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MODIFIED X-BAR RANGE CHART FOR SHORT RUN PRODUCTION PROCESS CONTROL

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Abstrak

Carta kawalan X-Bar dan R (X-BarR) digunakan secara meluas untuk memantau serta mengawal prestasi proses pengeluaran produk. Carta ini membantu untuk mengenal pasti sama ada sesuatu proses terkawal atau tidak terkawal. Carta ini amat berkesan dalam pemantauan pengeluaran jangka panjang, yang mana saiz sampel besar membolehkan anggaran min dan varians dibuat dengan lebih tepat. Namun, dalam pengeluaran jangka pendek, hanya sejumlah kecil produk dihasilkan dan perubahan tetapan pengeluaran berlaku dengan kerap. Keadaan ini menyebabkan saiz sampel menjadi kecil, sekali gus mengurangkan ketepatan anggaran dan melemahkan keberkesanan carta kawalan. Selain itu, data jangka pendek lebih cenderung mengandungi ralat dan pencilan, yang seterusnya mengurangkan kebolehpercayaan carta X-BarR. Bagi mengatasi masalah tersebut, kajian ini mengubah suai carta X-BarR dengan menggantikan kedua-dua penganggar lokasi dan skala. Min digantikan dengan median. Manakala, julat digantikan dengan tiga penganggar teguh: Sisihan mutlak median (MAD), sisihan mutlak median terhadap median (MADn), dan penganggar Sn untuk masing-masing menghasilkan carta kawalan Median-MAD, Median-MADn, dan Median-Sn. Kajian ini mengambil kira kedua-dua taburan normal dan tidak normal bagi meningkatkan kebolehpercayaan praktikal. Sepuluh ribu pengulangan analisis simulasi telah dijalankan dengan saiz sampel kecil, n = 5, 7, 9, bilangan subkumpulan yang berbeza, m = 10, 20, 30, 50, pelbagai magnitud peralihan min, serta 20% data yang tercemar. Seterusnya, prestasi carta kawalan telah diukur menggunakan Truncated Average Run Length (TARL) dan Truncated Standard Deviation Run Length (TSDRL). Untuk pengesahan, dataset sebenar daripada proses pengeluaran baja dan pengisian yogurt digunakan. Hasil daripada analisis simulasi dan set data sebenar menunjukkan bahawa carta Median–MAD mengatasi prestasi carta Median–MADn dan Median–Sn bagi taburan normal dan taburan tak-normal, menunjukan keteguhan serta keberkesanannya dalam mengendalikan saiz sampel kecil dan data yang tercemar. Kesimpulannya, carta Median-MAD disyorkan sebagai alat yang boleh dipercayai untuk pemantauan pengeluaran jangka pendek bagi memastikan kualiti produk.

Kata kunci: Kata kunci: Data tercemar, Carta kawalan, Pengeluaran jangka pendek, Saiz sampel kecil, Penganggar teguh.

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Abstract

The X-Bar and R (X-BarR) control chart is widely used to monitor and regulate process performance during production. It helps to identify whether a process is in-control or out-of-control. This chart is particularly effective in monitoring long-run production, where large sample sizes enable more accurate estimation of the mean and variance. However, in short-run production, only a limited number of products are manufactured, and frequent changes in production settings occur. These conditions lead to small sample sizes, which reduce the accuracy of estimates and weaken the effectiveness of the charts. Moreover, short-run data is more likely to contain errors and outliers, further diminishing the reliability of X-BarR charts. To address these limitations, this study modifies the X-BarR chart by replacing both the location and scale estimators. The mean is substituted with the median. While range is substituted with three robust estimators: the Median Absolute Deviation (MAD), the Median Absolute Deviation about the median (MADn), and the Sn estimator to produce Median-MAD, Median-MADn, and Median-Sn control charts accordingly. The study considered both normal and non-normal distributions to improve practical reliability. Ten-thousand repetitions of a simulation analysis were carried out with small sample sizes, n = 5, 7, 9, number of subgroups, m = 10, 20, 30, 50, various mean shift sizes and 20% contaminated data. Next, the control chart performance was measured using Truncated Average Run Length (TARL) and Truncated Standard Deviation Run Length (TSDRL). For validation, the real datasets from fertilizer and yogurt filling processes were used.The results from both the simulation analysis and real datasets showed that the Median–MAD chart outperformed the Median MADn and Median–Sn charts under normal and non-normal, demonstrating robustness and effectiveness in handling small sample sizes and contaminated data In conclusion, the Median-MAD chart is recommended as a reliable tool for short-run production monitoring to ensure product quality

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Keywords: Contaminated data, Control chart, Short run production, Small sample size, Robust estimator.

