \hat{K}_{2_{42}}^{[4]_3} & \hat{K}_{2_{43}}^{[4]_3} & \hat{K}_{2_{44}}^{[4]_3} \end{pmatrix}$$

where the entries of $\hat{E}_1^{[4]_3}$, $\hat{E}_2^{[4]_3}$, $\hat{K}_1^{[4]_3}$ and $\hat{K}_2^{[4]_3}$ are given as follows

$$\hat{E}^{[4]_3}_{1_{14}} = \frac{-r^4}{2162160s^3t^3}(-70r^7st - 70r^7s - 70r^7t + 260r^6s^2t + 260r^6s^2 + 260r^6st^2 + 645r^6st + 260r^6s + 260r^6t^2 + 260r^6t - 260r^5s^3t - 260r^5s^3 - 1040r^5s^2t^2 - 1820r^5s^2t - 1040r^5s^2 - 260r^5st^3 - 1820r^5st^2 - 1820r^5st - 260r^5s - 260r^5t^3 - 1040r^5t^2 - 260r^5t + 1144r^4s^3t^2 + 1742r^4s^3t + 1144r^4s^3 + 1144r^4s^2t^3 + 4680r^4s^2t^2 + 4680r^4s^2t + 1144r^4s^2 + 1742r^4st^3 + 4680r^4st^2 + 1742r^4st + 1144r^4t^3 + 1144r^4t^2 - 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3s^3t - 1430r^3s^3 - 4576r^3s^2t^3 - 6864r^3s^2t^2 - 4576r^3s^2t - 4576r^3st^3 - 4576r^3st^2 - 1430r^3t^3 + 5005r^2s^3t^3 + 2860r^2s^3t^2 + 5005r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 5005r^2st^3 + 8580rs^3t^3 + 8580rs^3t^2 + 8580rs^2t^3 - 77220s^3t^3),$$

$$\hat{E}^{[4]_3}_{1_{24}} = -\frac{s^4}{2162160r^3t^3}(-260r^3s^5t - 260r^3s^5 + 1144r^3s^4t^2 + 1742r^3s^4t + 1144r^3s^4 - 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3s^3t - 1430r^3s^3 + 5005r^3s^2t^3 + 2860r^3s^2t^2 + 5005r^3s^2t + 8580r^3st^3 + 8580r^3st^2 - 77220r^3t^3 + 260r^2s^6t + 260r^2s^6 - 1040r^2s^5t^2 - 1820r^2s^5t - 1040r^2s^5 + 1144r^2s^4t^3 + 4680r^2s^4t^2 + 4680r^2s^4t + 1144r^2s^4 - 4576r^2s^3t^3 - 6864r^2s^3t^2 - 4576r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 8580r^2st^3 - 70rs^7t - 70rs^7 + 260rs^6t^2 + 645rs^6t + 260rs^6 - 260rs^5t^3 - 1820rs^5t^2 - 1820rs^5t - 260rs^5 + 1742rs^4t^3 + 4680rs^4t^2 + 1742rs^4t - 4576rs^3t^3 - 4576rs^3t^2 + 5005rs^2t^3 - 70s^7t + 260s^6t^2 + 260s^6t - 260s^5t^3 - 1040s^5t^2 - 260s^5t + 1144s^4t^3 + 1144s^4t^2 - 1430s^3t^3),$$

$$\hat{E}^{[4]_3}_{1_{34}} = -\frac{t^4}{2162160r^3s^3}(-1430r^3s^3t^3 + 5005r^3s^3t^2 + 8580r^3s^3t - 77220r^3s^3 + 1144r^3s^2t^4 - 4576r^3s^2t^3 + 2860r^3s^2t^2 + 8580r^3s^2t - 260r^3st^5 + 1742r^3st^4 - 4576r^3st^3 + 5005r^3st^2 - 260r^3t^5 + 1144r^3t^4 - 1430r^3t^3 + 1144r^2s^3t^4 - 4576r^2s^3t^3 + 2860r^2s^3t^2 + 8580r^2s^3t - 1040r^2s^2t^5 + 4680r^2s^2t^4 - 6864r^2s^2t^3 + 2860r^2s^2t^2 + 260r^2st^6 - 1820r^2st^5 + 4680r^2st^4 - 4576r^2st^3 + 260r^2t^6 - 1040r^2t^5 + 1144r^2t^4 - 260rs^3t^5 + 1742rs^3t^4 - 4576rs^3t^3 + 5005rs^3t^2 + 260rs^2t^6 - 1820rs^2t^5 + 4680rs^2t^4 - 4576rs^2t^3 - 70rst^7 + 645rst^6 - 1820rst^5 + 1742rst^4 - 70rt^7 + 260rt^6 - 260rt^5 - 260s^3t^5 + 1144s^3t^4 - 1430s^3t^3 + 260s^2t^6 - 1040s^2t^5 + 1144s^2t^4 - 70st^7 + 260st^6 - 260st^5),$$

$$
\begin{aligned}
\hat{E}^{[4]_3}_{1_{44}} = & -\frac{1}{2162160r^3s^3t^3}(-77220r^3s^3t^3 + 8580r^3s^3t^2 + 5005r^3s^3t - 1430r^3s^3 + \\
& 8580r^3s^2t^3 + 2860r^3s^2t^2 - 4576r^3s^2t + 1144r^3s^2 + 5005r^3st^3 - 4576r^3st^2 + \\
& 1742r^3st - 260r^3s - 1430r^3t^3 + 1144r^3t^2 - 260r^3t + 8580r^2s^3t^3 + 2860r^2s^3t^2 - \\
& 4576r^2s^3t + 1144r^2s^3 + 2860r^2s^2t^3 - 6864r^2s^2t^2 + 4680r^2s^2t - 1040r^2s^2 - \\
& 4576r^2st^3 + 4680r^2st^2 - 1820r^2st + 260r^2s + 1144r^2t^3 - 1040r^2t^2 + 260r^2t + \\
& 5005rs^3t^3 - 4576rs^3t^2 + 1742rs^3t - 260rs^3 - 4576rs^2t^3 + 4680rs^2t^2 - 1820rs^2t + \\
& 260rs^2 + 1742rst^3 - 1820rst^2 + 645rst - 70rs - 260rt^3 + 260rt^2 - 70rt - 1430s^3t^3 + \\
& 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 + 260s^2t - 260st^3 + 260st^2 - 70st),
\end{aligned}
$$

$$
\begin{aligned}
\hat{E}^{[4]_3}_{2_{11}} = & \frac{r^4}{2162160(r-s)^3(r-t)^3(r-1)^3}(630r^9 - 2520r^8s - 2520r^8t - 2520r^8 + 3315r^7s^2 + \\
& 10390r^7st + 10390r^7s + 3315r^7t^2 + 10390r^7t + 3315r^7 - 1365r^6s^3 - 14105r^6s^2t - \\
& 14105r^6s^2 - 14105r^6st^2 - 44520r^6st - 14105r^6s - 1365r^6t^3 - 14105r^6t^2 - \\
& 14105r^6t - 1365r^6 + 5915r^5s^3t + 5915r^5s^3 + 19799r^5s^2t^2 + 63011r^5s^2t + \\
& 19799r^5s^2 + 5915r^5st^3 + 63011r^5st^2 + 63011r^5st + 5915r^5s + 5915r^5t^3 + \\
& 19799r^5t^2 + 5915r^5t - 8437r^4s^3t^2 - 27053r^4s^3t - 8437r^4s^3 - 8437r^4s^2t^3 - \\
& 93483r^4s^2t^2 - 93483r^4s^2t - 8437r^4s^2 - 27053r^4st^3 - 93483r^4st^2 - 27053r^4st - \\
& 8437r^4t^3 - 8437r^4t^2 + 3575r^3s^3t^3 + 41041r^3s^3t^2 + 41041r^3s^3t + 3575r^3s^3 + \\
& 41041r^3s^2t^3 + 148434r^3s^2t^2 + 41041r^3s^2t + 41041r^3st^3 + 41041r^3st^2 + 3575r^3t^3 - \\
& 17875r^2s^3t^3 - 67210r^2s^3t^2 - 17875r^2s^3t - 67210r^2s^2t^3 - 67210r^2s^2t^2 - \\
& 17875r^2st^3 + 30030rs^3t^3 + 30030rs^3t^2 + 30030rs^2t^3 - 12870s^3t^3),
\end{aligned}
$$

$$
\begin{aligned}
\hat{E}^{[4]_3}_{2_{21}} = & -\frac{s^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(-585r^4s^4 + 2340r^4s^3t + 2340r^4s^3 - 2574r^4s^2t^2 - \\
& 10296r^4s^2t - 2574r^4s^2 + 12870r^4st^2 + 12870r^4st - 19305r^4t^2 + 735r^3s^5 - \\
& 2405r^3s^4t - 2405r^3s^4 + 1222r^3s^3t^2 + 8528r^3s^3t + 1222r^3s^3 + 2002r^3s^2t^3 - \\
& 4576r^3s^2t^2 - 4576r^3s^2t + 2002r^3s^2 - 10010r^3st^3 + 715r^3st^2 - 10010r^3st + \\
& 15015r^3t^3 + 15015r^3t^2 - 210r^2s^6 + 245r^2s^5t + 245r^2s^5 + 1300r^2s^4t^2 + 715r^2s^4t + \\
& 1300r^2s^4 - 2210r^2s^3t^3 - 6318r^2s^3t^2 - 6318r^2s^3t - 2210r^2s^3 + 9152r^2s^2t^3 + \\
& 11154r^2s^2t^2 + 9152r^2s^2t - 7865r^2st^3 - 7865r^2st^2 - 10725r^2t^3 + 140rs^6t + 140rs^6 - \\
& 520rs^5t^2 - 705rs^5t - 520rs^5 + 520rs^4t^3 + 1300rs^4t^2 + 1300rs^4t + 520rs^4 - 910rs^3t^3 + \\
& 936rs^3t^2 - 910rs^3t - 3718rs^2t^3 - 3718rs^2t^2 + 9295rst^3 - 70s^6t + 260s^5t^2 + 260s^5t -
\end{aligned}
$$

$260s^4t^3 - 1040s^4t^2 - 260s^4t + 1144s^3t^3 + 1144s^3t^2 - 1430s^2t^3)$,

$\hat{E}^{[4]_3}_{2_{31}} = -\frac{t^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(-2574r^4s^2t^2 + 12870r^4s^2t - 19305r^4s^2 + 2340r^4st^3 - 10296r^4st^2 + 12870r^4st - 585r^4t^4 + 2340r^4t^3 - 2574r^4t^2 + 2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 + 1222r^3s^2t^3 - 4576r^3s^2t^2 + 715r^3s^2t + 15015r^3s^2 - 2405r^3st^4 + 8528r^3st^3 - 4576r^3st^2 - 10010r^3st + 735r^3t^5 - 2405r^3t^4 + 1222r^3t^3 + 2002r^3t^2 - 2210r^2s^3t^3 + 9152r^2s^3t^2 - 7865r^2s^3t - 10725r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t + 245r^2st^5 + 715r^2st^4 - 6318r^2st^3 + 9152r^2st^2 - 210r^2t^6 + 245r^2t^5 + 1300r^2t^4 - 2210r^2t^3 + 520rs^3t^4 - 910rs^3t^3 - 3718rs^3t^2 + 9295rs^3t - 520rs^2t^5 + 1300rs^2t^4 + 936rs^2t^3 - 3718rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 + 140rt^6 - 520rt^5 + 520rt^4 - 260s^3t^4 + 1144s^3t^3 - 1430s^3t^2 + 260s^2t^5 - 1040s^2t^4 + 1144s^2t^3 - 70st^6 + 260st^5 - 260st^4)$,

$\hat{E}^{[4]_3}_{2_{41}} = -\frac{1}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(-19305r^4s^2t^2 + 12870r^4s^2t - 2574r^4s^2 + 12870r^4st^2 - 10296r^4st + 2340r^4s - 2574r^4t^2 + 2340r^4t - 585r^4 + 15015r^3s^3t^2 - 10010r^3s^3t + 2002r^3s^3 + 15015r^3s^2t^3 + 715r^3s^2t^2 - 4576r^3s^2t + 1222r^3s^2 - 10010r^3st^3 - 4576r^3st^2 + 8528r^3st - 2405r^3s + 2002r^3t^3 + 1222r^3t^2 - 2405r^3t + 735r^3 - 10725r^2s^3t^3 - 7865r^2s^3t^2 + 9152r^2s^3t - 2210r^2s^3 - 7865r^2s^2t^3 + 11154r^2s^2t^2 - 6318r^2s^2t + 1300r^2s^2 + 9152r^2st^3 - 6318r^2st^2 + 715r^2st + 245r^2s - 2210r^2t^3 + 1300r^2t^2 + 245r^2t - 210r^2 + 9295rs^3t^3 - 3718rs^3t^2 - 910rs^3t + 520rs^3 - 3718rs^2t^3 + 936rs^2t^2 + 1300rs^2t - 520rs^2 - 910rst^3 + 1300rst^2 - 705rst + 140rs + 520rt^3 - 520rt^2 + 140rt - 1430s^3t^3 + 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 + 260s^2t - 260st^3 + 260st^2 - 70st)$,

$\hat{E}^{[4]_3}_{2_{12}} = \frac{r^8}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(-210r^6s^2 + 140r^6st + 140r^6s - 70r^6t + 735r^5s^3 + 245r^5s^2t + 245r^5s^2 - 520r^5st^2 - 705r^5st - 520r^5s + 260r^5t^2 + 260r^5t - 585r^4s^4 - 2405r^4s^3t - 2405r^4s^3 + 1300r^4s^2t^2 + 715r^4s^2t + 1300r^4s^2 + 520r^4st^3 + 1300r^4st^2 + 1300r^4st + 520r^4s - 260r^4t^3 - 1040r^4t^2 - 260r^4t + 2340r^3s^4t + 2340r^3s^4 + 1222r^3s^3t^2 + 8528r^3s^3t + 1222r^3s^3 - 2210r^3s^2t^3 - 6318r^3s^2t^2 - 6318r^3s^2t - 2210r^3s^2 - 910r^3st^3 + 936r^3st^2 - 910r^3st + 1144r^3t^3 + 1144r^3t^2 - 2574r^2s^4t^2 - 10296r^2s^4t - 2574r^2s^4 + 2002r^2s^3t^3 - 4576r^2s^3t^2 - 4576r^2s^3t + 2002r^2s^3 + 9152r^2$

$s^2t^3 + 11154r^2s^2t^2 + 9152r^2s^2t - 3718r^2st^3 - 3718r^2st^2 - 1430r^2t^3 + 12870rs^4t^2 + 12870rs^4t - 10010rs^3t^3 + 715rs^3t^2 - 10010rs^3t - 7865rs^2t^3 - 7865rs^2t^2 + 9295rst^3 - 19305s^4t^2 + 15015s^3t^3 + 15015s^3t^2 - 10725s^2t^3)$,

$\hat{E}_{2_{22}}^{[4]_3} = \frac{s^4}{2162160(r-s)^3(s-t)^3(s-1)^3}(1365r^3s^6 - 5915r^3s^5t - 5915r^3s^5 + 8437r^3s^4t^2 + 27053r^3s^4t + 8437r^3s^4 - 3575r^3s^3t^3 - 41041r^3s^3t^2 - 41041r^3s^3t - 3575r^3s^3 + 17875r^3s^2t^3 + 67210r^3s^2t^2 + 17875r^3s^2t - 30030r^3st^3 - 30030r^3st^2 + 12870r^3t^3 - 3315r^2s^7 + 14105r^2s^6t + 14105r^2s^6 - 19799r^2s^5t^2 - 63011r^2s^5t - 19799r^2s^5 + 8437r^2s^4t^3 + 93483r^2s^4t^2 + 93483r^2s^4t + 8437r^2s^4 - 41041r^2s^3t^3 - 148434r^2s^3t^2 - 41041r^2s^3t + 67210r^2s^2t^3 + 67210r^2s^2t^2 - 30030r^2st^3 + 2520rs^8 - 10390rs^7t - 10390rs^7 + 14105rs^6t^2 + 44520rs^6t + 14105rs^6 - 5915rs^5t^3 - 63011rs^5t^2 - 63011rs^5t - 5915rs^5 + 27053rs^4t^3 + 93483rs^4t^2 + 27053rs^4t - 41041rs^3t^3 - 41041rs^3t^2 + 17875rs^2t^3 - 630s^9 + 2520s^8t + 2520s^8 - 3315s^7t^2 - 10390s^7t - 3315s^7 + 1365s^6t^3 + 14105s^6t^2 + 14105s^6t + 1365s^6 - 5915s^5t^3 - 19799s^5t^2 - 5915s^5t + 8437s^4t^3 + 8437s^4t^2 - 3575s^3t^3)$,

$\hat{E}_{2_{32}}^{[4]_3} = \frac{t^8}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 - 2210r^3s^2t^3 + 9152r^3s^2t^2 - 7865r^3s^2t - 10725r^3s^2 + 520r^3st^4 - 910r^3st^3 - 3718r^3st^2 + 9295r^3st - 260r^3t^4 + 1144r^3t^3 - 1430r^3t^2 - 2574r^2s^4t^2 + 12870r^2s^4t - 19305r^2s^4 + 1222r^2s^3t^3 - 4576r^2s^3t^2 + 715r^2s^3t + 15015r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t - 520r^2st^5 + 1300r^2st^4 + 936r^2st^3 - 3718r^2st^2 + 260r^2t^5 - 1040r^2t^4 + 1144r^2t^3 + 2340rs^4t^3 - 10296rs^4t^2 + 12870rs^4t - 2405rs^3t^4 + 8528rs^3t^3 - 4576rs^3t^2 - 10010rs^3t + 245rs^2t^5 + 715rs^2t^4 - 6318rs^2t^3 + 9152rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 - 70rt^6 + 260rt^5 - 260rt^4 - 585s^4t^4 + 2340s^4t^3 - 2574s^4t^2 + 735s^3t^5 - 2405s^3t^4 + 1222s^3t^3 + 2002s^3t^2 - 210s^2t^6 + 245s^2t^5 + 1300s^2t^4 - 2210s^2t^3 + 140st^6 - 520st^5 + 520st^4)$,

$\hat{E}_{2_{42}}^{[4]_3} = \frac{1}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(15015r^3s^3t^2 - 10010r^3s^3t + 2002r^3s^3 - 10725r^3s^2t^3 - 7865r^3s^2t^2 + 9152r^3s^2t - 2210r^3s^2 + 9295r^3st^3 - 3718r^3st^2 - 910r^3st + 520r^3s - 1430r^3t^3 + 1144r^3t^2 - 260r^3t - 19305r^2s^4t^2 + 12870r^2s^4t -$

$2574r^2s^4 + 15015r^2s^3t^3 + 715r^2s^3t^2 - 4576r^2s^3t + 1222r^2s^3 - 7865r^2s^2t^3 + 11154r^2s^2t^2 - 6318r^2s^2t + 1300r^2s^2 - 3718r^2st^3 + 936r^2st^2 + 1300r^2st - 520r^2s + 1144r^2t^3 - 1040r^2t^2 + 260r^2t + 12870rs^4t^2 - 10296rs^4t + 2340rs^4 - 10010rs^3t^3 - 4576rs^3t^2 + 8528rs^3t - 2405rs^3 + 9152rs^2t^3 - 6318rs^2t^2 + 715rs^2t + 245rs^2 - 910rst^3 + 1300rst^2 - 705rst + 140rs - 260rt^3 + 260rt^2 - 70rt - 2574s^4t^2 + 2340s^4t - 585s^4 + 2002s^3t^3 + 1222s^3t^2 - 2405s^3t + 735s^3 - 2210s^2t^3 + 1300s^2t^2 + 245s^2t - 210s^2 + 520st^3 - 520st^2 + 140st)$,

$\hat{E}^{[4]_3}_{2_{13}} = \frac{-r^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(140r^6st - 70r^6s - 210r^6t^2 + 140r^6t - 520r^5s^2t + 260r^5s^2 + 245r^5st^2 - 705r^5st + 260r^5s + 735r^5t^3 + 245r^5t^2 - 520r^5t + 520r^4s^3t - 260r^4s^3 + 1300r^4s^2t^2 + 1300r^4s^2t - 1040r^4s^2 - 2405r^4st^3 + 715r^4st^2 + 1300r^4st - 260r^4s - 585r^4t^4 - 2405r^4t^3 + 1300r^4t^2 + 520r^4t - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 + 1222r^3s^2t^3 - 6318r^3s^2t^2 + 936r^3s^2t + 1144r^3s^2 + 2340r^3st^4 + 8528r^3st^3 - 6318r^3st^2 - 910r^3st + 2340r^3t^4 + 1222r^3t^3 - 2210r^3t^2 + 2002r^2s^3t^3 + 9152r^2s^3t^2 - 3718r^2s^3t - 1430r^2s^3 - 2574r^2s^2t^4 - 4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t - 10296r^2st^4 - 4576r^2st^3 + 9152r^2st^2 - 2574r^2t^4 + 2002r^2t^3 - 10010rs^3t^3 - 7865rs^3t^2 + 9295rs^3t + 12870rs^2t^4 + 715rs^2t^3 - 7865rs^2t^2 + 12870rst^4 - 10010rst^3 + 15015s^3t^3 - 10725s^3t^2 - 19305s^2t^4 + 15015s^2t^3)$,

$\hat{E}^{[4]_3}_{2_{23}} = \frac{-s^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(520r^3s^4t - 260r^3s^4 - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 + 2002r^3s^2t^3 + 9152r^3s^2t^2 - 3718r^3s^2t - 1430r^3s^2 - 10010r^3st^3 - 7865r^3st^2 + 9295r^3st + 15015r^3t^3 - 10725r^3t^2 - 520r^2s^5t + 260r^2s^5 + 1300r^2s^4t^2 + 1300r^2s^4t - 1040r^2s^4 + 1222r^2s^3t^3 - 6318r^2s^3t^2 + 936r^2s^3t + 1144r^2s^3 - 2574r^2s^2t^4 - 4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t + 12870r^2st^4 + 715r^2st^3 - 7865 r^2st^2 - 19305r^2t^4 + 15015r^2t^3 + 140rs^6t - 70rs^6 + 245rs^5t^2 - 705rs^5t + 260rs^5 - 2405rs^4t^3 + 715rs^4t^2 + 1300rs^4t - 260rs^4 + 2340rs^3t^4 + 8528rs^3t^3 - 6318rs^3t^2 - 910rs^3t - 10296rs^2t^4 - 4576rs^2t^3 + 9152rs^2t^2 + 12870rst^4 - 10010rst^3 - 210s^6t^2 + 140s^6t + 735s^5t^3 + 245s^5t^2 - 520s^5t - 585s^4t^4 - 2405s^4t^3 + 1300s^4t^2 + 520s^4t + 2340s^3t^4 + 1222s^3t^3 - 2210s^3t^2 - 2574s^2t^4 + 2002s^2t^3)$,

$$\hat{E}^{[4]_3}_{2_{33}} = \frac{-t^4}{2162160(r-t)^3(s-t)^3(t-1)^3}(-3575r^3s^3t^3 + 17875r^3s^3t^2 - 30030r^3s^3t + 12870r^3s^3 + 8437r^3s^2t^4 - 41041r^3s^2t^3 + 67210r^3s^2t^2 - 30030r^3s^2t - 5915r^3st^5 + 27053r^3st^4 - 41041r^3st^3 + 17875r^3st^2 + 1365r^3t^6 - 5915r^3t^5 + 8437r^3t^4 - 3575r^3t^3 + 8437r^2s^3t^4 - 41041r^2s^3t^3 + 67210r^2s^3t^2 - 30030r^2s^3t - 19799r^2s^2t^5 + 93483r^2s^2t^4 - 148434r^2s^2t^3 + 67210r^2s^2t^2 + 14105r^2st^6 - 63011r^2st^5 + 93483r^2st^4 - 41041r^2st^3 - 3315r^2t^7 + 14105r^2t^6 - 19799r^2t^5 + 8437r^2t^4 - 5915rs^3t^5 + 27053rs^3t^4 - 41041rs^3t^3 + 17875rs^3t^2 + 14105rs^2t^6 - 63011rs^2t^5 + 93483rs^2t^4 - 41041rs^2t^3 - 10390rst^7 + 44520rst^6 - 63011rst^5 + 27053rst^4 + 2520rt^8 - 10390rt^7 + 14105rt^6 - 5915rt^5 + 1365s^3t^6 - 5915s^3t^5 + 8437s^3t^4 - 3575s^3t^3 - 3315s^2t^7 + 14105s^2t^6 - 19799s^2t^5 + 8437s^2t^4 + 2520st^8 - 10390st^7 + 14105st^6 - 5915st^5 - 630t^9 + 2520t^8 - 3315t^7 + 1365t^6),$$

$$\hat{E}^{[4]_3}_{2_{43}} = \frac{-1}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(-10725r^3s^3t^2 + 9295r^3s^3t - 1430r^3s^3 + 15015r^3s^2t^3 - 7865r^3s^2t^2 - 3718r^3s^2t + 1144r^3s^2 - 10010r^3st^3 + 9152r^3st^2 - 910r^3st - 260r^3s + 2002r^3t^3 - 2210r^3t^2 + 520r^3t + 15015r^2s^3t^3 - 7865r^2s^3t^2 - 3718r^2s^3t + 1144r^2s^3 - 19305r^2s^2t^4 + 715r^2s^2t^3 + 11154r^2s^2t^2 + 936r^2s^2t - 1040r^2s^2 + 12870r^2st^4 - 4576r^2st^3 - 6318r^2st^2 + 1300r^2st + 260r^2s - 2574r^2t^4 + 1222r^2t^3 + 1300r^2t^2 - 520r^2t - 10010rs^3t^3 + 9152rs^3t^2 - 910rs^3t - 260rs^3 + 12870rs^2t^4 - 4576rs^2t^3 - 6318rs^2t^2 + 1300rs^2t + 260rs^2 - 10296rst^4 + 8528rst^3 + 715rst^2 - 705rst - 70rs + 2340rt^4 - 2405rt^3 + 245rt^2 + 140rt + 2002s^3t^3 - 2210s^3t^2 + 520s^3t - 2574s^2t^4 + 1222s^2t^3 + 1300s^2t^2 - 520s^2t + 2340st^4 - 2405st^3 + 245st^2 + 140st - 585t^4 + 735t^3 - 210t^2),$$

$$\hat{E}^{[4]_3}_{2_{14}} = \frac{r^8}{2162160(r-1)^3(s-1)^3(t-1)^3}(-70r^6st + 140r^6s + 140r^6t - 210r^6 + 260r^5s^2t - 520r^5s^2 + 260r^5st^2 - 705r^5st + 245r^5s - 520r^5t^2 + 245r^5t + 735r^5 - 260r^4s^3t + 520r^4s^3 - 1040r^4s^2t^2 + 1300r^4s^2t + 1300r^4s^2 - 260r^4st^3 + 1300r^4st^2 + 715r^4st - 2405r^4s + 520r^4t^3 + 1300r^4t^2 - 2405r^4t - 585r^4 + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 + 1144r^3s^2t^3 + 936r^3s^2t^2 - 6318r^3s^2t + 1222r^3s^2 - 910r^3st^3 - 6318r^3st^2 + 8528r^3st + 2340r^3s - 2210r^3t^3 + 1222r^3t^2 + 2340r^3t - 1430r^2s^3t^3 - 3718r^2s^3t^2 + 9152r^2s^3t + 2002r^2s^3 - 3718r^2s^2t^3 + 11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 + 9152$$

$$r^2st^3 - 4576r^2st^2 - 10296r^2st + 2002r^2t^3 - 2574r^2t^2 + 9295rs^3t^3 - 7865rs^3t^2 - 10010rs^3t - 7865rs^2t^3 + 715rs^2t^2 + 12870rs^2t - 10010rst^3 + 12870rst^2 - 10725s^3t^3 + 15015s^3t^2 + 15015s^2t^3 - 19305s^2t^2),$$

$$\hat{E}^{[4]_3}_{2_{24}} = \frac{s^8}{2162160(r-1)^3(s-1)^3(t-1)^3}(-260r^3s^4t + 520r^3s^4 + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 - 1430r^3s^2t^3 - 3718r^3s^2t^2 + 9152r^3s^2t + 2002r^3s^2 + 9295r^3st^3 - 7865r^3st^2 - 10010r^3st - 10725r^3t^3 + 15015r^3t^2 + 260r^2s^5t - 520r^2s^5 - 1040r^2s^4t^2 + 1300r^2s^4t + 1300r^2s^4 + 1144r^2s^3t^3 + 936r^2s^3t^2 - 6318r^2s^3t + 1222r^2s^3 - 3718r^2s^2t^3 + 11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 - 7865r^2st^3 + 715r^2st^2 + 12870r^2st + 15015r^2t^3 - 19305r^2t^2 - 70rs^6t + 140rs^6 + 260rs^5t^2 - 705rs^5t + 245rs^5 - 260rs^4t^3 + 1300rs^4t^2 + 715rs^4t - 2405rs^4 - 910rs^3t^3 - 6318rs^3t^2 + 8528rs^3t + 2340rs^3 + 9152rs^2t^3 - 4576rs^2t^2 - 10296rs^2t - 10010rst^3 + 12870rst^2 + 140s^6t - 210s^6 - 520s^5t^2 + 245s^5t + 735s^5 + 520s^4t^3 + 1300s^4t^2 - 2405s^4t - 585s^4 - 2210s^3t^3 + 1222s^3t^2 + 2340s^3t + 2002s^2t^3 - 2574s^2t^2),$$

$$\hat{E}^{[4]_3}_{2_{34}} = \frac{t^8}{2162160(r-1)^3(s-1)^3(t-1)^3}(-1430r^3s^3t^2 + 9295r^3s^3t - 10725r^3s^3 + 1144r^3s^2t^3 - 3718r^3s^2t^2 - 7865r^3s^2t + 15015r^3s^2 - 260r^3st^4 - 910r^3st^3 + 9152r^3st^2 - 10010r^3st + 520r^3t^4 - 2210r^3t^3 + 2002r^3t^2 + 1144r^2s^3t^3 - 3718r^2s^3t^2 - 7865r^2s^3t + 15015r^2s^3 - 1040r^2s^2t^4 + 936r^2s^2t^3 + 11154r^2s^2t^2 + 715r^2s^2t - 19305r^2s^2 + 260r^2st^5 + 1300r^2st^4 - 6318r^2st^3 - 4576r^2st^2 + 12870r^2st - 520r^2t^5 + 1300r^2t^4 + 1222r^2t^3 - 2574r^2t^2 - 260rs^3t^4 - 910rs^3t^3 + 9152rs^3t^2 - 10010rs^3t + 260rs^2t^5 + 1300rs^2t^4 - 6318rs^2t^3 - 4576rs^2t^2 + 12870rs^2t - 70rst^6 - 705rst^5 + 715rst^4 + 8528rst^3 - 10296rst^2 + 140rt^6 + 245rt^5 - 2405rt^4 + 2340rt^3 + 520s^3t^4 - 2210s^3t^3 + 2002s^3t^2 - 520s^2t^5 + 1300s^2t^4 + 1222s^2t^3 - 2574s^2t^2 + 140st^6 + 245st^5 - 2405st^4 + 2340st^3 - 210t^6 + 735t^5 - 585t^4),$$

$$\hat{E}^{[4]_3}_{2_{44}} = \frac{1}{2162160(r-1)^3(s-1)^3(t-1)^3}(12870r^3s^3t^3 - 30030r^3s^3t^2 + 17875r^3s^3t - 3575r^3s^3 - 30030r^3s^2t^3 + 67210r^3s^2t^2 - 41041r^3s^2t + 8437r^3s^2 + 17875r^3st^3 - 41041r^3st^2 + 27053r^3st - 5915r^3s - 3575r^3t^3 + 8437r^3t^2 - 5915r^3t + 1365r^3 - 30030r^2s^3t^3 + 67210r^2s^3t^2 - 41041r^2s^3t + 8437r^2s^3 + 67210r^2s^2t^3 - 148434r^2s^2t^2 +$$

$93483r^2s^2t - 19799r^2s^2 - 41041r^2st^3 + 93483r^2st^2 - 63011r^2st + 14105r^2s + 8437r^2t^3 - 19799r^2t^2 + 14105r^2t - 3315r^2 + 17875rs^3t^3 - 41041rs^3t^2 + 27053rs^3t - 5915rs^3 - 41041rs^2t^3 + 93483rs^2t^2 - 63011rs^2t + 14105rs^2 + 27053rst^3 - 63011rst^2 + 44520rst - 10390rs - 5915rt^3 + 14105rt^2 - 10390rt + 2520r - 3575s^3t^3 + 8437s^3t^2 - 5915s^3t + 1365s^3 + 8437s^2t^3 - 19799s^2t^2 + 14105s^2t - 3315s^2 - 5915st^3 + 14105st^2 - 10390st + 2520s + 1365t^3 - 3315t^2 + 2520t - 630)$,

$\hat{K}_{1_{14}}^{[4]_3} = \frac{r^5}{2162160s^2t^2}(35r^6 - 130r^5s - 130r^5t - 130r^5 + 130r^4s^2 + 520r^4st + 520r^4s + 130r^4t^2 + 520r^4t + 130r^4 - 572r^3s^2t - 572r^3s^2 - 572r^3st^2 - 2288r^3st - 572r^3s - 572r^3t^2 - 572r^3t + 715r^2s^2t^2 + 2860r^2s^2t + 715r^2s^2 + 2860r^2st^2 + 2860r^2st + 715r^2t^2 - 4290rs^2t^2 - 4290rs^2t - 4290rst^2 + 8580s^2t^2)$,

$\hat{K}_{1_{24}}^{[4]_3} = \frac{s^5}{2162160r^2t^2}(130r^2s^4 - 572r^2s^3t - 572r^2s^3 + 715r^2s^2t^2 + 2860r^2s^2t + 715r^2s^2 - 4290r^2st^2 - 4290r^2st + 8580r^2t^2 - 130rs^5 + 520rs^4t + 520rs^4 - 572rs^3t^2 - 2288rs^3t - 572rs^3 + 2860rs^2t^2 + 2860rs^2t - 4290rst^2 + 35s^6 - 130s^5t - 130s^5 + 130s^4t^2 + 520s^4t + 130s^4 - 572s^3t^2 - 572s^3t + 715s^2t^2)$,

$\hat{K}_{1_{34}}^{[4]_3} = \frac{t^5}{2162160r^2s^2}(715r^2s^2t^2 - 4290r^2s^2t + 8580r^2s^2 - 572r^2st^3 + 2860r^2st^2 - 4290r^2st + 130r^2t^4 - 572r^2t^3 + 715r^2t^2 - 572rs^2t^3 + 2860rs^2t^2 - 4290rs^2t + 520rst^4 - 2288rst^3 + 2860rst^2 - 130rt^5 + 520rt^4 - 572rt^3 + 130s^2t^4 - 572s^2t^3 + 715s^2t^2 - 130st^5 + 520st^4 - 572st^3 + 35t^6 - 130t^5 + 130t^4)$,

$\hat{K}_{1_{44}}^{[4]_3} = \frac{1}{2162160r^2s^2t^2}(8580r^2s^2t^2 - 4290r^2s^2t + 715r^2s^2 - 4290r^2st^2 + 2860r^2st - 572r^2s + 715r^2t^2 - 572r^2t + 130r^2 - 4290rs^2t^2 + 2860rs^2t - 572rs^2 + 2860rst^2 - 2288rst + 520rs - 572rt^2 + 520rt - 130r + 715s^2t^2 - 572s^2t + 130s^2 - 572st^2 + 520st - 130s + 130t^2 - 130t + 35)$,

$\hat{K}_{2_{11}}^{[4]_3} = \frac{-r^5}{1081080(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 91r^5s - 91r^5t - 91r^5 + 78r^4s^2 + 312r^4st + 312r^4s + 78r^4t^2 + 312r^4t + 78r^4 - 286r^3s^2t - 286r^3s^2 - 286r^3st^2 - 1144r^3st - 286r^3s - 286r^3t^2 - 286r^3t + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 + 1144r^2st^2 + 1144r^2st + 286r^2t^2 - 1287rs^2t^2 - 1287rs^2t - 1287rst^2 + 1716s^2t^2)$,

$\hat{K}_{2_{21}}^{[4]_3} = \frac{-s^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(572s^2t^2 - 130s^3t^2 + 286rs^2 - 260rs^3 + 65rs^4 + 2145rt^2 - 715st^2 + 572s^2t - 520s^3t + 130s^4t - 130s^3 + 130s^4 - 35s^5 - 1430rst^2 + 1144rs^2t - 260rs^3t + 286rs^2t^2 - 1430rst),$

$\hat{K}_{2_{31}}^{[4]_3} = \frac{-t^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(572s^2t^2 - 130s^2t^3 + 2145rs^2 + 286rt^2 - 260rt^3 + 65rt^4 + 572st^2 - 715s^2t - 520st^3 + 130st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430rs^2t - 260rst^3 + 286rs^2t^2 - 1430rst),$

$\hat{K}_{2_{41}}^{[4]_3} = \frac{-1}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(65r + 130s + 130t - 715s^2t^2 - 260rs - 260rt - 520st + 286rs^2 + 286rt^2 + 572st^2 + 572s^2t - 130s^2 - 130t^2 - 1430rst^2 - 1430rs^2t + 2145rs^2t^2 + 1144rst - 35),$

$\hat{K}_{2_{12}}^{[4]_3} = \frac{-r^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(572r^2t^2 - 130r^3t^2 + 286r^2s - 260r^3s + 65r^4s - 715rt^2 + 572r^2t - 520r^3t + 130r^4t + 2145st^2 - 130r^3 + 130r^4 - 35r^5 - 1430rst^2 + 1144r^2st - 260r^3st + 286r^2st^2 - 1430rst),$

$\hat{K}_{2_{22}}^{[4]_3} = \frac{-s^5}{1081080(r-s)^2(s-t)^2(s-1)^2}(78r^2s^4 - 286r^2s^3t - 286r^2s^3 + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 - 1287r^2st^2 - 1287r^2st + 1716r^2t^2 - 91rs^5 + 312rs^4t + 312rs^4 - 286rs^3t^2 - 1144rs^3t - 286rs^3 + 1144rs^2t^2 + 1144rs^2t - 1287rst^2 + 28s^6 - 91s^5t - 91s^5 + 78s^4t^2 + 312s^4t + 78s^4 - 286s^3t^2 - 286s^3t + 286s^2t^2),$

$\hat{K}_{2_{32}}^{[4]_3} = \frac{-t^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(572r^2t^2 - 130r^2t^3 + 2145r^2s + 572rt^2 - 715r^2t - 520rt^3 + 130rt^4 + 286st^2 - 260st^3 + 65st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430r^2st - 260rst^3 + 286r^2st^2 - 1430rst),$

$\hat{K}_{2_{42}}^{[4]_3} = \frac{-1}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(130r + 65s + 130t - 715r^2t^2 - 260rs - 520rt - 260st + 286r^2s + 572rt^2 + 572r^2t + 286st^2 - 130r^2 - 130t^2 - 1430rst^2 - 1430r^2st + 2145r^2st^2 + 1144rst - 35),$

$\hat{K}_{2_{13}}^{[4]_3} = \frac{-r^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(572r^2s^2 - 130r^3s^2 - 715rs^2 + 572r^2s - 520r^3s + 130r^4s + 286r^2t - 260r^3t + 65r^4t + 2145s^2t - 130r^3 + 130r^4 - 35r^5 - 1430rs^2t + 1144r^2st - 260r^3st + 286r^2s^2t - 1430rst),$

$\hat{K}_{2_{23}}^{[4]_3} = \frac{-s^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(572r^2s^2 - 130r^2s^3 + 572rs^2 - 715r^2s - 520rs^3 + 130rs^4 + 2145r^2t + 286s^2t - 260s^3t + 65s^4t - 130s^3 + 130s^4 - 35s^5 + 1144rs^2t - 1430r^2st - 260rs^3t + 286r^2s^2t - 1430rst),$

$\hat{K}_{2_{33}}^{[4]_3} = \frac{-t^5}{1081080(r-t)^2(s-t)^2(t-1)^2}(286r^2s^2t^2 - 1287r^2s^2t + 1716r^2s^2 - 286r^2st^3 + 1144r^2st^2 - 1287r^2st + 78r^2t^4 - 286r^2t^3 + 286r^2t^2 - 286rs^2t^3 + 1144rs^2t^2 - 1287rs^2t + 312rst^4 - 1144rst^3 + 1144rst^2 - 91rt^5 + 312rt^4 - 286rt^3 + 78s^2t^4 - 286s^2t^3 + 286s^2t^2 - 91st^5 + 312st^4 - 286st^3 + 28t^6 - 91t^5 + 78t^4),$

$\hat{K}_{2_{43}}^{[4]_3} = \frac{-1}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(130r + 130s + 65t - 715r^2s^2 - 520rs - 260rt - 260st + 572rs^2 + 572r^2s + 286r^2t + 286s^2t - 130r^2 - 130s^2 - 1430rs^2t - 1430r^2st + 2145r^2s^2t + 1144rst - 35),$

$\hat{K}_{2_{14}}^{[4]_3} = \frac{-r^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(-35r^5 + 130r^4s + 130r^4t + 65r^4 - 130r^3s^2 - 520r^3st - 260r^3s - 130r^3t^2 - 260r^3t + 572r^2s^2t + 286r^2s^2 + 572r^2st^2 + 1144r^2st + 286r^2t^2 - 715rs^2t^2 - 1430rs^2t - 1430rst^2 + 2145s^2t^2),$

$\hat{K}_{2_{24}}^{[4]_3} = \frac{-s^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(-130r^2s^3 + 572r^2s^2t + 286r^2s^2 - 715r^2st^2 - 1430r^2st + 2145r^2t^2 + 130rs^4 - 520rs^3t - 260rs^3 + 572rs^2t^2 + 1144rs^2t - 1430rst^2 - 35s^5 + 130s^4t + 65s^4 - 130s^3t^2 - 260s^3t + 286s^2t^2),$

$\hat{K}_{2_{34}}^{[4]_3} = \frac{-t^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(-715r^2s^2t + 2145r^2s^2 + 572r^2st^2 - 1430r^2st - 130r^2t^3 + 286r^2t^2 + 572rs^2t^2 - 1430rs^2t - 520rst^3 + 1144rst^2 + 130rt^4 - 260rt^3 - 130s^2t^3 + 286s^2t^2 + 130st^4 - 260st^3 - 35t^5 + 65t^4),$

$\hat{K}_{2_{44}}^{[4]_3} = \frac{-1}{1081080(r-1)^2(s-1)^2(t-1)^2}(1716r^2s^2t^2 - 1287r^2s^2t + 286r^2s^2 - 1287r^2st^2 + 1144r^2st - 286r^2s + 286r^2t^2 - 286r^2t + 78r^2 - 1287rs^2t^2 + 1144rs^2t - 286rs^2 + 1144rst^2 - 1144rst + 312rs - 286rt^2 + 312rt - 91r + 286s^2t^2 - 286s^2t + 78s^2 - 286st^2 + 312st - 91s + 78t^2 - 91t + 28).$

From the main block (5.11), we obtain the following equations

$$
\begin{aligned}
y_{n+r} = {} & y_n + hry'_n + \frac{h^2r^2y''_n}{2} + \frac{h^3r^3y'''_n}{6} + \frac{g_nh^5r^5}{2162160s^2t^2}(35r^6 - 130r^5s - 130r^5t - 130r^5 + \\
& 130r^4s^2 + 520r^4st + 520r^4s + 130r^4t^2 + 520r^4t + 130r^4 - 572r^3s^2t - 572r^3s^2 - \\
& 572r^3st^2 - 2288r^3st - 572r^3s - 572r^3t^2 - 572r^3t + 715r^2s^2t^2 + 2860r^2s^2t + \\
& 715r^2s^2 + 2860r^2st^2 + 2860r^2st + 715r^2t^2 - 4290rs^2t^2 - 4290rs^2t - 4290rst^2 + \\
& 8580s^2t^2) - \frac{f_nh^4r^4}{2162160s^3t^3}(260r^6s^2t - 70r^7s - 70r^7t - 70r^7st + 260r^6s^2 + 260r^6st^2 + \\
& 645r^6st + 260r^6s + 260r^6t^2 + 260r^6t - 260r^5s^3t - 260r^5s^3 - 1040r^5s^2t^2 - \\
& 1820r^5s^2t - 1040r^5s^2 - 260r^5st^3 - 1820r^5st^2 - 1820r^5st - 260r^5s - 260r^5t^3 - \\
& 1040r^5t^2 - 260r^5t + 1144r^4s^3t^2 + 1742r^4s^3t + 1144r^4s^3 + 1144r^4s^2t^3 + 4680r^4s^2t^2 + \\
& 4680r^4s^2t + 1144r^4s^2 + 1742r^4st^3 + 4680r^4st^2 + 1742r^4st + 1144r^4t^3 + 1144r^4t^2 - \\
& 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3s^3t - 1430r^3s^3 - 4576r^3s^2t^3 - 6864r^3s^2t^2 - \\
& 4576r^3s^2t - 4576r^3st^3 - 4576r^3st^2 - 1430r^3t^3 + 5005r^2s^3t^3 + 2860r^2s^3t^2 + \\
& 5005r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 5005r^2st^3 + 8580rs^3t^3 + 8580rs^3t^2 + \\
& 8580rs^2t^3 - 77220s^3t^3) - \frac{g_{n+1}h^5r^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(130r^4s - 35r^5 + 130r^4t + \\
& 65r^4 - 130r^3s^2 - 520r^3st - 260r^3s - 130r^3t^2 - 260r^3t + 572r^2s^2t + 286r^2s^2 + \\
& 572r^2st^2 + 1144r^2st + 286r^2t^2 - 715rs^2t^2 - 1430rs^2t - 1430rst^2 + 2145s^2t^2) - \\
& \frac{g_{n+r}h^5r^5}{1081080(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 91r^5s - 91r^5t - 91r^5 + 78r^4s^2 + 312r^4st + 312r^4s + \\
& 78r^4t^2 + 312r^4t + 78r^4 - 286r^3s^2t - 286r^3s^2 - 286r^3st^2 - 1144r^3st - 286r^3s - \\
& 286r^3t^2 - 286r^3t + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 + 1144r^2st^2 + 1144r^2st + \\
& 286r^2t^2 - 1287rs^2t^2 - 1287rs^2t - 1287rst^2 + 1716s^2t^2) + \frac{f_{n+1}h^4r^8}{2162160(r-1)^3(s-1)^3(t-1)^3} \\
& (140r^6s - 70r^6st + 140r^6t - 210r^6 + 260r^5s^2t - 520r^5s^2 + 260r^5st^2 - 705r^5st + \\
& 245r^5s - 520r^5t^2 + 245r^5t + 735r^5 - 260r^4s^3t + 520r^4s^3 - 1040r^4s^2t^2 + 1300r^4s^2t + \\
& 1300r^4s^2 - 260r^4st^3 + 1300r^4st^2 + 715r^4st - 2405r^4s + 520r^4t^3 + 1300r^4t^2 - \\
& 2405r^4t - 585r^4 + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 + 1144r^3s^2t^3 + 936r^3s^2t^2 - \\
& 6318r^3s^2t + 1222r^3s^2 - 910r^3st^3 - 6318r^3st^2 + 8528r^3st + 2340r^3s - 2210r^3t^3 + \\
& 1222r^3t^2 + 2340r^3t - 1430r^2s^3t^3 - 3718r^2s^3t^2 + 9152r^2s^3t + 2002r^2s^3 - \\
& 3718r^2s^2t^3 + 11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 + 9152r^2st^3 - 4576r^2st^2 - \\
& 10296r^2st + 2002r^2t^3 - 2574r^2t^2 + 9295rs^3t^3 - 7865rs^3t^2 - 10010rs^3t - 7865rs^2t^3 +
\end{aligned}
$$

$$715rs^2t^2 + 12870rs^2t - 10010rst^3 + 12870rst^2 - 10725s^3t^3 + 15015s^3t^2 +$$
$$15015s^2t^3 - 19305s^2t^2) + \frac{f_{n+r}h^4r^4}{2162160(r-s)^3(r-t)^3(r-1)^3}(630r^9 - 2520r^8s - 2520r^8t -$$
$$2520r^8 + 3315r^7s^2 + 10390r^7st + 10390r^7s + 3315r^7t^2 + 10390r^7t + 3315r^7 -$$
$$1365r^6s^3 - 14105r^6s^2t - 14105r^6s^2 - 14105r^6st^2 - 44520r^6st - 14105r^6s -$$
$$1365r^6t^3 - 14105r^6t^2 - 14105r^6t - 1365r^6 + 5915r^5s^3t + 5915r^5s^3 +$$
$$19799r^5s^2t^2 + 63011r^5s^2t + 19799r^5s^2 + 5915r^5st^3 + 63011r^5st^2 + 63011r^5st +$$
$$5915r^5s + 5915r^5t^3 + 19799r^5t^2 + 5915r^5t - 8437r^4s^3t^2 - 27053r^4s^3t -$$
$$8437r^4s^3 - 8437r^4s^2t^3 - 93483r^4s^2t^2 - 93483r^4s^2t - 8437r^4s^2 - 27053r^4st^3 -$$
$$93483r^4st^2 - 27053r^4st - 8437r^4t^3 - 8437r^4t^2 + 3575r^3s^3t^3 + 41041r^3s^3t^2 +$$
$$41041r^3s^3t + 3575r^3s^3 + 41041r^3s^2t^3 + 148434r^3s^2t^2 + 41041r^3s^2t + 41041r^3st^3 +$$
$$41041r^3st^2 + 3575r^3t^3 - 17875r^2s^3t^3 - 67210r^2s^3t^2 - 17875r^2s^3t - 67210r^2s^2t^3 -$$
$$67210r^2s^2t^2 - 17875r^2st^3 + 30030rs^3t^3 + 30030rs^3t^2 + 30030rs^2t^3 - 12870s^3t^3) -$$
$$\frac{g_{n+s}h^5r^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(572r^2t^2 - 130r^3t^2 + 286r^2s - 260r^3s + 65r^4s - 715rt^2 +$$
$$572r^2t - 520r^3t + 130r^4t + 2145st^2 - 130r^3 + 130r^4 - 35r^5 - 1430rst^2 +$$
$$1144r^2st - 260r^3st + 286r^2st^2 - 1430rst) - \frac{g_{n+t}h^5r^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(572r^2s^2 -$$
$$130r^3s^2 - 715rs^2 + 572r^2s - 520r^3s + 130r^4s + 286r^2t - 260r^3t + 65r^4t +$$
$$2145s^2t - 130r^3 + 130r^4 - 35r^5 - 1430rs^2t + 1144r^2st - 260r^3st + 286r^2s^2t -$$
$$1430rst) + \frac{f_{n+s}h^4r^8}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(140r^6st - 210r^6s^2 + 140r^6s - 70r^6t + 735r^5s^3 +$$
$$245r^5s^2t + 245r^5s^2 - 520r^5st^2 - 705r^5st - 520r^5s + 260r^5t^2 + 260r^5t - 585r^4s^4 -$$
$$2405r^4s^3t - 2405r^4s^3 + 1300r^4s^2t^2 + 715r^4s^2t + 1300r^4s^2 + 520r^4st^3 + 1300r^4st^2 +$$
$$1300r^4st + 520r^4s - 260r^4t^3 - 1040r^4t^2 - 260r^4t + 2340r^3s^4t + 2340r^3s^4 +$$
$$1222r^3s^3t^2 + 8528r^3s^3t + 1222r^3s^3 - 2210r^3s^2t^3 - 6318r^3s^2t^2 - 6318r^3s^2t -$$
$$2210r^3s^2 - 910r^3st^3 + 936r^3st^2 - 910r^3st + 1144r^3t^3 + 1144r^3t^2 - 2574r^2s^4t^2 -$$
$$10296r^2s^4t - 2574r^2s^4 + 2002r^2s^3t^3 - 4576r^2s^3t^2 - 4576r^2s^3t + 2002r^2s^3 +$$
$$9152r^2s^2t^3 + 11154r^2s^2t^2 + 9152r^2s^2t - 3718r^2st^3 - 3718r^2st^2 - 1430r^2t^3 +$$
$$12870rs^4t^2 + 12870rs^4t - 10010rs^3t^3 + 715rs^3t^2 - 10010rs^3t - 7865rs^2t^3 -$$
$$7865rs^2t^2 + 9295rst^3 - 19305s^4t^2 + 15015s^3t^3 + 15015s^3t^2 - 10725s^2t^3) -$$
$$\frac{f_{n+t}h^4r^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(140r^6st - 70r^6s - 210r^6t^2 + 140r^6t - 520r^5s^2t + 260r^5s^2 +$$
$$245r^5st^2 - 705r^5st + 260r^5s + 735r^5t^3 + 245r^5t^2 - 520r^5t + 520r^4s^3t - 260r^4s^3 +$$

$$1300r^4s^2t^2 + 1300r^4s^2t - 1040r^4s^2 - 2405r^4st^3 + 715r^4st^2 + 1300r^4st - 260r^4s - 585r^4t^4 - 2405r^4t^3 + 1300r^4t^2 + 520r^4t - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 + 1222r^3s^2t^3 - 6318r^3s^2t^2 + 936r^3s^2t + 1144r^3s^2 + 2340r^3st^4 + 8528r^3st^3 - 6318r^3st^2 - 910r^3st + 2340r^3t^4 + 1222r^3t^3 - 2210r^3t^2 + 2002r^2s^3t^3 + 9152r^2s^3t^2 - 3718r^2s^3t - 1430r^2s^3 - 2574r^2s^2t^4 - 4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t - 10296r^2st^4 - 4576r^2st^3 + 9152r^2st^2 - 2574r^2t^4 + 2002r^2t^3 - 10010rs^3t^3 - 7865rs^3t^2 + 9295rs^3t + 12870rs^2t^4 + 715rs^2t^3 - 7865rs^2t^2 + 12870rst^4 - 10010rs\,t^3 + 15015s^3t^3 - 10725s^3t^2 - 19305s^2t^4 + 15015s^2t^3), \tag{5.12}$$

$$y_{n+s} = y_n + hsy'_n + \frac{h^2s^2y''_n}{2} + \frac{h^3s^3y'''_n}{6} + \frac{g_nh^5s^5}{2162160r^2t^2}(130r^2s^4 - 572r^2s^3t - 572r^2s^3 + 715r^2s^2t^2 + 2860r^2s^2t + 715r^2s^2 - 4290r^2st^2 - 4290r^2st + 8580r^2t^2 - 130rs^5 + 520rs^4t + 520rs^4 - 572rs^3t^2 - 2288rs^3t - 572rs^3 + 2860rs^2t^2 + 2860rs^2t - 4290rst^2 + 35s^6 - 130s^5t - 130s^5 + 130s^4t^2 + 520s^4t + 130s^4 - 572s^3t^2 - 572s^3t + 715s^2t^2) - \frac{f_nh^4s^4}{2162160r^3t^3}(1144r^3s^4t^2 - 260r^3s^5 - 260r^3s^5t + 1742r^3s^4t + 1144r^3s^4 - 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3s^3t - 1430r^3s^3 + 5005r^3s^2t^3 + 2860r^3s^2t^2 + 5005r^3s^2t + 8580r^3st^3 + 8580r^3st^2 - 77220r^3t^3 + 260r^2s^6t + 260r^2s^6 - 1040r^2s^5t^2 - 1820r^2s^5t - 1040r^2s^5 + 1144r^2s^4t^3 + 4680r^2s^4t^2 + 4680r^2s^4t + 1144r^2s^4 - 4576r^2s^3t^3 - 6864r^2s^3t^2 - 4576r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 8580r^2st^3 - 70rs^7t - 70rs^7 + 260rs^6t^2 + 645rs^6t + 260rs^6 - 260rs^5t^3 - 1820rs^5t^2 - 1820rs^5t - 260rs^5 + 1742rs^4t^3 + 4680rs^4t^2 + 1742rs^4t - 4576rs^3t^3 - 4576rs^3t^2 + 5005rs^2t^3 - 70s^7t + 260s^6t^2 + 260s^6t - 260s^5t^3 - 1040s^5t^2 - 260s^5t + 1144s^4t^3 + 1144s^4t^2 - 1430s^3t^3) - \frac{g_{n+1}h^5s^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(572r^2s^2t - 130r^2s^3 + 286r^2s^2 - 715r^2st^2 - 1430r^2st + 2145r^2t^2 + 130rs^4 - 520rs^3t - 260rs^3 + 572rs^2t^2 + 1144rs^2t - 1430rst^2 - 35s^5 + 130s^4t + 65s^4 - 130s^3t^2 - 260s^3t + 286s^2t^2) - \frac{g_{n+s}h^5s^5}{1081080(r-s)^2(s-t)^2(s-1)^2}(78r^2s^4 - 286r^2s^3t - 286r^2s^3 + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 - 1287r^2st^2 - 1287r^2st + 1716r^2t^2 - 91rs^5 + 312rs^4t + 312rs^4 - 286rs^3t^2 - 1144rs^3t - 286rs^3 + 1144rs^2t^2 + 1144rs^2t - 1287rst^2 + 28s^6 - 91s^5t - 91s^5 + 78s^4t^2 + 312s^4t + 78s^4 - 286s^3t^2 - 286s^3t + 286s^2t^2) + \frac{f_{n+1}h^4s^8}{2162160(r-1)^3(s-1)^3(t-1)^3}(520r^3s^4 - 260r^3s^4t + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 -$$

