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MODELING FINANCIAL ENVIRONMENTS USING GEOMETRIC FRACTIONAL BROWNIAN MOTION MODEL WITH LONG MEMORY STOCHASTIC VOLATILITY

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Abstrak

Model Pergerakan Pecahan Geometrik Brownian (GFBM) digunakan dengan meluas dalam persekitaran kewangan. Model ini mengandungi parameter penting iaitu min, ruapan, dan indeks Hurst, yang bererti kepada kebanyakan masalah dalam bidang kewangan terutamanya bagi menentukan harga opsyen, nilai pada risiko, kadar tukaran, dan insuran cagaran. Kebanyakan penyelidikan terkini mengkaji *GFBM* dengan mengandaikan ruapannya adalah malar disebabkan keringkasannya. Walau bagaimanapun, anggapan ini selalunya disangkal dalam kebanyakan kajian empirikal. Oleh itu, kajian ini membangunkan model GFBM baharu yang mampu menerangkan dan menggambarkan situasi sebenar dengan lebih baik terutamanya dalam senario kewangan. Kesemua parameter yang terlibat dalam model yang dibangunkan dianggar menggunakan algoritma inovasi. Kajian simulasi seterusnya dilakukan untuk menentukan prestasi model baharu. Hasil simulasi mendedahkan bahawa penganggar yang disyorkan adalah cekap berdasarkan kepada kepincangan, varians, dan min kuasa dua ralat. Seterusnya, dua teorem berkaitan kewujudan dan keunikan penyelesaian bagi model baharu dan pengitlakannya dibina. Pengesahan bagi model yang dibangunkan kemudiannya dilakukan dengan membandingkannya dengan beberapa model lain bagi meramal harga terlaras Standard and Poor's 500, Shanghai Stock Exchange Composite Index, dan FTSE Kuala Lumpur Composite Index. Kajian empirikal terhadap empat aplikasi kewangan terpilih, iaitu penentuan harga opsyen, nilai risiko, kadar pertukaran, dan insuran gadai janji, menunjukkan bahawa model baharu mempamerkan keputusan yang lebih baik berbanding model sedia ada. Justeru itu, model baharu amat berpotensi untuk dijadikan model pendasar bagi sebarang aplikasi kewangan yang berupaya mencerminkan keadaan sebenar dengan lebih tepat.

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Kata kunci: Pergerakan Pecahan Geometrik Brownian, ruapan stokastik, memori panjang, senario kewangan.

Abstract

Geometric Fractional Brownian Motion (GFBM) model is widely used in financial environments. This model consists of important parameters i.e. mean, volatility, and Hurst index, which are significant to many problems in finance particularly option pricing, value at risk, exchange rate, and mortgage insurance. Most current works investigated GFBM under the assumption of its volatility that is constant due to its simplicity. However, such assumption is normally rejected in most empirical studies. Therefore, this research develops a new GFBM model that can better describe and reflect real life situations particularly in financial scenario. All parameters involved in the developed model are estimated by using innovation algorithm. A simulation study is then conducted to determine the performance of the new model. The results of simulation reveal that the proposed estimators are efficient based on the bias, variance, and mean square error. Subsequently, two theorems on existence and uniqueness of the solution for the new model and its generalisation are constructed. The validation of the developed model was then carried out by comparing with other models in forecasting adjusted prices of Standard and Poor 500, Shanghai Stock Exchange Composite Index, and FTSE Kuala Lumpur Composite Index. Empirical studies on four selected financial applications, i.e. option pricing, value at risk, exchange rate, and mortgage insurance, indicate that the new model performs better than the existing ones. Hence, the new model has strong potential to be employed as an underlying model for any financial applications that capable of reflecting the real situation more accurately.

Keywords: Geometric Fractional Brownian Motion, stochastic volatility, long memory, financial scenario.

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Universiti Utara Malaysia

Table of Contents

Permission to Use…………………………………………………………………….. i
Abstrak……………………………………………………………………………..ii
Abstract……………………………………………………………………………iii
Acknowledgement…………………………………………………………………... iv
Table of Contents…………………………………………………………………..v
List of Tables………………………………………………………………………viii
List of Figures……………………………………………………………………...x
List of Appendices…………………………………………………………………xiii
List of Abbreviations……………………………………………………………… xiv

CHAPTER ONE: INTRODUCTION 1

1.1 Research Background...1

1.1.1 Discrete Stochastic Volatility and Continuous Stochastic Volatility...........4

1.1.2 Long Memory in Financial Modeling 5

1.1.3 Geometric Fractional Brownian Motion (GFBM) 10

1.2 Problem Statement 13

1.3 Research Objective...14

1.4 Limitation of the Study 15

1.5 Significance of the Research 15

1.6 Outline of the Thesis 16

CHAPTER TWO: PRELIMINARIES AND LITERATURE REVIEW............ 18

2.1 Preliminaries 18

2.2 Stochastic Volatility Models in Financial Environment 20

2.3 Stochastic Volatility Models Perturbed by Long Memory 27

2.4 Content Analysis on Stochastic Volatility in the Literature 39

2.4.1 Jump 42

2.4.2 Multivariate Models 42

2.4.3 Long Memory Stochastic Volatility..43

2.4.4 Simulation Based Inference 44

2.4.5 Moment – Based Inference. 45

Universiti Utara Malaysia

CHAPTER THREE: NEW MODEL OF GEOMETRIC FRACTIONAL BROWNIAN MOTION PERTURBED BY LONG MEMORY STOCHASTIC VOLATILITY 46

3.1 Development of the Model........ 46

3.1.1 Deriving Geometric Fractional Brownian Motion Covariance........ 46

3.1.2 Estimating Geometric Fractional Brownian Motion Parameters........ 51

3.2 Simulation Study........ 58

3.2.1 Validation of Calculations........ 58

3.2.2 Results of the Simulation Study........ 62

3.2.2.1 Algorithm of Simulation........ 63

3.2.2.2 Numerical Results........ 66

3.3 Discussion........ 80

CHAPTER FOUR: EXISTENCE AND UNIQUENESS SOLUTION OF FRACTIONAL STOCHASTIC DIFFERENTIAL MODEL 82

4.1 Preliminaries Definitions and Theorems........ 82

4.2 Existence and Uniqueness Solution of Geometric Fractional Brownian Motion Model........ 85

4.3 Existence and Uniqueness solution of Fractional Stochastic Differential Model 95

4.4 Discussion........ 106

CHAPTER FIVE: VALIDATION OF THE DEVELOPED MODEL BASED ON DIFFERENT TYPES OF MARKET INDICES 107

5.1 Characteristic of Market Indices........ 109

5.1.1 Standard and Poor's 500........ 110

5.1.2 Shanghai Stock Exchange Composite Index........ 110

5.1.3 FTSE Bursa Malaysia KLCI........ 111

5.2 Validation of the Developed Model........ 111

5.2.1 Forecasting the Performance of Standard and Poor's 500........ 112

5.2.1.1 Description of Data........ 112

5.2.1.2 Forecasting Standard and Poor's 500........ 113

5.2.2 Forecasting the Performance of Shanghai Stock Exchange Composite Index........ 128

Universiti Utara Malaysia

5.2.2.1 Description of Data..128
5.2.2.2 Forecasting the Shanghai Stock Exchange Composite Index 129
5.2.3 Forecasting the Performance Kuala Lumpur Composite Index..............141
5.2.3.1 Description of Data..141
5.2.3.2 Forecasting of Kuala Lumpur Composite Index 142
5.3 Discussion 156