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CHAPTER ONE
INTRODUCTION

1.1 Background of study

The global manufacturing industry demands an excellent level of product quality, productivity, competitiveness, and continual improvement in order to remain competitive in today's market. The manufacturer can maximize their operations, costing, and maximize customer satisfaction by continually improving their products, processes, and operations (Sahoo & Sahu, 2020). One way to improve manufacturing process is by using quality control (Abbas, 2020). Quality control is a process used by organizations to ensure that the products they deliver meet specific standard quality (Papavasileiou, Michalos, & Makris, 2025). One common method for monitoring the quality is implementing the quality control charts which serve as a statistical tool (Hajej, Nyoungue, Abubakar, & Ali, 2021). Quality control charts help manufacturers to monitor process performance in real-time, allowing them to detect and correct problems quickly which can reduce variability and improve product quality (Clancy et al., 2020). Quality is a key feature to increase an efficiency in the manufacturing process in achieving zero defects in their production.

In area of statistics, Walter Shewhart proposed the prominent concept known as SPC which is stand for statistical process control in the 1920s. The concept has had a wide range of beneficiaries over the years, from manufacturers and consumers to businesses, employees, and society (Shewhart, 1931). The implementation of SPC has been included as a method of improving, monitoring, and boosting product quality as well

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APPENDIX A

Constant Table A_2, D_3, D_4

n	A_2	D_3	D_4	d_2
2	1.880	0.000	3.267	1.128
3	1.023	0.000	2.574	1.693
4	0.729	0.000	2.282	2.059
5	0.577	0.000	2.114	2.326
6	0.483	0.000	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

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Coding MATLAB *k* value for Median normal distribution

```
%How to get K for median mad chart
clc
clear

%parameter has been used
%=======================================
n=7; m=10; %n=sample size for each subgroup m=number of subgroup
mu=0;   %parameter for mu
sigma=1;  %parameter for standard deviation
%=======================================
itr=10000;%iteration
xchart1=[];%create a space for result
xchart2=[];
answer =[];
g=0;h=0;l=0;

%compute critical value based on simulated mean
for i=1:itr
x= random('Normal',mu,sigma,n,m);
xmedian=median(x); %median FOR EACH SUBGROUP
gmedian=median(xmedian); %grand median
xchart1=[xchart1;gmedian];
xmad=mad(x,1);
gmad=median(xmad);
xchart2=[xchart2;gmad];
end
y=sort(xchart2);
ggmad=mean(gmad);
lcl=y(13);
ucl=y(9987); %using 0.0027 (0.27%), then 10000x0.973 = 9973
k=((ucl-lcl)/2)/ggmad;
end
```

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Coding MATLAB for Median MAD normal distribution

```
%how to compute MEDIAN Mad chart
clc
clear

%parameter has been used
%=======================================
n=9; m=50; %n=sample size for each subgroup m=number of subgroup
mu=0.6;  %parameter for mu is actually the value of shift
sigma=1;  %parameter for standard deviation
k=0.2011;
%=======================================
itr=10000;%iteration
xchart1=[];
xchart2=[];
answer =[];
g=0;h=0;l=0;

%compute ARL TARL and TSDRL
for i=1:itr
x= random('Normal',mu,sigma,n,m);
xmedian=median(x); %median FOR EACH SUBGROUP
gmedian=median(xmedian); %grand median
xchart1=[xchart1;gmedian];
xmad=mad(x,1);
gmad=median(xmad);
xchart2=[xchart2;gmad];
end
ggmad=mean(gmad);
ucl=k*ggmad;
lcl=-(ucl);
%CROSSCHECK
for i=1:itr
if (xchart1(i)>= ucl)
   g=g+1;   %count ooc
elseif (xchart1(i)<= lcl)
   l=l+1;   %count 00c
else
   h=h+1;   %count in-control
end
end

g+l;              %no. of ooc must be 27
alpha=(g+l)/itr;   %alpha must be 0.0027
ARL=1/alpha;       %ARL must be 370.3704
%beta is probability in-control see Khoo equation 3
beta=h/itr;       %must be 99.735 at no shifted in mean value
```