$1430r^3s^2t^3 - 3718r^3s^2t^2 + 9152r^3s^2t + 2002r^3s^2 + 9295r^3st^3 - 7865r^3st^2 -$
$10010r^3st - 10725r^3t^3 + 15015r^3t^2 + 260r^2s^5t - 520r^2s^5 - 1040r^2s^4t^2 + 1300r^2s^4t +$
$1300r^2s^4 + 1144r^2s^3t^3 + 936r^2s^3t^2 - 6318r^2s^3t + 1222r^2s^3 - 3718r^2s^2t^3 +$
$11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 - 7865r^2st^3 + 715r^2st^2 + 12870r^2st +$
$15015r^2t^3 - 19305r^2t^2 - 70rs^6t + 140rs^6 + 260rs^5t^2 - 705rs^5t + 245rs^5 - 260rs^4t^3 +$
$1300rs^4t^2 + 715rs^4t - 2405rs^4 - 910rs^3t^3 - 6318rs^3t^2 + 8528rs^3t + 2340rs^3 +$
$9152rs^2t^3 - 4576rs^2t^2 - 10296rs^2t - 10010rst^3 + 12870rst^2 + 140s^6t - 210s^6 -$
$520s^5t^2 + 245s^5t + 735s^5 + 520s^4t^3 + 1300s^4t^2 - 2405s^4t - 585s^4 - 2210s^3t^3 +$
$1222s^3t^2 + 2340s^3t + 2002s^2t^3 - 2574s^2t^2) + \frac{f_{n+s}h^4s^4}{2162160(r-s)^3(s-t)^3(s-1)^3}(1365r^3s^6 -$
$5915r^3s^5t - 5915r^3s^5 + 8437r^3s^4t^2 + 27053r^3s^4t + 8437r^3s^4 - 3575r^3s^3t^3 -$
$41041r^3s^3t^2 - 41041r^3s^3t - 3575r^3s^3 + 17875r^3s^2t^3 + 67210r^3s^2t^2 + 17875r^3s^2t -$
$30030r^3st^3 - 30030r^3st^2 + 12870r^3t^3 - 3315r^2s^7 + 14105r^2s^6t + 14105r^2s^6 -$
$19799r^2s^5t^2 - 63011r^2s^5t - 19799r^2s^5 + 8437r^2s^4t^3 + 93483r^2s^4t^2 + 93483r^2s^4t +$
$8437r^2s^4 - 41041r^2s^3t^3 - 148434r^2s^3t^2 - 41041r^2s^3t + 67210r^2s^2t^3 + 67210r^2s^2t^2 -$
$30030r^2st^3 + 2520rs^8 - 10390rs^7t - 10390rs^7 + 14105rs^6t^2 + 44520rs^6t +$
$14105rs^6 - 5915rs^5t^3 - 63011rs^5t^2 - 63011rs^5t - 5915rs^5 + 27053rs^4t^3 +$
$93483rs^4t^2 + 27053rs^4t - 41041rs^3t^3 - 41041rs^3t^2 + 17875rs^2t^3 - 630s^9 + 2520s^8t +$
$2520s^8 - 3315s^7t^2 - 10390s^7t - 3315s^7 + 1365s^6t^3 + 14105s^6t^2 + 14105s^6t +$
$1365s^6 - 5915s^5t^3 - 19799s^5t^2 - 5915s^5t + 8437s^4t^3 + 8437s^4t^2 - 3575s^3t^3) -$
$\frac{f_{n+t}h^4s^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(520r^3s^4t - 260r^3s^4 - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 +$
$2002r^3s^2t^3 + 9152r^3s^2t^2 - 3718r^3s^2t - 1430r^3s^2 - 10010r^3st^3 - 7865r^3st^2 +$
$9295r^3st + 15015r^3t^3 - 10725r^3t^2 - 520r^2s^5t + 260r^2s^5 + 1300r^2s^4t^2 + 1300r^2s^4t -$
$1040r^2s^4 + 1222r^2s^3t^3 - 6318r^2s^3t^2 + 936r^2s^3t + 1144r^2s^3 - 2574r^2s^2t^4 -$
$4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t + 12870r^2st^4 + 715r^2st^3 - 7865r^2st^2 -$
$19305r^2t^4 + 15015r^2t^3 + 140rs^6t - 70rs^6 + 245rs^5t^2 - 705rs^5t + 260rs^5 -$
$2405rs^4t^3 + 715rs^4t^2 + 1300rs^4t - 260rs^4 + 2340rs^3t^4 + 8528rs^3t^3 - 6318rs^3t^2 -$
$910rs^3t - 10296rs^2t^4 - 4576rs^2t^3 + 9152rs^2t^2 + 12870rst^4 - 10010rst^3 - 210s^6t^2 +$
$140s^6t + 735s^5t^3 + 245s^5t^2 - 520s^5t - 585s^4t^4 - 2405s^4t^3 + 1300s^4t^2 + 520s^4t +$
$2340s^3t^4 + 1222s^3t^3 - 2210s^3t^2 - 2574s^2t^4 + 2002s^2t^3) - \frac{g_{n+r}h^5s^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}$

$(572s^2t^2-130s^3t^2+286rs^2-260rs^3+65rs^4+2145rt^2-715st^2+572s^2t-520s^3t+130s^4t-130s^3+130s^4-35s^5-1430rst^2+1144rs^2t-260rs^3t+286rs^2t^2-1430rst)-\frac{g_{n+t}h^5s^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(572r^2s^2-130r^2s^3+572rs^2-715r^2s-520rs^3+130rs^4+2145r^2t+286s^2t-260s^3t+65s^4t-130s^3+130s^4-35s^5+1144rs^2t-1430r^2st-260rs^3t+286r^2s^2t-1430rst)-\frac{f_{n+r}h^4s^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(2340r^4s^3t-585r^4s^4+2340r^4s^3-2574r^4s^2t^2-10296r^4s^2t-2574r^4s^2+12870r^4st^2+12870r^4st-19305r^4t^2+735r^3s^5-2405r^3s^4t-2405r^3s^4+1222r^3s^3t^2+8528r^3s^3t+1222r^3s^3+2002r^3s^2t^3-4576r^3s^2t^2-4576r^3s^2t+2002r^3s^2-10010r^3st^3+715r^3st^2-10010r^3st+15015r^3t^3+15015r^3t^2-210r^2s^6+245r^2s^5t+245r^2s^5+1300r^2s^4t^2+715r^2s^4t+1300r^2s^4-2210r^2s^3t^3-6318r^2s^3t^2-6318r^2s^3t-2210r^2s^3+9152r^2s^2t^3+11154r^2s^2t^2+9152r^2s^2t-7865r^2st^3-7865r^2st^2-10725r^2t^3+140rs^6t+140rs^6-520rs^5t^2-705rs^5t-520rs^5+520rs^4t^3+1300rs^4t^2+1300rs^4t+520rs^4-910rs^3t^3+936rs^3t^2-910rs^3t-3718rs^2t^3-3718rs^2t^2+9295rst^3-70s^6t+260s^5t^2+260s^5t-260s^4t^3-1040s^4t^2-260s^4t+1144s^3t^3+1144s^3t^2-1430s^2t^3),$ (5.13)

$y_{n+t}=y_n+hty'_n+\frac{h^2t^2y''_n}{2}+\frac{h^3t^3y'''_n}{6}+\frac{g_nh^5t^5}{2162160r^2s^2}(715r^2s^2t^2-4290r^2s^2t+8580r^2s^2-572r^2st^3+2860r^2st^2-4290r^2st+130r^2t^4-572r^2t^3+715r^2t^2-572rs^2t^3+2860rs^2t^2-4290rs^2t+520rst^4-2288rst^3+2860rst^2-130rt^5+520rt^4-572rt^3+130s^2t^4-572s^2t^3+715s^2t^2-130st^5+520st^4-572st^3+35t^6-130t^5+130t^4)-\frac{f_nh^4t^4}{2162160r^3s^3}(5005r^3s^3t^2-1430r^3s^3t^3+8580r^3s^3t-77220r^3s^3+1144r^3s^2t^4-4576r^3s^2t^3+2860r^3s^2t^2+8580r^3s^2t-260r^3st^5+1742r^3st^4-4576r^3st^3+5005r^3st^2-260r^3t^5+1144r^3t^4-1430r^3t^3+1144r^2s^3t^4-4576r^2s^3t^3+2860r^2s^3t^2+8580r^2s^3t-1040r^2s^2t^5+4680r^2s^2t^4-6864r^2s^2t^3+2860r^2s^2t^2+260r^2st^6-1820r^2st^5+4680r^2st^4-4576r^2st^3+260r^2t^6-1040r^2t^5+1144r^2t^4-260rs^3t^5+1742rs^3t^4-4576rs^3t^3+5005rs^3t^2+260rs^2t^6-1820rs^2t^5+4680rs^2t^4-4576rs^2t^3-70rst^7+645rst^6-1820rst^5+1742rst^4-70rt^7+260rt^6-260rt^5-260s^3t^5+1144s^3t^4-1430s^3t^3+260s^2t^6-1040s^2t^5+1144s^2t^4-70st^7+260st^6-260st^5)-\frac{g_{n+1}h^5t^8}{2162160(r-1)^2(s-1)^2(t-1)^2}(2145r^2s^2-715r^2s^2t+572r^2st^2-1430r^2st-130$

$r^2t^3 + 286r^2t^2 + 572rs^2t^2 - 1430rs^2t - 520rst^3 + 1144rst^2 + 130rt^4 - 260rt^3 - 130s^2t^3 + 286s^2t^2 + 130st^4 - 260st^3 - 35t^5 + 65t^4) - \frac{g_{n+t}h^5t^5}{1081080(r-t)^2(s-t)^2(t-1)^2}(286r^2s^2t^2 - 1287r^2s^2t + 1716r^2s^2 - 286r^2st^3 + 1144r^2st^2 - 1287r^2st + 78r^2t^4 - 286r^2t^3 + 286r^2t^2 - 286rs^2t^3 + 1144rs^2t^2 - 1287rs^2t + 312rst^4 - 1144rst^3 + 1144rst^2 - 91rt^5 + 312rt^4 - 286rt^3 + 78s^2t^4 - 286s^2t^3 + 286s^2t^2 - 91st^5 + 312st^4 - 286st^3 + 28t^6 - 91t^5 + 78t^4) + \frac{f_{n+1}h^4t^8}{2162160(r-1)^3(s-1)^3(t-1)^3}(9295r^3s^3t - 1430r^3s^3t^2 - 10725r^3s^3 + 1144r^3s^2t^3 - 3718r^3s^2t^2 - 7865r^3s^2t + 15015r^3s^2 - 260r^3st^4 - 910r^3st^3 + 9152r^3st^2 - 10010r^3st + 520r^3t^4 - 2210r^3t^3 + 2002r^3t^2 + 1144r^2s^3t^3 - 3718r^2s^3t^2 - 7865r^2s^3t + 15015r^2s^3 - 1040r^2s^2t^4 + 936r^2s^2t^3 + 11154r^2s^2t^2 + 715r^2s^2t - 19305r^2s^2 + 260r^2st^5 + 1300r^2st^4 - 6318r^2st^3 - 4576r^2st^2 + 12870r^2st - 520r^2t^5 + 1300r^2t^4 + 1222r^2t^3 - 2574r^2t^2 - 260rs^3t^4 - 910rs^3t^3 + 9152rs^3t^2 - 10010rs^3t + 260rs^2t^5 + 1300rs^2t^4 - 6318rs^2t^3 - 4576rs^2t^2 + 12870rs^2t - 70rst^6 - 705rst^5 + 715rst^4 + 8528rst^3 - 10296rst^2 + 140rt^6 + 245rt^5 - 2405rt^4 + 2340rt^3 + 520s^3t^4 - 2210s^3t^3 + 2002s^3t^2 - 520s^2t^5 + 1300s^2t^4 + 1222s^2t^3 - 2574s^2t^2 + 140st^6 + 245st^5 - 2405st^4 + 2340st^3 - 210t^6 + 735t^5 - 585t^4) - \frac{f_{n+t}h^4t^4}{2162160(r-t)^3(s-t)^3(t-1)^3}(17875r^3s^3t^2 - 3575r^3s^3t^3 - 30030r^3s^3t + 12870r^3s^3 + 8437r^3s^2t^4 - 41041r^3s^2t^3 + 67210r^3s^2t^2 - 30030r^3s^2t - 5915r^3st^5 + 27053r^3st^4 - 41041r^3st^3 + 17875r^3st^2 + 1365r^3t^6 - 5915r^3t^5 + 8437r^3t^4 - 3575r^3t^3 + 8437r^2s^3t^4 - 41041r^2s^3t^3 + 67210r^2s^3t^2 - 30030r^2s^3t - 19799r^2s^2t^5 + 93483r^2s^2t^4 - 148434r^2s^2t^3 + 67210r^2s^2t^2 + 14105r^2st^6 - 63011r^2st^5 + 93483r^2st^4 - 41041r^2st^3 - 3315r^2t^7 + 14105r^2t^6 - 19799r^2t^5 + 8437r^2t^4 - 5915rs^3t^5 + 27053rs^3t^4 - 41041rs^3t^3 + 17875rs^3t^2 + 14105rs^2t^6 - 63011rs^2t^5 + 93483rs^2t^4 - 41041rs^2t^3 - 10390rst^7 + 44520rst^6 - 63011rst^5 + 27053rst^4 + 2520rt^8 - 10390rt^7 + 14105rt^6 - 5915rt^5 + 1365s^3t^6 - 5915s^3t^5 + 8437s^3t^4 - 3575s^3t^3 - 3315s^2t^7 + 14105s^2t^6 - 19799s^2t^5 + 8437s^2t^4 + 2520st^8 - 10390st^7 + 14105st^6 - 5915st^5 - 630t^9 + 2520t^8 - 3315t^7 + 1365t^6) - \frac{f_{n+r}h^4t^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(12870r^4s^2t - 2574r^4s^2t^2 - 19305r^4s^2 + 2340r^4st^3 - 10296r^4st^2 + 12870r^4st - 585r^4t^4 + 2340r^4t^3 - 2574r^4t^2 + 2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 + 1222r^3s^2t^3 - 4576r^3s^2t^2 + 715r^3s^2t + 15015r^3s^2 - 2405$

$r^3st^4 + 8528r^3st^3 - 4576r^3st^2 - 10010r^3st + 735r^3t^5 - 2405r^3t^4 + 1222r^3t^3 + 2002r^3t^2 - 2210r^2s^3t^3 + 9152r^2s^3t^2 - 7865r^2s^3t - 10725r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t + 245r^2st^5 + 715r^2st^4 - 6318r^2st^3 + 9152r^2st^2 - 210r^2t^6 + 245r^2t^5 + 1300r^2t^4 - 2210r^2t^3 + 520rs^3t^4 - 910rs^3t^3 - 3718rs^3t^2 + 9295rs^3t - 520rs^2t^5 + 1300rs^2t^4 + 936rs^2t^3 - 3718rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 + 140rt^6 - 520rt^5 + 520rt^4 - 260s^3t^4 + 1144s^3t^3 - 1430s^3t^2 + 260s^2t^5 - 1040s^2t^4 + 1144s^2t^3 - 70st^6 + 260st^5 - 260st^4) + \frac{f_{n+s}h^4t^8}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 - 2210r^3s^2t^3 + 9152r^3s^2t^2 - 7865r^3s^2t - 10725r^3s^2 + 520r^3st^4 - 910r^3st^3 - 3718r^3st^2 + 9295r^3st - 260r^3t^4 + 1144r^3t^3 - 1430r^3t^2 - 2574r^2s^4t^2 + 12870r^2s^4t - 19305r^2s^4 + 1222r^2s^3t^3 - 4576r^2s^3t^2 + 715r^2s^3t + 15015r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t - 520r^2st^5 + 1300r^2st^4 + 936r^2st^3 - 3718r^2st^2 + 260r^2t^5 - 1040r^2t^4 + 1144r^2t^3 + 2340rs^4t^3 - 10296rs^4t^2 + 12870rs^4t - 2405rs^3t^4 + 8528rs^3t^3 - 4576rs^3t^2 - 10010rs^3t + 245rs^2t^5 + 715rs^2t^4 - 6318rs^2t^3 + 9152rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 - 70rt^6 + 260rt^5 - 260rt^4 - 585s^4t^4 + 2340s^4t^3 - 2574s^4t^2 + 735s^3t^5 - 2405s^3t^4 + 1222s^3t^3 + 2002s^3t^2 - 210s^2t^6 + 245s^2t^5 + 1300s^2t^4 - 2210s^2t^3 + 140st^6 - 520st^5 + 520st^4) - \frac{g_{n+r}h^5t^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(572s^2t^2 - 130s^2t^3 + 2145rs^2 + 286rt^2 - 260rt^3 + 65rt^4 + 572st^2 - 715s^2t - 520st^3 + 130st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430rs^2t - 260rst^3 + 286rs^2t^2 - 1430rst) - \frac{g_{n+s}h^5t^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(572r^2t^2 - 130r^2t^3 + 2145r^2s + 572rt^2 - 715r^2t - 520rt^3 + 130rt^4 + 286st^2 - 260st^3 + 65st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430r^2st - 260rst^3 + 286r^2st^2 - 1430rst),$ (5.14)

$y_{n+1} = y_n + hy'_n + \frac{h^2y''_n}{2} + \frac{h^3y'''_n}{6} + \frac{g_nh^5}{2162160r^2s^2t^2}(8580r^2s^2t^2 - 4290r^2s^2t + 715r^2s^2 - 4290r^2st^2 + 2860r^2st - 572r^2s + 715r^2t^2 - 572r^2t + 130r^2 - 4290rs^2t^2 + 2860rs^2t - 572rs^2 + 2860rst^2 - 2288rst + 520rs - 572rt^2 + 520rt - 130r + 715s^2t^2 - 572s^2t + 130s^2 - 572st^2 + 520st - 130s + 130t^2 - 130t + 35) + \frac{f_{n+1}h^4}{2162160(r-1)^3(s-1)^3(t-1)^3}(12870r^3s^3t^3 - 30030r^3s^3t^2 + 17875r^3s^3t - 3575r^3s^3 - 30030r^3s^2t^3 + 67210r^3s^2t^2 - 41041r^3s^2t + 8437r^3s^2 + 17875r^3st^3 - 41041r^3st^2 +$

$27053r^3st - 5915r^3s - 3575r^3t^3 + 8437r^3t^2 - 5915r^3t + 1365r^3 - 30030r^2s^3t^3 +$
$67210r^2s^3t^2 - 41041r^2s^3t + 8437r^2s^3 + 67210r^2s^2t^3 - 148434r^2s^2t^2 + 93483r^2s^2t -$
$19799r^2s^2 - 41041r^2st^3 + 93483r^2st^2 - 63011r^2st + 14105r^2s + 8437r^2t^3 -$
$19799r^2t^2 + 14105r^2t - 3315r^2 + 17875rs^3t^3 - 41041rs^3t^2 + 27053rs^3t - 5915rs^3 -$
$41041rs^2t^3 + 93483rs^2t^2 - 63011rs^2t + 14105rs^2 + 27053rst^3 - 63011rst^2 +$
$44520rst - 10390rs - 5915rt^3 + 14105rt^2 - 10390rt + 2520r - 3575s^3t^3 +$
$8437s^3t^2 - 5915s^3t + 1365s^3 + 8437s^2t^3 - 19799s^2t^2 + 14105s^2t - 3315s^2 -$
$5915st^3 + 14105st^2 - 10390st + 2520s + 1365t^3 - 3315t^2 + 2520t - 630) -$
$\frac{g_{n+1}h^5}{1081080(r-1)^2(s-1)^2(t-1)^2}(1716r^2s^2t^2 - 1287r^2s^2t + 286r^2s^2 - 1287r^2st^2 + 1144r^2st -$
$286r^2s + 286r^2t^2 - 286r^2t + 78r^2 - 1287rs^2t^2 + 1144rs^2t - 286rs^2 + 1144rst^2 -$
$1144rst + 312rs - 286rt^2 + 312rt - 91r + 286s^2t^2 - 286s^2t + 78s^2 - 286st^2 +$
$312st - 91s + 78t^2 - 91t + 28) - \frac{f_nh^4}{2162160r^3s^3t^3}(8580r^3s^3t^2 - 77220r^3s^3t^3 +$
$5005r^3s^3t - 1430r^3s^3 + 8580r^3s^2t^3 + 2860r^3s^2t^2 - 4576r^3s^2t + 1144r^3s^2 +$
$5005r^3st^3 - 4576r^3st^2 + 1742r^3st - 260r^3s - 1430r^3t^3 + 1144r^3t^2 - 260r^3t +$
$8580r^2s^3t^3 + 2860r^2s^3t^2 - 4576r^2s^3t + 1144r^2s^3 + 2860r^2s^2t^3 - 6864r^2s^2t^2 +$
$4680r^2s^2t - 1040r^2s^2 - 4576r^2st^3 + 4680r^2st^2 - 1820r^2st + 260r^2s + 1144r^2t^3 -$
$1040r^2t^2 + 260r^2t + 5005rs^3t^3 - 4576rs^3t^2 + 1742rs^3t - 260rs^3 - 4576rs^2t^3 +$
$4680rs^2t^2 - 1820rs^2t + 260rs^2 + 1742rst^3 - 1820rst^2 + 645rst - 70rs - 260rt^3 +$
$260rt^2 - 70rt - 1430s^3t^3 + 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 + 260s^2t -$
$260st^3 + 260st^2 - 70st) - \frac{f_{n+r}h^4}{2162160r^3(r-s)^3(r-t)^3(r-1)^3}(12870r^4s^2t - 19305r^4s^2t^2 -$
$2574r^4s^2 + 12870r^4st^2 - 10296r^4st + 2340r^4s - 2574r^4t^2 + 2340r^4t - 585r^4 +$
$15015r^3s^3t^2 - 10010r^3s^3t + 2002r^3s^3 + 15015r^3s^2t^3 + 715r^3s^2t^2 - 4576r^3s^2t +$
$1222r^3s^2 - 10010r^3st^3 - 4576r^3st^2 + 8528r^3st - 2405r^3s + 2002r^3t^3 + 1222r^3t^2 -$
$2405r^3t + 735r^3 - 10725r^2s^3t^3 - 7865r^2s^3t^2 + 9152r^2s^3t - 2210r^2s^3 - 7865r^2s^2t^3 +$
$11154r^2s^2t^2 - 6318r^2s^2t + 1300r^2s^2 + 9152r^2st^3 - 6318r^2st^2 + 715r^2st + 245r^2s -$
$2210r^2t^3 + 1300r^2t^2 + 245r^2t - 210r^2 + 9295rs^3t^3 - 3718rs^3t^2 - 910rs^3t + 520rs^3 -$
$3718rs^2t^3 + 936rs^2t^2 + 1300rs^2t - 520rs^2 - 910rst^3 + 1300rst^2 - 705rst + 140rs +$
$520rt^3 - 520rt^2 + 140rt - 1430s^3t^3 + 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 +$
$260s^2t - 260st^3 + 260st^2 - 70st) + \frac{f_{n+s}h^4}{2162160s^3(r-s)^3(s-t)^3(s-1)^3}(15015r^3s^3t^2 - 10010r^3$

$$s^3t+2002r^3s^3-10725r^3s^2t^3-7865r^3s^2t^2+9152r^3s^2t-2210r^3s^2+9295r^3st^3-3718r^3st^2-910r^3st+520r^3s-1430r^3t^3+1144r^3t^2-260r^3t-19305r^2s^4t^2+12870r^2s^4t-2574r^2s^4+15015r^2s^3t^3+715r^2s^3t^2-4576r^2s^3t+1222r^2s^3-7865r^2s^2t^3+11154r^2s^2t^2-6318r^2s^2t+1300r^2s^2-3718r^2st^3+936r^2st^2+1300r^2st-520r^2s+1144r^2t^3-1040r^2t^2+260r^2t+12870rs^4t^2-10296rs^4t+2340rs^4-10010rs^3t^3-4576rs^3t^2+8528rs^3t-2405rs^3+9152rs^2t^3-6318rs^2t^2+715rs^2t+245rs^2-910rst^3+1300rst^2-705rst+140rs-260rt^3+260rt^2-70rt-2574s^4t^2+2340s^4t-585s^4+2002s^3t^3+1222s^3t^2-2405s^3t+735s^3-2210s^2t^3+1300s^2t^2+245s^2t-210s^2+520st^3-520st^2+140st)-\frac{f_{n+t}h^4}{2162160t^3(r-t)^3(s-t)^3(t-1)^3}(9295r^3s^3t-10725r^3s^3t^2-1430r^3s^3+15015r^3s^2t^3-7865r^3s^2t^2-3718r^3s^2t+1144r^3s^2-10010r^3st^3+9152r^3st^2-910r^3st-260r^3s+2002r^3t^3-2210r^3t^2+520r^3t+15015r^2s^3t^3-7865r^2s^3t^2-3718r^2s^3t+1144r^2s^3-19305r^2s^2t^4+715r^2s^2t^3+11154r^2s^2t^2+936r^2s^2t-1040r^2s^2+12870r^2st^4-4576r^2st^3-6318r^2st^2+1300r^2st+260r^2s-2574r^2t^4+1222r^2t^3+1300r^2t^2-520r^2t-10010rs^3t^3+9152rs^3t^2-910rs^3t-260rs^3+12870rs^2t^4-4576rs^2t^3-6318rs^2t^2+1300rs^2t+260rs^2-10296rst^4+8528rst^3+715rst^2-705rst-70rs+2340rt^4-2405rt^3+245rt^2+140rt+2002s^3t^3-2210s^3t^2+520s^3t-2574s^2t^4+1222s^2t^3+1300s^2t^2-520s^2t+2340st^4-2405st^3+245st^2+140st-585t^4+735t^3-210t^2)-\frac{g_{n+r}h^5}{2162160r^2(r-s)^2(r-t)^2(r-1)^2}(65r+130s+130t-715s^2t^2-260rs-260rt-520st+286rs^2+286rt^2+572st^2+572s^2t-130s^2-130t^2-1430rst^2-1430rs^2t+2145rs^2t^2+1144rst-35)-\frac{g_{n+s}h^5}{2162160s^2(r-s)^2(s-t)^2(s-1)^2}(130r+65s+130t-715r^2t^2-260rs-520rt-260st+286r^2s+572rt^2+572r^2t+286st^2-130r^2-130t^2-1430rst^2-1430r^2st+2145r^2st^2+1144rst-35)-\frac{g_{n+t}h^5}{2162160t^2(r-t)^2(s-t)^2(t-1)^2}(130r+130s+65t-715r^2s^2-520rs-260rt-260st+572rs^2+572r^2s+286r^2t+286s^2t-130r^2-130s^2-1430rs^2t-1430r^2st+2145r^2s^2t+1144rst-35). \quad (5.15)$$

The first derivatives of (5.12) - (5.15) are given by

$$y'_{n+r}=y'_n+hry''_n+\frac{h^2r^2y'''_n}{2}+\frac{g_nh^4r^4}{55440s^2t^2}(7r^6-24r^5s-24r^5t-24r^5+22r^4s^2+88r^4st+88r^4s+22r^4t^2+88r^4t+22r^4-88r^3s^2t-88r^3s^2-88r^3st^2-352r^3st-88r^3s-88r^3t^2$$

$-88r^3t+99r^2s^2t^2+396r^2s^2t+99r^2s^2+396r^2st^2+396r^2st+99r^2t^2-528rs^2t^2-528rs^2t-528rst^2+924s^2t^2)-\frac{f_nh^3r^3}{27720s^3t^3}(24r^6s^2t-7r^7s-7r^7t-7r^7st+24r^6s^2+24r^6st^2+59r^6st+24r^6s+24r^6t^2+24r^6t-22r^5s^3t-22r^5s^3-88r^5s^2t^2-152r^5s^2t-88r^5s^2-22r^5st^3-152r^5st^2-152r^5st-22r^5s-22r^5t^3-88r^5t^2-22r^5t+88r^4s^3t^2+132r^4s^3t+88r^4s^3+88r^4s^2t^3+352r^4s^2t^2+352r^4s^2t+88r^4s^2+132r^4st^3+352r^4st^2+132r^4st+88r^4t^3+88r^4t^2-99r^3s^3t^3-308r^3s^3t^2-308r^3s^3t-99r^3s^3-308r^3s^2t^3-440r^3s^2t^2-308r^3s^2t-308r^3st^3-308r^3st^2-99r^3t^3+297r^2s^3t^3+132r^2s^3t^2+297r^2s^3t+132r^2s^2t^3+132r^2s^2t^2+297r^2st^3+528rs^3t^3+528rs^3t^2+528rs^2t^3-3696s^3t^3)-\frac{g_{n+1}h^4r^7}{55440(r-1)^2(s-1)^2(t-1)^2}(24r^4s-7r^5+24r^4t+12r^4-22r^3s^2-88r^3st-44r^3s-22r^3t^2-44r^3t+88r^2s^2t+44r^2s^2+88r^2st^2+176r^2st+44r^2t^2-99rs^2t^2-198rs^2t-198rst^2+264s^2t^2)-\frac{g_{n+r}h^4r^4}{55440(r-s)^2(r-t)^2(r-1)^2}(14r^6-42r^5s-42r^5t-42r^5+33r^4s^2+132r^4st+132r^4s+33r^4t^2+132r^4t+33r^4-110r^3s^2t-110r^3s^2-110r^3st^2-440r^3st-110r^3s-110r^3t^2-110r^3t+99r^2s^2t^2+396r^2s^2t+99r^2s^2+396r^2st^2+396r^2st+99r^2t^2-396rs^2t^2-396rs^2t-396rst^2+462s^2t^2)-\frac{f_{n+1}h^3r^7}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^6st-14r^6s-14r^6t+21r^6-24r^5s^2t+48r^5s^2-24r^5st^2+64r^5st-21r^5s+48r^5t^2-21r^5t-70r^5+22r^4s^3t-44r^4s^3+88r^4s^2t^2-106r^4s^2t-116r^4s^2+22r^4st^3-106r^4st^2-70r^4st+208r^4s-44r^4t^3-116r^4t^2+208r^4t+54r^4-88r^3s^3t^2+66r^3s^3t+176r^3s^3-88r^3s^2t^3-88r^3s^2t^2+506r^3s^2t-88r^3s^2+66r^3st^3+506r^3st^2-660r^3st-198r^3s+176r^3t^3-88r^3t^2-198r^3t+99r^2s^3t^3+275r^2s^3t^2-649r^2s^3t-154r^2s^3+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2t+198r^2s^2-649r^2st^3+275r^2st^2+792r^2st-154r^2t^3+198r^2t^2-594rs^3t^3+462rs^3t^2+693rs^3t+462rs^2t^3+66rs^2t^2-891rs^2t+693rst^3-891rst^2+660s^3t^3-924s^3t^2-924s^2t^3+1188s^2t^2)+\frac{f_{n+r}h^3r^3}{27720(r-s)^3(r-t)^3(r-1)^3}(84r^9-315r^8s-315r^8t-315r^8+390r^7s^2+1210r^7st+1210r^7s+390r^7t^2+1210r^7t+390r^7-154r^6s^3-1536r^6s^2t-1536r^6s^2-1536r^6st^2-4793r^6st-1536r^6s-154r^6t^3-1536r^6t^2-1536r^6t-154r^6+616r^5s^3t+616r^5s^3+2002r^5s^2t^2+6290r^5s^2t+2002r^5s^2+616r^5st^3+6290r^5st^2+6290r^5st+616r^5s+616r^5t^3+2002r^5t^2+616r^5t-814r^4s^3t^2-2574r^4s^3t-814r^4s^3-814r^4s^2t^3-8569r^4s^2t^2-8569r^4s^2t-814r^4s^2-2574r^4st^3-$

$8569r^4st^2 - 2574r^4st - 814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 + 3575r^3s^2t^3 + 12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 - 5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 + 2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3) - \frac{g_{n+s}h^4r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2 - 22r^3t^2 + 44r^2s - 44r^3s + 12r^4s - 99rt^2 + 88r^2t - 88r^3t + 24r^4t + 264st^2 - 22r^3 + 24r^4 - 7r^5 - 198rst^2 + 176r^2st - 44r^3st + 44r^2st^2 - 198rst) - \frac{g_{n+t}h^4r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 - 22r^3s^2 - 99rs^2 + 88r^2s - 88r^3s + 24r^4s + 44r^2t - 44r^3t + 12r^4t + 264s^2t - 22r^3 + 24r^4 - 7r^5 - 198rs^2t + 176r^2st - 44r^3st + 44r^2s^2t - 198rst) + \frac{f_{n+s}h^3r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t + 70r^5s^3 + 21r^5s^2t + 21r^5s^2 - 48r^5st^2 - 64r^5st - 48r^5s + 24r^5t^2 + 24r^5t - 54r^4s^4 - 208r^4s^3t - 208r^4s^3 + 116r^4s^2t^2 + 70r^4s^2t + 116r^4s^2 + 44r^4st^3 + 106r^4st^2 + 106r^4st + 44r^4s - 22r^4t^3 - 88r^4t^2 - 22r^4t + 198r^3s^4t + 198r^3s^4 + 88r^3s^3t^2 + 660r^3s^3t + 88r^3s^3 - 176r^3s^2t^3 - 506r^3s^2t^2 - 506r^3s^2t - 176r^3s^2 - 66r^3st^3 + 88r^3st^2 - 66r^3st + 88r^3t^3 + 88r^3t^2 - 198r^2s^4t^2 - 792r^2s^4t - 198r^2s^4 + 154r^2s^3t^3 - 275r^2s^3t^2 - 275r^2s^3t + 154r^2s^3 + 649r^2s^2t^3 + 781r^2s^2t^2 + 649r^2s^2t - 275r^2st^3 - 275r^2st^2 - 99r^2t^3 + 891rs^4t^2 + 891rs^4t - 693rs^3t^3 - 66rs^3t^2 - 693rs^3t - 462rs^2t^3 - 462rs^2t^2 + 594rst^3 - 1188s^4t^2 + 924s^3t^3 + 924s^3t^2 - 660s^2t^3) + \frac{f_{n+t}h^3r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(7r^6s - 14r^6st + 21r^6t^2 - 14r^6t + 48r^5s^2t - 24r^5s^2 - 21r^5st^2 + 64r^5st - 24r^5s - 70r^5t^3 - 21r^5t^2 + 48r^5t - 44r^4s^3t + 22r^4s^3 - 116r^4s^2t^2 - 106r^4s^2t + 88r^4s^2 + 208r^4st^3 - 70r^4st^2 - 106r^4st + 22r^4s + 54r^4t^4 + 208r^4t^3 - 116r^4t^2 - 44r^4t + 176r^3s^3t^2 + 66r^3s^3t - 88r^3s^3 - 88r^3s^2t^3 + 506r^3s^2t^2 - 88r^3s^2t - 88r^3s^2 - 198r^3st^4 - 660r^3st^3 + 506r^3st^2 + 66r^3st - 198r^3t^4 - 88r^3t^3 + 176r^3t^2 - 154r^2s^3t^3 - 649r^2s^3t^2 + 275r^2s^3t + 99r^2s^3 + 198r^2s^2t^4 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 792r^2st^4 + 275r^2st^3 - 649r^2st^2 + 198r^2t^4 - 154r^2t^3 + 693rs^3t^3 + 462rs^3t^2 - 594rs^3t - 891rs^2t^4 + 66rs^2t^3 + 462rs^2t^2 - 891rst^4 + 693rst^3 - 924s^3t^3 + 660s^3t^2 + 1188s^2t^4 - 924s^2t^3),$ (5.16)

$y'_{n+s} = y'_n + hsy''_n + \frac{h^2s^2y'''_n}{2} + \frac{g_nh^4s^4}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22$

$$s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2) - \frac{f_n h^3 s^3}{27720 r^3 t^3}(88r^3s^4t^2 - 22r^3s^5 -$$
$$22r^3s^5t + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 +$$
$$297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t +$$
$$24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 -$$
$$308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t -$$
$$7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t - 22rs^5 + 132rs^4t^3 +$$
$$352rs^4t^2 + 132rs^4t - 308rs^3t^3 - 308rs^3t^2 + 297rs^2t^3 - 7s^7t + 24s^6t^2 + 24s^6t -$$
$$22s^5t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3) - \frac{g_{n+1}h^4s^7}{55440(r-1)^2(s-1)^2(t-1)^2}(88r^2s^2t -$$
$$22r^2s^3 + 44r^2s^2 - 99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 +$$
$$88rs^2t^2 + 176rs^2t - 198rst^2 - 7s^5 + 24s^4t + 12s^4 - 22s^3t^2 - 44s^3t + 44s^2t^2) -$$
$$\frac{g_{n+s}h^4s^4}{55440(r-s)^2(s-t)^2(s-1)^2}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 -$$
$$396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t -$$
$$110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t +$$
$$33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2) - \frac{f_{n+1}h^3s^7}{27720(r-1)^3(s-1)^3(t-1)^3}(22r^3s^4t - 44r^3s^4 -$$
$$88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 -$$
$$594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 -$$
$$106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 -$$
$$781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 +$$
$$1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 -$$
$$70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 +$$
$$792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 -$$
$$116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2) +$$
$$\frac{f_{n+s}h^3s^3}{27720(r-s)^3(s-t)^3(s-1)^3}(154r^3s^6 - 616r^3s^5t - 616r^3s^5 + 814r^3s^4t^2 + 2574r^3s^4t +$$
$$814r^3s^4 - 330r^3s^3t^3 - 3575r^3s^3t^2 - 3575r^3s^3t - 330r^3s^3 + 1485r^3s^2t^3 + 5280r^3s^2t^2 +$$
$$1485r^3s^2t - 2244r^3st^3 - 2244r^3st^2 + 924r^3t^3 - 390r^2s^7 + 1536r^2s^6t + 1536r^2s^6 -$$
$$2002r^2s^5t^2 - 6290r^2s^5t - 2002r^2s^5 + 814r^2s^4t^3 + 8569r^2s^4t^2 + 8569r^2s^4t +$$
$$814r^2s^4 - 3575r^2s^3t^3 - 12320r^2s^3t^2 - 3575r^2s^3t + 5280r^2s^2t^3 + 5280r^2s^2t^2 -$$
$$2244r^2st^3 + 315rs^8 - 1210rs^7t - 1210rs^7 + 1536rs^6t^2 + 4793rs^6t + 1536rs^6 -$$
$$616rs^5t^3 - 6290rs^5t^2 - 6290rs^5t - 616rs^5 + 2574rs^4t^3 + 8569rs^4t^2 + 2574rs^4t - 3575$$

$rs^3t^3 - 3575rs^3t^2 + 1485rs^2t^3 - 84s^9 + 315s^8t + 315s^8 - 390s^7t^2 - 1210s^7t - 390s^7 + 154s^6t^3 + 1536s^6t^2 + 1536s^6t + 154s^6 - 616s^5t^3 - 2002s^5t^2 - 616s^5t + 814s^4t^3 + 814s^4t^2 - 330s^3t^3) - \frac{f_{n+t}h^3s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(44r^3s^4t - 22r^3s^4 - 176r^3s^3t^2 - 66r^3s^3t + 88r^3s^3 + 154r^3s^2t^3 + 649r^3s^2t^2 - 275r^3s^2t - 99r^3s^2 - 693r^3st^3 - 462r^3st^2 + 594r^3st + 924r^3t^3 - 660r^3t^2 - 48r^2s^5t + 24r^2s^5 + 116r^2s^4t^2 + 106r^2s^4t - 88r^2s^4 + 88r^2s^3t^3 - 506r^2s^3t^2 + 88r^2s^3t + 88r^2s^3 - 198r^2s^2t^4 - 275r^2s^2t^3 + 781r^2s^2t^2 - 275r^2s^2t + 891r^2st^4 - 66r^2st^3 - 462r^2st^2 - 1188r^2t^4 + 924r^2t^3 + 14rs^6t - 7rs^6 + 21rs^5t^2 - 64rs^5t + 24rs^5 - 208rs^4t^3 + 70rs^4t^2 + 106rs^4t - 22rs^4 + 198rs^3t^4 + 660rs^3t^3 - 506rs^3t^2 - 66rs^3t - 792rs^2t^4 - 275rs^2t^3 + 649rs^2t^2 + 891rst^4 - 693rst^3 - 21s^6t^2 + 14s^6t + 70s^5t^3 + 21s^5t^2 - 48s^5t - 54s^4t^4 - 208s^4t^3 + 116s^4t^2 + 44s^4t + 198s^3t^4 + 88s^3t^3 - 176s^3t^2 - 198s^2t^4 + 154s^2t^3) - \frac{g_{n+r}h^4s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst) - \frac{g_{n+t}h^4s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 - 22r^2s^3 + 88rs^2 - 99r^2s - 88rs^3 + 24rs^4 + 264r^2t + 44s^2t - 44s^3t + 12s^4t - 22s^3 + 24s^4 - 7s^5 + 176rs^2t - 198r^2st - 44rs^3t + 44r^2s^2t - 198rst) + \frac{f_{n+r}h^3s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3), \qquad (5.17)$

$y'_{n+t} = y'_n + hty''_n + \frac{h^2t^2y'''_n}{2} + \frac{g_nh^4t^4}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 - 528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 - 88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 - 352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 + 99s^2t^2 - 24st^5 + 88st^4 - 88st^3 + 7t^6 - 24t^5 + 22t^4) - \frac{f_nh^3t^3}{27720r^3s^3}(297r^3s^3t^2 - 99r^3s^3t^3 + 528r^3s^3t$

$-3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 + 132r^3st^4 - 308r^3st^3 + 297r^3st^2 - 22r^3t^5 + 88r^3t^4 - 99r^3t^3 + 88r^2s^3t^4 - 308r^2s^3t^3 + 132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 - 152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 - 22rs^3t^5 + 132rs^3t^4 - 308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 - 152rst^5 + 132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 - 88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5) - \frac{g_{n+1}h^4t^7}{55440(r-1)^2(s-1)^2(t-1)^2}(264r^2s^2 - 99r^2s^2t + 88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4) - \frac{g_{n+t}h^4t^4}{55440(r-t)^2(s-t)^2(t-1)^2}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4) - \frac{f_{n+1}h^3t^7}{27720(r-1)^3(s-1)^3(t-1)^3}(99r^3s^3t^2 - 594r^3s^3t + 660r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 462r^3s^2t - 924r^3s^2 + 22r^3st^4 + 66r^3st^3 - 649r^3st^2 + 693r^3st - 44r^3t^4 + 176r^3t^3 - 154r^3t^2 - 88r^2s^3t^3 + 275r^2s^3t^2 + 462r^2s^3t - 924r^2s^3 + 88r^2s^2t^4 - 88r^2s^2t^3 - 781r^2s^2t^2 + 66r^2s^2t + 1188r^2s^2 - 24r^2st^5 - 106r^2st^4 + 506r^2st^3 + 275r^2st^2 - 891r^2st + 48r^2t^5 - 116r^2t^4 - 88r^2t^3 + 198r^2t^2 + 22rs^3t^4 + 66rs^3t^3 - 649rs^3t^2 + 693rs^3t - 24rs^2t^5 - 106rs^2t^4 + 506rs^2t^3 + 275rs^2t^2 - 891rs^2t + 7rst^6 + 64rst^5 - 70rst^4 - 660rst^3 + 792rst^2 - 14rt^6 - 21rt^5 + 208rt^4 - 198rt^3 - 44s^3t^4 + 176s^3t^3 - 154s^3t^2 + 48s^2t^5 - 116s^2t^4 - 88s^2t^3 + 198s^2t^2 - 14st^6 - 21st^5 + 208st^4 - 198st^3 + 21t^6 - 70t^5 + 54t^4) - \frac{f_{n+t}h^3t^3}{27720(r-t)^3(s-t)^3(t-1)^3}(1485r^3s^3t^2 - 330r^3s^3t^3 - 2244r^3s^3t + 924r^3s^3 + 814r^3s^2t^4 - 3575r^3s^2t^3 + 5280r^3s^2t^2 - 2244r^3s^2t - 616r^3st^5 + 2574r^3st^4 - 3575r^3st^3 + 1485r^3st^2 + 154r^3t^6 - 616r^3t^5 + 814r^3t^4 - 330r^3t^3 + 814r^2s^3t^4 - 3575r^2s^3t^3 + 5280r^2s^3t^2 - 2244r^2s^3t - 2002r^2s^2t^5 + 8569r^2s^2t^4 - 12320r^2s^2t^3 + 5280r^2s^2t^2 + 1536r^2st^6 - 6290r^2st^5 + 8569r^2st^4 - 3575r^2st^3 - 390r^2t^7 + 1536r^2t^6 - 2002r^2t^5 + 814r^2t^4 - 616rs^3t^5 + 2574rs^3t^4 - 3575rs^3t^3 + 1485rs^3t^2 + 1536rs^2t^6 - 6290rs^2t^5 + 8569rs^2t^4 - 3575rs^2t^3 - 1210rst^7 + 4793rst^6 - 6290rst^5 + 2574rst^4 + 315rt^8 - 1210rt^7 + 1536rt^6 - 616rt^5 + 154s^3t^6 - 616s^3t^5 + 814s^3t^4 - 330s^3t^3 - 390s^2t^7 + 1536s^2t^6 - 2002s^2t^5 + 814s^2t^4 + 315st^8 - 1210st^7 + 1536st^6 - 616st^5 -$

$$
\begin{aligned}
&84t^9+315t^8-390t^7+154t^6)+\tfrac{f_{n+r}h^3t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(198r^4s^2t^2-891r^4s^2t+\\
&1188r^4s^2-198r^4st^3+792r^4st^2-891r^4st+54r^4t^4-198r^4t^3+198r^4t^2-\\
&154r^3s^3t^2+693r^3s^3t-924r^3s^3-88r^3s^2t^3+275r^3s^2t^2+66r^3s^2t-924r^3s^2+\\
&208r^3st^4-660r^3st^3+275r^3st^2+693r^3st-70r^3t^5+208r^3t^4-88r^3t^3-154r^3t^2+\\
&176r^2s^3t^3-649r^2s^3t^2+462r^2s^3t+660r^2s^3-116r^2s^2t^4+506r^2s^2t^3-781r^2s^2t^2+\\
&462r^2s^2t-21r^2st^5-70r^2st^4+506r^2st^3-649r^2st^2+21r^2t^6-21r^2t^5-116r^2t^4+\\
&176r^2t^3-44rs^3t^4+66rs^3t^3+275rs^3t^2-594rs^3t+48rs^2t^5-106rs^2t^4-88rs^2t^3+\\
&275rs^2t^2-14rst^6+64rst^5-106rst^4+66rst^3-14rt^6+48rt^5-44rt^4+\\
&22s^3t^4-88s^3t^3+99s^3t^2-24s^2t^5+88s^2t^4-88s^2t^3+7st^6-24st^5+22st^4)+\\
&\tfrac{f_{n+s}h^3t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(154r^3s^3t^2-693r^3s^3t+924r^3s^3-176r^3s^2t^3+649r^3s^2t^2-\\
&462r^3s^2t-660r^3s^2+44r^3st^4-66r^3st^3-275r^3st^2+594r^3st-22r^3t^4+88r^3t^3-\\
&99r^3t^2-198r^2s^4t^2+891r^2s^4t-1188r^2s^4+88r^2s^3t^3-275r^2s^3t^2-66r^2s^3t+\\
&924r^2s^3+116r^2s^2t^4-506r^2s^2t^3+781r^2s^2t^2-462r^2s^2t-48r^2st^5+106r^2st^4+\\
&88r^2st^3-275r^2st^2+24r^2t^5-88r^2t^4+88r^2t^3+198rs^4t^3-792rs^4t^2+891rs^4t-\\
&208rs^3t^4+660rs^3t^3-275rs^3t^2-693rs^3t+21rs^2t^5+70rs^2t^4-506rs^2t^3+649rs^2t^2+\\
&14rst^6-64rst^5+106rst^4-66rst^3-7rt^6+24rt^5-22rt^4-54s^4t^4+198s^4t^3-\\
&198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-\\
&176s^2t^3+14st^6-48st^5+44st^4)-\tfrac{g_{n+r}h^4t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2-22s^2t^3+\\
&264rs^2+44rt^2-44rt^3+12rt^4+88st^2-99s^2t-88st^3+24st^4-22t^3+24t^4-7t^5+\\
&176rst^2-198rs^2t-44rst^3+44rs^2t^2-198rst)-\tfrac{g_{n+s}h^4t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2-\\
&22r^2t^3+264r^2s+88rt^2-99r^2t-88rt^3+24rt^4+44st^2-44st^3+12st^4-22t^3+\\
&24t^4-7t^5+176rst^2-198r^2st-44rst^3+44r^2st^2-198rst),
\end{aligned}
\tag{5.18}
$$

$$
\begin{aligned}
y'_{n+1}&=y'_n+hy''_n+\tfrac{h^2y'''_n}{2}+\tfrac{g_nh^4}{55440r^2s^2t^2}(924r^2s^2t^2-528r^2s^2t+99r^2s^2-528r^2st^2+\\
&396r^2st-88r^2s+99r^2t^2-88r^2t+22r^2-528rs^2t^2+396rs^2t-88rs^2+396rst^2-\\
&352rst+88rs-88rt^2+88rt-24r+99s^2t^2-88s^2t+22s^2-88st^2+88st-\\
&24s+22t^2-24t+7)+\tfrac{f_{n+1}h^3}{27720(r-1)^3(s-1)^3(t-1)^3}(924r^3s^3t^3-2244r^3s^3t^2+1485r^3s^3t-\\
&330r^3s^3-2244r^3s^2t^3+5280r^3s^2t^2-3575r^3s^2t+814r^3s^2+1485r^3st^3-3575r^3st^2+\\
&2574r^3st-616r^3s-330r^3t^3+814r^3t^2-616r^3t+154r^3-2244r^2s^3t^3+5280r^2s^3t^2-
\end{aligned}
$$

$3575r^2s^3t + 814r^2s^3 + 5280r^2s^2t^3 - 12320r^2s^2t^2 + 8569r^2s^2t - 2002r^2s^2 -$
$3575r^2st^3 + 8569r^2st^2 - 6290r^2st + 1536r^2s + 814r^2t^3 - 2002r^2t^2 + 1536r^2t -$
$390r^2 + 1485rs^3t^3 - 3575rs^3t^2 + 2574rs^3t - 616rs^3 - 3575rs^2t^3 + 8569rs^2t^2 -$
$6290rs^2t + 1536rs^2 + 2574rst^3 - 6290rst^2 + 4793rst - 1210rs - 616rt^3 + 1536rt^2 -$
$1210rt + 315r - 330s^3t^3 + 814s^3t^2 - 616s^3t + 154s^3 + 814s^2t^3 - 2002s^2t^2 +$
$1536s^2t - 390s^2 - 616st^3 + 1536st^2 - 1210st + 315s + 154t^3 - 390t^2 + 315t -$
$84) - \frac{g_{n+1}h^4}{55440(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st -$
$110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 -$
$440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 +$
$132st - 42s + 33t^2 - 42t + 14) - \frac{f_nh^3}{27720r^3s^3t^3}(528r^3s^3t^2 - 3696r^3s^3t^3 + 297r^3s^3t -$
$99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2 - 308r^3s^2t + 88r^3s^2 + 297r^3st^3 - 308r^3st^2 +$
$132r^3st - 22r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t + 528r^2s^3t^3 + 132r^2s^3t^2 - 308r^2s^3t +$
$88r^2s^3 + 132r^2s^2t^3 - 440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 -$
$152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3t^2 + 132rs^3t -$
$22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs -$
$22rt^3 + 24rt^2 - 7rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 +$
$24st^2 - 7st) - \frac{f_{n+r}h^3}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t - 1188r^4s^2t^2 - 198r^4s^2 + 891r^4st^2 -$
$792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 +$
$924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st -$
$208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t -$
$176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 +$
$70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t +$
$44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs +$
$44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 +$
$24st^2 - 7st) + \frac{f_{n+s}h^3}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 - 660r^3s^2t^3 -$
$462r^3s^2t^2 + 649r^3s^2t - 176r^3s^2 + 594r^3st^3 - 275r^3st^2 - 66r^3st + 44r^3s - 99r^3t^3 +$
$88r^3t^2 - 22r^3t - 1188r^2s^4t^2 + 891r^2s^4t - 198r^2s^4 + 924r^2s^3t^3 - 66r^2s^3t^2 -$
$275r^2s^3t + 88r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 - 275r^2st^3 +$
$88r^2st^2 + 106r^2st - 48r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 891rs^4t^2 - 792rs^4t + 198rs^4 -$

$693rs^3t^3 - 275rs^3t^2 + 660rs^3t - 208rs^3 + 649rs^2t^3 - 506rs^2t^2 + 70rs^2t + 21rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs - 22rt^3 + 24rt^2 - 7rt - 198s^4t^2 + 198s^4t - 54s^4 + 154s^3t^3 + 88s^3t^2 - 208s^3t + 70s^3 - 176s^2t^3 + 116s^2t^2 + 21s^2t - 21s^2 + 44st^3 - 48st^2 + 14st) + \frac{f_{n+t}h^3}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(660r^3s^3t^2 - 594r^3s^3t + 99r^3s^3 - 924r^3s^2t^3 + 462r^3s^2t^2 + 275r^3s^2t - 88r^3s^2 + 693r^3st^3 - 649r^3st^2 + 66r^3st + 22r^3s - 154r^3t^3 + 176r^3t^2 - 44r^3t - 924r^2s^3t^3 + 462r^2s^3t^2 + 275r^2s^3t - 88r^2s^3 + 1188r^2s^2t^4 + 66r^2s^2t^3 - 781r^2s^2t^2 - 88r^2s^2t + 88r^2s^2 - 891r^2st^4 + 275r^2st^3 + 506r^2st^2 - 106r^2st - 24r^2s + 198r^2t^4 - 88r^2t^3 - 116r^2t^2 + 48r^2t + 693rs^3t^3 - 649rs^3t^2 + 66rs^3t + 22rs^3 - 891rs^2t^4 + 275rs^2t^3 + 506rs^2t^2 - 106rs^2t - 24rs^2 + 792rst^4 - 660rst^3 - 70rst^2 + 64rst + 7rs - 198rt^4 + 208rt^3 - 21rt^2 - 14rt - 154s^3t^3 + 176s^3t^2 - 44s^3t + 198s^2t^4 - 88s^2t^3 - 116s^2t^2 + 48s^2t - 198st^4 + 208st^3 - 21st^2 - 14st + 54t^4 - 70t^3 + 21t^2) - \frac{g_{n+r}h^4}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(12r + 24s + 24t - 99s^2t^2 - 44rs - 44rt - 88st + 44rs^2 + 44rt^2 + 88st^2 + 88s^2t - 22s^2 - 22t^2 - 198rst^2 - 198rs^2t + 264rs^2t^2 + 176rst - 7) - \frac{g_{n+s}h^4}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(24r + 12s + 24t - 99r^2t^2 - 44rs - 88rt - 44st + 44r^2s + 88rt^2 + 88r^2t + 44st^2 - 22r^2 - 22t^2 - 198rst^2 - 198r^2st + 264r^2st^2 + 176rst - 7) - \frac{g_{n+t}h^4}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(24r + 24s + 12t - 99r^2s^2 - 88rs - 44rt - 44st + 88rs^2 + 88r^2 s + 44r^2t + 44s^2t - 22r^2 - 22s^2 - 198rs^2t - 198r^2st + 264r^2s^2t + 176rst - 7) \quad (5.19)$

which can be expressed in block form as below :

$$
\begin{aligned}
I_4 Y_{n+1}^{'[4]_3} &= M_2^{'[4]_3} Y_{n-1}^{[4]_3} + hM_3^{'[4]_3} Y_{n-2}^{[4]_3} + h^2 M_4^{'[4]_3} Y_{n-3}^{[4]_3} + h^3 \left[E_1^{'[4]_3} F_n^{[4]_3} + E_2^{'[4]_3} F_{n+1}^{[4]_3} \right] \\
&\quad + h^4 \left[K_1^{'[4]_3} G_n^{[4]_3} + K_2^{'[4]_3} G_{n+1}^{[4]_3} \right] \qquad (5.20)
\end{aligned}
$$

where

$$
Y_{n+1}^{'[4]_3} = \begin{pmatrix} y'_{n+r} \\ y'_{n+s} \\ y'_{n+t} \\ y'_{n+1} \end{pmatrix}, \; M_2^{'[4]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; M_3^{'[4]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

$$M_4^{'[4]_3} = \begin{pmatrix} 0 & 0 & 0 & \frac{r^2}{2} \\ 0 & 0 & 0 & \frac{s^2}{2} \\ 0 & 0 & 0 & \frac{t^2}{2} \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix},\ E_1^{'[4]_3} = \begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{'[4]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{'[4]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{'[4]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{'[4]_3} \end{pmatrix},$$

$$E_2^{'[4]_3} = \begin{pmatrix} E_{2_{11}}^{'[4]_3} & E_{2_{12}}^{'[4]_3} & E_{2_{13}}^{'[4]_3} & E_{2_{14}}^{'[4]_3} \\ E_{2_{21}}^{'[4]_3} & E_{2_{22}}^{'[4]_3} & E_{2_{23}}^{'[4]_3} & E_{2_{24}}^{'[4]_3} \\ E_{2_{31}}^{'[4]_3} & E_{2_{32}}^{'[4]_3} & E_{2_{33}}^{'[4]_3} & E_{2_{34}}^{'[4]_3} \\ E_{2_{41}}^{'[4]_3} & E_{2_{42}}^{'[4]_3} & E_{2_{43}}^{'[4]_3} & E_{2_{44}}^{'[4]_3} \end{pmatrix},\ K_1^{'[4]_3} = \begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{'[4]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{'[4]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{'[4]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{'[4]_3} \end{pmatrix},$$

$$K_2^{'[4]_3} = \begin{pmatrix} K_{2_{11}}^{'[4]_3} & K_{2_{12}}^{'[4]_3} & K_{2_{13}}^{'[4]_3} & K_{2_{14}}^{'[4]_3} \\ K_{2_{21}}^{'[4]_3} & K_{2_{22}}^{'[4]_3} & K_{2_{23}}^{'[4]_3} & K_{2_{24}}^{'[4]_3} \\ K_{2_{31}}^{'[4]_3} & K_{2_{32}}^{'[4]_3} & K_{2_{33}}^{'[4]_3} & K_{2_{34}}^{'[4]_3} \\ K_{2_{41}}^{'[4]_3} & K_{2_{42}}^{'[4]_3} & K_{2_{43}}^{'[4]_3} & K_{2_{44}}^{'[4]_3} \end{pmatrix}$$

where the nonzero elements are as below :

$E_{1_{14}}^{'[4]_3} = \frac{-r^3}{27720s^3t^3}(-7r^7st - 7r^7s - 7r^7t + 24r^6s^2t + 24r^6s^2 + 24r^6st^2 + 59r^6st + 24r^6s + 24r^6t^2 + 24r^6t - 22r^5s^3t - 22r^5s^3 - 88r^5s^2t^2 - 152r^5s^2t - 88r^5s^2 - 22r^5st^3 - 152r^5st^2 - 152r^5st - 22r^5s - 22r^5t^3 - 88r^5t^2 - 22r^5t + 88r^4s^3t^2 + 132r^4s^3t + 88r^4s^3 + 88r^4s^2t^3 + 352r^4s^2t^2 + 352r^4s^2t + 88r^4s^2 + 132r^4st^3 + 352r^4st^2 + 132r^4st + 88r^4t^3 + 88r^4t^2 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 - 308r^3s^2t^3 - 440r^3s^2t^2 - 308r^3s^2t - 308r^3st^3 - 308r^3st^2 - 99r^3t^3 + 297r^2s^3t^3 + 132r^2s^3t^2 + 297r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 297r^2st^3 + 528rs^3t^3 + 528rs^3t^2 + 528rs^2t^3 - 3696s^3t^3)$,

$E_{1_{24}}^{'[4]_3} = \frac{-s^3}{27720r^3t^3}(-22r^3s^5t - 22r^3s^5 + 88r^3s^4t^2 + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 + 297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t + 24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t - 7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2$

$-152rs^5t-22rs^5+132rs^4t^3+352rs^4t^2+132rs^4t-308rs^3t^3-308rs^3t^2+297rs^2t^3-7s^7t+24s^6t^2+24s^6t-22s^5t^3-88s^5t^2-22s^5t+88s^4t^3+88s^4t^2-99s^3t^3)$,

$E_{1_{34}}^{'[4]_3} = \frac{-t^3}{27720r^3s^3}(-99r^3s^3t^3+297r^3s^3t^2+528r^3s^3t-3696r^3s^3+88r^3s^2t^4-308r^3s^2t^3+132r^3s^2t^2+528r^3s^2t-22r^3st^5+132r^3st^4-308r^3st^3+297r^3st^2-22r^3t^5+88r^3t^4-99r^3t^3+88r^2s^3t^4-308r^2s^3t^3+132r^2s^3t^2+528r^2s^3t-88r^2s^2t^5+352r^2s^2t^4-440r^2s^2t^3+132r^2s^2t^2+24r^2st^6-152r^2st^5+352r^2st^4-308r^2st^3+24r^2t^6-88r^2t^5+88r^2t^4-22rs^3t^5+132rs^3t^4-308rs^3t^3+297rs^3t^2+24rs^2t^6-152rs^2t^5+352rs^2t^4-308rs^2t^3-7rst^7+59rst^6-152rst^5+132rst^4-7rt^7+24rt^6-22rt^5-22s^3t^5+88s^3t^4-99s^3t^3+24s^2t^6-88s^2t^5+88s^2t^4-7st^7+24st^6-22st^5)$,

$E_{1_{44}}^{'[4]_3} = \frac{-1}{27720r^3s^3t^3}(-3696r^3s^3t^3+528r^3s^3t^2+297r^3s^3t-99r^3s^3+528r^3s^2t^3+132r^3s^2t^2-308r^3s^2t+88r^3s^2+297r^3st^3-308r^3st^2+132r^3st-22r^3s-99r^3t^3+88r^3t^2-22r^3t+528r^2s^3t^3+132r^2s^3t^2-308r^2s^3t+88r^2s^3+132r^2s^2t^3-440r^2s^2t^2+352r^2s^2t-88r^2s^2-308r^2st^3+352r^2st^2-152r^2st+24r^2s+88r^2t^3-88r^2t^2+24r^2t+297rs^3t^3-308rs^3t^2+132rs^3t-22rs^3-308rs^2t^3+352rs^2t^2-152rs^2t+24rs^2+132rst^3-152rst^2+59rst-7rs-22rt^3+24rt^2-7rt-99s^3t^3+88s^3t^2-22s^3t+88s^2t^3-88s^2t^2+24s^2t-22st^3+24st^2-7st)$.

