EXTENDED MULTIPLE MODELS SELECTION ALGORITHMS BASED ON ITERATIVE FEASIBLE GENERALIZED LEAST SQUARES (IFGLS) AND EXPECTATION-MAXIMIZATION (EM) ALGORITHM

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Abstrak

Pemilihan model automatik telah digunakan untuk merapatkan jurang antara pakar dan pengguna akhir sejak tahun 1960-an bermula dengan *Stepwise* dan baru-baru ini dengan *Autometrics* untuk satu persamaan. Pelanjutan *Autometrics* dalam pemilihan model ini juga dibangunkan untuk persamaan berganda dengan mengintegrasikannya dengan persamaan regresi seolah-olah tak terhubung (SURE) dan dianggarkan menggunakan penganggaran kuasa dua terkecil teritlak boleh-laksana (FGLS), yang dikenali sebagai algoritma *SURE-Autometrics*. Walau bagaimanapun, *SURE-Autometrics* tidak pernah dianggar menggunakan anggaran kebolehjadian maksimum (MLE). Oleh itu, dalam kajian ini, *SURE-Autometrics* ditambah baik menggunakan dua kaedah MLE iaitu kuasa dua terkecil teritlak boleh-laksana secara lelaran (IFGLS) dan algoritma pemaksimuman-jangkaan (EM), dikenali sebagai algoritma *SURE*(*IFGLS*)-*Autometrics* dan *SURE(EM)-Autometrics*. Kajian simulasi dan empirik dijalankan untuk mengesahkan prestasi dua algoritma tersebut. Dalam kajian simulasi, saiz sampel yang berbeza, kekuatan korelasi di antara persamaan, saiz model tanpa batas umum (GUMS), bilangan persamaan, paras keertian dan model spesifikasi benar digunakan untuk menilai peratusan dalam mencari GUMS yang sebenar. Manakala, dalam kajian empirik, dua set data empirik iaitu kadar pertumbuhan negara dan indeks kualiti air (WQI) dinilai menggunakan punca min ralat kuasa dua dan punca min ralat kuasa dua geometri, di mana 18 prosedur pemilihan model secara manual dan automatik dibandingkan. Keputusan simulasi menunjukkan bahawa prestasi algoritma *SURE(IFGLS)-Autometrics* dan *SURE(EM)-Autometrics* bertambah baik dalam keadaan sampel yang besar, korelasi yang kuat antara persamaan, GUMS kecil, bilangan persamaan yang kecil, paras keertian yang ketat dan di dalam model kosong (tanpa pembolehubah peramal). Keputusan empirik bagi kedua-dua algoritma berprestasi baik berbanding prosedur pemilihan model yang lain, terutamanya menggunakan data WQI di mana saiz sampel lebih besar dan mempunyai data yang berkualiti. Kesimpulannya, *SURE*(*IFGLS*)-*Autometrics* dan *SURE(EM)-Autometrics* boleh digunakan sebagai algoritma pemilihan model. Sebagai tambahan, kedua-dua algoritma adalah sesuai untuk meningkatkan prestasi prosedur pemilihan model automatik. Penemuan umum menyokong idea bahawa prosedur automatik mengatasi prosedur manual.

Kata kunci: Pemilihan model, persamaan regresi seolah-olah tak terhubung, anggaran kebolehjadian maksimum, kuasa dua terkecil teritlak boleh-laksana secara lelaran, algoritma pemaksimuman-jangkaan.

Abstract

Automated model selection has been used to bridge the gap between experts and end users since 1960s starting with *Stepwise* and recently with *Autometrics* for single equation. This extension of *Autometrics* for model selection was also developed for multiple equations by integrating it with seemingly unrelated regressions equations (SURE) and estimated using feasible generalized least squares (FGLS), known as *SURE-Autometrics* algorithm. However, *SURE-Autometrics* has not been estimated using maximum likelihood estimation (MLE). Therefore, in this study *SURE-Autometrics* is improvised using two MLE methods, which are iterative feasible generalized least squares (IFGLS) and expectation-maximization (EM) algorithm, named as *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* algorithms. Simulation and empirical studies are conducted in validating the performance of the two algorithms. In the simulation study, different sample sizes, strength of correlation among equations, size of general unrestricted model (GUMS), number of equations, significance levels and true specification models are incorporated by evaluating the percentages of finding the true GUMS. While in the empirical study, two empirical data sets which are national growth rates and water quality index (WQI) are assessed using root mean square error and geometric root mean square error where 18 models selection procedures of manual and automated approaches are compared. The simulation results indicated that performance of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* algorithms improved in conditions of large sample, strong correlation among equations, small GUMS, a smaller number of equations, tight significance level and in an empty model (without predictor variables). The empirical results for both algorithms performed well as compared to other models selection procedures, particularly using WQI data where the sample size is bigger and has good quality data. In conclusion, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* can be used as models selection algorithms. Additionally, both algorithms are suitable in improving performance of automated models selection procedures. General findings support the idea that automated procedures surpass the manual procedures.

