STUDY OF MUSLIMS IN MARITAL SYSTEM USING MARKOV CHAIN SIMPLE EXPONENTIAL SMOOTHING (MC_{SES}) TECHNIQUE

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Abstrak

Model Rantai Markov (MC) merupakan model m atik yang popular digunakan untuk melihat aliran data dalam sistem. Model ini boleh digunakan untuk meramal nilai masa depan bagi tempoh jangka pendek. Walau bagaimanapun, kebanyakan kajian lalu tidak memberi tumpuan kepada ketepatan nilai ramalan. Integrasi model MC dan teknik Purata Bergerak Mudah (SMA) diketahui dapat menghasilkan ketepatan ramalan yang lebih tinggi daripada model MC yang asal bagi kes unjuran jangka masa panjang di mana data bagi tempoh sebelumnya diketahui. Waiau bagaimanapun, teknik pelicinan eksponen mudah (SES) adalah lebih fleksibel daripada SMA kerana ia menggunakan pemalar kelicinan. Oleh yang demikian, kajian ini membangunkan langkah model bagi model MC_{SES} bagi data terhad danunjuran jangka masa pendek dengan mengintegrasikan model MC dengan SES. Model MC_{SES} hibrid digunakan untuk mengukuhkan model MC dan meningkatkan ketepatan nilai ramalan. Empat pengukur ralat yang digunakan untuk menentukan ketepatan model ini adalah min sisihan mutlak, min peratusan sisihan mutlak , min peratusan ralat mutlak dan min kuasa dua ralat. Kajian ini menggunakan sampel sebanyak 6061 pasangan Islam di daerah Pendang, Kedah yang terdapat dalam pangkalan data sistem perkahwinan bagi tahun 2013 dan 2014. Jangkaan bilangan umat Islam di Pendang berdasarkan jantina dan kategori umur bagi tahun yang berikutnya dibuat menggunakan model hibrid MC_{SES}. Perbandingan dengan model MC dan model MC_{SMA} menunjukkan bahawa model MC_{SES} kacukan menghasilkan ketepatan ramalan yang lebih baik. Oleh itu, dicadangkan model MC_{SES} adalah model yang paling sesuai dalam membuat jangkaan bilangan umat Islam yang terdapat dalam sistem perkahwinan berdasarkan jantina dan kategori umur bagi tahun 2014. Model ini boleh menyumbang dalam kes-kes yang terdapatnya data yang terhad dan untuk unjuran jangka masa pendek serta boleh juga digunakan dalam pelbagai bidang lain.

Kata kunci: Model hibrid rantai markov, Ketepatan ramalan, Pelicinan eksponen mudah

Abstract

The Markov Chain (MC) model is a popular mathematical model used to observe the flow of data in a system. It can also be used to forecast future values for short-term period. However, most previous studies do not focus on the accuracy of the forecast values. Integration of the MC model and Simple Moving Average (SMA) technique is known to produce higher forecast accuracy than the classical MC model for the case of long-term projection with known previous data. However, Simple Exponential Smoothing (SES) technique is more flexible than SMA because it uses a smoothing constant. Therefore, this study develops modeling steps for MC model in the case of limited data and short-term projection by integrating MC model with SES (MC_{SES}). The MC_{SES} hybrid model is used to enhance the MC model and improve the accuracy of the forecast values. Four error measures used to determine the accuracy of this model are mean absolute deviation, mean absolute percentage deviation, mean absolute percentage error and mean square error. This study uses a sample of 6061 Muslim couples data in Pendang, Kedah who are in the marital system for the year 2013 and 2014. The number of Muslims in subsequence year according to gender and age categories is forecasted using proposed MC_{SES} hybrid model. Comparison with MC and MC_{SMA} models indicates that the developed MC_{SES} hybrid model has better forecast accuracy. Therefore, the MC_{SES} hybrid model is the most appropriate model to forecast the number of Muslims in the marital system according to gender and age categories for the year 2014. This model can be used for short-term projection in cases with limited data and is applicable in various fields.

Keywords: Hybrid markov chain model, Forecast accuracy, Simple exponential smoothing

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List of Abbreviations

ARMA	autoregressive moving average
ARRES	adaptive response rate exponential smoothing
MAD	mean absolute deviation
MAE	mean absolute error
MAPD	mean absolute percentage deviation
MAPE	mean absolute percentage error
MSE	mean square error
RMSE	root mean square error
SSE	sum of squared errors

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

A stochastic process is a random process which can be used to predict a set of possible outcomes weighted by their probabilities. It considers some indeterminacy in the future evolution which can be described by probability distributions. A stochastic process is the opposite of a deterministic process which deals with only possible reality of how the process might evolve overtime (Adeleke, Oguntuase, & Ogunsakin, 2014).

A Markov process can also be defined as a stochastic process since it involves probabilities. This process is only concerned with the current data without references to historical data. Markov process deals with state space and a Markov process that is in a discrete state is known as a Markov Chain.

A Markov Chain (MC) model, which is named after Andrei Andreyevich Markov, a Russian mathematician, is a mathematical system that undergoes transitions from one state to another. It involves a finite or countable number of possible states (Borah & Kakaty, 2012). It is modeled using current data which is in discrete state where the historical data are not necessary in order to predict the future values (Taylor & Karlin, 1998).

MC model is a popular mathematical model that is used to examine the flow of data using the stochastic processes and also forecasting future values for long-term and short- term projection. It has been used for long-term projection in various fields including management (Igboanugo & Onifade, 2011), education (Adeleke et al., 2014; Awadhi & Konsowa, 2007; Nicholls, 2009; Saad, 2012), military (Skulj, Vehovar, & Stamfelj, 2008), health administration (Borah & Kakaty, 2012; Lagarde & Cairns, 2012), biology (Liang, Cheng, Wixon, & Balser, 2011), accounting (Saibeni, 2010), and banking (Hassan, 2010).

1.2 Problem Statement

MC model is a popular model used among practitioners for examining data flow and forecasting future values. However, most of the previous studies do not consider the forecast accuracy of a model. Measuring forecast accuracy of the model is very important because a forecast is never completely accurate. The study on forecast accuracy aims to determine the deviation of forecast value from the actual value. Thus it is crucial for practitioners to study the forecast accuracy of the model before completely relying on the forecast value produced.

Lately, there have been studies on MC model that take into account the forecast accuracy. The most recent study utilizes smoothing technique in MC model (Adnan, 2012). A model is constructed using simple moving average (SMA), one of the smoothing techniques, to modify the projected transition probability matrix using MC model, to forecast the enrolment of postgraduate students at the College of Arts and Sciences in Universiti Utara Malaysia. The research uses 4 years data to produce a steady-state transition probability matrix for long-term projection using MC model. Besides, the study use smoothed transition probability matrix where the integration between MC model and simple moving average is made in order to predict the future number of students' enrolment. In addition, the study measures the accuracy of the model used. The result shows the forecast values using the MC model with SMA is more accurate than the forecast values produced using the classical MC model. The accuracy of the MC model has been determined using the validation of data by considering the error measure of mean absolute percentage error (MAPE). The study is limited for the case of long-term projection only where the previous data are available. However, the model occurs for short-term or immediate years forecasting has not been tested.

SMA gives equal weight to all the observations. The most recent data and earlier past data are assumed to have equal chance to occur. In real practice, recent data has more chances to occur. Therefore, the exponential smoothing is suitable to be used compared to SMA. The simple exponential smoothing (SES) is the simplest short-term forecasting technique among others in terms of the calculations and it is the most widely used in the family of exponential smoothing. The advantage of SES technique over SMA is that SES gives the most weight to the most recent observation. This technique, which introduces a smoothing constant, α is more flexible than SMA. The SES method strongly depends on the optimal value of the smoothing constant, α in order to ensure its accuracy. Besides that, SES is used for data with no trend and only requires a limited amount of data.

Therefore, this research proposes to integrate SES and MC model to develop modeling steps for MC model in the case of limited data and short-term projection. The model known as MC_{SES} hybrid model is used for the purpose of observing the flow of data as well as to forecast future values. The forecast accuracy of the proposed MC_{SES} hybrid model will be measured and compared to the classical MC model and the MC model with simple moving average. The research focuses on modeling the flow of Muslims in the marital system and obtaining the projected number of Muslims in the marital system for the following future period.

1.3 Objectives of the Study

The main objective of this study is to develop modeling steps for Markov Chain model in the case of limited data and short-term projection, called the MC_{SES} hybrid model. In order to achieve the main objective, we have the following objectives:

i. To generate values for previous periods for transition probability matrix in the case of missing records
ii. To construct a projected transition probability matrix using simple exponential smoothing
iii. To forecast the number of Muslims in the marital system using the developed MC_{SES} hybrid model
iv. To compare the forecast accuracy of the MC_{SES} hybrid model with the classical Markov Chain model and the MC model with simple moving average in terms of four error measures
v. To determine the most appropriate model in estimating the forecast values for the case of Muslims in the marital system for the following future year

1.4 Scope of the Study

This study uses data of the number of Muslims in the marital system in Kota Setar, one of the districts of Kedah. The data is categorized into gender and age category.

This study deals with data of two years, which is of 2013 and 2014. The study utilizes the data of 2013-2014 for the purpose of observing the flow and projects the values for the following future year. The forecast accuracy is measured by using data of 2014-2015.

1.5 Significance of the Study

This study is beneficial for mathematical practitioners and applied researchers. The developed MC_{SES} hybrid model would contribute towards the area of mathematics as well as towards the application of the mathematical model. For mathematical practitioners, the study serves as their reference or as a guide to improve the MC model by integrating smoothing technique. For applied researchers, the study provides them with a guideline in order to apply the hybrid model in various fields. In addition, this hybrid model can be utilized in cases where there is limited historical data.

1.6 Organization of the Report

Chapter 1 briefly introduces the MC model and some of its applications in various fields. The discussion continues on to the forecast accuracy of MC model. It has been stated that the integration of MC model and SMA is known to produce higher forecast accuracy than the classical MC model. However, simple exponential smoothing (SES) technique, which introduces a smoothing constant, is more flexible than SMA. The discussion then leads to the main objectives of this study which is to develop modeling steps for Markov Chain model in the case of limited data and short-term projection. The model, known MC_{SES} hybrid model, is used to both observe the data flow and forecast future values for the case study of Muslims in the marital system.

Chapter 2 gives a literature review on the theories of the MC model, exponential smoothing (ES) technique and four types of error measures for measuring forecast accuracy. Besides that, the applications of MC model and ES technique in real world problem based on past studies are discussed. Reviews on studies regarding marriage and divorce in Malaysia are also given in this chapter.

Chapter 3 describes the case study, the collection of data, the framework of the study and the modeling steps of the proposed MC_{SES} hybrid model. This is followed by a description of the procedure of measuring forecast accuracy of the proposed model and comparing the forecast accuracy with other MC models.

Chapter 4 gives the results of the proposed MC_{SES} hybrid model for the case of Muslims in the marital system. The forecast accuracy of the proposed MC_{SES} hybrid model is measured using the error measures.

In addition, the comparison of the forecast accuracy between the proposed MC_{SES} hybrid model and the other two models which are the MC model and MC_{SMA} model has also been done in this study. Thus, Chapter 4 reports the model that yields the best results in forecasting the number of Muslims in the marital system for the following future year.

Finally, Chapter 5 begins with the summary and contributions of the whole study. This is followed by the limitations of the study as well as some suggestions for future research.

CHAPTER TWO

LITERATURE REVIEW

This chapter briefly describes the theories of the Markov Chain model and exponential smoothing (ES). In addition, four types of error measures used to measure the forecast accuracy are also briefly described. Besides that, the applications of MC model and ES from previous researches in various fields are also being reviewed. Reviews on studies regarding marriage and divorce in Malaysia are also provided in this chapter.

2.1 Stochastic Process

Stochastic is one of the Greek words which means "random" or "chance". In a stochastic model, a set of possible weighted outcomes is being predicted by probabilities. A Markov process is one of the stochastic processes that deals with current data. Thus, the historical data are not necessary in order to predict data for the next time period (Taylor & Karlin, 1998).

Markov process can be classified according to the nature of the time parameter and the nature of the state space (Ibe, 2009). There are four basic types of Markov processes: Discrete-time Markov Chain (DTMC), Continuous-time Markov Chain (CTMC), Discrete-time Markov process (DTMP) and Continuous-time Markov process (CTMP). The illustration of the classification of the Markov processes is shown in the diagram in Figure 2.1.

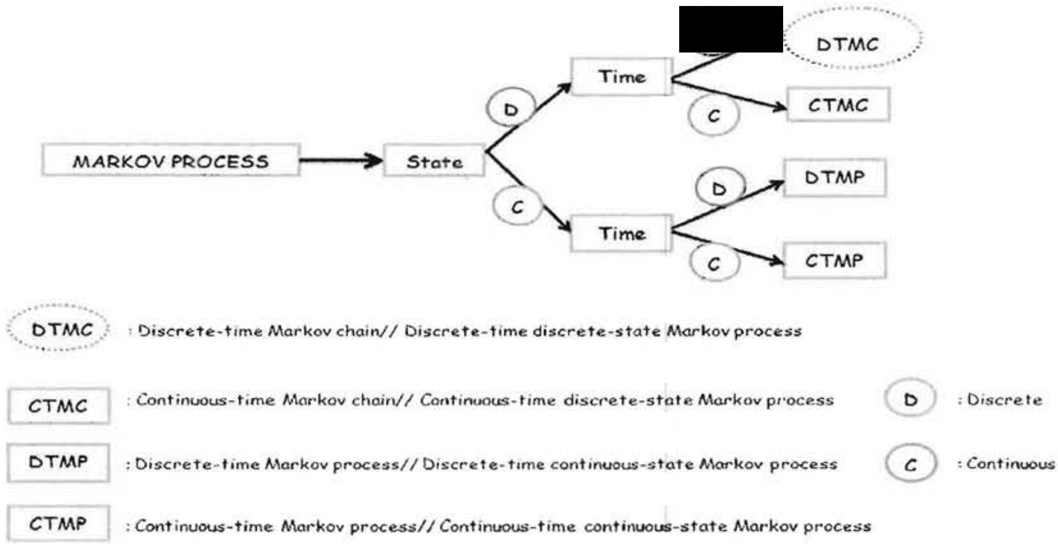

Figure 2.1. Classification of Markov processes

2.2 Markov Chain (MC) Model

A Markov process is a Markov Chain model if the number of states is finite or countable (Adeleke et al., 2014). A Markov Chain model can be classified into Discrete-time Markov Chain model (or discrete-time discrete-state Markov process) and also Continuous-time Markov Chain model (or continuous-time discrete-state Markov process). A Discrete-time Markov Chain (DTMC) is a model that deals with discrete time steps. It means that the changes to the model can only happen at one of those discrete time values. On the other hand, a Continuous-time Markov Chain (CTMC) model is one in which changes to the system can happen at any time along a continuous interval whereas in DTMC model, the transitions can happen at a specific time (Castaneda, Arunachalam, & Dharmaraja, 2012).

2.2.1 Terminologies in Markov Chain Model

There are several important terminologies in MC model to be explained for the purpose of the study. The definitions of transition probability matrix, transient state, absorbing state and absorbing Markov Chain model are given as follows.