CHAPTER SIX: APPLICATIONS OF DEVELOPED LONG MEMORY STOCHASTIC VOLATILITY MODEL IN FINANCE 158

6.1 Pricing the Options...158
6.1.1 Fractional Black–Sholes Model for European Option Pricing 159
6.1.2 Description of Data 160
6.1.3 Pricing the Options 162
6.2 Value at Risk and Long Memory 168
6.2.1 Description of Data 172
6.2.2 Calculating Value at Risk with Self-similarity 174
6.3 Exchange Rate...177
6.3.1 Description of Data 178
6.3.2 Forecasting Exchange Rates 180
6.4 Mortgage Insurance...189
6.4.1 Description of Data 190
6.4.2 Valuing Insurer Potential Loss..192
6.5 Discussion 196

CHAPTER SEVEN: CONCLUSIONS AND RECOMMENDATIONS........... 199

7.1 Conclusion 199
7.2 Future Research Problems and Recommendations.................................201

REFERENCES...202

Universiti Utara Malaysia

List of Tables

Table 1.1 Representation of memory dependence families 9

Table 1.2 Models of stochastic processes describing Y_t in SV models 11

Table 1.3 Popular SV models 12

Table 2.1 Current Stochastic Volatility Models.. 32

Table 2.2 The evolution of stochastic volatility from 2001–2011........................ 40

Table 3.1 Calculation of the proposed method for sample size $n = 1$ 59

Table 3.2 Parameters estimates for sample size $n = 1$. 59

Table 3.3 Calculation of the proposed method for sample size $n = 2$................ 60

Table 3.4 Parameters estimates for sample size $n = 2$. 60

Table 3.5 Calculation of the proposed method for sample size $n = 3$ 61

Table 3.6 Parameters estimates for sample size $n = 3$. 62

Table 3.7 Brief summary of the procedure for selected simulation algorithms.... 63

Table 3.8 Simulation Based on Simulated Annealing Algorithm......................... 69

Table 3.9 Simulation Based on Nelder–Mead Algorithm.................................... 70

Table 3.10 Simulation Based on Random Search Algorithm 71

Table 3.11 Simulation Based on Differential Evolution Algorithm 72

Table 3.12 Simulation of mean reverting parameter (α) for different sizes and different methods, with best variance, bias, and mean square error in $\{\cdot\}$, $(\cdot)$ and $[\cdot]$ respectively. 75

Table 3.13 Simulation of mean of volatility parameter (m) for different sizes and different methods, with best variance, bias, and mean square error in $\{\cdot\}$, $(\cdot)$ and $[\cdot]$ respectively. 76

Table 3.14 Simulation of volatility of volatility parameter (β) for different sizes and different methods, with best variance, bias, and mean square error in $\{\cdot\}$, $(\cdot)$ and $[\cdot]$ respectively. 77

Table 3.15 Simulation of drift parameter (μ) for different sizes and different methods, with best variance, bias, and mean square error in $\{\cdot\}$, $(\cdot)$ and $[\cdot]$ respectively.. 78

Table 5.1 Formulas of Volatility.. 108

Universiti Utara Malaysia

Table 5.2 The scale of judgment of forecast accuracy using MAPE 109
Table 5.3 The values of volatilities according to the formulas of Simple (S), Log (L), High-Low-Close (HLC) and Stochastic (STO).................... 114
Table 5.4 Forecasted Prices and Actual Prices of S&P 500 with MAPE 115
Table 5.5 The level of accuracy ranking for forecasting model of S&P 500...... 122
Table 5.6 The values of volatilities according to the formulas of Simple (S), Log (L), and Stochastic (STO) 129
Table 5.7 Forecasted prices and actual prices of SSE with MAPE 131
Table 5.8 The level of accuracy ranking for forecasting model of SSE 137
Table 5.9 The values of volatilities according to the formulas of Simple (S), Log (L), High-Low-Close (HLC) and Stochastic (STO).................... 142
Table 5.10 Forecasted Prices and Actual Prices of KLCI with MAPE 144
Table 5.11 The level of accuracy ranking for forecasting model of KLCI........... 150
Table 6.1 Summary of parameters 162
Table 6.2 Comparison of the European call option prices using different methods with H in ( ) and σ^2 in [ ]. 164
Table 6.3 Parameters summary of PRPFX 174
Table 6.4 PRPFX :VaR model with memory and stochastic volatility versus VaR model with no memory and constant volatility 175
Table 6.5 Formulas of volatility... 180
Table 6.6 Volatility values according to different formulas 181
Table 6.7 Forecast value for exchange rate USD/MYR with MSE 182
Table 6.8 The level of accuracy ranking for forecasting models. 185
Table 6.9 The models under consideration 192
Table 6.10 Involved parameters value 193
Table 6.11 Collaterals values and their corresponding potential loss. 194

Universiti Utara Malaysia

List of Figures

Figure 2.1. Numbers of articles in long memory stochastic, volatility, jumps, moment–based inference, and simulation–based inference. 41

Figure 5.1. Daily adjusted price series of S&P 500 from 1st January 2015 to 31st December 2015 112

Figure 5.2. Daily returns series of S&P 500 from 1st January 2015 to 31st December 2015 113

Figure 5.3. Forecast prices of S&P 500 by using GBM-S and GFBM-S vs actual prices 124

Figure 5.4. Forecast prices of S&P 500 by using GBM-L and GFBM-L vs actual prices 125

Figure 5.5. Forecast prices of S&P 500 by using GBM-HLC and GFBM-HLC vs actual prices 126

Figure 5.6. Forecast prices of S&P 500 by using GBM-STO and GFBM-STO vs actual prices 127

Figure 5.7. Daily adjust price series of SSE from 5th January 2015 to 31st December 2015 128

Figure 5.8. Daily returns SSE from 5th January 2015 to 31st December 2015.... 129

Figure 5.9. Forecast prices of SSE by using GBM-S and GFBM-S vs actual prices 138

Figure 5.10. Forecast prices of SSE by using GBM-L and GFBM-L vs actual prices 139

Figure 5.11. Forecast prices of SSE by using GBM-STO and GFBM-STO vs actual prices. 140

Figure 5.12. Daily adjust price series of KLCI from 2nd January 2015 to 31st December 2015 141

Figure 5.13. Daily returns SSE KLCI from 2nd January 2015 to 31st December 2015 142

Figure 5.14. Forecast prices of KLCI by using GBM-S and GFBM-S vs actual prices 152

Universiti Utara Malaysia

Figure 5.15. Forecast prices of KLCI by using GBM-L and GFBM-L vs actual prices 153
Figure 5.16. Forecast prices of KLCI using by GBM-HLC and GFBM-HLC vs actual prices. 154
Figure 5.17. Forecast prices of KLCI by using GBM-STO and GFBM-STO vs actual prices. 155
Figure 6.1. Daily closed price series of KLCI from 3rd January 2005 to 29th December 2006 161
Figure 6.2. Daily return series of KLCI from 3rd January 2005 to 29th December 2006. 161
Figure 6.3. European call option prices using different methods with maturity time 15 days. 165
Figure 6.4. European call option prices using different methods with maturity time 30 days. 166
Figure 6.5. European call option prices using different methods with maturity time 45 days. 166
Figure 6.6. European call option prices using different methods with maturity time 60 days. 167
Figure 6.7. Daily adjust price series of PRPFX from 1st January 2015 to 31st December 2015. 173
Figure 6.8. Daily returns series of PRPFX from 1st January 2015 to 31st December 2015. 173
Figure 6.9. VaR of PRPFX with $q = 0.01$ 175
Figure 6.10. VaR of PRPFX with $q = 0.05$ 176
Figure 6.11. VaR of PRPFX with $q = 0.1$ 176
Figure 6.12. Historical exchange rates between USD and MYR from 2nd January 2015 to 31st December 2015 179
Figure 6.13. Daily returns series of exchange rate between USD and MYR from 2nd January 2015 to 31st December 2015 179
Figure 6.14. Forecast exchange rates vs actual price (constant volatility case). 186
Figure 6.15. Forecast exchange rates vs actual price (STO-1 case). 187
Figure 6.16. Forecast exchange rates vs actual price (STO-2 case). 187