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```
TARLnot= (1-((1-alpha)^(m+1)))/alpha; %this value will be similar to m value
c=m+1;
TARLooc= (1-(beta^c))/(1-beta);
w=(2*m)+1;
aa=beta*(1-(beta^w));
bb=(1-beta)*(beta^c)*(1+(2*m));
TSDRL=sqrt(aa-bb)/(1-beta);
answer=[ARL TARLooc TSDRL]
```

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Coding MATLAB *k* value for Median Gamma distribution

```
%How to get K for median mad chart
clc
clear

%parameter has been used
%=======================================
n=5; m=10; %n=sample size for each subgroup m=number of subgroup
mu=0;   %parameter for mu
sigma=1;  %parameter for standard deviation
%=======================================
itr=10000;%iteration
xchart1=[];%create a space for result
xchart2=[];
answer =[];
sz = [n 1]; %size of data matrix, n x p
g=0;h=0;l=0;

%compute critical value based on simulated mean
for i=1:itr
x = gamrnd(3,1,sz); % alpha = 3, beta = 1
xmedian=median(x); %median FOR EACH SUBGROUP
gmedian=median(xmedian); %grand median
xchart1=[xchart1;gmedian];
xmad=mad(x,1);
gmad=median(xmad);
xchart2=[xchart2;gmad];
end
y=sort(xchart2);
ggmad=mean(gmad);
lcl=y(13);
ucl=y(9987); %using 0.0027 (0.27%), then 10000x0.973 = 9973
k=((ucl-lcl)/2)/ggmad;
end
```

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Coding MATLAB for Median MAD Gamma distribution

```
%how to compute MEDIAN Mad chart
clc
clear

%parameter has been used
%=======================================
n=9; m=10; %n=sample size for each subgroup m=number of subgroup
mu=2;  %parameter for mu is actually the value of shift
sigma=1;  %parameter for standard deviation
k=1.0518;
%=======================================
itr=10000;%iteration
xchart1=[];
xchart2=[];
answer =[];
sz = [n 1]; %size of data matrix, n x p
g=0;h=0;l=0;

%compute ARL TARL and TSDRL
for i=1:itr
x = gamrnd(3,1,sz); % alpha = 3, beta = 1
xmedian=median(x); %median FOR EACH SUBGROUP
gmedian=median(xmedian); %grand median
xchart1=[xchart1;gmedian];
xmad=mad(x,1);
gmad=median(xmad);
xchart2=[xchart2;gmad];
end
y=sort(xchart2);
ggmad=mean(gmad);
ucl=k*ggmad;
lcl=-(ucl);
%CROSSCHECK
for i=1:itr
if (xchart2(i)>= ucl)
   g=g+1;   %count ooc
elseif (xchart2(i)<= lcl)
   l=l+1;   %count 00c
else
   h=h+1;   %count in-control
end
end

g+l;              %no. of ooc must be 27
alpha=(g+l)/itr;   %alpha must be 0.0027
ARL=1/alpha;       %ARL must be 370.3704
```

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```
%beta is probability in-control see Khoo equation 3
beta=h/itr;       %must be 99.735 at no shifted in mean value
TARLnot= (1-((1-alpha)^(m+1)))/alpha; %this value will be similar to m value
c=m+1;
TARLooc= (1-(beta^c))/(1-beta);
w=(2*m)+1;
aa=beta*(1-(beta^w));
bb=(1-beta)*(beta^c)*(1+(2*m));
TSDRL=sqrt(aa-bb)/(1-beta);
answer=[ARL TARLooc TSDRL]
```

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