Keywords: Models selection, seemingly unrelated regression equations, maximum likelihood estimation, iterative feasible generalised least squares, expectation-maximization algorithm.

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Declaration Associated with the Thesis

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List of Abbreviations

ARCH	autoregressive conditional heteroscedasticity
DGP	data-generating process
e.g.	for example
EM	expectation-maximization
et al.	and others
etc	and so forth
FGLS	feasible generalised least squares
FIML	full information maximum likelihood
IFGLS	iterative feasible generalised least squares
GDP	gross domestic product
GETS	general-to-specific
GLS	general least squares
GRMSE	geometric root mean square error
GUM	general unrestricted model for single equation
GUMS	general unrestricted model for multiple equations
i.e.	that is
i.i.d	independent and identical
LM	Lagrange multiplier
LR	likelihood ratio
ML	maximum likelihood
MLE	maximum likelihood estimation
MC-QLR	Monte Carlo quasi-likelihood ratio
MI	multivariate independent
OLS	ordinary least squares
RMSE	root mean square error
SUM	specific unrestricted model
SURE	seemingly unrelated regression equations
SURR	seemingly unrelated restricted residuals
SUUR	seemingly unrelated unrestricted residuals
WQI	water quality index

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

A model is generally used as an instrument in understanding a concept or phenomenon of the real world. In other words, a model plays a vital role in assessing interactions among variables involved and in forecasting the effect of changes in some variables towards the future course of others. A good model is therefore needed to test the necessary hypothesis and forecast accurately which lead to good decision making for the future either for planning or controlling (Hendry & Pretis, 2016).

Some criteria have been identified in judging a good model which includes the parsimony of the model. This is important as a model is functioned to capture the essence of an event. Hence, a simpler model is more favoured compared to unreasonably large one when other things are equal (Zucchini, 2000). Apart from being parsimonious, goodness of fit with a high adjusted R-square where the sample data fitted to the model relatively well is also advantageous in a statistical modelling.

Any models should be consistent with the theory related. Significant variables are supposedly retained, while the irrelevant ones are to be excluded. The coefficients in model are expected to have right signs, especially when the model is used for forecasting. This predictive power of the model, which is referred to the capability of a model to produce testable predictions, is also taken into account (Harrell, 2015). One way to examine this is through comparison of the model's forecast with experience. Practically, these criteria need to be considered critically in order to select the right model so as accurate estimation with consistent and reliable forecasts is attained.

Basically, the model is determined through an identification and specification of the variables first. The modeller will identify the variable that will be forecasted, and all the potential variables that are related to it. However, model simplification may lead to difficulties in determining which set of predictors that best explain the observed responses because there could be more than one possible specification for one modelling situation. Therefore, model selection which is a procedure of choosing an adequate model instead of making a random choice of model is being done. It involves the inclusion or removal of variables until some termination criterion is satisfied (Kadane & Lazar, 2004).

The modelling selections process begins with an estimation of a model that initially specified by the modeller. Then it is re-specified according to the results of hypothesis testing of single parameters to determine significant variables, or diagnostic checking of model's assumptions (Greene, 2012). Sometimes the modellers only implement the diagnostic tests for the initial model or the final model. This whole process basically can be done manually or automatically. In manual selections procedure, the decision on how the model should be re-specified is determined by the modellers. For instance, relevant or irrelevant variables can be determined with the use of *p*-value through significance or alpha level which has been set in prior, normally at 0.05 (5%). If a *p*-value is less than 0.05, it is concluded that result is significant and the null hypothesis is not accepted.