Universiti Utara Malaysia

Figure 6.17. Forecast exchange rates vs actual price (STO-3 case)...................... 188
Figure 6.18. Quarterly house price index in the UK from 4Q73 to 1Q17. 191
Figure 6.19. Quarterly return of house price index in the UK from 4Q73 to 1Q17. 191
Figure 6.20. Comparison between the levels of potential losses in the first six years... 194

Universiti Utara Malaysia

List of Appendices

Appendix A Flowchart of Coding 214
Appendix B Standard Simulation 218
Appendix C Simulation with Segmentation 224

Universiti Utara Malaysia

List of Abbreviations

AR	Autoregressive Process
ARCH	Autoregressive Conditional Heteroscedasticity
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARSV	Autoregressive Stochastic Volatility
BM	Brownian Motion
BS	Black–Scholes
CIR	Cox–Ingersoll–Ross
CMLE	Complete Maximum Likelihood Estimation
DE	Differential Evolution
EBSCO	Elton Bryson Stephens Company
EMM	Efficient Method of Moments
FBM	Fractional Brownian Motion
FBS	Fractional Black–Scholes
GFBM	Geometric Fractional Brownian Motion
FGN	Fractional Gaussian Noise
FOU	Fractional Ornstein Uhlenbeck
FSDE	Fractional Stochastic Differential Equation
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GBM	Geometric Brownian Motion
GMM	Generalized method of moment
GS	Google Scholar
H	Hurst
JP	Jump
KLCI	Kuala Lumpur Composite Index
LMSV	Long Memory Stochastic Volatility
MAPE	Mean Absolute Percentage Error
MBI	Moment Based Inference
MCMC	Markov Chain Monte Carlo

MLE	Maximum Likelihood Estimation
MM	Method of Moments
MSE	Mean Square Error
OU	Ornstein– Uhlenbeck
R/S	Rescale / Range
RS	Random Search
S &P 500	Standard and Poor's 500
S.V	SciVerse
SA	Simulated Annealing
SABR	Stochastic Alpha–Beta–Rho
SBI	Simulation Based Inference
SDE	Stochastic Differential Equation
SV	Stochastic Volatility
Var	Variance

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CHAPTER ONE
INTRODUCTION

1.1 Research Background

Volatility has been actively discussed in time series econometrics and economic forecasting in recent years. Volatility explains the variations witnessed in some phenomena over time. In economics, it is used to describe variability of random component of a time series. In financial economics, volatility is defined as the standard deviation of a random Wiener driven component in a continuous time diffusion model.

In the last decades, two main classes of volatility models have been developed: the generalized autoregressive conditional Heteroscedasticity (GARCH) and the stochastic volatility (SV) model. These classes were developed in order to capture time–varying autocorrelation, i.e. the correlation between values of the process at different points in time.

To begin, in 1982 Engle introduced autoregressive conditional heteroscedasticity (ARCH) model to estimate conditional variance of the sequence of increasing price of the United Kingdom's financial environment. This model was developed by prior assumption that the variance of random errors was related to the previous random, with inclusion of the conditional variance and mean in equation. Four years later, the extension of ARCH was proposed by Bollerslev (1986), known as generalized autoregressive conditional heteroscedasticity (GARCH) model. This model adds the memory of past variances to the model which is useful in modeling and forecasting

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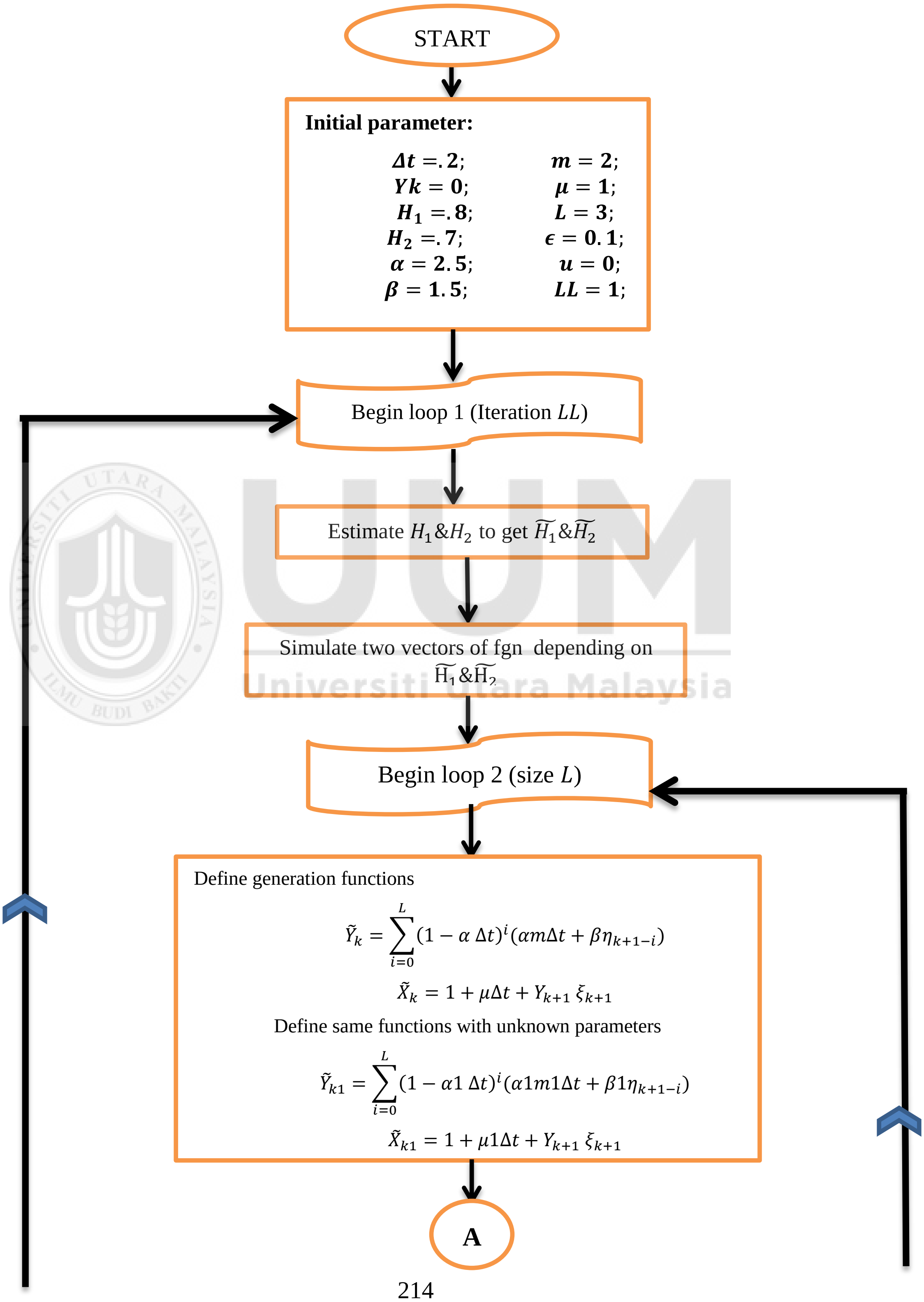
Appendix A