$E_{2_{11}}^{'[4]_3} = \frac{r^3}{27720(r-s)^3(r-t)^3(r-1)^3}(84r^9-315r^8s-315r^8t-315r^8+390r^7s^2+1210r^7st+1210r^7s+390r^7t^2+1210r^7t+390r^7-154r^6s^3-1536r^6s^2t-1536r^6s^2-1536r^6st^2-4793r^6st-1536r^6s-154r^6t^3-1536r^6t^2-1536r^6t-154r^6+616r^5s^3t+616r^5s^3+2002r^5s^2t^2+6290r^5s^2t+2002r^5s^2+616r^5st^3+6290r^5st^2+6290r^5st+616r^5s+616r^5t^3+2002r^5t^2+616r^5t-814r^4s^3t^2-2574r^4s^3t-814r^4s^3-814r^4s^2t^3-8569r^4s^2t^2-8569r^4s^2t-814r^4s^2-2574r^4st^3-8569r^4st^2-2574r^4st-814r^4t^3-814r^4t^2+330r^3s^3t^3+3575r^3s^3t^2+3575r^3s^3t+330r^3s^3+3575r^3s^2t^3+12320r^3s^2t^2+3575r^3s^2t+3575r^3st^3+3575r^3st^2+330r^3t^3-1485r^2s^3t^3-5280r^2s^3t^2-1485r^2s^3t-5280r^2s^2t^3-5280r^2s^2t^2-1485r^2st^3+2244rs^3t^3+2244rs^3t^2+2244rs^2t^3-924s^3t^3)$,

$E_{2_{21}}^{'[4]_3} = \frac{s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3)$,

$E_{2_{31}}^{'[4]_3} = \frac{t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(198r^4s^2t^2 - 891r^4s^2t + 1188r^4s^2 - 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 - 198r^4t^3 + 198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t - 924r^3s^2 + 208r^3st^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 - 154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 + 462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 - 781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 - 649r^2st^2 + 21r^2t^6 - 21r^2t^5 - 116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594rs^3t + 48rs^2t^5 - 106rs^2t^4 - 88rs^2t^3 + 275rs^2t^2 - 14rst^6 + 64rst^5 - 106rst^4 + 66rst^3 - 14rt^6 + 48rt^5 - 44rt^4 + 22s^3t^4 - 88s^3t^3 + 99s^3t^2 - 24s^2t^5 + 88s^2t^4 - 88s^2t^3 + 7st^6 - 24st^5 + 22st^4)$,

$E_{2_{41}}^{'[4]_3} = \frac{-1}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-1188r^4s^2t^2 + 891r^4s^2t - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs + 44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st)$,

$$E_{2_{12}}^{'[4]_3} = \frac{r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2+14r^6st+14r^6s-7r^6t+70r^5s^3+21r^5s^2t+21r^5s^2-48r^5st^2-64r^5st-48r^5s+24r^5t^2+24r^5t-54r^4s^4-208r^4s^3t-208r^4s^3+116r^4s^2t^2+70r^4s^2t+116r^4s^2+44r^4st^3+106r^4st^2+106r^4st+44r^4s-22r^4t^3-88r^4t^2-22r^4t+198r^3s^4t+198r^3s^4+88r^3s^3t^2+660r^3s^3t+88r^3s^3-176r^3s^2t^3-506r^3s^2t^2-506r^3s^2t-176r^3s^2-66r^3st^3+88r^3st^2-66r^3st+88r^3t^3+88r^3t^2-198r^2s^4t^2-792r^2s^4t-198r^2s^4+154r^2s^3t^3-275r^2s^3t^2-275r^2s^3t+154r^2s^3+649r^2s^2t^3+781r^2s^2t^2+649r^2s^2t-275r^2st^3-275r^2st^2-99r^2t^3+891rs^4t^2+891rs^4t-693rs^3t^3-66rs^3t^2-693rs^3t-462rs^2t^3-462rs^2t^2+594rst^3-1188s^4t^2+924s^3t^3+924s^3t^2-660s^2t^3),$$

$$E_{2_{22}}^{'[4]_3} = \frac{s^3}{27720(r-s)^3(s-t)^3(s-1)^3}(154r^3s^6-616r^3s^5t-616r^3s^5+814r^3s^4t^2+2574r^3s^4t+814r^3s^4-330r^3s^3t^3-3575r^3s^3t^2-3575r^3s^3t-330r^3s^3+1485r^3s^2t^3+5280r^3s^2t^2+1485r^3s^2t-2244r^3st^3-2244r^3st^2+924r^3t^3-390r^2s^7+1536r^2s^6t+1536r^2s^6-2002r^2s^5t^2-6290r^2s^5t-2002r^2s^5+814r^2s^4t^3+8569r^2s^4t^2+8569r^2s^4t+814r^2s^4-3575r^2s^3t^3-12320r^2s^3t^2-3575r^2s^3t+5280r^2s^2t^3+5280r^2s^2t^2-2244r^2st^3+315rs^8-1210rs^7t-1210rs^7+1536rs^6t^2+4793rs^6t+1536rs^6-616rs^5t^3-6290rs^5t^2-6290rs^5t-616rs^5+2574rs^4t^3+8569rs^4t^2+2574rs^4t-3575rs^3t^3-3575rs^3t^2+1485rs^2t^3-84s^9+315s^8t+315s^8-390s^7t^2-1210s^7t-390s^7+154s^6t^3+1536s^6t^2+1536s^6t+154s^6-616s^5t^3-2002s^5t^2-616s^5t+814s^4t^3+814s^4t^2-330s^3t^3),$$

$$E_{2_{32}}^{'[4]_3} = \frac{t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(154r^3s^3t^2-693r^3s^3t+924r^3s^3-176r^3s^2t^3+649r^3s^2t^2-462r^3s^2t-660r^3s^2+44r^3st^4-66r^3st^3-275r^3st^2+594r^3st-22r^3t^4+88r^3t^3-99r^3t^2-198r^2s^4t^2+891r^2s^4t-1188r^2s^4+88r^2s^3t^3-275r^2s^3t^2-66r^2s^3t+924r^2s^3+116r^2s^2t^4-506r^2s^2t^3+781r^2s^2t^2-462r^2s^2t-48r^2st^5+106r^2st^4+88r^2st^3-275r^2st^2+24r^2t^5-88r^2t^4+88r^2t^3+198rs^4t^3-792rs^4t^2+891rs^4t-208rs^3t^4+660rs^3t^3-275rs^3t^2-693rs^3t+21rs^2t^5+70rs^2t^4-506rs^2t^3+649rs^2t^2+14rst^6-64rst^5+106rst^4-66rst^3-7rt^6+24rt^5-22rt^4-54s^4t^4+198s^4t^3-198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-176s^2t^3+14st^6-48st^5+44st^4),$$

$E_{2_{42}}^{'[4]_3} = \frac{1}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 - 660r^3s^2t^3 - 462r^3s^2t^2 + 649r^3s^2t - 176r^3s^2 + 594r^3st^3 - 275r^3st^2 - 66r^3st + 44r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t - 1188r^2s^4t^2 + 891r^2s^4t - 198r^2s^4 + 924r^2s^3t^3 - 66r^2s^3t^2 - 275r^2s^3t + 88r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 - 275r^2st^3 + 88r^2st^2 + 106r^2st - 48r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 891rs^4t^2 - 792rs^4t + 198rs^4 - 693rs^3t^3 - 275rs^3t^2 + 660rs^3t - 208rs^3 + 649rs^2t^3 - 506rs^2t^2 + 70rs^2t + 21rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs - 22rt^3 + 24rt^2 - 7rt - 198s^4t^2 + 198s^4t - 54s^4 + 154s^3t^3 + 88s^3t^2 - 208s^3t + 70s^3 - 176s^2t^3 + 116s^2t^2 + 21s^2t - 21s^2 + 44st^3 - 48st^2 + 14st)$,

$E_{2_{13}}^{'[4]_3} = \frac{r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st + 7r^6s + 21r^6t^2 - 14r^6t + 48r^5s^2t - 24r^5s^2 - 21r^5st^2 + 64r^5st - 24r^5s - 70r^5t^3 - 21r^5t^2 + 48r^5t - 44r^4s^3t + 22r^4s^3 - 116r^4s^2t^2 - 106r^4s^2t + 88r^4s^2 + 208r^4st^3 - 70r^4st^2 - 106r^4st + 22r^4s + 54r^4t^4 + 208r^4t^3 - 116r^4t^2 - 44r^4t + 176r^3s^3t^2 + 66r^3s^3t - 88r^3s^3 - 88r^3s^2t^3 + 506r^3s^2t^2 - 88r^3s^2t - 88r^3s^2 - 198r^3st^4 - 660r^3st^3 + 506r^3st^2 + 66r^3st - 198r^3t^4 - 88r^3t^3 + 176r^3t^2 - 154r^2s^3t^3 - 649r^2s^3t^2 + 275r^2s^3t + 99r^2s^3 + 198r^2s^2t^4 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 792r^2st^4 + 275r^2st^3 - 649r^2st^2 + 198r^2t^4 - 154r^2t^3 + 693rs^3t^3 + 462rs^3t^2 - 594rs^3t - 891rs^2t^4 + 66rs^2t^3 + 462rs^2t^2 - 891rst^4 + 693rst^3 - 924s^3t^3 + 660s^3t^2 + 1188s^2t^4 - 924s^2t^3)$,

$E_{2_{23}}^{'[4]_3} = \frac{-s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(44r^3s^4t - 22r^3s^4 - 176r^3s^3t^2 - 66r^3s^3t + 88r^3s^3 + 154r^3s^2t^3 + 649r^3s^2t^2 - 275r^3s^2t - 99r^3s^2 - 693r^3st^3 - 462r^3st^2 + 594r^3st + 924r^3t^3 - 660r^3t^2 - 48r^2s^5t + 24r^2s^5 + 116r^2s^4t^2 + 106r^2s^4t - 88r^2s^4 + 88r^2s^3t^3 - 506r^2s^3t^2 + 88r^2s^3t + 88r^2s^3 - 198r^2s^2t^4 - 275r^2s^2t^3 + 781r^2s^2t^2 - 275r^2s^2t + 891r^2st^4 - 66r^2st^3 - 462r^2st^2 - 1188r^2t^4 + 924r^2t^3 + 14rs^6t - 7rs^6 + 21rs^5t^2 - 64rs^5t + 24rs^5 - 208rs^4t^3 + 70rs^4t^2 + 106rs^4t - 22rs^4 + 198rs^3t^4 + 660rs^3t^3 - 506rs^3t^2 - 66rs^3t - 792rs^2t^4 - 275rs^2t^3 + 649rs^2t^2 + 891rst^4 - 693rst^3 - 21s^6t^2 + 14s^6t + 70s^5t^3 + 21s^5t^2 - 48s^5t - 54s^4t^4 - 208s^4t^3 + 116s^4t^2 + 44s^4t + 198s^3t^4 + 88s^3t^3 - 176s^3t^2 - 198s^2t^4 + 154s^2t^3)$,

$E_{2_{33}}^{'[4]_3} = \frac{-t^3}{27720(r-t)^3(s-t)^3(t-1)^3}(-330r^3s^3t^3 + 1485r^3s^3t^2 - 2244r^3s^3t + 924r^3s^3 + 814r^3s^2t^4 - 3575r^3s^2t^3 + 5280r^3s^2t^2 - 2244r^3s^2t - 616r^3st^5 + 2574r^3st^4 - 3575r^3st^3 + 1485r^3st^2 + 154r^3t^6 - 616r^3t^5 + 814r^3t^4 - 330r^3t^3 + 814r^2s^3t^4 - 3575r^2s^3t^3 + 5280r^2s^3t^2 - 2244r^2s^3t - 2002r^2s^2t^5 + 8569r^2s^2t^4 - 12320r^2s^2t^3 + 5280r^2s^2t^2 + 1536r^2st^6 - 6290r^2st^5 + 8569r^2st^4 - 3575r^2st^3 - 390r^2t^7 + 1536r^2t^6 - 2002r^2t^5 + 814r^2t^4 - 616rs^3t^5 + 2574rs^3t^4 - 3575rs^3t^3 + 1485rs^3t^2 + 1536rs^2t^6 - 6290rs^2t^5 + 8569rs^2t^4 - 3575rs^2t^3 - 1210rst^7 + 4793rst^6 - 6290rst^5 + 2574rst^4 + 315rt^8 - 1210rt^7 + 1536rt^6 - 616rt^5 + 154s^3t^6 - 616s^3t^5 + 814s^3t^4 - 330s^3t^3 - 390s^2t^7 + 1536s^2t^6 - 2002s^2t^5 + 814s^2t^4 + 315st^8 - 1210st^7 + 1536st^6 - 616st^5 - 84t^9 + 315t^8 - 390t^7 + 154t^6)$,

$E_{2_{43}}^{'[4]_3} = \frac{1}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(660r^3s^3t^2 - 594r^3s^3t + 99r^3s^3 - 924r^3s^2t^3 + 462r^3s^2t^2 + 275r^3s^2t - 88r^3s^2 + 693r^3st^3 - 649r^3st^2 + 66r^3st + 22r^3s - 154r^3t^3 + 176r^3t^2 - 44r^3t - 924r^2s^3t^3 + 462r^2s^3t^2 + 275r^2s^3t - 88r^2s^3 + 1188r^2s^2t^4 + 66r^2s^2t^3 - 781r^2s^2t^2 - 88r^2s^2t + 88r^2s^2 - 891r^2st^4 + 275r^2st^3 + 506r^2st^2 - 106r^2st - 24r^2s + 198r^2t^4 - 88r^2t^3 - 116r^2t^2 + 48r^2t + 693rs^3t^3 - 649rs^3t^2 + 66rs^3t + 22rs^3 - 891rs^2t^4 + 275rs^2t^3 + 506rs^2t^2 - 106rs^2t - 24rs^2 + 792rst^4 - 660rst^3 - 70rst^2 + 64rst + 7rs - 198rt^4 + 208rt^3 - 21rt^2 - 14rt - 154s^3t^3 + 176s^3t^2 - 44s^3t + 198s^2t^4 - 88s^2t^3 - 116s^2t^2 + 48s^2t - 198st^4 + 208st^3 - 21st^2 - 14st + 54t^4 - 70t^3 + 21t^2)$,

$E_{2_{14}}^{'[4]_3} = \frac{-r^7}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 24r^5s^2t + 48r^5s^2 - 24r^5st^2 + 64r^5st - 21r^5s + 48r^5t^2 - 21r^5t - 70r^5 + 22r^4s^3t - 44r^4s^3 + 88r^4s^2t^2 - 106r^4s^2t - 116r^4s^2 + 22r^4st^3 - 106r^4st^2 - 70r^4st + 208r^4s - 44r^4t^3 - 116r^4t^2 + 208r^4t + 54r^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 - 88r^3s^2t^3 - 88r^3s^2t^2 + 506r^3s^2t - 88r^3s^2 + 66r^3st^3 + 506r^3st^2 - 660r^3st - 198r^3s + 176r^3t^3 - 88r^3t^2 - 198r^3t + 99r^2s^3t^3 + 275r^2s^3t^2 - 649r^2s^3t - 154r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 - 649r^2st^3 + 275r^2st^2 + 792r^2st - 154r^2t^3 + 198r^2t^2 - 594rs^3t^3 + 462rs^3t^2 + 693rs^3t + 462rs^2t^3 + 66rs^2t^2 - 891rs^2t + 693rst^3 - 891rst^2 + 660s^3t^3 - 924s^3t^2 - 924s^2t^3 + 1188s^2t^2)$,

$E_{2_{24}}^{'[4]_3} = \frac{-s^7}{27720(r-1)^3(s-1)^3(t-1)^3}(22r^3s^4t - 44r^3s^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 - 594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 - 106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 + 1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 - 70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 + 792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 - 116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2)$,

$E_{2_{34}}^{'[4]_3} = \frac{-t^7}{27720(r-1)^3(s-1)^3(t-1)^3}(99r^3s^3t^2 - 594r^3s^3t + 660r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 462r^3s^2t - 924r^3s^2 + 22r^3st^4 + 66r^3st^3 - 649r^3st^2 + 693r^3st - 44r^3t^4 + 176r^3t^3 - 154r^3t^2 - 88r^2s^3t^3 + 275r^2s^3t^2 + 462r^2s^3t - 924r^2s^3 + 88r^2s^2t^4 - 88r^2s^2t^3 - 781r^2s^2t^2 + 66r^2s^2t + 1188r^2s^2 - 24r^2st^5 - 106r^2st^4 + 506r^2st^3 + 275r^2st^2 - 891r^2st + 48r^2t^5 - 116r^2t^4 - 88r^2t^3 + 198r^2t^2 + 22rs^3t^4 + 66rs^3t^3 - 649rs^3t^2 + 693rs^3t - 24rs^2t^5 - 106rs^2t^4 + 506rs^2t^3 + 275rs^2t^2 - 891rs^2t + 7rst^6 + 64rst^5 - 70rst^4 - 660rst^3 + 792rst^2 - 14rt^6 - 21rt^5 + 208rt^4 - 198rt^3 - 44s^3t^4 + 176s^3t^3 - 154s^3t^2 + 48s^2t^5 - 116s^2t^4 - 88s^2t^3 + 198s^2t^2 - 14st^6 - 21st^5 + 208st^4 - 198st^3 + 21t^6 - 70t^5 + 54t^4)$,

$E_{2_{44}}^{'[4]_3} = \frac{1}{27720(r-1)^3(s-1)^3(t-1)^3}(924r^3s^3t^3 - 2244r^3s^3t^2 + 1485r^3s^3t - 330r^3s^3 - 2244r^3s^2t^3 + 5280r^3s^2t^2 - 3575r^3s^2t + 814r^3s^2 + 1485r^3st^3 - 3575r^3st^2 + 2574r^3st - 616r^3s - 330r^3t^3 + 814r^3t^2 - 616r^3t + 154r^3 - 2244r^2s^3t^3 + 5280r^2s^3t^2 - 3575r^2s^3t + 814r^2s^3 + 5280r^2s^2t^3 - 12320r^2s^2t^2 + 8569r^2s^2t - 2002r^2s^2 - 3575r^2st^3 + 8569r^2st^2 - 6290r^2st + 1536r^2s + 814r^2t^3 - 2002r^2t^2 + 1536r^2t - 390r^2 + 1485rs^3t^3 - 3575rs^3t^2 + 2574rs^3t - 616rs^3 - 3575rs^2t^3 + 8569rs^2t^2 - 6290rs^2t + 1536rs^2 + 2574rst^3 - 6290rst^2 + 4793rst - 1210rs - 616rt^3 + 1536rt^2 - 1210rt + 315r - 330s^3t^3 + 814s^3t^2 - 616s^3t + 154s^3 + 814s^2t^3 - 2002s^2t^2 + 1536s^2t - 390s^2 - 616st^3 + 1536st^2 - 1210st + 315s + 154t^3 - 390t^2 + 315t - 84)$,

$$K_{1_{14}}^{'[4]_3} = \frac{r^4}{55440s^2t^2}(7r^6 - 24r^5s - 24r^5t - 24r^5 + 22r^4s^2 + 88r^4st + 88r^4s + 22r^4t^2 + 88r^4t + 22r^4 - 88r^3s^2t - 88r^3s^2 - 88r^3st^2 - 352r^3st - 88r^3s - 88r^3t^2 - 88r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 528rs^2t^2 - 528rs^2t - 528rst^2 + 924s^2t^2),$$

$$K_{1_{24}}^{'[4]_3} = \frac{s^4}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2),$$

$$K_{1_{34}}^{'[4]_3} = \frac{t^4}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 - 528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 - 88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 - 352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 + 99s^2t^2 - 24st^5 + 88st^4 - 88st^3 + 7t^6 - 24t^5 + 22t^4),$$

$$K_{1_{44}}^{'[4]_3} = \frac{1}{55440r^2s^2t^2}(924r^2s^2t^2 - 528r^2s^2t + 99r^2s^2 - 528r^2st^2 + 396r^2st - 88r^2s + 99r^2t^2 - 88r^2t + 22r^2 - 528rs^2t^2 + 396rs^2t - 88rs^2 + 396rst^2 - 352rst + 88rs - 88rt^2 + 88rt - 24r + 99s^2t^2 - 88s^2t + 22s^2 - 88st^2 + 88st - 24s + 22t^2 - 24t + 7),$$

$$K_{2_{11}}^{'[4]_3} = \frac{-r^4}{55440(r-s)^2(r-t)^2(r-1)^2}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2),$$

$$K_{2_{21}}^{'[4]_3} = \frac{-s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst),$$

$$K_{2_{31}}^{'[4]_3} = \frac{-t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^2t^3 + 264rs^2 + 44rt^2 - 44rt^3 + 12rt^4 + 88st^2 - 99s^2t - 88st^3 + 24st^4 - 22t^3 + 24t^4 - 7t^5 + 176rst^2 - 198rs^2t - 44rst^3 + 44rs^2t^2 - 198rst),$$

$K_{2_{41}}^{'[4]_3} = \frac{-1}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(12r + 24s + 24t - 99s^2t^2 - 44rs - 44rt - 88st + 44rs^2 + 44rt^2 + 88st^2 + 88s^2t - 22s^2 - 22t^2 - 198rst^2 - 198rs^2t + 264rs^2t^2 + 176rst - 7)$,

$K_{2_{12}}^{'[4]_3} = \frac{-r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2 - 22r^3t^2 + 44r^2s - 44r^3s + 12r^4s - 99rt^2 + 88r^2t - 88r^3t + 24r^4t + 264st^2 - 22r^3 + 24r^4 - 7r^5 - 198rst^2 + 176r^2st - 44r^3st + 44r^2st^2 - 198rst)$,

$K_{2_{22}}^{'[4]_3} = \frac{-s^4}{55440(r-s)^2(s-t)^2(s-1)^2}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2)$,

$K_{2_{32}}^{'[4]_3} = \frac{-t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2 - 22r^2t^3 + 264r^2s + 88rt^2 - 99r^2t - 88rt^3 + 24rt^4 + 44st^2 - 44st^3 + 12st^4 - 22t^3 + 24t^4 - 7t^5 + 176rst^2 - 198r^2st - 44rst^3 + 44r^2st^2 - 198rst)$,

$K_{2_{42}}^{'[4]_3} = \frac{-1}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(24r + 12s + 24t - 99r^2t^2 - 44rs - 88rt - 44st + 44r^2s + 88rt^2 + 88r^2t + 44st^2 - 22r^2 - 22t^2 - 198rst^2 - 198r^2st + 264r^2st^2 + 176rst - 7)$,

$K_{2_{13}}^{'[4]_3} = \frac{-r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 - 22r^3s^2 - 99rs^2 + 88r^2s - 88r^3s + 24r^4s + 44r^2t - 44r^3t + 12r^4t + 264s^2t - 22r^3 + 24r^4 - 7r^5 - 198rs^2t + 176r^2st - 44r^3st + 44r^2s^2t - 198rst)$,

$K_{2_{23}}^{'[4]_3} = \frac{-s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2 - 22r^2s^3 + 88rs^2 - 99r^2s - 88rs^3 + 24rs^4 + 264r^2t + 44s^2t - 44s^3t + 12s^4t - 22s^3 + 24s^4 - 7s^5 + 176rs^2t - 198r^2st - 44rs^3t + 44r^2s^2t - 198rst)$,

$K_{2_{33}}^{'[4]_3} = \frac{-t^4}{55440(r-t)^2(s-t)^2(t-1)^2}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4)$,

$K_{2_{43}}^{'[4]_3} = \frac{-1}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(24r + 24s + 12t - 99r^2s^2 - 88rs - 44rt - 44st + 88rs^2 + 88r^2s + 44r^2t + 44s^2t - 22r^2 - 22s^2 - 198rs^2t - 198r^2st + 264r^2s^2t + 176rst - 7)$,

$K_{2_{14}}^{'[4]_3} = \frac{-r^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-7r^5 + 24r^4s + 24r^4t + 12r^4 - 22r^3s^2 - 88r^3st - 44r^3s - 22r^3t^2 - 44r^3t + 88r^2s^2t + 44r^2s^2 + 88r^2st^2 + 176r^2st + 44r^2t^2 - 99rs^2t^2 - 198rs^2t - 198rst^2 + 264s^2t^2)$,

$K_{2_{24}}^{'[4]_3} = \frac{-s^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-22r^2s^3 + 88r^2s^2t + 44r^2s^2 - 99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 + 88rs^2t^2 + 176rs^2t - 198rst^2 - 7s^5 + 24s^4t + 12s^4 - 22s^3t^2 - 44s^3t + 44s^2t^2)$,

$K_{2_{34}}^{'[4]_3} = \frac{-t^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-99r^2s^2t + 264r^2s^2 + 88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4)$,

$K_{2_{44}}^{'[4]_3} = \frac{-1}{55440(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st - 110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 - 440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14)$.

The second derivative of (5.12) - (5.15) are as below :

$y''_{n+r} = y''_n + hry'''_n + \frac{g_nh^3r^3}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2) - \frac{f_nh^2r^2}{27720s^3t^3}(132r^6s^2t - 42r^7s - 42r^7t - 42r^7st + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 -$

$$1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 +$$
$$990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 -$$
$$9702s^3t^3) - \frac{g_{n+1}h^3r^6}{27720(r-1)^2(s-1)^2(t-1)^2}(66r^4s - 21r^5 + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st -$$
$$110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 -$$
$$198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2) - \frac{g_{n+r}h^3r^3}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 77r^5s -$$
$$77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t -$$
$$165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t +$$
$$132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2) -$$
$$\frac{f_{n+1}h^2r^6}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^6st - 84r^6s - 84r^6t + 126r^6 - 132r^5s^2t + 264r^5s^2 -$$
$$132r^5st^2 + 345r^5st - 105r^5s + 264r^5t^2 - 105r^5t - 399r^5 + 110r^4s^3t - 220r^4s^3 +$$
$$440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 - 506r^4st^2 - 407r^4st + 1067r^4s -$$
$$220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 - 396r^3s^2t^3 -$$
$$484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 + 2387r^3st^2 - 2992r^3st - 990r^3s +$$
$$825r^3t^3 - 363r^3t^2 - 990r^3t + 396r^2s^3t^3 + 1188r^2s^3t^2 - 2673r^2s^3t - 693r^2s^3 +$$
$$1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st -$$
$$693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 + 726rs^2t^2 -$$
$$3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2) +$$
$$\frac{f_{n+r}h^2r^2}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^9 - 2646r^8s - 2646r^8t - 2646r^8 + 3069r^7s^2 + 9420r^7st +$$
$$9420r^7s + 3069r^7t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 -$$
$$11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 - 11121r^6t^2 - 11121r^6t - 1155r^6 +$$
$$4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 +$$
$$41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t - 5148r^4s^3t^2 -$$
$$16027r^4s^3t - 5148r^4s^3 - 5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 -$$
$$16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 - 5148r^4t^2 + 1980r^3s^3t^3 +$$
$$20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t +$$
$$20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t -$$
$$26598r^2s^2t^3 - 26598r^2s^2t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 -$$
$$4158s^3t^3) - \frac{g_{n+s}h^3r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s -$$
$$198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 + 396r^2st -$$

$110r^3st + 99r^2st^2 - 396rst) - \frac{g_{n+t}h^3r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst) + \frac{f_{n+s}h^2r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(84r^6st - 126r^6s^2 + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 - 345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 + 616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s - 110r^4t^3 - 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 + 2992r^3s^3t + 363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3 + 484r^3st^2 - 275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 + 693r^2s^3t^3 - 891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t - 1188r^2st^3 - 1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 - 2772rs^3t - 1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 - 2310s^2t^3) + \frac{f_{n+t}h^2r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(42r^6s - 84r^6st + 126r^6t^2 - 84r^6t + 264r^5s^2t - 132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t + 110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st + 110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 - 363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 + 275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t + 396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 + 891r^2st^3 - 2673r^2st^2 + 891r^2t^4 - 693r^2t^3 + 2772rs^3t^3 + 1518rs^3t^2 - 2178rs^3t - 3564rs^2t^4 + 726rs^2t^3 + 1518rs^2t^2 - 3564rst^4 + 2772rst^3 - 3234s^3t^3 + 2310s^3t^2 + 4158s^2t^4 - 3234s^2t^3), \quad (5.21)$

$y''_{n+s} = y''_n + hsy'''_n + \frac{g_nh^3s^3}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st + 1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 + 21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2) - \frac{f_nh^2s^2}{27720r^3t^3}(396r^3s^4t^2 - 110r^3s^5 - 110r^3s^5t + 583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 + 264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t +$

$$132r^2s^6 - 440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 - 1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t - 110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t + 132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3) - \frac{g_{n+1}h^3s^6}{27720(r-1)^2(s-1)^2(t-1)^2}(198r^2s^2t - 55r^2s^3 + 99r^2s^2 - 198r^2st^2 - 396r^2st + 462r^2t^2 + 66rs^4 - 220rs^3t - 110rs^3 + 198rs^2t^2 + 396rs^2t - 396rst^2 - 21s^5 + 66s^4t + 33s^4 - 55s^3t^2 - 110s^3t + 99s^2t^2) - \frac{g_{n+s}h^3s^3}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2) - \frac{f_{n+1}h^2s^6}{27720(r-1)^3(s-1)^3(t-1)^3}(110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + 440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 + 2387rs^3t^2 - 2992rs^3t - 990rs^3 - 2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - 616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2) + \frac{f_{n+s}h^2s^2}{27720(r-s)^3(s-t)^3(s-1)^3}(1155r^3s^6 - 4235r^3s^5t - 4235r^3s^5 + 5148r^3s^4t^2 + 16027r^3s^4t + 5148r^3s^4 - 1980r^3s^3t^3 - 20196r^3s^3t^2 - 20196r^3s^3t - 1980r^3s^3 + 7920r^3s^2t^3 + 26598r^3s^2t^2 + 7920r^3s^2t - 10626r^3st^3 - 10626r^3st^2 + 4158r^3t^3 - 3069r^2s^7 + 11121r^2s^6t + 11121r^2s^6 - 13376r^2s^5t^2 - 41437r^2s^5t - 13376r^2s^5 + 5148r^2s^4t^3 + 51436r^2s^4t^2 + 51436r^2s^4t + 5148r^2s^4 - 20196r^2s^3t^3 - 66330r^2s^3t^2 - 20196r^2s^3t + 26598r^2s^2t^3 + 26598r^2s^2t^2 - 10626r^2st^3 + 2646rs^8 - 9420rs^7t - 9420rs^7 + 11121rs^6t^2 + 34278rs^6t + 11121rs^6 - 4235rs^5t^3 - 41437rs^5t^2 - 41437rs^5t - 4235rs^5 + 16027rs^4t^3 + 51436rs^4t^2 + 16027rs^4t - 20196rs^3t^3 - 20196rs^3t^2 + 7920rs^2t^3 - 756s^9 + 2646s^8t + 2646s^8 - 3069s^7t^2 - 9420s^7t - 3069s^7$$

$$+1155s^6t^3+11121s^6t^2+11121s^6t+1155s^6-4235s^5t^3-13376s^5t^2-4235s^5t+$$

$$5148s^4t^3+5148s^4t^2-1980s^3t^3)-\frac{f_{n+t}h^2s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t-110r^3s^4-$$

$$825r^3s^3t^2-275r^3s^3t+396r^3s^3+693r^3s^2t^3+2673r^3s^2t^2-1188r^3s^2t-396r^3s^2-$$

$$2772r^3st^3-1518r^3st^2+2178r^3st+3234r^3t^3-2310r^3t^2-264r^2s^5t+132r^2s^5+$$

$$616r^2s^4t^2+506r^2s^4t-440r^2s^4+363r^2s^3t^3-2387r^2s^3t^2+484r^2s^3t+396r^2s^3-$$

$$891r^2s^2t^4-891r^2s^2t^3+3168r^2s^2t^2-1188r^2s^2t+3564r^2st^4-726r^2st^3-$$

$$1518r^2st^2-4158r^2t^4+3234r^2t^3+84rs^6t-42rs^6+105rs^5t^2-345rs^5t+132rs^5-$$

$$1067rs^4t^3+407rs^4t^2+506rs^4t-110rs^4+990rs^3t^4+2992rs^3t^3-2387rs^3t^2-$$

$$275rs^3t-3564rs^2t^4-891rs^2t^3+2673rs^2t^2+3564rst^4-2772rst^3-126s^6t^2+$$

$$84s^6t+399s^5t^3+105s^5t^2-264s^5t-297s^4t^4-1067s^4t^3+616s^4t^2+220s^4t+$$

$$990s^3t^4+363s^3t^3-825s^3t^2-891s^2t^4+693s^2t^3)-\frac{g_{n+r}h^3s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2-$$

$$55s^3t^2+99rs^2-110rs^3+33rs^4+462rt^2-198st^2+198s^2t-220s^3t+66s^4t-$$

$$55s^3+66s^4-21s^5-396rst^2+396rs^2t-110rs^3t+99rs^2t^2-396rst)-$$

$$\frac{g_{n+t}h^3s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2-55r^2s^3+198rs^2-198r^2s-220rs^3+66rs^4+$$

$$462r^2t+99s^2t-110s^3t+33s^4t-55s^3+66s^4-21s^5+396rs^2t-396r^2st-$$

$$110rs^3t+99r^2s^2t-396rst)+\frac{f_{n+r}h^2s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(297r^4s^4-990r^4s^3t-$$

$$990r^4s^3+891r^4s^2t^2+3564r^4s^2t+891r^4s^2-3564r^4st^2-3564r^4st+4158r^4t^2-$$

$$399r^3s^5+1067r^3s^4t+1067r^3s^4-363r^3s^3t^2-2992r^3s^3t-363r^3s^3-693r^3s^2t^3+$$

$$891r^3s^2t^2+891r^3s^2t-693r^3s^2+2772r^3st^3+726r^3st^2+2772r^3st-3234r^3t^3-$$

$$3234r^3t^2+126r^2s^6-105r^2s^5t-105r^2s^5-616r^2s^4t^2-407r^2s^4t-616r^2s^4+$$

$$825r^2s^3t^3+2387r^2s^3t^2+2387r^2s^3t+825r^2s^3-2673r^2s^2t^3-3168r^2s^2t^2-$$

$$2673r^2s^2t+1518r^2st^3+1518r^2st^2+2310r^2t^3-84rs^6t-84rs^6+264rs^5t^2+$$

$$345rs^5t+264rs^5-220rs^4t^3-506rs^4t^2-506rs^4t-220rs^4+275rs^3t^3-484rs^3t^2+$$

$$275rs^3t+1188rs^2t^3+1188rs^2t^2-2178rst^3+42s^6t-132s^5t^2-132s^5t+110s^4t^3+$$

$$440s^4t^2+110s^4t-396s^3t^3-396s^3t^2+396s^2t^3), \tag{5.22}$$

$$y''_{n+t}=y''_n+hty'''_n+\frac{g_nh^3t^3}{27720r^2s^2}(198r^2s^2t^2-924r^2s^2t+1386r^2s^2-198r^2st^3+792r^2st^2-$$

$$924r^2st+55r^2t^4-198r^2t^3+198r^2t^2-198rs^2t^3+792rs^2t^2-924rs^2t+220rst^4-$$

$$792rst^3+792rst^2-66rt^5+220rt^4-198rt^3+55s^2t^4-198s^2t^3+198s^2t^2-66st^5+$$

$$220st^4-198st^3+21t^6-66t^5+55t^4)-\frac{f_nh^2t^2}{27720r^3s^3}(990r^3s^3t^2-396r^3s^3t^3+1848r^3s^3t-$$
$$9702r^3s^3+396r^3s^2t^4-1188r^3s^2t^3+264r^3s^2t^2+1848r^3s^2t-110r^3st^5+583r^3st^4-$$
$$1188r^3st^3+990r^3st^2-110r^3t^5+396r^3t^4-396r^3t^3+396r^2s^3t^4-1188r^2s^3t^3+$$
$$264r^2s^3t^2+1848r^2s^3t-440r^2s^2t^5+1540r^2s^2t^4-1584r^2s^2t^3+264r^2s^2t^2+$$
$$132r^2st^6-748r^2st^5+1540r^2st^4-1188r^2st^3+132r^2t^6-440r^2t^5+396r^2t^4-$$
$$110rs^3t^5+583rs^3t^4-1188rs^3t^3+990rs^3t^2+132rs^2t^6-748rs^2t^5+1540rs^2t^4-$$
$$1188rs^2t^3-42rst^7+321rst^6-748rst^5+583rst^4-42rt^7+132rt^6-110rt^5-$$
$$110s^3t^5+396s^3t^4-396s^3t^3+132s^2t^6-440s^2t^5+396s^2t^4-42st^7+132st^6-$$
$$110st^5)-\frac{g_{n+1}h^3t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2-198r^2s^2t+198r^2st^2-396r^2st-$$
$$55r^2t^3+99r^2t^2+198rs^2t^2-396rs^2t-220rst^3+396rst^2+66rt^4-110rt^3-$$
$$55s^2t^3+99s^2t^2+66st^4-110st^3-21t^5+33t^4)-\frac{g_{n+t}h^3t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2-$$
$$462r^2s^2t+462r^2s^2-165r^2st^3+528r^2st^2-462r^2st+55r^2t^4-165r^2t^3+132r^2t^2-$$
$$165rs^2t^3+528rs^2t^2-462rs^2t+220rst^4-660rst^3+528rst^2-77rt^5+220rt^4-$$
$$165rt^3+55s^2t^4-165s^2t^3+132s^2t^2-77st^5+220st^4-165st^3+28t^6-77t^5+$$
$$55t^4)-\frac{f_{n+1}h^2t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(396r^3s^3t^2-2178r^3s^3t+2310r^3s^3-396r^3s^2t^3+$$
$$1188r^3s^2t^2+1518r^3s^2t-3234r^3s^2+110r^3st^4+275r^3st^3-2673r^3st^2+2772r^3st-$$
$$220r^3t^4+825r^3t^3-693r^3t^2-396r^2s^3t^3+1188r^2s^3t^2+1518r^2s^3t-3234r^2s^3+$$
$$440r^2s^2t^4-484r^2s^2t^3-3168r^2s^2t^2+726r^2s^2t+4158r^2s^2-132r^2st^5-506r^2st^4+$$
$$2387r^2st^3+891r^2st^2-3564r^2st+264r^2t^5-616r^2t^4-363r^2t^3+891r^2t^2+$$
$$110rs^3t^4+275rs^3t^3-2673rs^3t^2+2772rs^3t-132rs^2t^5-506rs^2t^4+2387rs^2t^3+$$
$$891rs^2t^2-3564rs^2t+42rst^6+345rst^5-407rst^4-2992rst^3+3564rst^2-$$
$$84rt^6-105rt^5+1067rt^4-990rt^3-220s^3t^4+825s^3t^3-693s^3t^2+264s^2t^5-$$
$$616s^2t^4-363s^2t^3+891s^2t^2-84st^6-105st^5+1067st^4-990st^3+126t^6-$$
$$399t^5+297t^4)-\frac{f_{n+t}h^2t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(7920r^3s^3t^2-1980r^3s^3t^3-10626r^3s^3t+$$
$$4158r^3s^3+5148r^3s^2t^4-20196r^3s^2t^3+26598r^3s^2t^2-10626r^3s^2t-4235r^3st^5+$$
$$16027r^3st^4-20196r^3st^3+7920r^3st^2+1155r^3t^6-4235r^3t^5+5148r^3t^4-$$
$$1980r^3t^3+5148r^2s^3t^4-20196r^2s^3t^3+26598r^2s^3t^2-10626r^2s^3t-13376r^2s^2t^5+$$
$$51436r^2s^2t^4-66330r^2s^2t^3+26598r^2s^2t^2+11121r^2st^6-41437r^2st^5+51436r^2st^4-$$
$$20196r^2st^3-3069r^2t^7+11121r^2t^6-13376r^2t^5+5148r^2t^4-4235rs^3t^5+16027rs^3$$

$$
\begin{aligned}
&t^4-20196rs^3t^3+7920rs^3t^2+11121rs^2t^6-41437rs^2t^5+51436rs^2t^4-20196rs^2t^3-\\
&9420rst^7+34278rst^6-41437rst^5+16027rst^4+2646rt^8-9420rt^7+11121rt^6-\\
&4235rt^5+1155s^3t^6-4235s^3t^5+5148s^3t^4-1980s^3t^3-3069s^2t^7+11121s^2t^6-\\
&13376s^2t^5+5148s^2t^4+2646st^8-9420st^7+11121st^6-4235st^5-756t^9+2646t^8-\\
&3069t^7+1155t^6)+\tfrac{f_{n+r}h^2t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2-3564r^4s^2t+4158r^4s^2-\\
&990r^4st^3+3564r^4st^2-3564r^4st+297r^4t^4-990r^4t^3+891r^4t^2-693r^3s^3t^2+\\
&2772r^3s^3t-3234r^3s^3-363r^3s^2t^3+891r^3s^2t^2+726r^3s^2t-3234r^3s^2+1067r^3st^4-\\
&2992r^3st^3+891r^3st^2+2772r^3st-399r^3t^5+1067r^3t^4-363r^3t^3-693r^3t^2+\\
&825r^2s^3t^3-2673r^2s^3t^2+1518r^2s^3t+2310r^2s^3-616r^2s^2t^4+2387r^2s^2t^3-\\
&3168r^2s^2t^2+1518r^2s^2t-105r^2st^5-407r^2st^4+2387r^2st^3-2673r^2st^2+126r^2t^6-\\
&105r^2t^5-616r^2t^4+825r^2t^3-220rs^3t^4+275rs^3t^3+1188rs^3t^2-2178rs^3t+\\
&264rs^2t^5-506rs^2t^4-484rs^2t^3+1188rs^2t^2-84rst^6+345rst^5-506rst^4+\\
&275rst^3-84rt^6+264rt^5-220rt^4+110s^3t^4-396s^3t^3+396s^3t^2-132s^2t^5+\\
&440s^2t^4-396s^2t^3+42st^6-132st^5+110st^4)+\tfrac{f_{n+s}h^2t^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^3s^3t^2-\\
&2772r^3s^3t+3234r^3s^3-825r^3s^2t^3+2673r^3s^2t^2-1518r^3s^2t-2310r^3s^2+220r^3st^4-\\
&275r^3st^3-1188r^3st^2+2178r^3st-110r^3t^4+396r^3t^3-396r^3t^2-891r^2s^4t^2+\\
&3564r^2s^4t-4158r^2s^4+363r^2s^3t^3-891r^2s^3t^2-726r^2s^3t+3234r^2s^3+616r^2s^2t^4-\\
&2387r^2s^2t^3+3168r^2s^2t^2-1518r^2s^2t-264r^2st^5+506r^2st^4+484r^2st^3-1188r^2st^2+\\
&132r^2t^5-440r^2t^4+396r^2t^3+990rs^4t^3-3564rs^4t^2+3564rs^4t-1067rs^3t^4+\\
&2992rs^3t^3-891rs^3t^2-2772rs^3t+105rs^2t^5+407rs^2t^4-2387rs^2t^3+2673rs^2t^2+\\
&84rst^6-345rst^5+506rst^4-275rst^3-42rt^6+132rt^5-110rt^4-297s^4t^4+\\
&990s^4t^3-891s^4t^2+399s^3t^5-1067s^3t^4+363s^3t^3+693s^3t^2-126s^2t^6+105s^2t^5+\\
&616s^2t^4-825s^2t^3+84st^6-264st^5+220st^4)-\tfrac{g_{n+r}h^3t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2-\\
&55s^2t^3+462rs^2+99rt^2-110rt^3+33rt^4+198st^2-198s^2t-220st^3+66st^4-\\
&55t^3+66t^4-21t^5+396rst^2-396rs^2t-110rst^3+99rs^2t^2-396rst)-\\
&\tfrac{g_{n+s}h^3t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2-55r^2t^3+462r^2s+198rt^2-198r^2t-220rt^3+\\
&66rt^4+99st^2-110st^3+33st^4-55t^3+66t^4-21t^5+396rst^2-396r^2st-110rst^3+\\
&99r^2st^2-396rst),
\end{aligned}
\tag{5.23}
$$

$$
\begin{aligned}
y''_{n+1} &= y''_n + hy'''_n + \frac{g_n h^3}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - \\
&198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + \\
&220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + \\
&55t^2 - 66t + 21) + \frac{f_{n+1}h^2}{27720(r-1)^3(s-1)^3(t-1)^3}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - \\
&1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - \\
&20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - \\
&10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + \\
&51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + \\
&5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - \\
&4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - \\
&41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - \\
&1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - \\
&3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - \\
&756) - \frac{g_{n+1}h^3}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - \\
&165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + \\
&220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + \\
&55t^2 - 77t + 28) - \frac{f_n h^2}{27720r^3s^3t^3}(1848r^3s^3t^2 - 9702r^3s^3t^3 + 990r^3s^3t - 396r^3s^3 + \\
&1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 + 990r^3st^3 - 1188r^3st^2 + 583r^3st - \\
&110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t + \\
&396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - \\
&748r^2st + 132r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - \\
&110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 + 583rst^3 - 748rst^2 + 321rst - \\
&42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + \\
&132s^2t - 110st^3 + 132st^2 - 42st) - \frac{f_{n+r}h^2}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(3564r^4s^2t - 4158r^4s^2t^2 - \\
&891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - \\
&2772r^3s^3t + 693r^3s^3 + 3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - \\
&891r^3st^2 + 2992r^3st - 1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - \\
&1518r^2s^3t^2 + 2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + \\
&616r^2s^2 + 2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t -
\end{aligned}
$$

$$
\begin{aligned}
&126r^2 + 2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + \\
&506rs^2t - 264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - \\
&396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - \\
&42st) + \tfrac{f_{n+s}h^2}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - \\
&1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - \\
&396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - \\
&726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + \\
&616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + \\
&132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - \\
&1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - \\
&345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + \\
&363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - \\
&264st^2 + 84st) + \tfrac{f_{n+t}h^2}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2 - 2178r^3s^3t + 396r^3s^3 - \\
&3234r^3s^2t^3 + 1518r^3s^2t^2 + 1188r^3s^2t - 396r^3s^2 + 2772r^3st^3 - 2673r^3st^2 + 275r^3st + \\
&110r^3s - 693r^3t^3 + 825r^3t^2 - 220r^3t - 3234r^2s^3t^3 + 1518r^2s^3t^2 + 1188r^2s^3t - \\
&396r^2s^3 + 4158r^2s^2t^4 + 726r^2s^2t^3 - 3168r^2s^2t^2 - 484r^2s^2t + 440r^2s^2 - 3564r^2st^4 + \\
&891r^2st^3 + 2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t + \\
&2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 - \\
&506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 + \\
&1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 - \\
&616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2) - \\
&\tfrac{g_{n+r}h^3}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + \\
&99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - \\
&21) - \tfrac{g_{n+s}h^3}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + \\
&99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + \\
&396rst - 21) - \tfrac{g_{n+t}h^3}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - \\
&110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t \\
&+396rst - 21)
\end{aligned} \tag{5.24}
$$

which also can be represented in block form as below

$$\begin{aligned} I_4 Y_{n+1}^{''[4]_3} &= M_3^{''[4]_3} Y_{n-2}^{[4]_3} + h M_4^{''[4]_3} Y_{n-3}^{[4]_3} + h^2 \left[E_1^{''[4]_3} F_n^{[4]_3} + E_2^{''[4]_3} F_{n+1}^{[4]_3} \right] \\ &\quad + h^3 \left[K_1^{''[4]_3} G_n^{[4]_3} + K_2^{''[4]_3} G_{n+1}^{[4]_3} \right] \end{aligned} \tag{5.25}$$

where

$$Y_{n+1}^{''[4]_3} = \begin{pmatrix} y_{n+r}^{''} \\ y_{n+s}^{''} \\ y_{n+t}^{''} \\ y_{n+1}^{''} \end{pmatrix}, \; M_3^{''[4]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; M_4^{''[4]_3} = \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & 0 & s \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$E_1^{''[4]_3} = \begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{''[4]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{''[4]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{''[4]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{''[4]_3} \end{pmatrix}, \; E_2^{''[4]_3} = \begin{pmatrix} E_{2_{11}}^{''[4]_3} & E_{2_{12}}^{''[4]_3} & E_{2_{13}}^{''[4]_3} & E_{2_{14}}^{''[4]_3} \\ E_{2_{21}}^{''[4]_3} & E_{2_{22}}^{''[4]_3} & E_{2_{23}}^{''[4]_3} & E_{2_{24}}^{''[4]_3} \\ E_{2_{31}}^{''[4]_3} & E_{2_{32}}^{''[4]_3} & E_{2_{33}}^{''[4]_3} & E_{2_{34}}^{''[4]_3} \\ E_{2_{41}}^{''[4]_3} & E_{2_{42}}^{''[4]_3} & E_{2_{43}}^{''[4]_3} & E_{2_{44}}^{''[4]_3} \end{pmatrix},$$

$$K_1^{''[4]_3} = \begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{''[4]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{''[4]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{''[4]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{''[4]_3} \end{pmatrix}, \; K_2^{''[4]_3} = \begin{pmatrix} K_{2_{11}}^{''[4]_3} & K_{2_{12}}^{''[4]_3} & K_{2_{13}}^{''[4]_3} & K_{2_{14}}^{''[4]_3} \\ K_{2_{21}}^{''[4]_3} & K_{2_{22}}^{''[4]_3} & K_{2_{23}}^{''[4]_3} & K_{2_{24}}^{''[4]_3} \\ K_{2_{31}}^{''[4]_3} & K_{2_{32}}^{''[4]_3} & K_{2_{33}}^{''[4]_3} & K_{2_{34}}^{''[4]_3} \\ K_{2_{41}}^{''[4]_3} & K_{2_{42}}^{''[4]_3} & K_{2_{43}}^{''[4]_3} & K_{2_{44}}^{''[4]_3} \end{pmatrix}$$

where the non zero entries are as follows

$E_{1_{14}}^{''[4]_3} = \frac{-r^2}{27720s^3t^3}(-42r^7st - 42r^7s - 42r^7t + 132r^6s^2t + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 - 1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 31188r^3st^2 - 96r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2+$

$990r^2st^3+1848rs^3t^3+1848rs^3t^2+1848rs^2t^3-9702s^3t^3)$,

$E_{1_{24}}^{''[4]_3} = \frac{-s^2}{27720r^3t^3}(-110r^3s^5t - 110r^3s^5 + 396r^3s^4t^2 + 583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 + 264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t + 132r^2s^6 - 440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 - 1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t - 110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t + 132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3)$,

$E_{1_{34}}^{''[4]_3} = \frac{-t^2}{27720r^3s^3}(-396r^3s^3t^3 + 990r^3s^3t^2 + 1848r^3s^3t - 9702r^3s^3 + 396r^3s^2t^4 - 1188r^3s^2t^3 + 264r^3s^2t^2 + 1848r^3s^2t - 110r^3st^5 + 583r^3st^4 - 1188r^3st^3 + 990r^3st^2 - 110r^3t^5 + 396r^3t^4 - 396r^3t^3 + 396r^2s^3t^4 - 1188r^2s^3t^3 + 264r^2s^3t^2 + 1848r^2s^3t - 440r^2s^2t^5 + 1540r^2s^2t^4 - 1584r^2s^2t^3 + 264r^2s^2t^2 + 132r^2st^6 - 748r^2st^5 + 1540r^2st^4 - 1188r^2st^3 + 132r^2t^6 - 440r^2t^5 + 396r^2t^4 - 110rs^3t^5 + 583rs^3t^4 - 1188rs^3t^3 + 990rs^3t^2 + 132rs^2t^6 - 748rs^2t^5 + 1540rs^2t^4 - 1188rs^2t^3 - 42rst^7 + 321rst^6 - 748rst^5 + 583rst^4 - 42rt^7 + 132rt^6 - 110rt^5 - 110s^3t^5 + 396s^3t^4 - 396s^3t^3 + 132s^2t^6 - 440s^2t^5 + 396s^2t^4 - 42st^7 + 132st^6 - 110st^5)$,

$E_{1_{44}}^{''[4]_3} = \frac{-1}{27720r^3s^3t^3}(-9702r^3s^3t^3 + 1848r^3s^3t^2 + 990r^3s^3t - 396r^3s^3 + 1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 + 990r^3st^3 - 1188r^3st^2 + 583r^3st - 110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t + 396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - 748r^2st + 132r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - 110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 + 583rst^3 - 748rst^2 + 321rst - 42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st)$,

$E_{2_{11}}^{''[4]_3} = \frac{r^2}{27720(r-s)^3(r-t)^3(r-1)^3}(756r^9 - 2646r^8s - 2646r^8t - 2646r^8 + 3069r^7s^2 + 9420r^7st + 9420r^7s + 3069r^7t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 - 11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 - 11121r^6t^2 - 11121$

$r^6t - 1155r^6 + 4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 + 41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t - 5148r^4s^3t^2 - 16027r^4s^3t - 5148r^4s^3 - 5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 - 16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 - 5148r^4t^2 + 1980r^3s^3t^3 + 20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t + 20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t - 26598r^2s^2t^3 - 26598r^2s^2t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 - 4158s^3t^3)$,

$E''^{[4]_3}_{2_{21}} = \frac{s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(297r^4s^4 - 990r^4s^3t - 990r^4s^3 + 891r^4s^2t^2 + 3564r^4s^2t + 891r^4s^2 - 3564r^4st^2 - 3564r^4st + 4158r^4t^2 - 399r^3s^5 + 1067r^3s^4t + 1067r^3s^4 - 363r^3s^3t^2 - 2992r^3s^3t - 363r^3s^3 - 693r^3s^2t^3 + 891r^3s^2t^2 + 891r^3s^2t - 693r^3s^2 + 2772r^3st^3 + 726r^3st^2 + 2772r^3st - 3234r^3t^3 - 3234r^3t^2 + 126r^2s^6 - 105r^2s^5t - 105r^2s^5 - 616r^2s^4t^2 - 407r^2s^4t - 616r^2s^4 + 825r^2s^3t^3 + 2387r^2s^3t^2 + 2387r^2s^3t + 825r^2s^3 - 2673r^2s^2t^3 - 3168r^2s^2t^2 - 2673r^2s^2t + 1518r^2st^3 + 1518r^2st^2 + 2310r^2t^3 - 84rs^6t - 84rs^6 + 264rs^5t^2 + 345rs^5t + 264rs^5 - 220rs^4t^3 - 506rs^4t^2 - 506rs^4t - 220rs^4 + 275rs^3t^3 - 484rs^3t^2 + 275rs^3t + 1188rs^2t^3 + 1188rs^2t^2 - 2178rst^3 + 42s^6t - 132s^5t^2 - 132s^5t + 110s^4t^3 + 440s^4t^2 + 110s^4t - 396s^3t^3 - 396s^3t^2 + 396s^2t^3)$,