Definition 2.1: Transition Probability Matrix

Transition probability matrix, P is defined as a matrix where the probability of the system being in a given state in a particular period depends only on its state in the preceding period and it is independent of all earlier periods. P is presented as follows:

$$P=\begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1k} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ p_{k1} & p_{k2} & p_{k3} & \cdots & p_{kk} \end{pmatrix}=\left(p_{ij}\right) \tag{2.1}$$

where p_{ij} is the probability of transition from state 'i' to state 'j' in just one step which satisfy $p_{ij}\geq 0$ and $\sum_{j=1}^{k} p_{ij}=1$ for all i .

Definition 2.2: Transient State

A state is called a transient state if it is of a non-zero probability. Once the entity leaves the state, it cannot return. In other words, a state i is said to be transient if $p_{ii}<1$ which means returning to state i is uncertain. The transient state has a property that a state j is accessible from i but state i is not accessible from j which means there is no possible return (Ibe, 2009; Shah & Burke, 1999).

Definition 2.3: Absorbing State

A state i is said to be an absorbing state if $p_{ii} = 1$ or equivalently $p_{ij} = 0$ for all $j \neq i$. An absorbing state is a state in which once an entity enters the state, it is impossible for the entity to leave the state (Ibe, 2009; Shah & Burke, 1999).

Definition 2.4: Absorbing Markov Chain Model

An absorbing Markov Chain is a Markov Chain with at least one absorbing state. Listing the states such that the m transient (nonabsorbing) states come first and followed by s absorbing states, we obtain matrix P. The $(m+s)\times(m+s)$ matrix P is in canonical form with

[redacted] (2.2)

where Q is $m\times m$ submatrix that gives the probability of transition from one transient state to another transient state, R is $m\times s$ submatrix that gives the probabilities of transition from transient states to absorbing states, 0 is $s\times m$ zero matrix and I is $s\times s$ identity matrix.

2.3 Exponential Smoothing (ES)

ES technique is a widely used technique in smoothing discrete and univariate time series and also in forecasting short-term period of data (Maia & Carvalho, 2011). It is originally formulated by Robert G. Brown during 1959 and C.C. Holt in the late 1950s in order to forecast inventory in the inventory control systems (Chat, Koehler, Ord, & Snyder, 2001).

ES is very popular because of its relative simplicity and low cost. In addition, it is computationally friendly where it can be integrated easily with computer software such as Excel to generate new forecasts. Moreover, the ease of adjusting its responsiveness to changes in the process of forecasting and its reasonable short-term accuracy makes this method relevant until now (Lazim, 2012; Maia & Carvalho, 2011; Ostertagova & Ostertag, 2011, 2012).

In ES, we want to allow the more recent values of the series to have greater influence on the forecast of future values than the more distant observations whereby the forecast value is constructed from an exponentially weighted average of past observations. This condition can be achieved by selecting an appropriate value for the coefficient known as "smoothing constant" (Tersine & Green, 1979). In comparison to other projection methods, ES can produce the most accurate forecast value for real data sets (Chat et al., 2001; Lazim, 2012).

There are four popular types of ES:

i) Simple Exponential Smoothing (SES)

ii) Double Exponential Smoothing (DES)

iii) Holt's Method (HM)

iv) Holt-Winter's Trend and Seasonality

2.3.1 Simple Exponential Smoothing (SES)

The SES is a short-term forecasting technique used for the data with no trend (no consistent growth or decline). In other words, it is usually based on the premise that the level of time series should fluctuate about a constant level or change slowly over time (Anusha, Alok, & Shaik, 2014; Ostertagova & Ostertag, 2012).

This technique is said to be the simplest form and the most widely used in the family of ES (Ismail & Hassan, 2013; Lazim, 2012; Nazim & Afthanorhan, 2014; Ostertagova & Ostertag, 2012). It only requires a limited amount of data. This technique gives the most recent observation the most weight instead of giving equal weights to each observation (Bowerman, O'Connell, & Koehler, 2005). This SES method strongly depends on the optimal value of the smoothing constant, α in order to determine its accuracy (Lazim, 2012; Ostertagova & Ostertag, 2012).

The value for α may range between zero and one, where zero means assigning no weight at all to recent data and one means assigning all the weights to actual recent data. In fact, a large smoothing constant value places greater weight on current demand conditions, while a smaller value lends more weight on long-term demand conditions and has a greater smoothing effect. One can simply choose the value that gives the smallest mean squared error (MSE) in order to determine the best value of α. Thus, trial and error method can be applied through retrospective testing on a sample of actual past data in choosing the value for α (Gaynor & Kirkpatrick, 1994).

2.3.2 Double Exponential Smoothing (DES)

DES or Brown's method is suitable for series that has a linear trend characteristic (Choudhury & Jones, 2014; Ravinder, 2013). This technique has the ability to generate multiple-ahead forecasts compared to SES. It uses only one parameter, which is α to generate forecast value (Lazim, 2012).

2.3.3 Holt's Method (HM)

HM also known as Holt's linear exponential smoothing or Holt's two-parameter method, is an alternative for handling forecasting where there is linear trend (Choudhury & Jones, 2014). This technique can overcome the problem of sensitivity at linear trend values to random influences that is faced in DES technique. It provides more flexibility in selecting the rates at which the trend and slopes are tracked besides smoothing them directly using different smoothing constants which are α and β. The smoothing constants are usually restricted to the range $(0,1)$ (Anusha et al., 2014; Lazim, 2012).

2.3.4 Holt-Winter's Trend and Seasonality

Holt-Winter's Trend and Seasonality is the most suitable technique to be applied when dealing with data that consists of seasonality because it considers the trend and seasonality factors (Anusha et al., 2014).

2.4 Measuring Forecast Accuracy

A forecast may not be completely accurate. The principle objective of forecasting is that the deviation of the forecast from the actual value is as small as possible. The error measures can be used to determine which forecasting technique is the most accurate. In addition, the error measures also allow researchers to compare the forecasting techniques used and to determine the better technique. There are four types of error measures used for the purpose of this study. The error measures are mean absolute deviation (MAD), mean absolute percent deviation (MAPD), mean absolute percentage error (MAPE) and mean squared error (MSE) (Anusha et al., 2014; Lazim, 2012; Taylor III, 2013).

2.4.1 MAD

MAD is one of the most popular and simplest-to-use measures of error measures. It is defined as an average of the difference between the forecast and actual value. The lower the value of MAD relative to the magnitude of the data, the more accurate the forecast and vice versa. MAD is calculated using the following formula:

$$MAD = \frac{\sum_{i=1}^{n} \left| A_i - F_i \right|}{n} \tag{2.3}$$

where i is the state, $i = 1,2,3,...,n$, A_i is the actual value of state i for the following future period and F_i is the following future period.

2.4.2 MAPD

MAPD is the variation of MAD. It measures the absolute error as a percentage of demand rather than per period. Thus, it eliminates the problem of interpreting the measure of accuracy relative to the magnitude of the actual and forecast values, as MAD does. The lower the MAPD value the more accurate the forecast value. The computation of MAPD is shown as follows:

$$MAPD = \sum_{i=1}^{n} \frac{\left| A_i - F_i \right|}{A_i} \times 100 \tag{2.4}$$

where i is the state, $i = 1,2,3,...,n$, A_i is the actual value of state i for the following future period and F_i is the following future period.

2.4.2 MAPE

MAPE is another measure of forecast error. MAPE measures the size of the error in percentage terms. As with other measures of forecast accuracy, the smaller the MAPE value, the better the model is. It is calculated as the average of the unsigned percentage error, as shown in the following formula:

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right|}{n} \times 100 \tag{2.5}$$

where i is the state, $i = 1,2,3,...,n$, A_i is the actual value of state i for the following future period and F_i is the following future period.

2.4.4 MSE

MSE is an error measure defined as individual error values which are first squared and then summed and averaged. The smaller the MSE value, the better the model is. The formula for MSE is as follows:

$$MSE = \frac{\sum_{i=1}^{n} \left(\left|A_i - F_i\right|\right)^2}{n} \tag{2.6}$$

where i is the state, $i = 1,2,3,...,n$, A_i is the actual value of state i for the following future period and F_i is the following future period.

2.5 Application of the MC Model in Various Fields

There have been several attempts at applying MC model to a wide range of problems including in management, education, military, health administration, biology, accounting and banking.

Igboanugo & Onifade (2011) uses the Markovian statistical tool to see the staff stock flow in order to describe the existing manpower policy of the first generation Nigerian University. They use a forty-year data of staff transition to construct transition probability matrix. From the transition probability matrix, they obtain the probability of newly recruited staff who exit the employment system through normal retirement, voluntary withdrawal or wastage. Another study applies Markov Chain model in military system to see the flow of the Slovenian armed forces. At first, 120 types of military segments, including civil servants, are selected to observe the transitions between the segments for the 2001 to 2005 period of data. Then, MC model is applied to project the structures of the military segments for 5, 10 and 20 years ahead (Skulj et al., 2008).

A study on Markov Chain model in education employs undergraduate students at Ekiti State University, Nigeria to see the flow of undergraduate student admission and academic performance and also forecast the data. In using MC model, transition probability has been developed using data of ten academic sessions where graduating and withdrawing are defined as absorption. The probabilities of students graduating and students withdrawing as well as the expected length of students remaining in the system before graduating are determined and this is followed by the prediction on the enrolment and academic students for four academic sessions. The application of the

MC model is very good for education planning and can be used by government policy makers to check with respect to a particular educational policy of the institution (Adeleke et al., 2014).

Another application of Markov Chain in education is the study of the flow of undergraduate students in Kuwait University (Awadhi & Konsowa, 2007). The research uses data consisting of academic status and grade reports of a sample of 250 students from the Faculty of Science. Using the MC model on the data collected from the academic year of 1996-1997 until 2004-2005, the probabilities of a student graduating, staying, withdrawing and also progressing to a higher level of the study are determined.

A study proposes MC model in education to study the postgraduate students flow at the College of Arts and Sciences in Universiti Utara Malaysia. An enrolment projection model based on the Markov Chain is developed based on four years of data of student enrolments at the college. In addition, the model is helpful for the college future planning in matters regarding postgraduate student enrolments (Adnan, 2012).

A paper presents an application of Markov Chain to observe the flow of students enrolment of a DBA Program in a Graduate School in Australia in order to facilitate planning for that program. As a result, using this model the short-term and long-term forecasting facilities of the expected numbers in the DBA Program can be predicted (Nicholls, 2009).

Researchers also use MC model in the field of health administration. A research utilizes Markov Chain to see the flow of South African nurses of the professional labor market over time between different segments. The results indicate that MC model has the ability to model the effects of human resources policy interventions in the short and long run in order to attract and retain nurses in rural areas (Lagarde & Cairns, 2012).

An article uses Markov Chain model in health to study the flow of cardiac patients within the hospital by observing the dynamics of changes of the care units in which the patient stays, the probability of discharge and state of death among the patients. This research uses the data of cardiac patients between the year 2007 to 2010 which are collected from Guwahati Neurological Research Centre (GNRC), India (Borah & Kakaty, 2012).

In biology, Markov Chain is applied to model the flow of soil C transformations among microbial states. Using this method, the researchers are able to predict the soil C transformation towards the microbial states in the system by using the transition probability matrix (Liang et al., 2011).

In accounting, MC model is being used to see the flow of accounts receivable collections and also to forecast the collections by monthly basis of a company (Saibeni, 2010). Moreover, MC model has been applied in banking to model the flow of the duration of retail loans in order to make the management of the bank's assets become effective. The results also demonstrate the prediction of its contingent assets status by using MC model (Hassan, 2010).

2.6 Application of Exponential Smoothing in Various Fields

Exponential smoothing (ES) is the most popular used class of forecasting techniques to predict data in short-term planning especially in health administration, business management, demographic, finance, and agriculture.

Application of three ES which are SES, DES, and HM with another two univariate models which are naïve with trend model and average percent change model (ACPM) has been made to forecast the rate of deaths due to cancer, which is one of the major causes of death in Malaysia, at a Public University Hospital in Malaysia. The result indicates that SES is found to be the best method to predict the rate of cancer deaths for the next two years (2012 to 2013) since it gives the lowest value of MSE with $\alpha = 0.10$ for estimation and evaluation part compared to other approaches (Ismail & Hassan, 2013).

An article demonstrates the application and screening of simple forecast models including simple moving average, simple exponential smoothing, Holt-Winter's Trend and Seasonality to forecast demand for the Indian Pharmaceutical Retail. Okacet and Stamlo Beta are chosen as the tested products for the study. The result shows that Holt-Winter's Trend and Seasonality provides the most accurate sales forecast compared to the other three models with MAD, MAPE and MSE values of 92.28, 27.50 and 14635.74 respectively. On the other hand, six-month simple moving average best predicts the demand for Stamlo Beta, which accurately provides sales forecast with error measure values of 47.19, 394719.07 and 45.62 for MAD, MSE and MAPE respectively compared to the other models (Anusha et al., 2014).

The application of ES (SES, DES, and HM) and also moving average is being conducted to forecast air traffic volume data (passenger traffic, flight traffic and freight traffic) in Turkey. Using the data between 2007 until 2013, DES and HM are employed to predict the two types of air traffic volume data which are passenger traffic variable and flight traffic variable and to be used to predict the data for year 2014 to 2023 (Onder & Kuzu, 2014).

A research proposes SES to predict the number of personnel in industrial production for the year 2011 in Slovakia. This research uses yearly data from 2001 to 2010. The accuracy measure of the forecast values has been made using MSE and root mean squared error (RMSE) (Ostertagova & Ostertag, 2011).

Holt-Winter's Trend and Seasonality, SES, DES and HM are used in predicting newspaper demand in Turkey in order to meet customer need with minimum number of returns, missed sales and oversupply. As a result, Holt-Winter's Trend and Seasonality, which yields the lowest RMSE value, is found to be the most suitable approach in forecasting values for the newspaper demand for the next 4 days (Incesu, 2012).

SES, which is a powerful technique for forecasting time series data, has also been applied to predict primary production of electricity in Slovakia for the year 2010. Prediction is based on the data of primary production of electricity covering the years of 2001-2009 (Ostertagova & Ostertag, 2012).

A research done in China aims to forecast the American Electric Vehicles (EV) demand and its proportion to the whole car sales based on the historical 37 EVs monthly sales and cars monthly sales spanning from December 2010 to December 2013. There are three types of ES used in the study which are SES, DES and HM. It is found that HM, which gives the smallest MSE, is the best method to forecast the American EV (Peng, Yu, Wang, & Yang, 2015).

In Malaysia, a study employs four types of ES which are SES, DES, HM and adaptive response rate exponential smoothing (ARRES) in predicting the country's population by using secondary data of Malaysia's population between the periods of 1957 up to 2013. The finding indicates that HM is the most suitable method for predicting the Malaysia's population by 2020 since it produces the lowest MSE value (Nazim & Afthanorhan, 2014).