In model building, tacit or personal knowledge is normally used, including by econometricians and applied economists, where theoretical and judgemental knowledge are combined together in their studies. Senker (2016) and Ryan and O'Connor (2013) emphasized the pivotal roles of tacit knowledge acquired and shared during scientific experiments including data assessment, as well as methodology in using the tools such as statistical techniques. Only learning experience would provide the right skills in formulating scientific problems and develop approaches in finding the answers.

This intuitive judgment in manual model selection however is very difficult to master. Different researchers may come up with different modelling paradigms or different strategies adopted in the model construction process. Consequently, several different models are produced for a given data set even though they are within the same methodological approach (Magnus & Morgan, 1999). Out of the several models produced, there could exist models with weak predictive accuracy which then results in questionable conclusions and lead to poor decision making. Hendry and Krolzig (2004) too criticized that manual search for a good model is a hard work. The iterative process in this kind of search is labour intensive since a lot of aspects must be taken into account at the same time and also possibilities of going around in circles.

The resultant different models indicate that variability may exist when the model specification process is manually employed. Ultimately this condition contributes to the existence of the gap between the experts and novice users, where these novice users may include beginners in statistical or econometric modelling who face difficulties in understanding the model itself. This knowledge discrepancy had motivated the needs for a more convenient and conclusive solution that is through automated approach. The use of expert systems methods appears to be an ideal answer in bridging the gap, including in model selection. This kind of system would have the feature to let the non-expert users to stick to a pre-determined "best-practice" path. Oxley (1995) described the progress to an expert systems approach in model selection, based around the computerisation of a specific rules-based decision procedure, is a natural advancement given currently accessible computing power and the demand for modelling facilities by non-expert users. This system is assumed to deliver performance at least as good as models developed by the experts, but in a much lesser time taken.

The work by Hoover and Perez (1999) was a pioneer in automated model selection. Moving from there, it had sparked a great interest from empirical modellers. They returned to the Lovell's analysis in 1983 and demonstrated that a general-to-specific (GETS*)* strategy which initiated by Hendry (1993) recovered the correct specification or a closely related specification most of the time. More details of GETS strategy can be found in Chapter Two. Hendry and Krolzig (2001) continued Hoover Perez effort by enhancing algorithm of data mining (Hendry & Krolzig, 1999, 2005) and produced *PcGets*, a programme meant for empirical modeller. *Autometrics* was then introduced by Doornik and Hendry (2007) applying the concept of tree search strategy.

Autometrics and its predecessor procedures however focussed on single equation only. There are many settings in which single equation models apply to a group of related variables. In these contexts, it makes sense to consider the several models jointly and treat them as a system of equations. The word "*system*" means that the equations are to be considered collectively, instead of individually. Examples of this kind of system include simultaneous equations model, seemingly unrelated regression equations (SURE) model and vector autoregression model. At macroeconomic level, the vector autoregression model is a specific form of SURE model.

The SURE model as suggested by Arnold Zellner in 1962, which consists of multiple equations, are generalizations of linear regression model. Even though the error terms are assumed to be correlated across the equations, every equation can be estimated individually. This is because each equation stands on its own with dependent variable and possibly different sets of regressors. Hence, these equations seemed to be unrelated due to their individualities, but still connected through the error terms.

SURE model has been widely applied in areas of econometric, financial and sociological modelling (Fildes, Wei, & Ismail, 2011; Srivastava & Giles, 1987; Zainudin & Nordin, 2017; Zellner, 1962). Nonetheless, it can also be used in other fields, for example forest ecology (Fu, Sharma, Wang, & Tang, 2017), accident research (Anastasopoulos & Mannering, 2016) and neuroimaging (Jahanshad et al., 2015). Improvements and extensions from the original equations had also prompted more researches (Fu et al., 2016; Kakarantza & Symeonides, 2017; Zhao & Xu, 2017). This means SURE model is pertinent in almost all aspects of life.

With the advancement of *PcGets* and *Autometrics* in automated model selection, Ismail (2005) and Yusof (2016) had come out with automated models selection algorithms for multiple equations, namely *SURE-PcGets* and *SURE-Autometrics* respectively. These two algorithms have exploited the advantages of automated models selection and system of equations in SURE model concept.