Flowchart for Parameters Estimation



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A

Compute the following covariance function:

$$\gamma_{\xi}(n) = \frac{1}{2}\left(|(n+1)\Delta t|^{2H_1} + |(n-1)\Delta t|^{2H_1} - 2|n\Delta t|^{2H_1}\right)$$

$$\gamma_{\eta}(n) = \frac{1}{2}\left(|(n+1)\Delta t|^{2H_2} + |(n-1)\Delta t|^{2H_2} - 2|n\Delta t|^{2H_2}\right)$$

$$\gamma_{\tilde{Y}}(n) = \beta^2 \sum_{i=0}^{L}\sum_{j=0}^{L}(1-\alpha\,\Delta t)^{i+j}\gamma_{\eta}(n+i-j)$$

Construct the following matrices:

$$\Gamma_{n-1} = \{\gamma(k-j)\}_{j,k=1}^{n-1} \quad \& \quad {\Gamma_{n-1}}^{-1}$$

$$\tilde{\gamma}_{n-1} = (\gamma_{\tilde{X}}(n-1), \ldots, \gamma_{\tilde{X}}(1))' \quad \& \quad {\tilde{\gamma}_{n-1}}'$$

$$\gamma_n = (\gamma(1), \ldots, \gamma(n))'$$

$$\Gamma_n = \begin{bmatrix} \Gamma_{n-1} & \tilde{\gamma}_{n-1} \\ {\tilde{\gamma}_{n-1}}' & \gamma_{\tilde{X}}(0) \end{bmatrix}$$

$$\phi_n = \Gamma_n^{-1}\gamma_n$$

$$\Gamma_n^{-1} = \begin{bmatrix} I & -\Gamma_{n-1}^{-1}\tilde{\gamma}_{n-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Gamma_{n-1}^{-1} & 0 \\ 0 & (\gamma_{\tilde{X}}(0) - \tilde{\gamma}'_{n-1}\Gamma_{n-1}^{-1}\tilde{\gamma}_{n-1})^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -\tilde{\gamma}'_{n-1}\Gamma_{n-1}^{-1} & 1 \end{bmatrix}$$

Compute $v_T^2 = \gamma(0) - \gamma_{T'}\Gamma_T^{-1}\gamma_T$

B

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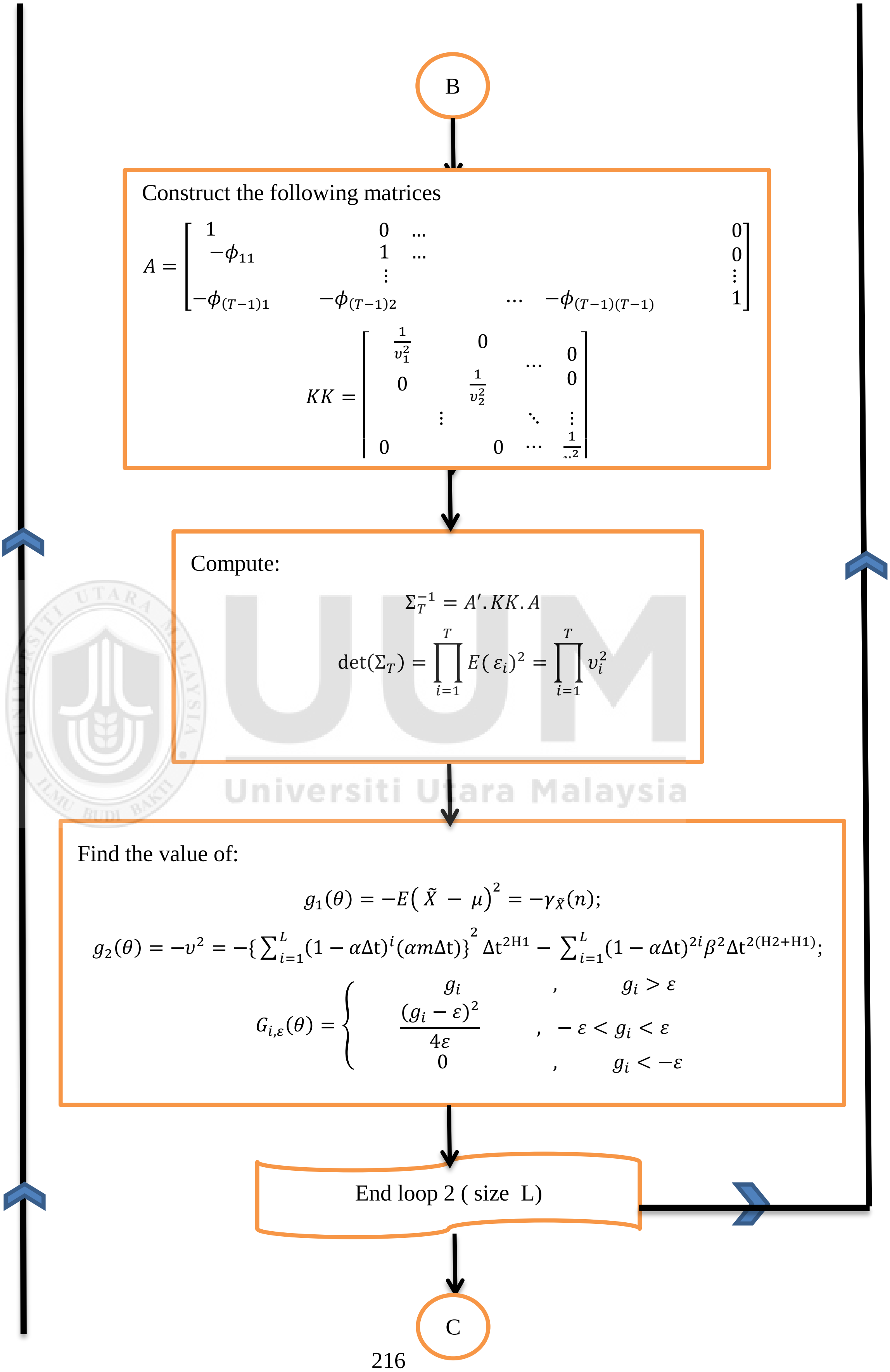
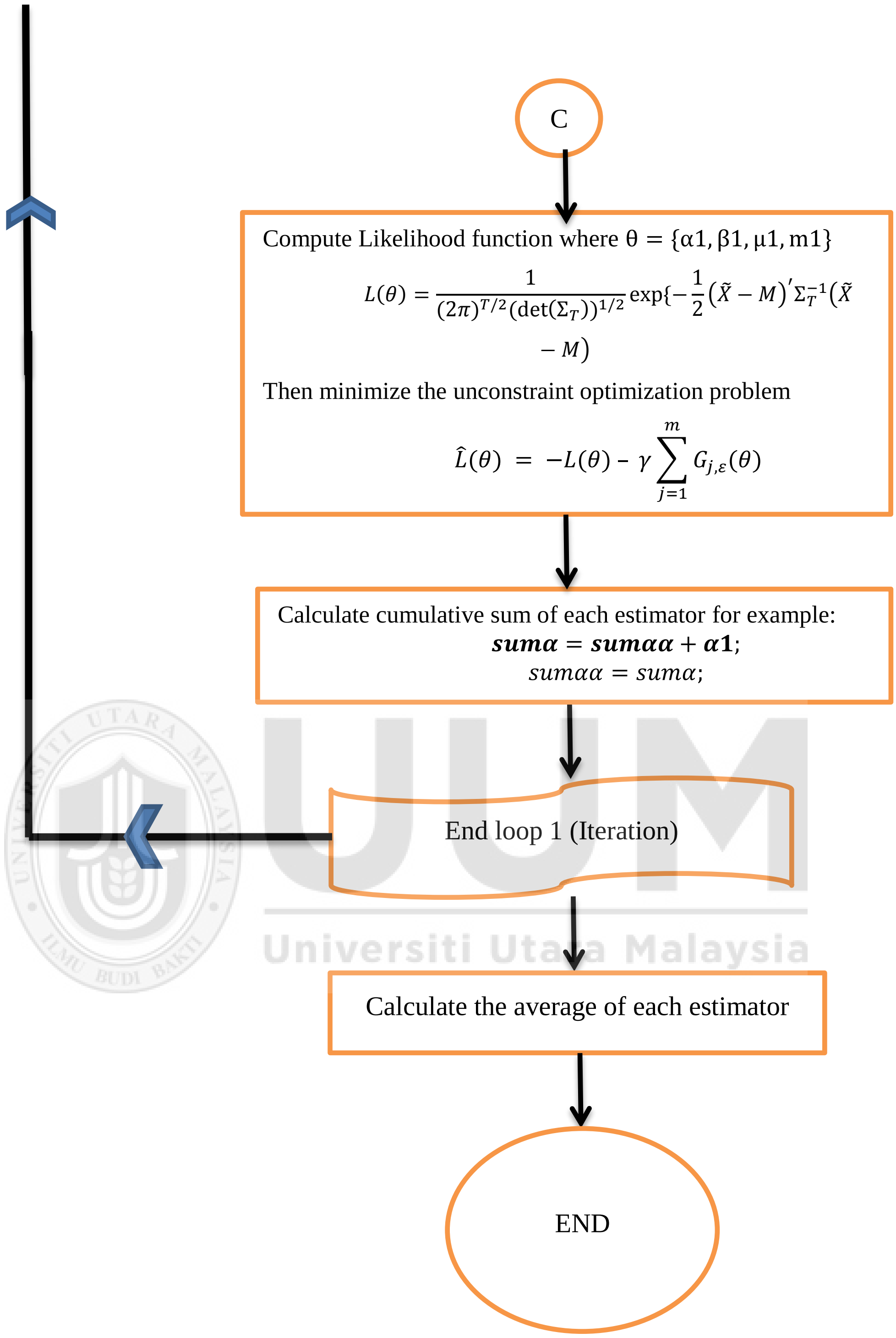