ES is also used in finance for the prediction of the inflation values. SES model, DES model and Holt-Winter model are chosen to forecast the 24 months of Nigeria's CPI inflation sample data. The results show that DES with α = 0.68 is the best performing model based on the lowest standard forecasting accuracy measures of MSE, mean absolute error (MAE), RMSE, sum of squared errors (SSE) and MAPE (Osarumwense, 2014).

In the field of agriculture, a research applies several forecasting methods for evaluating crop yield estimates in Ghana. Crop yield forecasting, which provides information for decision makers, is important in many ways to Ghana's economy. Comparison of yield forecast values has been made using SES, DES, damped-trend linear exponential smoothing and autoregressive moving average (ARMA) models. In the study, the ARMA model proved to be the best time-series model among the other exponential smoothing methods for predicting crop yield where it gives the smallest MSE value of 0.0787606 (Choudhury & Jones, 2014).

2.7 Studies on Marriage and Divorce in Malaysia

There are various issues that have been studied by researchers regarding marriage and divorce in Malaysia including purpose of marriage, adjustments after marriage, underage marriage governance, divorce rate, causes of divorce and consequences of divorce.

A research has been conducted in Malaysia concerning the purpose of marriage among single Malaysian youths where data is collected based on the lists of purposes for marriage taken from 27 undergraduate students where all of them are single and at the age of 23 to 34 years old. In addition, 10 university students are chosen for in-depth interviews. The results indicate that the individual personal belief, life culture and life expectations are listed as the purposes of marriage among the undergraduate students (Manap, Kassim, Hoesni, Nen, Idris, & Ghazali, 2013).

In an attempt to investigate the issue of marital adjustment among married postgraduate students in two public universities in Malaysia, a study collects random samples of 176 students selected from University Putra Malaysia (UPM) and International Islamic University of Malaysia (IIUM) (Horany & Hassan, 2011). The study utilizes a set of demographic variables and employs the adjusted Locke-Wallace Marital Adjustment. In addition, ANOVA 2x3x2 is used to investigate the difference in the means of marital adjustment according to gender, university and nationality among the participants. Results indicate significant difference in marital adjustment mean scores between gender, between the two universities as well as among the three nationalities.

A study investigating the loopholes of the existing system has been done to develop a new model of underage marriage governance for Muslims in Malaysia (Saidon, Adil, Sahri, Alias, Daud, & Murad, 2015). The study uses qualitative method where Islamic Family Law Federal Territories Act 1984 and other related laws as well as administrative procedures and formalities are studied and analyzed. It also employs semi-structured interviews conducted on several respondents from related institutions. The results indicate five weaknesses of the present governance. They are the absence of pre-marital counseling, loopholes in Islamic family enactments, the absence of guidelines on giving permission to marry, the absence of a support system and lack of Research and Development (R&D) on marriage.

The issue of divorce in Malaysia is studied in a research that analyzes the divorce situation in Malaysia (Chlen & Mustaffa, 2008). Secondary data about divorce are collected from two reports from National Registration Department and Jabatan Kemajuan Islam Malaysia. The analysis is concerned with questions on how divorce happens, why divorce happens, what are the impacts of divorce on a family and what are the ways for a community to facilitate in preventing divorce.

Another research on the issue of divorce in Malaysia explores several theoretical perspectives on divorce and the consequences on the changing societal view of divorce from a very bad one to a healthy concept in the Malaysian context (Nor, Jaladin, Karim, & Ahmad, 2013). As a result of the research, a marriage and family therapy model that works effectively with divorcing Malay-Muslim couples is developed.

With regard to the issue of divorce rate, a study has been done to describe the incidence of divorce in Muslim community in Malaysia (Zainab, Wan-Ibrahim, & Asyraf, 2014). The study utilizes a content analysis approach using secondary data that has been obtained from reports from Population and Housing Census of Malaysia as well as from filed records from the Department of Islamic Development Malaysia. The study shows that the divorce rates among Malaysian Muslims aged between 18 and 50 in the country is increasing significantly. In addition, among the states in Malaysia, it is found that the highest divorce rate is in the Federal Territory Kuala Lumpur, followed by Terengganu, Perlis and Kedah.

2.8 Summary

In this chapter, the theories of the Markov Chain (MC) model and exponential smoothing (ES) technique have been described in detail. Four types of error measures used to measure the forecast accuracy are also briefly described. Besides that, the applications of using MC model and ES from previous researches in various fields are also being reviewed. In addition, this chapter gives a review on marriage and divorce in Malaysia. The methodology of this research for the case of Muslims in the marital system is described in the next chapter.

CHAPTER THREE

RESEARCH METHODOLOGY

This chapter describes the case study, the collection of data, the framework of the study, modeling steps of the proposed MC_{SES} hybrid model and measurement of forecast accuracy of the proposed model. The comparison of forecast accuracy of the MC_{SES} hybrid model with both the classical Markov Chain model and the Markov Chain model with simple moving average, MC_{SMA} model in terms of four error measures is given towards the end of the chapter.

3.1 The Case Study

Marriage is an important starting point of a family life cycle. In Malaysia, a total of 112, 262 Muslim couples got married in 2004, and the number that has risen steadily in general over the years to 148, 806 marriages in 2012, according to the Department of Islamic Development Malaysia (JAKIM). Meanwhile, the statistics provided by the Syariah Judiciary Department Malaysia (JKSM) indicates that the number of Muslim couples getting divorced has increased from 20,916 in 2004 to 47,740 in 2011 to 49,311 in 2012 (Su, 2014).

It is important to further study the flow of Muslims in the marital system instead of just stating the number of marriages and divorces in general. Therefore, this study employs the Markov Chain model to observe the flow of Muslims in the marital system.

3.2 Data Collection

This study uses secondary data of 3093 Muslim couples in a district in Kedah who either married (2566) or divorced (527) in the current period which is in year 2013 and 2968 Muslim couples in a district in Kedah who either married (2439) or divorced (529) in the following future period which is in year 2014 from records provided by Pejabat Agama Daerah Kota Setar.

3.3 Framework of the Study

Figure 3.1 shows the framework of the study. It includes 5 stages including input, the modeling steps for the proposed MC_{SES} hybrid model, measurement of forecast accuracy, comparison of forecast accuracy and findings of this study.

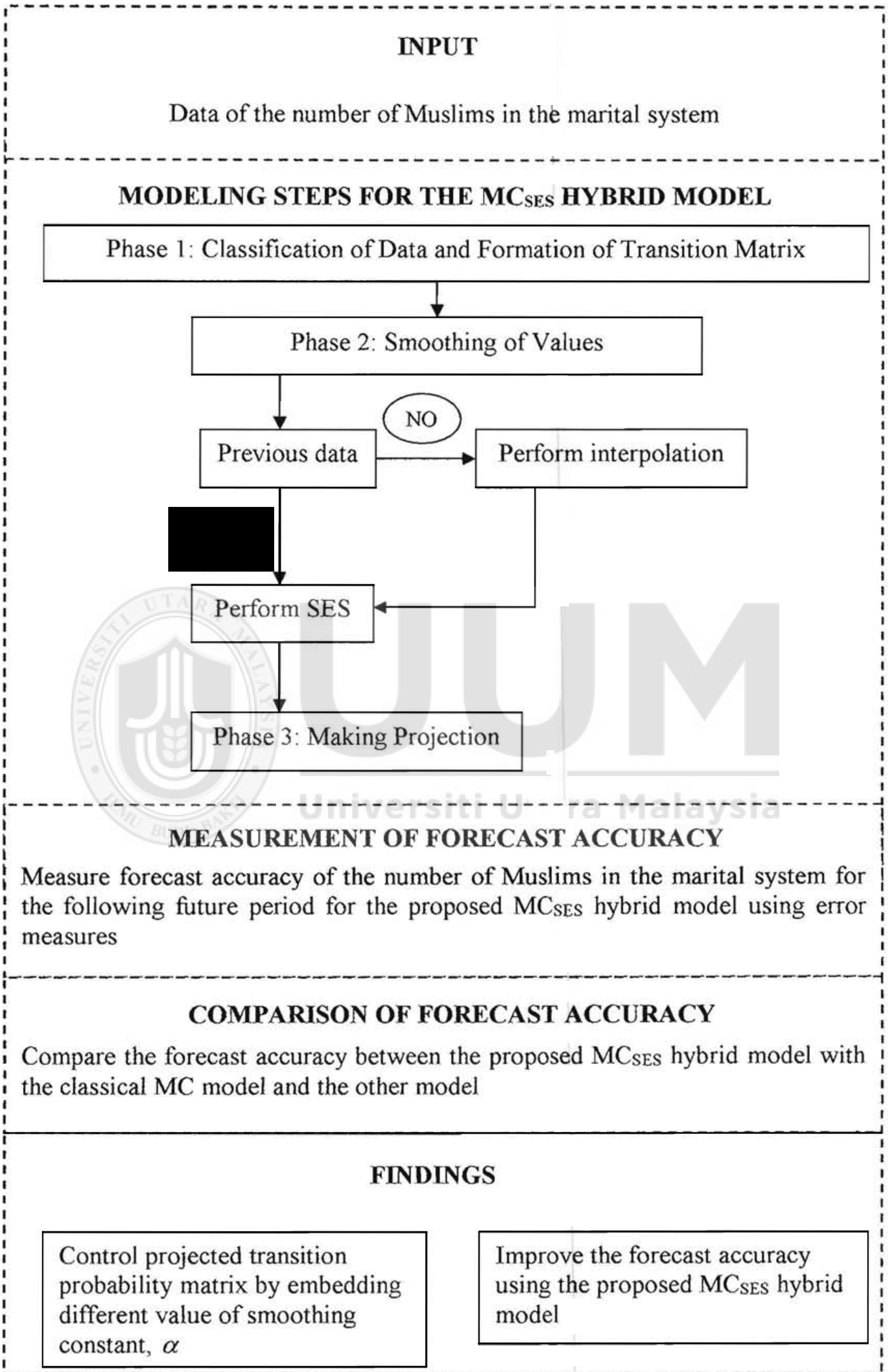

Figure 3.1. Framework of the study

Referring to Figure 3.1, the first stage is the input stage. The stage consists of data of the number of Muslims in the marital system for the current period which is in year 2013. The second stage is the modeling steps for the proposed MC_{SES} hybrid model. There are 3 phases in modeling steps for the proposed model. In phase 1, we classify the data into transient states (marriage) and absorbing state (divorce) according to gender and age category. Then, we form a transition matrix of the current period, T^c where c refers to the current period which is 2013 to explain the frequency transition of the number of Muslims in the marital system according to states for the current period. Then, we proceed to Phase 2 which is smoothing the values where we apply interpolation technique and also SES technique. Generally, in order to apply SES we need to use data values for the previous few years. Since in this study, the available data for the modeling is two years data of 2013-2014 and 2014-2015, we apply interpolation technique which is another smoothing technique, to obtain values for the previous periods. Then, we proceed to Phase 3 which is making projection where we project the number of Muslims in the marital system for the following future period which is in year 2014. The details on modeling steps of the proposed MC_{SES} hybrid model will be discussed in the next section.

In the third stage, we measure forecast accuracy of the number of Muslims in the marital system for the following future period which is in year 2014 of the proposed MC_{SES} hybrid model using error measures. There are four error measures applied in this study which are MAD, MAPD, MAPE, and MSE.

The fourth stage is the comparison of forecast accuracy. The forecast accuracy of the proposed MC_{SES} hybrid model will be compared to the forecast values of the classical Markov Chain model and the other model which is MC_{SMA} model. MC_{SMA} model refers to the integration of the MC model and simple moving average (SMA) technique. Based on the comparison of the forecast accuracy, we will then determine the most appropriate model in estimating the forecast values for the case of Muslims in the marital system for the year 2014.

The last stage in the framework of this study is the findings of the study. First, we can control projected transition probability matrix by embedding different value of smoothing constant, α using the proposed MC_{SES} hybrid model. Furthermore, we can improve the forecast accuracy using the proposed MC_{SES} hybrid model.

3.4 Modeling Steps of the Proposed MC_{SES} Hybrid Model

This section shows the modeling steps of the proposed MC_{SES} hybrid model. It consists of 3 phases with 8 steps. As shown in Figure 3.2, the first phase is the classification of data and formation of transition matrix which includes Step 1 and Step 2. The second phase is the smoothing of values phase which involves Step 3 to Step 4. The third phase is known as making projection phase which involves Step 5 to Step 8.

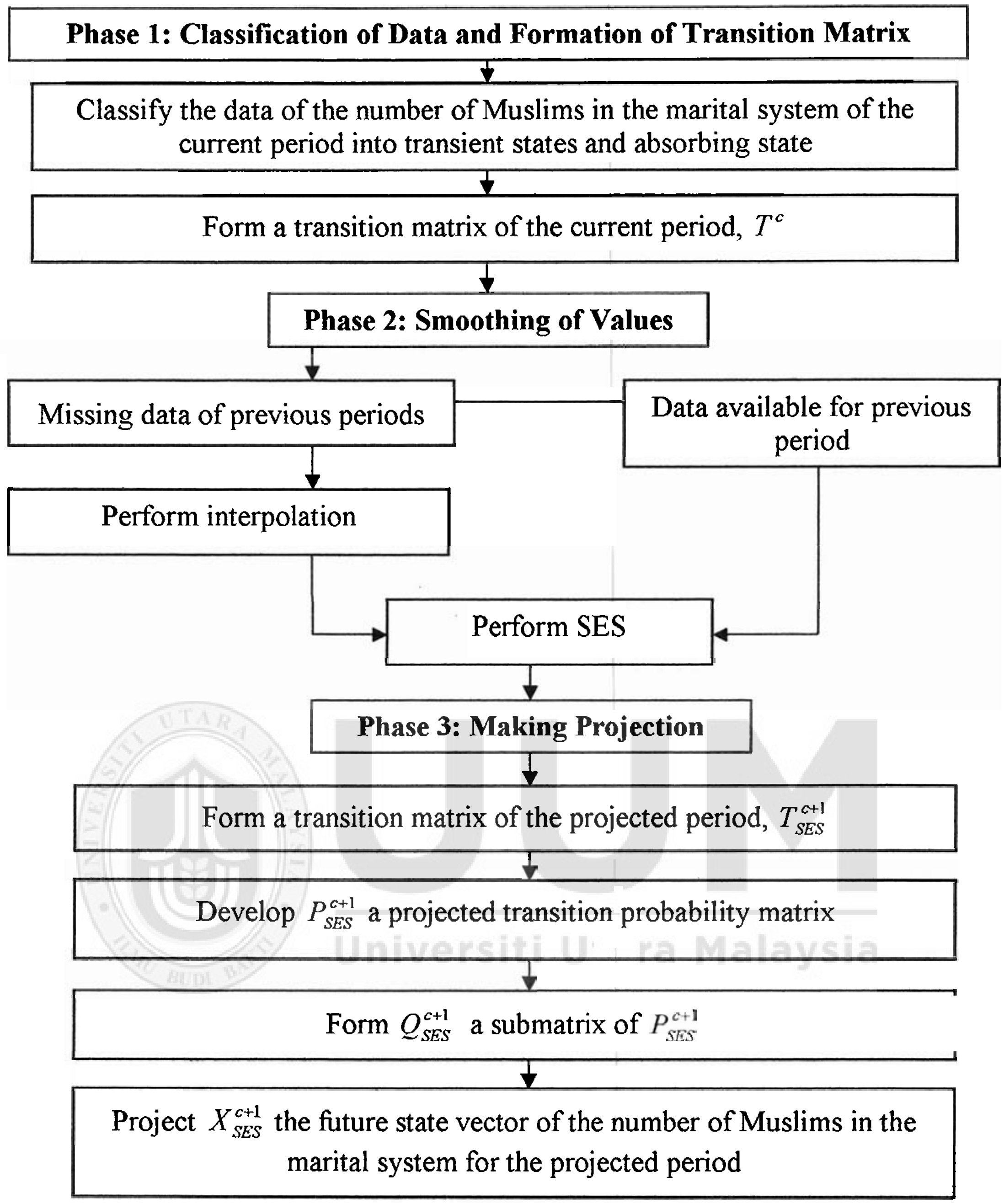

Figure 3.2. Modeling steps of the proposed MC$_{SES}$ hybrid model

3.4.1 Phase 1: Classification of Data and Formation of Transition Matrix

Phase 1 involves the classification of data and formation of transition matrix. This phase consists of Step 1 and Step 2. In Step 1, we classify the data of the number of Muslims in the marital system of the current period into transient states and absorbing state. In this study, we define current period as current year which is year 2013, following future period as following future year which is year 2014, transient state as marriage and absorbing state as divorce. There are 19 states utilized to model the flow of the number of Muslims in the marital system in Kota Setar, Kedah. They include 18 transient states and one absorbing state. For transient states, Muslims are modelled through the one year of marriage (From 2013 to 2014). They are categorized by two gender categories, where M represents male and F represents female and by nine age categories ranging from Under 26 to Above 60 as shown in Table 3.1.