The development of *SURE-PcGets* by Ismail (2005) primarily focused on an extension of *PcGets* algorithm for multiple equations of SURE model. The basic concept of this algorithm is combining the selection stages in *PcGets* with SURE model. The *SURE-PcGets* selections steps are similar to those of *PcGets*, but the differences are in the first step where the general unrestricted model (GUM) is formulated and the testing for contemporaneous correlation disturbances. A feasible generalized least squares (FGLS) estimator was used in *SURE-PcGets* due to its efficiency in a SURE model as shown by Zellner (1962), in which regressors were stationary and errors were independent and identical (i.i.d) over time. Correlation information among individual regression equations were exploited in achieving this efficiency. Meanwhile, the models used in evaluating the algorithm were based on Hoover and Perez (1999). The simulation results proved the success of implementing the algorithm in identifying the true model. Comparing with other selections methods based on error measures, *SURE-PcGets* was however outperformed by a more personalised approach when using problem data.

By adapting the work of Ismail (2005), Yusof (2016) developed another algorithm for SURE model using *Autometrics*, known as *SURE-Autometrics*. Similar to *SURE-PcGets*, *SURE-Autometrics* too had utilized FGLS as the estimation method of the SURE model. Two approaches had been considered in that study, which were (i) model selected by ordinary least squares (OLS) weighed against model selected by FGLS and (ii) model selected by OLS and then estimated by FGLS against model selected and estimated by FGLS simultaneously. The first approach was aimed to see how strongly the performance of single equation estimation and system estimation differ, whereas the second approach intended to show the differences between single and simultaneous process of selections. Yusof (2016) showed that *SURE-Autometrics* managed to exclude more irrelevant variables when the correlation strength was high, if the model was to be selected and estimated simultaneously compared to separate selections and estimation in the pre-search reduction.

The use of FGLS in both *SURE-PcGets* and *SURE-Autometrics* has signified the customary role of least squares method in SURE model. The principle of least squares in this type of model provides us with two major procedures. One is OLS that is to estimate the coefficients of every equation in the SURE model. Another is joint estimation, which may also be done by adopting generalized least squares (GLS) estimator. An obvious difference is the first one does not concern the joints in SURE model making it as unbiased but inefficient estimator. A model that consists of independent regression equations is assumed here. Meanwhile, the second approach takes the combination in SURE model into account and information on non-zero correlations between disturbances of the different equations of the model is utilized (Greene, 2012).

This information from correlations between disturbances is an essential part of SURE specification. It is expected that this extra information should improve the efficiency of GLS estimation method compared to OLS, at least asymptotically. However, the GLS estimator is not a feasible estimator since the elements of variance-covariance matrix of disturbances, Σ for the whole equations is not known (Srivastava & Giles, 1987). Whenever Σ is not known, two-stage method based on OLS residuals, which is also known as FGLS estimation, is considered a suitable choice. It is also recognized as Zellner's SURE estimator. FGLS estimator is also known as seemingly unrelated restricted residuals (SURR).

Even though estimation efficiency is expected to be realized through FGLS, this method may also generate other complications. Early finding by Kmenta and Gilbert (1968) showed that FGLS was not always efficient in small samples, while Kuan (2004) raised the issue of not knowing the finite-sample properties of FGLS estimator. Obtaining statistical inferences from FGLS estimation results is not easy since an FGLS estimator is not known for its unbiasedness or efficiency compared to the OLS estimator. In addition, a suitable FGLS estimator is accessible with more conditions on Σ. But, the performance of the FGLS estimator may be jeopardized if these simplifying conditions are wrongly executed.

FGLS and seemingly unrelated unrestricted residuals (SUUR) estimators as proposed by (Zellner, 1962, 1963) had been studied together with OLS estimators in terms of the efficiency properties. These three estimators were examined, but it turned out that none of the estimators had consistently stood out among others. From this outcome, Srivastava and Giles (1987) had questioned whether FGLS and SUUR estimators could be made better or different estimator could be created. They even recommended iteration with respect to the choice of an observable replacement matrix for Σ.

Apart from two-stage methods based on OLS residuals which is the FGLS method, maximum likelihood (ML) has been used to find system estimators (Bivand & Piras, 2015; Griffiths, Skeels, & Chotikapanich, 2002; Lai & Huang, 2013). According to Drton (2006), the parameters of a SURE model can be estimated efficiently, i.e. with small variance, by maximizing the likelihood function. The ML estimator of unidentified coefficients comprises coefficient values which maximize likelihood function.