B
Construct the following matrices
$A = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ -\phi_{11} & 1 & \cdots & & 0 \\ & \vdots & & & \vdots \\ -\phi_{(T-1)1} & -\phi_{(T-1)2} & \cdots & -\phi_{(T-1)(T-1)} & 1 \end{bmatrix}$
$KK = \begin{bmatrix} \frac{1}{v_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{v_2^2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{v_T^2} \end{bmatrix}$
Compute:
$\Sigma_T^{-1} = A'.KK.A$
$\det(\Sigma_T) = \prod_{i=1}^{T} E(\varepsilon_i)^2 = \prod_{i=1}^{T} v_i^2$
Find the value of:
$g_1(\theta) = -E(\tilde{X} - \mu)^2 = -\gamma_{\tilde{X}}(n);$
$g_2(\theta) = -v^2 = -\{\sum_{i=1}^{L}(1-\alpha\Delta t)^i(\alpha m\Delta t)\}^2 \Delta t^{2H1} - \sum_{i=1}^{L}(1-\alpha\Delta t)^{2i}\beta^2\Delta t^{2(H2+H1)};$
$G_{i,\varepsilon}(\theta) = \begin{cases} g_i & , \quad g_i > \varepsilon \\ \frac{(g_i-\varepsilon)^2}{4\varepsilon} & , \quad -\varepsilon < g_i < \varepsilon \\ 0 & , \quad g_i < -\varepsilon \end{cases}$
End loop 2 (size L)
C



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Appendix B

Standard Simulation for Parameters Estimation

```
Δt=.2;
H1=.8;
H2=.7;
α=2.5;
m=2;
β=1.5;
μ=1;
ϵ=0.1;
l=0;
t=0;
u=0;
v=0;
c=0;
e=0;
f=0;
RR=100;
NN=10;
sumαhut1=0;
summhut1=0;
sumβhut1=0;
sumμhut1=0;
H11=0.8;
H22=0.7;
For[r=1,r<=RR,r++,
"Simulate a fractional Brownian motion process–1st one  ";
data1=RandomFunction[FractionalBrownianMotionProcess[H1],{0,1,0.001}];
"finds the parameter estimates for the fbm process(H1) from data1";
EH1=FindProcessParameters[data1,FractionalBrownianMotionProcess[h]];
H11=EH1[[1,2]];
"Simulate a fractional Brownian motion process–2nd one ";
data2=RandomFunction[FractionalBrownianMotionProcess[H2],{0,1,0.001}];
"finds the parameter estimates for the fbm process(H2) from data2";
```

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```
EH2=FindProcessParameters[data2,FractionalBrownianMotionProcess[h]];
H22=EH2[[1,2]];
“simulate fgn”;
SimulateFGN[H_,n_]:=Module[{𝓝,ac}, 𝓝 =2^Ceiling[Log[2,n–1]];
ac=Table[FGNAcf[k,H],{k,0, 𝓝 }];
Take[SimulateGLP[ac],n]];SimulateGLP[γ_]:=Module[{m=Length[γ],n,c,g,Z,
Ncap},n=2^Ceiling[Log[2,m–1]];acvf=If[n==m–
1,γ,PadRight[γ,n+1]];Ncap=2*n;
c=Join[acvf,Rest[Reverse[Rest[acvf]]]];
g=Re[Fourier[c,FourierParameters–>{1,–1}]];
Z=RandomVariate[NormalDistribution[0,1],Ncap–2];
Z=(Complex[Sequence@@#1]&)/@Partition[Z,2];
Z=Flatten[{RandomVariate[NormalDistribution[0,Sqrt[2]]],Z,RandomVariate
[NormalDistribution[0,Sqrt[2]]],Reverse[Conjugate[Z]]}];
Take[Re[InverseFourier[Sqrt[g]*Z,FourierParameters–>{0,–1}]],m]/Sqrt[2]];
FGNAcf[k_,H_]:=Module[{},0.5*(Abs[k+1.0]^(2.0*H)–
2*Abs[k]^(2.0*H)+Abs[k–1.0]^(2.0*H))];
“fgn at H=.65 and n=20”;
SmH111=SimulateFGN[H11,RR+1];
“fgn at H=.9 and n=20”;
SmH222=SimulateFGN[H22,RR+1];