Table 3.1

Transient States

Under 26	26-30	31-35	36-40	41-45	46-50	51-55	56-60	Above 60
M F	M F	M F	M F	M F	M F	M F	M F	M F

In Step 2, we form a transition matrix of the current period, T^c where c refers to the current period which is 2013 to explain the frequency transition of the number of Muslims in the marital system according to states for the current period. The period of transition between states used is one year. The basic template of T^c is shown in Table 3.2.

Table 3.2

T^c, A Transition Matrix of the Current Period (from 2013 to 2014)

To 2014 From 2013	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60	Divorce	Row Total
M : Under 26	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 26-30	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 31-35	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 36-40	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 41-45	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 46-50	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	x	x
M : 51-55	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	x	x
M : 56-60	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	x	x
M : Above 60	0	0	0	0	0	0	0	0	x	0	0	0	0	0	0	0	0	0	x	x
F : Under 26	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	x	x
F : 26-30	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	x	x
F : 31-35	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	x	x
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	x	x
F : 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	x	x
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	x	x
F : 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	x	x
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	x	x
F : Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	x
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

3.4.2 Phase 2: Smoothing of Values

Phase 2 involves the smoothing of values. This phase consists of Step 3 and Step 4. Step 3 involves interpolation technique while Step 4 involves SES technique. Throughout this writing, case item will be used quite often whereby it refers to the state transition. In this phase, coding has been done first using T^c to ease reference to the transient cases and absorbing cases for the process of smoothing the values using the techniques of interpolation and SES. Table 3.3 shows the coding of cases, $C_{i,j}$. $C_{1,1}$ until $C_{18,18}$ indicates 34 transient cases and $C_{1,19}$ until $C_{18,19}$ indicates 18 absorbing cases. For instance, $C_{1,1}$ refers to the case of males who are in state (M: Under 26) in the current year and remain in the same state in the following year. $C_{1,2}$ refers to the case of males who are in state (M: Under 26) in the current year but move to state (M: 26-30) in the next year. On other hand, $C_{1,19}$ refers to the case of males of the age category Under 26 who are divorced in the current year.

Table 3.3

Coding of Cases

	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60	Divorce	Row Total
M : Under 26	$C_{1,1}$	$C_{1,2}$																	$C_{1,19}$	
M : 26-30		$C_{2,2}$	$C_{2,3}$																$C_{2,19}$	
M : 31-35			$C_{3,3}$	$C_{3,4}$															$C_{3,19}$	
M : 36-40				$C_{4,4}$	$C_{4,5}$														$C_{4,19}$	
M : 41-45					$C_{5,5}$	$C_{5,6}$													$C_{5,19}$	
M : 46-50						$C_{6,6}$	$C_{6,7}$												$C_{6,19}$	
M : 51-55							$C_{7,7}$	$C_{7,8}$											$C_{7,19}$	
M : 56-60								$C_{8,8}$	$C_{8,9}$										$C_{8,19}$	
M : Above 60									$C_{9,9}$										$C_{9,19}$	
F : Under 26										$C_{10,10}$	$C_{10,11}$								$C_{10,19}$	
F : 26-30											$C_{11,11}$	$C_{11,12}$							$C_{11,19}$	
F : 31-35												$C_{12,12}$	$C_{12,13}$						$C_{12,19}$	
F : 36-40													$C_{13,13}$	$C_{13,14}$					$C_{13,19}$	
F : 41-45														$C_{14,14}$	$C_{14,15}$				$C_{14,19}$	
F : 46-50															$C_{15,15}$	$C_{15,16}$			$C_{15,19}$	
F : 51-55																$C_{16,16}$	$C_{16,17}$		$C_{16,19}$	
F : 56-60																	$C_{17,17}$	$C_{17,18}$	$C_{17,19}$	
F : Above 60																		$C_{18,18}$	$C_{18,19}$	
Divorce																				

Simple exponential smoothing (SES) is the most suitable technique to be used in this study because there is no trend of data found in the case of the number of Muslims in the marital system. Generally, in order to apply SES we need to use data values for the previous few years. Since in this study, the available data for the modeling is two years data of 2013 and 2014, we apply interpolation technique to obtain values for the previous 4 periods.

Step 3 involves interpolation where we perform this technique on the cases in the transition matrix of the current period, T^c. T^c is obtained in Phase 1 which explains the frequency transition of the number of Muslims in the marital system according to states for the current period. Values of all cases are denoted as V^c.

The quadratic polynomial interpolation, which is one type of interpolation is performed to interpolate values of the 52 cases for the previous 4 periods, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} where $c-1, c-2, c-3$ and $c-4$ refers to the year 2012, 2011, 2010 and 2009 respectively. The quadratic polynomial interpolation is a simple technique used to obtain a continuous path about three points (Chang, 2014). Since it is generally impossible to force a quadratic to pass through n data points, it must be decided which quadratic function best fits the data. This process is called smoothing (Giordano, 2003).

To perform the quadratic polynomial interpolation, we use three data points (x_0, y_0), (x_1, y_1) and (x_2, y_2) with x_0, x_1, x_2 being the minimum, mean and maximum value of the data respectively. Assume that $y = f(x), y_0 = f(x_0), y_1 = f(x_1)$, and $y_2 = f(x_2)$.

Then, we can construct the quadratic polynomial, $f_2(x)$ that passes through these points using the formula shown below:

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) \tag{3.1}$$

where coefficient $b_0 = f(x_0)$, $b_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$ and

$$b_2 = \frac{1}{x_2 - x_0}\left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{(f(x_1) - f(x_0))}{x_1 - x_0}\right)$$

The smoothing process using quadratic polynomial interpolation in Step 3 yields the interpolated values for all the cases of the previous 4 periods, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} where $c-1, c-2, c-3$ and $c-4$ refers to the year 2012, 2011, 2010 and 2009 respectively.

In Step 4, we perform SES technique on V^c the actual values of all cases for the current period which is in year 2013 and on the interpolated values, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} for 2009 until 2012. First, using $V^c, V_I^{c-1}, V_I^{c-2}, V_I^{c-3}$ and V_I^{c-4}, we proceed to obtain fitted values for all the cases of the current period, V_{SES}^{c} and the previous 4 periods, $V_{SES}^{c-1}, V_{SES}^{c-2}, V_{SES}^{c-3}$ and V_{SES}^{c-4} . The calculation of the fitted values for all cases using SES is shown below:

$$V_{SES}^{c-k}(C_{i,j}) = \alpha V^{c-k}(C_{i,j}) + (1-\alpha)V_{SES}^{c-k-1}(C_{i,j}) \tag{3.2}$$

where V_{SES}^{c-k} is the fitted values for all cases at period $c-k$, V^{c-k} is the values for all cases at period $c-k$, V_{SES}^{c-k-1} is the fitted values for all cases at period $c-k-1$, α is the smoothing constant, $C_{i,j}$ is the coding of cases and k is the number of year, $k = 1,2,3,4$.

We use the interpolated values, V^{c-4} and fitted values, V_{SES}^{c-4} of all cases of 2009 to get the fitted values, V_{SES}^{c-3} of all cases for 2010. In SES, the fitted values of 2009 are generally initialized to be the same as the actual values of 2009. Since the actual values of all cases of 2009 are not available in this study, we initialize the fitted values, V_{SES}^{c-4} of all cases of 2009 to be the same as the interpolated values, V^{c-4} of 2009. Subsequently, using the obtained fitted values, V_{SES}^{c-3} of all cases of 2010, we get the fitted values, V_{SES}^{c-2} of all cases of 2011. Repeating the procedure of SES, we obtain $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}, V_{SES}^{c-1}$ the fitted values of all cases of the previous 4 periods (2009, 2010, 2011, 2012) and V_{SES}^{c}, the fitted values for all cases of the current period (2013).

After we obtain $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}, V_{SES}^{c-1}$ and V_{SES}^{c} we proceed to obtain V_{SES}^{c+1} consisting of the projected values of all cases for the following future period, using $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}, V_{SES}^{c-1}$ and V_{SES}^{c}. The calculation of the projected values for all cases, V_{SES}^{c+1} using SES is shown below:

$$V_{SES}^{c+1}(C_{i,j}) = \alpha V^{c}(C_{i,j}) + (1-\alpha) V_{SES}^{c}(C_{i,j}) \tag{3.3}$$

where V_{SES}^{c+1} is the projected values for all cases at period $c+1$, V^{c} is the actual values for all cases of the current period, V_{SES}^{c} is the fitted values for all cases of the current period, α is the smoothing constant and $C_{i,j}$ is the coding of cases.

3.4.3 Phase 3: Making Projection

Phase 3 involves making projection of the number of Muslims in the marital system for the following future period. Phase 3 consists of steps 5 to 8.

In Step 5, we form a transition matrix of the following future period, T_{SES}^{c+1} using the obtained V_{SES}^{c+1} in Phase 2 to explain the frequency transition of the number of Muslims in the marital system according to states for the following future period as shown in Table 3.4.

T_{SES}^{c+1} is then used in Step 6 to develop P_{SES}^{c+1} a projected transition probability matrix for the following future period. The row total from Table 3.4 is used to construct the corresponding probabilities. The matrix P_{SES}^{c+1} is obtained by dividing each frequency by the appropriate row total and is shown in Table 3.5.

Step 7 is the formation of Q_{SES}^{c+1} which is a submatrix of P_{SES}^{c+1} . It is formed by deleting the row and column in P_{SES}^{c+1} containing the absorbing state. The matrix Q_{SES}^{c+1} is shown in Table 3.6.

Table 3.4

T_{SES}^{c+1} *A Transition Matrix of the Following Future Period (from 2014 to 2015)*

To 2015 From 2014	M: Under 26	M: 26-30	M: 31-35	M: 36-40	M: 41-45	M 46-50	M: 51-55	M 56-60	M: Above 60	F: Under 26	F: 26-30	F: 31-35	F: 36-40	F: 41-45	F: 46-50	F: 51-55	F: 56-60	F: Above 60	Divorce	Row Total
M: Under 26	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M: 26-30	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M: 31-35	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M 36-40	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M 41-45	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	0	x	x
M: 46-50	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	0	x	x
M: 51-55	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	0	x	x
M: 56-60	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	0	0	x	x
M: Above 60	0	0	0	0	0	0	0	0	x	0	0	0	0	0	0	0	0	0	x	x
F: Under 26	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	0	x	x
F: 26-30	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	0	x	x
F: 31-35	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	0	x	x
F 36-40	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	0	x	x
F: 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	0	x	x
F: 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	0	x	x
F: 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	0	x	x
F: 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	x	x
F: Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x	x	x
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Tab e 3 5

P_{SES}^{c+1} *A Transition Probability Matrix of Muslims in the Marital System for the Following Future Period (from 2014 to 2015)*

To 20 5 From 20 4	M: Under 26	M: 26-30	M: 3 -35	M: 36-40	M: 4 -45	M: 46-50	M: 5 -55	M: 56-60	M: Above 60	F: Under 26	F: 26-30	F: 3 -35	F: 36-40	F 4 -45	F: 46-50	F: 5 -55	F: 56-60	F: Above 60	Divorce
M: Under 26	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 26-30	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 3 -35	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 36-40	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 4 -45	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 46-50	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 5 -55	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0.xxxx
M: 56-60	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0.xxxx
M: Above 60	0	0	0	0	0	0	0	0	0.xxxx	0	0	0	0	0	0	0	0	0	0.xxxx
F: Under 26	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0.xxxx
F: 26-30	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0.xxxx
F: 3 -35	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0.xxxx
F: 36-40	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0.xxxx
F: 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0.xxxx
F: 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0.xxxx
F: 5 -55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0.xxxx
F: 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0.xxxx
F: Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x

Table 3.6

Q_{SES}^{c+1} *A Submatrix of* P_{SES}^{c+1}

To 20 5	M	M	M	M	M	M	M	M	M	F	F	F	F	F	F	F	F	F
From 2014	Under 26	26-30	3 -35	36-40	41-45	46-50	51-55	56-60	Above 60	Under 26	26-30	31-35	36-40	41-45	46-50	51-55	56-60	Above 60
M : Under 26	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 26-30	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M 31-35	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 36-40	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0	0
M 41-45	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0	0
M : 46-50	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0	0
M 5--55	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0	0
M : 56-60	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0	0	0	0	0	0	0	0
M : Above 60	0	0	0	0	0	0	0	0	0.xxxx	0	0	0	0	0	0	0	0	0
F Under 26	0	0	0	0	0	0	0	0	0	0.xxxx	0 xxxx	0	0	0	0	0	0	0
F 26-30	0	0	0	0	0	0	0	0	0	0	0 xxxx	0.xxxx	0	0	0	0	0	0
F 3 -35	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0 xxxx	0	0	0	0	0
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	0 xxxx	0.xxxx	0	0	0	0
F 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0 xxxx	0.xxxx	0	0	0
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0	0
F 5 -55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx	0
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx	0.xxxx
F Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.xxxx

Step 8 is the last step for Phase 3 where we project X_{SES}^{c+1} the future state vector of the number of Muslims in the marital system of the 18 transient states for the following future period which is year 2014. X_{SES}^{c+1} is calculated using the following equation

$$X_{SES}^{c+1} = X^0 Q_{SES}^{c+1} \tag{3.4}$$

where X_{SES}^{c+1} is the future state vector of the number of Muslims in the marital system for the following future period, X^0 is the initial state vector of the number of Muslims in the marital system and Q_{SES}^{c+1} is the submatrix of P_{SES}^{c+1}. X^0 is given in equation (3.5).

$$X^0 = (914\ \ 1132\ \ 282\ \ 96\ \ 34\ \ 28\ \ 22\ \ 25\ \ 42\ \ 1414\ \ 788\ \ 169\ \ 75\ \ 38\ \ 23\ \ 22\ \ 13\ \ 15) \tag{3.5}$$

3.5 Measurement of Forecast Accuracy

The forecast accuracy of the number of Muslims in the marital system for 2014 using the developed proposed MC_{SES} hybrid model is measured using four error measures. They are MAD, MAPD, MAPE and MSE.