In order to find ML estimator in the context of SURE model, repeated measures analysis of two-stage GLS estimation is used to obtain regression parameters and variance-covariance matrix. The ML estimator of the regression parameters can be obtained by performing the two-stage estimation iteratively. It is through iterative procedure that yields iterative FGLS (IFGLS). The idea of IFGLS was proposed by Zellner (1962) in order to create robust standard errors or parameters with better estimates. IFGLS also has the similar asymptotic properties as the FGLS (Kmenta & Gilbert, 1968). Oberhofer and Kmenta (1974) showed that for certain models, including SURE model, one can iterate back and forth between two estimators. Thus, the ML estimators are obtained by iterating to convergence between ML estimator of β and Σ. The process may begin with OLS estimator.

With the supplementary iteration in IFGLS, more information on how the estimates change can be gained through this simultaneous method and at the same time the efficiency of estimators can also be ascertained. One factor which influences the extent of estimation changes from one iteration into another is the level of multicollinearity among the explanatory variables within a certain equation and also among the explanatory variables of the different equations. Therefore, iterative estimates of IFGLS can indirectly show how much multicollinearity is present and therefore enables modeller to make decisions on utilizing the simultaneous estimates or only the one-by-one equation estimator like OLS (Draper & Smith, 2014; Telser, 1964).

Dufour and Khalaf (2002) used several estimators including IFGLS for experiments of exact tests for contemporaneous correlation of disturbances in SURE, while Beasley (2008) too employed IFGLS for solving path analysis model using SURE.

Phillips (2010, 2014) had utilized the use of IFGLS in SURE model focusing on dynamic panel data model. Matched panel data using IFGLS was also investigated by Nilsen, Raknerud and Skjerpen (2016). Even until recently, the iterative character in IFGLS still managed to improve its performance in seemingly unrelated system of econometric equations (Kakarantza & Symeonides, 2017). However, the use of MLE method in this study is not just limited to IFGLS, but also with the implementation of Expectation-Maximization (EM) algorithm.

EM algorithm is generally known to obtain ML estimator of an unrestricted mean vector and covariance matrix. The EM algorithm by Dempster, Laird, and Rubin (1977) is rooted on the idea that a complicated model for some data observed can be formulated in a more direct way with an additional hidden or latent data. If the additional data were observed, the computation of the ML estimator would be quite simple. Each iteration of the EM algorithm is made up of two iterative steps, the Expectation (E) step and the Maximization (M) step. The E-step takes the expectation of the log-likelihood function with respect to the additional data. The M-step maximizes this expected log-likelihood with respect to the parameters (Hoogerheide, Opschoor, & van Dijk, 2012).

The iterative SURE two-stage estimation procedure is shown to be equivalent to the EM algorithm proposed by Jennrich and Schluchter (1986) and Laird, Lange, and Stram (1987) for repeated measures data (Park, 1993). The use of EM algorithm was also applied in other scopes of SURE model (Galimberti, Scardovi, & Soffritti, 2016; Hoogerheide et al., 2012; Huang & Sloan, 1987; Li, 2014; Wang, 2010).

The EM algorithm has some interesting properties which makes it relevant in numerous applications. The algorithm is numerically steady with the increasing likelihood from every iteration and is considered simple in terms of its implementation. Due to its simplicity, each iteration's process is basically low despite the total number of iterations which could be larger than other procedures. Moreover, it can also be exploited in estimation of missing data and consequently puts it on par or even better than other estimation methods (Ng, Krishnan, & McLachlan, 2012). In terms of the computing hours, the EM algorithm offers significantly fewer hours, thanks to the absence of complicated derivatives and the Hessian of likelihood function (Sohn, 2016). This is supported by van Ryzin and Vulcano (2017) who proved that EM algorithm is between twice and six times faster and also produced equal quality of the estimates, in comparison with direct MLE methods.

One approach of MLE is through direct maximization by inserting a special form of matrix of unknown constants in the log-likelihood function for generalized regression model. The model, however, is re-examined in a somewhat different formulation (Greene, 2012). Furthermore, this kind of maximization by using gradient methods is sensitive to starting values, besides not being able to find the maximizer if there are too many parameters. The method may even stop before finding the maximizer in situations where the likelihood function is very flat near its maximum (Park & Lee, 2012). Thus, direct maximization is not the MLE chosen method for this study.