$E''^{[4]_3}_{2_{31}} = \frac{t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2 - 3564r^4s^2t + 4158r^4s^2 - 990r^4st^3 + 3564r^4st^2 - 3564r^4st + 297r^4t^4 - 990r^4t^3 + 891r^4t^2 - 693r^3s^3t^2 + 2772r^3s^3t - 3234r^3s^3 - 363r^3s^2t^3 + 891r^3s^2t^2 + 726r^3s^2t - 3234r^3s^2 + 1067r^3st^4 - 2992r^3st^3 + 891r^3st^2 + 2772r^3st - 399r^3t^5 + 1067r^3t^4 - 363r^3t^3 - 693r^3t^2 + 825r^2s^3t^3 - 2673r^2s^3t^2 + 1518r^2s^3t + 2310r^2s^3 - 616r^2s^2t^4 + 2387r^2s^2t^3 - 3168r^2s^2t^2 + 1518r^2s^2t - 105r^2st^5 - 407r^2st^4 + 2387r^2st^3 - 2673r^2st^2 + 126r^2t^6 - 105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t + 264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 - 84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 - 396s^2t^3 + 42st^6 - 132st^5 + 110st^4)$,

$$E''^{[4]_3}_{2_{41}} = \frac{-1}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-4158r^4s^2t^2+3564r^4s^2t-891r^4s^2+3564r^4st^2-$$

$$3564r^4st+990r^4s-891r^4t^2+990r^4t-297r^4+3234r^3s^3t^2-2772r^3s^3t+693r^3s^3+$$

$$3234r^3s^2t^3-726r^3s^2t^2-891r^3s^2t+363r^3s^2-2772r^3st^3-891r^3st^2+2992r^3st-$$

$$1067r^3s+693r^3t^3+363r^3t^2-1067r^3t+399r^3-2310r^2s^3t^3-1518r^2s^3t^2+$$

$$2673r^2s^3t-825r^2s^3-1518r^2s^2t^3+3168r^2s^2t^2-2387r^2s^2t+616r^2s^2+2673r^2st^3-$$

$$2387r^2st^2+407r^2st+105r^2s-825r^2t^3+616r^2t^2+105r^2t-126r^2+2178rs^3t^3-$$

$$1188rs^3t^2-275rs^3t+220rs^3-1188rs^2t^3+484rs^2t^2+506rs^2t-264rs^2-275rst^3+$$

$$506rst^2-345rst+84rs+220rt^3-264rt^2+84rt-396s^3t^3+396s^3t^2-110s^3t+$$

$$396s^2t^3-440s^2t^2+132s^2t-110st^3+132st^2-42st),$$

$$E''^{[4]_3}_{2_{12}} = \frac{r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-126r^6s^2+84r^6st+84r^6s-42r^6t+399r^5s^3+$$

$$105r^5s^2t+105r^5s^2-264r^5st^2-345r^5st-264r^5s+132r^5t^2+132r^5t-297r^4s^4-$$

$$1067r^4s^3t-1067r^4s^3+616r^4s^2t^2+407r^4s^2t+616r^4s^2+220r^4st^3+506r^4st^2+$$

$$506r^4st+220r^4s-110r^4t^3-440r^4t^2-110r^4t+990r^3s^4t+990r^3s^4+363r^3s^3t^2+$$

$$2992r^3s^3t+363r^3s^3-825r^3s^2t^3-2387r^3s^2t^2-2387r^3s^2t-825r^3s^2-275r^3st^3+$$

$$484r^3st^2-275r^3st+396r^3t^3+396r^3t^2-891r^2s^4t^2-3564r^2s^4t-891r^2s^4+$$

$$693r^2s^3t^3-891r^2s^3t^2-891r^2s^3t+693r^2s^3+2673r^2s^2t^3+3168r^2s^2t^2+2673r^2s^2t-$$

$$1188r^2st^3-1188r^2st^2-396r^2t^3+3564rs^4t^2+3564rs^4t-2772rs^3t^3-726rs^3t^2-$$

$$2772rs^3t-1518rs^2t^3-1518rs^2t^2+2178rst^3-4158s^4t^2+3234s^3t^3+3234s^3t^2-$$

$$2310s^2t^3),$$

$$E''^{[4]_3}_{2_{22}} = \frac{s^2}{27720(r-s)^3(s-t)^3(s-1)^3}(1155r^3s^6-4235r^3s^5t-4235r^3s^5+5148r^3s^4t^2+$$

$$16027r^3s^4t+5148r^3s^4-1980r^3s^3t^3-20196r^3s^3t^2-20196r^3s^3t-1980r^3s^3+$$

$$7920r^3s^2t^3+26598r^3s^2t^2+7920r^3s^2t-10626r^3st^3-10626r^3st^2+4158r^3t^3-$$

$$3069r^2s^7+11121r^2s^6t+11121r^2s^6-13376r^2s^5t^2-41437r^2s^5t-13376r^2s^5+$$

$$5148r^2s^4t^3+51436r^2s^4t^2+51436r^2s^4t+5148r^2s^4-20196r^2s^3t^3-66330r^2s^3t^2-$$

$$20196r^2s^3t+26598r^2s^2t^3+26598r^2s^2t^2-10626r^2st^3+2646rs^8-9420rs^7t-$$

$$9420rs^7+11121rs^6t^2+34278rs^6t+11121rs^6-4235rs^5t^3-41437rs^5t^2-$$

$$41437rs^5t-4235rs^5+16027rs^4t^3+51436rs^4t^2+16027rs^4t-20196rs^3t^3-$$

$$20196rs^3t^2+7920rs^2t^3-756s^9+2646s^8t+2646s^8-3069s^7t^2-9420s^7t-3069s^7+$$

$1155s^6t^3 + 11121s^6t^2 + 11121s^6t + 1155s^6 - 4235s^5t^3 - 13376s^5t^2 - 4235s^5t + 5148s^4t^3 + 5148s^4t^2 - 1980s^3t^3)$,

$$E^{''[4]_3}_{2_{32}} = \frac{t^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(693r^3s^3t^2 - 2772r^3s^3t + 3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 - 275r^3st^3 - 1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 + 3564r^2s^4t - 4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 - 2387r^2s^2t^3 + 3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 + 132r^2t^5 - 440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 + 2992rs^3t^3 - 891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 + 84rst^6 - 345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 - 891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + 616s^2t^4 - 825s^2t^3 + 84st^6 - 264st^5 + 220st^4),$$

$$E^{''[4]_3}_{2_{42}} = \frac{1}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - 726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st),$$

$$E^{''[4]_3}_{2_{13}} = \frac{r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-84r^6st + 42r^6s + 126r^6t^2 - 84r^6t + 264r^5s^2t - 132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t - 220r^4s^3t + 110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 - 506r^4st + 110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t - 396r^3s^3 - 363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 + 2387r^3st^2 + 275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 + 1188r^2s^3t + 396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 + 891r^2st^3 -$$

$2673r^2st^2+891r^2t^4-693r^2t^3+2772rs^3t^3+1518rs^3t^2-2178rs^3t-3564rs^2t^4+726rs^2t^3+1518rs^2t^2-3564rst^4+2772rst^3-3234s^3t^3+2310s^3t^2+4158s^2t^4-3234s^2t^3)$,

$E''^{[4]_3}_{2_{23}}=\frac{-s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t-110r^3s^4-825r^3s^3t^2-275r^3s^3t+396r^3s^3+693r^3s^2t^3+2673r^3s^2t^2-1188r^3s^2t-396r^3s^2-2772r^3st^3-1518r^3st^2+2178r^3st+3234r^3t^3-2310r^3t^2-264r^2s^5t+132r^2s^5+616r^2s^4t^2+506r^2s^4t-440r^2s^4+363r^2s^3t^3-2387r^2s^3t^2+484r^2s^3t+396r^2s^3-891r^2s^2t^4-891r^2s^2t^3+3168r^2s^2t^2-1188r^2s^2t+3564r^2st^4-726r^2st^3-1518r^2st^2-4158r^2t^4+3234r^2t^3+84rs^6t-42rs^6+105rs^5t^2-345rs^5t+132rs^5-1067rs^4t^3+407rs^4t^2+506rs^4t-110rs^4+990rs^3t^4+2992rs^3t^3-2387rs^3t^2-275rs^3t-3564rs^2t^4-891rs^2t^3+2673rs^2t^2+3564rst^4-2772rst^3-126s^6t^2+84s^6t+399s^5t^3+105s^5t^2-264s^5t-297s^4t^4-1067s^4t^3+616s^4t^2+220s^4t+990s^3t^4+363s^3t^3-825s^3t^2-891s^2t^4+693s^2t^3)$,

$E''^{[4]_3}_{2_{33}}=\frac{-t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(-1980r^3s^3t^3+7920r^3s^3t^2-10626r^3s^3t+4158r^3s^3+5148r^3s^2t^4-20196r^3s^2t^3+26598r^3s^2t^2-10626r^3s^2t-4235r^3st^5+16027r^3st^4-20196r^3st^3+7920r^3st^2+1155r^3t^6-4235r^3t^5+5148r^3t^4-1980r^3t^3+5148r^2s^3t^4-20196r^2s^3t^3+26598r^2s^3t^2-10626r^2s^3t-13376r^2s^2t^5+51436r^2s^2t^4-66330r^2s^2t^3+26598r^2s^2t^2+11121r^2st^6-41437r^2st^5+51436r^2st^4-20196r^2st^3-3069r^2t^7+11121r^2t^6-13376r^2t^5+5148r^2t^4-4235rs^3t^5+16027rs^3t^4-20196rs^3t^3+7920rs^3t^2+11121rs^2t^6-41437rs^2t^5+51436rs^2t^4-20196rs^2t^3-9420rst^7+34278rst^6-41437rst^5+16027rst^4+2646rt^8-9420rt^7+11121rt^6-4235rt^5+1155s^3t^6-4235s^3t^5+5148s^3t^4-1980s^3t^3-3069s^2t^7+11121s^2t^6-13376s^2t^5+5148s^2t^4+2646st^8-9420st^7+11121st^6-4235st^5-756t^9+2646t^8-3069t^7+1155t^6)$,

$E''^{[4]_3}_{2_{43}}=\frac{1}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2-2178r^3s^3t+396r^3s^3-3234r^3s^2t^3+1518r^3s^2t^2+1188r^3s^2t-396r^3s^2+2772r^3st^3-2673r^3st^2+275r^3st+110r^3s-693r^3t^3+825r^3t^2-220r^3t-3234r^2s^3t^3+1518r^2s^3t^2+1188r^2s^3t-396r^2s^3+4158r^2s^2t^4+726r^2s^2t^3-3168r^2s^2t^2-484r^2s^2t+440r^2s^2-3564r^2st^4+891r^2st^3+$

$2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t + 2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 - 506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 + 1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 - 616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2)$,

$E''^{[4]_3}_{2_{14}} = \frac{-r^6}{27720(r-1)^3(s-1)^3(t-1)^3}(42r^6st - 84r^6s - 84r^6t + 126r^6 - 132r^5s^2t + 264r^5s^2 - 132r^5st^2 + 345r^5st - 105r^5s + 264r^5t^2 - 105r^5t - 399r^5 + 110r^4s^3t - 220r^4s^3 + 440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 - 506r^4st^2 - 407r^4st + 1067r^4s - 220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 - 396r^3s^2t^3 - 484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 + 2387r^3st^2 - 2992r^3st - 990r^3s + 825r^3t^3 - 363r^3t^2 - 990r^3t + 396r^2s^3t^3 + 1188r^2s^3t^2 - 2673r^2s^3t - 693r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 + 726rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2)$,

$E''^{[4]_3}_{2_{24}} = \frac{-s^6}{27720(r-1)^3(s-1)^3(t-1)^3}(110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + 440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 + 2387rs^3t^2 - 2992rs^3t - 990rs^3 - 2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - 616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2)$,

$E''^{[4]_3}_{2_{34}} = \frac{-t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 + 1188r^3s^2t^2 + 1518r^3s^2t - 3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st - 220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + 1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 + 440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - 506r^2st^4 +$

$2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 + 275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 - 3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4)$,

$E_{2_{44}}^{''[4]_3} = \frac{1}{27720(r-1)^3(s-1)^3(t-1)^3}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - 10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - 41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756)$,

$K_{1_{14}}^{''[4]_3} = \frac{r^3}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2)$,

$K_{1_{24}}^{''[4]_3} = \frac{s^3}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st + 1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 + 21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2)$,

$K_{1_{34}}^{''[4]_3} = \frac{t^3}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4)$,

$$K''^{[4]_3}_{1_{44}} = \frac{1}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21).$$

$$K''^{[4]_3}_{2_{11}} = \frac{-r^3}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2),$$

$$K''^{[4]_3}_{2_{21}} = \frac{-s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^3t^2 + 99rs^2 - 110rs^3 + 33rs^4 + 462rt^2 - 198st^2 + 198s^2t - 220s^3t + 66s^4t - 55s^3 + 66s^4 - 21s^5 - 396rst^2 + 396rs^2t - 110rs^3t + 99rs^2t^2 - 396rst),$$

$$K''^{[4]_3}_{2_{31}} = \frac{-t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2 - 55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst),$$

$$K''^{[4]_3}_{2_{41}} = \frac{-1}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21),$$

$$K''^{[4]_3}_{2_{12}} = \frac{-r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s - 198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 + 396r^2st - 110r^3st + 99r^2st^2 - 396rst),$$

$$K''^{[4]_3}_{2_{22}} = \frac{-s^3}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2),$$

$$K''^{[4]_3}_{2_{32}} = \frac{-t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + 66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + 99r^2st^2 - 396rst),$$

$$K''^{[4]_3}_{2_{42}} = \frac{-1}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r+33s+66t-198r^2t^2-110rs-220rt-110st+99r^2s+198rt^2+198r^2t+99st^2-55r^2-55t^2-396rst^2-396r^2st+462r^2st^2+396rst-21),$$

$$K''^{[4]_3}_{2_{13}} = \frac{-r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2-55r^3s^2-198rs^2+198r^2s-220r^3s+66r^4s+99r^2t-110r^3t+33r^4t+462s^2t-55r^3+66r^4-21r^5-396rs^2t+396r^2st-110r^3st+99r^2s^2t-396rst),$$

$$K''^{[4]_3}_{2_{23}} = \frac{-s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2-55r^2s^3+198rs^2-198r^2s-220rs^3+66rs^4+462r^2t+99s^2t-110s^3t+33s^4t-55s^3+66s^4-21s^5+396rs^2t-396r^2st-110rs^3t+99r^2s^2t-396rst),$$

$$K''^{[4]_3}_{2_{33}} = \frac{-t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2-462r^2s^2t+462r^2s^2-165r^2st^3+528r^2st^2-462r^2st+55r^2t^4-165r^2t^3+132r^2t^2-165rs^2t^3+528rs^2t^2-462rs^2t+220rst^4-660rst^3+528rst^2-77rt^5+220rt^4-165rt^3+55s^2t^4-165s^2t^3+132s^2t^2-77st^5+220st^4-165st^3+28t^6-77t^5+55t^4),$$

$$K''^{[4]_3}_{2_{43}} = \frac{-1}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r+66s+33t-198r^2s^2-220rs-110rt-110st+198rs^2+198r^2s+99r^2t+99s^2t-55r^2-55s^2-396rs^2t-396r^2st+462r^2s^2t+396rst-21),$$

$$K''^{[4]_3}_{2_{14}} = \frac{-r^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-21r^5+66r^4s+66r^4t+33r^4-55r^3s^2-220r^3st-110r^3s-55r^3t^2-110r^3t+198r^2s^2t+99r^2s^2+198r^2st^2+396r^2st+99r^2t^2-198rs^2t^2-396rs^2t-396rst^2+462s^2t^2),$$

$$K''^{[4]_3}_{2_{24}} = \frac{-s^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-55r^2s^3+198r^2s^2t+99r^2s^2-198r^2st^2-396r^2st+462r^2t^2+66rs^4-220rs^3t-110rs^3+198rs^2t^2+396rs^2t-396rst^2-21s^5+66s^4t+33s^4-55s^3t^2-110s^3t+99s^2t^2),$$

$$K''^{[4]_3}_{2_{34}} = \frac{-t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-198r^2s^2t+462r^2s^2+198r^2st^2-396r^2st-55r^2t^3+99r^2t^2+198rs^2t^2-396rs^2t-220rst^3+396rst^2+66rt^4-110rt^3-55s^2t^3+99s^2t^2+66st^4-110st^3-21t^5+33t^4),$$

$K''^{[4]_3}_{2_{44}} = \frac{-1}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28).$

Again, the third derivatives of (5.12) - (5.15) are as below

$$y'''_{n+r} = y'''_n + \frac{f_n hr}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 - 20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t + 60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 - 69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 - 69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 + 144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 - 168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3) + \frac{g_n h^2 r^2}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2) - \frac{g_{n+1}h^2r^5}{2520(r-1)^2(s-1)^2(t-1)^2}(20r^4s - 7r^5 + 20r^4t + 10r^4 - 15r^3s^2 - 60r^3st - 30r^3s - 15r^3t^2 - 30r^3t + 48r^2s^2t + 24r^2s^2 + 48r^2st^2 + 96r^2st + 24r^2t^2 - 42rs^2t^2 - 84rs^2t - 84rst^2 + 84s^2t^2) - \frac{g_{n+r}h^2r^2}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 70r^5s - 70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t - 120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2) + \frac{f_{n+r}hr}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9 - 819r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t + 885r^7 - 315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s - 315r^6t^3 - 2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 + 9847r^5s^2t + 3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 + 3228r^5t^2 + 1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 - 11034r^4s^2t^2 - 11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st - 1164r^4t^3 - 1164r^4t^2 + 420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 + 12618r^3s^2t^2 + 4026$$

$r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 - 4662r^2s^3t^2 - 1470r^2s^3t -$
$4662r^2s^2t^3 - 4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 + 1722rs^3t^2 + 1722rs^2t^3 -$
$630s^3t^3) - \frac{f_{n+1}hr^5}{1260(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 20r^5s^2t + 40r^5s^2 -$
$20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t - 63r^5 + 15r^4s^3t - 30r^4s^3 + 60r^4s^2t^2 -$
$65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st + 150r^4s - 30r^4t^3 - 90r^4t^2 + 150r^4t +$
$45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3 - 72r^3s^2t^2 + 306r^3s^2t - 39r^3s^2 +$
$30r^3st^3 + 306r^3st^2 - 366r^3st - 135r^3s + 105r^3t^3 - 39r^3t^2 - 135r^3t + 42r^2s^3t^3 +$
$138r^2s^3t^2 - 294r^2s^3t - 84r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 -$
$294r^2st^3 + 66r^2st^2 + 432r^2st - 84r^2t^3 + 108r^2t^2 - 210rs^3t^3 + 126rs^3t^2 + 294rs^3t +$
$126rs^2t^3 + 126rs^2t^2 - 378rs^2t + 294rst^3 - 378rst^2 + 210s^3t^3 - 294s^3t^2 - 294s^2t^3 +$
$378s^2t^2) - \frac{g_{n+s}h^2r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^3t^2 + 24r^2s - 30r^3s + 10r^4s - 42rt^2 +$
$48r^2t - 60r^3t + 20r^4t + 84st^2 - 15r^3 + 20r^4 - 7r^5 - 84rst^2 + 96r^2st - 30r^3st +$
$24r^2st^2 - 84rst) - \frac{g_{n+t}h^2r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^3s^2 - 42rs^2 + 48r^2s - 60r^3s +$
$20r^4s + 24r^2t - 30r^3t + 10r^4t + 84s^2t - 15r^3 + 20r^4 - 7r^5 - 84rs^2t + 96r^2st -$
$30r^3st + 24r^2s^2t - 84rst) + \frac{f_{n+s}hr^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t +$
$63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 + 20r^5t - 45r^4s^4 -$
$150r^4s^3t - 150r^4s^3 + 90r^4s^2t^2 + 65r^4s^2t + 90r^4s^2 + 30r^4st^3 + 65r^4st^2 + 65r^4st +$
$30r^4s - 15r^4t^3 - 60r^4t^2 - 15r^4t + 135r^3s^4t + 135r^3s^4 + 39r^3s^3t^2 + 366r^3s^3t +$
$39r^3s^3 - 105r^3s^2t^3 - 306r^3s^2t^2 - 306r^3s^2t - 105r^3s^2 - 30r^3st^3 + 72r^3st^2 - 30r^3st +$
$48r^3t^3 + 48r^3t^2 - 108r^2s^4t^2 - 432r^2s^4t - 108r^2s^4 + 84r^2s^3t^3 - 66r^2s^3t^2 - 66r^2s^3t +$
$84r^2s^3 + 294r^2s^2t^3 + 342r^2s^2t^2 + 294r^2s^2t - 138r^2st^3 - 138r^2st^2 - 42r^2t^3 +$
$378rs^4t^2 + 378rs^4t - 294rs^3t^3 - 126rs^3t^2 - 294rs^3t - 126rs^2t^3 - 126rs^2t^2 +$
$210rst^3 - 378s^4t^2 + 294s^3t^3 + 294s^3t^2 - 210s^2t^3) + \frac{f_{n+t}hr^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(7r^6s -$
$14r^6st + 21r^6t^2 - 14r^6t + 40r^5s^2t - 20r^5s^2 - 14r^5st^2 + 51r^5st - 20r^5s - 63r^5t^3 -$
$14r^5t^2 + 40r^5t - 30r^4s^3t + 15r^4s^3 - 90r^4s^2t^2 - 65r^4s^2t + 60r^4s^2 + 150r^4st^3 -$
$65r^4st^2 - 65r^4st + 15r^4s + 45r^4t^4 + 150r^4t^3 - 90r^4t^2 - 30r^4t + 105r^3s^3t^2 + 30r^3s^3t -$
$48r^3s^3 - 39r^3s^2t^3 + 306r^3s^2t^2 - 72r^3s^2t - 48r^3s^2 - 135r^3st^4 - 366r^3st^3 + 306r^3st^2 +$
$30r^3st - 135r^3t^4 - 39r^3t^3 + 105r^3t^2 - 84r^2s^3t^3 - 294r^2s^3t^2 + 138r^2s^3t + 42r^2s^3 +$
$108r^2s^2t^4 + 66r^2s^2t^3 - 342r^2s^2t^2 + 138r^2s^2t + 432r^2st^4 + 66r^2st^3 - 294r^2st^2 +$

$108r^2t^4-84r^2t^3+294rs^3t^3+126rs^3t^2-210rs^3t-378rs^2t^4+126rs^2t^3+126rs^2t^2-$
$378rst^4+294rst^3-294s^3t^3+210s^3t^2+378s^2t^4-294s^2t^3),\qquad (5.26)$

$y'''_{n+s}=y'''_n+\frac{f_nhs}{1260r^3t^3}(15r^3s^5t+15r^3s^5-48r^3s^4t^2-69r^3s^4t-48r^3s^4+42r^3s^3t^3+$
$120r^3s^3t^2+120r^3s^3t+42r^3s^3-84r^3s^2t^3-84r^3s^2t-168r^3st^3-168r^3st^2+$
$630r^3t^3-20r^2s^6t-20r^2s^6+60r^2s^5t^2+100r^2s^5t+60r^2s^5-48r^2s^4t^3-180r^2s^4t^2-$
$180r^2s^4t-48r^2s^4+120r^2s^3t^3+144r^2s^3t^2+120r^2s^3t-168r^2st^3+7rs^7t+7rs^7-$
$20rs^6t^2-48rs^6t-20rs^6+15rs^5t^3+100rs^5t^2+100rs^5t+15rs^5-69rs^4t^3-$
$180rs^4t^2-69rs^4t+120rs^3t^3+120rs^3t^2-84rs^2t^3+7s^7t-20s^6t^2-20s^6t+$
$15s^5t^3+60s^5t^2+15s^5t-48s^4t^3-48s^4t^2+42s^3t^3)+\frac{g_nh^2s^2}{2520r^2t^2}(15r^2s^4-48r^2s^3t-$
$48r^2s^3+42r^2s^2t^2+168r^2s^2t+42r^2s^2-168r^2st^2-168r^2st+210r^2t^2-20rs^5+$
$60rs^4t+60rs^4-48rs^3t^2-192rs^3t-48rs^3+168rs^2t^2+168rs^2t-168rst^2+$
$7s^6-20s^5t-20s^5+15s^4t^2+60s^4t+15s^4-48s^3t^2-48s^3t+42s^2t^2)-$
$\frac{g_{n+1}h^2s^5}{2520(r-1)^2(s-1)^2(t-1)^2}(48r^2s^2t-15r^2s^3+24r^2s^2-42r^2st^2-84r^2st+84r^2t^2+$
$20rs^4-60rs^3t-30rs^3+48rs^2t^2+96rs^2t-84rst^2-7s^5+20s^4t+10s^4-15s^3t^2-$
$30s^3t+24s^2t^2)-\frac{g_{n+s}h^2s^2}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4-120r^2s^3t-120r^2s^3+84r^2s^2t^2+$
$336r^2s^2t+84r^2s^2-252r^2st^2-252r^2st+210r^2t^2-70rs^5+180rs^4t+180rs^4-$
$120rs^3t^2-480rs^3t-120rs^3+336rs^2t^2+336rs^2t-252rst^2+28s^6-70s^5t-70s^5+$
$45s^4t^2+180s^4t+45s^4-120s^3t^2-120s^3t+84s^2t^2)+\frac{f_{n+s}hs}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6-$
$1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-$
$4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-$
$1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-$
$9847r^2s^5t-3228r^2s^5+1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-$
$4026r^2s^3t^3-12618r^2s^3t^2-4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+$
$819rs^8-2686rs^7t-2686rs^7+2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-$
$9847rs^5t^2-9847rs^5t-1050rs^5+3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-$
$4026rs^3t^2+1470rs^2t^3-252s^9+819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+$
$315s^6t^3+2930s^6t^2+2930s^6t+315s^6-1050s^5t^3-3228s^5t^2-1050s^5t+1164s^4t^3+$
$1164s^4t^2-420s^3t^3)-\frac{f_{n+1}hs^5}{1260(r-1)^3(s-1)^3(t-1)^3}(15r^3s^4t-30r^3s^4-48r^3s^3t^2+30r^3s^3t$

$$+105r^3s^3+42r^3s^2t^3+138r^3s^2t^2-294r^3s^2t-84r^3s^2-210r^3st^3+126r^3st^2+294r^3st+210r^3t^3-294r^3t^2-20r^2s^5t+40r^2s^5+60r^2s^4t^2-65r^2s^4t-90r^2s^4-48r^2s^3t^3-72r^2s^3t^2+306r^2s^3t-39r^2s^3+138r^2s^2t^3-342r^2s^2t^2+66r^2s^2t+108r^2s^2+126r^2st^3+126r^2st^2-378r^2st-294r^2t^3+378r^2t^2+7rs^6t-14rs^6-20rs^5t^2+51rs^5t-14rs^5+15rs^4t^3-65rs^4t^2-65rs^4t+150rs^4+30rs^3t^3+306rs^3t^2-366rs^3t-135rs^3-294rs^2t^3+66rs^2t^2+432rs^2t+294rst^3-378rst^2-14s^6t+21s^6+40s^5t^2-14s^5t-63s^5-30s^4t^3-90s^4t^2+150s^4t+45s^4+105s^3t^3-39s^3t^2-135s^3t-84s^2t^3+108s^2t^2)-\frac{g_{n+r}h^2s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2-15s^3t^2+24rs^2-30rs^3+10rs^4+84rt^2-42st^2+48s^2t-60s^3t+20s^4t-15s^3+20s^4-7s^5-84rst^2+96rs^2t-30rs^3t+24rs^2t^2-84rst)-\frac{g_{n+t}h^2s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2-15r^2s^3+48rs^2-42r^2s-60rs^3+20rs^4+84r^2t+24s^2t-30s^3t+10s^4t-15s^3+20s^4-7s^5+96rs^2t-84r^2st-30rs^3t+24r^2s^2t-84rst)+\frac{f_{n+r}hs^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4-135r^4s^3t-135r^4s^3+108r^4s^2t^2+432r^4s^2t+108r^4s^2-378r^4st^2-378r^4st+378r^4t^2-63r^3s^5+150r^3s^4t+150r^3s^4-39r^3s^3t^2-366r^3s^3t-39r^3s^3-84r^3s^2t^3+66r^3s^2t^2+66r^3s^2t-84r^3s^2+294r^3st^3+126r^3st^2+294r^3st-294r^3t^3-294r^3t^2+21r^2s^6-14r^2s^5t-14r^2s^5-90r^2s^4t^2-65r^2s^4t-90r^2s^4+105r^2s^3t^3+306r^2s^3t^2+306r^2s^3t+105r^2s^3-294r^2s^2t^3-342r^2s^2t^2-294r^2s^2t+126r^2st^3+126r^2st^2+210r^2t^3-14rs^6t-14rs^6+40rs^5t^2+51rs^5t+40rs^5-30rs^4t^3-65rs^4t^2-65rs^4t-30rs^4+30rs^3t^3-72rs^3t^2+30rs^3t+138rs^2t^3+138rs^2t^2-210rst^3+7s^6t-20s^5t^2-20s^5t+15s^4t^3+60s^4t^2+15s^4t-48s^3t^3-48s^3t^2+42s^2t^3)-\frac{f_{n+t}hs^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t-15r^3s^4-105r^3s^3t^2-30r^3s^3t+48r^3s^3+84r^3s^2t^3+294r^3s^2t^2-138r^3s^2t-42r^3s^2-294r^3st^3-126r^3st^2+210r^3st+294r^3t^3-210r^3t^2-40r^2s^5t+20r^2s^5+90r^2s^4t^2+65r^2s^4t-60r^2s^4+39r^2s^3t^3-306r^2s^3t^2+72r^2s^3t+48r^2s^3-108r^2s^2t^4-66r^2s^2t^3+342r^2s^2t^2-138r^2s^2t+378r^2st^4-126r^2st^3-126r^2st^2-378r^2t^4+294r^2t^3+14rs^6t-7rs^6+14rs^5t^2-51rs^5t+20rs^5-150rs^4t^3+65rs^4t^2+65rs^4t-15rs^4+135rs^3t^4+366rs^3t^3-306rs^3t^2-30rs^3t-432rs^2t^4-66rs^2t^3+294rs^2t^2+378rst^4-294rst^3-21s^6t^2+14s^6t+63s^5t^3+14s^5t^2-40s^5t-45s^4t^4-150s^4t^3+90s^4t^2+30s^4t+135s^3t^4+39s^3t^3-105s^3t^2-108s^2t^4+84s^2t^3), \tag{5.27}$$

$y'''_{n+t} = y'''_n + \frac{f_n ht}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5) + \frac{g_n h^2t^2}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4) - \frac{g_{n+1}h^2t^5}{2520(r-1)^2(s-1)^2(t-1)^2}(84r^2s^2 - 42r^2s^2t + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4) - \frac{g_{n+t}h^2t^2}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4) - \frac{f_{n+t}ht}{1260(r-t)^3(s-t)^3(t-1)^3}(1470r^3s^3t^2 - 420r^3s^3t^3 - 1722r^3s^3t + 630r^3s^3 + 1164r^3s^2t^4 - 4026r^3s^2t^3 + 4662r^3s^2t^2 - 1722r^3s^2t - 1050r^3st^5 + 3561r^3st^4 - 4026r^3st^3 + 1470r^3st^2 + 315r^3t^6 - 1050r^3t^5 + 1164r^3t^4 - 420r^3t^3 + 1164r^2s^3t^4 - 4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686rst^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 + 315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 + 1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6) - \frac{f_{n+1}ht^5}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st$

$$+40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t -$$
$$20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 -$$
$$366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 +$$
$$40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 +$$
$$45t^4) - \frac{g_{n+r}h^2t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 +$$
$$48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 +$$
$$24rs^2t^2 - 84rst) - \frac{g_{n+s}h^2t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^2t^3 + 84r^2s + 48rt^2 - 42r^2t -$$
$$60rt^3 + 20rt^4 + 24st^2 - 30st^3 + 10st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84r^2st -$$
$$30rst^3 + 24r^2st^2 - 84rst) + \frac{f_{n+r}ht^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 -$$
$$135r^4st^3 + 432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t -$$
$$294r^3s^3 - 39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 +$$
$$66r^3st^2 + 294r^3st - 63r^3t^5 + 150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 +$$
$$126r^2s^3t + 210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 -$$
$$65r^2st^4 + 306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 +$$
$$30rs^3t^3 + 138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 +$$
$$51rst^5 - 65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 -$$
$$20s^2t^5 + 60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4) + \frac{f_{n+s}ht^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2 -$$
$$294r^3s^3t + 294r^3s^3 - 105r^3s^2t^3 + 294r^3s^2t^2 - 126r^3s^2t - 210r^3s^2 + 30r^3st^4 -$$
$$30r^3st^3 - 138r^3st^2 + 210r^3st - 15r^3t^4 + 48r^3t^3 - 42r^3t^2 - 108r^2s^4t^2 + 378r^2s^4t -$$
$$378r^2s^4 + 39r^2s^3t^3 - 66r^2s^3t^2 - 126r^2s^3t + 294r^2s^3 + 90r^2s^2t^4 - 306r^2s^2t^3 +$$
$$342r^2s^2t^2 - 126r^2s^2t - 40r^2st^5 + 65r^2st^4 + 72r^2st^3 - 138r^2st^2 + 20r^2t^5 - 60r^2t^4 +$$
$$48r^2t^3 + 135rs^4t^3 - 432rs^4t^2 + 378rs^4t - 150rs^3t^4 + 366rs^3t^3 - 66rs^3t^2 - 294rs^3t +$$
$$14rs^2t^5 + 65rs^2t^4 - 306rs^2t^3 + 294rs^2t^2 + 14rst^6 - 51rst^5 + 65rst^4 - 30rst^3 - 7rt^6 +$$
$$20rt^5 - 15rt^4 - 45s^4t^4 + 135s^4t^3 - 108s^4t^2 + 63s^3t^5 - 150s^3t^4 + 39s^3t^3 + 84s^3t^2 -$$
$$21s^2t^6 + 14s^2t^5 + 90s^2t^4 - 105s^2t^3 + 14st^6 - 40st^5 + 30st^4), \tag{5.28}$$

$$y'''_{n+1} = y'''_n + \frac{f_{n+1}h}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3 - 1722r^3s^3t^2 + 1470r^3s^3t - 420r^3s^3 -$$
$$1722r^3s^2t^3 + 4662r^3s^2t^2 - 4026r^3s^2t + 1164r^3s^2 + 1470r^3st^3 - 4026r^3st^2 +$$
$$3561r^3st - 1050r^3s - 420r^3t^3 + 1164r^3t^2 - 1050r^3t + 315r^3 - 1722r^2s^3t^3 + 4662r^2$$

$s^3t^2 - 4026r^2s^3t + 1164r^2s^3 + 4662r^2s^2t^3 - 12618r^2s^2t^2 + 11034r^2s^2t - 3228r^2s^2 -$
$4026r^2st^3 + 11034r^2st^2 - 9847r^2st + 2930r^2s + 1164r^2t^3 - 3228r^2t^2 + 2930r^2t -$
$885r^2 + 1470rs^3t^3 - 4026rs^3t^2 + 3561rs^3t - 1050rs^3 - 4026rs^2t^3 + 11034rs^2t^2 -$
$9847rs^2t + 2930rs^2 + 3561rst^3 - 9847rst^2 + 8913rst - 2686rs - 1050rt^3 + 2930rt^2 -$
$2686rt + 819r - 420s^3t^3 + 1164s^3t^2 - 1050s^3t + 315s^3 + 1164s^2t^3 - 3228s^2t^2 +$
$2930s^2t - 885s^2 - 1050st^3 + 2930st^2 - 2686st + 819s + 315t^3 - 885t^2 + 819t -$
$252) + \frac{g_n h^2}{2520r^2s^2t^2}(210r^2s^2t^2 - 168r^2s^2t + 42r^2s^2 - 168r^2st^2 + 168r^2st - 48r^2s +$
$42r^2t^2 - 48r^2t + 15r^2 - 168rs^2t^2 + 168rs^2t - 48rs^2 + 168rst^2 - 192rst + 60rs -$
$48rt^2 + 60rt - 20r + 42s^2t^2 - 48s^2t + 15s^2 - 48st^2 + 60st - 20s + 15t^2 - 20t +$
$7) - \frac{g_{n+1}h^2}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2 - 252r^2s^2t + 84r^2s^2 - 252r^2st^2 + 336r^2st -$
$120r^2s + 84r^2t^2 - 120r^2t + 45r^2 - 252rs^2t^2 + 336rs^2t - 120rs^2 + 336rst^2 - 480rst +$
$180rs - 120rt^2 + 180rt - 70r + 84s^2t^2 - 120s^2t + 45s^2 - 120st^2 + 180st - 70s +$
$45t^2 - 70t + 28) + \frac{f_n h}{1260r^3s^3t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - 168r^3s^2t^3 +$
$120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - 48r^3t^2 +$
$15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + 120r^2st^3 -$
$180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t - 84rs^3t^3 + 120rs^3t^2 -$
$69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + 100rst^2 - 48rst +$
$7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + 60s^2t^2 - 20s^2t +$
$15st^3 - 20st^2 + 7st) - \frac{g_{n+r}h^2}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r + 20s + 20t - 42s^2t^2 - 30rs -$
$30rt - 60st + 24rs^2 + 24rt^2 + 48st^2 + 48s^2t - 15s^2 - 15t^2 - 84rst^2 - 84rs^2t +$
$84rs^2t^2 + 96rst - 7) - \frac{g_{n+s}h^2}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r + 10s + 20t - 42r^2t^2 - 30rs - 60rt -$
$30st + 24r^2s + 48rt^2 + 48r^2t + 24st^2 - 15r^2 - 15t^2 - 84rst^2 - 84r^2st + 84r^2st^2 +$
$96rst - 7) - \frac{g_{n+t}h^2}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r + 20s + 10t - 42r^2s^2 - 60rs - 30rt - 30st +$
$48rs^2 + 48r^2s + 24r^2t + 24s^2t - 15r^2 - 15s^2 - 84rs^2t - 84r^2st + 84r^2s^2t + 96rst -$
$7) - \frac{f_{n+r}h}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(378r^4s^2t - 378r^4s^2t^2 - 108r^4s^2 + 378r^4st^2 - 432r^4st +$
$135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + 294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 + 294r^3s^2t^3 -$
$126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - 294r^3st^3 - 66r^3st^2 + 366r^3st - 150r^3s + 84r^3t^3 +$
$39r^3t^2 - 150r^3t + 63r^3 - 210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - 105r^2s^3 - 126r^2s^2t^3 +$
$342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 + 294r^2st^3 - 306r^2st^2 + 65r^2st + 14r^2s - 105r^2t^3 +$

$90r^2t^2 + 14r^2t - 21r^2 + 210rs^3t^3 - 138rs^3t^2 - 30rs^3t + 30rs^3 - 138rs^2t^3 + 72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs + 30rt^3 - 40rt^2 + 14rt - 42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - 60s^2t^2 + 20s^2t - 15st^3 + 20st^2 - 7st) + \frac{f_{n+s}h}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 - 210r^3s^2t^3 - 126r^3s^2t^2 + 294r^3s^2t - 105r^3s^2 + 210r^3st^3 - 138r^3st^2 - 30r^3st + 30r^3s - 42r^3t^3 + 48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - 108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + 39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - 40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + 378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - 66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - 306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - 108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + 39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + 14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st) + \frac{f_{n+t}h}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2 - 210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + 138r^3s^2t - 48r^3s^2 + 294r^3st^3 - 294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - 30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + 138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - 342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - 378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + 108r^2t^4 - 39r^2t^3 - 90r^2t^2 + 40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - 378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - 65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + 7rs - 135rt^4 + 150rt^3 - 14rt^2 - 14rt - 84s^3t^3 + 105s^3t^2 - 30s^3t + 108s^2t^4 - 39s^2t^3 - 90s^2t^2 + 40s^2t - 135st^4 + 150st^3 - 14st^2 - 14st + 45t^4 - 63t^3 + 21t^2). \quad (5.29)$

Equations (5.26) - (5.29) can be written in block form :

$$I_4 Y_{n+1}^{'''[4]_3} = M_4^{'''[4]_3} Y_{n-3}^{[4]_3} + h[E_1^{'''[4]_3} F_n^{[4]_3} + E_2^{'''[4]_3} F_{n+1}^{[4]_3}] + h^2[K_1^{'''[4]_3} G_n^{[4]_3} + K_2^{'''[4]_3} G_{n+1}^{[4]_3}] \tag{5.30}$$

where

$$Y_{n+1}^{'''[4]_3} = \begin{pmatrix} y_{n+r}''' \\ y_{n+s}''' \\ y_{n+t}''' \\ y_{n+1}''' \end{pmatrix}, \ M_4^{'''[4]_3} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$E_{1}^{'''[4]_3}=\begin{pmatrix} 0 & 0 & 0 & E_{1_{14}}^{'''[4]_3} \\ 0 & 0 & 0 & E_{1_{24}}^{'''[4]_3} \\ 0 & 0 & 0 & E_{1_{34}}^{'''[4]_3} \\ 0 & 0 & 0 & E_{1_{44}}^{'''[4]_3} \end{pmatrix},\ E_{2}^{'''[4]_3}=\begin{pmatrix} E_{2_{11}}^{'''[4]_3} & E_{2_{12}}^{'''[4]_3} & E_{2_{13}}^{'''[4]_3} & E_{2_{14}}^{'''[4]_3} \\ E_{2_{21}}^{'''[4]_3} & E_{2_{22}}^{'''[4]_3} & E_{2_{23}}^{'''[4]_3} & E_{2_{24}}^{'''[4]_3} \\ E_{2_{31}}^{'''[4]_3} & E_{2_{32}}^{'''[4]_3} & E_{2_{33}}^{'''[4]_3} & E_{2_{34}}^{'''[4]_3} \\ E_{2_{41}}^{'''[4]_3} & E_{2_{42}}^{'''[4]_3} & E_{2_{43}}^{'''[4]_3} & E_{2_{44}}^{'''[4]_3} \end{pmatrix},$$

$$K_{1}^{'''[4]_3}=\begin{pmatrix} 0 & 0 & 0 & K_{1_{14}}^{'''[4]_3} \\ 0 & 0 & 0 & K_{1_{24}}^{'''[4]_3} \\ 0 & 0 & 0 & K_{1_{34}}^{'''[4]_3} \\ 0 & 0 & 0 & K_{1_{44}}^{'''[4]_3} \end{pmatrix},\ K_{2}^{'''[4]_3}=\begin{pmatrix} K_{2_{11}}^{'''[4]_3} & K_{2_{12}}^{'''[4]_3} & K_{2_{13}}^{'''[4]_3} & K_{2_{14}}^{'''[4]_3} \\ K_{2_{21}}^{'''[4]_3} & K_{2_{22}}^{'''[4]_3} & K_{2_{23}}^{'''[4]_3} & K_{2_{24}}^{'''[4]_3} \\ K_{2_{31}}^{'''[4]_3} & K_{2_{32}}^{'''[4]_3} & K_{2_{33}}^{'''[4]_3} & K_{2_{34}}^{'''[4]_3} \\ K_{2_{41}}^{'''[4]_3} & K_{2_{42}}^{'''[4]_3} & K_{2_{43}}^{'''[4]_3} & K_{2_{44}}^{'''[4]_3} \end{pmatrix}.$$

where the entries are given as below

$E_{1_{14}}^{'''[4]_3}=\frac{r}{1260s^3t^3}(7r^7st+7r^7s+7r^7t-20r^6s^2t-20r^6s^2-20r^6st^2-48r^6st-20r^6s-20r^6t^2-20r^6t+15r^5s^3t+15r^5s^3+60r^5s^2t^2+100r^5s^2t+60r^5s^2+15r^5st^3+100r^5st^2+100r^5st+15r^5s+15r^5t^3+60r^5t^2+15r^5t-48r^4s^3t^2-69r^4s^3t-48r^4s^3-48r^4s^2t^3-180r^4s^2t^2-180r^4s^2t-48r^4s^2-69r^4st^3-180r^4st^2-69r^4st-48r^4t^3-48r^4t^2+42r^3s^3t^3+120r^3s^3t^2+120r^3s^3t+42r^3s^3+120r^3s^2t^3+144r^3s^2t^2+120r^3s^2t+120r^3st^3+120r^3st^2+42r^3t^3-84r^2s^3t^3-84r^2s^3t-84r^2st^3-168rs^3t^3-168rs^3t^2-168rs^2t^3+630s^3t^3)$,

$E_{1_{24}}^{'''[4]_3}=\frac{s}{1260r^3t^3}(15r^3s^5t+15r^3s^5-48r^3s^4t^2-69r^3s^4t-48r^3s^4+42r^3s^3t^3+120r^3s^3t^2+120r^3s^3t+42r^3s^3-84r^3s^2t^3-84r^3s^2t-168r^3st^3-168r^3st^2+630r^3t^3-20r^2s^6t-20r^2s^6+60r^2s^5t^2+100r^2s^5t+60r^2s^5-48r^2s^4t^3-180r^2s^4t^2-180r^2s^4t-48r^2s^4+120r^2s^3t^3+144r^2s^3t^2+120r^2s^3t-168r^2st^3+7rs^7t+7rs^7-20rs^6t^2-48rs^6t-20rs^6+15rs^5t^3+100rs^5t^2+100rs^5t+15rs^5-69rs^4t^3-180rs^4t^2-69rs^4t+120rs^3t^3+120rs^3t^2-84rs^2t^3+7s^7t-20s^6t^2-20s^6t+15s^5t^3+60s^5t^2+15s^5t-48s^4t^3-48s^4t^2+42s^3t^3)$,

$E_{1_{34}}^{'''[4]_3}=\frac{t}{1260r^3s^3}(42r^3s^3t^3-84r^3s^3t^2-168r^3s^3t+630r^3s^3-48r^3s^2t^4+120r^3s^2t^3-168r^3s^2t+15r^3st^5-69r^3st^4+120r^3st^3-84r^3st^2+15r^3t^5-48r^3t^4+42r^3t^3-48r^2s^3t^4+120r^2s^3t^3-168r^2s^3t+60r^2s^2t^5-180r^2s^2t^4+144r^2s^2t^3-20r^2st^6+$

$100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5)$,

$E_{1_{44}}^{'''[4]_3} = \frac{1}{1260r^3s^3t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - 168r^3s^2t^3 + 120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - 48r^3t^2 + 15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + 120r^2st^3 - 180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t - 84rs^3t^3 + 120rs^3t^2 - 69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + 100rst^2 - 48rst + 7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + 60s^2t^2 - 20s^2t + 15st^3 - 20st^2 + 7st)$,

$E_{2_{11}}^{'''[4]_3} = \frac{r}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9 - 819r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t + 885r^7 - 315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s - 315r^6t^3 - 2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 + 9847r^5s^2t + 3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 + 3228r^5t^2 + 1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 - 11034r^4s^2t^2 - 11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st - 1164r^4t^3 - 1164r^4t^2 + 420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 + 12618r^3s^2t^2 + 4026r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 - 4662r^2s^3t^2 - 1470r^2s^3t - 4662r^2s^2t^3 - 4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 + 1722rs^3t^2 + 1722rs^2t^3 - 630s^3t^3)$,

$E_{2_{21}}^{'''[4]_3} = \frac{s^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4 - 135r^4s^3t - 135r^4s^3 + 108r^4s^2t^2 + 432r^4s^2t + 108r^4s^2 - 378r^4st^2 - 378r^4st + 378r^4t^2 - 63r^3s^5 + 150r^3s^4t + 150r^3s^4 - 39r^3s^3t^2 - 366r^3s^3t - 39r^3s^3 - 84r^3s^2t^3 + 66r^3s^2t^2 + 66r^3s^2t - 84r^3s^2 + 294r^3st^3 + 126r^3st^2 + 294r^3st - 294r^3t^3 - 294r^3t^2 + 21r^2s^6 - 14r^2s^5t - 14r^2s^5 - 90r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 + 105r^2s^3t^3 + 306r^2s^3t^2 + 306r^2s^3t + 105r^2s^3 - 294r^2s^2t^3 - 342r^2s^2t^2 - 294r^2s^2t + 126r^2st^3 + 126r^2st^2 + 210r^2t^3 - 14rs^6t - 14rs^6 + 40rs^5t^2 + 51rs^5t + 40rs^5 - 30rs^4t^3 - 65rs^4t^2 - 65rs^4t - 30rs^4 + 30rs^3t^3 - 72rs^3t^2 + 30rs^3t + 138rs^2t^3 +$

$138rs^2t^2 - 210rst^3 + 7s^6t - 20s^5t^2 - 20s^5t + 15s^4t^3 + 60s^4t^2 + 15s^4t - 48s^3t^3 - 48s^3t^2 + 42s^2t^3)$,

$E_{2_{31}}^{'''[4]_3} = \frac{t^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 - 135r^4st^3 + 432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t - 294r^3s^3 - 39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 + 66r^3st^2 + 294r^3st - 63r^3t^5 + 150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 + 126r^2s^3t + 210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 - 65r^2st^4 + 306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 + 30rs^3t^3 + 138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 + 51rst^5 - 65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 - 20s^2t^5 + 60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4)$,

$E_{2_{41}}^{'''[4]_3} = \frac{-1}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(-378r^4s^2t^2 + 378r^4s^2t - 108r^4s^2 + 378r^4st^2 - 432r^4st + 135r^4s - 108r^4t^2 + 135r^4t - 45r^4 + 294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 + 294r^3s^2t^3 - 126r^3s^2t^2 - 66r^3s^2t + 39r^3s^2 - 294r^3st^3 - 66r^3st^2 + 366r^3st - 150r^3s + 84r^3t^3 + 39r^3t^2 - 150r^3t + 63r^3 - 210r^2s^3t^3 - 126r^2s^3t^2 + 294r^2s^3t - 105r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 + 294r^2st^3 - 306r^2st^2 + 65r^2st + 14r^2s - 105r^2t^3 + 90r^2t^2 + 14r^2t - 21r^2 + 210rs^3t^3 - 138rs^3t^2 - 30rs^3t + 30rs^3 - 138rs^2t^3 + 72rs^2t^2 + 65rs^2t - 40rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs + 30rt^3 - 40rt^2 + 14rt - 42s^3t^3 + 48s^3t^2 - 15s^3t + 48s^2t^3 - 60s^2t^2 + 20s^2t - 15st^3 + 20st^2 - 7st)$,

$E_{2_{12}}^{'''[4]_3} = \frac{r^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2 + 14r^6st + 14r^6s - 7r^6t + 63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 + 20r^5t - 45r^4s^4 - 150r^4s^3t - 150r^4s^3 + 90r^4s^2t^2 + 65r^4s^2t + 90r^4s^2 + 30r^4st^3 + 65r^4st^2 + 65r^4st + 30r^4s - 15r^4t^3 - 60r^4t^2 - 15r^4t + 135r^3s^4t + 135r^3s^4 + 39r^3s^3t^2 + 366r^3s^3t + 39r^3s^3 - 105r^3s^2t^3 - 306r^3s^2t^2 - 306r^3s^2t - 105r^3s^2 - 30r^3st^3 + 72r^3st^2 - 30r^3st + 48r^3t^3 + 48r^3t^2 - 108r^2s^4t^2 - 432r^2s^4t - 108r^2s^4 + 84r^2s^3t^3 - 66r^2s^3t^2 - 66r^2s^3t + 84r^2s^3 + 294r^2s^2t^3 + 342r^2s^2t^2 + 294r^2s^2t - 138r^2st^3 - 138r^2st^2 - 42r^2t^3 + 378rs^4t^2 + 378rs^4t - 294rs^3t^3 - 126rs^3t^2 - 294rs^3t - 126rs^2t^3 - 126rs^2t^2 + 210rst^3 - 378s^4t^2 + 294s^3t^3 + 294s^3t^2 - 210s^2t^3)$,

$$E_{2_{22}}^{'''[4]_3} = \frac{s}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6 - 1050r^3s^5t - 1050r^3s^5 + 1164r^3s^4t^2 + 3561r^3s^4t + 1164r^3s^4 - 420r^3s^3t^3 - 4026r^3s^3t^2 - 4026r^3s^3t - 420r^3s^3 + 1470r^3s^2t^3 + 4662r^3s^2t^2 + 1470r^3s^2t - 1722r^3st^3 - 1722r^3st^2 + 630r^3t^3 - 885r^2s^7 + 2930r^2s^6t + 2930r^2s^6 - 3228r^2s^5t^2 - 9847r^2s^5t - 3228r^2s^5 + 1164r^2s^4t^3 + 11034r^2s^4t^2 + 11034r^2s^4t + 1164r^2s^4 - 4026r^2s^3t^3 - 12618r^2s^3t^2 - 4026r^2s^3t + 4662r^2s^2t^3 + 4662r^2s^2t^2 - 1722r^2st^3 + 819rs^8 - 2686rs^7t - 2686rs^7 + 2930rs^6t^2 + 8913rs^6t + 2930rs^6 - 1050rs^5t^3 - 9847rs^5t^2 - 9847rs^5t - 1050rs^5 + 3561rs^4t^3 + 11034rs^4t^2 + 3561rs^4t - 4026rs^3t^3 - 4026rs^3t^2 + 1470rs^2t^3 - 252s^9 + 819s^8t + 819s^8 - 885s^7t^2 - 2686s^7t - 885s^7 + 315s^6t^3 + 2930s^6t^2 + 2930s^6t + 315s^6 - 1050s^5t^3 - 3228s^5t^2 - 1050s^5t + 1164s^4t^3 + 1164s^4t^2 - 420s^3t^3),$$

$$E_{2_{32}}^{'''[4]_3} = \frac{t^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2 - 294r^3s^3t + 294r^3s^3 - 105r^3s^2t^3 + 294r^3s^2t^2 - 126r^3s^2t - 210r^3s^2 + 30r^3st^4 - 30r^3st^3 - 138r^3st^2 + 210r^3st - 15r^3t^4 + 48r^3t^3 - 42r^3t^2 - 108r^2s^4t^2 + 378r^2s^4t - 378r^2s^4 + 39r^2s^3t^3 - 66r^2s^3t^2 - 126r^2s^3t + 294r^2s^3 + 90r^2s^2t^4 - 306r^2s^2t^3 + 342r^2s^2t^2 - 126r^2s^2t - 40r^2st^5 + 65r^2st^4 + 72r^2st^3 - 138r^2st^2 + 20r^2t^5 - 60r^2t^4 + 48r^2t^3 + 135rs^4t^3 - 432rs^4t^2 + 378rs^4t - 150rs^3t^4 + 366rs^3t^3 - 66rs^3t^2 - 294rs^3t + 14rs^2t^5 + 65rs^2t^4 - 306rs^2t^3 + 294rs^2t^2 + 14rst^6 - 51rst^5 + 65rst^4 - 30rst^3 - 7rt^6 + 20rt^5 - 15rt^4 - 45s^4t^4 + 135s^4t^3 - 108s^4t^2 + 63s^3t^5 - 150s^3t^4 + 39s^3t^3 + 84s^3t^2 - 21s^2t^6 + 14s^2t^5 + 90s^2t^4 - 105s^2t^3 + 14st^6 - 40st^5 + 30st^4),$$

$$E_{2_{42}}^{'''[4]_3} = \frac{1}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2 - 294r^3s^3t + 84r^3s^3 - 210r^3s^2t^3 - 126r^3s^2t^2 + 294r^3s^2t - 105r^3s^2 + 210r^3st^3 - 138r^3st^2 - 30r^3st + 30r^3s - 42r^3t^3 + 48r^3t^2 - 15r^3t - 378r^2s^4t^2 + 378r^2s^4t - 108r^2s^4 + 294r^2s^3t^3 - 126r^2s^3t^2 - 66r^2s^3t + 39r^2s^3 - 126r^2s^2t^3 + 342r^2s^2t^2 - 306r^2s^2t + 90r^2s^2 - 138r^2st^3 + 72r^2st^2 + 65r^2st - 40r^2s + 48r^2t^3 - 60r^2t^2 + 20r^2t + 378rs^4t^2 - 432rs^4t + 135rs^4 - 294rs^3t^3 - 66rs^3t^2 + 366rs^3t - 150rs^3 + 294rs^2t^3 - 306rs^2t^2 + 65rs^2t + 14rs^2 - 30rst^3 + 65rst^2 - 51rst + 14rs - 15rt^3 + 20rt^2 - 7rt - 108s^4t^2 + 135s^4t - 45s^4 + 84s^3t^3 + 39s^3t^2 - 150s^3t + 63s^3 - 105s^2t^3 + 90s^2t^2 + 14s^2t - 21s^2 + 30st^3 - 40st^2 + 14st),$$

$$
\begin{aligned}
E^{'''[4]_3}_{2_{13}} = {} & \tfrac{r^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st+7r^6s+21r^6t^2-14r^6t+40r^5s^2t-20r^5s^2- \\
& 14r^5st^2+51r^5st-20r^5s-63r^5t^3-14r^5t^2+40r^5t-30r^4s^3t+15r^4s^3-90r^4s^2t^2- \\
& 65r^4s^2t+60r^4s^2+150r^4st^3-65r^4st^2-65r^4st+15r^4s+45r^4t^4+150r^4t^3-90r^4t^2- \\
& 30r^4t+105r^3s^3t^2+30r^3s^3t-48r^3s^3-39r^3s^2t^3+306r^3s^2t^2-72r^3s^2t-48r^3s^2- \\
& 135r^3st^4-366r^3st^3+306r^3st^2+30r^3st-135r^3t^4-39r^3t^3+105r^3t^2-84r^2s^3t^3- \\
& 294r^2s^3t^2+138r^2s^3t+42r^2s^3+108r^2s^2t^4+66r^2s^2t^3-342r^2s^2t^2+138r^2s^2t+ \\
& 432r^2st^4+66r^2st^3-294r^2st^2+108r^2t^4-84r^2t^3+294rs^3t^3+126rs^3t^2-210rs^3t- \\
& 378rs^2t^4+126rs^2t^3+126rs^2t^2-378rst^4+294rst^3-294s^3t^3+210s^3t^2+378s^2t^4- \\
& 294s^2t^3),
\end{aligned}
$$

$$
\begin{aligned}
E^{'''[4]_3}_{2_{23}} = {} & -\tfrac{s^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t-15r^3s^4-105r^3s^3t^2-30r^3s^3t+48r^3s^3+ \\
& 84r^3s^2t^3+294r^3s^2t^2-138r^3s^2t-42r^3s^2-294r^3st^3-126r^3st^2+210r^3st+ \\
& 294r^3t^3-210r^3t^2-40r^2s^5t+20r^2s^5+90r^2s^4t^2+65r^2s^4t-60r^2s^4+39r^2s^3t^3- \\
& 306r^2s^3t^2+72r^2s^3t+48r^2s^3-108r^2s^2t^4-66r^2s^2t^3+342r^2s^2t^2-138r^2s^2t+ \\
& 378r^2st^4-126r^2st^3-126r^2st^2-378r^2t^4+294r^2t^3+14rs^6t-7rs^6+14rs^5t^2- \\
& 51rs^5t+20rs^5-150rs^4t^3+65rs^4t^2+65rs^4t-15rs^4+135rs^3t^4+366rs^3t^3- \\
& 306rs^3t^2-30rs^3t-432rs^2t^4-66rs^2t^3+294rs^2t^2+378rst^4-294rst^3-21s^6t^2+ \\
& 14s^6t+63s^5t^3+14s^5t^2-40s^5t-45s^4t^4-150s^4t^3+90s^4t^2+30s^4t+135s^3t^4+ \\
& 39s^3t^3-105s^3t^2-108s^2t^4+84s^2t^3),
\end{aligned}
$$

$$
\begin{aligned}
E^{'''[4]_3}_{2_{33}} = {} & -\tfrac{t}{1260(r-t)^3(s-t)^3(t-1)^3}(-420r^3s^3t^3+1470r^3s^3t^2-1722r^3s^3t+630r^3s^3+ \\
& 1164r^3s^2t^4-4026r^3s^2t^3+4662r^3s^2t^2-1722r^3s^2t-1050r^3st^5+3561r^3st^4- \\
& 4026r^3st^3+1470r^3st^2+315r^3t^6-1050r^3t^5+1164r^3t^4-420r^3t^3+1164r^2s^3t^4- \\
& 4026r^2s^3t^3+4662r^2s^3t^2-1722r^2s^3t-3228r^2s^2t^5+11034r^2s^2t^4-12618r^2s^2t^3+ \\
& 4662r^2s^2t^2+2930r^2st^6-9847r^2st^5+11034r^2st^4-4026r^2st^3-885r^2t^7+ \\
& 2930r^2t^6-3228r^2t^5+1164r^2t^4-1050rs^3t^5+3561rs^3t^4-4026rs^3t^3+1470rs^3t^2+ \\
& 2930rs^2t^6-9847rs^2t^5+11034rs^2t^4-4026rs^2t^3-2686rst^7+8913rst^6-9847rst^5+ \\
& 3561rst^4+819rt^8-2686rt^7+2930rt^6-1050rt^5+315s^3t^6-1050s^3t^5+1164s^3t^4- \\
& 420s^3t^3-885s^2t^7+2930s^2t^6-3228s^2t^5+1164s^2t^4+819st^8-2686st^7+2930st^6- \\
& 1050st^5-252t^9+819t^8-885t^7+315t^6),
\end{aligned}
$$