3.6 Comparison of Forecast Accuracy

The forecast accuracy of the proposed MC_{SES} hybrid model will be compared to the forecast values of both the Markov Chain model and the MC_{SMA} model in terms of four error measures. Based on comparison of the forecast accuracy, we will then determine the most appropriate model in estimating the forecast values for the case of Muslims in the marital system for the year 2014.

3.7 Summary

This chapter explains the case study, the collection of data, framework of the study, modeling steps of the proposed MC_{SES} hybrid model which involves 3 phases and measurement of forecast accuracy of the proposed model. Lastly, the proposed MC_{SES} hybrid model is compared in terms of forecast accuracy with the classical Markov Chain model and the MC_{SMA}. The next chapter presents the analysis and discussion of results of this study.

CHAPTER FOUR

FINDINGS AND DISCUSSIONS

This study emphasizes on the development of modeling steps for Markov Chain model in case of limited data and short-term projection. Thus the model is known as a MC_{SES} hybrid model. The accuracy of the forecast values using the proposed MC_{SES} hybrid model is evaluated using error measures. The error measures used in this study are MAPD, MAPE, MAD and MSE. Then, the proposed MC_{SES} hybrid model is compared in terms of the forecast accuracy of the model with the MC model and also with the MC_{SMA} model. Thus, the model which produces the highest forecast accuracy is found to be the most appropriate model in forecasting values in short-term period. This model is applied to the case study of Muslims in the marital system where we use data of 3093 Muslim couples who either got married or got divorced in the year 2013 in order to see the flow of Muslims in the marital system and obtaining the projected number of Muslims in the marital system for the following future period. For the purpose of measuring the forecast accuracy of the model, we use data of 2968 Muslim couples in the same district who got married in 2014.

4.1 Modeling Steps of the Proposed MC_{SES} Hybrid Model

In this study, modeling steps of the proposed MC_{SES} hybrid model are developed to model the flow of Muslims in the marital system and to forecast future number of Muslims in the system. It consists of 3 phases with 8 steps. The first phase is the classification of data and formation of transition matrix which includes Step 1 and Step 2. The second phase is the smoothing of values phase which involves Step 3 to Step 4. The third phase is known as making projection phase which involves Step 5 to Step 8. The explanation and results of each of the phases and steps is given in this section.

4.1.1 Phase 1: Classification of States and Tabulation of States

Phase 1 involves the classification of data and formation of transition matrix. This phase consists of Step 1 and Step 2. In Step 1, we classify the data of the number of Muslims in the marital system of the current period into transient states and absorbing state as shown in Table 3.1. In Step 2, we form a transition matrix of the current period, T^c. The transition matrix T^c of Muslims in the marital system is shown in Table 4.1.

Table 4.1

T^c, A Transition Matrix of Muslims in the Marital System for the Current Period (from 2013 to 2014)

To 2014 / From 2013	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60	Divorce	Row Total
M : Under 26	662	252	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	938
M : 26-30	0	1015	117	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	105	1237
M : 31-35	0	0	250	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	378
M : 36-40	0	0	0	84	12	0	0	0	0	0	0	0	0	0	0	0	0	0	72	168
M : 41-45	0	0	0	0	30	4	0	0	0	0	0	0	0	0	0	0	0	0	70	104
M : 46-50	0	0	0	0	0	23	5	0	0	0	0	0	0	0	0	0	0	0	54	82
M : 51-55	0	0	0	0	0	0	18	4	0	0	0	0	0	0	0	0	0	0	34	56
M : 56-60	0	0	0	0	0	0	0	20	5	0	0	0	0	0	0	0	0	0	19	44
M : Above 60	0	0	0	0	0	0	0	0	42	0	0	0	0	0	0	0	0	0	53	95
F : Under 26	0	0	0	0	0	0	0	0	0	1111	303	0	0	0	0	0	0	0	58	1472
F : 26-30	0	0	0	0	0	0	0	0	0	0	726	62	0	0	0	0	0	0	125	913
F : 31-35	0	0	0	0	0	0	0	0	0	0	0	149	20	0	0	0	0	0	124	293
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	65	10	0	0	0	0	61	136
F : 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	29	9	0	0	0	52	90
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	21	2	0	0	34	57
F : 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	3	0	26	48
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	4	19	32
F : Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	28	43
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4.1 presents the transition process of Muslims in the marital system from one state to another in one year duration which is from 2013 to 2014. From the table, the first row indicates that out of 938 Muslims, 662 male Muslims at the age category of Under 26 which refers to the state (M:Under 26) who are in the marital system in the year 2013 will be in the same state (M:Under 26) in the following year of 2014. Meanwhile, 252 male Muslims at the age category Under 26 which refers to the state (M:Under 26) who are in the marital system in 2013 but are moved to the next state (M:26-30) in the following year of 2014. However, 24 male Muslims are divorced in the year 2013. Similar interpretation can be made for the other rows.

4.1.2 Phase 2: Smoothing of Values

Phase 2 is the smoothing of values phase. It consists of Step 3 and Step 4. The quadratic polynomial interpolation is performed in Step 3 to interpolate values of the 52 cases for the previous 4 periods, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} where $c-1, c-2, c-3$ and $c-4$ refers to the year 2012, 2011, 2010 and 2009 respectively. The interpolated values of all cases are calculated based on the difference in values between year 2013 and 2014. We assume the state transition in the past 4 periods follow the similar pattern. Interpolation of the values for the year 2009 to 2012 is done separately for 34 transient cases and 18 absorbing cases. The calculation of the interpolated values of 34 transient cases and 18 absorbing cases for the period $c-1$ which is in the year 2012 is done first using V^c.

Table 4.2

V^c, The Actual Values of All Cases for the Current Period (Year 2013)

Coding of Cases	V^c	Coding of Cases	V^c
$C_{1,1}$	662	$C_{14,15}$	9
$C_{1,2}$	252	$C_{15,15}$	21
$C_{2,2}$	1015	$C_{15,16}$	2
$C_{2,3}$	117	$C_{16,16}$	19
$C_{3,3}$	250	$C_{16,17}$	3
$C_{3,4}$	32	$C_{17,17}$	9
$C_{4,4}$	84	$C_{17,18}$	4
$C_{4,5}$	12	$C_{18,18}$	15
$C_{5,5}$	30	$C_{1,19}$	24
$C_{5,6}$	4	$C_{2,19}$	105
$C_{6,6}$	23	$C_{3,19}$	96
$C_{6,7}$	5	$C_{4,19}$	72
$C_{7,7}$	18	$C_{5,19}$	70
$C_{7,8}$	4	$C_{6,19}$	54
$C_{8,8}$	20	$C_{7,19}$	34
$C_{8,9}$	5	$C_{8,19}$	19
$C_{9,9}$	42	$C_{9,19}$	53
$C_{10,10}$	1111	$C_{10,19}$	58
$C_{10,11}$	303	$C_{11,19}$	125
$C_{11,11}$	726	$C_{12,19}$	124
$C_{11,12}$	62	$C_{13,19}$	61
$C_{12,12}$	149	$C_{14,19}$	52
$C_{12,13}$	20	$C_{15,19}$	34
$C_{13,13}$	65	$C_{16,19}$	26
$C_{13,14}$	10	$C_{17,19}$	19
$C_{14,14}$	29	$C_{18,19}$	28

To perform the quadratic polynomial interpolation, we use three data points (x_0, y_0), (x_1, y_1) and (x_2, y_2) with x_0, x_1, x_2 being the minimum, mean and maximum value of the data respectively. In this study, x_0, x_1, x_2 refers to the minimum, mean and maximum value of cases in the current period which is in year 2013. V^c is arranged in ascending order so that we can select the value for x_0, x_1, x_2.

By using the equation (3.1), we get the interpolated values for each of the 34 transient cases, V_{tran}^{c-1} as shown in Table 4.3. In addition, by using the same equations, we get the values for each of the absorbing cases, V_{abs}^{c-1} as shown in Table 4.4. The calculations for the interpolated values of 34 transient cases and 18 absorbing cases for the other periods which are $c-2, c-3$, and $c-4$ follow the same calculation procedure.

Table 4.3

V_{tran}^{c-1}, *Interpolated Values of 34 Transient Cases for the Period of* $c-1$, *with Coefficient* $b_0 = 1, b_1 = 1.0068, b_2 = 0.0001$

Coding of Transient Cases	V_{tran}^{c}	V_{tran}^{c-1}	Coding of Transient Cases	V_{tran}^{c}	V_{tran}^{c-1}
$C_{15,16}$	2	1	$C_{6,6}$	23	22
$C_{16,17}$	3	2	$C_{14,14}$	29	28
$C_{5,6}$	4	3	$C_{5,5}$	30	29
$C_{7,8}$	4	3	$C_{3,4}$	32	31
$C_{17,18}$	4	3	$C_{9,9}$	42	40
$C_{6,7}$	5	4	$C_{11,12}$	62	60
$C_{8,9}$	5	4	$C_{13,13}$	65	63
$C_{14,15}$	9	8	$C_{4,4}$	84	82
$C_{17,17}$	9	8	$C_{2,3}$	117	114
$C_{13,14}$	10	9	$C_{12,12}$	149	145
$C_{4,5}$	12	11	$C_{3,3}$	250	241
$C_{18,18}$	15	14	$C_{1,2}$	252	243
$C_{7,7}$	18	17	$C_{10,11}$	303	290
$C_{16,16}$	19	18	$C_{1,1}$	662	612
$C_{8,8}$	20	19	$C_{11,11}$	726	666
$C_{12,13}$	20	19	$C_{2,2}$	1015	903
$C_{15,15}$	21	20	$C_{18,18}$	1111	977

Table 4.4

V_{abs}^{c-1}, Interpolated Values of 18 Absorbing Cases for the Period of $c-1$, with Coefficient $b_0 = 23, b_1 = 0.9487, b_2 = -0.0021$

Coding of Absorbing Cases	V_{abs}^{c}	V_{abs}^{c-1}
$C_{8,19}$	19	23
$C_{17,19}$	19	23
$C_{1,19}$	24	28
$C_{16,19}$	26	30
$C_{18,19}$	28	32
$C_{7,19}$	34	38
$C_{15,19}$	34	38
$C_{14,19}$	52	55
$C_{9,19}$	53	56
$C_{6,19}$	54	56
$C_{10,19}$	58	60
$C_{13,19}$	61	63
$C_{5,19}$	70	70
$C_{4,19}$	72	72
$C_{3,19}$	96	90
$C_{2,19}$	105	96
$C_{12,19}$	124	108
$C_{11,19}$	125	109

Using the interpolated values of 34 transient cases, V_{tran}^{c-1} and 18 absorbing cases, V_{abs}^{c-1} for the period of $c-1$ which is in year 2012, we obtain the interpolated values of 34 transient cases, V_{tran}^{c-2} and 18 absorbing cases, V_{abs}^{c-2} for the period of $c-2$ which is in year 2011. The process is repeated for the interpolated values of 34 transient cases, V_{tran}^{c-3} and 18 absorbing cases, V_{abs}^{c-3} for the period of $c-3$ which is in year 2010 and for the interpolated values of 34 transient cases, V_{tran}^{c-4} and 18 absorbing cases, V_{abs}^{c-4} for the period of $c-4$ which is in year 2009.

Table 4.5 shows the interpolated values of 5 transient cases. For instance, the third row shows the interpolated values, $V_{tran}^{c-1}, V_{tran}^{c-2}, V_{tran}^{c-3}$, and V_{tran}^{c-4} of 903, 718, 844 and 679 for 2012, 2011, 2010 and 2009 respectively for the case of $C_{2,2}$. On the other hand, Table 4.6 shows the interpolation results for 5 absorbing cases. For instance, the third row shows the interpolated values of 90, 83, 75 and 63 for the period of $c-1, c-2, c-3$ and $c-4$ respectively for the case of $C_{3,19}$. The interpolation results of 34 transient cases are shown by Table 1 in Appendix A where as the interpolation results of 18 absorbing cases are shown by Table 2 in Appendix A. The smoothing process using interpolation in Step 3 yields the interpolated values for all cases of the previous 4 periods, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} as shown in Table 3 Appendix A.

Table 4.5

Interpolated Values of 5 Transient Cases $(C_{1,1},\ldots,C_{3,3})$ *for the Previous 4 Periods,* $V_{tran}^{c-1}, V_{tran}^{c-2}, V_{tran}^{c-3}$, and V_{tran}^{c-4} *(Full Result Refers to Table 2 in Appendix B)*

Coding of Transient Cases	V_{tran}^{c-1}	V_{tran}^{c-2}	V_{tran}^{c-3}	V_{tran}^{c-4}
$C_{1,1}$	612	522	583	499
$C_{1,2}$	243	224	230	213
$C_{2,2}$	903	718	844	679
$C_{2,3}$	114	108	107	101
$C_{3,3}$	241	223	228	212

Table 4.6

Interpolated Values of 5 Absorbing Cases $(C_{1,19},\ldots,C_{15,19})$ *for the Previous 4 Periods,* $V_{abs}^{c-1}, V_{abs}^{c-2}, V_{abs}^{c-3}$, and V_{abs}^{c-4} *(Full Result Refers to Table 3 in Appendix B)*

Coding of Absorbing Cases	V_{abs}^{c-1}	V_{abs}^{c-2}	V_{abs}^{c-3}	V_{abs}^{c-4}
$C_{1,19}$	28	32	37	46
$C_{2,19}$	96	87	76	62
$C_{3,19}$	90	83	75	63
$C_{4,19}$	72	71	69	65
$C_{5,19}$	70	70	69	65

In Step 4, we perform SES technique on V^c the actual values of all cases for the current period which is in year 2013 and on the interpolated values of all cases, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} for 2009 until 2012. The fitted values for all 52 cases of the current period, V_{SES}^{c} and the previous 4 periods, $V_{SES}^{c-1}, V_{SES}^{c-2}, V_{SES}^{c-3}$ and V_{SES}^{c-4} are obtained using equation (3.2).

In order to find the fitted values with the best value of α of all cases, we apply $\alpha = 0.1$ until $\alpha = 0.9$ for each of the cases. Then, α that gives the smallest MSE value of each of the cases is chosen as the best value of α . The fitted values with the best value of α are used in obtaining the projected values of each of the cases for the following future period, 2014. For example, for the case of $C_{1,1}$ based on the results in Table 4 in Appendix A, we choose $\alpha = 0.8$ because it gives the smallest MSE value of 3973.4434.