In addition, MLE for system of equations can also be executed using full information ML (FIML), which analyses full set of equations at one step. The FIML method presumes that errors are normally distributed and likelihood function is maximized with condition that restrictions on all parameters in a model are considered, not only in the equation being estimated. Nevertheless, the FIML involves more computational process and is more complicated because of the Jacobian matrix that appears in the log-likelihood function (Hendry, Neale, & Srba, 1988). Moreover, FIML is more suitable in simultaneous equations model, where at least one of right-hand side variables will be endogenous, and consequently error term will be correlated with at least one of those variables. Unlike in SURE model, regressor variables are assumed of being uncorrelated with error term (Dagenais, 1978). Therefore, the FIML is not appropriate to be employed in this analysis for system of SURE models selection.

1.2 Problem Statement

A system of SURE models usually requires higher computational technology than a single equation, especially when the calculations involve iterations. Nevertheless, with technological advancement available nowadays, computational burden is no longer a major concern for researchers. This aspect similarly goes for selecting best regression model automatically. Ismail (2005) showed that *PcGets* can be extended for multiple equations, particularly SURE model, using *SURE-PcGets*. Nevertheless, *PcGets* only conducts multi-path searches, but lack of tree search strategy as found in *Autometrics*. The emergence of *Autometrics* after a few years since development of *PcGets*, with the former searches for more candidate models had prompted Yusof (2016) to introduce *SURE-Autometrics* with aim of selecting SURE model using *Autometrics*. These first attempts using *PcGets* and *Autometrics* to merge the joint estimation and models selection of the entire SURE system has opened up more possibilities in the research of SURE itself. However, the algorithms have only used FGLS estimator in the modelling process.

There have been some issues with FGLS including being less efficient mainly whenever correlation coefficients of disturbances in SURE model are weak, in small samples and situations where every equation has same regressors (Srivastava and Giles, 1987). Moreover, if there is even one error in one equation, estimations in all equations tend to be biased since the equations are managed as a system (Murray, 2006). These factors indicate that FGLS is not essentially an appropriate estimator for SURE models. Instead, the parameters of a SURE model can be estimated efficiently through MLE (Drton, 2006). Unlike FGLS, MLE may overcome the problem when all SURE equations in the model cannot be estimated simultaneously. ML estimator is known for being sufficient, consistent and efficient Myung (2003). Therefore, two MLE methods are employed in this study, which are IFGLS and EM algorithm.

The use of IFGLS gives benefits in obtaining further information on the change of the estimates, which comes from its additional iterations and also display the presence of multicollinearity (Draper & Smith, 2014). Furthermore, IFGLS estimates can facilitate to enhance forecasting performance in simulations based on root mean square error (RMSE) (Phillips, 2010). At the same time, EM algorithm too is beneficial in models estimations, especially for missing data cases (Ng, Krishnan, & McLachlan, 2012). The simplicity in EM algorithm without having any complicated derivatives and the Hessian of likelihood function contributes to low iterations process (Sohn, 2016). Thus, every iteration in EM algorithm is found to be numerically steady with the increasing likelihood. Therefore, IFGLS and EM algorithm are chosen in this study due to their advantages and promising results in previous researches.

All in all, this study is directed towards extending the *SURE-Autometrics*, but with modification on estimation method used in models for the whole algorithm. Two MLE methods i.e. IFGLS and EM algorithm are embedded in automated models selection algorithms for multiple equations of SURE model, resulting two different modified algorithms. These ML estimators are given concentration because of their widespread use in statistical inference. The general good properties of ML estimators would be the central factor of proper model building and are likely to contribute to successful automated models selection. In addition, it is also crucial to measure performance of models selection procedures of different estimation methods from both manual and automated approaches. The one that performs the 'best' could be largely due to appropriate estimation method chosen. With the 'best' final model selected, this means the tasks of including relevant variables and removing irrelevant ones have been simplified.

1.3 Objectives of the Study

The essential purpose of this study is to present new estimation methods based on ML approach for *SURE-Autometrics*, which is an automated models selection algorithm for the application of SURE within the GETS strategy. In addition, this study shows different ways of estimating SURE model under different conditions. The objectives of this study are:

i. to develop two extended models selection algorithms for SURE model using two different MLE methods, which are IFGLS and EM algorithm. The newly-built algorithms are known as *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*, respectively.

ii. to evaluate the performance of the *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* using simulated data under different conditions.

iii. to make comparisons of SURE model selection procedures using empirical data, including the use of other packages and personalized manual selections.