n=2;
While[n<=NN,
Clear[α1,m1,β1,μ1];
sumαα=0;
sumββ=0;
sumμμ=0;
summm=0;
```

Yk11[n_]:= $\sum_{i=0}^{\infty}((\mathbf{1}-\boldsymbol{\alpha}\mathbf{1}\ \boldsymbol{\Delta}\mathbf{t})^{\mathbf{i}}(\boldsymbol{\alpha}\mathbf{1}\ \mathbf{m1}\ \boldsymbol{\Delta}\mathbf{t}+\boldsymbol{\beta}\mathbf{1}\ \mathbf{SmH2}[[\mathbf{Abs}[\mathbf{i}+\mathbf{1}]]])$;

Xk1[n_]:=1+μ1 Δt+SmH1[[n]]*

$\sum_{i=0}^{\infty}((\mathbf{1}-\boldsymbol{\alpha}\mathbf{1}\ \boldsymbol{\Delta}\mathbf{t})^{\mathbf{i}}(\boldsymbol{\alpha}\mathbf{1}\ \mathbf{m1}\ \boldsymbol{\Delta}\mathbf{t}+\boldsymbol{\beta}\mathbf{1}\ \mathbf{SmH2}[[\mathbf{Abs}[\mathbf{i}+\mathbf{1}]]])$;

Yk[n_]:= $\sum_{i=0}^{\infty}(\mathbf{1}-\boldsymbol{\alpha}\ \boldsymbol{\Delta}\mathbf{t})^{\mathbf{i}}(\boldsymbol{\alpha}\mathbf{m}\boldsymbol{\Delta}\mathbf{t}+\boldsymbol{\beta}\ \mathbf{SmH2}[[\mathbf{Abs}[\mathbf{i}+\mathbf{1}]]])$;

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```
Xk[n_]:=1+μ Δt+Yk[n]*SmH1[[n]];
mu[n_]:=Mean[Table[Xk1[i],{i,1,n}]];
X[n_]:=Table[Xk1[i],{i,1,n}]; XX[n_]:=Table[Xk[i],{i,1,n–
1}]~Join~{Xk1[n]};
γξ[n_]:=1/2 (Abs[(n–1) Δt]^(2 H11)+Abs[(n+1) Δt]^(2 H11)–2 Abs[n Δt]^(2 H11));
γη[n_]:=1/2 (Abs[(n–1) Δt]^(2 H22)+Abs[(n+1) Δt]^(2 H22)–2 Abs[n Δt]^(2 H22));
γX[n_]:=Yk[n]^2 γξ[n];
γY[n_]:= β^2 ∑_{i=0}^∞ ∑_{j=0}^∞ (1 − α Δt)^(i+j) γη[n + 1 − j];

Γn1[n_]:=Table[γX[i–j],{i,1,n–1},{j,1,n–1}];"Γn–1";
γn1[n_]:=Table[γX[n–i],{i,1,n–1},{j,1,1}];"γn–1";
γTn1[n_]:=Transpose[γn1[n]];"γn–1 transpose";
γX0=γX[0];"γX(0)";
Γ[n_]:=ArrayFlatten[{{Γn1[n],γn1[n]},{γTn1[n],γX0}}];
invΓ[n_]:=Inverse[Γ[n]];
γ[n_]:=Table[{γX[i]},{i,1,n}] ;"γT in 3.27";
VT2[n_]:=Abs[γX[0]–Transpose[γ[n]].invΓ[n].γ[n]];"3.27";
VT22[n_]:=ToExpression[StringReplace[ToString[VT2[n]],{"{"–>"","}"–
>""}]];
invΓ[1]=1/γX0;
For[i=1,i<=n,i++,
ϕ[i_]:=invΓ[i].γ[i];
ϕ1[i_]:=Flatten[ϕ[i]];
For[j=1,j<=n,j++,
If[i==j,a[i,j]=1];
If[i<j,a[i,j]=0];
If[i>j,a[i,j]=– ϕ1[i–1][[j]]]
      ];
a[1,1]=1;
a[2,1]=–γX[1]/γX0;
    ];
A=Table[a[i,j],{i,1,n},{j,1,n}];
For[i=2,i<=n,i++,
For[j=2,j<=n,j++,
VT1=Abs[γX0–γX[1]* 1/γX0 *γX[1]];
d[1,1]=(1/VT1);
```

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```
If[i==j,d[i,j]=1/VT22[i]];
If[i<j,d[i,j]=0];
If[i>j,d[i,j]=0];
      ]
     ];

KK=Table[If[i==1,d[i,j]=0,d[i,j]],{i,1,n},
{j,1,n}];
KK=Table[If[j==1,d[i,j]=0,d[i,j]],{i,1,n},
{j,1,n}];
d[1,1]=Abs[1/VT1];
KK=Table[d[i,j],{i,1,n},{j,1,n}];
segmainv[n_]:=SetPrecision[Transpose[A].KK.A ,5];
"segma[n_]:=PaddedForm[SetPrecision[Inverse[segmainv[n]],5],{5,5}]";
```

DetKKK[n_]:=VT1 * $\prod_{i=1}^{T} \mathbf{VT2}[\mathbf{i}]$;

```
"define penalty function ";
g1[n_]:=–γX[n];
```

g2[n_]:= $-\sum_{i=1}^{L}(\mathbf{1}-\boldsymbol{\alpha\Delta t})^{\mathbf{i}}(\boldsymbol{\alpha m \Delta t})\}^{2}\,\boldsymbol{\Delta t}^{\mathbf{2H11}}-\sum_{i=1}^{L}(\mathbf{1}-\boldsymbol{\alpha\Delta t})^{\mathbf{2i}}\boldsymbol{\beta}^{\mathbf{2}}\boldsymbol{\Delta t}^{\mathbf{2(H22+H11)}}$;

```
G1[n_]:=Piecewise[{{g1[n],g1[n]>ϵ},{(g1[n]–ϵ)^2/(4 ϵ),–
ϵ<g1[n]<ϵ},{0,g1[n]<– ϵ}}];"3.36 w.r.t g1";
G2[n_]:=Piecewise[{{g2[n],g2[n]>ϵ},{(g2[n]–ϵ)^2/(4 ϵ),–
ϵ<g2[n]<ϵ},{0,g2[n]<– ϵ}}];"3.36w.r.t g2";
;n++];
"lhood[NN_]:=Log[PDF[MultinormalDistribution[Flatten[Table[mu[i],{i,1,N
N}]],segma[NN]],X[NN]]]";
lhood1[NN_]:=Log[Exp[–0.5* Transpose[Table[Xk1[i]–
mu[i],{i,1,NN}],{1}].segmainv[NN].Table[Xk1[i]–
mu[i],{i,1,NN}]]/ToExpression[StringReplace[ToString[(2 Pi)^(NN/2)
DetKKK [NN]^(1/2) ],{"{"–>"","}"–>""}]]]];
```

op=NMinimize[– lhood1[NN]–ϵ/4 $\sum_{i=1}^{NN} \mathbf{G1}[\mathbf{i}]$ –ϵ/4 $\sum_{i=1}^{NN} \mathbf{G2}[\mathbf{i}]$

