MULTIVARIATE SYNTHETIC CONTROL CHARTS BASED ON HIGH BREAKDOWN POINT ROBUST LOCATION ESTIMATORS

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Abstrak

Carta kawalan sintetik multivariat membolehkan pemantauan serentak terhadap pemboleh ubah proses berganda, seterusnya membendung peningkatan kadar penggera palsu yang biasanya diperhatikan apabila menggunakan carta kawalan individu yang berasingan. Walau bagaimanapun, carta kawalan sintetik tradisional berdasarkan min, C_{mean}, gagal menghasilkan anggaran parameter yang boleh dipercayai apabila data tercemar dengan pencilan. Tambahan pula, anggaran tidak dapat diperoleh dalam tetapan berdimensi tinggi. Oleh itu, kajian ini menangani jurang tersebut dengan membina tiga carta kawalan sintetik multivariat teguh baharu, C_{MRCD}, C_{WS}, dan C_{WP}, menggunakan penentu kovarian teratur minimum (MRCD) dan penganggar M-satu langkah terubah suai terwinsor (WMOM). Semua carta kawalan dinilai melalui kajian simulasi yang meluas menggunakan R, merangkumi dimensi p = 2, 10, dan 15 serta saiz sampel n = 5, 10, 15, 25, 50, dan 100, 200, 500. Prestasi dinilai menggunakan kadar penggera palsu (α) dan kebarangkalian pengesanan. Untuk menunjukkan kebolehgunaan dalam keadaan dunia sebenar, carta kawalan yang dicadangkan diaplikasikan pada data kualiti air sungai dari Kampung Medan, Selangor. Keputusan simulasi menunjukkan bahawa carta kawalan yang baharu didapati mengatasi C_{mean} secara bererti dalam kedua-dua kestabilan dan pengesanan peralihan. Secara khusus, ketika C_{mean} gagal mengawal α di bawah pencemaran yang tinggi, C_{MRCD} kekal stabil dalam kriteria ketat Bradley (dalam lingkungan 0.045 hingga 0.055). Dari segi kebarangkalian pengesanan, C_{MRCD} menunjukkan prestasi yang lebih baik berbanding C_{mean}, dengan mencapai kebarangkalian sekurang-kurangnya 0.80 dalam 174 keadaan, manakala C_{mean} hanya memenuhi kriteria ini dalam 63 keadaan. Berdasarkan matriks kekeliruan, data sebenar menunjukkan bahawa carta kawalan baharu ini boleh diimplimentasi dengan berkesan dalam konteks pemantauan kualiti air. Secara keseluruhan, hasil kajian mendapati bahawa C_{MRCD}, C_{WS} dan C_{WP} berprestasi lebih baik daripada C_{mean} dalam keadaan tercemar. C_{MRCD} dikenal pasti sebagai alat pemantauan yang paling teguh dan cekap dalam kes berdimensi tinggi apabila data tercemar.

Kata kunci: Carta kawalan; Kadar penggera palsu; Sintetik multivariat; Kebarangkalian pengesanan; Penganggar teguh; Kawalan proses statistik

Abstract

Multivariate synthetic control charts enable simultaneous monitoring of multiple process variables, thereby mitigating the inflation of false alarm rates typically observed when using separate individual control charts. However, the traditional synthetic control chart based on mean, C_{mean}, fails to produce reliable parameter estimates when data are contaminated with outliers. Moreover, the estimates cannot be obtained in a high-dimensional setting. Thus, this study addressed the gap by constructing three new robust multivariate synthetic control charts, C_{MRCD}, C_{WS}, and C_{WP}, using the Minimum Regularized Covariance Determinant (MRCD) and Winsorized Modified One-step M-estimator (WMOM). The control charts were evaluated through extensive simulation studies using R, covering dimensions p = 2, 10, and 15 and sample sizes n = 5, 10, 15, 25, 50, 100, 200, and 500. The performances were assessed using the false alarm rate (α) and probability of detection. To demonstrate the applicability to real-world conditions, the proposed control charts were applied to river water quality data from Kampung Medan, Selangor. The simulation results demonstrate that the new control charts significantly outperform the C_{mean} in both stability and shift detection. Specifically, while the C_{mean} fails to control the α under high contamination, the C_{MRCD} remains stable within Bradley's stringent criteria (within 0.045 to 0.055). In terms of the probability of detection, the C_{MRCD} outperforms the C_{mean}, achieving a probability of at least 0.80 in 174 conditions, whereas the C_{mean} meets this criterion in only 63 conditions. Based on the confusion matrix, real data indicate that new control charts can be effectively implemented in water quality monitoring contexts. Overall, the findings indicate that the C_{MRCD}, C_{WS}, and C_{WP} perform better than the C_{mean} in contaminated conditions. The C_{MRCD} is identified as the most robust and efficient monitoring tool in high-dimensional cases when data are contaminated.