$E_{2_{43}}^{'''[4]_3} = \frac{1}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2 - 210r^3s^3t + 42r^3s^3 - 294r^3s^2t^3 + 126r^3s^2t^2 + 138r^3s^2t - 48r^3s^2 + 294r^3st^3 - 294r^3st^2 + 30r^3st + 15r^3s - 84r^3t^3 + 105r^3t^2 - 30r^3t - 294r^2s^3t^3 + 126r^2s^3t^2 + 138r^2s^3t - 48r^2s^3 + 378r^2s^2t^4 + 126r^2s^2t^3 - 342r^2s^2t^2 - 72r^2s^2t + 60r^2s^2 - 378r^2st^4 + 66r^2st^3 + 306r^2st^2 - 65r^2st - 20r^2s + 108r^2t^4 - 39r^2t^3 - 90r^2t^2 + 40r^2t + 294rs^3t^3 - 294rs^3t^2 + 30rs^3t + 15rs^3 - 378rs^2t^4 + 66rs^2t^3 + 306rs^2t^2 - 65rs^2t - 20rs^2 + 432rst^4 - 366rst^3 - 65rst^2 + 51rst + 7rs - 135rt^4 + 150rt^3 - 14rt^2 - 14rt - 84s^3t^3 + 105s^3t^2 - 30s^3t + 108s^2t^4 - 39s^2t^3 - 90s^2t^2 + 40s^2t - 135st^4 + 150st^3 - 14st^2 - 14st + 45t^4 - 63t^3 + 21t^2)$,

$E_{2_{14}}^{'''[4]_3} = -\frac{r^5}{1260(r-1)^3(s-1)^3(t-1)^3}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 20r^5s^2t + 40r^5s^2 - 20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t - 63r^5 + 15r^4s^3t - 30r^4s^3 + 60r^4s^2t^2 - 65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st + 150r^4s - 30r^4t^3 - 90r^4t^2 + 150r^4t + 45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3 - 72r^3s^2t^2 + 306r^3s^2t - 39r^3s^2 + 30r^3st^3 + 306r^3st^2 - 366r^3st - 135r^3s + 105r^3t^3 - 39r^3t^2 - 135r^3t + 42r^2s^3t^3 + 138r^2s^3t^2 - 294r^2s^3t - 84r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 - 294r^2st^3 + 66r^2st^2 + 432r^2st - 84r^2t^3 + 108r^2t^2 - 210rs^3t^3 + 126rs^3t^2 + 294rs^3t + 126rs^2t^3 + 126rs^2t^2 - 378rs^2t + 294rst^3 - 378rst^2 + 210s^3t^3 - 294s^3t^2 - 294s^2t^3 + 378s^2t^2)$,

$E_{2_{24}}^{'''[4]_3} = -\frac{s^5}{1260(r-1)^3(s-1)^3(t-1)^3}(15r^3s^4t - 30r^3s^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - 84r^3s^2 - 210r^3st^3 + 126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 - 48r^2s^3t^3 - 72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 + 126r^2st^3 + 126r^2st^2 - 378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + 40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2)$,

$E_{2_{34}}^{'''[4]_3} = -\frac{t^5}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 - 366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 + 40s^2t^5 - 90s^2t^4 - 39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 + 45t^4)$,

$E_{2_{44}}^{'''[4]_3} = \frac{1}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3 - 1722r^3s^3t^2 + 1470r^3s^3t - 420r^3s^3 - 1722r^3s^2t^3 + 4662r^3s^2t^2 - 4026r^3s^2t + 1164r^3s^2 + 1470r^3st^3 - 4026r^3st^2 + 3561r^3st - 1050r^3s - 420r^3t^3 + 1164r^3t^2 - 1050r^3t + 315r^3 - 1722r^2s^3t^3 + 4662r^2s^3t^2 - 4026r^2s^3t + 1164r^2s^3 + 4662r^2s^2t^3 - 12618r^2s^2t^2 + 11034r^2s^2t - 3228r^2s^2 - 4026r^2st^3 + 11034r^2st^2 - 9847r^2st + 2930r^2s + 1164r^2t^3 - 3228r^2t^2 + 2930r^2t - 885r^2 + 1470rs^3t^3 - 4026rs^3t^2 + 3561rs^3t - 1050rs^3 - 4026rs^2t^3 + 11034rs^2t^2 - 9847rs^2t + 2930rs^2 + 3561rst^3 - 9847rst^2 + 8913rst - 2686rs - 1050r t^3 + 2930rt^2 - 2686rt + 819r - 420s^3t^3 + 1164s^3t^2 - 1050s^3t + 315s^3 + 1164s^2t^3 - 3228s^2t^2 + 2930s^2t - 885s^2 - 1050st^3 + 2930st^2 - 2686st + 819s + 315t^3 - 885t^2 + 819t - 252)$,

$K_{1_{14}}^{'''[4]_3} = \frac{r^2}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2)$,

$K_{1_{24}}^{'''[4]_3} = \frac{s^2}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t - 48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 - 48s^3t^2 - 48s^3t + 42s^2t^2)$,

$$K^{'''[4]_3}_{1_{34}} = \frac{t^2}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4),$$

$$K^{'''[4]_3}_{1_{44}} = \frac{1}{2520r^2s^2t^2}(210r^2s^2t^2 - 168r^2s^2t + 42r^2s^2 - 168r^2st^2 + 168r^2st - 48r^2s + 42r^2t^2 - 48r^2t + 15r^2 - 168rs^2t^2 + 168rs^2t - 48rs^2 + 168rst^2 - 192rst + 60rs - 48rt^2 + 60rt - 20r + 42s^2t^2 - 48s^2t + 15s^2 - 48st^2 + 60st - 20s + 15t^2 - 20t + 7),$$

$$K^{'''[4]_3}_{2_{11}} = -\frac{r^2}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6 - 70r^5s - 70r^5t - 70r^5 + 45r^4s^2 + 180r^4st + 180r^4s + 45r^4t^2 + 180r^4t + 45r^4 - 120r^3s^2t - 120r^3s^2 - 120r^3st^2 - 480r^3st - 120r^3s - 120r^3t^2 - 120r^3t + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 + 336r^2st^2 + 336r^2st + 84r^2t^2 - 252rs^2t^2 - 252rs^2t - 252rst^2 + 210s^2t^2),$$

$$K^{'''[4]_3}_{2_{21}} = -\frac{s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^3t^2 + 24rs^2 - 30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 + 96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst),$$

$$K^{'''[4]_3}_{2_{31}} = -\frac{t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2 - 15s^2t^3 + 84rs^2 + 24rt^2 - 30rt^3 + 10rt^4 + 48st^2 - 42s^2t - 60st^3 + 20st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84rs^2t - 30rst^3 + 24rs^2t^2 - 84rst),$$

$$K^{'''[4]_3}_{2_{41}} = -\frac{1}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r + 20s + 20t - 42s^2t^2 - 30rs - 30rt - 60st + 24rs^2 + 24rt^2 + 48st^2 + 48s^2t - 15s^2 - 15t^2 - 84rst^2 - 84rs^2t + 84rs^2t^2 + 96rst - 7),$$

$$K^{'''[4]_3}_{2_{12}} = -\frac{r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^3t^2 + 24r^2s - 30r^3s + 10r^4s - 42rt^2 + 48r^2t - 60r^3t + 20r^4t + 84st^2 - 15r^3 + 20r^4 - 7r^5 - 84rst^2 + 96r^2st - 30r^3st + 24r^2st^2 - 84rst),$$

$$K^{'''[4]_3}_{2_{22}} = -\frac{s^2}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4 - 120r^2s^3t - 120r^2s^3 + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 - 252r^2st^2 - 252r^2st + 210r^2t^2 - 70rs^5 + 180rs^4t + 180rs^4 - 120rs^3t^2 - 480rs^3t - 120rs^3 + 336rs^2t^2 + 336rs^2t - 252rst^2 + 28s^6 - 70s^5t - 70s^5 + 45s^4t^2 + 180s^4t + 45s^4 - 120s^3t^2 - 120s^3t + 84s^2t^2),$$

$K_{2_{32}}^{'''[4]_3} = -\frac{t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2 - 15r^2t^3 + 84r^2s + 48rt^2 - 42r^2t - 60rt^3 + 20rt^4 + 24st^2 - 30st^3 + 10st^4 - 15t^3 + 20t^4 - 7t^5 + 96rst^2 - 84r^2st - 30rst^3 + 24r^2st^2 - 84rst),$

$K_{2_{42}}^{'''[4]_3} = -\frac{1}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r + 10s + 20t - 42r^2t^2 - 30rs - 60rt - 30st + 24r^2s + 48rt^2 + 48r^2t + 24st^2 - 15r^2 - 15t^2 - 84rst^2 - 84r^2st + 84r^2st^2 + 96rst - 7),$

$K_{2_{13}}^{'''[4]_3} = -\frac{r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^3s^2 - 42rs^2 + 48r^2s - 60r^3s + 20r^4s + 24r^2t - 30r^3t + 10r^4t + 84s^2t - 15r^3 + 20r^4 - 7r^5 - 84rs^2t + 96r^2st - 30r^3st + 24r^2s^2t - 84rst),$

$K_{2_{23}}^{'''[4]_3} = -\frac{s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 + 84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t + 24r^2s^2t - 84rst),$

$K_{2_{33}}^{'''[4]_3} = -\frac{t^2}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4),$

$K_{2_{43}}^{'''[4]_3} = -\frac{1}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r + 20s + 10t - 42r^2s^2 - 60rs - 30rt - 30st + 48rs^2 + 48r^2s + 24r^2t + 24s^2t - 15r^2 - 15s^2 - 84rs^2t - 84r^2st + 84r^2s^2t + 96rst - 7),$

$K_{2_{14}}^{'''[4]_3} = -\frac{r^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-7r^5 + 20r^4s + 20r^4t + 10r^4 - 15r^3s^2 - 60r^3st - 30r^3s - 15r^3t^2 - 30r^3t + 48r^2s^2t + 24r^2s^2 + 48r^2st^2 + 96r^2st + 24r^2t^2 - 42rs^2t^2 - 84rs^2t - 84rst^2 + 84s^2t^2),$

$K_{2_{24}}^{'''[4]_3} = -\frac{s^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-15r^2s^3 + 48r^2s^2t + 24r^2s^2 - 42r^2st^2 - 84r^2st + 84r^2t^2 + 20rs^4 - 60rs^3t - 30rs^3 + 48rs^2t^2 + 96rs^2t - 84rst^2 - 7s^5 + 20s^4t + 10s^4 - 15s^3t^2 - 30s^3t + 24s^2t^2),$

$K_{2_{34}}^{'''[4]_3} = -\frac{t^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-42r^2s^2t + 84r^2s^2 + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4),$

$K'''^{[4]_3}_{2_{44}} = -\frac{1}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2 - 252r^2s^2t + 84r^2s^2 - 252r^2st^2 + 336r^2st - 120r^2s + 84r^2t^2 - 120r^2t + 45r^2 - 252rs^2t^2 + 336rs^2t - 120rs^2 + 336rst^2 - 480rst + 180rs - 120rt^2 + 180rt - 70r + 84s^2t^2 - 120s^2t + 45s^2 - 120st^2 + 180st - 70s + 45t^2 - 70t + 28).$

5.2.1 Properties of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

The basic properties as examined in Section 4.3.1 are also investigated in this section.

5.2.1.1 Order of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

The order of the main block Equation (5.11) is obtained using the linear difference operator ∇ which can be expressed as follows

$$\begin{aligned}\nabla[y(x),h] &= Y^{[4]_3}_{n+1} - \hat{M}^{[4]_3}_1 Y^{[3]_3}_n - h\hat{M}^{[4]_3}_2 Y^{[4]_3}_{n-1} - h^2\hat{M}^{[4]_3}_3 Y^{[4]_3}_{n-2} - h^3\hat{M}^{[4]_3}_4 Y^{[4]_3}_{n-3} \\ &\quad -h^4\left[\hat{E}^{[4]_3}_1 F^{[4]_3}_n + \hat{E}^{[4]_3}_2 F^{[4]_3}_{n+1}\right] - h^5\left[\hat{K}^{[4]_3}_1 G^{[4]_3}_n + \hat{K}^{[4]_3}_2 G^{[4]_3}_{n+1}\right]. \quad (5.31)\end{aligned}$$

Expanding the functions $Y^{[4]_3}_{n+1}, F^{[4]_3}_{n+1}$ and $G^{[4]_3}_{n+1}$ in Taylor series expansion about x_n and set to **0** yields

$$\left[Q^{[4]_3}_{11}\ Q^{[4]_3}_{21}\ Q^{[4]_3}_{31}\ Q^{[4]_3}_{41}\right]^T = \left[0\ 0\ 0\ 0\right]^T$$

where

$Q^{[4]_3}_{11} = \sum_{i=0}^{\infty}\frac{(rh)^i}{i!}y^{(i)}_n - y_n - rhy'_n - \frac{h^2r^2y''_n}{2} - \frac{h^3r^3y'''_n}{6} + \frac{h^4r^4y^{iv}_n}{2162160s^3t^3}(260r^6s^2t - 70r^7s - 70r^7t - 70r^7st + 260r^6s^2 + 260r^6st^2 + 645r^6st + 260r^6s + 260r^6t^2 + 260r^6t - 260r^5s^3t - 260r^5s^3 - 1040r^5s^2t^2 - 1820r^5s^2t - 1040r^5s^2 - 260r^5st^3 - 1820r^5st^2 - 1820r^5st - 260r^5s - 260r^5t^3 - 1040r^5t^2 - 260r^5t + 1144r^4s^3t^2 + 1742r^4s^3t + 1144r^4s^3 + 1144r^4s^2t^3 + 4680r^4s^2t^2 + 4680r^4s^2t + 1144r^4s^2 + 1742r^4st^3 + 4680r^4st^2 + 1742r^4st + 1144r^4t^3 + 1144r^4t^2 - 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3$

$s^3t - 1430r^3s^3 - 4576r^3s^2t^3 - 6864r^3s^2t^2 - 4576r^3s^2t - 4576r^3st^3 - 4576r^3st^2 - 1430r^3t^3 + 5005r^2s^3t^3 + 2860r^2s^3t^2 + 5005r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 5005r^2st^3 + 8580rs^3t^3 + 8580rs^3t^2 + 8580rs^2t^3 - 77220s^3t^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} r^4}{2162160(r-s)^3(r-t)^3(r-1)^3!}(630r^9 - 2520r^8s - 2520r^8t - 2520r^8 + 3315r^7s^2 + 10390r^7st + 10390r^7s + 3315r^7t^2 + 10390r^7t + 3315r^7 - 1365r^6s^3 - 14105r^6s^2t - 14105r^6s^2 - 14105r^6st^2 - 44520r^6st - 14105r^6s - 1365r^6t^3 - 14105r^6t^2 - 14105r^6t - 1365r^6 + 5915r^5s^3t + 5915r^5s^3 + 19799r^5s^2t^2 + 63011r^5s^2t + 19799r^5s^2 + 5915r^5st^3 + 63011r^5st^2 + 63011r^5st + 5915r^5s + 5915r^5t^3 + 19799r^5t^2 + 5915r^5t - 8437r^4s^3t^2 - 27053r^4s^3t - 8437r^4s^3 - 8437r^4s^2t^3 - 93483r^4s^2t^2 - 93483r^4s^2t - 8437r^4s^2 - 27053r^4st^3 - 93483r^4st^2 - 27053r^4st - 8437r^4t^3 - 8437r^4t^2 + 3575r^3s^3t^3 + 41041r^3s^3t^2 + 41041r^3s^3t + 3575r^3s^3 + 41041r^3s^2t^3 + 148434r^3s^2t^2 + 41041r^3s^2t + 41041r^3st^3 + 41041r^3st^2 + 3575r^3t^3 - 17875r^2s^3t^3 - 67210r^2s^3t^2 - 17875r^2s^3t - 67210r^2s^2t^3 - 67210r^2s^2t^2 - 17875r^2st^3 + 30030rs^3t^3 + 30030rs^3t^2 + 30030rs^2t^3 - 12870s^3t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)} r^8}{2162160s^3(r-s)^3(s-t)^3(s-1)^3!}(140\ \ r^6st - 210r^6s^2 + 140r^6s - 70r^6t + 735r^5s^3 + 245r^5s^2t + 245r^5s^2 - 520r^5st^2 - 705r^5st - 520r^5s + 260r^5t^2 + 260r^5t - 585r^4s^4 - 2405r^4s^3t - 2405r^4s^3 + 1300r^4s^2t^2 + 715r^4s^2t + 1300r^4s^2 + 520r^4st^3 + 1300r^4st^2 + 1300r^4st + 520r^4s - 260r^4t^3 - 1040r^4t^2 - 260r^4t + 2340r^3s^4t + 2340r^3s^4 + 1222r^3s^3t^2 + 8528r^3s^3t + 1222r^3s^3 - 2210r^3s^2t^3 - 6318r^3s^2t^2 - 6318r^3s^2t - 2210r^3s^2 - 910r^3st^3 + 936r^3st^2 - 910r^3st + 1144r^3t^3 + 1144r^3t^2 - 2574r^2s^4t^2 - 10296r^2s^4t - 2574r^2s^4 + 2002r^2s^3t^3 - 4576r^2s^3t^2 - 4576r^2s^3t + 2002r^2s^3 + 9152r^2s^2t^3 + 11154r^2s^2t^2 + 9152r^2s^2t - 3718r^2st^3 - 3718r^2st^2 - 1430r^2t^3 + 12870rs^4t^2 + 12870rs^4t - 10010rs^3t^3 + 715rs^3t^2 - 10010rs^3t - 7865rs^2t^3 - 7865rs^2t^2 + 9295rst^3 - 19305s^4t^2 + 15015s^3t^3 + 15015s^3t^2 - 10725s^2t^3) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+4)} r^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3!}(140r^6st - 70r^6s - 210r^6t^2 + 140r^6t - 520r^5s^2t + 260r^5s^2 + 245r^5st^2 - 705r^5st + 260r^5s + 735r^5t^3 + 245r^5t^2 - 520r^5t + 520r^4s^3t - 260r^4s^3 + 1300r^4s^2t^2 + 1300r^4s^2t - 1040r^4s^2 - 2405r^4st^3 + 715r^4st^2 + 1300r^4st - 260r^4s - 585r^4t^4 - 2405r^4t^3 + 1300r^4t^2 + 520r^4t - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 + 1222r^3s^2t^3 - 6318r^3s^2t^2 + 936r^3s^2t + 1144r^3s^2 + 2340r^3st^4 + 8528r^3$

$st^3 - 6318r^3st^2 - 910r^3st + 2340r^3t^4 + 1222r^3t^3 - 2210r^3t^2 + 2002r^2s^3t^3 + 9152r^2s^3t^2 - 3718r^2s^3t - 1430r^2s^3 - 2574r^2s^2t^4 - 4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t - 10296r^2st^4 - 4576r^2st^3 + 9152r^2st^2 - 2574r^2t^4 + 2002r^2t^3 - 10010rs^3t^3 - 7865rs^3t^2 + 9295rs^3t + 12870rs^2t^4 + 715rs^2t^3 - 7865rs^2t^2 + 12870rst^4 - 10010rst^3 + 15015s^3t^3 - 10725s^3t^2 - 19305s^2t^4 + 15015s^2t^3) - \sum_{i=0}^{\infty} \frac{h^{i+4}y_n^{(i+4)}r^8}{2162160(r-1)^3(s-1)^3(t-1)^3!}(140r^6s - 70r^6st + 140r^6t - 210r^6 + 260r^5s^2t - 520r^5s^2 + 260r^5st^2 - 705r^5st + 245r^5s - 520r^5t^2 + 245r^5t + 735r^5 - 260r^4s^3t + 520r^4s^3 - 1040r^4s^2t^2 + 1300r^4s^2t + 1300r^4s^2 - 260r^4st^3 + 1300r^4st^2 + 715r^4st - 2405r^4s + 520r^4t^3 + 1300r^4t^2 - 2405r^4t - 585r^4 + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 + 1144r^3s^2t^3 + 936r^3s^2t^2 - 6318r^3s^2t + 1222r^3s^2 - 910r^3st^3 - 6318r^3st^2 + 8528r^3st + 2340r^3s - 2210r^3t^3 + 1222r^3t^2 + 2340r^3t - 1430r^2s^3t^3 - 3718r^2s^3t^2 + 9152r^2s^3t + 2002r^2s^3 - 3718r^2s^2t^3 + 11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 + 9152r^2st^3 - 4576r^2st^2 - 10296r^2st + 2002r^2t^3 - 2574r^2t^2 + 9295rs^3t^3 - 7865rs^3t^2 - 10010rs^3t - 7865rs^2t^3 + 715rs^2t^2 + 12870rs^2t - 10010rst^3 + 12870rst^2 - 10725s^3t^3 + 15015s^3t^2 + 15015s^2t^3 - 19305s^2t^2) - \frac{h^5r^5y_n^{v}}{2162160s^2t^2}(35r^6 - 130r^5s - 130r^5t - 130r^5 + 130r^4s^2 + 520r^4st + 520r^4s + 130r^4t^2 + 520r^4t + 130r^4 - 572r^3s^2t - 572r^3s^2 - 572r^3st^2 - 2288r^3st - 572r^3s - 572r^3t^2 - 572r^3t + 715r^2s^2t^2 + 2860r^2s^2t + 715r^2s^2 + 2860r^2st^2 + 2860r^2st + 715r^2t^2 - 4290rs^2t^2 - 4290rs^2t - 4290rst^2 + 8580s^2t^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+5}y_n^{(i+5)}r^5}{1081080(r-s)^2(r-t)^2(r-1)^2i!}(28r^6 - 91r^5s - 91r^5t - 91r^5 + 78r^4s^2 + 312r^4st + 312r^4s + 78r^4t^2 + 312r^4t + 78r^4 - 286r^3s^2t - 286r^3s^2 - 286r^3st^2 - 1144r^3st - 286r^3s - 286r^3t^2 - 286r^3t + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 + 1144r^2st^2 + 1144r^2st + 286r^2t^2 - 1287rs^2t^2 - 1287rs^2t - 1287rst^2 + 1716s^2t^2) + \sum_{i=0}^{\infty} \frac{s^ih^{i+5}y_n^{(i+5)}r^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2i!}(572r^2t^2 - 130r^3t^2 + 286r^2s - 260r^3s + 65r^4s - 715rt^2 + 572r^2t - 520r^3t + 130r^4t + 2145st^2 - 130r^3 + 130r^4 - 35r^5 - 1430rst^2 + 1144r^2st - 260r^3st + 286r^2st^2 - 1430rst) + \sum_{i=0}^{\infty} \frac{t^ih^{i+5}y_n^{(i+5)}r^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2i!} (572r^2s^2 - 130r^3s^2 - 715rs^2 + 572r^2s - 520r^3s + 130r^4s + 286r^2t - 260r^3t + 65r^4t + 2145s^2t - 130r^3 + 130r^4 - 35r^5 - 1430rs^2t + 1144r^2st - 260r^3st + 286r^2s^2t - 1430rst) + \sum_{i=0}^{\infty} \frac{h^{i+5}y_n^{(i+5)}r^8}{2162160(r-1)^2(s-1)^2(t-1)^2i!}(130r^4s - 35r^5 + 130r^4t + 65r^4 - 130r^3s^2 - 520r^3st - 260r^3s - 130r^3t^2 - 260r^3t + 572r^2s^2t +$

$286r^2s^2 + 572r^2st^2 + 1144r^2st + 286r^2t^2 - 715rs^2t^2 - 1430rs^2t - 1430rst^2 + 2145s^2t^2)$,

$Q_{21}^{[4]_3} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i)} - y_n - shy_n' - \frac{h^2s^2y_n''}{2} - \frac{h^3s^3y_n'''}{6} + \frac{h^4s^4y_n^{iv}}{2162160r^3t^3}(1144r^3s^4t^2 - 260r^3s^5 - 260r^3s^5t + 1742r^3s^4t + 1144r^3s^4 - 1430r^3s^3t^3 - 4576r^3s^3t^2 - 4576r^3s^3t - 1430r^3s^3 + 5005r^3s^2t^3 + 2860r^3s^2t^2 + 5005r^3s^2t + 8580r^3st^3 + 8580r^3st^2 - 77220r^3t^3 + 260r^2s^6t + 260r^2s^6 - 1040r^2s^5t^2 - 1820r^2s^5t - 1040r^2s^5 + 1144r^2s^4t^3 + 4680r^2s^4t^2 + 4680r^2s^4t + 1144r^2s^4 - 4576r^2s^3t^3 - 6864r^2s^3t^2 - 4576r^2s^3t + 2860r^2s^2t^3 + 2860r^2s^2t^2 + 8580r^2st^3 - 70rs^7t - 70rs^7 + 260rs^6t^2 + 645rs^6t + 260rs^6 - 260rs^5t^3 - 1820rs^5t^2 - 1820rs^5t - 260rs^5 + 1742rs^4t^3 + 4680rs^4t^2 + 1742rs^4t - 4576rs^3t^3 - 4576rs^3t^2 + 5005rs^2t^3 - 70s^7t + 260s^6t^2 + 260s^6t - 260s^5t^3 - 1040s^5t^2 - 260s^5t + 1144s^4t^3 + 1144s^4t^2 - 1430s^3t^3) + \sum_{i=0}^{\infty} \frac{r^ih^{i+4}y_n^{(i+4)}s^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3!}(2340r^4s^3t - 585r^4s^4 + 2340r^4s^3 - 2574r^4s^2t^2 - 10296r^4s^2t - 2574r^4s^2 + 12870r^4st^2 + 12870r^4st - 19305r^4t^2 + 735r^3s^5 - 2405r^3s^4t - 2405r^3s^4 + 1222r^3s^3t^2 + 8528r^3s^3t + 1222r^3s^3 + 2002r^3s^2t^3 - 4576r^3s^2t^2 - 4576r^3s^2t + 2002r^3s^2 - 10010r^3st^3 + 715r^3st^2 - 10010r^3st + 15015r^3t^3 + 15015r^3t^2 - 210r^2s^6 + 245r^2s^5t + 245r^2s^5 + 1300r^2s^4t^2 + 715r^2s^4t + 1300r^2s^4 - 2210r^2s^3t^3 - 6318r^2s^3t^2 - 6318r^2s^3t - 2210r^2s^3 + 9152r^2s^2t^3 + 11154r^2s^2t^2 + 9152r^2s^2t - 7865r^2st^3 - 7865r^2st^2 - 10725r^2t^3 + 140rs^6t + 140rs^6 - 520rs^5t^2 - 705rs^5t - 520rs^5 + 520rs^4t^3 + 1300rs^4t^2 + 1300rs^4t + 520rs^4 - 910rs^3t^3 + 936rs^3t^2 - 910rs^3t - 3718rs^2t^3 - 3718rs^2t^2 + 9295rst^3 - 70s^6t + 260s^5t^2 + 260s^5t - 260s^4t^3 - 1040s^4t^2 - 260s^4t + 1144s^3t^3 + 1144s^3t^2 - 1430s^2t^3) - \sum_{i=0}^{\infty} \frac{s^ih^{i+4}y_n^{(i+4)}s^4}{2162160(r-s)^3(s-t)^3(s-1)^3!}(1365r^3s^6 - 5915r^3s^5t - 5915r^3s^5 + 8437r^3s^4t^2 + 27053r^3s^4t + 8437r^3s^4 - 3575r^3s^3t^3 - 41041r^3s^3t^2 - 41041r^3s^3t - 3575r^3s^3 + 17875r^3s^2t^3 + 67210r^3s^2t^2 + 17875r^3s^2t - 30030r^3st^3 - 30030r^3st^2 + 12870r^3t^3 - 3315r^2s^7 + 14105r^2s^6t + 14105r^2s^6 - 19799r^2s^5t^2 - 63011r^2s^5t - 19799r^2s^5 + 8437r^2s^4t^3 + 93483r^2s^4t^2 + 93483r^2s^4t + 8437r^2s^4 - 41041r^2s^3t^3 - 148434r^2s^3t^2 - 41041r^2s^3t + 67210r^2s^2t^3 + 67210r^2s^2t^2 - 30030r^2st^3 + 2520rs^8 - 10390rs^7t - 10390rs^7 + 14105rs^6t^2 + 44520rs^6t + 14105rs^6 - 5915rs^5t^3 - 63011rs^5t^2 - 63011rs^5t - 5915rs^5 + 27053rs^4t^3 + 93483rs^4t^2 + 27053rs^4t - 41041rs^3t^3 - 41041$

$rs^3t^2 + 17875rs^2t^3 - 630s^9 + 2520s^8t + 2520s^8 - 3315s^7t^2 - 10390s^7t - 3315s^7 + 1365s^6t^3 + 14105s^6t^2 + 14105s^6t + 1365s^6 - 5915s^5t^3 - 19799s^5t^2 - 5915s^5t + 8437s^4t^3 + 8437s^4t^2 - 3575s^3t^3) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+4)} s^8}{2162160t^3(r-t)^3(s-t)^3(t-1)^3!}(520r^3s^4t - 260r^3s^4 - 2210r^3s^3t^2 - 910r^3s^3t + 1144r^3s^3 + 2002r^3s^2t^3 + 9152r^3s^2t^2 - 3718r^3s^2t - 1430r^3s^2 - 10010r^3st^3 - 7865r^3st^2 + 9295r^3st + 15015r^3t^3 - 10725r^3t^2 - 520r^2s^5t + 260r^2s^5 + 1300r^2s^4t^2 + 1300r^2s^4t - 1040r^2s^4 + 1222r^2s^3t^3 - 6318r^2s^3t^2 + 936r^2s^3t + 1144r^2s^3 - 2574r^2s^2t^4 - 4576r^2s^2t^3 + 11154r^2s^2t^2 - 3718r^2s^2t + 12870r^2st^4 + 715r^2st^3 - 7865r^2st^2 - 19305r^2t^4 + 15015r^2t^3 + 140rs^6t - 70rs^6 + 245rs^5t^2 - 705rs^5t + 260rs^5 - 2405rs^4t^3 + 715rs^4t^2 + 1300rs^4t - 260rs^4 + 2340rs^3t^4 + 8528rs^3t^3 - 6318rs^3t^2 - 910rs^3t - 10296rs^2t^4 - 4576rs^2t^3 + 9152rs^2t^2 + 12870rst^4 - 10010rst^3 - 210s^6t^2 + 140s^6t + 735s^5t^3 + 245s^5t^2 - 520s^5t - 585s^4t^4 - 2405s^4t^3 + 1300s^4t^2 + 520s^4t + 2340s^3t^4 + 1222s^3t^3 - 2210s^3t^2 - 2574s^2t^4 + 2002s^2t^3) - \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+4)} s^8}{2162160(r-1)^3(s-1)^3(t-1)^3!}(520r^3s^4 - 260r^3s^4t + 1144r^3s^3t^2 - 910r^3s^3t - 2210r^3s^3 - 1430r^3s^2t^3 - 3718r^3s^2t^2 + 9152r^3s^2t + 2002r^3s^2 + 9295r^3st^3 - 7865r^3st^2 - 10010r^3st - 10725r^3t^3 + 15015r^3t^2 + 260r^2s^5t - 520r^2s^5 - 1040r^2s^4t^2 + 1300r^2s^4t + 1300r^2s^4 + 1144r^2s^3t^3 + 936r^2s^3t^2 - 6318r^2s^3t + 1222r^2s^3 - 3718r^2s^2t^3 + 11154r^2s^2t^2 - 4576r^2s^2t - 2574r^2s^2 - 7865r^2st^3 + 715r^2st^2 + 12870r^2st + 15015r^2t^3 - 19305r^2t^2 - 70rs^6t + 140rs^6 + 260rs^5t^2 - 705rs^5t + 245rs^5 - 260rs^4t^3 + 1300rs^4t^2 + 715rs^4t - 2405rs^4 - 910rs^3t^3 - 6318rs^3t^2 + 8528rs^3t + 2340rs^3 + 9152rs^2t^3 - 4576rs^2t^2 - 10296rs^2t - 10010rst^3 + 12870rst^2 + 140s^6t - 210s^6 - 520s^5t^2 + 245s^5t + 735s^5 + 520s^4t^3 + 1300s^4t^2 - 2405s^4t - 585s^4 - 2210s^3t^3 + 1222s^3t^2 + 2340s^3t + 2002s^2t^3 - 2574s^2t^2) - \frac{h^5s^5y_n^v}{2162160r^2t^2}(130r^2s^4 - 572r^2s^3t - 572r^2s^3 + 715r^2s^2t^2 + 2860r^2s^2t + 715r^2s^2 - 4290r^2st^2 - 4290r^2st + 8580r^2t^2 - 130rs^5 + 520rs^4t + 520rs^4 - 572rs^3t^2 - 2288rs^3t - 572rs^3 + 2860rs^2t^2 + 2860rs^2t - 4290rst^2 + 35s^6 - 130s^5t - 130s^5 + 130s^4t^2 + 520s^4t + 130s^4 - 572s^3t^2 - 572s^3t + 715s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+5} y_n^{(i+5)} s^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2 i!}(572s^2t^2 - 130s^3t^2 + 286rs^2 - 260rs^3 + 65rs^4 + 2145rt^2 - 715st^2 + 572s^2t - 520s^3t + 130s^4t - 130s^3 + 130s^4 - 35s^5 - 1430rst^2 + 1144rs^2t - 260rs^3t + 286rs^2t^2 - 1430rst) + \sum_{i=0}^{\infty} \frac{s^i h^{i+5} y_n^{(i+5)} s^5}{1081080(r-s)^2(s-t)^2(s-1)^2 i!}(78r^2s^4 -$

$286r^2s^3t - 286r^2s^3 + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 - 1287r^2st^2 - 1287r^2st + 1716r^2t^2 - 91rs^5 + 312rs^4t + 312rs^4 - 286rs^3t^2 - 1144rs^3t - 286rs^3 + 1144rs^2t^2 + 1144rs^2t - 1287rst^2 + 28s^6 - 91s^5t - 91s^5 + 78s^4t^2 + 312s^4t + 78s^4 - 286s^3t^2 - 286s^3t + 286s^2t^2) + \sum_{i=0}^{\infty} \frac{t^i h^{i+5} y_n^{(i+5)} s^8}{2162160t^2(r-t)^2(s-t)^2(t-1)^2 i!}(572r^2s^2 - 130r^2s^3 + 572rs^2 - 715r^2s - 520rs^3 + 130rs^4 + 2145r^2t + 286s^2t - 260s^3t + 65s^4t - 130s^3 + 130s^4 - 35s^5 + 1144rs^2t - 1430r^2st - 260rs^3t + 286r^2s^2t - 1430rst) + \sum_{i=0}^{\infty} \frac{h^{i+5} y_n^{(i+5)} s^8}{2162160(r-1)^2(s-1)^2(t-1)^2 i!}(572r^2s^2t - 130r^2s^3 + 286r^2s^2 - 715r^2st^2 - 1430r^2st + 2145r^2t^2 + 130rs^4 - 520rs^3t - 260rs^3 + 572rs^2t^2 + 1144rs^2t - 1430rst^2 - 35s^5 + 130s^4t + 65s^4 - 130s^3t^2 - 260s^3t + 286s^2t^2),$

$Q_{31}^{[4]_3} = \sum_{i=0}^{\infty} \frac{(th)^i}{i!} y_n^{(i)} - y_n - thy_n' - \frac{h^2t^2y_n''}{2} - \frac{h^3t^3y_n'''}{6} + \frac{h^4t^4y_n^{iv}}{2162160r^3s^3}(5005r^3s^3t^2 - 1430r^3s^3t^3 + 8580r^3s^3t - 77220r^3s^3 + 1144r^3s^2t^4 - 4576r^3s^2t^3 + 2860r^3s^2t^2 + 8580r^3s^2t - 260r^3st^5 + 1742r^3st^4 - 4576r^3st^3 + 5005r^3st^2 - 260r^3t^5 + 1144r^3t^4 - 1430r^3t^3 + 1144r^2s^3t^4 - 4576r^2s^3t^3 + 2860r^2s^3t^2 + 8580r^2s^3t - 1040r^2s^2t^5 + 4680r^2s^2t^4 - 6864r^2s^2t^3 + 2860r^2s^2t^2 + 260r^2st^6 - 1820r^2st^5 + 4680r^2st^4 - 4576r^2st^3 + 260r^2t^6 - 1040r^2t^5 + 1144r^2t^4 - 260rs^3t^5 + 1742rs^3t^4 - 4576rs^3t^3 + 5005rs^3t^2 + 260rs^2t^6 - 1820rs^2t^5 + 4680rs^2t^4 - 4576rs^2t^3 - 70rst^7 + 645rst^6 - 1820rst^5 + 1742rst^4 - 70rt^7 + 260rt^6 - 260rt^5 - 260s^3t^5 + 1144s^3t^4 - 1430s^3t^3 + 260s^2t^6 - 1040s^2t^5 + 1144s^2t^4 - 70st^7 + 260st^6 - 260st^5) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+4)} t^8}{2162160r^3(r-s)^3(r-t)^3(r-1)^3!}(12870r^4s^2t - 2574r^4s^2t^2 - 19305r^4s^2 + 2340r^4st^3 - 10296\ r^4st^2 + 12870r^4st - 585r^4t^4 + 2340r^4t^3 - 2574r^4t^2 + 2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 + 1222r^3s^2t^3 - 4576r^3s^2t^2 + 715r^3s^2t + 15015r^3s^2 - 2405r^3st^4 + 8528r^3st^3 - 4576r^3st^2 - 10010r^3st + 735r^3t^5 - 2405r^3t^4 + 1222r^3t^3 + 2002r^3t^2 - 2210r^2s^3t^3 + 9152r^2s^3t^2 - 7865r^2s^3t - 10725r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t + 245r^2st^5 + 715r^2st^4 - 6318r^2st^3 + 9152r^2st^2 - 210r^2t^6 + 245r^2t^5 + 1300r^2t^4 - 2210r^2t^3 + 520rs^3t^4 - 910rs^3t^3 - 3718rs^3t^2 + 9295rs^3t - 520rs^2t^5 + 1300rs^2t^4 + 936rs^2t^3 - 3718rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 + 140rt^6 - 520rt^5 + 520rt^4 - 260s^3t^4 + 1144s^3t^3 - 1430s^3t^2 + 260s^2t^5 - 1040s^2t^4 + 1144s^2t^3 - 70st^6 + 260st^5 - 260st^4) -$

$\sum_{i=0}^{\infty}\frac{s^i h^{i+4} y_n^{(i+4)} t^8}{2162160 s^3(r-s)^3(s-t)^3(s-1)^3!}(2002r^3s^3t^2 - 10010r^3s^3t + 15015r^3s^3 - 2210r^3s^2t^3 + 9152r^3s^2t^2 - 7865r^3s^2t - 10725r^3s^2 + 520r^3st^4 - 910r^3st^3 - 3718r^3st^2 + 9295r^3st - 260r^3t^4 + 1144r^3t^3 - 1430r^3t^2 - 2574r^2s^4t^2 + 12870r^2s^4t - 19305r^2s^4 + 1222r^2s^3t^3 - 4576r^2s^3t^2 + 715r^2s^3t + 15015r^2s^3 + 1300r^2s^2t^4 - 6318r^2s^2t^3 + 11154r^2s^2t^2 - 7865r^2s^2t - 520r^2st^5 + 1300r^2st^4 + 936r^2st^3 - 3718r^2st^2 + 260r^2t^5 - 1040r^2t^4 + 1144r^2t^3 + 2340rs^4t^3 - 10296rs^4t^2 + 12870rs^4t - 2405rs^3t^4 + 8528rs^3t^3 - 4576rs^3t^2 - 10010rs^3t + 245rs^2t^5 + 715rs^2t^4 - 6318rs^2t^3 + 9152rs^2t^2 + 140rst^6 - 705rst^5 + 1300rst^4 - 910rst^3 - 70rt^6 + 260rt^5 - 260rt^4 - 585s^4t^4 + 2340s^4t^3 - 2574s^4t^2 + 735s^3t^5 - 2405s^3t^4 + 1222s^3t^3 + 2002s^3t^2 - 210s^2t^6 + 245s^2t^5 + 1300s^2t^4 - 2210s^2t^3 + 140st^6 - 520st^5 + 520st^4) + \sum_{i=0}^{\infty}\frac{t^i h^{i+4} y_n^{(i+4)} t^4}{2162160(r-t)^3(s-t)^3(t-1)^3!}(17875r^3s^3t^2 - 3575r^3s^3t^3 - 30030r^3s^3t + 12870r^3s^3 + 8437r^3\ s^2t^4 - 41041r^3s^2t^3 + 67210r^3s^2t^2 - 30030r^3s^2t - 5915r^3st^5 + 27053r^3st^4 - 41041r^3st^3 + 17875r^3st^2 + 1365r^3t^6 - 5915r^3t^5 + 8437r^3t^4 - 3575r^3t^3 + 8437r^2s^3t^4 - 41041r^2s^3t^3 + 67210r^2s^3t^2 - 30030r^2s^3t - 19799r^2s^2t^5 + 93483r^2s^2t^4 - 148434r^2s^2t^3 + 67210r^2s^2t^2 + 14105r^2st^6 - 63011r^2st^5 + 93483r^2st^4 - 41041r^2st^3 - 3315r^2t^7 + 14105r^2t^6 - 19799r^2t^5 + 8437r^2t^4 - 5915rs^3t^5 + 27053rs^3t^4 - 41041rs^3t^3 + 17875rs^3t^2 + 14105rs^2t^6 - 63011rs^2t^5 + 93483rs^2t^4 - 41041rs^2t^3 - 10390rst^7 + 44520rst^6 - 63011rst^5 + 27053rst^4 + 2520rt^8 - 10390rt^7 + 14105rt^6 - 5915rt^5 + 1365s^3t^6 - 5915s^3t^5 + 8437s^3t^4 - 3575s^3t^3 - 3315s^2t^7 + 14105s^2t^6 - 19799s^2t^5 + 8437s^2t^4 + 2520st^8 - 10390st^7 + 14105st^6 - 5915st^5 - 630t^9 + 2520t^8 - 3315t^7 + 1365t^6) - \sum_{i=0}^{\infty}\frac{h^{i+4} y_n^{(i+4)} t^8}{2162160(r-1)^3(s-1)^3(t-1)^3!}(9295r^3s^3t - 1430r^3s^3t^2 - 10725r^3s^3 + 1144r^3s^2t^3 - 3718r^3s^2t^2 - 7865r^3s^2t + 15015r^3s^2 - 260r^3st^4 - 910r^3st^3 + 9152r^3st^2 - 10010r^3st + 520r^3t^4 - 2210r^3t^3 + 2002r^3t^2 + 1144r^2s^3t^3 - 3718r^2s^3t^2 - 7865r^2s^3t + 15015r^2s^3 - 1040r^2s^2t^4 + 936r^2s^2t^3 + 11154r^2s^2t^2 + 715r^2s^2t - 19305r^2s^2 + 260r^2st^5 + 1300r^2st^4 - 6318r^2st^3 - 4576r^2st^2 + 12870r^2st - 520r^2t^5 + 1300r^2t^4 + 1222r^2t^3 - 2574r^2t^2 - 260rs^3t^4 - 910rs^3t^3 + 9152rs^3t^2 - 10010rs^3t + 260rs^2t^5 + 1300rs^2t^4 - 6318rs^2t^3 - 4576rs^2t^2 + 12870rs^2t - 70rst^6 - 705rst^5 + 715rst^4 + 8528rst^3 - 10296rst^2 + 140rt^6 + 245rt^5 - 2405rt^4 + 2340rt^3 + 520s^3t^4 - 2210s^3t^3 +$

$2002s^3t^2 - 520s^2t^5 + 1300s^2t^4 + 1222s^2t^3 - 2574s^2t^2 + 140st^6 + 245st^5 - 2405st^4 + 2340st^3 - 210t^6 + 735t^5 - 585t^4) - \frac{h^5t^5y_n^{(v)}}{2162160r^2s^2}(715r^2s^2t^2 - 4290r^2s^2t + 8580r^2s^2 - 572r^2st^3 + 2860r^2st^2 - 4290r^2st + 130r^2t^4 - 572r^2t^3 + 715r^2t^2 - 572rs^2t^3 + 2860rs^2t^2 - 4290rs^2t + 520rst^4 - 2288rst^3 + 2860rst^2 - 130rt^5 + 520rt^4 - 572rt^3 + 130s^2t^4 - 572s^2t^3 + 715s^2t^2 - 130st^5 + 520st^4 - 572st^3 + 35t^6 - 130t^5 + 130t^4) + \sum_{i=0}^{\infty}\frac{r^ih^{i+5}y_n^{(i+5)}t^8}{2162160r^2(r-s)^2(r-t)^2(r-1)^2i!}(572s^2t^2 - 130s^2t^3 + 2145rs^2 + 286rt^2 - 260rt^3 + 65rt^4 + 572st^2 - 715s^2t - 520st^3 + 130st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430rs^2t - 260rst^3 + 286rs^2t^2 - 1430rst) + \sum_{i=0}^{\infty}\frac{s^ih^{i+5}y_n^{(i+5)}t^8}{2162160s^2(r-s)^2(s-t)^2(s-1)^2i!}(572r^2t^2 - 130r^2t^3 + 2145r^2s + 572rt^2 - 715r^2t - 520rt^3 + 130rt^4 + 286st^2 - 260st^3 + 65st^4 - 130t^3 + 130t^4 - 35t^5 + 1144rst^2 - 1430r^2st - 260rst^3 + 286r^2st^2 - 1430rst) + \sum_{i=0}^{\infty}\frac{t^ih^{i+5}y_n^{(i+5)}t^5}{1081080(r-t)^2(s-t)^2(t-1)^2i!}(286r^2s^2t^2 - 1287r^2s^2t + 1716r^2s^2 - 286r^2st^3 + 1144r^2st^2 - 1287r^2st + 78r^2t^4 - 286r^2t^3 + 286r^2t^2 - 286rs^2t^3 + 1144rs^2t^2 - 1287rs^2t + 312rst^4 - 1144rst^3 + 1144rst^2 - 91rt^5 + 312rt^4 - 286rt^3 + 78s^2t^4 - 286s^2t^3 + 286s^2t^2 - 91st^5 + 312st^4 - 286st^3 + 28t^6 - 91t^5 + 78t^4) + \sum_{i=0}^{\infty}\frac{h^{i+5}y_n^{(i+5)}t^8}{2162160(r-1)^2(s-1)^2(t-1)^2i!}(2145\quad r^2s^2 - 715r^2s^2t + 572r^2st^2 - 1430r^2st - 130r^2t^3 + 286r^2t^2 + 572rs^2t^2 - 1430rs^2t - 520rst^3 + 1144rst^2 + 130rt^4 - 260rt^3 - 130s^2t^3 + 286s^2t^2 + 130st^4 - 260st^3 - 35t^5 + 65t^4),$

$Q_{41}^{[4]_3} = \sum_{i=0}^{\infty}\frac{(h)^i}{i!}y_n^{(i)} - y_n - hy_n' - \frac{h^2y_n''}{2} - \frac{h^3y_n'''}{6} + \frac{h^4y_n^{iv}}{2162160r^3s^3t^3}(8580r^3s^3t^2 - 77220r^3s^3t^3 + 5005r^3s^3t - 1430r^3s^3 + 8580r^3s^2t^3 + 2860r^3s^2t^2 - 4576r^3s^2t + 1144r^3s^2 + 5005r^3st^3 - 4576r^3st^2 + 1742r^3st - 260r^3s - 1430r^3t^3 + 1144r^3t^2 - 260r^3t + 8580r^2s^3t^3 + 2860r^2s^3t^2 - 4576r^2s^3t + 1144r^2s^3 + 2860r^2s^2t^3 - 6864r^2s^2t^2 + 4680r^2s^2t - 1040r^2s^2 - 4576r^2st^3 + 4680r^2st^2 - 1820r^2st + 260r^2s + 1144r^2t^3 - 1040r^2t^2 + 260r^2t + 5005rs^3t^3 - 4576rs^3t^2 + 1742rs^3t - 260rs^3 - 4576rs^2t^3 + 4680rs^2t^2 - 1820rs^2t + 260rs^2 + 1742rst^3 - 1820rst^2 + 645rst - 70rs - 260rt^3 + 260rt^2 - 70rt - 1430s^3t^3 + 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 + 260s^2t - 260st^3 + 260st^2 - 70st) + \sum_{i=0}^{\infty}\frac{r^ih^{i+4}y_n^{(i+4)}}{2162160r^3(r-s)^3(r-t)^3(r-1)^3!}(12870r^4s^2t - 19305r^4s^2t^2 - 2574r^4s^2 + 12870r^4st^2 - 10296r^4st + 2340r^4s - 2574r^4t^2 + 2340r^4t -$

$585r^4 + 15015r^3s^3t^2 - 10010r^3s^3t + 2002r^3s^3 + 15015r^3s^2t^3 + 715r^3s^2t^2 - 4576r^3s^2t + 1222r^3s^2 - 10010r^3st^3 - 4576r^3st^2 + 8528r^3st - 2405r^3s + 2002r^3t^3 + 1222r^3t^2 - 2405r^3t + 735r^3 - 10725r^2s^3t^3 - 7865r^2s^3t^2 + 9152r^2s^3t - 2210r^2s^3 - 7865r^2s^2t^3 + 11154r^2s^2t^2 - 6318r^2s^2t + 1300r^2s^2 + 9152r^2st^3 - 6318r^2st^2 + 715r^2st + 245r^2s - 2210r^2t^3 + 1300r^2t^2 + 245r^2t - 210r^2 + 9295rs^3t^3 - 3718rs^3t^2 - 910rs^3t + 520rs^3 - 3718rs^2t^3 + 936rs^2t^2 + 1300rs^2t - 520rs^2 - 910rst^3 + 1300rst^2 - 705rst + 140rs + 520rt^3 - 520rt^2 + 140rt - 1430s^3t^3 + 1144s^3t^2 - 260s^3t + 1144s^2t^3 - 1040s^2t^2 + 260s^2t - 260st^3 + 260st^2 - 70st) - \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+4)}}{2162160s^3(r-s)^3(s-t)^3(s-1)^3!}(15015r^3s^3t^2 - 10010r^3s^3t + 2002r^3s^3 - 10725r^3s^2t^3 - 7865r^3s^2t^2 + 9152r^3s^2t - 2210r^3s^2 + 9295r^3st^3 - 3718r^3st^2 - 910r^3st + 520r^3s - 1430r^3t^3 + 1144r^3t^2 - 260r^3t - 19305r^2s^4t^2 + 12870r^2s^4t - 2574r^2s^4 + 15015r^2s^3t^3 + 715r^2s^3t^2 - 4576r^2s^3t + 1222r^2s^3 - 7865r^2s^2t^3 + 11154r^2s^2t^2 - 6318r^2s^2t + 1300r^2s^2 - 3718r^2st^3 + 936r^2st^2 + 1300r^2st - 520r^2s + 1144r^2t^3 - 1040r^2t^2 + 260r^2t + 12870rs^4t^2 - 10296rs^4t + 2340rs^4 - 10010rs^3t^3 - 4576rs^3t^2 + 8528rs^3t - 2405rs^3 + 9152rs^2t^3 - 6318rs^2t^2 + 715rs^2t + 245rs^2 - 910rst^3 + 1300rst^2 - 705rst + 140rs - 260rt^3 + 260rt^2 - 70rt - 2574s^4t^2 + 2340s^4t - 585s^4 + 2002s^3t^3 + 1222s^3t^2 - 2405s^3t + 735s^3 - 2210s^2t^3 + 1300s^2t^2 + 245s^2t - 210s^2 + 520st^3 - 520st^2 + 140st) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+4)}}{2162160t^3(r-t)^3(s-t)^3(t-1)^3!}(9295r^3s^3t - 10725r^3s^3t^2 - 1430r^3s^3 + 15015r^3s^2t^3 - 7865r^3s^2t^2 - 3718r^3s^2t + 1144r^3s^2 - 10010r^3st^3 + 9152r^3st^2 - 910r^3st - 260r^3s + 2002r^3t^3 - 2210r^3t^2 + 520r^3t + 15015r^2s^3t^3 - 7865r^2s^3t^2 - 3718r^2s^3t + 1144r^2s^3 - 19305r^2s^2t^4 + 715r^2s^2t^3 + 11154r^2s^2t^2 + 936r^2s^2t - 1040r^2s^2 + 12870r^2st^4 - 4576r^2st^3 - 6318r^2st^2 + 1300r^2st + 260r^2s - 2574r^2t^4 + 1222r^2t^3 + 1300r^2t^2 - 520r^2t - 10010rs^3t^3 + 9152rs^3t^2 - 910rs^3t - 260rs^3 + 12870rs^2t^4 - 4576rs^2t^3 - 6318rs^2t^2 + 1300rs^2t + 260rs^2 - 10296rst^4 + 8528rst^3 + 715rst^2 - 705rst - 70rs + 2340rt^4 - 2405rt^3 + 245rt^2 + 140rt + 2002s^3t^3 - 2210s^3t^2 + 520s^3t - 2574s^2t^4 + 1222s^2t^3 + 1300s^2t^2 - 520s^2t + 2340st^4 - 2405st^3 + 245st^2 + 140st - 585t^4 + 735t^3 - 210t^2) - \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+4)}}{2162160(r-1)^3(s-1)^3(t-1)^3!}(12870r^3s^3t^3 - 30030r^3s^3t^2 + 17875r^3s^3t - 3575r^3s^3 - 30030r^3s^2t^3 + 67210r^3s^2t^2 - 41041r^3s^2t + 8437r^3s^2 + 17875r^3st^3 - 41041r^3st^2$

$$
\begin{aligned}
&+27053r^3st-5915r^3s-3575r^3t^3+8437r^3t^2-5915r^3t+1365r^3-30030r^2s^3t^3+\\
&67210r^2s^3t^2-41041r^2s^3t+8437r^2s^3+67210r^2s^2t^3-148434r^2s^2t^2+93483r^2s^2t-\\
&19799r^2s^2-41041r^2st^3+93483r^2st^2-63011r^2st+14105r^2s+8437r^2t^3-\\
&19799r^2t^2+14105r^2t-3315r^2+17875rs^3t^3-41041rs^3t^2+27053rs^3t-\\
&5915rs^3-41041rs^2t^3+93483rs^2t^2-63011rs^2t+14105rs^2+27053rst^3-\\
&63011rst^2+44520rst-10390rs-5915rt^3+14105rt^2-10390rt+2520r-\\
&3575s^3t^3+8437s^3t^2-5915s^3t+1365s^3+8437s^2t^3-19799s^2t^2+14105s^2t-\\
&3315s^2-5915st^3+14105st^2-10390st+2520s+1365t^3-3315t^2+2520t-\\
&630)-\tfrac{h^5y_n^v}{2162160r^2s^2t^2}(8580r^2s^2t^2-4290r^2s^2t+715r^2s^2-4290r^2st^2+2860r^2st-\\
&572r^2s+715r^2t^2-572r^2t+130r^2-4290rs^2t^2+2860rs^2t-572rs^2+2860rst^2-\\
&2288rst+520rs-572rt^2+520rt-130r+715s^2t^2-572s^2t+130s^2-572st^2+\\
&520st-130s+130t^2-130t+35)+\textstyle\sum_{i=0}^{\infty}\tfrac{r^ih^{i+5}y_n^{(i+5)}}{2162160r^2(r-s)^2(r-t)^2(r-1)^2i!}(65r+\\
&130s+130t-715s^2t^2-260rs-260rt-520st+286rs^2+286rt^2+572st^2+\\
&572s^2t-130s^2-130t^2-1430rst^2-1430rs^2t+2145rs^2t^2+1144rst-35)+\\
&\textstyle\sum_{i=0}^{\infty}\tfrac{s^ih^{i+5}y_n^{(i+5)}}{2162160s^2(r-s)^2(s-t)^2(s-1)^2i!}(130r+65s+130t-715r^2t^2-260rs-520rt-\\
&260st+286r^2s+572rt^2+572r^2t+286st^2-130r^2-130t^2-1430rst^2-\\
&1430r^2st+2145r^2st^2+1144rst-35)+\textstyle\sum_{i=0}^{\infty}\tfrac{t^ih^{i+5}y_n^{(i+5)}}{2162160t^2(r-t)^2(s-t)^2(t-1)^2i!}(130r+\\
&130s+65t-715r^2s^2-520rs-260rt-260st+572rs^2+572r^2s+286r^2t+\\
&286s^2t-130r^2-130s^2-1430rs^2t-1430r^2st+2145r^2s^2t+1144rst-35)+\\
&\textstyle\sum_{i=0}^{\infty}\tfrac{h^{i+5}y_n^{(i+5)}}{1081080(r-1)^2(s-1)^2(t-1)^2i!}(1716r^2s^2t^2-1287r^2s^2t+286r^2s^2-1287r^2st^2+\\
&1144r^2st-286r^2s+286r^2t^2-286r^2t+78r^2-1287rs^2t^2+1144rs^2t-286rs^2+\\
&1144rst^2-1144rst+312rs-286rt^2+312rt-91r+286s^2t^2-286s^2t+78s^2-\\
&286st^2+312st-91s+78t^2-91t+28).
\end{aligned}
$$

Like terms of h^j and $y^{(j)}$ are compared to produce $\bar{D}_0=\bar{D}_1=...=\bar{D}_{13}=0$ and $\bar{D}_{14}\neq 0$. Thus, it is found that the order of the main block is $[10,10,10,10]^T$ along with vector of error constants

$$\bar{D}_{14} = \begin{bmatrix} \bar{D}_{14_1} \; \bar{D}_{14_2} \; \bar{D}_{14_3} \; \bar{D}_{14_4} \end{bmatrix}^T$$

where

$\bar{D}_{14_1} = \frac{r^8}{7846046208000}(20r^6 - 70r^5s - 70r^5t - 70r^5 + 65r^4s^2 + 260r^4st + 260r^4s + 65r^4t^2 + 260r^4t + 65r^4 - 260r^3s^2t - 260r^3s^2 - 260r^3st^2 - 1040r^3st - 260r^3s - 260r^3t^2 - 260r^3t + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 + 1144r^2st^2 + 1144r^2st + 286r^2t^2 - 1430rs^2t^2 - 1430rs^2t - 1430rst^2 + 2145s^2t^2)$,

$\bar{D}_{14_2} = \frac{s^8}{7846046208000}(65r^2s^4 - 260r^2s^3t - 260r^2s^3 + 286r^2s^2t^2 + 1144r^2s^2t + 286r^2s^2 - 1430r^2st^2 - 1430r^2st + 2145r^2t^2 - 70rs^5 + 260rs^4t + 260rs^4 - 260rs^3t^2 - 1040rs^3t - 260rs^3 + 1144rs^2t^2 + 1144rs^2t - 1430rst^2 + 20s^6 - 70s^5t - 70s^5 + 65s^4t^2 + 260s^4t + 65s^4 - 260s^3t^2 - 260s^3t + 286s^2t^2)$,

$\bar{D}_{14_3} = \frac{t^8}{7846046208000}(286r^2s^2t^2 - 1430r^2s^2t + 2145r^2s^2 - 260r^2st^3 + 1144r^2st^2 - 1430r^2st + 65r^2t^4 - 260r^2t^3 + 286r^2t^2 - 260rs^2t^3 + 1144rs^2t^2 - 1430rs^2t + 260rst^4 - 1040rst^3 + 1144rst^2 - 70rt^5 + 260rt^4 - 260rt^3 + 65s^2t^4 - 260s^2t^3 + 286s^2t^2 - 70st^5 + 260st^4 - 260st^3 + 20t^6 - 70t^5 + 65t^4)$,

$\bar{D}_{14_4} = \frac{1}{7846046208000}(2145r^2s^2t^2 - 1430r^2s^2t + 286r^2s^2 - 1430r^2st^2 + 1144r^2st - 260r^2s + 286r^2t^2 - 260r^2t + 65r^2 - 1430rs^2t^2 + 1144rs^2t - 260rs^2 + 1144rst^2 - 1040rst + 260rs - 260rt^2 + 260rt - 70r + 286s^2t^2 - 260s^2t + 65s^2 - 260st^2 + 260st - 70s + 65t^2 - 70t + 20)$.