```
,{α1,m1,β1,μ1},WorkingPrecision–>15,Method–>"DifferentialEvolution"];
dd=op;
answer=op[[1]];
```

optimall$_{u=u+1}$=answer;

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var=dd[[2]];”the answer”;

αα=var[[1]];
ααα=αα[[2]];
sumα=sumαα+ααα;
sumαα=sumα;

mm=var[[2]];
mmm=mm[[2]];
summ=summm+mmm;
summm=summ;

ββ=var[[3]];
βββ=ββ[[2]];
sumβ=sumββ+βββ;
sumββ=sumβ;

μμ0=var[[4]];
μμμ=μμ0[[2]];
sumμ=sumμμ+μμμ;
sumμμ=sumμ;

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“Print[dd];”;
sepvar=Table[var[[i,2]],{i,1,4}];
{ α1,m1,β1,μ1}=sepvar;
Clear[α1,m1,β1,μ1];
Table[$optimall_{i}$,{i,1,RR}];
H1hut=H11;
$H1hutt_{l=l+1}$=H1hut;
H2hut=H22;
$H2hutt_{t=t+1}$=H2hut;
αhut=sumαα/RR;
$\alpha hutt_{v=v+1}$=αhut;
mhut=summm/RR;
$mhutt_{c=c+1}$=mhut;
βhut=sumββ/RR;

```
βhutt_{e=e+1}=βhut;
μhut=sumμμ/RR;
μhutt_{f=f+1}=μhut;
Print[H1hut];
Print[H2hut];
Print[αhut];
Print[mhut];
Print[βhut];
Print[μhut];
Print[">>>>>>>>>>>>>>>>>>>>>>>>>>>>>"];
  ];
```

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Appendix C

Simulation with Segmentation for Parameters Estimation

```
Δt=.2;
H1=.8;
H2=.7;
α=2.5;
m=2;
β=1.5;
μ=1;
ϵ=0.1;
l=0;
t=0;
u=0;
v=0;
c=0;
e=0;
f=0;
RR=100;
JJ=10;
NN=10;
sumαhut1=0;
summhut1=0;
sumβhut1=0;
sumμhut1=0;
H11=0.8;
H22=0.7;
For[r=1,r<=RR,r++,
“Simulate a fractional Brownian motion process –1st one “;
data1=RandomFunction[FractionalBrownianMotionProcess[H1],{0,1,0.001}];
“finds the parameter estimates for the fbm process(H1) from data1”;
EH1=FindProcessParameters[data1,FractionalBrownianMotionProcess[h]];
H11=EH1[[1,2]];
“Simulate a fractional Brownian motion process–2nd one “;
```

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```
data2=RandomFunction[FractionalBrownianMotionProcess[H2],{0,1,0.001}];
”finds the parameter estimates for the fbm process(H2) from data2”;
EH2=FindProcessParameters[data2,FractionalBrownianMotionProcess[h]];
H22=EH2[[1,2]];
“simulate fgn”;
SimulateFGN[H_,n_]:=Module[{𝓝,ac}, 𝓝 =2^Ceiling[Log[2,n–1]];
ac=Table[FGNAcf[k,H],{k,0, 𝓝 }];
Take[SimulateGLP[ac],n]];SimulateGLP[γ_]:=Module[{m=Length[γ],n,c,g,Z,
Ncap},n=2^Ceiling[Log[2,m–1]];acvf=If[n==m–
1,γ,PadRight[γ,n+1]];Ncap=2*n;
c=Join[acvf,Rest[Reverse[Rest[acvf]]]];
g=Re[Fourier[c,FourierParameters–>{1,–1}]];
Z=RandomVariate[NormalDistribution[0,1],Ncap–2];
Z=(Complex[Sequence@@#1]&)/@Partition[Z,2];
Z=Flatten[{RandomVariate[NormalDistribution[0,Sqrt[2]]],Z,RandomVariate
[NormalDistribution[0,Sqrt[2]]],Reverse[Conjugate[Z]]}];
Take[Re[InverseFourier[Sqrt[g]*Z,FourierParameters–>{0,–1}]],m]/Sqrt[2]];
FGNAcf[k_,H_]:=Module[{},0.5*(Abs[k+1.0]^(2.0*H)–
2*Abs[k]^(2.0*H)+Abs[k–1.0]^(2.0*H))];
“fgn at H=.65 and n=20”;
SmH111=SimulateFGN[H11,110];
“fgn at H=.9 and n=20”;
SmH222=SimulateFGN[H22,110];
Do[
sumαα=0;
sumββ=0;
sumμμ=0;
summm=0;
w[j_]:=j+10(j–1);
SmH11[j_]:=Table[SmH111[[i]],{i,w[j],w[j]+10}];
SmH22[j_]:=Table[SmH222[[i]],{i,w[j],w[j]+10}];
SmH1=SmH11[j];
SmH2=SmH22[j];

n=2;
While[n<=NN,
```

Clear[α1,m1,β1,μ1];
Yk11[n_]:= $\sum_{i=0}^{\infty}((\mathbf{1-\alpha 1\ \Delta t})^{\mathbf{i}}(\mathbf{\alpha 1\ m1\ \Delta t+\beta 1\ SmH2}[[\mathbf{Abs}[\mathbf{i+1}]]])$;
Xk1[n_]:=1+μ1 Δt+SmH1[[n]]*
$\sum_{i=0}^{\infty}((\mathbf{1-\alpha 1\ \Delta t})^{\mathbf{i}}(\mathbf{\alpha 1\ m1\ \Delta t+\beta 1\ SmH2}[[\mathbf{Abs}[\mathbf{i+1}]]])$;
Yk[n_]:= $\sum_{i=0}^{\infty}(\mathbf{1-\alpha\ \Delta t})^{\mathbf{i}}(\mathbf{\alpha m \Delta t+\beta\ SmH2}[[\mathbf{Abs}[\mathbf{i+1}]]])$;
Xk[n_]:=1+μ Δt+Yk[n]*SmH1[[n]];
mu[n_]:=Mean[Table[Xk1[i],{i,1,n}]];
X[n_]:=Table[Xk1[i],{i,1,n}]; XX[n_]:=Table[Xk[i],{i,1,n–1}]~Join~{Xk1[n]};
γξ[n_]:=1/2 (Abs[(n–1) Δt]$^{2\ H11}$+Abs[(n+1) Δt]$^{2\ H11}$–2 Abs[n Δt]$^{2\ H11}$);
γη[n_]:=1/2 (Abs[(n–1) Δt]$^{2\ H22}$+Abs[(n+1) Δt]$^{2\ H22}$–2 Abs[n Δt]$^{2\ H22}$);
γX[n_]:=Yk[n]^2 γξ[n];
γY[n_]:= $\boldsymbol{\beta^2} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} (\mathbf{1}-\boldsymbol{\alpha\ \Delta t})^{\boldsymbol{i+j}}\ \boldsymbol{\gamma\eta}[\mathbf{n+1-j}]$;

Γn1[n_]:=Table[γX[i–j],{i,1,n–1},{j,1,n–1}];"Γn–1";
γn1[n_]:=Table[γX[n–i],{i,1,n–1},{j,1,1}];"γn–1";
γTn1[n_]:=Transpose[γn1[n]];"γn–1 transpose";
γX0=γX[0];"γX(0)";
Γ[n_]:=ArrayFlatten[{{Γn1[n],γn1[n]},{γTn1[n],γX0}}];
invΓ[n_]:=Inverse[Γ[n]];
γ[n_]:=Table[{γX[i]},{i,1,n}] ;"γT in 3.27";
VT2[n_]:=Abs[γX[0]–Transpose[γ[n]].invΓ[n].γ[n]];"3.27";
VT22[n_]:=ToExpression[StringReplace[ToString[VT2[n]],{"{"–>"","}"–>""}]];
invΓ[1]=1/γX0;
For[i=1,i<=n,i++,
ϕ[i_]:=invΓ[i].γ[i];
ϕ1[i_]:=Flatten[ϕ[i]];
For[j=1,j<=n,j++,
If[i==j,a[i,j]=1];
If[i<j,a[i,j]=0];
If[i>j,a[i,j]=– ϕ1[i–1][[j]]]
];
a[1,1]=1;
a[2,1]=–γX[1]/γX0;