Keywords: Control chart; False alarm rate; Multivariate synthetic; Probability of detection; Robust estimators; Statistical process control

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List of Abbreviations

ARL	Average Run Length
$ARL_{in\text{-}control}$	Average Run Length for an in-control process
$ARL_{out\text{-}of\text{-}control}$	Average Run Length for an out-of-control process
CL	Control Limit
COD	Chemical Oxygen Demand
CRL	Conforming Run Length
DO	Dissolved Oxygen
EWMA	Exponentially Weighted Moving Average
EWMA-SN	EWMA Sign
FMCD	Fast Minimum Covariance Determinant
FN	False Negative
FP	False Positive
KDE	Kernel Density Estimation
MAD*n*	Median Absolute Deviation about the Mean
MCD	Minimum Covariance Determinant
MCUSUM	Multivariate Cumulative Sum
MEWMA	Multivariate Exponentially Weighted Moving Average
MOM	Modified One-Step M-estimator
MRCD	Minimum Regularized Covariance Determinant
MVE	Minimum Volume Ellipsoid
NH3N	Ammoniacal Nitrogen
NPV	Negative Predictive Value
p	Number of quality characteristics/variables
PCA	Principal Component Analysis
pH	Potential of Hydrogen
PPV	Positive Predictive Value
Qn	A robust scale estimator
S	Covariance matrix
S-estimator	Scale estimators
SPC	Statistical Process Control
TSS	Total Suspended Solids

TN	True Negative
TP	True Positive
WMOM	Winsorized Modified One-step M-estimator
WQI	Water Quality Index
WV	Weighted Variance
X	Standard mean vector
α	False alarm rate/Type I error
β	Type II error probability
δ^*	Likely shift of the location process
ε	Proportion of contamination
μ	Mean
μ_1	Shift of mean for generating contaminated data
μ_{11}	Shift of mean for generating out-of-control data
ρ	Pearson correlation coefficient
ρpb	Percentage Bend correlation
ρs	Spearman's correlation
σ	Standard deviation

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CHAPTER ONE

INTRODUCTION

1.1 Background of Statistical Process Monitoring

Control charts serve as a real-time monitoring tool for processes, employing statistical techniques to track and manage the quality characteristics of a process (Yeganeh & Shongwe, 2023; Zwetsloot, Jones-Farmer, & Woodall, 2024). Hence, utilizing this tool for process monitoring guarantees that the output quality of the product aligns with the desired standards. The monitoring process typically encompasses multiple quality characteristics concurrently, rendering the univariate control chart inadequate to meet these requirements. This separate monitoring of quality characteristics through a univariate control chart can significantly increase the overall false alarm rate (α) in the process (Bersimis, Psarakis, & Panaretos, 2007). Therefore, multivariate control charts are advised for effective process monitoring to address this issue.

1.2 Process Shifts in Statistical Process Monitoring

Distinguishing between common cause and special cause variations is essential in process monitoring. These common causes are intrinsic to the system and signify typical process variation. Meanwhile, special causes suggest unexpected disturbances and signal an issue necessitating immediate action (Montgomery, 2020). A special cause typically results in a shift in the process means or variability. This shift is denoted as a prolonged departure from the initial state of the process (Montgomery, 2020). Thus, shift detection is essential, as delayed detection identification can result in ongoing processes operating under abnormal conditions (Lei & MacKenzie, 2020; Montgomery, 2020).

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Appendix A

Representative Real Data Sample

No	NH_3N	COD	DO	PH	TSS	Date Time	Location	Class
1	0.16	17	0.04	7.77	21	10/08/2023 1815	1. KAMPUNG MEDAN	III
2	0.26	21	0.01	7.68	66	11/08/2023 1130	1. KAMPUNG MEDAN	III
3	0.27	21	0.03	7.63	66	11/08/2023 1145	1. KAMPUNG MEDAN	III
4	0.27	21	0.05	7.55	66	11/08/2023 1245	1. KAMPUNG MEDAN	III
5	0.32	21	0.03	7.43	66	11/08/2023 1415	1. KAMPUNG MEDAN	III
6	0.31	21	0.05	7.44	66	11/08/2023 1445	1. KAMPUNG MEDAN	III
7	0.32	16	0.04	7.56	15	11/08/2023 1545	1. KAMPUNG MEDAN	III
8	0.3	16	0.03	7.51	17	11/08/2023 1645	1. KAMPUNG MEDAN	III
9	0.3	16	0.02	7.87	22	11/08/2023 1745	1. KAMPUNG MEDAN	III
10	0.28	16	0.02	7.48	23	11/08/2023 1900	1. KAMPUNG MEDAN	III
11	0.06	20	0	8.68	33	14/08/2023 1145	1. KAMPUNG MEDAN	III
12	0.07	20	0.01	8.67	32	14/08/2023 1245	1. KAMPUNG MEDAN	III
13	0.1	21	0.01	8.6	64	14/08/2023 1345	1. KAMPUNG MEDAN	III
14	0.14	20	0.01	8.71	41	14/08/2023 1445	1. KAMPUNG MEDAN	III
15	0.2	23	0.01	8.45	63	14/08/2023 1545	1. KAMPUNG MEDAN	III