Table 4.7

Fitted Values for $C_{1,1}$ of the Previous k Periods, $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} and the Current Period of 2013, V_{SES}^{c} with $\alpha = 0.8$

α	k	V^{c-k}	V_{SES}^{c-k}	e $\left(V^{c-k} - V_{SES}^{c-k}\right)$	e^2
0.8	4	499	499	0.0000	0.0000
	3	583	499	83.7531	7014.5861
	2	522	566	-43.7476	1913.8513
	1	612	531	80.7619	6522.4882
	0	662	596	66.4552	4416.2916
				SSQ	19867.2172
				MSE	**3973.4434**

Table 4.7 shows the fitted values for the case of $C_{1,1}$ for the previous 4 periods, $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} and for the current period, V_{SES}^{c} with $\alpha = 0.8$. The fitted

values for all 52 cases of the previous k periods, $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} and the current period, V_{SES}^{c} is shown in Table 5 in Appendix A. Next, we obtain V_{SES}^{c+1} consisting of the projected values of all cases for the following future period using equation (3.3). For instance, Table 4.8 shows the projected value of $C_{1,1}$ for the following future period (2014) with $\alpha = 0.8$.

Table 4.8

Projected Value of $C_{1,1}$ for the Following Future Period, V_{SES}^{c+1} where $k = -1$

α	k	V^{c-k}	V_{SES}^{c-k}
0.8	4	499	499
	3	583	499
	2	522	566
	1	612	531
	0	662	596
	-1		**649**

Table 4.8 shows the results of $C_{1,1}$ indicating the projected number of male Muslims at the age category Under 26 who are in state (M: Under 26) in the marital system for the period of $k = -1$ which is in 2014 and remains in the same state in the following year which is in year 2015 with $\alpha = 0.8$ is 649. The same calculation procedure is used for the other cases. The smoothing process using SES in Step 4 yields V_{SES}^{c+1} the projected values of all cases (52 cases) for the following future period as shown in Table 6 in Appendix A.

4.1.3 Phase 3: Making Projection

Phase 3 involves making projection of the number of Muslims in the marital system for the following future period. Phase 3 consists of steps 5 to 8. In Step 5, we form a transition matrix of the following future period, T_{SES}^{c+1}. The period of transition between states used is one year which starts from year 2014 to 2015. The transition matrix T_{SES}^{c+1} of Muslims in the marital system is shown in Table 4.9.

Table 4.9 presents the transition process of Muslims in the marital system from one state to another in one year duration which is from 2014 to 2015. From the table, the first row indicates that out of 924 Muslims, 649 male Muslims at the age category of Under 26 which refers to the state (M:Under 26) who are in the marital system in the year 2014 will be in the same state (M:Under 26) in the following year of 2015. Meanwhile, 251 male Muslims at the age category Under 26 which refers to the state (M:Under 26) who are in the marital system in 2014 but are moved to the next state (M:26-30) in the following year of 2015. However, 24 male Muslims are out of the marital system which means divorce in year 2014. Similar interpretation can be made for the other rows.

In Step 6, we develop P_{SES}^{c+1} a projected transition probability matrix of Muslims in the marital system for the following future period which is from 2014 to 2015. Table 4.10 shows the 19×19 matrix of transition probabilities P_{SES}^{c+1} . Step 7 is the formation of Q_{SES}^{c+1} which is a submatrix of P_{SES}^{c+1} . The matrix Q_{SES}^{c+1} is shown in Table 4.11.

Table 4.9

T_{SES}^{c+1} , *A Transition Matrix of the Following Future Period (from 2014 to 2015)*

To 2015 From 2014	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60	Divorce	Row Total
M : Under 26	649	251	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	924
M : 26-30	0	986	117	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	104	1207
M : 31-35	0	0	249	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	95	376
M : 36-40	0	0	0	84	12	0	0	0	0	0	0	0	0	0	0	0	0	0	72	168
M : 41-45	0	0	0	0	30	4	0	0	0	0	0	0	0	0	0	0	0	0	70	104
M : 46-50	0	0	0	0	0	23	5	0	0	0	0	0	0	0	0	0	0	0	54	82
M : 51-55	0	0	0	0	0	0	18	4	0	0	0	0	0	0	0	0	0	0	34	56
M : 56-60	0	0	0	0	0	0	0	20	5	0	0	0	0	0	0	0	0	0	19	44
M : Above 60	0	0	0	0	0	0	0	0	42	0	0	0	0	0	0	0	0	0	53	95
F : Under 26	0	0	0	0	0	0	0	0	0	1077	301	0	0	0	0	0	0	0	58	1436
F : 26-30	0	0	0	0	0	0	0	0	0	0	710	62	0	0	0	0	0	0	123	895
F : 31-35	0	0	0	0	0	0	0	0	0	0	0	148	20	0	0	0	0	0	122	290
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	65	10	0	0	0	0	61	136
F : 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	29	6	0	0	0	52	87
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	21	2	0	0	34	57
F : 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	3	0	26	48
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	4	19	32
F : Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	28	43
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4.10

P_{SES}^{c+1} , A Transition Probability Matrix of Muslims in the Marital System for the Following Future Period (from 2014 to 2015)

To 2015 From 2014	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60	Divorce
M : Under 26	0.7024	0.2716	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0260
M : 26-30	0	0.8169	0.0969	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0862
M : 31-35	0	0	0.6622	0.0851	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2527
M : 36-40	0	0	0	0.5000	0.0714	0	0	0	0	0	0	0	0	0	0	0	0	0	0.4286
M : 41-45	0	0	0	0	0.2885	0.0385	0	0	0	0	0	0	0	0	0	0	0	0	0.6731
M : 46-50	0	0	0	0	0	0.2805	0.0610	0	0	0	0	0	0	0	0	0	0	0	0.6585
M : 51-55	0	0	0	0	0	0	0.3214	0.0714	0	0	0	0	0	0	0	0	0	0	0.6071
M : 56-60	0	0	0	0	0	0	0	0.4545	0.1136	0	0	0	0	0	0	0	0	0	0.4318
M : Above 60	0	0	0	0	0	0	0	0	0.4421	0	0	0	0	0	0	0	0	0	0.5579
F : Under 26	0	0	0	0	0	0	0	0	0	0.7500	0.2096	0	0	0	0	0	0	0	0.0404
F : 26-30	0	0	0	0	0	0	0	0	0	0	0.7933	0.0693	0	0	0	0	0	0	0.1374
F : 31-35	0	0	0	0	0	0	0	0	0	0	0	0.5103	0.0690	0	0	0	0	0	0.4207
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	0.4779	0.0735	0	0	0	0	0.4485
F : 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3333	0.0690	0	0	0	0.5977
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3684	0.0351	0	0	0.5965
F : 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3958	0.0625	0	0.5417
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2813	0.1250	0.5938
F : Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3488	0.6512
Divorce	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 4.11

Q_{SES}^{c+1}, A Submatrix of P_{SES}^{c+1}

To 2015 From 2014	M : Under 26	M : 26-30	M : 31-35	M : 36-40	M : 41-45	M : 46-50	M : 51-55	M : 56-60	M : Above 60	F : Under 26	F : 26-30	F : 31-35	F : 36-40	F : 41-45	F : 46-50	F : 51-55	F : 56-60	F : Above 60
M : Under 26	0.7024	0.2716	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 26-30	0	0.8169	0.0969	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 31-35	0	0	0.6622	0.0851	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 36-40	0	0	0	0.5000	0.0714	0	0	0	0	0	0	0	0	0	0	0	0	0
M : 41-45	0	0	0	0	0.2885	0.0385	0	0	0	0	0	0	0	0	0	0	0	0
M : 46-50	0	0	0	0	0	0.2805	0.0610	0	0	0	0	0	0	0	0	0	0	0
M : 51-55	0	0	0	0	0	0	0.3214	0.0714	0	0	0	0	0	0	0	0	0	0
M : 56-60	0	0	0	0	0	0	0	0.4545	0.1136	0	0	0	0	0	0	0	0	0
M : Above 60	0	0	0	0	0	0	0	0	0.4421	0	0	0	0	0	0	0	0	0
F : Under 26	0	0	0	0	0	0	0	0	0	0.7500	0.2096	0	0	0	0	0	0	0
F : 26-30	0	0	0	0	0	0	0	0	0	0	0.7933	0.0693	0	0	0	0	0	0
F : 31-35	0	0	0	0	0	0	0	0	0	0	0	0.5103	0.0690	0	0	0	0	0
F : 36-40	0	0	0	0	0	0	0	0	0	0	0	0	0.4779	0.0735	0	0	0	0
F : 41-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3333	0.0690	0	0	0
F : 46-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3684	0.0351	0	0
F : 51-55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3958	0.0625	0
F : 56-60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2813	0.1250
F : Above 60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3488

Step 8 is the last step for Phase 3 where we project X_{SES}^{c+1} the future state vector of the number of Muslims in the marital system for the following future year which is in the year 2014 for each of the transient states (marriage). The result of X_{SES}^{c+1} is shown in equation (4.1). X_{SES}^{c+1} is then tabulated in Table 4.12

$$X_{SES}^{c+1} = (642\ \ 1173\ \ 296\ \ 72\ \ 17\ \ 9\ \ 9\ \ 13\ \ 21\ \ 1061\ \ 922\ \ 141\ \ 48\ \ 18\ \ 11\ \ 10\ \ 5\ \ 7) \tag{4.1}$$

Table 4.12

Projected Number of Muslims in the Marital System for the Following Future Year (2014) for Each of the Transient States Using the Proposed MC_{SES} Hybrid Model

Transient States	Projected Number of Muslims in the Marital System for the Year 2014
M: Under 26	642
M: 26-30	1173
M: 31-35	296
M: 36-40	72
M: 41-45	17
M: 46-50	9
M: 51-55	9
M: 56-60	13
M: Above 60	21
F: Under 26	1061
F: 26-30	922
F: 31-35	141
F: 36-40	48
F: 41-45	18
F: 46-50	11
F: 51-55	10
F: 56-60	5
F: Above 60	7

Table 4.12 shows the projected number of Muslims in the marital system for the year 2014 for each of the transient states. Based on this table, for instance we can see the projected number of male Muslims who are in the marital system at the age category of Under 26 which refers to the state (M: Under 26) using the proposed MC_{SES} hybrid model is 642 compared to the actual number, 755 as referred in Table 1 in Appendix B. The difference between the projected values and the actual values of each of the transient states are then being used to calculate the error measures to measure the forecast accuracy the proposed MC_{SES} hybrid model. The projected number of Muslims in the marital system for the year 2014 for each of the transient states using MC model and MC_{SMA} model are shown in Table 2 and Table 3 in Appendix B.

4.2 Measurement of Forecast Accuracy

In order to measure the forecast accuracy of the proposed MC_{SES} hybrid model, the four error measures which are MAD, MAPD, MAPE and MSE are applied in this study using equation (2.3) through equation (2.6). The proposed MC_{SES} hybrid model produces values of 42.43, 15.66%, 38.43% and 4672.22 for MAD, MAPD, MAPE and MSE respectively.

4.3 Comparison of Forecast Accuracy

In addition, the comparison of the forecast accuracy between the proposed MC_{SES} hybrid model and the other two models which are the MC model and MC_{SMA} model has also been done in this study.

Table 4.13

Comparison of the Forecast Accuracy for the Proposed MC_{SES} Hybrid Model, MC Model and MC_{SMA} Model

Model	MAD	MAPD (%)	MAPE (%)	MSE
MC_{SES} Hybrid	42.43	15.66	38.43	4672.22
MC	71.3	26.31	55.36	18913.52
MC_{SMA}	47.11	17.38	43.52	6122.87

Table 4.13 shows the comparison of the three models use in this study which is the proposed MC_{SES} hybrid model, the MC model, and also the MC model with simple moving average which is denoted as MC_{SMA} model. From the result obtained, it can be summarised that the proposed MC_{SES} hybrid model is found to be the most appropriate model in forecasting the number of Muslims in the marital system for the year 2014 since it produces the highest forecast accuracy with values of 42.43, 15.66%, 38.43% and 4672.22 for MAD, MAPD, MAPE and MSE, respectively. MC_{SMA} model gives the next highest accuracy with values of 47.11, 17.38%, 43.52% and 6122.87, respectively. Meanwhile, MC model produces the lowest accuracy for all the error measures with values of 71.3, 26.31%, 55.36% and 18913.52, respectively. Therefore, the proposed MC_{SES} hybrid model is the most appropriate model in forecasting the number of Muslims in the marital system for the year 2014.

4.4 Summary

Chapter 4 displays the results of this study. Firstly, we collect the data. Next, the modeling steps are developed. Then, the forecast accuracy of the MC_{SES} hybrid model is determined. It is based on four error measures used in this study which are MAD, MAPD, MAPE and MSE. Comparison of forecast accuracy between the proposed MC_{SES} hybrid model with MC model and MC_{SMA} model indicates that proposed MC_{SES} hybrid model produces the highest forecast accuracy with values of 42.43, 15.66%, 38.43% and 4672.22 for MAD, MAPD, MAPE and MSE, respectively. Therefore, the proposed MC_{SES} hybrid model is the most appropriate model to forecast the number of Muslims in the marital system for the year 2014.

CHAPTER FIVE

CONCLUSION

Chapter 5 begins with the summary and contributions of the whole study. This is followed by the limitations of the study as well as some suggestions for future research.

5.1 Summary

In this research, we develop modeling steps for Markov Chain model where the model is known as the MC_{SES} hybrid model. This is done by integrating MC model with SES to enhance the MC model and improve the accuracy of the forecast values in case of limited data and short-term projection. Four error measures used to determine the accuracy of this model are MAD, MAPD, MSE and MAPE. This study uses a sample of 6061 Muslim couples in a district in Kedah who are in the marital system for the year 2013 and 2014. The number of Muslims in the system for the following year is forecast using the proposed MC_{SES} hybrid model, MC model and MC_{SMA} model. It is found that the proposed MC_{SES} hybrid model produces the highest forecast accuracy with values of 42.43, 15.66%, 38.43% and 4672.22 for MAD, MAPD, MAPE and MSE, respectively. MC_{SMA} model gives the next highest accuracy while MC model produces the lowest forecast accuracy for all the error measures. Therefore, it is believed that the proposed MC_{SES} hybrid model is the most appropriate model to forecast the number of Muslims in the marital system for the year 2014.

The results obtained in this study manage to achieve objectives, thus the main objective of this study is achieved. The main objective of this study is to develop modeling steps for Markov Chain model in case of limited data and short-term projection, called the MC_{SES} hybrid model.

In order to achieve the first objective which is to generic values for previous periods for transition probability matrix for previous periods in case of missing records, the interpolation technique is applied in this study. We perform quadratic polynomial interpolation on V^c, the actual values of all cases for the current period which is the year 2013 by using equation (3.1). This is conducted in Step 3 of modeling steps. The interpolation results for 34 transient cases are shown by Table 1 in Appendix A where as the interpolation results for 18 absorbing cases are shown by Table 2 in Appendix A. Step 3 yields the interpolated values of the 52 cases for the previous 4 periods, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} where $c-1, c-2, c-3$ and $c-4$ refers to the year 2012, 2011, 2010 and 2009 respectively.