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```
    ];
A=Table[a[i,j],{i,1,n},{j,1,n}];
For[i=2,i<=n,i++,
For[j=2,j<=n,j++,
VT1=Abs[γX0–γX[1]* 1/γX0 *γX[1]];
d[1,1]=(1/VT1);
If[i==j,d[i,j]=1/VT22[i]];
If[i<j,d[i,j]=0];
If[i>j,d[i,j]=0];
     ]
    ];

KK=Table[If[i==1,d[i,j]=0,d[i,j]],{i,1,n},
{j,1,n}];
KK=Table[If[j==1,d[i,j]=0,d[i,j]],{i,1,n},
{j,1,n}];
d[1,1]=Abs[1/VT1];
KK=Table[d[i,j],{i,1,n},{j,1,n}];
segmainv[n_]:=SetPrecision[Transpose[A].KK.A ,5];
"segma[n_]:=PaddedForm[SetPrecision[Inverse[segmainv[n]],5],{5,5}]";
```

DetKKK[n_]:=VT1 * $\prod_{\mathbf{i=1}}^{\mathbf{T}} \mathbf{VT2[i]}$;

```
"define penalty function ";
g1[n_]:=–γX[n];
```

g2[n_]:= $-\sum_{i=1}^{L}(\mathbf{1}-\boldsymbol{\alpha\Delta t})^{\mathbf{i}}(\boldsymbol{\alpha m \Delta t})\}^{\mathbf{2}}\,\boldsymbol{\Delta t}^{\mathbf{2H11}}-\sum_{i=1}^{L}(\mathbf{1}-\boldsymbol{\alpha\Delta t})^{\mathbf{2i}}\boldsymbol{\beta}^{\mathbf{2}}\boldsymbol{\Delta t}^{\mathbf{2(H22+H11)}}$;

```
G1[n_]:=Piecewise[{{g1[n],g1[n]>ϵ},{(g1[n]–ϵ)^2/(4 ϵ),–
ϵ<g1[n]<ϵ},{0,g1[n]<– ϵ}}];"3.36 w.r.t g1";
G2[n_]:=Piecewise[{{g2[n],g2[n]>ϵ},{(g2[n]–ϵ)^2/(4 ϵ),–
ϵ<g2[n]<ϵ},{0,g2[n]<– ϵ}}];"3.36w.r.t g2";
;n++];
"lhood[NN_]:=Log[PDF[MultinormalDistribution[Flatten[Table[mu[i],{i,1,N
N}]],segma[NN]],X[NN]]]";
lhood1[NN_]:=Log[Exp[–0.5* Transpose[Table[Xk1[i]–
mu[i],{i,1,NN}],{1}].segmainv[NN].Table[Xk1[i]–
```

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```
mu[i],{i,1,NN}]]/ToExpression[StringReplace[ToString[(2 Pi)^(NN/2)
DetKKK [NN]^(1/2) ],{"{"–>"","}"–>""}]]]];
op=NMinimize[– lhood1[NN]–ϵ/4 ∑_{i=1}^{NN} G1[i] –ϵ/4 ∑_{i=1}^{NN} G2[i]
,{α1,m1,β1,μ1},WorkingPrecision–>15,Method–>"DifferentialEvolution"];
dd=op;
answer=op[[1]];
optimall_{u=u+1}=answer;
var=dd[[2]];"the answer";

αα=var[[1]];
ααα=αα[[2]];
sumα=sumαα+ααα;
sumαα=sumα;

mm=var[[2]];
mmm=mm[[2]];
summ=summm+mmm;
summm=summ;

ββ=var[[3]];
βββ=ββ[[2]];
sumβ=sumββ+βββ;
sumββ=sumβ;

μμ0=var[[4]];
μμμ=μμ0[[2]];
sumμ=sumμμ+μμμ;
sumμμ=sumμ;

"Print[dd];";
sepvar=Table[var[[i,2]],{i,1,4}];
{ α1,m1,β1,μ1}=sepvar;
Clear[ α1,m1,β1,μ1];
,{j,JJ}];
Table[optimall_i,{i,1,JJ}];
```

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```
H1hut=H11;
H1hutt_{l=l+1}=H1hut;
H2hut=H22;
H2hutt_{t=t+1}=H2hut;
αhut=sumαα/JJ;
αhutt_{v=v+1}=αhut;
mhut=summm/JJ;
mhutt_{c=c+1}=mhut;
βhut=sumββ/JJ;
βhutt_{e=e+1}=βhut;
μhut=sumμμ/JJ;
μhutt_{f=f+1}=μhut;
Print[H1hut];
Print[H2hut];
Print[αhut];
Print[mhut];
Print[βhut];
Print[μhut];
Print[">>>>>>>>>>>>>>>>>>>>>>>>>>>>"];
  ];
H1est=Table [H1hutt_i,{i,1,RR}];
H2est=Table [H2hutt_i,{i,1,RR}];
αest=Table[αhutt_i,{i,1,RR}];
mest=Table[mhutt_i,{i,1,RR}];
βest=Table[βhutt_i,{i,1,RR}];
μest=Table[μhutt_i,{i,1,RR}];
H1hutav=Mean[H1est];
H2hutav=Mean[H2est];
αhutav=Mean[αest];
mhutav=Mean[mest];
βhutav=Mean[βest];
μhutav=Mean[μest];
"αvar=Variance[αest];
mvar=Variance[mest];
βvar=Variance[βest];
μvar=Variance[μest]";
```


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```
Print["H1hut–average = ",H1hutav];
Print["H2hut–average = ",H2hutav];
Print["αhut–average = ",αhutav];
Print["mhut–average = ",mhutav];
Print["βhut–average = ",βhutav];
Print["μhut–average = ",μhutav];
Print["H1–variance = ",Variance[H1est]];
Print["H2–variance = ",Variance[H2est]];
Print["α–variance = ",Variance[αest]];
Print["m–variance = ",Variance[mest]];
Print["β–variance = ",Variance[βest]];
Print["μ–variance = ",Variance[μest]];
Print["H1–bias = ",Abs[H1–H1hutav]];
Print["H2–bias = ",Abs[H2–H1hutav]];
Print["α–bias = ",Abs[α–αhutav]];
Print["m–bias = ",Abs[m–mhutav]];
Print["β–bias = ",Abs[β–βhutav]];
Print["μ–bias = ",Abs[μ–μhutav]];
Print["H1–MSE = ",Variance[H1est]+(Abs[H1–H1hutav])^2];
Print["H2–MSE = ",Variance[H2est]+(Abs[H2–H2hutav])^2];
Print["α–MSE = ",Variance[αest]+(Abs[α–αhutav])^2];
Print["m–MSE = ",Variance[mest]+(Abs[m–mhutav])^2];
Print["β–MSE = ",Variance[βest]+(Abs[β–βhutav])^2];
Print["μ–MSE = ",Variance[μest]+(Abs[μ–μhutav])^2];
```

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