Similarly, to find the order of the first derivative block (5.20) the linear difference operator ∇ is produced and each function of $Y'^{[4]_3}_{n+1}, F^{[4]_3}_{n+1}$ and $G^{[4]_3}_{n+1}$ are expanded in Taylor series about x_n after this setting to $\mathbf{0}$ yields

$$\left[Q_{11}^{'[4]_3}\ Q_{21}^{'[4]_3}\ Q_{31}^{'[4]_3}\ Q_{41}^{'[4]_3}\right]^T = \left[0\ 0\ 0\ 0\right]^T$$

where

$$Q_{11}^{'[4]_3} = \sum_{i=0}^{\infty} \frac{(rh)^i}{i!} y_n^{(i)} - y_n' - hry_n'' - \frac{h^2 r^2 y_n'''}{2} + \frac{h^3 r^3 y_n^{iv}}{27720 s^3 t^3}(24r^6s^2t - 7r^7s - 7r^7t - 7r^7st + 24r^6s^2 + 24r^6st^2 + 59r^6st + 24r^6s + 24r^6t^2 + 24r^6t - 22r^5s^3t - 22r^5s^3 - 88r^5s^2t^2 - 152r^5s^2t - 88r^5s^2 - 22r^5st^3 - 152r^5st^2 - 152r^5st - 22r^5s - 22r^5t^3 - 88r^5t^2 - 22r^5t + 88r^4s^3t^2 + 132r^4s^3t + 88r^4s^3 + 88r^4s^2t^3 + 352r^4s^2t^2 + 352r^4s^2t + 88r^4s^2 + 132r^4st^3 + 352r^4st^2 + 132r^4st + 88r^4t^3 + 88r^4t^2 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 - 308r^3s^2t^3 - 440r^3s^2t^2 - 308r^3s^2t - 308r^3st^3 - 308r^3st^2 - 99r^3t^3 + 297r^2s^3t^3 + 132r^2s^3t^2 + 297r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 297r^2st^3 + 528rs^3t^3 + 528rs^3t^2 + 528rs^2t^3 - 3696s^3t^3) - \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} r^3}{27720(r-s)^3(r-t)^3(r-1)^3 i!}(84r^9 - 315r^8s - 315r^8t - 315r^8 + 390r^7s^2 + 1210r^7st + 1210r^7s + 390r^7t^2 + 1210r^7t + 390r^7 - 154r^6s^3 - 1536r^6s^2t - 1536r^6s^2 - 1536r^6st^2 - 4793r^6st - 1536r^6s - 154r^6t^3 - 1536r^6t^2 - 1536r^6t - 154r^6 + 616r^5s^3t + 616r^5s^3 + 2002r^5s^2t^2 + 6290r^5s^2t + 2002r^5s^2 + 616r^5st^3 + 6290r^5st^2 + 6290r^5st + 616r^5s + 616r^5t^3 + 2002r^5t^2 + 616r^5t - 814r^4s^3t^2 - 2574r^4s^3t - 814r^4s^3 - 814r^4s^2t^3 - 8569r^4s^2t^2 - 8569r^4s^2t - 814r^4s^2 - 2574r^4st^3 - 8569r^4st^2 - 2574r^4st - 814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 + 3575r^3s^2t^3 + 12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 - 5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 + 2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} r^7}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3 i!}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t + 70r^5s^3 + 21r^5s^2t + 21r^5s^2 - 48r^5st^2 - 64r^5st - 48r^5s + 24r^5t^2 + 24r^5t - 54r^4s^4 - 208r^4s^3t - 208r^4s^3 + 116r^4s^2t^2 + 70r^4s^2t + 116r^4s^2 + 44r^4st^3 + 106r^4st^2 + 106r^4st + 44r^4s - 22r^4t^3 - 88r^4t^2 - 22r^4t + 198r^3s^4t + 198r^3s^4 + 88r^3s^3t^2 + 660r^3s^3t + 88r^3s^3 - 176r^3s^2t^3 -$$

$506r^3s^2t^2 - 506r^3s^2t - 176r^3s^2 - 66r^3st^3 + 88r^3st^2 - 66r^3st + 88r^3t^3 + 88r^3t^2 -$

$198r^2s^4t^2 - 792r^2s^4t - 198r^2s^4 + 154r^2s^3t^3 - 275r^2s^3t^2 - 275r^2s^3t + 154r^2s^3 +$

$649r^2s^2t^3 + 781r^2s^2t^2 + 649r^2s^2t - 275r^2st^3 - 275r^2st^2 - 99r^2t^3 + 891rs^4t^2 +$

$891rs^4t - 693rs^3t^3 - 66rs^3t^2 - 693rs^3t - 462rs^2t^3 - 462rs^2t^2 + 594rst^3 -$

$1188s^4t^2 + 924s^3t^3 + 924s^3t^2 - 660s^2t^3) - \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+4)} r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(7r^6s -$

$14r^6st + 21r^6t^2 - 14r^6t + 48r^5s^2t - 24r^5s^2 - 21r^5st^2 + 64r^5st - 24r^5s - 70r^5t^3 -$

$21r^5t^2 + 48r^5t - 44r^4s^3t + 22r^4s^3 - 116r^4s^2t^2 - 106r^4s^2t + 88r^4s^2 + 208r^4st^3 -$

$70r^4st^2 - 106r^4st + 22r^4s + 54r^4t^4 + 208r^4t^3 - 116r^4t^2 - 44r^4t + 176r^3s^3t^2 +$

$66r^3s^3t - 88r^3s^3 - 88r^3s^2t^3 + 506r^3s^2t^2 - 88r^3s^2t - 88r^3s^2 - 198r^3st^4 - 660r^3st^3 +$

$506r^3st^2 + 66r^3st - 198r^3t^4 - 88r^3t^3 + 176r^3t^2 - 154r^2s^3t^3 - 649r^2s^3t^2 + 275r^2s^3t +$

$99r^2s^3 + 198r^2s^2t^4 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 792r^2st^4 + 275r^2st^3 -$

$649r^2st^2 + 198r^2t^4 - 154r^2t^3 + 693rs^3t^3 + 462rs^3t^2 - 594rs^3t - 891rs^2t^4 +$

$66rs^2t^3 + 462rs^2t^2 - 891rst^4 + 693rst^3 - 924s^3t^3 + 660s^3t^2 + 1188s^2t^4 - 924s^2t^3) +$

$\sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)} r^7}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(7r^6st - 14r^6s - 14r^6t + 21r^6 - 24r^5s^2t + 48r^5s^2 -$

$24r^5st^2 + 64r^5st - 21r^5s + 48r^5t^2 - 21r^5t - 70r^5 + 22r^4s^3t - 44r^4s^3 + 88r^4s^2t^2 -$

$106r^4s^2t - 116r^4s^2 + 22r^4st^3 - 106r^4st^2 - 70r^4st + 208r^4s - 44r^4t^3 - 116r^4t^2 +$

$208r^4t + 54r^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 - 88r^3s^2t^3 - 88r^3s^2t^2 + 506r^3s^2t -$

$88r^3s^2 + 66r^3st^3 + 506r^3st^2 - 660r^3st - 198r^3s + 176r^3t^3 - 88r^3t^2 - 198r^3t +$

$99r^2s^3t^3 + 275r^2s^3t^2 - 649r^2s^3t - 154r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t +$

$198r^2s^2 - 649r^2st^3 + 275r^2st^2 + 792r^2st - 154r^2t^3 + 198r^2t^2 - 594rs^3t^3 + 462rs^3t^2 +$

$693rs^3t + 462rs^2t^3 + 66rs^2t^2 - 891rs^2t + 693rst^3 - 891rst^2 + 660s^3t^3 - 924s^3t^2 -$

$924s^2t^3 + 1188s^2t^2) - \frac{h^4 r^4 y_n^v}{55440s^2t^2}(7r^6 - 24r^5s - 24r^5t - 24r^5 + 22r^4s^2 + 88r^4st +$

$88r^4s + 22r^4t^2 + 88r^4t + 22r^4 - 88r^3s^2t - 88r^3s^2 - 88r^3st^2 - 352r^3st - 88r^3s -$

$88r^3t^2 - 88r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 -$

$528rs^2t^2 - 528rs^2t - 528rst^2 + 924s^2t^2) + \sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+5)} r^4}{55440(r-s)^2(r-t)^2(r-1)^2 i!}(14r^6 -$

$42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 -$

$110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 +$

$396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 +$

$462s^2t^2) + \sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+5)} r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2 i!}(88r^2t^2 - 22r^3t^2 + 44r^2s - 44r^3s + 12r^4s -$

$99rt^2 + 88r^2t - 88r^3t + 24r^4t + 264st^2 - 22r^3 + 24r^4 - 7r^5 - 198rst^2 + 176r^2st -$
$44r^3st + 44r^2st^2 - 198rst) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+5)} r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2 i!}(88r^2s^2 - 22r^3s^2 - 99rs^2 +$
$88r^2s - 88r^3s + 24r^4s + 44r^2t - 44r^3t + 12r^4t + 264s^2t - 22r^3 + 24r^4 - 7r^5 -$
$198rs^2t + 176r^2st - 44r^3st + 44r^2s^2t - 198rst) + \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+5)} r^7}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(24r^4s -$
$7r^5 + 24r^4t + 12r^4 - 22r^3s^2 - 88r^3st - 44r^3s - 22r^3t^2 - 44r^3t + 88r^2s^2t + 44r^2s^2 +$
$88r^2st^2 + 176r^2st + 44r^2t^2 - 99rs^2t^2 - 198rs^2t - 198rst^2 + 264s^2t^2),$

$Q_{21}^{'[4]_3} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i)} - y_n' - hsy_n'' - \frac{h^2s^2y_n'''}{2} + \frac{h^3s^3y_n^{iv}}{27720r^3t^3}(88r^3s^4t^2 - 22r^3s^5 -$
$22r^3s^5t + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 +$
$297r^3s^2t^3 + 132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t +$
$24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 + 352r^2s^4t^2 + 352r^2s^4t +$
$88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 -$
$7rs^7t - 7rs^7 + 24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t -$
$22rs^5 + 132rs^4t^3 + 352rs^4t^2 + 132rs^4t - 308rs^3t^3 - 308rs^3t^2 + 297rs^2t^3 -$
$7s^7t + 24s^6t^2 + 24s^6t - 22s^5t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3) -$
$\sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t +$
$198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 -$
$660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 +$
$66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 -$
$70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 -$
$781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 +$
$64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t +$
$275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 + 88s^4t^2 + 22s^4t -$
$88s^3t^3 - 88s^3t^2 + 99s^2t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} s^3}{27720(r-s)^3(s-t)^3(s-1)^3 i!}(154r^3s^6 - 616r^3s^5t -$
$616r^3s^5 + 814r^3s^4t^2 + 2574r^3s^4t + 814r^3s^4 - 330r^3s^3t^3 - 3575r^3s^3t^2 - 3575r^3s^3t -$
$330r^3s^3 + 1485r^3s^2t^3 + 5280r^3s^2t^2 + 1485r^3s^2t - 2244r^3st^3 - 2244r^3st^2 + 924r^3t^3 -$
$390r^2s^7 + 1536r^2s^6t + 1536r^2s^6 - 2002r^2s^5t^2 - 6290r^2s^5t - 2002r^2s^5 + 814r^2s^4t^3 +$
$8569r^2s^4t^2 + 8569r^2s^4t + 814r^2s^4 - 3575r^2s^3t^3 - 12320r^2s^3t^2 - 3575r^2s^3t +$
$5280r^2s^2t^3 + 5280r^2s^2t^2 - 2244r^2st^3 + 315rs^8 - 1210rs^7t - 1210rs^7 + 1536rs^6t^2 +$

$4793rs^6t + 1536rs^6 - 616rs^5t^3 - 6290rs^5t^2 - 6290rs^5t - 616rs^5 + 2574rs^4t^3 + 8569rs^4t^2 + 2574rs^4t - 3575rs^3t^3 - 3575rs^3t^2 + 1485rs^2t^3 - 84s^9 + 315s^8t + 315s^8 - 390s^7t^2 - 1210s^7t - 390s^7 + 154s^6t^3 + 1536s^6t^2 + 1536s^6t + 154s^6 - 616s^5t^3 - 2002s^5t^2 - 616s^5t + 814s^4t^3 + 814s^4t^2 - 330s^3t^3) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+4)}s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3i!}(44r^3s^4t - 22r^3s^4 - 176r^3s^3t^2 - 66r^3s^3t + 88r^3s^3 + 154r^3s^2t^3 + 649r^3s^2t^2 - 275r^3s^2t - 99r^3s^2 - 693r^3st^3 - 462r^3st^2 + 594r^3st + 924r^3t^3 - 660r^3t^2 - 48r^2s^5t + 24r^2s^5 + 116r^2s^4t^2 + 106r^2s^4t - 88r^2s^4 + 88r^2s^3t^3 - 506r^2s^3t^2 + 88r^2s^3t + 88r^2s^3 - 198r^2s^2t^4 - 275r^2s^2t^3 + 781r^2s^2t^2 - 275r^2s^2t + 891r^2st^4 - 66r^2st^3 - 462r^2st^2 - 1188r^2t^4 + 924r^2t^3 + 14rs^6t - 7rs^6 + 21rs^5t^2 - 64rs^5t + 24rs^5 - 208rs^4t^3 + 70rs^4t^2 + 106rs^4t - 22rs^4 + 198rs^3t^4 + 660rs^3t^3 - 506rs^3t^2 - 66rs^3t - 792rs^2t^4 - 275rs^2t^3 + 649rs^2t^2 + 891rst^4 - 693rst^3 - 21s^6t^2 + 14s^6t + 70s^5t^3 + 21s^5t^2 - 48s^5t - 54s^4t^4 - 208s^4t^3 + 116s^4t^2 + 44s^4t + 198s^3t^4 + 88s^3t^3 - 176s^3t^2 - 198s^2t^4 + 154s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+4)}s^7}{27720(r-1)^3(s-1)^3(t-1)^3i!}(22r^3s^4t - 44r^3s^4 - 88r^3s^3t^2 + 66r^3s^3t + 176r^3s^3 + 99r^3s^2t^3 + 275r^3s^2t^2 - 649r^3s^2t - 154r^3s^2 - 594r^3st^3 + 462r^3st^2 + 693r^3st + 660r^3t^3 - 924r^3t^2 - 24r^2s^5t + 48r^2s^5 + 88r^2s^4t^2 - 106r^2s^4t - 116r^2s^4 - 88r^2s^3t^3 - 88r^2s^3t^2 + 506r^2s^3t - 88r^2s^3 + 275r^2s^2t^3 - 781r^2s^2t^2 + 275r^2s^2t + 198r^2s^2 + 462r^2st^3 + 66r^2st^2 - 891r^2st - 924r^2t^3 + 1188r^2t^2 + 7rs^6t - 14rs^6 - 24rs^5t^2 + 64rs^5t - 21rs^5 + 22rs^4t^3 - 106rs^4t^2 - 70rs^4t + 208rs^4 + 66rs^3t^3 + 506rs^3t^2 - 660rs^3t - 198rs^3 - 649rs^2t^3 + 275rs^2t^2 + 792rs^2t + 693rst^3 - 891rst^2 - 14s^6t + 21s^6 + 48s^5t^2 - 21s^5t - 70s^5 - 44s^4t^3 - 116s^4t^2 + 208s^4t + 54s^4 + 176s^3t^3 - 88s^3t^2 - 198s^3t - 154s^2t^3 + 198s^2t^2) - \frac{h^4s^4y_n^v}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 - 24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 + 22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+4}y_n^{(i+5)}s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2i!}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst) + \sum_{i=0}^{\infty} \frac{s^ih^{i+4}y_n^{(i+5)}s^4}{55440(r-s)^2(s-t)^2(s-1)^2i!}(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14$

$s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2) + \sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+5)} s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2 i!}(88r^2s^2 - 22r^2s^3 + 88rs^2 - 99r^2s - 88rs^3 + 24rs^4 + 264r^2t + 44s^2t - 44s^3t + 12s^4t - 22s^3 + 24s^4 - 7s^5 + 176rs^2t - 198r^2st - 44rs^3t + 44r^2s^2t - 198rst) + \sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+5)} s^7}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(88r^2s^2t - 22r^2s^3 + 44r^2s^2 - 99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 + 88rs^2t^2 + 176rs^2t - 198rst^2 - 7s^5 + 24s^4t + 12s^4 - 22s^3t^2 - 44s^3t + 44s^2t^2),$

$Q_{31}^{'[4]_3} = \sum_{i=0}^{\infty} \frac{(th)^i}{i!} y_n^{(i)} - y_n' - hty_n'' - \frac{h^2t^2y_n'''}{2} + \frac{h^3t^3y_n^{iv}}{27720r^3s^3}(297r^3s^3t^2 - 99r^3s^3t^3 + 528r^3s^3t - 3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 + 132r^3st^4 - 308r^3st^3 + 297r^3st^2 - 22r^3t^5 + 88r^3t^4 - 99r^3t^3 + 88r^2s^3t^4 - 308r^2s^3t^3 + 132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 - 152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 - 22rs^3t^5 + 132rs^3t^4 - 308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 - 152rst^5 + 132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 - 88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5) - \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)} t^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(198r^4s^2t^2 - 891r^4s^2t + 1188r^4s^2 - 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 - 198r^4t^3 + 198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t - 924r^3s^2 + 208r^3st^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 - 154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 + 462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 - 781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 - 649r^2st^2 + 21r^2t^6 - 21r^2t^5 - 116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594rs^3t + 48rs^2t^5 - 106rs^2t^4 - 88rs^2t^3 + 275rs^2t^2 - 14rst^6 + 64rst^5 - 106rst^4 + 66rst^3 - 14rt^6 + 48rt^5 - 44rt^4 + 22s^3t^4 - 88s^3t^3 + 99s^3t^2 - 24s^2t^5 + 88s^2t^4 - 88s^2t^3 + 7st^6 - 24st^5 + 22st^4) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)} t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}(154r^3s^3t^2 - 693r^3s^3t + 924r^3s^3 - 176r^3s^2t^3 + 649r^3s^2t^2 - 462r^3s^2t - 660r^3s^2 + 44r^3st^4 - 66r^3st^3 - 275r^3st^2 + 594r^3st - 22r^3t^4 + 88r^3t^3 - 99r^3t^2 - 198r^2s^4t^2 + 891r^2s^4t - 1188r^2s^4 + 88r^2s^3t^3 - 275r^2s^3t^2 - 66r^2s^3t + 924r^2s^3 + 116r^2s^2t^4 - 506r^2s^2t^3 + 781r^2s^2t^2 - 462r^2s^2t - 48r^2st^5 + 106r^2st^4 + 88r^2st^3 - 275r^2st^2 + 24r^2t^5 - 88r^2t^4 + 88r^2t^3 + 198rs^4t^3 - 792rs^4t^2 + 891rs^4t - 208rs^3t^4 + 660rs^3t^3 - 275rs^3t^2 - 693rs^3t + 21rs^2t^5 + 70rs^2t^4 - 506rs^2t^3$

$$+649rs^2t^2+14rst^6-64rst^5+106rst^4-66rst^3-7rt^6+24rt^5-$$
$$22rt^4-54s^4t^4+198s^4t^3-198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+$$
$$154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-176s^2t^3+14st^6-48st^5+44st^4)+$$
$$\textstyle\sum_{i=0}^{\infty}\frac{t^ih^{i+3}y_n^{(i+4)}t^3}{27720(r-t)^3(s-t)^3(t-1)^3i!}(1485r^3s^3t^2-330r^3s^3t^3-2244r^3s^3t+924r^3s^3+$$
$$814r^3s^2t^4-3575r^3s^2t^3+5280r^3s^2t^2-2244r^3s^2t-616r^3st^5+2574r^3st^4-$$
$$3575r^3st^3+1485r^3st^2+154r^3t^6-616r^3t^5+814r^3t^4-330r^3t^3+814r^2s^3t^4-$$
$$3575r^2s^3t^3+5280r^2s^3t^2-2244r^2s^3t-2002r^2s^2t^5+8569r^2s^2t^4-12320r^2s^2t^3+$$
$$5280r^2s^2t^2+1536r^2st^6-6290r^2st^5+8569r^2st^4-3575r^2st^3-390r^2t^7+1536r^2t^6-$$
$$2002r^2t^5+814r^2t^4-616rs^3t^5+2574rs^3t^4-3575rs^3t^3+1485rs^3t^2+1536rs^2t^6-$$
$$6290rs^2t^5+8569rs^2t^4-3575rs^2t^3-1210rst^7+4793rst^6-6290rst^5+2574rst^4+$$
$$315rt^8-1210rt^7+1536rt^6-616rt^5+154s^3t^6-616s^3t^5+814s^3t^4-330s^3t^3-$$
$$390s^2t^7+1536s^2t^6-2002s^2t^5+814s^2t^4+315st^8-1210st^7+1536st^6-616st^5-$$
$$84t^9+315t^8-390t^7+154t^6)+\textstyle\sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+4)}t^7}{27720(r-1)^3(s-1)^3(t-1)^3i!}(99r^3s^3t^2-594r^3s^3t+$$
$$660r^3s^3-88r^3s^2t^3+275r^3s^2t^2+462r^3s^2t-924r^3s^2+22r^3st^4+66r^3st^3-$$
$$649r^3st^2+693r^3st-44r^3t^4+176r^3t^3-154r^3t^2-88r^2s^3t^3+275r^2s^3t^2+462r^2s^3t-$$
$$924r^2s^3+88r^2s^2t^4-88r^2s^2t^3-781r^2s^2t^2+66r^2s^2t+1188r^2s^2-24r^2st^5-$$
$$106r^2st^4+506r^2st^3+275r^2st^2-891r^2st+48r^2t^5-116r^2t^4-88r^2t^3+198r^2t^2+$$
$$22rs^3t^4+66rs^3t^3-649rs^3t^2+693rs^3t-24rs^2t^5-106rs^2t^4+506rs^2t^3+275rs^2t^2-$$
$$891rs^2t+7rst^6+64rst^5-70rst^4-660rst^3+792rst^2-14rt^6-21rt^5+208rt^4-$$
$$198rt^3-44s^3t^4+176s^3t^3-154s^3t^2+48s^2t^5-116s^2t^4-88s^2t^3+198s^2t^2-$$
$$14st^6-21st^5+208st^4-198st^3+21t^6-70t^5+54t^4)-\frac{h^4t^4y_n^{v}}{55440r^2s^2}(99r^2s^2t^2-$$
$$528r^2s^2t+924r^2s^2-88r^2st^3+396r^2st^2-528r^2st+22r^2t^4-88r^2t^3+99r^2t^2-$$
$$88rs^2t^3+396rs^2t^2-528rs^2t+88rst^4-352rst^3+396rst^2-24rt^5+88rt^4-$$
$$88rt^3+22s^2t^4-88s^2t^3+99s^2t^2-24st^5+88st^4-88st^3+7t^6-24t^5+22t^4)+$$
$$\textstyle\sum_{i=0}^{\infty}\frac{r^ih^{i+4}y_n^{(i+5)}t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2i!}(88s^2t^2-22s^2t^3+264rs^2+44rt^2-44rt^3+12rt^4+$$
$$88st^2-99s^2t-88st^3+24st^4-22t^3+24t^4-7t^5+176rst^2-198rs^2t-44rst^3+$$
$$44rs^2t^2-198rst)+\textstyle\sum_{i=0}^{\infty}\frac{s^ih^{i+4}y_n^{(i+5)}t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2i!}(88r^2t^2-22r^2t^3+264r^2s+88rt^2-$$
$$99r^2t-88rt^3+24rt^4+44st^2-44st^3+12st^4-22t^3+24t^4-7t^5+176rst^2-$$
$$198r^2st-44rst^3+44r^2st^2-198rst)+\textstyle\sum_{i=0}^{\infty}\frac{t^ih^{i+4}y_n^{(i+5)}t^4}{55440(r-t)^2(s-t)^2(t-1)^2i!}(99r^2s^2t^2-396$$

$r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4) + \sum_{i=0}^{\infty} \frac{h^{i+4}y_n^{(i+5)}t^7}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(264r^2s^2 - 99r^2s^2t + 88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4)$,

$Q_{41}^{'[4]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!} y_n^{(i)} - y_n' - hy_n'' - \frac{h^2 y_n'''}{2} + \frac{h^3 y_n^{iv}}{27720r^3s^3t^3}(528r^3s^3t^2 - 3696r^3s^3t^3 + 297r^3s^3t - 99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2 - 308r^3s^2t + 88r^3s^2 + 297r^3st^3 - 308r^3st^2 + 132r^3st - 22r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t + 528r^2s^3t^3 + 132r^2s^3t^2 - 308r^2s^3t + 88r^2s^3 + 132r^2s^2t^3 - 440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 - 152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3t^2 + 132rs^3t - 22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs - 22rt^3 + 24rt^2 - 7rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st) + \sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+4)}}{27720r^3(r-s)^3(r-t)^3(r-1)^3 i!}(891r^4s^2t - 1188r^4s^2t^2 - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 + 198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 - 275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 - 660r^2s^3t^3 - 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 + 21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs + 44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st) - \sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+4)}}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}(924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 - 660r^3s^2t^3 - 462r^3s^2t^2 + 649r^3s^2t - 176r^3s^2 + 594r^3st^3 - 275r^3st^2 - 66r^3st + 44r^3s - 99r^3t^3 + 88r^3t^2 - 22r^3t - 1188r^2s^4t^2 + 891r^2s^4t - 198r^2s^4 + 924r^2s^3t^3 - 66r^2s^3t^2 - 275r^2s^3t + 88r^2s^3 - 462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 - 275r^2st^3 + 88r^2st^2 + 106r^2st - 48r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 891rs^4t^2 - 792rs^4t + 198rs^4 - 693rs^3t^3 - 275rs^3t^2 + 660rs^3t - 208rs^3 + 649rs^2t^3 - 506rs^2t^2 + 70rs^2t + 21rs^2 - 66rst^3 + 106rst^2 - 64rst + 14rs - 22rt^3 +$

$$24rt^2 - 7rt - 198s^4t^2 + 198s^4t - 54s^4 + 154s^3t^3 + 88s^3t^2 - 208s^3t +$$

$$70s^3 - 176s^2t^3 + 116s^2t^2 + 21s^2t - 21s^2 + 44st^3 - 48st^2 + 14st) -$$

$$\sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+4)}}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(660r^3s^3t^2 - 594r^3s^3t + 99r^3s^3 - 924r^3s^2t^3 +$$

$$462r^3s^2t^2 + 275r^3s^2t - 88r^3s^2 + 693r^3st^3 - 649r^3st^2 + 66r^3st + 22r^3s - 154r^3t^3 +$$

$$176r^3t^2 - 44r^3t - 924r^2s^3t^3 + 462r^2s^3t^2 + 275r^2s^3t - 88r^2s^3 + 1188r^2s^2t^4 +$$

$$66r^2s^2t^3 - 781r^2s^2t^2 - 88r^2s^2t + 88r^2s^2 - 891r^2st^4 + 275r^2st^3 + 506r^2st^2 -$$

$$106r^2st - 24r^2s + 198r^2t^4 - 88r^2t^3 - 116r^2t^2 + 48r^2t + 693rs^3t^3 - 649rs^3t^2 +$$

$$66rs^3t + 22rs^3 - 891rs^2t^4 + 275rs^2t^3 + 506rs^2t^2 - 106rs^2t - 24rs^2 + 792rst^4 -$$

$$660rst^3 - 70rst^2 + 64rst + 7rs - 198rt^4 + 208rt^3 - 21rt^2 - 14rt - 154s^3t^3 +$$

$$176s^3t^2 - 44s^3t + 198s^2t^4 - 88s^2t^3 - 116s^2t^2 + 48s^2t - 198st^4 + 208st^3 - 21st^2 -$$

$$14st + 54t^4 - 70t^3 + 21t^2) - \sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+4)}}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(924r^3s^3t^3 - 2244r^3s^3t^2 +$$

$$1485r^3s^3t - 330r^3s^3 - 2244r^3s^2t^3 + 5280r^3s^2t^2 - 3575r^3s^2t + 814r^3s^2 + 1485r^3st^3 -$$

$$3575r^3st^2 + 2574r^3st - 616r^3s - 330r^3t^3 + 814r^3t^2 - 616r^3t + 154r^3 - 2244r^2s^3t^3 +$$

$$5280r^2s^3t^2 - 3575r^2s^3t + 814r^2s^3 + 5280r^2s^2t^3 - 12320r^2s^2t^2 + 8569r^2s^2t -$$

$$2002r^2s^2 - 3575r^2st^3 + 8569r^2st^2 - 6290r^2st + 1536r^2s + 814r^2t^3 - 2002r^2t^2 +$$

$$1536r^2t - 390r^2 + 1485rs^3t^3 - 3575rs^3t^2 + 2574rs^3t - 616rs^3 - 3575rs^2t^3 +$$

$$8569rs^2t^2 - 6290rs^2t + 1536rs^2 + 2574rst^3 - 6290rst^2 + 4793rst - 1210rs -$$

$$616rt^3 + 1536rt^2 - 1210rt + 315r - 330s^3t^3 + 814s^3t^2 - 616s^3t + 154s^3 + 814s^2t^3 -$$

$$2002s^2t^2 + 1536s^2t - 390s^2 - 616st^3 + 1536st^2 - 1210st + 315s + 154t^3 - 390t^2 +$$

$$315t - 84) - \frac{h^4 y_n^{v}}{55440r^2s^2t^2}(924r^2s^2t^2 - 528r^2s^2t + 99r^2s^2 - 528r^2st^2 + 396r^2st - 88r^2s +$$

$$99r^2t^2 - 88r^2t + 22r^2 - 528rs^2t^2 + 396rs^2t - 88rs^2 + 396rst^2 - 352rst + 88rs -$$

$$88rt^2 + 88rt - 24r + 99s^2t^2 - 88s^2t + 22s^2 - 88st^2 + 88st - 24s + 22t^2 - 24t + 7) +$$

$$\sum_{i=0}^{\infty} \frac{r^i h^{i+4} y_n^{(i+5)}}{55440r^2(r-s)^2(r-t)^2(r-1)^2 i!}(12r + 24s + 24t - 99s^2t^2 - 44rs - 44rt - 88st + 44rs^2 +$$

$$44rt^2 + 88st^2 + 88s^2t - 22s^2 - 22t^2 - 198rst^2 - 198rs^2t + 264rs^2t^2 + 176rst - 7) +$$

$$\sum_{i=0}^{\infty} \frac{s^i h^{i+4} y_n^{(i+5)}}{55440s^2(r-s)^2(s-t)^2(s-1)^2 i!}(24r + 12s + 24t - 99r^2t^2 - 44rs - 88rt - 44st + 44r^2s +$$

$$88rt^2 + 88r^2t + 44st^2 - 22r^2 - 22t^2 - 198rst^2 - 198r^2st + 264r^2st^2 + 176rst - 7) +$$

$$\sum_{i=0}^{\infty} \frac{t^i h^{i+4} y_n^{(i+5)}}{55440t^2(r-t)^2(s-t)^2(t-1)^2 i!}(24r + 24s + 12t - 99r^2s^2 - 88rs - 44rt - 44st + 88rs^2 +$$

$$88r^2s + 44r^2t + 44s^2t - 22r^2 - 22s^2 - 198rs^2t - 198r^2st + 264r^2s^2t + 176rst - 7) +$$

$$\sum_{i=0}^{\infty} \frac{h^{i+4} y_n^{(i+5)}}{55440(r-1)^2(s-1)^2(t-1)^2 i!}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st -$$

$110r^2s+99r^2t^2-110r^2t+33r^2-396rs^2t^2+396rs^2t-110rs^2+396rst^2-440rst+132rs-110rt^2+132rt-42r+99s^2t^2-110s^2t+33s^2-110st^2+132st-42s+33t^2-42t+14)$.

Likewise, the coefficients of h^j and $y^{(j)}$ are then compared which yields $\bar{D}'_0=\bar{D}'_1=...=\bar{D}'_{13}=0$ and $\bar{D}'_{14}\neq 0$. Consequently, the order of first derivative of the main block is $[10,10,10,10]^T$ with vector of error constants
$\bar{D}'_{14}=[\bar{D}'_{14_1},\bar{D}'_{14_2},\bar{D}'_{14_3},\bar{D}'_{14_4}]^T$ where

$\bar{D}'_{14_1}=\frac{r^7}{1307674368000}(28r^6-91r^5s-91r^5t-91r^5+78r^4s^2+312r^4st+312r^4s+78r^4t^2+312r^4t+78r^4-286r^3s^2t-286r^3s^2-286r^3st^2-1144r^3st-286r^3s-286r^3t^2-286r^3t+286r^2s^2t^2+1144r^2s^2t+286r^2s^2+1144r^2st^2+1144r^2st+286r^2t^2-1287rs^2t^2-1287rs^2t-1287rst^2+1716s^2t^2)$,

$\bar{D}'_{14_2}=\frac{s^7}{1307674368000}(78r^2s^4-286r^2s^3t-286r^2s^3+286r^2s^2t^2+1144r^2s^2t+286r^2s^2-1287r^2st^2-1287r^2st+1716r^2t^2-91rs^5+312rs^4t+312rs^4-286rs^3t^2-1144rs^3t-286rs^3+1144rs^2t^2+1144rs^2t-1287rst^2+28s^6-91s^5t-91s^5+78s^4t^2+312s^4t+78s^4-286s^3t^2-286s^3t+286s^2t^2)$,

$\bar{D}'_{14_3}=\frac{t^7}{1307674368000}(286r^2s^2t^2-1287r^2s^2t+1716r^2s^2-286r^2st^3+1144r^2st^2-1287r^2st+78r^2t^4-286r^2t^3+286r^2t^2-286rs^2t^3+1144rs^2t^2-1287rs^2t+312rst^4-1144rst^3+1144rst^2-91rt^5+312rt^4-286rt^3+78s^2t^4-286s^2t^3+286s^2t^2-91st^5+312st^4-286st^3+28t^6-91t^5+78t^4)$,

$\bar{D}'_{14_4}=\frac{1}{1307674368000}(1716r^2s^2t^2-1287r^2s^2t+286r^2s^2-1287r^2st^2+1144r^2st-286r^2s+286r^2t^2-286r^2t+78r^2-1287rs^2t^2+1144rs^2t-286rs^2+1144rst^2-1144rst+312rs-286rt^2+312rt-91r+286s^2t^2-286s^2t+78s^2-286st^2+312st-91s+78t^2-91t+28)$.

The order of second derivative block (5.25) is found in a similar manner as before using the linear difference operator ∇. Then, the functions of $Y''^{[4]_3}_{n+1}, F^{[4]_3}_{n+1}$, $G^{[4]_3}_{n+1}$ are expanded and set to $\mathbf{0}$ which produces

$$\left[Q_{11}^{''[4]_3}\ Q_{21}^{''[4]_3}\ Q_{31}^{''[4]_3}\ Q_{41}^{''[4]_3}\right]^T = \left[0\ \ 0\ \ 0\ \ 0\right]^T$$

where

$$Q_{11}^{''[4]_3} = \sum_{i=0}^{\infty}\frac{(rh)^i}{i!}y_n^{(i+2)} - y_n'' - hry_n''' + \frac{h^2r^2y_n^{iv}}{27720s^3t^3}(132r^6s^2t - 42r^7s - 42r^7t - 42r^7st + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t - 110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 - 440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 + 1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 - 1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 - 9702s^3t^3) - \sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+4)}r^2}{27720(r-s)^3(r-t)^3(r-1)^3i!}(756r^9 - 2646r^8s - 2646r^8t - 2646r^8 + 3069r^7s^2 + 9420r^7st + 9420r^7s + 3069r^7t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 - 11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 - 11121r^6t^2 - 11121r^6t - 1155r^6 + 4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 + 41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t - 5148r^4s^3t^2 - 16027r^4s^3t - 5148r^4s^3 - 5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 - 16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 - 5148r^4t^2 + 1980r^3s^3t^3 + 20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t + 20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t - 26598r^2s^2t^3 - 26598r^2s^2t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 - 4158s^3t^3) - \sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+4)}r^6}{27720s^3(r-s)^3(s-t)^3(s-1)^3i!}(84r^6st - 126r^6s^2 + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 - 345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 + 616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s - 110r^4t^3 - 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 +$$

$$2992r^3s^3t + 363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3 +$$
$$484r^3st^2 - 275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 +$$
$$693r^2s^3t^3 - 891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t -$$
$$1188r^2st^3 - 1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 -$$
$$2772rs^3t - 1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 -$$
$$2310s^2t^3) - \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+4)} r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(42r^6s - 84r^6st + 126r^6t^2 - 84r^6t +$$
$$264r^5s^2t - 132r^5s^2 - 105r^5st^2 + 345r^5st - 132r^5s - 399r^5t^3 - 105r^5t^2 + 264r^5t -$$
$$220r^4s^3t + 110r^4s^3 - 616r^4s^2t^2 - 506r^4s^2t + 440r^4s^2 + 1067r^4st^3 - 407r^4st^2 -$$
$$506r^4st + 110r^4s + 297r^4t^4 + 1067r^4t^3 - 616r^4t^2 - 220r^4t + 825r^3s^3t^2 + 275r^3s^3t -$$
$$396r^3s^3 - 363r^3s^2t^3 + 2387r^3s^2t^2 - 484r^3s^2t - 396r^3s^2 - 990r^3st^4 - 2992r^3st^3 +$$
$$2387r^3st^2 + 275r^3st - 990r^3t^4 - 363r^3t^3 + 825r^3t^2 - 693r^2s^3t^3 - 2673r^2s^3t^2 +$$
$$1188r^2s^3t + 396r^2s^3 + 891r^2s^2t^4 + 891r^2s^2t^3 - 3168r^2s^2t^2 + 1188r^2s^2t + 3564r^2st^4 +$$
$$891r^2st^3 - 2673r^2st^2 + 891r^2t^4 - 693r^2t^3 + 2772rs^3t^3 + 1518rs^3t^2 - 2178rs^3t -$$
$$3564rs^2t^4 + 726rs^2t^3 + 1518rs^2t^2 - 3564rst^4 + 2772rst^3 - 3234s^3t^3 + 2310s^3t^2 +$$
$$4158s^2t^4 - 3234s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)} r^6}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(42r^6st - 84r^6s - 84r^6t +$$
$$126r^6 - 132r^5s^2t + 264r^5s^2 - 132r^5st^2 + 345r^5st - 105r^5s + 264r^5t^2 - 105r^5t -$$
$$399r^5 + 110r^4s^3t - 220r^4s^3 + 440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 -$$
$$506r^4st^2 - 407r^4st + 1067r^4s - 220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 +$$
$$275r^3s^3t + 825r^3s^3 - 396r^3s^2t^3 - 484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 +$$
$$2387r^3st^2 - 2992r^3st - 990r^3s + 825r^3t^3 - 363r^3t^2 - 990r^3t + 396r^2s^3t^3 +$$
$$1188r^2s^3t^2 - 2673r^2s^3t - 693r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2t +$$
$$891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 +$$
$$1518rs^3t^2 + 2772rs^3t + 1518rs^2t^3 + 726rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 +$$
$$2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2) - \frac{h^3r^3y_n^{v}}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t -$$
$$66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 198r^3s^2t - 198r^3s^2 -$$
$$198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 +$$
$$792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2) +$$
$$\sum_{i=0}^{\infty} \frac{r^i h^{i+3} y_n^{(i+5)} r^3}{13860(r-s)^2(r-t)^2(r-1)^2 i!}(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st +$$
$$220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165$$

$r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 +$
$528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2) +$
$\sum_{i=0}^{\infty} \frac{s^i h^{i+3} y_n^{(i+5)} r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2 i!}(198r^2t^2 - 55r^3t^2 + 99r^2s - 110r^3s + 33r^4s -$
$198rt^2 + 198r^2t - 220r^3t + 66r^4t + 462st^2 - 55r^3 + 66r^4 - 21r^5 - 396rst^2 +$
$396r^2st - 110r^3st + 99r^2st^2 - 396rst) + \sum_{i=0}^{\infty} \frac{t^i h^{i+3} y_n^{(i+5)} r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2 i!}(198r^2s^2 -$
$55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t -$
$55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst) +$
$\sum_{i=0}^{\infty} \frac{h^{i+3} y_n^{(i+5)} r^6}{27720(r-1)^2(s-1)^2(t-1)^2 i!}(66r^4s - 21r^5 + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st -$
$110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 -$
$198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2),$

$Q_{21}^{''[4]_3} = \sum_{i=0}^{\infty} \frac{(sh)^i}{i!} y_n^{(i+2)} - y_n'' - hsy_n''' + \frac{h^2s^2y_n^{iv}}{27720r^3t^3}(396r^3s^4t^2 - 110r^3s^5 - 110r^3s^5t +$
$583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 + 990r^3s^2t^3 +$
$264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 + 132r^2s^6t + 132r^2s^6 -$
$440r^2s^5t^2 - 748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 -$
$1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t + 264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t -$
$42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 - 748rs^5t - 110rs^5 +$
$583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t +$
$132s^6t^2 + 132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3) -$
$\sum_{i=0}^{\infty} \frac{r^i h^{i+2} y_n^{(i+4)} s^2}{27720(r-s)^3(s-t)^3(s-1)^3 i!}(1155r^3s^6 - 4235r^3s^5t - 4235r^3s^5 + 5148r^3s^4t^2 +$
$16027r^3s^4t + 5148r^3s^4 - 1980r^3s^3t^3 - 20196r^3s^3t^2 - 20196r^3s^3t - 1980r^3s^3 +$
$7920r^3s^2t^3 + 26598r^3s^2t^2 + 7920r^3s^2t - 10626r^3st^3 - 10626r^3st^2 + 4158r^3t^3 -$
$3069r^2s^7 + 11121r^2s^6t + 11121r^2s^6 - 13376r^2s^5t^2 - 41437r^2s^5t - 13376r^2s^5 +$
$5148r^2s^4t^3 + 51436r^2s^4t^2 + 51436r^2s^4t + 5148r^2s^4 - 20196r^2s^3t^3 - 66330r^2s^3t^2 -$
$20196r^2s^3t + 26598r^2s^2t^3 + 26598r^2s^2t^2 - 10626r^2st^3 + 2646rs^8 - 9420rs^7t -$
$9420rs^7 + 11121rs^6t^2 + 34278rs^6t + 11121rs^6 - 4235rs^5t^3 - 41437rs^5t^2 -$
$41437rs^5t - 4235rs^5 + 16027rs^4t^3 + 51436rs^4t^2 + 16027rs^4t - 20196rs^3t^3 -$
$20196rs^3t^2 + 7920rs^2t^3 - 756s^9 + 2646s^8t + 2646s^8 - 3069s^7t^2 - 9420s^7t -$
$3069s^7 + 1155s^6t^3 + 11121s^6t^2 + 11121s^6t + 1155s^6 - 4235s^5t^3 - 13376s^5t^2 -$

$4235s^5t+5148s^4t^3+5148s^4t^2-1980s^3t^3)-\sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+4)}s^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3i!}(297r^4s^4-$
$990r^4s^3t-990r^4s^3+891r^4s^2t^2+3564r^4s^2t+891r^4s^2-3564r^4st^2-3564r^4st+$
$4158r^4t^2-399r^3s^5+1067r^3s^4t+1067r^3s^4-363r^3s^3t^2-2992r^3s^3t-363r^3s^3-$
$693r^3s^2t^3+891r^3s^2t^2+891r^3s^2t-693r^3s^2+2772r^3st^3+726r^3st^2+2772r^3st-$
$3234r^3t^3-3234r^3t^2+126r^2s^6-105r^2s^5t-105r^2s^5-616r^2s^4t^2-407r^2s^4t-$
$616r^2s^4+825r^2s^3t^3+2387r^2s^3t^2+2387r^2s^3t+825r^2s^3-2673r^2s^2t^3-$
$3168r^2s^2t^2-2673r^2s^2t+1518r^2st^3+1518r^2st^2+2310r^2t^3-84rs^6t-84rs^6+$
$264rs^5t^2+345rs^5t+264rs^5-220rs^4t^3-506rs^4t^2-506rs^4t-220rs^4+$
$275rs^3t^3-484rs^3t^2+275rs^3t+1188rs^2t^3+1188rs^2t^2-2178rst^3+42s^6t-$
$132s^5t^2-132s^5t+110s^4t^3+440s^4t^2+110s^4t-396s^3t^3-396s^3t^2+396s^2t^3)+$
$\sum_{i=0}^{\infty}\frac{t^ih^{i+2}y_n^{(i+4)}s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3i!}(220r^3s^4t-110r^3s^4-825r^3s^3t^2-275r^3s^3t+396r^3s^3+$
$693r^3s^2t^3+2673r^3s^2t^2-1188r^3s^2t-396r^3s^2-2772r^3st^3-1518r^3st^2+2178r^3st+$
$3234r^3t^3-2310r^3t^2-264r^2s^5t+132r^2s^5+616r^2s^4t^2+506r^2s^4t-440r^2s^4+$
$363r^2s^3t^3-2387r^2s^3t^2+484r^2s^3t+396r^2s^3-891r^2s^2t^4-891r^2s^2t^3+3168r^2s^2t^2-$
$1188r^2s^2t+3564r^2st^4-726r^2st^3-1518r^2st^2-4158r^2t^4+3234r^2t^3+84rs^6t-$
$42rs^6+105rs^5t^2-345rs^5t+132rs^5-1067rs^4t^3+407rs^4t^2+506rs^4t-110rs^4+$
$990rs^3t^4+2992rs^3t^3-2387rs^3t^2-275rs^3t-3564rs^2t^4-891rs^2t^3+2673rs^2t^2+$
$3564rst^4-2772rst^3-126s^6t^2+84s^6t+399s^5t^3+105s^5t^2-264s^5t-297s^4t^4-$
$1067s^4t^3+616s^4t^2+220s^4t+990s^3t^4+363s^3t^3-825s^3t^2-891s^2t^4+693s^2t^3)+$
$\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+4)}s^6}{27720(r-1)^3(s-1)^3(t-1)^3i!}(110r^3s^4t-220r^3s^4-396r^3s^3t^2+275r^3s^3t+825r^3s^3+$
$396r^3s^2t^3+1188r^3s^2t^2-2673r^3s^2t-693r^3s^2-2178r^3st^3+1518r^3st^2+2772r^3st+$
$2310r^3t^3-3234r^3t^2-132r^2s^5t+264r^2s^5+440r^2s^4t^2-506r^2s^4t-616r^2s^4-$
$396r^2s^3t^3-484r^2s^3t^2+2387r^2s^3t-363r^2s^3+1188r^2s^2t^3-3168r^2s^2t^2+$
$891r^2s^2t+891r^2s^2+1518r^2st^3+726r^2st^2-3564r^2st-3234r^2t^3+4158r^2t^2+$
$42rs^6t-84rs^6-132rs^5t^2+345rs^5t-105rs^5+110rs^4t^3-506rs^4t^2-407rs^4t+$
$1067rs^4+275rs^3t^3+2387rs^3t^2-2992rs^3t-990rs^3-2673rs^2t^3+891rs^2t^2+$
$3564rs^2t+2772rst^3-3564rst^2-84s^6t+126s^6+264s^5t^2-105s^5t-399s^5-$
$220s^4t^3-616s^4t^2+1067s^4t+297s^4+825s^3t^3-363s^3t^2-990s^3t-693s^2t^3+$
$891s^2t^2)-\frac{h^3s^3y_n^v}{27720r^2t^2}(55r^2s^4-198r^2s^3t-198r^2s^3+198r^2s^2t^2+792r^2s^2t+198r^2s^2$

$$
\begin{aligned}
&-924r^2st^2-924r^2st+1386r^2t^2-66rs^5+220rs^4t+220rs^4-198rs^3t^2-792rs^3t-\\
&198rs^3+792rs^2t^2+792rs^2t-924rst^2+21s^6-66s^5t-66s^5+55s^4t^2+220s^4t+\\
&55s^4-198s^3t^2-198s^3t+198s^2t^2)+\sum_{i=0}^{\infty}\frac{r^ih^{i+3}y_n^{(i+5)}s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2i!}(198s^2t^2-\\
&55s^3t^2+99rs^2-110rs^3+33rs^4+462rt^2-198st^2+198s^2t-220s^3t+66s^4t-\\
&55s^3+66s^4-21s^5-396rst^2+396rs^2t-110rs^3t+99rs^2t^2-396rst)+\\
&\sum_{i=0}^{\infty}\frac{s^ih^{i+3}y_n^{(i+5)}s^3}{13860(r-s)^2(s-t)^2(s-1)^2i!}(55r^2s^4-165r^2s^3t-165r^2s^3+132r^2s^2t^2+\\
&528r^2s^2t+132r^2s^2-462r^2st^2-462r^2st+462r^2t^2-77rs^5+220rs^4t+\\
&220rs^4-165rs^3t^2-660rs^3t-165rs^3+528rs^2t^2+528rs^2t-462rst^2+\\
&28s^6-77s^5t-77s^5+55s^4t^2+220s^4t+55s^4-165s^3t^2-165s^3t+132s^2t^2)+\\
&\sum_{i=0}^{\infty}\frac{t^ih^{i+3}y_n^{(i+5)}s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2i!}(198r^2s^2-55r^2s^3+198rs^2-198r^2s-220rs^3+\\
&66rs^4+462r^2t+99s^2t-110s^3t+33s^4t-55s^3+66s^4-21s^5+396rs^2t-396r^2st-\\
&110rs^3t+99r^2s^2t-396rst)+\sum_{i=0}^{\infty}\frac{h^{i+3}y_n^{(i+5)}s^6}{27720(r-1)^2(s-1)^2(t-1)^2i!}(198r^2s^2t-55r^2s^3+\\
&99r^2s^2-198r^2st^2-396r^2st+462r^2t^2+66rs^4-220rs^3t-110rs^3+198rs^2t^2+\\
&396rs^2t-396rst^2-21s^5+66s^4t+33s^4-55s^3t^2-110s^3t+99s^2t^2),
\end{aligned}
$$

$$
\begin{aligned}
&Q''^{[4]_3}_{31}=\sum_{i=0}^{\infty}\frac{(th)^i}{i!}y_n^{(i+2)}-y_n''-hsy_n'''+\frac{h^2t^2y_n^{iv}}{27720r^3s^3}(990r^3s^3t^2-396r^3s^3t^3+1848r^3s^3t-\\
&9702r^3s^3+396r^3s^2t^4-1188r^3s^2t^3+264r^3s^2t^2+1848r^3s^2t-110r^3st^5+583r^3st^4-\\
&1188r^3st^3+990r^3st^2-110r^3t^5+396r^3t^4-396r^3t^3+396r^2s^3t^4-1188r^2s^3t^3+\\
&264r^2s^3t^2+1848r^2s^3t-440r^2s^2t^5+1540r^2s^2t^4-1584r^2s^2t^3+264r^2s^2t^2+\\
&132r^2st^6-748r^2st^5+1540r^2st^4-1188r^2st^3+132r^2t^6-440r^2t^5+396r^2t^4-\\
&110rs^3t^5+583rs^3t^4-1188rs^3t^3+990rs^3t^2+132rs^2t^6-748rs^2t^5+1540rs^2t^4-\\
&1188rs^2t^3-42rst^7+321rst^6-748rst^5+583rst^4-42rt^7+132rt^6-110rt^5-\\
&110s^3t^5+396s^3t^4-396s^3t^3+132s^2t^6-440s^2t^5+396s^2t^4-42st^7+132st^6-\\
&110st^5)-\sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+4)}t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3i!}(891r^4s^2t^2-3564r^4s^2t+4158r^4s^2-\\
&990r^4st^3+3564r^4st^2-3564r^4st+297r^4t^4-990r^4t^3+891r^4t^2-693r^3s^3t^2+\\
&2772r^3s^3t-3234r^3s^3-363r^3s^2t^3+891r^3s^2t^2+726r^3s^2t-3234r^3s^2+1067r^3st^4-\\
&2992r^3st^3+891r^3st^2+2772r^3st-399r^3t^5+1067r^3t^4-363r^3t^3-693r^3t^2+\\
&825r^2s^3t^3-2673r^2s^3t^2+1518r^2s^3t+2310r^2s^3-616r^2s^2t^4+2387r^2s^2t^3-\\
&3168r^2s^2t^2+1518r^2s^2t-105r^2st^5-407r^2st^4+2387r^2st^3-2673r^2st^2+126r^2t^6-
\end{aligned}
$$

$105r^2t^5 - 616r^2t^4 + 825r^2t^3 - 220rs^3t^4 + 275rs^3t^3 + 1188rs^3t^2 - 2178rs^3t +$
$264rs^2t^5 - 506rs^2t^4 - 484rs^2t^3 + 1188rs^2t^2 - 84rst^6 + 345rst^5 - 506rst^4 + 275rst^3 -$
$84rt^6 + 264rt^5 - 220rt^4 + 110s^3t^4 - 396s^3t^3 + 396s^3t^2 - 132s^2t^5 + 440s^2t^4 -$
$396s^2t^3 + 42st^6 - 132st^5 + 110st^4) - \sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+4)} t^6}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3 i!}(693r^3s^3t^2 -$
$2772r^3s^3t + 3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 -$
$275r^3st^3 - 1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 +$
$3564r^2s^4t - 4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 -$
$2387r^2s^2t^3 + 3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 +$
$132r^2t^5 - 440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 +$
$2992rs^3t^3 - 891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 +$
$84rst^6 - 345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 -$
$891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + 616s^2t^4 -$
$825s^2t^3 + 84st^6 - 264st^5 + 220st^4) + \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+4)} t^2}{27720 (r-t)^3 (s-t)^3 (t-1)^3 i!}(7920r^3s^3t^2 -$
$1980r^3s^3t^3 - 10626r^3s^3t + 4158r^3s^3 + 5148r^3s^2t^4 - 20196r^3s^2t^3 + 26598r^3s^2t^2 -$
$10626r^3s^2t - 4235r^3st^5 + 16027r^3st^4 - 20196r^3st^3 + 7920r^3st^2 + 1155r^3t^6 -$
$4235r^3t^5 + 5148r^3t^4 - 1980r^3t^3 + 5148r^2s^3t^4 - 20196r^2s^3t^3 + 26598r^2s^3t^2 -$
$10626r^2s^3t - 13376r^2s^2t^5 + 51436r^2s^2t^4 - 66330r^2s^2t^3 + 26598r^2s^2t^2 +$
$11121r^2st^6 - 41437r^2st^5 + 51436r^2st^4 - 20196r^2st^3 - 3069r^2t^7 + 11121r^2t^6 -$
$13376r^2t^5 + 5148r^2t^4 - 4235rs^3t^5 + 16027rs^3t^4 - 20196rs^3t^3 + 7920rs^3t^2 +$
$11121rs^2t^6 - 41437rs^2t^5 + 51436rs^2t^4 - 20196rs^2t^3 - 9420rst^7 + 34278rst^6 -$
$41437rst^5 + 16027rst^4 + 2646rt^8 - 9420rt^7 + 11121rt^6 - 4235rt^5 + 1155s^3t^6 -$
$4235s^3t^5 + 5148s^3t^4 - 1980s^3t^3 - 3069s^2t^7 + 11121s^2t^6 - 13376s^2t^5 + 5148s^2t^4 +$
$2646st^8 - 9420st^7 + 11121st^6 - 4235st^5 - 756t^9 + 2646t^8 - 3069t^7 + 1155t^6) +$
$\sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)} t^6}{27720 (r-1)^3 (s-1)^3 (t-1)^3 i!}(396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 +$
$1188r^3s^2t^2 + 1518r^3s^2t - 3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st -$
$220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + 1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 +$
$440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - 506r^2st^4 +$
$2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 +$
$110rs^3t^4 + 275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 +$