Next, the second objective which is to construct a projected transition probability matrix using simple exponential smoothing (SES) is done by performing SES technique on V^c the actual values of all cases for the current period which is in year 2013 and on the interpolated values, $V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4} for 2009 until 2012. This is conducted in Step 4 of modeling steps. First, using $V^c, V^{c-1}, V^{c-2}, V^{c-3}$ and V^{c-4}, we obtain fitted values for all the cases of the current period, V^c_{SES} and the previous 4 periods, $V^{c-1}_{SES}, V^{c-2}_{SES}, V^{c-3}_{SES}$ and V^{c-4}_{SES} using equation (3.2). The fitted values for all 52 cases of the current period and the previous k periods,

$V_{SES}^{c}, V_{SES}^{c-1}, V_{SES}^{c-2}, V_{SES}^{c-3}$ and V_{SES}^{c-4} is shown in Table 5 in Appendix A. Next, we obtain V_{SES}^{c+1} consisting of the projected values of cases for the following future period using equation (3.3). The smoothing process using SES in Step 4 yields V_{SES}^{c+1} the projected values of cases for the following future period, using $V_{SES}^{c}, V_{SES}^{c-1}, V_{SES}^{c-2}, V_{SES}^{c-3}$ and V_{SES}^{c-4} as shown in Table 6 in Appendix A. In Step 5, we form a transition matrix of the following future period, T_{SES}^{c+1} using the obtained V_{SES}^{c+1}. The transition matrix T_{SES}^{c+1} of Muslims in the marital system is shown in Table 4.9. T_{SES}^{c+1} is then used in Step 6 to develop P_{SES}^{c+1} a projected transition probability matrix of Muslims in the marital system for the following future period which is from 2014 to 2015. P_{SES}^{c+1} is obtained by dividing each frequency by the appropriate row total of T_{SES}^{c+1} and is shown in Table 4.10.

Next, the third objective is achieved by projecting the number of Muslims in the marital system using equation (3.4). Q_{SES}^{c+1} which is a submatrix of P_{SES}^{c+1} is multiplied with X^{0}, the initial state vector of the number of Muslims in the marital system in 2013 for each transient states (marriage) in order to get X_{SES}^{c+1}, the future state vector of the number of Muslims in the marital system for the following future year which is in the year 2014 for each transient states as shown in Table 4.12.

The fourth objective is achieved by comparing the forecast accuracy of the proposed MC$_{SES}$ hybrid model with the classical Markov Chain model and the MC$_{SMA}$ model for the number of Muslims in a marital system for 2014 by using four error measures as shown in Table 4.13. They are MAD, MAPD, MAPE and MSE. From the result obtained in Table 4.13, the proposed MC_{SES} hybrid model produces the highest forecast accuracy with values of 42.43, 15.66%, 38.43% and 4672.22 for MAD, MAPD, MAPE and MSE, respectively. MC_{SMA} model gives the next highest accuracy with values of 47.11, 17.38%, 43.52% and 6122.87 respectively. Meanwhile, MC model produces the lowest accuracy for all the error measures with values of 71.3, 26.31%, 55.36% and 18913.52 respectively.

Since the proposed MC_{SES} hybrid model gives the highest forecast accuracy, it is found to be the most appropriate model to forecast the number of Muslims in the marital system for the following future year which is in year 2014. Thus, the fifth objective is achieved. Therefore, the main objective of this study is achieved. The modeling steps for Markov Chain are developed in case of limited data and short-term projection and the model is known as a MC_{SES} hybrid model.

5.2 Contributions of the Study

There is no doubt that developing the modeling steps for Markov Chain model by embedding SES technique capable to increase the forecast accuracy of the forecast values for a short-term period. This study tries to bring forward the use of SES to obtain a projected transition probability matrix, P_{SES}^{c+1} thus forecasts the future values for the number of Muslims in a marital system for the year 2014. The proposed model which is MC_{SES} hybrid model can be served as an alternative in studying the flow of the data and also forecasting future values in a short-term period. In addition, this model can contribute in cases where there is limited historical data. Moreover, this model can be applied in various fields. Therefore, researchers can use this model to study flow of the data and also forecast future values based on their related fields.

5.3 Limitations of the Study

There are two limitations found in this study. Firstly, this study only focuses on Muslims in a marital system. Secondly, this study only caters for data which has no trend. Moreover, the mean time and also average time of Muslims in the marital system are not taken into consideration. In addition, the study is only focused on the transient states.

5.4 Suggestions for Future Research

In future, it is suggested that researchers use different types of data. This is because, in reality there could be cases where the data may have trend and certain criteria that should be considered. Therefore, researchers should concern on other types of exponential smoothing that can be integrated to MC model based on the data collected. In addition, researchers could use this approach in various fields. Moreover, it is good for researchers to take into account the mean time, average time and also absorbing states.

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Appendix A

Smoothing of Values

Table 1

Interpolated Values of 34 Transient Cases for the Previous 4 Periods, $V_{tran}^{c-1}, V_{tran}^{c-2}, V_{tran}^{c-3}$, and V_{tran}^{c-4}

Coding of Transient Cases	V_{tran}^{c-1}	V_{tran}^{c-2}	V_{tran}^{c-3}	V_{tran}^{c-4}
$C_{1,1}$	612	522	583	499
$C_{1,2}$	243	224	230	213
$C_{2,2}$	903	718	844	679
$C_{2,3}$	114	108	107	101
$C_{3,3}$	241	223	228	212
$C_{3,4}$	31	29	28	27
$C_{4,4}$	82	77	76	73
$C_{4,5}$	11	10	9	9
$C_{5,5}$	29	27	26	25
$C_{5,6}$	2	2	2	2
$C_{6,6}$	22	20	19	19
$C_{6,7}$	3	3	3	3
$C_{7,7}$	17	15	15	14
$C_{7,8}$	2	2	2	2
$C_{8,8}$	19	17	17	16
$C_{8,9}$	4	3	3	3
$C_{9,9}$	40	38	37	36
$C_{10,10}$	977	763	907	718
$C_{10,11}$	290	266	276	254
$C_{11,11}$	666	562	633	536
$C_{11,12}$	60	57	56	54
$C_{12,12}$	145	136	136	129
$C_{12,13}$	19	17	17	16
$C_{13,13}$	63	60	58	56
$C_{13,14}$	9	8	7	7
$C_{14,14}$	28	26	25	24
$C_{14,15}$	4	7	6	6
$C_{15,15}$	20	18	18	17

Table 1 continued.

$C_{15,16}$	1	0	0	0
$C_{16,16}$	18	16	16	15
$C_{16,17}$	1	1	1	1
$C_{17,17}$	8	7	6	6
$C_{17,18}$	2	2	2	2
$C_{18,18}$	14	12	12	12

Table 2

Interpolated Values of 18 Absorbing Cases for the Previous 4 Periods, V_{abs}^{c-1}, V_{abs}^{c-2}, V_{abs}^{c-3}, and V_{abs}^{c-4}

Coding of Absorbing Cases	V_{abs}^{c-1}	V_{abs}^{c-2}	V_{abs}^{c-3}	V_{abs}^{c-4}
$C_{1,19}$	28	32	37	46
$C_{2,19}$	96	87	76	62
$C_{3,19}$	90	83	75	63
$C_{4,19}$	72	71	69	65
$C_{5,19}$	70	70	69	65
$C_{6,19}$	56	59	62	66
$C_{7,19}$	38	42	48	59
$C_{8,19}$	23	27	31	35
$C_{9,19}$	56	58	61	66
$C_{10,19}$	60	62	64	66
$C_{11,19}$	109	93	77	61
$C_{12,19}$	108	93	77	61
$C_{13,19}$	63	64	65	66
$C_{14,19}$	55	58	61	66
$C_{15,19}$	38	42	48	59
$C_{16,19}$	30	34	40	49
$C_{17,19}$	23	27	31	35
$C_{18,19}$	32	37	42	52

Table 3

Interpolated Values of all Cases for the Previous 4 Periods, V^{c-1}, V^{c-2}, V^{c-3}, and V^{c-4}

Coding of Cases	V^{c-1}	V^{c-2}	V^{c-3}	V^{c-4}
$C_{1,1}$	612	522	583	499
$C_{1,2}$	243	224	230	213
$C_{2,2}$	903	718	844	679
$C_{2,3}$	114	108	107	101
$C_{3,3}$	241	223	228	212
$C_{3,4}$	31	29	28	27
$C_{4,4}$	82	77	76	73
$C_{4,5}$	11	10	9	9
$C_{5,5}$	29	27	26	25
$C_{5,6}$	2	2	2	2
$C_{6,6}$	22	20	19	19
$C_{6,7}$	3	3	3	3
$C_{7,7}$	17	15	15	14
$C_{7,8}$	2	2	2	2
$C_{8,8}$	19	17	17	16
$C_{8,9}$	4	3	3	3
$C_{9,9}$	40	38	37	36
$C_{10,10}$	977	763	907	718
$C_{10,11}$	290	266	276	254
$C_{11,11}$	666	562	633	536
$C_{11,12}$	60	57	56	54
$C_{12,12}$	145	136	136	129
$C_{12,13}$	19	17	17	16
$C_{13,13}$	63	60	58	56
$C_{13,14}$	9	8	7	7
$C_{14,14}$	28	26	25	24
$C_{14,15}$	4	7	6	6
$C_{15,15}$	20	18	18	17
$C_{15,16}$	1	0	0	0
$C_{16,16}$	18	16	16	15
$C_{16,17}$	1	1	1	1
$C_{17,17}$	8	7	6	6
$C_{17,18}$	2	2	2	2

Table 3 continued.

$C_{18,18}$	14	12	12	12
$C_{1,19}$	28	32	37	46
$C_{2,19}$	96	87	76	62
$C_{3,19}$	90	83	75	63
$C_{4,19}$	72	71	69	65
$C_{5,19}$	70	70	69	65
$C_{6,19}$	56	59	62	66
$C_{7,19}$	38	42	48	59
$C_{8,19}$	23	27	31	35
$C_{9,19}$	56	58	61	66
$C_{10,19}$	60	62	64	66
$C_{11,19}$	109	93	77	61
$C_{12,19}$	108	93	77	61
$C_{13,19}$	63	64	65	66
$C_{14,19}$	55	58	61	66
$C_{15,19}$	38	42	48	59
$C_{16,19}$	30	34	40	49
$C_{17,19}$	23	27	31	35
$C_{18,19}$	32	37	42	52

Table 4

Fitted Values for $C_{1,1}$ *of the Previous k Periods,* $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} *and the Current Period of 2013,* V_{SES}^{c} *with* $\alpha = 0.1$ until $\alpha = 0.9$

α	k	V^{c-k}	V_{SES}^{c-k}	e $\left(V^{c-k} - V_{SES}^{c-k}\right)$	e^2
0.1	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	507	14.8796	221.403
	1	612	509	102.903	10589
	0	662	519	142.916	20424.9
				SSQ	38249.9
				MSE	7649.98
0.2	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	516	6.5043	42.3058
	1	612	517	94.7149	8970.91
	0	662	536	126.075	15894.8
				SSQ	31922.6
				MSE	6384.53
0.3	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	524	-1.871	3.5007
	1	612	523	88.2017	7779.54
	0	662	550	112.044	12553.9
				SSQ	27351.5
				MSE	5470.3
0.4	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	532	-10.246	104.987
	1	612	528	83.3636	6949.5
	0	662	562	100.321	10064.3
				SSQ	24133.4
				MSE	4826.67
0.5	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	541	-18.622	346.766
	1	612	531	80.2006	6432.14
	0	662	572	90.4031	8172.72
				SSQ	21966.2
				MSE	4393.24

Table 4 continued.

0.6	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	549	-26.997	728.836
	1	612	533	78.7127	6195.68
	0	662	580	81.7879	6689.25
				SSQ	20628.4
				MSE	4125.67
0.7	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	558	-35.372	1251.2
	1	612	533	78.8998	6225.17
	0	662	588	73.9727	5471.96
				SSQ	19962.9
				MSE	3992.58
0.8	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	566	-43.748	1913.85
	1	612	531	80.7619	6522.49
	0	662	596	66.4552	4416.29
				SSQ	19867.2
				MSE	**3973.44**
0.9	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	574	-52.123	2716.8
	1	612	527	84.2992	7106.35
	0	662	603	58.7327	3449.53
				SSQ	20287.3
				MSE	4057.45

Table 5

The fitted values for all 52 cases of the previous k periods, $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} and the current period, V_{SES}^{c}

Coding of Cases	α	k	V^{c-k}	V_{SES}^{c-k}	e $\left(V^{c-k} - V_{SES}^{c-k}\right)$	e^2
$C_{1,1}$	0.8	4	499	499	0	0
		3	583	499	83.7531	7014.59
		2	522	566	-43.748	1913.85
		1	612	531	80.7619	6522.49
		0	662	596	66.4552	4416.29
					SSQ	19867.2
					MSE	3973.44
$C_{1,2}$	0.9	4	213	213	0	0
		3	230	213	16.8727	284.688
		2	224	229	-4.2601	18.1485
		1	243	225	17.7877	316.404
		0	252	241	11.2046	125.542
					SSQ	744.783
					MSE	148.957
$C_{2,2}$	0.8	4	679	679	0	0
		3	844	679	165.435	27368.7
		2	718	811	-92.463	8549.41
		1	903	737	165.597	27422.3
		0	1015	869	145.55	21184.9
					SSQ	84525.3
					MSE	16905.1
$C_{2,3}$	0.9	4	101	101	0	0
		3	107	101	5.17239	26.7536
		2	108	106	1.43587	2.06172
		1	114	107	6.29892	39.6764
		0	117	113	3.95369	15.6317
					SSQ	84.1234
					MSE	16.8247
$C_{3,3}$	0.9	4	212	212	0	0
		3	228	212	16.6569	277.454
		2	223	227	-4.1269	17.031
		1	241	223	17.5771	308.954
		0	250	239	11.0665	122.468
					SSQ	725.906
					MSE	145.181

Table 5 continued.

$C_{3,4}$	0.9	4	27	27	0	0
		3	28	27	0.91135	0.83055
		2	29	28	1.01595	1.03216
		1	31	29	2.00881	4.03533
		0	32	30	1.55268	2.41082
					SSQ	8.30886
					MSE	1.66177
$C_{4,4}$	0.9	4	73	73	0	0
		3	76	73	3.22613	10.4079
		2	77	76	1.68316	2.83303
		1	82	77	4.35634	18.9777
		0	84	81	2.82223	7.96501
					SSQ	40.1837
					MSE	8.03674
$C_{4,5}$	0.9	4	9	9	0	0
		3	9	9	0.26826	0.07196
		2	10	9	0.39084	0.15275
		1	11	10	1.34187	1.80063
		0	12	11	1.23199	1.51779
					SSQ	3.54314
					MSE	0.70863
$C_{5,5}$	0.9	4	25	25	0	0
		3	26	25	0.84084	0.70701
		2	27	26	0.96297	0.92732
		1	29	27	1.93618	3.74877
		0	30	28	1.51642	2.29952
					SSQ	7.68262
					MSE	1.53652
$C_{5,6}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222

Table 5 continued.

