A SIMPLE MODIFICATION OF TRINOMIAL DISTRIBUTION

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Abstrak

Taburan statistik memainkan peranan penting dalam menganalisis data dan merupakan asas dalam sains data. Ia menyediakan kerangka kerja untuk memahami dan memodelkan pelbagai fenomena data, yang membolehkan ramalan yang tepat, pemahaman yang bermakna, serta membuat keputusan yang efektif. Taburan kebarangkalian diskret klasik seperti taburan binomial, trinomial dan multinomial memerlukan kesudahan yang mungkin bagi setiap percubaan adalah saling ekslusif, di mana kesudahan tidak boleh berlaku secara serentak. Dalam situasi biasa, apabila melibatkan proses membuat keputusan, jawapan sepatutnya "Ya" atau "Tidak" sahaja. Walau bagaimanapun, ketidakpastian mungkin timbul yang membawa kepada keputusan "Tidak Pasti" dan ini bukanlah kesudahan yang saling eksklusif dengan "Ya" dan "Tidak". Oleh itu, kajian ini mengambil inisiatif untuk mengubahsuai taburan trinomial klasik bagi menyesuaikan kesudahan yang tidak saling eksklusif. Fungsi jisim kebarangkalian (PMF) Taburan Trinomial Terubahsuai (MoTD) diperoleh melalui kembangan trinomial. Parameter MoTD dianggarkan menggunakan kaedah kebolehjadian maksimum dan kaedah momen untuk memastikan penganggar yang diperolehi adalah tidak bias, cukup, cekap dan konsisten. MoTD telah diuji pada data keseimbangan kehidupan-kerja, dan prestasi taburan dari segi sisihan piawai dibandingkan dengan taburan binomial dan trinomial. Dapatan kajian menunjukkan bahawa parameter MoTD memenuhi sifat penganggar yang baik. Ujian ke atas data sebenar menunjukkan bahawa MoTD berprestasi lebih baik berbanding taburan binomial dan trinomial kerana ia menghasilkan sisihan piawai yang lebih rendah, menandakan serakan lebih kecil. Kesimpulannya, MoTD memperluas taburan trinomial klasik dengan membenarkan "Tidak Pasti" menjadi kesudahan yang tidak saling eksklusif. MoTD dapat membantu pembuat keputusan yang berhadapan dengan ketidakpastian dengan meminimumkan sisihan piawai dalam analisis data, sekali gus membawa kepada pembuatan keputusan yang lebih baik.

Kata Kunci: Pembuatan keputusan, Taburan diskret, Taburan Trinomial Terubahsuai, Kesudahan tidak saling eksklusif, Sisihan piawai.

Abstract

Statistical distributions play an important role in analyzing data and are fundamental to data science. They provide a framework for understanding and modeling various data phenomena, which enable accurate predictions, meaningful insights, and effective decision-making. Classical discrete probability distributions, such as binomial, trinomial, and multinomial distributions, require the possible outcomes for each trial to be mutually exclusive, where the outcomes cannot occur simultaneously. In normal situation, when involving a decision-making process, the answer should be “Yes” or “No” only. However, the uncertainty may arise leading the outcome of “Not Sure” and this is not a mutually exclusive outcome with “Yes” and “No”. Therefore, this study took the initiative to modify the classical trinomial distribution to accommodate a non-mutually exclusive outcome. The probability mass function (PMF) of the Modification of Trinomial Distribution (MoTD) is derived through trinomial expansion. The parameters of MoTD estimated using the maximum likelihood method and the method of moment to ensure those obtained estimators are unbiased, sufficient, efficient, and consistent. The MoTD was tested on work-life balance data, and its performance in terms of standard deviation was compared to the binomial and trinomial distributions. The findings indicate that the parameters of MoTD fulfil the properties of good estimators. A test on real data showed that MoTD performed better compared to the binomial and trinomial distributions, as it produced lower standard deviations, signifying smaller dispersion. In conclusion, the MoTD extends the classical trinomial distribution by allowing “Not Sure” to be non-mutually exclusive outcome. MoTD can help decision-makers dealing with uncertainty by minimizing the standard deviation in data analysis, leading to better decision-making.

Keywords: Decision-making, Discrete distribution, Modified Trinomial Distribution, Non-mutually exclusive outcomes, Standard deviation.

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List of Abbreviations

CDF	Cumulative Distribution Function
COM-Poisson	Conway–Maxwell–Poisson
CR	Cramér–Rao
eGFP-gp16	enhanced Green Fluorescent Protein-Glycoprotein
GIT	Generalized shifted Inverse Trinomial
GPD	Generalized Poisson Distribution
gp16	Glycoprotein
i.e.	that is
MSE	Mean Squared Error
MGF	Moment Generating Function
ML	Maximum Likelihood
MoTD	Modified Trinomial Distribution
MVUE	Minimum-Variance Unbiased Estimator
PMF	Probability Mass Function
QMUL	Queen Mary University of London
SRS	Simple Random Sampling
ϕ29	phi29

CHAPTER ONE
INTRODUCTION

1.1 Background of the Study

Statistical distributions play an important role in analyzing data and are fundamental to data science. They provide a framework for understanding and modeling various data phenomena, enabling data scientists to make accurate predictions, draw meaningful insights, and make decisions. A statistical model is a set of probability distributions on the possible outcomes of an experiment (Evans & Rosenthal, 2009). In other words, statistical distribution is a broader term that includes both probability distributions and empirical distributions. An empirical distribution is based on observed data from real-world phenomena. It represents the true distribution of data points in a sample or population. Empirical distributions can take many forms, and they may or may not correspond to any particular theoretical distribution (Dekking, 2007). A probability distribution is a statistical function that describes the probability of obtaining all possible values that a random variable can take (Ugoni & Walker, 1995).

So, the statistical distribution is appropriate for analysing the frequency, period of each occurrence, and impact of most natural events. Most natural events or other scientific fields have their characteristics. For example, discrete probability distributions are suitable to model earthquakes, road accidents, the number of deaths from disease, and the number and costs of landslides (Banik & Kibria, 2009; Crovelli, 2000). The existing well-known discrete probability distributions, such as binomial, trinomial, and multinomial distributions, require the possible outcomes for each trial to be mutually exclusive (Zhu, 2020). Mutually exclusive is a statistical term

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Appendix A
Academicians' Work-Life Balance Data
from Universities in Northern Region of Malaysia

F1	F2	F3	F4
28	1	20	24
92	90	89	67
84	85	71	84
98	99	99	86
100	100	100	100
98	93	83	50
39	85	14	29
42	10	43	40
50	76	83	76
13	100	18	100
23	5	2	33
52	80	46	29
100	91	100	78
59	16	74	18
97	100	53	31
74	51	45	72
59	23	15	15
61	69	68	39
77	88	84	14
51	8	27	7
100	79	99	73
100	66	84	70
49	61	65	59
56	46	90	78
10	13	15	22
85	46	14	16
100	94	100	87
60	59	48	39
73	37	59	27
50	85	84	86
31	100	29	1
50	17	90	23
55	43	50	19
18	30	30	1
19	1	28	9
81	83	37	15
76	36	35	52
71	76	75	55
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88	29	1	17
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75	78	64	58
18	14	19	20
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76	74	69	87
99	89	100	81
80	93	83	53
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27	34	46	42
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5	7	91	41
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87	83	74	67
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1	81	27	1
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61	69	83	57
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36	39	31	86
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81	39	85	52
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94	19	67	68
81	28	77	24
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