$891rs^2t^2 - 3564rs^2t + 42rst^6 + 345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4) - \frac{h^3t^3y_n^{v}}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4) + \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+5)}t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2i!}(198s^2t^2 - 55s^2t^3 + 462rs^2 + 99rt^2 - 110rt^3 + 33rt^4 + 198st^2 - 198s^2t - 220st^3 + 66st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396rs^2t - 110rst^3 + 99rs^2t^2 - 396rst) + \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+5)}t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2i!}(198r^2t^2 - 55r^2t^3 + 462r^2s + 198rt^2 - 198r^2t - 220rt^3 + 66rt^4 + 99st^2 - 110st^3 + 33st^4 - 55t^3 + 66t^4 - 21t^5 + 396rst^2 - 396r^2st - 110rst^3 + 99r^2st^2 - 396rst) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+5)}t^3}{13860(r-t)^2(s-t)^2(t-1)^2i!}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+5)}t^6}{27720(r-1)^2(s-1)^2(t-1)^2i!}(462r^2s^2 - 198r^2s^2t + 198r^2st^2 - 396r^2st - 55r^2t^3 + 99r^2t^2 + 198rs^2t^2 - 396rs^2t - 220rst^3 + 396rst^2 + 66rt^4 - 110rt^3 - 55s^2t^3 + 99s^2t^2 + 66st^4 - 110st^3 - 21t^5 + 33t^4),$

$Q_{41}^{''[4]_3} = \sum_{i=0}^{\infty} \frac{(h)^i}{i!}y_n^{(i+2)} - y_n'' - hy_n''' + \frac{h^2y_n^{iv}}{27720r^3s^3t^3}(1848r^3s^3t^2 - 9702r^3s^3t^3 + 990r^3s^3t - 396r^3s^3 + 1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 + 990r^3st^3 - 1188r^3st^2 + 583r^3st - 110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t + 396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - 748r^2st + 132r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - 110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 + 583rst^3 - 748rst^2 + 321rst - 42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) + \sum_{i=0}^{\infty} \frac{r^ih^{i+2}y_n^{(i+4)}}{27720r^3(r-s)^3(r-t)^3(r-1)^3i!}(3564r^4s^2t - 4158r^4s^2t^2 - 891r^4s^2 + 3564r^4st^2 - 3564r^4st + 990r^4s - 891r^4t^2 + 990r^4t - 297r^4 + 3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3$

$+3234r^3s^2t^3 - 726r^3s^2t^2 - 891r^3s^2t + 363r^3s^2 - 2772r^3st^3 - 891r^3st^2 + 2992r^3st -$
$1067r^3s + 693r^3t^3 + 363r^3t^2 - 1067r^3t + 399r^3 - 2310r^2s^3t^3 - 1518r^2s^3t^2 +$
$2673r^2s^3t - 825r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 +$
$2673r^2st^3 - 2387r^2st^2 + 407r^2st + 105r^2s - 825r^2t^3 + 616r^2t^2 + 105r^2t - 126r^2 +$
$2178rs^3t^3 - 1188rs^3t^2 - 275rs^3t + 220rs^3 - 1188rs^2t^3 + 484rs^2t^2 + 506rs^2t -$
$264rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs + 220rt^3 - 264rt^2 + 84rt - 396s^3t^3 +$
$396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 + 132s^2t - 110st^3 + 132st^2 - 42st) -$
$\sum_{i=0}^{\infty} \frac{s^i h^{i+2} y_n^{(i+4)}}{27720s^3(r-s)^3(s-t)^3(s-1)^3 i!}(3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 -$
$1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s -$
$396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 -$
$726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3 - 1518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t +$
$616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 +$
$132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t -$
$1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 -$
$345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 +$
$363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 -$
$264st^2 + 84st) - \sum_{i=0}^{\infty} \frac{t^i h^{i+2} y_n^{(i+4)}}{27720t^3(r-t)^3(s-t)^3(t-1)^3 i!}(2310r^3s^3t^2 - 2178r^3s^3t + 396r^3s^3 -$
$3234r^3s^2t^3 + 1518r^3s^2t^2 + 1188r^3s^2t - 396r^3s^2 + 2772r^3st^3 - 2673r^3st^2 + 275r^3st +$
$110r^3s - 693r^3t^3 + 825r^3t^2 - 220r^3t - 3234r^2s^3t^3 + 1518r^2s^3t^2 + 1188r^2s^3t -$
$396r^2s^3 + 4158r^2s^2t^4 + 726r^2s^2t^3 - 3168r^2s^2t^2 - 484r^2s^2t + 440r^2s^2 - 3564r^2st^4 +$
$891r^2st^3 + 2387r^2st^2 - 506r^2st - 132r^2s + 891r^2t^4 - 363r^2t^3 - 616r^2t^2 + 264r^2t +$
$2772rs^3t^3 - 2673rs^3t^2 + 275rs^3t + 110rs^3 - 3564rs^2t^4 + 891rs^2t^3 + 2387rs^2t^2 -$
$506rs^2t - 132rs^2 + 3564rst^4 - 2992rst^3 - 407rst^2 + 345rst + 42rs - 990rt^4 +$
$1067rt^3 - 105rt^2 - 84rt - 693s^3t^3 + 825s^3t^2 - 220s^3t + 891s^2t^4 - 363s^2t^3 -$
$616s^2t^2 + 264s^2t - 990st^4 + 1067st^3 - 105st^2 - 84st + 297t^4 - 399t^3 + 126t^2) -$
$\sum_{i=0}^{\infty} \frac{h^{i+2} y_n^{(i+4)}}{27720(r-1)^3(s-1)^3(t-1)^3 i!}(4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 -$
$10626r^3s^2t^3 + 26598r^3s^2t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 +$
$16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3t + 1155r^3 - 10626r^2s^3t^3 +$
$26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2t -$

$13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + 7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - 41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + 5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756) - \frac{h^3y_n^v}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21) + \sum_{i=0}^{\infty} \frac{r^ih^{i+3}y_n^{(i+5)}}{27720r^2(r-s)^2(r-t)^2(r-1)^2i!}(33r + 66s + 66t - 198s^2t^2 - 110rs - 110rt - 220st + 99rs^2 + 99rt^2 + 198st^2 + 198s^2t - 55s^2 - 55t^2 - 396rst^2 - 396rs^2t + 462rs^2t^2 + 396rst - 21) + \sum_{i=0}^{\infty} \frac{s^ih^{i+3}y_n^{(i+5)}}{27720s^2(r-s)^2(s-t)^2(s-1)^2i!}(66r + 33s + 66t - 198r^2t^2 - 110rs - 220rt - 110st + 99r^2s + 198rt^2 + 198r^2t + 99st^2 - 55r^2 - 55t^2 - 396rst^2 - 396r^2st + 462r^2st^2 + 396rst - 21) + \sum_{i=0}^{\infty} \frac{t^ih^{i+3}y_n^{(i+5)}}{27720t^2(r-t)^2(s-t)^2(t-1)^2i!}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t + 396rst - 21) + \sum_{i=0}^{\infty} \frac{h^{i+3}y_n^{(i+5)}}{13860(r-1)^2(s-1)^2(t-1)^2i!}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28).$

The coefficients of h^j and $y^{(j)}$ are also compared this yields $\bar{D''}_0 = \bar{D''}_1 = ... = \bar{D''}_{13} = 0$ and $\bar{D''}_{14} \neq 0$. Thus, the second derivative of the main block possess an order $[10,10,10,10]^T$ with vector of error constants
$\bar{D''}_{14} = [\bar{D''}_{14_1}, \bar{D''}_{14_2}, \bar{D''}_{14_3}, \bar{D''}_{14_4}]^T$ where

$\bar{D''}_{14_1} = \frac{r^6}{100590336000}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 -$

$396rs^2t^2-396rs^2t-396rst^2+462s^2t^2)$,

$\bar{D}''_{14_2}=\frac{s^6}{100590336000}(33r^2s^4-110r^2s^3t-110r^2s^3+99r^2s^2t^2+396r^2s^2t+99r^2s^2-396r^2st^2-396r^2st+462r^2t^2-42rs^5+132rs^4t+132rs^4-110rs^3t^2-440rs^3t-110rs^3+396rs^2t^2+396rs^2t-396rst^2+14s^6-42s^5t-42s^5+33s^4t^2+132s^4t+33s^4-110s^3t^2-110s^3t+99s^2t^2)$,

$\bar{D}''_{14_3}=\frac{t^6}{100590336000}(99r^2s^2t^2-396r^2s^2t+462r^2s^2-110r^2st^3+396r^2st^2-396r^2st+33r^2t^4-110r^2t^3+99r^2t^2-110rs^2t^3+396rs^2t^2-396rs^2t+132rst^4-440rst^3+396rst^2-42rt^5+132rt^4-110rt^3+33s^2t^4-110s^2t^3+99s^2t^2-42st^5+132st^4-110st^3+14t^6-42t^5+33t^4)$,

$\bar{D}''_{14_4}=\frac{1}{100590336000}(462r^2s^2t^2-396r^2s^2t+99r^2s^2-396r^2st^2+396r^2st-110r^2s+99r^2t^2-110r^2t+33r^2-396rs^2t^2+396rs^2t-110rs^2+396rst^2-440rst+132rs-110rt^2+132rt-42r+99s^2t^2-110s^2t+33s^2-110st^2+132st-42s+33t^2-42t+14)$.

The order of the third derivative (5.30) is found by expanding all function in $Y'''^{[4]_3}_{n+1}, F^{[4]_3}_{n+1}$ and $G^{[4]_3}_{n+1}$ in Taylor series and setting to $\mathbf{0}$. This gives

$$\begin{bmatrix} Q'''^{[4]_3}_{11} & Q'''^{[4]_3}_{21} & Q'''^{[4]_3}_{31} & Q'''^{[4]_3}_{41} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

where

$Q'''^{[4]_3}_{11}=\sum_{i=0}^{\infty}\frac{(rh)^i}{i!}y_n^{(i+3)}-y''_n-\frac{hry_n^{iv}}{1260s^3t^3}(7r^7st+7r^7s+7r^7t-20r^6s^2t-20r^6s^2-20r^6st^2-48r^6st-20r^6s-20r^6t^2-20r^6t+15r^5s^3t+15r^5s^3+60r^5s^2t^2+100r^5s^2t+60r^5s^2+15r^5st^3+100r^5st^2+100r^5st+15r^5s+15r^5t^3+60r^5t^2+15r^5t-48r^4s^3t^2-69r^4s^3t-48r^4s^3-48r^4s^2t^3-180r^4s^2t^2-180r^4s^2t-48r^4s^2-69r^4st^3-180r^4st^2-69r^4st-48r^4t^3-48r^4t^2+42r^3s^3t^3+120r^3s^3t^2+120r^3s^3t+42r^3s^3+120r^3s^2t^3+144r^3s^2t^2+120r^3s^2t+120r^3st^3+120r^3st^2+42r^3t^3-84r^2s^3t^3-84r^2s^3t-84r^2st^3-168rs^3t^3-168rs^3t^2-168rs^2t^3+630s^3t^3)-\sum_{i=0}^{\infty}\frac{r^ih^{i+1}y_n^{(i+4)}r}{1260(r-s)^3(r-t)^3(r-1)^3i!}(252r^9-819$

$r^8s - 819r^8t - 819r^8 + 885r^7s^2 + 2686r^7st + 2686r^7s + 885r^7t^2 + 2686r^7t + 885r^7 -$
$315r^6s^3 - 2930r^6s^2t - 2930r^6s^2 - 2930r^6st^2 - 8913r^6st - 2930r^6s - 315r^6t^3 -$
$2930r^6t^2 - 2930r^6t - 315r^6 + 1050r^5s^3t + 1050r^5s^3 + 3228r^5s^2t^2 + 9847r^5s^2t +$
$3228r^5s^2 + 1050r^5st^3 + 9847r^5st^2 + 9847r^5st + 1050r^5s + 1050r^5t^3 + 3228r^5t^2 +$
$1050r^5t - 1164r^4s^3t^2 - 3561r^4s^3t - 1164r^4s^3 - 1164r^4s^2t^3 - 11034r^4s^2t^2 -$
$11034r^4s^2t - 1164r^4s^2 - 3561r^4st^3 - 11034r^4st^2 - 3561r^4st - 1164r^4t^3 - 1164r^4t^2 +$
$420r^3s^3t^3 + 4026r^3s^3t^2 + 4026r^3s^3t + 420r^3s^3 + 4026r^3s^2t^3 + 12618r^3s^2t^2 +$
$4026r^3s^2t + 4026r^3st^3 + 4026r^3st^2 + 420r^3t^3 - 1470r^2s^3t^3 - 4662r^2s^3t^2 -$
$1470r^2s^3t - 4662r^2s^2t^3 - 4662r^2s^2t^2 - 1470r^2st^3 + 1722rs^3t^3 + 1722rs^3t^2 +$
$1722rs^2t^3 - 630s^3t^3) - \sum_{i=0}^{\infty} \frac{s^i h^{i+1} y_n^{(i+4)} r^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3 i!}(14r^6st - 21r^6s^2 + 14r^6s - 7r^6t +$
$63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 + 20r^5t - 45r^4s^4 -$
$150r^4s^3t - 150r^4s^3 + 90r^4s^2t^2 + 65r^4s^2t + 90r^4s^2 + 30r^4st^3 + 65r^4st^2 + 65r^4st +$
$30r^4s - 15r^4t^3 - 60r^4t^2 - 15r^4t + 135r^3s^4t + 135r^3s^4 + 39r^3s^3t^2 + 366r^3s^3t +$
$39r^3s^3 - 105r^3s^2t^3 - 306r^3s^2t^2 - 306r^3s^2t - 105r^3s^2 - 30r^3st^3 + 72r^3st^2 - 30r^3st +$
$48r^3t^3 + 48r^3t^2 - 108r^2s^4t^2 - 432r^2s^4t - 108r^2s^4 + 84r^2s^3t^3 - 66r^2s^3t^2 - 66r^2s^3t +$
$84r^2s^3 + 294r^2s^2t^3 + 342r^2s^2t^2 + 294r^2s^2t - 138r^2st^3 - 138r^2st^2 - 42r^2t^3 +$
$378rs^4t^2 + 378rs^4t - 294rs^3t^3 - 126rs^3t^2 - 294rs^3t - 126rs^2t^3 - 126rs^2t^2 + 210rst^3 -$
$378s^4t^2 + 294s^3t^3 + 294s^3t^2 - 210s^2t^3) - \sum_{i=0}^{\infty} \frac{t^i h^{i+1} y_n^{(i+4)} r^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3 i!}(7r^6s - 14r^6st +$
$21r^6t^2 - 14r^6t + 40r^5s^2t - 20r^5s^2 - 14r^5st^2 + 51r^5st - 20r^5s - 63r^5t^3 - 14r^5t^2 +$
$40r^5t - 30r^4s^3t + 15r^4s^3 - 90r^4s^2t^2 - 65r^4s^2t + 60r^4s^2 + 150r^4st^3 - 65r^4st^2 -$
$65r^4st + 15r^4s + 45r^4t^4 + 150r^4t^3 - 90r^4t^2 - 30r^4t + 105r^3s^3t^2 + 30r^3s^3t - 48r^3s^3 -$
$39r^3s^2t^3 + 306r^3s^2t^2 - 72r^3s^2t - 48r^3s^2 - 135r^3st^4 - 366r^3st^3 + 306r^3st^2 + 30r^3st -$
$135r^3t^4 - 39r^3t^3 + 105r^3t^2 - 84r^2s^3t^3 - 294r^2s^3t^2 + 138r^2s^3t + 42r^2s^3 + 108r^2s^2t^4 +$
$66r^2s^2t^3 - 342r^2s^2t^2 + 138r^2s^2t + 432r^2st^4 + 66r^2st^3 - 294r^2st^2 + 108r^2t^4 - 84r^2t^3 +$
$294rs^3t^3 + 126rs^3t^2 - 210rs^3t - 378rs^2t^4 + 126rs^2t^3 + 126rs^2t^2 - 378rst^4 + 294rst^3 -$
$294s^3t^3 + 210s^3t^2 + 378s^2t^4 - 294s^2t^3) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+4)} r^5}{1260(r-1)^3(s-1)^3(t-1)^3 i!}(7r^6st - 14r^6s -$
$14r^6t + 21r^6 - 20r^5s^2t + 40r^5s^2 - 20r^5st^2 + 51r^5st - 14r^5s + 40r^5t^2 - 14r^5t -$
$63r^5 + 15r^4s^3t - 30r^4s^3 + 60r^4s^2t^2 - 65r^4s^2t - 90r^4s^2 + 15r^4st^3 - 65r^4st^2 - 65r^4st +$
$150r^4s - 30r^4t^3 - 90r^4t^2 + 150r^4t + 45r^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 - 48r^3s^2t^3$

$-72r^3s^2t^2+306r^3s^2t-39r^3s^2+30r^3st^3+306r^3st^2-366r^3st-135r^3s+105r^3t^3-$
$39r^3t^2-135r^3t+42r^2s^3t^3+138r^2s^3t^2-294r^2s^3t-84r^2s^3+138r^2s^2t^3-$
$342r^2s^2t^2+66r^2s^2t+108r^2s^2-294r^2st^3+66r^2st^2+432r^2st-84r^2t^3+$
$108r^2t^2-210rs^3t^3+126rs^3t^2+294rs^3t+126rs^2t^3+126rs^2t^2-378rs^2t+$
$294rst^3-378rst^2+210s^3t^3-294s^3t^2-294s^2t^3+378s^2t^2)-\frac{h^2r^2y_n^{v}}{2520s^2t^2}(7r^6-$
$20r^5s-20r^5t-20r^5+15r^4s^2+60r^4st+60r^4s+15r^4t^2+60r^4t+15r^4-$
$48r^3s^2t-48r^3s^2-48r^3st^2-192r^3st-48r^3s-48r^3t^2-48r^3t+42r^2s^2t^2+$
$168r^2s^2t+42r^2s^2+168r^2st^2+168r^2st+42r^2t^2-168rs^2t^2-168rs^2t-$
$168rst^2+210s^2t^2)+\sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+5)}r^2}{2520(r-s)^2(r-t)^2(r-1)^2i!}(28r^6-70r^5s-70r^5t-70r^5+$
$45r^4s^2+180r^4st+180r^4s+45r^4t^2+180r^4t+45r^4-120r^3s^2t-120r^3s^2-$
$120r^3st^2-480r^3st-120r^3s-120r^3t^2-120r^3t+84r^2s^2t^2+336r^2s^2t+84r^2s^2+$
$336r^2st^2+336r^2st+84r^2t^2-252rs^2t^2-252rs^2t-252rst^2+210s^2t^2)+$
$\sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+5)}r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2i!}(48r^2t^2-15r^3t^2+24r^2s-30r^3s+10r^4s-42rt^2+$
$48r^2t-60r^3t+20r^4t+84st^2-15r^3+20r^4-7r^5-84rst^2+96r^2st-30r^3st+$
$24r^2st^2-84rst)+\sum_{i=0}^{\infty}\frac{t^ih^{i+2}y_n^{(i+5)}r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2i!}(48r^2s^2-15r^3s^2-42rs^2+48r^2s-$
$60r^3s+20r^4s+24r^2t-30r^3t+10r^4t+84s^2t-15r^3+20r^4-7r^5-84rs^2t+96r^2st-$
$30r^3st+24r^2s^2t-84rst)+\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+5)}r^5}{2520(r-1)^2(s-1)^2(t-1)^2i!}(20r^4s-7r^5+20r^4t+10r^4-$
$15r^3s^2-60r^3st-30r^3s-15r^3t^2-30r^3t+48r^2s^2t+24r^2s^2+48r^2st^2+96r^2st+$
$24r^2t^2-42rs^2t^2-84rs^2t-84rst^2+84s^2t^2),$

$Q_{21}^{'''[4]_3}=\sum_{i=0}^{\infty}\frac{(sh)^i}{i!}y_n^{(i+3)}-y_n''-\frac{hsy_n^{iv}}{1260r^3t^3}(15r^3s^5t+15r^3s^5-48r^3s^4t^2-69r^3s^4t-$
$48r^3s^4+42r^3s^3t^3+120r^3s^3t^2+120r^3s^3t+42r^3s^3-84r^3s^2t^3-84r^3s^2t-$
$168r^3st^3-168r^3st^2+630r^3t^3-20r^2s^6t-20r^2s^6+60r^2s^5t^2+100r^2s^5t+$
$60r^2s^5-48r^2s^4t^3-180r^2s^4t^2-180r^2s^4t-48r^2s^4+120r^2s^3t^3+144r^2s^3t^2+$
$120r^2s^3t-168r^2st^3+7rs^7t+7rs^7-20rs^6t^2-48rs^6t-20rs^6+15rs^5t^3+100rs^5t^2+$
$100rs^5t+15rs^5-69rs^4t^3-180rs^4t^2-69rs^4t+120rs^3t^3+120rs^3t^2-84rs^2t^3+$
$7s^7t-20s^6t^2-20s^6t+15s^5t^3+60s^5t^2+15s^5t-48s^4t^3-48s^4t^2+42s^3t^3)-$
$\sum_{i=0}^{\infty}\frac{r^ih^{i+1}y_n^{(i+4)}s^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3i!}(45r^4s^4-135r^4s^3t-135r^4s^3+108r^4s^2t^2+432r^4s^2t+$
$108r^4s^2-378r^4st^2-378r^4st+378r^4t^2-63r^3s^5+150r^3s^4t+150r^3s^4-39r^3s^3t^2-$

$$366r^3s^3t-39r^3s^3-84r^3s^2t^3+66r^3s^2t^2+66r^3s^2t-84r^3s^2+294r^3st^3+126r^3st^2+$$

$$294r^3st-294r^3t^3-294r^3t^2+21r^2s^6-14r^2s^5t-14r^2s^5-90r^2s^4t^2-65r^2s^4t-$$

$$90r^2s^4+105r^2s^3t^3+306r^2s^3t^2+306r^2s^3t+105r^2s^3-294r^2s^2t^3-342r^2s^2t^2-$$

$$294r^2s^2t+126r^2st^3+126r^2st^2+210r^2t^3-14rs^6t-14rs^6+40rs^5t^2+51rs^5t+$$

$$40rs^5-30rs^4t^3-65rs^4t^2-65rs^4t-30rs^4+30rs^3t^3-72rs^3t^2+30rs^3t+138rs^2t^3+$$

$$138rs^2t^2-210rst^3+7s^6t-20s^5t^2-20s^5t+15s^4t^3+60s^4t^2+15s^4t-48s^3t^3-$$

$$48s^3t^2+42s^2t^3)-\sum_{i=0}^{\infty}\frac{s^ih^{i+1}y_n^{(i+4)}s}{1260(r-s)^3(s-t)^3(s-1)^3i!}(315r^3s^6-1050r^3s^5t-1050r^3s^5+$$

$$1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-4026r^3s^3t^2-4026r^3s^3t-$$

$$420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-1722r^3st^3-1722r^3st^2+$$

$$630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-9847r^2s^5t-3228r^2s^5+$$

$$1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-4026r^2s^3t^3-12618r^2s^3t^2-$$

$$4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+819rs^8-2686rs^7t-2686rs^7+$$

$$2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-9847rs^5t^2-9847rs^5t-1050rs^5+$$

$$3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-4026rs^3t^2+1470rs^2t^3-252s^9+$$

$$819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+315s^6t^3+2930s^6t^2+2930s^6t+$$

$$315s^6-1050s^5t^3-3228s^5t^2-1050s^5t+1164s^4t^3+1164s^4t^2-420s^3t^3)+$$

$$\sum_{i=0}^{\infty}\frac{t^ih^{i+1}y_n^{(i+4)}s^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3i!}(30r^3s^4t-15r^3s^4-105r^3s^3t^2-30r^3s^3t+48r^3s^3+$$

$$84r^3s^2t^3+294r^3s^2t^2-138r^3s^2t-42r^3s^2-294r^3st^3-126r^3st^2+210r^3st+$$

$$294r^3t^3-210r^3t^2-40r^2s^5t+20r^2s^5+90r^2s^4t^2+65r^2s^4t-60r^2s^4+39r^2s^3t^3-$$

$$306r^2s^3t^2+72r^2s^3t+48r^2s^3-108r^2s^2t^4-66r^2s^2t^3+342r^2s^2t^2-138r^2s^2t+$$

$$378r^2st^4-126r^2st^3-126r^2st^2-378r^2t^4+294r^2t^3+14rs^6t-7rs^6+14rs^5t^2-$$

$$51rs^5t+20rs^5-150rs^4t^3+65rs^4t^2+65rs^4t-15rs^4+135rs^3t^4+366rs^3t^3-$$

$$306rs^3t^2-30rs^3t-432rs^2t^4-66rs^2t^3+294rs^2t^2+378rst^4-294rst^3-21s^6t^2+$$

$$14s^6t+63s^5t^3+14s^5t^2-40s^5t-45s^4t^4-150s^4t^3+90s^4t^2+30s^4t+135s^3t^4+$$

$$39s^3t^3-105s^3t^2-108s^2t^4+84s^2t^3)+\sum_{i=0}^{\infty}\frac{h^{i+1}y_n^{(i+4)}s^5}{1260(r-1)^3(s-1)^3(t-1)^3i!}(15r^3s^4t-$$

$$30r^3s^4-48r^3s^3t^2+30r^3s^3t+105r^3s^3+42r^3s^2t^3+138r^3s^2t^2-294r^3s^2t-84r^3s^2-$$

$$210r^3st^3+126r^3st^2+294r^3st+210r^3t^3-294r^3t^2-20r^2s^5t+40r^2s^5+60r^2s^4t^2-$$

$$65r^2s^4t-90r^2s^4-48r^2s^3t^3-72r^2s^3t^2+306r^2s^3t-39r^2s^3+138r^2s^2t^3-342r^2s^2t^2+$$

$$66r^2s^2t+108r^2s^2+126r^2st^3+126r^2st^2-378r^2st-294r^2t^3+378r^2t^2+7rs^6t-14r$$

$s^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + 30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + 40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2) - \frac{h^2s^2y_n^{v}}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t - 48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 - 48s^3t^2 - 48s^3t + 42s^2t^2) + \sum_{i=0}^{\infty} \frac{r^ih^{i+2}y_n^{(i+5)}s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2i!}(48s^2t^2 - 15s^3t^2 + 24rs^2 - 30rs^3 + 10rs^4 + 84rt^2 - 42st^2 + 48s^2t - 60s^3t + 20s^4t - 15s^3 + 20s^4 - 7s^5 - 84rst^2 + 96rs^2t - 30rs^3t + 24rs^2t^2 - 84rst) + \sum_{i=0}^{\infty} \frac{s^ih^{i+2}y_n^{(i+5)}s^2}{2520(r-s)^2(s-t)^2(s-1)^2i!}(45r^2s^4 - 120r^2s^3t - 120r^2s^3 + 84r^2s^2t^2 + 336r^2s^2t + 84r^2s^2 - 252r^2st^2 - 252r^2st + 210r^2t^2 - 70rs^5 + 180rs^4t + 180rs^4 - 120rs^3t^2 - 480rs^3t - 120rs^3 + 336rs^2t^2 + 336rs^2t - 252rst^2 + 28s^6 - 70s^5t - 70s^5 + 45s^4t^2 + 180s^4t + 45s^4 - 120s^3t^2 - 120s^3t + 84s^2t^2) + \sum_{i=0}^{\infty} \frac{t^ih^{i+2}y_n^{(i+5)}s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2i!}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 + 84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t + 24r^2s^2t - 84rst) + \sum_{i=0}^{\infty} \frac{h^{i+2}y_n^{(i+5)}s^5}{2520(r-1)^2(s-1)^2(t-1)^2i!}(48r^2s^2t - 15r^2s^3 + 24r^2s^2 - 42r^2st^2 - 84r^2st + 84r^2t^2 + 20rs^4 - 60rs^3t - 30rs^3 + 48rs^2t^2 + 96rs^2t - 84rst^2 - 7s^5 + 20s^4t + 10s^4 - 15s^3t^2 - 30s^3t + 24s^2t^2),$

$Q_{31}^{'''[4]_3} = \sum_{i=0}^{\infty} \frac{(th)^i}{i!}y_n^{(i+3)} - y_n'' - \frac{hty_n^{iv}}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5) - \sum_{i=0}^{\infty} \frac{r^ih^{i+1}y_n^{(i+4)}t^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3i!}(108r^4s^2t^2 - 378r^4s^2t + 378r^4s^2 - 135r^4st^3 + 432r^4st^2 - 378r^4st + 45r^4t^4 - 135r^4t^3 + 108r^4t^2 - 84r^3s^3t^2 + 294r^3s^3t - 294r^3s^3 - 39r^3s^2t^3 + 66r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 150r^3st^4 - 366r^3st^3 + 66r^3st^2 + 294r^3st - 63r^3t^5 +$

$150r^3t^4 - 39r^3t^3 - 84r^3t^2 + 105r^2s^3t^3 - 294r^2s^3t^2 + 126r^2s^3t + 210r^2s^3 - 90r^2s^2t^4 + 306r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t - 14r^2st^5 - 65r^2st^4 + 306r^2st^3 - 294r^2st^2 + 21r^2t^6 - 14r^2t^5 - 90r^2t^4 + 105r^2t^3 - 30rs^3t^4 + 30rs^3t^3 + 138rs^3t^2 - 210rs^3t + 40rs^2t^5 - 65rs^2t^4 - 72rs^2t^3 + 138rs^2t^2 - 14rst^6 + 51rst^5 - 65rst^4 + 30rst^3 - 14rt^6 + 40rt^5 - 30rt^4 + 15s^3t^4 - 48s^3t^3 + 42s^3t^2 - 20s^2t^5 + 60s^2t^4 - 48s^2t^3 + 7st^6 - 20st^5 + 15st^4) - \sum_{i=0}^{\infty} \frac{s^i h^{i+1} y_n^{(i+4)} t^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3 i!}(84r^3s^3t^2 - 294r^3s^3t + 294r^3s^3 - 105r^3s^2t^3 + 294r^3s^2t^2 - 126r^3s^2t - 210r^3s^2 + 30r^3st^4 - 30r^3st^3 - 138r^3st^2 + 210r^3st - 15r^3t^4 + 48r^3t^3 - 42r^3t^2 - 108r^2s^4t^2 + 378r^2s^4t - 378r^2s^4 + 39r^2s^3t^3 - 66r^2s^3t^2 - 126r^2s^3t + 294r^2s^3 + 90r^2s^2t^4 - 306r^2s^2t^3 + 342r^2s^2t^2 - 126r^2s^2t - 40r^2st^5 + 65r^2st^4 + 72r^2st^3 - 138r^2st^2 + 20r^2t^5 - 60r^2t^4 + 48r^2t^3 + 135rs^4t^3 - 432rs^4t^2 + 378rs^4t - 150rs^3t^4 + 366rs^3t^3 - 66rs^3t^2 - 294rs^3t + 14rs^2t^5 + 65rs^2t^4 - 306rs^2t^3 + 294rs^2t^2 + 14rst^6 - 51rst^5 + 65rst^4 - 30rst^3 - 7rt^6 + 20rt^5 - 15rt^4 - 45s^4t^4 + 135s^4t^3 - 108s^4t^2 + 63s^3t^5 - 150s^3t^4 + 39s^3t^3 + 84s^3t^2 - 21s^2t^6 + 14s^2t^5 + 90s^2t^4 - 105s^2t^3 + 14st^6 - 40st^5 + 30st^4) + \sum_{i=0}^{\infty} \frac{t^i h^{i+1} y_n^{(i+4)} t}{1260(r-t)^3(s-t)^3(t-1)^3 i!}(1470r^3s^3t^2 - 420r^3s^3t^3 - 1722r^3s^3t + 630r^3s^3 + 1164r^3s^2t^4 - 4026r^3s^2t^3 + 4662r^3s^2t^2 - 1722r^3s^2t - 1050r^3st^5 + 3561r^3st^4 - 4026r^3st^3 + 1470r^3st^2 + 315r^3t^6 - 1050r^3t^5 + 1164r^3t^4 - 420r^3t^3 + 1164r^2s^3t^4 - 4026r^2s^3t^3 + 4662r^2s^3t^2 - 1722r^2s^3t - 3228r^2s^2t^5 + 11034r^2s^2t^4 - 12618r^2s^2t^3 + 4662r^2s^2t^2 + 2930r^2st^6 - 9847r^2st^5 + 11034r^2st^4 - 4026r^2st^3 - 885r^2t^7 + 2930r^2t^6 - 3228r^2t^5 + 1164r^2t^4 - 1050rs^3t^5 + 3561rs^3t^4 - 4026rs^3t^3 + 1470rs^3t^2 + 2930rs^2t^6 - 9847rs^2t^5 + 11034rs^2t^4 - 4026rs^2t^3 - 2686rst^7 + 8913rst^6 - 9847rst^5 + 3561rst^4 + 819rt^8 - 2686rt^7 + 2930rt^6 - 1050rt^5 + 315s^3t^6 - 1050s^3t^5 + 1164s^3t^4 - 420s^3t^3 - 885s^2t^7 + 2930s^2t^6 - 3228s^2t^5 + 1164s^2t^4 + 819st^8 - 2686st^7 + 2930st^6 - 1050st^5 - 252t^9 + 819t^8 - 885t^7 + 315t^6) + \sum_{i=0}^{\infty} \frac{h^{i+1} y_n^{(i+4)} t^5}{1260(r-1)^3(s-1)^3(t-1)^3 i!}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 + 30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 - 72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 + 108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 + 51rst^5 - 65rst^4 - 366rs$

$t^3+432rst^2-14rt^6-14rt^5+150rt^4-135rt^3-30s^3t^4+105s^3t^3-84s^3t^2+40s^2t^5-90s^2t^4-39s^2t^3+108s^2t^2-14st^6-14st^5+150st^4-135st^3+21t^6-63t^5+45t^4)-\frac{h^2t^2y_n^{v}}{2520r^2s^2}(42r^2s^2t^2-168r^2s^2t+210r^2s^2-48r^2st^3+168r^2st^2-168r^2st+15r^2t^4-48r^2t^3+42r^2t^2-48rs^2t^3+168rs^2t^2-168rs^2t+60rst^4-192rst^3+168rst^2-20rt^5+60rt^4-48rt^3+15s^2t^4-48s^2t^3+42s^2t^2-20st^5+60st^4-48st^3+7t^6-20t^5+15t^4)+\sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+5)}t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2i!}(48s^2t^2-15s^2t^3+84rs^2+24rt^2-30rt^3+10rt^4+48st^2-42s^2t-60st^3+20st^4-15t^3+20t^4-7t^5+96rst^2-84rs^2t-30rst^3+24rs^2t^2-84rst)+\sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+5)}t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2i!}(48r^2t^2-15r^2t^3+84r^2s+48rt^2-42r^2t-60rt^3+20rt^4+24st^2-30st^3+10st^4-15t^3+20t^4-7t^5+96rst^2-84r^2st-30rst^3+24r^2st^2-84rst)+\sum_{i=0}^{\infty}\frac{t^ih^{i+2}y_n^{(i+5)}t^2}{2520(r-t)^2(s-t)^2(t-1)^2i!}(84r^2s^2t^2-252r^2s^2t+210r^2s^2-120r^2st^3+336r^2st^2-252r^2st+45r^2t^4-120r^2t^3+84r^2t^2-120rs^2t^3+336rs^2t^2-252rs^2t+180rst^4-480rst^3+336rst^2-70rt^5+180rt^4-120rt^3+45s^2t^4-120s^2t^3+84s^2t^2-70st^5+180st^4-120st^3+28t^6-70t^5+45t^4)+\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+5)}t^5}{2520(r-1)^2(s-1)^2(t-1)^2i!}(84r^2s^2-42r^2s^2t+48r^2st^2-84r^2st-15r^2t^3+24r^2t^2+48rs^2t^2-84rs^2t-60rst^3+96rst^2+20rt^4-30rt^3-15s^2t^3+24s^2t^2+20st^4-30st^3-7t^5+10t^4)$,

$Q_{41}^{'''[4]_3}=\sum_{i=0}^{\infty}\frac{(h)^i}{i!}y_n^{(i+3)}-y_n''-\frac{hy_n^{iv}}{1260r^3s^3t^3}(630r^3s^3t^3-168r^3s^3t^2-84r^3s^3t+42r^3s^3-168r^3s^2t^3+120r^3s^2t-48r^3s^2-84r^3st^3+120r^3st^2-69r^3st+15r^3s+42r^3t^3-48r^3t^2+15r^3t-168r^2s^3t^3+120r^2s^3t-48r^2s^3+144r^2s^2t^2-180r^2s^2t+60r^2s^2+120r^2st^3-180r^2st^2+100r^2st-20r^2s-48r^2t^3+60r^2t^2-20r^2t-84rs^3t^3+120rs^3t^2-69rs^3t+15rs^3+120rs^2t^3-180rs^2t^2+100rs^2t-20rs^2-69rst^3+100rst^2-48rst+7rs+15rt^3-20rt^2+7rt+42s^3t^3-48s^3t^2+15s^3t-48s^2t^3+60s^2t^2-20s^2t+15st^3-20st^2+7st)+\sum_{i=0}^{\infty}\frac{r^ih^{i+1}y_n^{(i+4)}}{1260r^3(r-s)^3(r-t)^3(r-1)^3i!}(378r^4s^2t-378r^4s^2t^2-108r^4s^2+378r^4st^2-432r^4st+135r^4s-108r^4t^2+135r^4t-45r^4+294r^3s^3t^2-294r^3s^3t+84r^3s^3+294r^3s^2t^3-126r^3s^2t^2-66r^3s^2t+39r^3s^2-294r^3st^3-66r^3st^2+366r^3st-150r^3s+84r^3t^3+39r^3t^2-150r^3t+63r^3-210r^2s^3t^3-126r^2s^3t^2+294r^2s^3t-105r^2s^3-126r^2s^2t^3+342r^2s^2t^2-306r^2s^2t+90r^2s^2+294r^2st^3-306r^2st^2+65r^2st+14r^2s-105r^2t^3+90r^2t^2+14r^2t-21r^2+$

$210rs^3t^3-138rs^3t^2-30rs^3t+30rs^3-138rs^2t^3+72rs^2t^2+65rs^2t-40rs^2-30rst^3+$
$65rst^2-51rst+14rs+30rt^3-40rt^2+14rt-42s^3t^3+48s^3t^2-15s^3t+48s^2t^3-$
$60s^2t^2+20s^2t-15st^3+20st^2-7st)-\sum_{i=0}^{\infty}\frac{s^ih^{i+1}y_n^{(i+4)}}{1260s^3(r-s)^3(s-t)^3(s-1)^3i!}(294r^3s^3t^2-$
$294r^3s^3t+84r^3s^3-210r^3s^2t^3-126r^3s^2t^2+294r^3s^2t-105r^3s^2+210r^3st^3-$
$138r^3st^2-30r^3st+30r^3s-42r^3t^3+48r^3t^2-15r^3t-378r^2s^4t^2+378r^2s^4t-$
$108r^2s^4+294r^2s^3t^3-126r^2s^3t^2-66r^2s^3t+39r^2s^3-126r^2s^2t^3+342r^2s^2t^2-$
$306r^2s^2t+90r^2s^2-138r^2st^3+72r^2st^2+65r^2st-40r^2s+48r^2t^3-60r^2t^2+20r^2t+$
$378rs^4t^2-432rs^4t+135rs^4-294rs^3t^3-66rs^3t^2+366rs^3t-150rs^3+294rs^2t^3-$
$306rs^2t^2+65rs^2t+14rs^2-30rst^3+65rst^2-51rst+14rs-15rt^3+20rt^2-7rt-$
$108s^4t^2+135s^4t-45s^4+84s^3t^3+39s^3t^2-150s^3t+63s^3-105s^2t^3+90s^2t^2+$
$14s^2t-21s^2+30st^3-40st^2+14st)-\sum_{i=0}^{\infty}\frac{t^ih^{i+1}y_n^{(i+4)}}{1260t^3(r-t)^3(s-t)^3(t-1)^3i!}(210r^3s^3t^2-$
$210r^3s^3t+42r^3s^3-294r^3s^2t^3+126r^3s^2t^2+138r^3s^2t-48r^3s^2+294r^3st^3-$
$294r^3st^2+30r^3st+15r^3s-84r^3t^3+105r^3t^2-30r^3t-294r^2s^3t^3+126r^2s^3t^2+$
$138r^2s^3t-48r^2s^3+378r^2s^2t^4+126r^2s^2t^3-342r^2s^2t^2-72r^2s^2t+60r^2s^2-$
$378r^2st^4+66r^2st^3+306r^2st^2-65r^2st-20r^2s+108r^2t^4-39r^2t^3-90r^2t^2+40r^2t+$
$294rs^3t^3-294rs^3t^2+30rs^3t+15rs^3-378rs^2t^4+66rs^2t^3+306rs^2t^2-65rs^2t-$
$20rs^2+432rst^4-366rst^3-65rst^2+51rst+7rs-135rt^4+150rt^3-14rt^2-14rt-$
$84s^3t^3+105s^3t^2-30s^3t+108s^2t^4-39s^2t^3-90s^2t^2+40s^2t-135st^4+150st^3-$
$14st^2-14st+45t^4-63t^3+21t^2)-\sum_{i=0}^{\infty}\frac{h^{i+1}y_n^{(i+4)}}{1260(r-1)^3(s-1)^3(t-1)^3i!}(630r^3s^3t^3-$
$1722r^3s^3t^2+1470r^3s^3t-420r^3s^3-1722r^3s^2t^3+4662r^3s^2t^2-4026r^3s^2t+$
$1164r^3s^2+1470r^3st^3-4026r^3st^2+3561r^3st-1050r^3s-420r^3t^3+1164r^3t^2-$
$1050r^3t+315r^3-1722r^2s^3t^3+4662r^2s^3t^2-4026r^2s^3t+1164r^2s^3+4662r^2s^2t^3-$
$12618r^2s^2t^2+11034r^2s^2t-3228r^2s^2-4026r^2st^3+11034r^2st^2-9847r^2st+$
$2930r^2s+1164r^2t^3-3228r^2t^2+2930r^2t-885r^2+1470rs^3t^3-4026rs^3t^2+$
$3561rs^3t-1050rs^3-4026rs^2t^3+11034rs^2t^2-9847rs^2t+2930rs^2+3561rst^3-$
$9847rst^2+8913rst-2686rs-1050rt^3+2930rt^2-2686rt+819r-420s^3t^3+$
$1164s^3t^2-1050s^3t+315s^3+1164s^2t^3-3228s^2t^2+2930s^2t-885s^2-1050st^3+$
$2930st^2-2686st+819s+315t^3-885t^2+819t-252)-\frac{h^2y_n^{v}}{2520r^2s^2t^2}(210r^2s^2t^2-$
$168r^2s^2t+42r^2s^2-168r^2st^2+168r^2st-48r^2s+42r^2t^2-48r^2t+15r^2-168rs^2t^2+$

$$168rs^2t-48rs^2+168rst^2-192rst+60rs-48rt^2+60rt-20r+42s^2t^2-48s^2t+15s^2-48st^2+60st-20s+15t^2-20t+7)+\sum_{i=0}^{\infty}\frac{r^ih^{i+2}y_n^{(i+5)}}{2520r^2(r-s)^2(r-t)^2(r-1)^2i!}(10r+20s+20t-42s^2t^2-30rs-30rt-60st+24rs^2+24rt^2+48st^2+48s^2t-15s^2-15t^2-84rst^2-84rs^2t+84rs^2t^2+96rst-7)+\sum_{i=0}^{\infty}\frac{s^ih^{i+2}y_n^{(i+5)}}{2520s^2(r-s)^2(s-t)^2(s-1)^2i!}(20r+10s+20t-42r^2t^2-30rs-60rt-30st+24r^2s+48rt^2+48r^2t+24st^2-15r^2-15t^2-84rst^2-84r^2st+84r^2st^2+96rst-7)+\sum_{i=0}^{\infty}\frac{t^ih^{i+2}y_n^{(i+5)}}{2520t^2(r-t)^2(s-t)^2(t-1)^2i!}(20r+20s+10t-42r^2s^2-60rs-30rt-30st+48rs^2+48r^2s+24r^2t+24s^2t-15r^2-15s^2-84rs^2t-84r^2st+84r^2s^2t+96rst-7)+\sum_{i=0}^{\infty}\frac{h^{i+2}y_n^{(i+5)}}{2520(r-1)^2(s-1)^2(t-1)^2i!}(210r^2s^2t^2-252r^2s^2t+84r^2s^2-252r^2st^2+336r^2st-120r^2s+84r^2t^2-120r^2t+45r^2-252rs^2t^2+336rs^2t-120rs^2+336rst^2-480rst+180rs-120rt^2+180rt-70r+84s^2t^2-120s^2t+45s^2-120st^2+180st-70s+45t^2-70t+28).$$

Similar to the procedure of finding the order in first and second derivative cases we obtain $\bar{D'''}_0=\bar{D'''}_1=...=\bar{D'''}_{13}=0$ and $\bar{D'''}_{14}\neq 0$ by comparing the coefficients of h^j and $y^{(j)}$. Therefore, the third derivative of the main block has order $[10,10,10,10]^T$ with the following vector of error constants $\bar{D'''}_{14}=[\bar{D'''}_{14_1},\bar{D'''}_{14_2},\bar{D'''}_{14_3},\bar{D'''}_{14_4}]^T$ where

$$\bar{D'''}_{14_1}=\frac{r^5}{50295168000}(28r^6-77r^5s-77r^5t-77r^5+55r^4s^2+220r^4st+220r^4s+55r^4t^2+220r^4t+55r^4-165r^3s^2t-165r^3s^2-165r^3st^2-660r^3st-165r^3s-165r^3t^2-165r^3t+132r^2s^2t^2+528r^2s^2t+132r^2s^2+528r^2st^2+528r^2st+132r^2t^2-462rs^2t^2-462rs^2t-462rst^2+462s^2t^2),$$

$$\bar{D'''}_{14_2}=\frac{s^5}{50295168000}(55r^2s^4-165r^2s^3t-165r^2s^3+132r^2s^2t^2+528r^2s^2t+132r^2s^2-462r^2st^2-462r^2st+462r^2t^2-77rs^5+220rs^4t+220rs^4-165rs^3t^2-660rs^3t-165rs^3+528rs^2t^2+528rs^2t-462rst^2+28s^6-77s^5t-77s^5+55s^4t^2+220s^4t+55s^4-165s^3t^2-165s^3t+132s^2t^2),$$

$\bar{D'''}_{14_3} = \frac{t^5}{50295168000}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)$,

$\bar{D'''}_{14_4} = \frac{1}{50295168000}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$.

5.2.1.2 Zero-Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

Referring to the main block, first, second and third derivatives block. Substituting $m = 4$ and $z = 3$ in (5.11), (5.20), (5.25) and(5.30) respectively, the following first characteristic polynomials are produced

$$\begin{aligned}
\psi^{[4]_3}(q) &= |qI_4 - \hat{M}_1^{[4]_3}| \\
&= \left| q\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1)
\end{aligned}$$

$$\begin{aligned}
\psi'^{[4]_3}(q) &= |qI_4 - M_2'^{[4]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1) \\
\psi''^{[4]_3}(q) &= |qI_4 - M_3''^{[4]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1) \\
\psi'''^{[4]_3}(q) &= |qI_4 - M_3'''^{[4]_3}| \\
&= \left| q \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| \\
&= q^3(q-1).
\end{aligned}$$

After that, each of these characteristic polynomials is equated to 0, and this gives $q = \{0,0,0,1\}$, which implies that one-step HBM with three off-step points is zero-stable.

5.2.1.3 Consistency of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

Following from Definition 2.4.4 that the main block (5.11) and its derivatives (5.20), (5.25) and (5.30) are consistent.

5.2.1.4 Convergence of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

Using Theorem 2.1 we conclude that the main block (5.11) and its derivatives (5.20), (5.25) and (5.30) are convergent.

5.2.1.5 Region of Absolute Stability of One-Step Hybrid Block Method with Generalised Three Off-Step Points for Solving Fourth Order ODEs

Substituting $m = 4$ and $z = 3$ in (3.27) gives

$$\begin{aligned} M^{[4]_3}(q) &= (I_4 - q^4\hat{E}_2^{[4]_3} - q^5\hat{K}_2^{[4]_3})^{-1}(\hat{M}_1^{[4]_3} + q\hat{M}_2^{[4]_3} + q^2\hat{M}_3^{[4]_3} + q^3\hat{M}_4^{[4]_3} \\ &\quad + q^4\hat{E}_1^{[4]_3} + q^5\hat{K}_1^{[4]_3}). \end{aligned}$$

Calculating the eigenvalues of the matrix $M^{[4]_3}(q)$ produces the values $\{0, 0, 0, \eta_4^{[4]_3}\}$, where $\eta_4^{[4]_3}$ is a function of q as shown below

$$\eta_4^{[4]_3} = eig(M^{[4]_3}(q)). \tag{5.32}$$

Similarly, substituting $r = \frac{1}{4}, s = \frac{1}{2}$ and $t = \frac{3}{4}$ into Equation (5.32), this yields

$$\eta_4^{[4]_3} = \frac{\sum_{i=0}^{20} c_i q^i}{K\sum_{j=0}^{21} d_j q^j}$$

where $K = 1037629354146162784$ and the values c_i and d_j are shown in tables 5.1-5.2.

Table 5.1
Coefficients of the Eigenvalue $(\eta_4^{[4]_3})$ *for the Matrix* $M^{[4]_3}$

c-value	q^i Coefficients
c_0	-1999125943378661369976823728703803104428032000
c_1	-1999125943378661369976823728703803104428032000
c_2	-999562971689330684988411864351901552214016000
c_3	-333187657229776894996137288117300517404672000
c_4	-78083522390790487043756727396404132472422400
c_5	-12270936643443632883203625005043197725900800
c_6	-994813550529796720162894578620563036569600
c_7	59773830578769195640752956172148093747200
c_8	31534047193241297392512273919387851816960
c_9	4794949827573668397142419401567455150080
c_{10}	359049264395827172053492731789485015040
c_{11}	-3356754007386062537367597774117273600
c_{12}	-4527291309580798166245018591931924480
c_{13}	-632549152682909096776561402023559168
c_{14}	-50983394698341254755895386446446592
c_{15}	-2347009723724383338997462912988928
c_{16}	-35813599709623920574227277044576
c_{17}	8897056249190436050277154788032
c_{18}	741826115832426188473604897088
c_{19}	30585824514148316274163930311
c_{20}	582915589719914249658541800
d-value	q^j Coefficients
d_0	0
d_1	19266281696740250537164800 0
d_2	0
d_3	0
d_4	0
d_5	5024329637381642649600 00
d_6	- 7950292609901631897600 0
d_7	0
d_8	0
d_9	133171866570211983360
d_{10}	- 14458011284982988800
d_{11}	268452685173657600
d_{12}	0
d_{13}	5775969118252032
d_{14}	- 659482112163840
d_{15}	26223671235440
d_{16}	- 529590387200
d_{17}	67083520896
d_{18}	- 8638140672
d_{19}	506275974
d_{20}	- 15909210
d_{21}	189567

The region of absolute stability can be obtained by sketching the function $(\eta_4^{[4]_3})$ in Matlab as represented by the dark area in Figure 5.2.

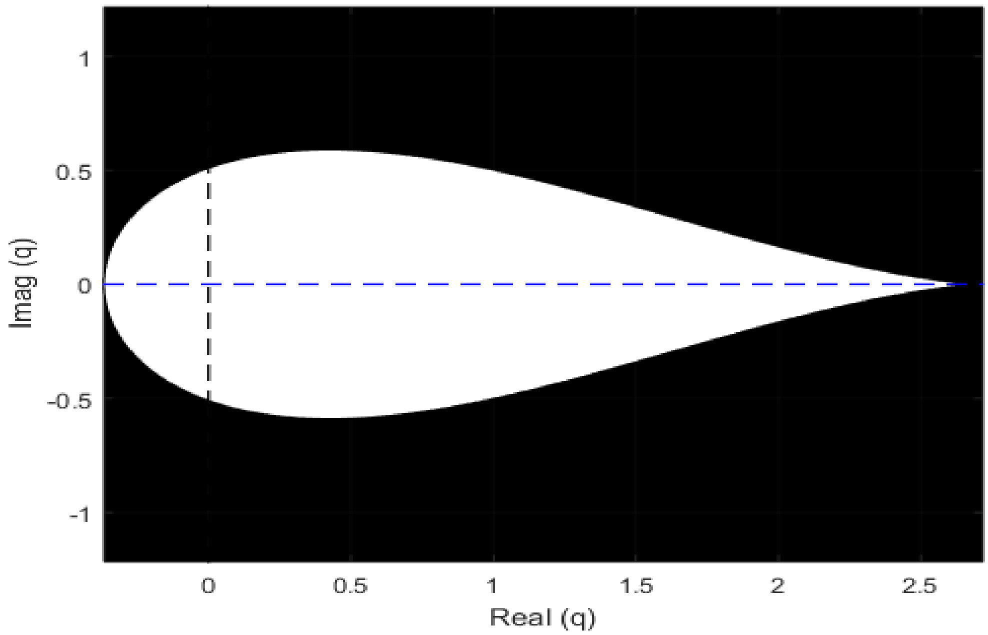


Figure 5.2. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{4}, s = \frac{1}{2}$ and $t = \frac{3}{4}$ for Fourth Order ODEs

Following the same argument as before we can obtain asimilar graph of the function $(\eta_4^{[4]_3})$ for different values of r, s and t. The region of absolute stability again is shown in dark area in Figure 5.3.

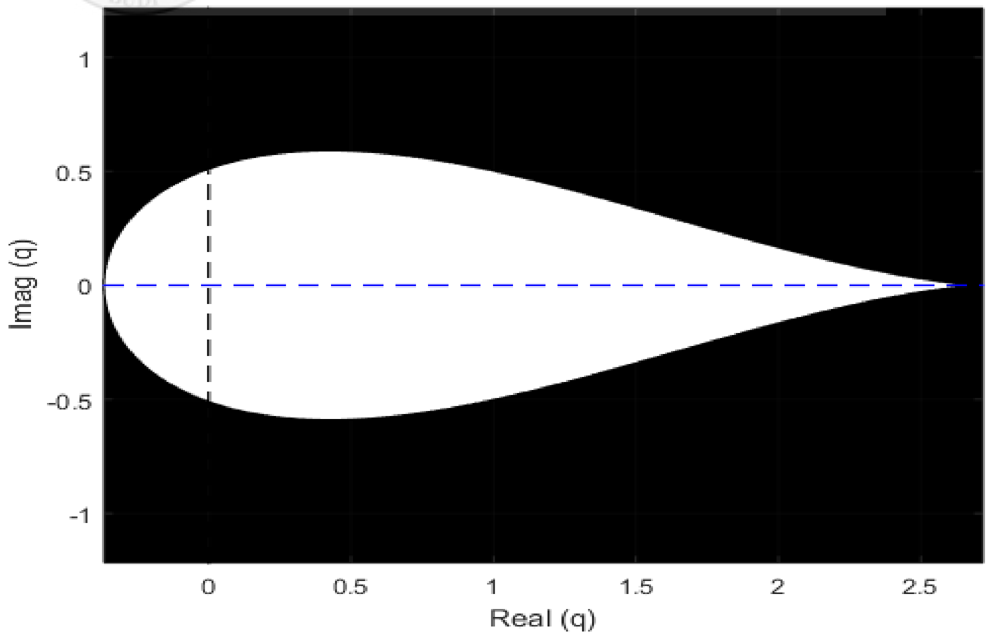


Figure 5.3. Region of Absolute Stability of One-Step HBM with Three Off-Step Points $r = \frac{1}{7}, s = \frac{3}{7}$ and $t = \frac{5}{7}$ for Fourth Order ODEs

Thereupon, the region of absolute stability is plotted for different sets of off-step points and yet we still obtain the same area as detected above.

In the next section, we use a similar argument to third order ODEs and described the procedure of transforming a BVPs of fourth order ODEs to the equivalent IVPs.

5.3 Transforming Boundary Value Problems of Fourth Order ODEs to the Equivalent Initial Value Problems Using Shooting Method

As considered in Sections 3.5 and 4.4, a similar procedure is applied, with some changes to Steps 1 and 2. As for the general fourth order BVPs involving only one missing condition the problem will be similar to the third order BVPs and can be described as follows

$$y^{(iv)} = f(x,y,y',y'',y''') \text{ for } a \leq x \leq b, \tag{5.33}$$

subject to one of the boundary conditions

$$y(a) = \alpha,\ y'(a) = \alpha_1,\ y''(a) = \alpha_2,\ y(b) = \beta. \tag{5.34}$$

Equation (5.33) will be converted to a fourth order IVP with initial conditions

$$y(a) = \alpha,\ y'(a) = \alpha_1,\ y''(a) = \alpha_2,\ y'''(a) = t, \tag{5.35}$$

and the derivative of Equation (5.33) with respect to t, i.e

$$\frac{\partial y^{(iv)}(x,t)}{\partial t} = v^{(iv)}(x,t) = f_y(x,y,y',y'',y''')v(x,t) + f_{y'}(x,y,y',y'',y''')v'(x,t) +$$

$$f_{y''}(x,y,y',y'',y''')v''(x,t) + f_{y'''}(x,y,y',y'',y''')v'''(x,t)$$

with initial conditions

$$v(a,t) = 0,\ v'(a,t) = 0,\ v''(a,t) = 0,\ v'''(a,t) = 1\ .$$

It is observed that not all boundary conditions for fourth order BVPs were considered. Thus, any future case encountered in the following section, we shall deal with it according to the given boundary conditions.