$C_{6,6}$	0.9	4	19	19	0	0
		3	19	19	0.6049	0.3659
		2	20	19	0.76098	0.57909
		1	22	20	1.69233	2.86398
		0	23	22	1.39683	1.95114
					SSQ	5.76012
					MSE	1.15202
$C_{6,7}$	0.9	4	3	3	0	0
		3	3	3	0.07577	0.00574
		2	3	3	0.12121	0.01469
		1	3	3	0.14896	0.02219
		0	5	3	2.0335	4.1351
					SSQ	4.17773
					MSE	0.83555
$C_{7,7}$	0.9	4	14	14	0	0
		3	15	14	0.44671	0.19955
		2	15	15	0.60078	0.36094
		1	17	15	1.52806	2.33498
		0	18	17	1.31841	1.7382
					SSQ	4.63367
					MSE	0.92673
$C_{7,8}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222
$C_{8,8}$	0.9	4	16	16	0	0
		3	17	16	0.50895	0.25903
		2	17	17	0.66646	0.44417
		1	19	17	1.59278	2.53694
		0	20	19	1.34908	1.82001
					SSQ	5.06016
					MSE	1.01203

Table 5 continued.

$C_{8,9}$	0.9	4	3	3	0	0
		3	3	3	0.07577	0.00574
		2	3	3	0.12121	0.01469
		1	4	3	1.13976	1.29904
		0	5	4	1.14178	1.30365
					SSQ	2.62313
					MSE	0.52463
$C_{9,9}$	0.9	4	36	36	0	0
		3	37	36	1.28454	1.65004
		2	38	37	1.24968	1.5617
		1	40	38	2.39173	5.72037
		0	42	40	1.74797	3.05541
					SSQ	11.9875
					MSE	2.3975
$C_{10,10}$	0.8	4	718	718	0	0
		3	907	718	189.281	35827.2
		2	763	869	-105.87	11207.6
		1	977	785	192.705	37135.4
		0	1111	939	172.178	29645.4
					SSQ	113816
					MSE	22763.1
$C_{10,11}$	0.9	4	254	254	0	0
		3	276	254	22.7867	519.232
		2	266	274	-8.0898	65.4444
		1	290	267	23.5583	554.991
		0	303	288	15.0354	226.064
					SSQ	1365.73
					MSE	273.146
$C_{11,11}$	0.8	4	536	536	0	0
		3	633	536	97.4418	9494.9
		2	562	614	-52.217	2726.66
		1	666	572	94.2371	8880.63
		0	726	647	78.5646	6172.4
					SSQ	27274.6
					MSE	5454.92

Table 5 continued.

$C_{11,12}$	0.9	4	54	54	0	0
		3	56	54	2.1339	4.55352
		2	57	55	1.56441	2.44737
		1	60	57	3.25577	10.6
		0	62	60	2.20838	4.87693
					SSQ	22.4779
					MSE	4.49557
$C_{12,12}$	0.9	4	129	129	0	0
		3	136	129	7.40862	54.8877
		2	136	135	0.73595	0.54162
		1	145	136	8.51265	72.4653
		0	149	144	5.29187	28.0038
					SSQ	155.898
					MSE	31.1797
$C_{12,13}$	0.9	4	16	16	0	0
		3	17	16	0.50895	0.25903
		2	17	17	0.66646	0.44417
		1	19	17	1.59278	2.53694
		0	20	19	1.34908	1.82001
					SSQ	5.06016
					MSE	1.01203
$C_{13,13}$	0.9	4	56	56	0	0
		3	58	56	2.27312	5.16706
		2	60	58	1.59442	2.54219
		1	63	60	3.39662	11.537
		0	65	63	2.28546	5.22334
					SSQ	24.4696
					MSE	4.89392
$C_{13,14}$	0.9	4	7	7	0	0
		3	7	7	0.21154	0.04475
		2	8	7	0.31653	0.10019
		1	9	8	1.28246	1.64472
		0	10	9	1.20505	1.45214
					SSQ	3.24179
					MSE	0.64836

Table 5 continued.

$C_{14,14}$	0.9	4	24	24	0	0
		3	25	24	0.8061	0.6498
		2	26	25	0.9357	0.87553
		1	28	26	1.90035	3.61133
		0	29	28	1.49864	2.24591
					SSQ	7.38257
					MSE	1.47651
$C_{14,15}$	0.1	4	6	6	0	0
		3	6	6	0.1837	0.03374
		2	7	6	0.42551	0.18106
		1	4	6	-2.3528	5.53584
		0	9	6	2.91025	8.46954
					SSQ	14.2202
					MSE	2.84404
$C_{15,15}$	0.9	4	17	17	0	0
		3	18	17	0.54059	0.29224
		2	18	18	0.6985	0.4879
		1	20	18	1.62563	2.64268
		0	21	20	1.36476	1.86258
					SSQ	5.2854
					MSE	1.05708
$C_{15,16}$	0.9	4	0	0	0	0
		3	0	0	-0.0015	2.4E-06
		2	0	0	-0.0026	6.6E-06
		1	1	0	1.05812	1.11963
		0	2	1	1.10661	1.22459
					SSQ	2.34423
					MSE	0.46885
$C_{16,16}$	0.9	4	15	15	0	0
		3	16	15	0.47766	0.22816
		2	16	16	0.63389	0.40182
		1	18	16	1.56026	2.4344
		0	19	18	1.33363	1.77856
					SSQ	4.84293
					MSE	0.96859

Table 5 continued.

$C_{16,17}$	0.9	4	1	1	0	0
		3	1	1	0.02388	0.00057
		2	1	1	0.03924	0.00154
		1	1	1	0.0938	0.0088
		0	3	1	2.01018	4.04082
					SSQ	4.05173
					MSE	0.81035
$C_{17,17}$	0.9	4	6	6	0	0
		3	6	6	0.1837	0.03374
		2	7	6	0.27856	0.07759
		1	8	7	1.25326	1.57066
		0	9	8	1.19193	1.42069
					SSQ	3.10268
					MSE	0.62054
$C_{17,18}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222
$C_{18,18}$	0.9	4	12	12	0	0
		3	12	12	0.35593	0.12669
		2	12	12	0.49824	0.24824
		1	14	12	1.43348	2.05486
		0	15	14	1.27415	1.62345
					SSQ	4.05324
					MSE	0.81065
$C_{1,19}$	0.9	4	46	46	0	0
		3	37	46	-8.4962	72.1851
		2	32	38	-5.8101	33.7572
		1	28	33	-4.8977	23.9872
		0	24	29	-4.582	20.995
					SSQ	150.925
					MSE	30.1849

Table 5 continued.

$C_{2,19}$	0.9	4	62	62	0	0
		3	76	62	13.5554	183.749
		2	87	74	12.423	154.332
		1	96	85	10.8975	118.756
		0	105	95	9.79015	95.847
					SSQ	552.684
					MSE	110.537
$C_{3,19}$	0.9	4	63	63	0	0
		3	75	63	11.7835	138.852
		2	83	73	9.76363	95.3285
		1	90	82	7.94082	63.0566
		0	96	89	6.74402	45.4818
					SSQ	342.718
					MSE	68.5437
$C_{4,19}$	0.9	4	65	65	0	0
		3	69	65	4.24711	18.0379
		2	71	69	2.09705	4.39761
		1	72	71	0.9344	0.8731
		0	72	72	0.33323	0.11104
					SSQ	23.4197
					MSE	4.68393
$C_{5,19}$	0.9	4	65	65	0	0
		3	69	65	3.42257	11.714
		2	70	68	1.4556	2.11878
		1	70	70	0.43224	0.18683
		0	70	70	-0.0862	0.00743
					SSQ	14.027
					MSE	2.80541
$C_{6,19}$	0.9	4	66	66	0	0
		3	62	66	-3.8958	15.1773
		2	59	62	-3.2908	10.8296
		1	56	59	-2.9467	8.68291
		0	54	57	-2.7869	7.76702
					SSQ	42.4568
					MSE	8.49136

Table 5 continued.

$C_{7,19}$	0.9	4	59	59	0	0
		3	48	59	-10.809	116.84
		2	42	49	-6.596	43.5076
		1	38	43	-5.0519	25.5213
		0	34	38	-4.4743	20.0195
					SSQ	205.889
					MSE	41.1778
$C_{8,19}$	0.9	4	35	35	0	0
		3	31	35	-4	16
		2	27	31	-4.4	19.36
		1	23	27	-4.44	19.7136
		0	19	23	-4.444	19.7491
					SSQ	74.8227
					MSE	14.9645
$C_{9,19}$	0.9	4	66	66	0	0
		3	61	66	-4.3663	19.0645
		2	58	62	-3.5488	12.5938
		1	56	59	-3.1145	9.69993
		0	53	56	-2.9165	8.50614
					SSQ	49.8644
					MSE	9.97287
$C_{10,19}$	0.9	4	66	66	0	0
		3	64	66	-2	4.00002
		2	62	64	-2.2	4.83997
		1	60	62	-2.22	4.9284
		0	58	60	-2.222	4.93729
					SSQ	18.7057
					MSE	3.74114
$C_{11,19}$	0.9	4	61	61	0	0
		3	77	61	15.9992	255.974
		2	93	75	17.5992	309.733
		1	109	91	17.759	315.383
		0	125	107	17.778	316.057
					SSQ	1197.15
					MSE	239.43

Table 5 continued.

$C_{12,19}$	0.9	4	61	61	0	0
		3	77	61	15.9153	253.296
		2	93	75	17.3667	301.601
		1	108	91	17.4118	303.17
		0	124	107	17.3392	300.648
					SSQ	1158.71
					MSE	231.743
$C_{13,19}$	0.9	4	66	66	0	0
		3	65	66	-0.5878	0.34551
		2	64	65	-1.3301	1.76921
		1	63	64	-1.6199	2.6242
		0	61	63	-1.7497	3.06153
					SSQ	7.80046
					MSE	1.56009
$C_{14,19}$	0.9	4	66	66	0	0
		3	61	66	-4.8333	23.3612
		2	58	61	-3.8	14.44
		1	55	58	-3.2765	10.7353
		0	52	55	-3.0414	9.25035
					SSQ	57.7869
					MSE	11.5574
$C_{15,19}$	0.9	4	59	59	0	0
		3	48	59	-10.809	116.84
		2	42	49	-6.596	43.5076
		1	38	43	-5.0519	25.5213
		0	34	38	-4.4743	20.0195
					SSQ	205.889
					MSE	41.1778
$C_{16,19}$	0.9	4	49	49	0	0
		3	40	49	-9.5564	91.324
		2	34	41	-6.162	37.9706
		1	30	35	-5.0058	25.0581
		0	26	31	-4.601	21.1695
					SSQ	175.522
					MSE	35.1044

Table 5 continued.

$C_{17,19}$	0.9	4	35	35	0	0
		3	31	35	-4	16
		2	27	31	-4.4	19.36
		1	23	27	-4.44	19.7136
		0	19	23	-4.444	19.7491
					SSQ	74.8227
					MSE	14.9645
$C_{18,19}$	0.9	4	52	52	0	0
		3	42	52	-10.274	105.563
		2	37	43	-6.4079	41.0606
		1	32	37	-5.0737	25.7425
		0	28	33	-4.5996	21.1563
					SSQ	193.522
					MSE	38.7045

Table 6

V_{SES}^{c+1} The Projected Values of All Cases (52 Cases) for the Following Future Period (2014) with the best value of α

Coding of Cases	α	V_{SES}^{c+1}
$C_{1,1}$	0.8	649
$C_{1,2}$	0.9	251
$C_{2,2}$	0.8	986
$C_{2,3}$	0.9	117
$C_{3,3}$	0.9	249
$C_{3,4}$	0.9	32
$C_{4,4}$	0.9	84
$C_{4,5}$	0.9	12
$C_{5,5}$	0.9	30
$C_{5,6}$	0.9	4
$C_{6,6}$	0.9	23
$C_{6,7}$	0.9	5
$C_{7,7}$	0.9	18
$C_{7,8}$	0.9	4
$C_{8,8}$	0.9	20
$C_{8,9}$	0.9	5
$C_{9,9}$	0.9	42
$C_{10,10}$	0.8	1077
$C_{10,11}$	0.9	301
$C_{11,11}$	0.8	710
$C_{11,12}$	0.9	62
$C_{12,12}$	0.9	148
$C_{12,13}$	0.9	20
$C_{13,13}$	0.9	65
$C_{13,14}$	0.9	10
$C_{14,14}$	0.9	29
$C_{14,15}$	0.1	6
$C_{15,15}$	0.9	21
$C_{15,16}$	0.9	2
$C_{16,16}$	0.9	19
$C_{16,17}$	0.9	3

Table 6 continued.

$C_{17,17}$	0.9	9
$C_{17,18}$	0.9	4
$C_{18,18}$	0.9	15
$C_{1,19}$	0.9	24
$C_{2,19}$	0.9	104
$C_{3,19}$	0.9	95
$C_{4,19}$	0.9	72
$C_{5,19}$	0.9	70
$C_{6,19}$	0.9	54
$C_{7,19}$	0.9	34
$C_{8,19}$	0.9	19
$C_{9,19}$	0.9	53
$C_{10,19}$	0.9	58
$C_{11,19}$	0.9	123
$C_{12,19}$	0.9	122
$C_{13,19}$	0.9	61
$C_{14,19}$	0.9	52
$C_{15,19}$	0.9	34
$C_{16,19}$	0.9	26
$C_{17,19}$	0.9	19
$C_{18,19}$	0.9	28

Appendix B

Making Projection

Table 1

Actual Number of Muslims in a Marital System for the Year 2014 for 18 Transient States

Transient States	Actual Number of Muslims in the Marital System for the Year 2014
M: Under 26	755
M: 26-30	1058
M: 31-35	309
M: 36-40	99
M: 41-45	39
M: 46-50	33
M: 51-55	34
M: 56-60	17
M: Above 60	47
F: Under 26	1281
F: 26-30	857
F: 31-35	169
F: 36-40	62
F: 41-45	40
F: 46-50	24
F: 51-55	26
F: 56-60	13
F: Above 60	15

Table 2

Projected Number of Muslims in the Marital System for the Following Future Year (2014) for Each of the Transient States Using MC Model

Transient States	Projected Number of Muslims in the Marital System for the Year 2014
M: Under 26	927
M: 26-30	802
M: 31-35	133
M: 36-40	26
M: 41-45	3
M: 46-50	3
M: 51-55	4
M: 56-60	9
M: Above 60	8
F: Under 26	1059
F: 26-30	514
F: 31-35	49
F: 36-40	20
F: 41-45	6
F: 46-50	4
F: 51-55	4
F: 56-60	2
F: Above 60	2

Table 3

Projected Number of Muslims in the Marital System for the Following Future Year (2014) for Each of the Transient States Using MC_{SMA} Model

Transient States	Projected Number of Muslims in the Marital System for the Year 2014
M: Under 26	624
M: 26-30	1171
M: 31-35	311
M: 36-40	71
M: 41-45	15
M: 46-50	8
M: 51-55	7
M: 56-60	10
M: Above 60	18
F: Under 26	1025
F: 26-30	953
F: 31-35	153
F: 36-40	46
F: 41-45	16
F: 46-50	9
F: 51-55	7
F: 56-60	3
F: Above 60	5