5.4 Numerical Results for Solving Fourth Order ODEs

To solve fourth order ODEs, in this section, we introduced a fifth derivative to show the numerical results attained from driving the one-step HBMs through the approach of interpolation and collocation. The performance of HBM and the accuracy of these results are examined by comparing the HBMs with the existing numerical methods. Hence, a specific set of three off-step points are randomly chosen. The values $r = \frac{1}{4}, s = \frac{1}{2}$ and $t = \frac{3}{4}$ are substituted into Equations (5.12)-(5.15), (5.16)-(5.19), (5.21)-(5.24) and (5.26)-(5.29) to obtain the block and derivatives as below

$$
\begin{aligned}
y_{n+\frac{1}{4}} &= y_n + \frac{hy'_n}{4} + \frac{h^2y''_n}{32} + \frac{h^3y'''_n}{384} + \frac{h^4}{1912940789760}(228729193f_n + 4309327f_{n+1} \\
&\quad -9940960f_{n+\frac{1}{4}} + 44902728f_{n+\frac{1}{2}} + 43350752f_{n+\frac{3}{4}}) - \frac{h^5}{1275293859840}(161893g_{n+1} \\
&\quad -2925919g_n + 14016352g_{n+\frac{1}{4}} + 12680604g_{n+\frac{1}{2}} + 3547712g_{n+\frac{3}{4}})
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{1}{2}} &= y_n + \frac{hy'_n}{2} + \frac{h^2y''_n}{8} + \frac{h^3y'''_n}{48} + \frac{h^4}{622702080}(815739f_n + 23099f_{n+1} \\
&\quad +276736f_{n+\frac{1}{4}} + 270270f_{n+\frac{1}{2}} + 235776f_{n+\frac{3}{4}}) - \frac{h^5}{830269440}(1733g_{n+1} \\
&\quad -24293g_n + 166464g_{n+\frac{1}{4}} + 140544g_{n+\frac{1}{2}} + 38336g_{n+\frac{3}{4}})
\end{aligned}
$$

$$
\begin{aligned}
y_{n+\frac{3}{4}} &= y_n + \frac{3hy'_n}{4} + \frac{9h^2y''_n}{32} + \frac{9h^3y'''_n}{128} + \frac{27h^4}{2624061440}(481701f_n + 15939f_{n+1} \\
&\quad +356256f_{n+\frac{1}{4}} + 261144f_{n+\frac{1}{2}} + 166240f_{n+\frac{3}{4}}) - \frac{27h^5}{5248122880}(1791g_{n+1} \\
&\quad -22597g_n + 165312g_{n+\frac{1}{4}} + 151524g_{n+\frac{1}{2}} + 40096g_{n+\frac{3}{4}})
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} &= y_n + hy'_n + \frac{h^2y''_n}{2} + \frac{h^3y'''_n}{6} + \frac{h^4}{29189160}(363988f_n + 13675f_{n+1} \\
&\quad +372992f_{n+\frac{1}{4}} + 306072f_{n+\frac{1}{2}} + 159488f_{n+\frac{3}{4}}) - \frac{h^5}{4864860}(127g_{n+1} \\
&\quad -1459g_n + 10768g_{n+\frac{1}{4}} + 10422g_{n+\frac{1}{2}} + 2864g_{n+\frac{3}{4}})
\end{aligned}
$$

$$y'_{n+\frac{1}{4}} = y'_n + \frac{hy''_n}{4} + \frac{h^2 y'''_n}{32} + \frac{h^3}{12262440960}(21033953 f_n + 487679 f_{n+1} + 231520 f_{n+\frac{1}{4}} + 5248800 f_{n+\frac{1}{2}} + 4931488 f_{n+\frac{3}{4}}) - \frac{h^4}{8174960640}(18311 g_{n+1} - 292457 g_n + 1669520 g_{n+\frac{1}{4}} + 1450476 g_{n+\frac{1}{2}} + 402640 g_{n+\frac{3}{4}})$$

$$y'_{n+\frac{1}{2}} = y'_n + \frac{hy''_n}{2} + \frac{h^2 y'''_n}{8} + \frac{h^3}{15966720}(139513 f_n + 4455 f_{n+1} + 85248 f_{n+\frac{1}{4}} + 57600 f_{n+\frac{1}{2}} + 45824 f_{n+\frac{3}{4}}) - \frac{h^4}{10644480}(167 g_{n+1} - 2169 g_n + 16000 g_{n+\frac{1}{4}} + 13860 g_{n+\frac{1}{2}} + 3712 g_{n+\frac{3}{4}})$$

$$y'_{n+\frac{3}{4}} = y'_n + \frac{3hy''_n}{4} + \frac{9h^2 y'''_n}{32} + \frac{9h^3}{50462720}(119843 f_n + 4413 f_{n+1} + 126240 f_{n+\frac{1}{4}} + 95904 f_{n+\frac{1}{2}} + 47840 f_{n+\frac{3}{4}}) - \frac{27h^4}{100925440}(165 g_{n+1} - 1931 g_n + 14448 g_{n+\frac{1}{4}} + 14148 g_{n+\frac{1}{2}} + 3760 g_{n+\frac{3}{4}})$$

$$y'_{n+1} = y'_n + hy''_n + \frac{h^2 y'''_n}{2} + \frac{h^3}{26730}(1061 f_n + 50 f_{n+1} + 1408 f_{n+\frac{1}{4}} + 1296 f_{n+\frac{1}{2}} + 640 f_{n+\frac{3}{4}}) - \frac{h^4}{249480}(25 g_{n+1} - 244 g_n + 1792 g_{n+\frac{1}{4}} + 1800 g_{n+\frac{1}{2}} + 512 g_{n+\frac{3}{4}})$$

$$y''_{n+\frac{1}{4}} = y''_n + \frac{hy'''_n}{4} + \frac{h^2}{766402560}(13011695 f_n + 382169 f_{n+1} + 2371696 f_{n+\frac{1}{4}} + 4293432 f_{n+\frac{1}{2}} + 3891088 f_{n+\frac{3}{4}}) - \frac{h^3}{510935040}(14339 g_{n+1} - 198401 g_n + 1410560 g_{n+\frac{1}{4}} + 1152972 g_{n+\frac{1}{2}} + 316720 g_{n+\frac{3}{4}})$$

$$y''_{n+\frac{1}{2}} = y''_n + \frac{hy'''_n}{2} + \frac{h^2}{35640}(1400 f_n + 52 f_{n+1} + 1568 f_{n+\frac{1}{4}} + 891 f_{n+\frac{1}{2}} + 544 f_{n+\frac{3}{4}}) - \frac{h^3}{1330560}(109 g_{n+1} - 1277 g_n + 9840 g_{n+\frac{1}{4}} + 9600 g_{n+\frac{1}{2}} + 2448 g_{n+\frac{3}{4}})$$

$$y''_{n+\frac{3}{4}} = y''_n + \frac{3hy'''_n}{4} + \frac{9h^2}{3153920}(21675 f_n + 941 f_{n+1} + 32944 f_{n+\frac{1}{4}} + 30888 f_{n+\frac{1}{2}} + 12112 f_{n+\frac{3}{4}}) - \frac{9h^3}{6307840}(105 g_{n+1} - 1083 g_n + 8016 g_{n+\frac{1}{4}} + 8532 g_{n+\frac{1}{2}} + 2560 g_{n+\frac{3}{4}})$$

$$y''_{n+1} = y''_n + hy'''_n + \frac{h^2}{374220}(31765f_n + 3457f_{n+1} + 55808f_{n+\frac{1}{4}} + 61776f_{n+\frac{1}{2}} + 34304f_{n+\frac{3}{4}}) - \frac{h^3}{124740}(50g_{n+1} - 269g_n + 1856g_{n+\frac{1}{4}} + 1800g_{n+\frac{1}{2}} + 448g_{n+\frac{3}{4}})$$

$$y'''_{n+\frac{1}{4}} = y'''_n + \frac{h}{17418240}(1539551f_n + 59681f_{n+1} + 1429936f_{n+\frac{1}{4}} + 711936f_{n+\frac{1}{2}} + 613456f_{n+\frac{3}{4}}) - \frac{h^2}{11612160}(2237g_{n+1} - 26051g_n + 249656g_{n+\frac{1}{4}} + 183708g_{n+\frac{1}{2}} + 49720g_{n+\frac{3}{4}})$$

$$y'''_{n+\frac{1}{2}} = y'''_n + \frac{h}{272160}(24463f_n + 1153f_{n+1} + 52928f_{n+\frac{1}{4}} + 44928f_{n+\frac{1}{2}} + 12608f_{n+\frac{3}{4}}) - \frac{h^2}{181440}(43g_{n+1} - 421g_n + 3040g_{n+\frac{1}{4}} + 4536g_{n+\frac{1}{2}} + 992g_{n+\frac{3}{4}})$$

$$y'''_{n+\frac{3}{4}} = y'''_n + \frac{3h}{71680}(2167f_n + 137f_{n+1} + 4912f_{n+\frac{1}{4}} + 6912f_{n+\frac{1}{2}} + 3792f_{n+\frac{3}{4}}) - \frac{3h^2}{143360}(15g_{n+1} - 113g_n + 744g_{n+\frac{1}{4}} + 756g_{n+\frac{1}{2}} + 488g_{n+\frac{3}{4}})$$

$$y'''_{n+1} = y'''_n + \frac{h}{17010}(1601f_n + 1601f_{n+1} + 4096f_{n+\frac{1}{4}} + 5616f_{n+\frac{1}{2}} + 4096f_{n+\frac{3}{4}}) + \frac{h^2}{11340}(29g_n - 29g_{n+1} - 128g_{n+\frac{1}{4}} + 128g_{n+\frac{3}{4}}).$$

Also in this section the new HBM method produced is an examples of HBM with three specific off-step points by replacing the values $r = \frac{1}{4}$, $s = \frac{1}{2}$ and $t = \frac{3}{4}$ are substituted into Equations (5.12)-(5.15), (5.16)-(5.19), (5.21)-(5.24) and (5.26)-(5.29) respectively. For different values of r, s and t in the previous equations we shall achieve a HBM with different specific off-step points.

5.5 Test Problems and Numerical Results

Following Section 4.6, the same notations were also applied, and different types of IVPs and BVPs were tested using the developed HBM in this chapter as shown below:

Problem 15: $y^{(iv)} + xy = -(8+7x+x^3)e^x,\ y(0) = y(1) = 0,$

$y''(0) = 0, y''(1) = -4e,\ x \in [0,1].$

Exact solution: $y(x) = x(1-x)e^x$

Source: (Chen & Li, 2012)

Problem 16: $EIy^{(iv)} = k(x),\ y(0) = 0, y'(0) = 0,$

$y''(L) = 0, y'''(L) = 0,\ x \in [0,L].$

This cantilever beam of length L with both ends fixed, distributed load, $k(x)$, modulus of elasticity E and the moment of inertia I. The problem is solved for $k(x) = x$, $L = 1$ and $EI = 1$.

Exact solution: $y(x) = \dfrac{1}{120}(20x^2 - 10x^3 + x^5)$

Source: (Jator, 2008a)

Problem 17: $y^{(iv)} - (y^{'})^2 + yy^{''} + 4x^2 - e^x(1-4x+x^2) = 0, \quad y(0) = 1,$

$y^{'}(0) = 1,\ y^{''}(0) = 3,\ y^{'''}(0) = 1,\ x \in [0,1].$

Exact solution: $y(x) = x^2 + e^x$

Source: (Olabode & Omole, 2015)

Problem 18: $y^{(iv)} - 4y^{''} = 0,\ y(0) = 1,\ y^{'}(0) = 3, \quad y^{''}(0) = 0,$

$y^{'''}(0) = 16,\ x \in [0,1].$

Exact solution: $y(x) = 1 - x + e^{2x} - e^{-2x}$

Source: (Awoyemi, Kayode, & Adoghe, 2015)

Problem 19: $y^{(iv)} = 2y''' - y'',\ y(0) = y''(0) = 1,\ y(1) = y''(1) = e,\ x \in [0,1].$

Exact solution: $y(x) = e^x$

Source: (Taiwo & Ogunlaran, 2011)

Problem 20: $y^{(iv)} = 2y'' - y - 8e^x,\ y(0) = y(1) = 0,\ y''(0) = 0,$

$y''(1) = -4e,\ x \in [0,1].$

Exact solution: $y(x) = x(1-x)e^x$

Source: (Kelesoglu, 2014)

The developed HBM is capable of working for larger intervals for solving the test problems in this section. In order to compare with the existing methods, only the numerical results on the same intervals are considered. The numerical results for the test problems are obtained using the HBMs with three specific off-step points in the presence of fifth derivative and the existing methods for the purpose of comparison. The recent notations used in Chapters 3-4 are also adopted in the following tables.

Table 5.2

Comparison of One-Step HBM with HAFDS of order 8 for solving Problem 15

h		HBM with $r=\frac{1}{5}, s=\frac{2}{5}, t=\frac{3}{5}$	HAFDS
2^{-3}	Exact solution	0.412180317675032050	N/A
	Computed solution	0.412180317674994750	N/A
	Max Error	3.73E(-14)	6.90E(-08)
2^{-4}	Exact solution	0.412180317675032050	N/A
	Computed Solution	0.412180317674978870	N/A
	Max Error	5.31E(-14)	6.09E(-10)
2^{-5}	Exact solution	0.412180317675032050	N/A
	Computed Solution	0.412180317674971150	N/A
	Max Error	6.08E(-14)	4.55E(-12)

Table 5.3

Comparison of One-Step HBM with NIFOBVP of order 6 for solving Problem 16

x	Exact Solution	Computed Solution (HBM)	HBM with $r=\frac{1}{4}, s=\frac{1}{2}, t=\frac{3}{4}$	NIFOBVP
0	0.000000000000000000	0.000000000000000000	0.00000E(+00)	0.00000E(+00)
$\frac{1}{8}$	0.002441660563151041	0.002441660563151041	4.33680E(-19)	2.38524E(-17)
$\frac{2}{8}$	0.009122721354166667	0.009122721354166665	1.73472E(-18)	3.46945E(-17)
$\frac{3}{8}$	0.019104766845703124	0.019104766845703121	3.46944E(-18)	1.11022E(-16)
$\frac{4}{8}$	0.031510416666666666	0.031510416666666659	6.93889E(-18)	2.28983E(-16)
$\frac{5}{8}$	0.045553843180338539	0.045553843180338532	6.93889E(-18)	3.12250E(-16)
$\frac{6}{8}$	0.060571289062499999	0.060571289062499985	1.38777E(-17)	3.67761E(-16)
$\frac{7}{8}$	0.076051584879557294	0.076051584879557280	1.38777E(-17)	4.44089E(-16)
1	0.091666666666666660	0.091666666666666646	1.38777E(-17)	5.13478E(-16)

Table 5.4

Comparison of One-Step HBM with THOSM of order 5 and IHBNTM of order 5 for Solving Problem 17

x		HBM with $r=\frac{1}{7}, s=\frac{3}{7}, t=\frac{5}{7}$	THOSM	IHBNTM
0.0031250	Exact solution	1.003139653527739000	1.003139653527739000	1.003139653527739000
	Computed solution	1.003139653527739000	1.003139653527739300	1.003139653526590265
	Error	0000000E(+00)	2.220446E(-16)	1.148884E(-12)
0.0062500	Exact solution	1.006308634503762000	1.006308634503762000	1.006308634503762000
	Computed Solution	1.006308634503762000	1.006308634503762200	1.006308634484910542
	Error	0000000E(+00)	2.220446E(-16)	1.8851468E(-11)
0.0093750	Exact solution	1.009506973589070900	1.009506973589070900	1.009506973589070900
	Computed solution	1.009506973589070900	1.009506973589071200	1.009506973491318106
	Error	0000000E(+00)	2.220446E(-16)	9.7752980E(-11)
0.0125000	Exact solution	1.012734701540634500	1.012734701540634500	1.012734701540634500
	Computed solution	1.012734701540634300	1.012734701540634300	1.012734701224875248
	Error	2.220446E(-16)	2.220446E(-16)	3.15759129E(-10)
0.0156250	Exact Solution	1.015991849211685900	1.015991849211685900	1.015991849211685900
	Computed solution	1.015991849211685900	1.015991849211680800	1.015991848424806972
	Error	0000000E(+00)	2.220446E(-16)	1.15463E(-10)
0.0187500	Exact Solution	1.019278447552026500	1.019278447552026500	1.019278447552026500
	Computed solution	1.019278447552026500	1.019278447552020500	1.019278445888007693
	Error	0000000E(+00)	2.220446E(-16)	1.66401853E(-9)
0.0218750	Exact solution	1.022594527608326400	1.022594527608326400	1.022594527608326400
	Computed solution	1.022594527608326400	1.022594527608326400	1.022594524466586058
	Error	0000000E(+00)	0000000E(+00)	3.14174019E(-9)
0.0250000	Exact solution	1.025940120524429000	1.025940120524429000	1.025940120524429000
	Computed solution	1.025940120524429000	1.025940120524429200	1.025940115065457591
	Error	0000000E(+00)	2.220446E(-16)	5.45897125E(-9)
0.0281250	Exact solution	1.029315257541653800	1.029315257541653800	1.029315257541653800
	Computed Solution	1.029315257541654000	1.029315257541654000	1.029315248639974225
	Error	2.220446E(-16)	2.220446E(-16)	8.90167956E(-9)
0.0312500	Exact solution	1.032719969999102800	1.032719969999102800	1.032719969999102800
	Computed solution	1.032719969999103000	1.032719969999102800	1.032719956193600508
	Error	2.220446E(-16)	0000000E(+00)	1.38055022E(-8)

Table 5.5

Comparison of One-Step HBM with OSTHBM of order 5 for Solving Problem 18

x		HBM with $r=\frac{2}{9}, s=\frac{4}{9}, t=\frac{7}{9}$	OSTHBM
0.0031250	Exact solution	1.009375081380367200	1.009375081380367200
	Computed solution	1.009375081380367200	1.009375081380367000
	Error	0000000E(+00)	2.22044E(-16)
0.0062500	Exact solution	1.018750651046753200	1.018750651046753200
	Computed Solution	1.018750651046753000	1.018750651046753000
	Error	2.220446E(-16)	2.22044E(-16)
0.0093750	Exact solution	1.028127197304249400	1.028127197304249400
	Computed solution	1.028127197304249200	1.028127197304249000
	Error	2.220446E(-16)	4.440892E(-16)
0.0125000	Exact solution	1.037505208496096300	1.037505208496096300
	Computed solution	1.037505208496096100	1.037505208496095800
	Error	2.220446E(-16)	4.440892E(-16)
0.0156250	Exact Solution	1.046885173022758400	1.046885173022758400
	Computed solution	1.046885173022758600	1.046885173022757700
	Error	2.220446E(-16)	6.661338E(-16)
0.0187500	Exact Solution	1.056267579361003000	1.056267579361003000
	Computed solution	1.056267579361003200	1.056267579361001900
	Error	2.220446E(-16)	1.110223E(-15)
0.0218750	Exact solution	1.065652916082981100	1.065652916082981100
	Computed solution	1.065652916082980900	1.065652916082978400
	Error	2.220446E(-16)	2.664535E(-15)
0.0250000	Exact solution	1.075041671875310000	-1.075041671875310000
	Computed solution	1.075041671875310000	1.075041671875305500
	Error	0000000E(+00)	4.440892E(-15)
0.0281250	Exact solution	1.084434335558167600	-1.084434335558167600
	Computed Solution	1.084434335558167800	1.084434335558160700
	Error	2.220446E(-16)	6.883383E(-15)
0.0312500	Exact solution	1.093831396104383300	1.093831396104383300
	Computed solution	1.093831396104383700	1.093831396104372900
	Error	4.440892E(-16)	1.043610E(-14)

Table 5.6

Comparison of One-Step HBM with NPSM of order 4 for solving Problem 19

x	Exact Solution	Computed Solution (HBM)	HBM with $r = \frac{1}{8}, s = \frac{1}{4}, t = \frac{5}{8}$	NPSM
0.0	0.000000000000000000	0.000000000000000000	0.000E(+00)	0.000E(+00)
0.2	1.221402758160169900	1.221402758160169900	0.000E(+00)	2.008E(-05)
0.4	1.491824697641270300	1.491824697641270300	0.000E(+00)	3.373E(-05)
0.6	1.822118800390508900	1.822118800390509100	2.220E(-16)	3.564E(-05)
0.8	2.225540928492467400	2.225540928492467900	4.440E(-16)	2.339E(-05)
1.0	2.718281828459045100	2.718281828459045500	4.440E(-16)	N/A

Table 5.7

Comparison of One-Step HBM with ADM for solving Problem 20

x	Exact Solution	Computed Solution (HBM)	HBM with $r = \frac{1}{10}, s = \frac{3}{10}, t = \frac{1}{2}$	ADM
0	0.000000000000000000	0.000000000000000000	0.00000E(+00)	0.00000E+00)
0.1	0.099465382626808305	0.195424441305626390	4.44089E(-16)	2.92821E(-15)
0.2	0.195424441305627220	0.009122721354166665	8.32667E(-16)	2.16493E(-15)
0.3	0.283470349590960680	0.283470349590959570	1.11022E(-15)	6.10623E(-16)
0.4	0.358037927433904890	0.358037927433903560	1.33226E(-15)	1.30451E(-13)
0.5	0.412180317675032050	0.412180317675030550	1.49880E(-15)	2.37055E(-12)
0.6	0.437308512093722120	0.437308512093720570	1.55431E(-15)	2.58711E(-11)
0.7	0.422888068568800120	0.422888068568798510	1.60982E(-15)	1.94957E(-10)
0.8	0.356086548558794850	0.356086548558793460	1.38777E(-15)	1.12373E(-09)
0.9	0.221364280004125610	0.221364280004124480	1.13797E(-15)	5.27758E(-09)
1.0	0.000000000000000302	-0.000000000000000411	7.13065E(-16)	2.10884E(-08)

5.6 Comments on the Results

In order to compare the performance of the developed method for solving IVPs and BVPs of fourth order ODEs, we must use the same step sizes and points as employed in the existing methods. Otherwise, the comparison is not valid. Problem 15 is a linear BVP of fourth order ODE. The numerical results show that HBM with three off-step points in the presence of fifth derivative produces better results in terms of accuracy if compared to the existing method. It is interesting to observe that even though the order of accuracy is the same for HBM for solving this problem, there is a little increase in maximum error as the step sizes decrease.

HBM with three off-step points continue to show its advantage when solving a linear BVP in Problem 16. It can be seen from Table 5.3 that the proposed HBM produce less errors than NIFOBVB method.

Problem 17 is a nonlinear IVP of fourth order ODE. Based on the numerical results in Table 5.4, it was found that the accuracy of HBM is comparable with THOSM. The results also reveal that both methods perform better than the existing IHBNTM method. To investigate the performance of HBM further, a linear IVP of fourth order ODEs was considered in Problem 18. Again, the new developed HBM demonstrates its superiority over OSTHBM method, particularly when the values of x increase (refer to Table 5.5).

The superiority of HBM with three off-step points in the presence of fifth derivative is too obvious in solving a linear BVP in Problem 19. The numerical results produced by this new method in Table 5.6 outshines the existing NPSM method. This claim is supported by the numerical results obtained in Table 5.7 for solving linear BVP of fourth order ODE as stated in Problem 20. The numerical results also suggest that the performance of the new HBM follows the similar trend as in Problem 18. HBM outperforms ADM method as the values of x increase.

5.7 Summary

In this chapter, HBM with three generalised off-steps points in the presence of fifth derivative for solving fourth order ODEs directly have been developed using interpolation and collocation approach. The numerical results confirm the superiority of the new method over the existing methods in term of accuracy besides possessing good properties of numerical methods. Furthermore, it can also be concluded from the numerical results that HBM with three off-step points is the best option for solving both IVPs and BVPs of fourth ODEs directly.

CHAPTER SIX
CONCLUSION AND AREA OF FUTURE WORK

This last chapter concludes this thesis by summarising the main contributions that have been accomplished in our study. At the end of this chapter, suggestions for future potential research work to be explored are made.

6.1 Conclusion

Several new methods for solving both IVPs and BVPs of high order ODEs directly have been successfully developed in this study. Based on the research gaps in the literature, three new direct methods for solving second order ODEs were proposed in Chapter Three. These methods are one-step HBMs with generalised one, two and three off-step points. In deriving these methods, interpolation and collocation approach was adopted. In addition, to further improve the accuracy of the developed methods, higher derivative was also included in the derivation. There are several advantages of the new methods. First, these methods are capable of solving both IVPs and BVPs of second order ODEs directly without requiring them to be converted into their equivalent first order systems. Therefore, the number of equations remains the same and thus avoiding the evaluation of extra functions which may contribute to computational burden and increase the complexity of writing the computer codes. Second, the newly developed HBMs have the ability to overcome the zero-stability barrier encountered in block methods by using information at off-step points. As a result, the methods are more reliable. Third, the developed methods are more flexible if compared to the existing ones since generalised off-step points are considered in the derivation. It was discovered that most of the existing methods only focus on specific off-step points. Consequently, new derivations have to be made each time different off-step points are considered. On the other hand, the proposed HBMs do not require new derivations for different off-step points as all possible off-step points have already been taken into consideration. Finally, by taking the full advantage of the existence of higher derivative in the derivation, the proposed methods are able to approximate the solutions of high ODEs more accurately. The investigation on the theoretical properties of the new methods revealed that they are consistent and zero-stable which subsequently implies they are also convergent. The superiority of the proposed methods was proven by their numerical results obtained in solving several IVPs and BVPs of second order ODEs and then comparing with the existing methods. It was also noticed that when more off-step points included in the given interval, the methods produce better accuracy. Impressed by the capabilities and the superb performances of the HBMs developed for solving second order ODEs, similar approach was adopted and extended to solve third and fourth order ODEs. In Chapter Four, two new methods were successfully developed for solving both IVPs and BVPs of third order ODEs directly. These two methods are one-step HBMs with two and three generalised off-step points in the presence of fourth derivative. Meanwhile, in Chapter Five, one-step HBM with three generalised off-step points was proposed to solve fourth order ODEs. In deriving this method, fifth derivative were included. The methods introduced in Chapter Four and Chapter Five also share the same advantages demonstrated by the methods developed earlier in Chapter Three. Hence, the methods developed in this study are viable alternatives for solving high order ODEs.

6.2 Areas for Future Work

This research work has considered the development of one-step HBMs with one, two and three off-step point(s) for solving both IVPs and BVPs of second, third and fourth order ODEs directly. Therefore, future research can consider the development of direct multi-step HBMs for solving high order ODEs. The derivation of the methods in this study is based on constant step size. It would be very interesting to consider variable step size in deriving HBMs for high order ODEs. Future research work can also extend the proposed methods in this study by solving other types of differential equations such as fractional and partial differential equations.

REFERENCES

Abdelrahim, R. (2016). *One-step hybrid block methods with generalised off-step points for solving directly higher order ordinary differential equations* (Unpublished doctoral dissertation). Universiti Utara Malaysia, Malaysia.

Abdelrahim, R., & Omar, Z. (2015). Uniform order one-step hybrid block method with two generalized off-step points for solving third order ordinary differential equations directly. *Global Journal of Pure and Applied Mathematics*, *11*(6), 4809–4823.

Abdullah, A. S., Majid, Z., & Senu, N. (2013). Solving third order boundary value problem using fourth order block method. *Applied Mathematical Sciences*, *7*(53-56), 2629–2645.

Ademiluyi, R. D. M., & Bolarinwa, B. (2014). Modified block method for the direct solution of initial value problems of fourth order ordinary differential equations. *Australian Journal of Basic and Applied Sciences*, *8*(10), 89-394.

Adeniran, O., & Ogundare, B. (2015). An efficient hybrid numerical scheme for solving general second order initial value problems. *International Journal of Applied Mathematical Research*, *4*(2), 411–419.

Adeniyi, R., & Adeyefa, E. (2013). Chebyshev collocation approach for a continuous formulation of implicit hybrid methods for VIPs in second order ODEs. *Journal of Mathematics*, *12*, 9–12.

Adesanya, A., Abdulqadri, B., & Ibrahimb, Y. (2014). Hybrid one-step block method for the solution of third order initial value problems of ordinary differential equations. *International Journal of Applied Mathematics and Computation*, *6*(1), 10–16.

Adesanya, A., Fasasi, M. K., & Anake, T. (2013). Three-steps hybrid block method for the solution of general second order ordinary differential equations. *Journal of Applied Mathematics*, *8*(1 , 2), 1–11.

Adesanya, A., Odekunle, M., & Adeyeye, O. (2012). Continuous block hybrid predictor corrector method for the solution of $y'' = f(x,y,y')$. *International Journal of Mathematics and Soft Computing*, *2*(2), 35–42.

Adesanya, A., Udoh, D. M., & Ajileye, A. M. (2013). A new hybrid block method for the solution of general third order initial value problems of ordinary differential equations. *International Journal of Pure and Applied Mathematics*, *86*(2), 365–375.

Adeyeye, O. (2013). Two-step two-point hybrid methods for general second order differential equations. *African Journal of Mathematics and Computer Science Research*, *6*(10), 191–196.

Adeyeye, O., & Omar, Z. (2016). New uniform order eight hybrid third derivative block method for solving second order initial value problems. *Far East Journal of Mathematical Sciences*, *100*(9), 1515.

Adeyeye, O., & Omar, Z. (2017). Linear block method derived from direct Taylor series expansions for solving linear third order boundary value problems. *Advanced Science, Engineering and Medicine*, *9*(10), 895–900.

Anake, T., Awoyemi, D. O., & Adesanya, A. (2012a). One-step implicit hybrid block method for the direct solution of general second order ordinary differential equations. *International Journal of Applied Mathematics*, *42*(4), 1-5.

Anake, T., Awoyemi, D. O., & Adesanya, A. (2012b). A one-step method for the solution of general second order ordinary differential equations. *International Journal of Science and Technology*, *2*(4), 159–163.

Atkinson, K. (1989). *An introduction to numerical analysis*. New York, NY: John Wiley and Sons.

Atkinson, K., & Han, W. (2004). *Elementary numerical analysis*. New York, NY: John Wiley and Sons.

Awoyemi, D. (2003). A p-stable linear multi-step method for solving general third order ordinary differential equations. *International Journal of Computer Mathematics*, *80*(8), 985–991.

Awoyemi, D., Adebile, E., Adesanya, A., & Anake, T. A. (2011). Modified block method for the direct solution of second order ordinary differential equations. *International Journal of Applied Mathematics and Computation*, *3*(3), 181–188.

Awoyemi, D., Kayode, S. J., & Adoghe, L. (2015). A six-step continuous multistep method for the solution of general fourth order initial value problems of ordinary differential equations. *Journal of Natural Sciences Research*, *5*(5), 131–138.

Badmus, A. (2014). An efficient seven-point hybrid block method for the direct solution of $y'' = f(x,y,y')$. *British Journal of Mathematics & Computer Science*, *4*(19), 2840–2852.

Burden, R., & Faires, J. (2011). *Numerical analysis*. Boston, Massachusetts: Brooks/Cole Cengage Learning.

Butcher, J. C. (1965). A modified multi-step method for the numerical integration of ordinary differential equations. *Journal of the ACM*, *12*(1), 124–135.

Chen, J. (2011). Fast multilevel augmentation methods for nonlinear boundary value problems. *Computers & Mathematics with Applications*, *61*(3), 612–619.

Chen, J., & Li, C. (2012). High accuracy finite difference schemes for linear fourth order boundary value problem and derivatives. *Journal of Information and Computational Science*, *9*(10), 2751–2759.

Chu, M. T., & Hamilton, H. (1987). Parallel solution of ODEs by multiblock methods. *SIAM Journal on Scientific and Statistical Computing*, *8*(3), 342–353.

Enright, W. (1974). Second derivative multi-step methods for stiff ordinary differential equations. *SIAM Journal on Numerical Analysis*, *11*(2), 321–331.

Faires, J., & Burden, R. (2002). *Numerical methods*. Boston, Massachusetts: Brooks/Cole Cengage Learning.

Fatunla, S. (1991). Block methods for second order ODEs. *International journal of computer mathematics*, *41*(1-2), 55–63.

Gbenga, O. B., Olaoluwa, O. E., & Olayemi, O. O. (2015). Hybrid and non-hybrid implicit schemes for solving third order ODEs using block method as predictors. *Mathematical Theory and Modeling*, *5*(3), 10–25.

Gear, C. W. (1965). Hybrid methods for initial value problems in ordinary differential equations. *Journal of the Society for Industrial and Applied Mathematics, Series B: Numerical Analysis*, *2*(1), 69–86.

Gragg, W. B., & Stetter, H. J. (1964). Generalized multi-step predictor-corrector methods. *Journal of the ACM*, *11*(2), 188–209.

Henrici, P. (1962). *Discrete variable methods in ordinary differential equations*. New York, NY: John Wiley and Sons.

Hijazi, M., & Abdelrahim, R. (2017). The numerical computation of three-step hybrid block method for directly solving third order ordinary differential equations. *Global Journal of Pure and Applied Mathematics*, *13*(1), 89–103.

James, A., Adesanya, A., & Joshua, S. (2013). Continuous block method for the solution of second order initial value problems of ordinary differential equation. *International Journal of Pure and Applied Mathematics*, *83*(3), 405–416.

Jator, S. (2008a). Numerical integrators for fourth order initial and boundary value problems. *International Journal of Pure and Applied Mathematics*, *47*(4), 563–576.

Jator, S. (2008b). On the numerical integration of third order boundary value problems by a linear multi-step method. *International Journal of Pure and Applied Mathematics*, *46*(3), 375–388.

Jator, S., Akinfenwa, A., Okunuga, S. A., & Sofoluwe, A. B. (2013). High-order continuous third derivative formulas with block extensions for $y'' = f(x,y,y')$. *International Journal of Computer Mathematics*, *90*(9), 1899–1914.

Jator, S., & Li, J. (2012). An algorithm for second order initial and boundary value problems with an automatic error estimate based on a third derivative method. *Numerical Algorithms*, *59*(3), 333–346.

Kayode, S. (2011). A class of one-point zero-stable continuous hybrid methods for direct solution of second-order differential equations. *African Journal of Mathematics and Computer Science Research*, *4*(3), 93–99.

Kayode, S., & Adeyeye, O. (2011). A 3-step hybrid method for direct solution of second order initial value problems. *Australian Journal of Basic and Applied Sciences*, *5*(12), 2121–2126.

Kayode, S., Duromola, M., & Bolaji, B. (2014). Direct solution of initial value problems of fourth order ordinary differential equations using modified implicit hybrid block method. *Journal of Scientific Research & Reports*, *3*(21), 2792-2800.

Kayode, S., & Obarhua, F. (2015). 3-step y- function hybrid methods for direct numerical integration of second order IVPs in ODEs. *Theoretical Mathematics & Applications*, *5*(1), 39–51.

Kelesoglu, O. (2014). The solution of fourth order boundary value problem arising out of the beam-column theory using Adomian decomposition method. *Mathematical Problems in Engineering*, *2014*, 1–6.

Keller, H. B. (1976). *Numerical solution of two point boundary value problems*. Philadelphia, Pennsylvania: Society for Industrial and Applied Mathematics.

Kolawole, F., Adesanya, A., Momoh, A., & Emmanuel, N. (2014). Continuous hybrid block Stomer Cowell methods for solutions of second order ordinary differential equations. *Journal of Mathematical and Computational Science*, *4*(1), 118-127.

Kolawole, F., Fasansi, M., Remilekun, O. M., & Adesanya, A. (2013). One-step, three hybrid block predictor-corrector method for the solution of $y''' = f(x,y,y',y'')$. *Journal of Applied & Computational Mathematics*, *2*(4), 137-142.

Kuboye, J., & Omar, Z. (2015a). Numerical solution of third order ordinary differential equations using a seven-step block method. *International Journal of Mathematical Analysis*, *9*(15), 743–745.

Kuboye, J., & Omar, Z. (2015b). Numerical solution of third order ordinary differential equations using a seven-step block method. *International Journal of Mathematical Analysis*, *9*(15), 743–745.

Lambert, J. (1973). *Computational methods in ordinary differential equations*. Chichester, West Sussex: John Wiley and Sons.

Lambert, J. (1991). *Numerical methods for ordinary differential systems:the initial value problem*. New York, NY: John Wiley and Sons.

Lambert, J., & Watson, I. (1976). Symmetric multistip methods for periodic initial value problems. *IMA Journal of Applied Mathematics*, *18*(2), 189–202.

Liu, L. B., Liu, H. W., & Chen, Y. (2011). Polynomial spline approach for solving second-order boundary-value problems with Neumann conditions. *Applied Mathematics and Computation*, *217*(16), 6872–6882.

Milne, W. (1953). *Numerical solution of differential equations*. New York, NY: Wiley.

Mohammed, U., & Adeniyi, R. (2014). A three-step implicit hybrid linear multi-step method for the solution of third order ordinary differential equations. *Gen. Math. Notes*, *25*(1), 62–74.

Morrison, D. D., Riley, J. D., & Zancanaro, J. F. (1962). Multiple shooting method for two-point boundary value problems. *Communications of the ACM*, *5*(12), 613–614.

Ngwane, F., & Jator, S. (2012). Block hybrid-second derivative method for stiff systems. *International Journal of Pure and Applied Mathematics*, *80*(4), 543–559.

Olabode, B. (2007). *Some linear multi-step methods for special and general third order initial value problems of ordinary differential equations* (Unpublished doctoral dissertation). Federal University of Technology, Akure, Nigeria.

Olabode, B. (2014). Block multi-step method for the direct solution of third order of ordinary differential equations. *Futa Journal of Research in Sciences*, *9*(2), 194–200.

Olabode, B., & Omole, E. (2015). Implicit hybrid block numerov-type method for the direct solution of fourth-order ordinary differential equations. *American Journal of Computational and Applied Mathematics*, *5*(5), 129–139.

Olabode, B., & Yusuph, Y. (2009). A new block method for special third order ordinary differential equations. *Journal of Mathematics and statistics*, *5*(3), 167.

Omar, Z. (1999). *Parallel block methods for solving higher order ordinary differential equations directly* (Unpublished doctoral dissertation). Universiti Putra Malaysia, Malaysia.

Omar, Z. (2004). Developing parallel 3-point implicit block method for solving second order ordinary differential equations directly. *International Journal of Management Studies*, *11*(1), 91–103.

Omar, Z., & Abdelrahim, R. (2015). Developing a single-step hybrid block method with generalized three off-step points for the direct solution of second order ordinary differential equations. *International Journal of Mathematical Analysis*, *9*(46), 2257–2272.

Omar, Z., & Abdelrahim, R. (2016). Direct solution of fourth order ordinary differential equations using a one-step hybrid block method of order five. *International Journal of Pure and Applied Mathematics*, *109*(4), 763–777.

Omar, Z., & Abdelrahim, R. (2017). A four-step implicit block method with three generalized off-step points for solving fourth order initial value problems directly. *Journal of King Saud University-Science*, *29*(2017), 401–412.

Omar, Z., & Adeyeye, O. (2016). Solving two-point second order boundary value problems using two-step block method with starting and non-starting values. *International Journal of Applied Engineering Research*, *11*(4), 2407–2410.

Omar, Z., & Kuboye, J. (2015). Derivation of block methods for solving second order ordinary differential equations directly using direct integration and collocation approaches. *Indian Journal of Science and Technology*, *8*(12), 1–4.

Omar, Z., & Suleiman, M. (1999). Solving second order ODEs directly using parallel 2-point explicit block method. *Prosiding Kolokium Kebangsaan Pengintegrasian Teknologi Dalam Sains Matematik, Universiti Sains Malaysia*, 390–395.

Phang, P., Majid, Z., Suleiman, M., & Ismail, F. (2013). Solving boundary value problems with neumann conditions using direct method. *World Applied Sciences Journal*, 129–133. doi: 10.5829/idosi.wasj.2013.21.mae.99937

Ramos, H., Kalogiratou, Z., Monovasilis, T., & Simos, T. E. (2015). An optimized two-step hybrid block method for solving general second order initial-value problems. *Numerical Algorithms*, 1–14. doi: 10.1007/s11075-015-0081-8

Sagir, A. (2013a). 2–block 3-point modified numerov block methods for solving ordinary differential equations. *World Academy of Science, Engineering and Technology*, *7*(1), 700–704.

Sagir, A. (2013b). An accurate computation of block hybrid method for solving stiff ordinary differential equations. *World Academy of Science, Engineering and Technology*, *7*(4), 321–324.

Sahi, R., Jator, S., & Khan, N. (2013). Continuous fourth derivative method for third order boundary value problems. *International Journal of Pure and Applied Mathematics*, *85*(5), 907–923.

Taiwo, O. A., & Ogunlaran, O. M. (2011). A non-polynomial spline method for solving linear fourth-order boundary-value problems. *International Journal of Physical Sciences*, *6*(13), 3246–3254.

Vigo-Aguiar, J., & Ramos, H. (2006). Variable stepsize implementation of multi-step methods for $y'' = f(x,y,y')$. *Journal of Computational and Applied Mathematics*, *192*(1), 114–131.

Wend, D. V. (1969). Existence and uniqueness of solutions of ordinary differential equations. *Proceedings of the American Mathematical Society*, *23*(1), 27–33.

Yahaya, Y., Sagir, A., & Tech, M. (2013). An order five implicit 3-stepblock method for solving ordinary differential equations. *The Pacific Journal of Science and Technology*, *14*, 176–181.

Yap, L. K., & Ismail, F. (2015). Block hybrid collocation method with application to fourth order differential equations. *Mathematical Problems in Engineering*, *2015*, 1–6. doi: 10.1155/2015/561489

Yap, L. K., Ismail, F., & Senu, N. (2014). An accurate block hybrid collocation method for third order ordinary differential equations. *Journal of Applied Mathematics*, *2014*, 1-9. doi: 10.1155/2014/549597

Yun, J. H. (2008). A note on three-step iterative method for nonlinear equations. *Applied Mathematics and Computation*, *202*(1), 401–405.

Zhang, W. (2012). *Improved implementation of multiple shooting for BVPs*. (Master's thesis, Computer Science Department, University of Toronto). Retrieved from http://www.cs.toronto.edu/pub/reports/na/Weidong_Zhang_Thesis.pdf.

APPENDIX A

SAMPLE MATLAB CODE OF ONE-STEP HBM FOR SOLVING SECOND ORDER ODES

%This MATLAB code employs the one-step with one-HBM for solving a second order ODE

syms $y1$ $y2$ $z1$ $z2$

$x0 = 0; y0 = 0; z0 = -1; h = 1/10;$

disp ('x – value exact – solution computed – solution error')

$for \quad j = 0 : h : 1;$

$x1 = x0 + 2/5 * h; \; x2 = x0 + h;$

$eqn1 = y0 - y1 + (2*h*z0)/5 + (2389*h^2*z0)/46875 + (397*h^2*z1)/14175 + (794*h^3*z0)/328125 + (9104*h^2*z2)/8859375 - (452*h^3*z1)/118125 - (464*h^3*z2)/2953125 == 0;$

$eqn2 = y0 - y2 + h*z0 + (11*h^2*z0)/60 + (625*h^2*z1)/2268 + (19*h^3*z0)/1680 + (233*h^2*z2)/5670 + (25*h^3*z1)/3024 - (4*h^3*z2)/945 == 0;$

$eqn3 = z0 - z1 + (1643*h*z0)/9375 + (89*h*z1)/405 + (1264*h*z2)/253125 + (89*h^2*z0)/9375 - (73*h^2*z1)/3375 - (64*h^2*z2)/84375 == 0;$

$eqn4 = z0 - z2 + (13*h*z0)/48 + (625*h*z1)/1296 + (20*h*z2)/81 + (h^2*z0)/48 + (25*h^2*z1)/432 - (h^2*z2)/54 == 0;$

$[A, B] = equationsToMatrix([eqn1, eqn2, eqn3, eqn4], [y1, y2, z1, z2]);$

$K = (A \setminus B); err1 = abs(1 - exp(x1) - double(K(1)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x1, double(1 - exp(x1)), double(K(1)), double(err1));$

$err2 = abs(1 - exp(x2) - double(K(2)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x2, double(1 - exp(x2)), double(K(2)), double(err2));$

$x0 = x2; y0 = K(2); z0 = K(4);$end

APPENDIX B

SAMPLE MATLAB CODE OF ONE-STEP HBM FOR SOLVING THIRD ORDER ODES

%This MATLAB code employs the one-step with Two-HBM for solving a third order ODE

clear

clc

syms $y1 \quad y2 \quad y3 \quad z1 \quad z2 \quad z3 \quad v1 \quad v2 \quad v3$

$x0 = 0; y0 = 1; z0 = 0; v0 = -2; N = 10; h = 1/N;$

disp ('x – value exact – solution computed – solution error')

$for \quad j = linspace(0,1,N)$

$x1 = x0 + 1/3*h;$

$x2 = x0 + 2/3*h;$

$x3 = x0 + (h);$

$f = 3*sin(x0);$

$fr = 3*sin(x1);$

$fs = 3*sin(x2);$

$f1 = 3*sin(x3);$

$g = 3*cos(x0);$

$gr = 3*cos(x1);$

$gs = 3*cos(x2);$

$g1 = 3*cos(x3);$

$eqn1 = y0 - y1 + (h*z0)/3 + (62387*f*h^3)/14696640 + (1031*f1*h^3)/7348320 + (89*fr*h^3)/90720 + (439*fs*h^3)/544320 + (1879*g*h^4)/14696640 - (17*g1*h^4)/1469664 - (359*gr*h^4)/816480 - (13*gs*h^4)/77760 + (h^2*v0)/18 == 0;$

$eqn2 = y0 - y2 + (2*h*z0)/3 + (5048*f*h^3)/229635 + (244*f1*h^3)/229635 + (164*fr*h^3)/8505 + (138999988499079*fs*h^3)/19703248369745920 + (172*$

$g*h^4)/229635-(4*g1*h^4)/45927-(4*gr*h^4)/1215-(34*gs*h^4)/25515+$

$(2*h^2*v0)/9 == 0;$

$eqn3 = y0-y3+h*z0+(73*f*h^3)/1344+(609862449539781*f1*h^3)$

$/157625986957967360+(81*fr*h^3)/1120+(182395784908512459*fs*h^3)$

$/5044031582654955520+(13*g*h^4)/6720-(g1*h^4)/3360-(9*gr*h^4)/1120-$

$(9*gs*h^4)/2240+(h^2*v0)/2 == 0;$

$eqn4 = z0-z1+(h*v0)/3+(7536502154204791*f*h^2)/236438980436951040+$

$(3329*f1*h^2)/2449440+(1301*fr*h^2)/90720+(181*fs*h^2)/22680+(371*$

$g*h^3)/349920-(137*g1*h^3)/1224720-(313*gr*h^3)/68040-(89*gs*h^3)/$

$54432 == 0;$

$eqn5 = z0-z2+(2*h*v0)/3+(5731*f*h^2)/76545+(344*f1*h^2)/76545+$

$(296*fr*h^2)/2835+(75754993731993*fs*h^2)/1970324836974592+(206*g*$

$h^3)/76545-(4*g1*h^3)/10935-(20*gr*h^3)/1701-(52*gs*h^3)/8505 == 0;$

$eqn6 = z0-z3+h*v0+(67*f*h^2)/560+(295548725546181*f1*h^2)/$

$15762598695796736+(243*fr*h^2)/1120+(18239578490849715*fs*h^2)/$

$126100789566373888+(g*h^3)/224-(g1*h^3)/840-(9*gr*h^3)/560-(9*gs*$

$h^3)/1120 == 0;$

$eqn7 = v0-v1+(6893*f*h)/54432+(397*f1*h)/54432+(313*fr*h)/2016+$

$(89*fs*h)/2016+(1283*g*h^2)/272160-(163*g1*h^2)/272160-(851*gr*$

$h^2)/30240-(269*gs*h^2)/30240 == 0;$

$eqn8 = v0-v2+(223*f*h)/1701+(20*f1*h)/1701+(20*fr*h)/63+$

$(406574966359773*fs*h)/1970324836974592+(43*g*h^2)/$

$8505-(8*g1*h^2)/8505-(16*gr*h^2)/945-(19*gs*h^2)/945 == 0;$

$eqn9 = v0-v3+(31*f*h)/224+(389541262412043*f1*h)/2814749767106560+$

$(81*fr*h)/224+(81*fs*h)/224+(19*g*h^2)/3360-(19*g1*h^2)/3360-(9*$

$gr*h^2)/1120+(9*gs*h^2)/1120 == 0;$

$[A,B] = equationsToMatrix$

$([eqn1, eqn2, eqn3, eqn4, eqn5, eqn6, eqn7, eqn8, eqn9],$

$[y1, y2, y3, z1, z2, z3, v1, v2, v3]);$

$K = (A \setminus B);$

$err1 = abs(3*cos(x1) + x1^2/2 - 2 - double(K(1)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x1, double(3*cos(x1) + x1^2/2 - 2),$

$double(K(1)), double(err1));$

$err2 = abs(3*cos(x2) + x2^2/2 - 2 - double(K(2)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x2, double(3*cos(x2) + x2^2/2 - 2),$

$double(K(2)), double(err2));$

$err3 = abs(3*cos(x3) + x3^2/2 - 2 - double(K(3)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x3, double(3*cos(x3) + x3^2/2 - 2),$

$double(K(3)), double(err3));$

$x0 = x3; y0 = K(3); z0 = K(6); v0 = K(9);$

end

APPENDIX C

SAMPLE MATLAB CODE OF ONE-STEP HBM FOR SOLVING FOURTH ORDER ODES

%This MATLAB code employs the one-step with Three-HBM for solving fourth order ODE

clear

clc

$syms y1 \quad y2 \quad y3 \quad y4 \quad z1 \quad z2 \quad z3 \quad z4 \quad v1 \quad v2 \quad v3 \quad v4 \quad w1 \quad w2 \quad w3 \quad w4$

$x0 = 0; y0 = 1; z0 = 3; v0 = 0; w0 = 16; N = 10; h = 1/N;$

disp ('x – value exact – solution computed – solution error')

$for j = linspace(0,1,N)$

$x1 = x0 + 1/4 * h;$

$x2 = x0 + 1/2 * h; \;\; x3 = x0 + 3/4 * h; \;\; x4 = x0 + 1 * h;$

$f = 4 * v0; \;\; fr = 4 * v1; \;\; fs = 4 * v2; \;\; ft = 4 * v3; \;\; f1 = 4 * v4;$

$g = 4 * w0; \;\; gr = 4 * w1; \;\; gs = 4 * w2; \;\; gt = 4 * w3; \;\; g1 = 4 * w4;$

$eqn1 = y0 - y1 + (h * z0)/4 + (2970509 * f * h^4)/24843386880 + (391757 * f1 * h^4)/173903708160 - (62131 * fr * h^4)/11955879936 + (15991 * fs * h^4)/681246720 + (1354711 * ft * h^4)/59779399680 + (2925919 * g * h^5)/1275293859840 - (161893 * g1 * h^5)/1275293859840 - (62573 * gr * h^5)/5693276160 - (117413 * gs * h^5)/11808276480 - (7919 * gt * h^5)/2846638080 + (h^2 * v0)/32 + (h^3 * w0)/384 == 0;$

$eqn2 = y0 - y2 + (h * z0)/2 + (271913 * f * h^4)/207567360 + (23099 * f1 * h^4)/622702080 + (1081 * fr * h^4)/2432430 + (fs * h^4)/2304 + (307 * ft * h^4)/810810 + (24293 * g * h^5)/830269440 - (1733 * g1 * h^5)/830269440 - (289 * gr * h^5)/1441440 - (61 * gs * h^5)/360360 - (599 * gt * h^5)/12972960 + (h^2 * v0)/8 + (h^3 * w0)/48 == 0;$

$eqn3 = y0 - y3 + (3 * h * z0)/4 + (1182357 * f * h^4)/238551040 + (5589 * f1 * h^4)/34078720 + (300591 * fr * h^4)/82001920 + (67797 * fs * h^4)/25231360 + (28053 * ft * h^4)/16400384 + (610119 * g * h^5)/5248122880 - (48357 * g1 * h^5)/5248122880 -$

$(9963*gr*h^5)/11714560-(1022787*gs*h^5)/1312030720-(4833*gt*h^5)/$

$23429120+(9*h^2*v0)/32+(9*h^3*w0)/128==0;$

$eqn4=y0-y4+h*z0+(90997*f*h^4)/7297290+(2735*f1*h^4)/5837832+$

$(46624*fr*h^4)/3648645+(109*fs*h^4)/10395+(2848*ft*h^4)/521235+(1459*$

$g*h^5)/4864860-(127*g1*h^5)/4864860-(2692*gr*h^5)/1216215-(193*gs*$

$h^5)/90090-(716*gt*h^5)/1216215+(h^2*v0)/2+(h^3*w0)/6==0;$

$eqn5=z0-z1+(h*v0)/4+(21033953*f*h^3)/12262440960+(487679*f1*$

$h^3)/12262440960+(1447*fr*h^3)/76640256+(135*fs*h^3)/315392+(154109*$

$ft*h^3)/383201280+(26587*g*h^4)/743178240-(18311*g1*h^4)/8174960640-$

$(20869*gr*h^4)/102187008-(40291*gs*h^4)/227082240-(719*gt*h^4)/14598144+$

$(h^2*w0)/32==0;$

$eqn6=z0-z2+(h*v0)/2+(12683*f*h^3)/1451520+(f1*h^3)/3584+(37*fr*$

$h^3)/6930+(5*fs*h^3)/1386+(179*ft*h^3)/62370+(241*g*h^4)/1182720-$

$(167*g1*h^4)/10644480-(25*gr*h^4)/16632-(gs*h^4)/768-(29*gt*h^4)/83160+$

$(h^2*w0)/8==0;$

$eqn7=z0-z3+(3*h*v0)/4+(1078587*f*h^3)/50462720+(39717*f1*h^3)/$

$50462720+(7101*fr*h^3)/315392+(26973*fs*h^3)/1576960+(2691*ft*h^3)/$

$315392+(52137*g*h^4)/100925440-(81*g1*h^4)/1835008-(3483*gr*h^4)/$

$901120-(95499*gs*h^4)/25231360-(1269*gt*h^4)/1261568+(9*h^2*w0)/32==$

$0;$

$eqn8=z0-z4+h*v0+(1061*f*h^3)/26730+(5*f1*h^3)/2673+(64*fr*$

$h^3)/1215+(8*fs*h^3)/165+(64*ft*h^3)/2673+(61*g*h^4)/62370-(5*g1*$

$h^4)/49896-(32*gr*h^4)/4455-(5*gs*h^4)/693-(64*gt*h^4)/31185+(h^2*$

$w0)/2==0;$

$eqn9=v0-v1+(h*w0)/4+(2602339*f*h^2)/153280512+(382169*f1*h^2)/$

$766402560+(148231*fr*h^2)/47900160+(1807*fs*h^2)/322560+(243193*$

$ft*h^2)/47900160+(28343*g*h^3)/72990720-(14339*g1*h^3)/510935040-$

$(551*gr*h^3)/199584-(32027*gs*h^3)/14192640-(3959*gt*h^3)/6386688==$

$0;$

$eqn10=v0-v2+(h*w0)/2+(35*f*h^2)/891+(13*f1*h^2)/8910+(196*fr*$

$h^2)/4455+(fs*h^2)/40+(68*ft*h^2)/4455+(1277*g*h^3)/1330560-(109*g1*$

$h^3)/1330560-(41*gr*h^3)/5544-(5*gs*h^3)/693-(17*gt*h^3)/9240==0;$

$eqn11=v0-v3+(3*h*w0)/4+(39015*f*h^2)/630784+(8469*f1*h^2)/3153920+$

$(18531*fr*h^2)/197120+(3159*fs*h^2)/35840+(6813*ft*h^2)/197120+(9747*$

$g*h^3)/6307840-(27*g1*h^3)/180224-(4509*gr*h^3)/394240-(19197*gs*$

$h^3)/1576960-(9*gt*h^3)/2464==0;$

$eqn12=v0-v4+h*w0+(6353*f*h^2)/74844+(3457*f1*h^2)/374220+(13952*$

$fr*h^2)/93555+(52*fs*h^2)/315+(8576*ft*h^2)/93555+(269*g*h^3)/124740-$

$(5*g1*h^3)/12474-(464*gr*h^3)/31185-(10*gs*h^3)/693-(16*gt*h^3)/4455==$

$0;$

$eqn13=w0-w1+(1539551*f*h)/17418240+(59681*f1*h)/17418240+(89371*$

$fr*h)/1088640+(103*fs*h)/2520+(38341*ft*h)/1088640+(26051*g*$

$h^2)/11612160-(2237*g1*h^2)/11612160-(31207*gr*h^2)/1451520-(81*gs*$

$h^2)/5120-(1243*gt*h^2)/290304==0;$

$eqn14=w0-w2+(24463*f*h)/272160+(1153*f1*h)/272160+(1654*fr*$

$h)/8505+(52*fs*h)/315+(394*ft*h)/8505+(421*g*h^2)/181440-(43*g1*$

$h^2)/181440-(19*gr*h^2)/1134-(gs*h^2)/40-(31*gt*h^2)/5670==0;$

$eqn15=w0-w3+(6501*f*h)/71680+(411*f1*h)/71680+(921*fr*h)/4480+$

$(81*fs*h)/280+(711*ft*h)/4480+(339*g*h^2)/143360-(9*g1*h^2)/28672-$

$(279*gr*h^2)/17920-(81*gs*h^2)/5120-(183*gt*h^2)/17920==0;$

$eqn16=w0-w4+(1601*f*h)/17010+(1601*f1*h)/17010+(2048*fr*$

$h)/8505+(104*fs*h)/315+(2048*ft*h)/8505+(29*g*h^2)/11340-(29*$

$g1*h^2)/11340-(32*gr*h^2)/2835+(32*gt*h^2)/2835==0;$

$[A,B]=equationsToMatrix([eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,$

$eqn7, eqn8, eqn9, eqn10, eqn11, eqn12, eqn13, eqn14, eqn15, eqn16], [y1,$

$y2, y3, y4, z1, z2, z3, z4, v1, v2, v3, v4, w1, w2, w3, w4]);$

$K = (A \setminus B);$

$err1 = abs(1 - x1 + exp(2 * x1) - exp(-2 * x1) - double(K(1)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x1, double(1 - x1 + exp(2 * x1) - exp(-2 *$

$x1)), double(K(1)), double(err1));$

$err2 = abs(1 - x2 + exp(2 * x2) - exp(-2 * x2) - double(K(2)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x2, double(1 - x2 + exp(2 * x2) - exp(-2 *$

$x2)), double(K(2)), double(err2));$

$err3 = abs(1 - x3 + exp(2 * x3) - exp(-2 * x3) - double(K(3)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x3, double(1 - x3 + exp(2 * x3) - exp(-2 *$

$x3)), double(K(3)), double(err3));$

$err4 = abs(1 - x4 + exp(2 * x4) - exp(-2 * x4) - double(K(4)));$

$fprintf('\%2.7f\%3.18f\%3.18f\%1.6e', x4, double(1 - x4 + exp(2 * x4) - exp(-2 *$

$x4)), double(K(4)), double(err4));$

$x0 = x4; y0 = K(4); z0 = K(8); v0 = K(12); w0 = K(16);$ end