1.4 Significance of the Study

This study intends to add new knowledge in estimation methods for automated models selection for SURE model. In particular, two different MLE methods are embedded into two different algorithms in this study. With regard to model selection approaches, this study provides modellers, especially end users with a more convenient selections procedure by steering them through algorithms, instead of relying on manual approaches which may lead to different outcomes. By having these automated selections, the gaps between applied and theoretical modelling as well as between the experts and end users could be bridged accordingly. This will eventually lessen the dependency on tacit knowledge.

Furthermore, this study also benefits modellers with these alternative estimation methods. They are presented with recommendations on appropriate estimation methods in automated multiple equations modelling, depending on their features of data and research setting. Researchers in various fields will have more options to choose from in estimating models by utilizing more structured algorithms. Therefore, it is hoped that this study will be an added value to the body of knowledge in statistics and econometrics.

1.5 Thesis Outline

This thesis comprises six chapters as described below:

Chapter One gives a general idea on model selection approaches, GETS automated procedure, introductions to SURE model and its methods of estimation. This chapter is divided into sections of background, problem statement, objectives and significance of the study along with outline of thesis.

Chapter Two reviews the pertinent literature on model selection and their specification strategies, and how the automated selection had evolved throughout the time. Some issues in terms of single against multiple equations selections and explanation on SURE model with its estimation methods are also looked into.

Chapter Three contains all methods and framework involved, including *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* development in order to achieve the first objective of this study. Description of these algorithms has been split into five phases accordingly.

Chapter Four consists of details on simulation process in order to assess the performance of the algorithms, so that the second objective of study is accomplished. Data generation process and simulations conditions, which are different sample sizes, strength of correlation among equations, size of general unrestricted model (GUMS), number of equations, significance levels and true specification models, are elaborated.

Chapter Five shows application of the algorithms by exploiting two sets of empirical data with the aim to fulfil the third objective of study. Different manual and automated models selection procedures are compared in terms of their forecasting performances based on error measures by using *GAUSS 15*, *PcGive 14* and *IBM SPSS Statistics 21* packages.

Chapter Six ends the thesis with summary of study, guidelines on choosing SURE model, limitations of study and suggestions for future researches.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter contains review on topics that are related to this study. The structure begins with Section 2.2 on models selection. Section 2.3 explains the specification strategies. Section 2.4 describes the automated single equation selection procedures within GETS modelling. Then, Section 2.5 provides the SURE model and its estimations using least squares and MLE, while Section 2.6 touches on automated multiple equations selections. Finally, Section 2.7 presents the summary.

2.2 Model Selection

A model which fits data better than other models is a model that encapsulates original process and consequently represents a close approximation to the true model that generated the data. However, whenever there is more than one model to be considered, a model's goodness of fit only is not sufficient. Hence, selections decision needs to be based on its capability to generalize, which is defined as a model's ability to fit not only current data but also to forecast future data (Myung, 2003).

The selections decision remains the same regardless of the complexity of models. The fundamental process is difficult to be identified, besides having numerous variables. Thus, model selection is a way in dealing with this situation. Myung (2000) and Zucchini (2000) explained the elementary ideas on model selection. Other aspects of model selection were also investigated, including Müller, Scealy, and Welsh (2013) who discussed model selection in linear mixed models. Castle, Qin, and Reed (2013) reviewed model selection algorithms, while Doornik, Hendry, and Cook (2015) explored model selection in big data. Piironen and Vehtari (2017) compared predictive methods in Bayesian model selection.

Model selection too provide some issues. Information criteria, stepwise regression and shrinkage methods are selections procedures that intend to select a set of relevant variables from a candidate set. These selections include procedures derived from penalized model fit: (e.g.) the C_p criterion of Mallows (1973) and various information criteria, such as Akaike (1973), Schwarz (1978), Hannan and Quinn (1979), and Chow (1981). On the other hand, these procedures do not guarantee congruency which may lead to a misspecified model (Bontemps & Mizon, 2003). Stepwise regression is also another popular method, but it is inclined to negative reliance since it depends on the path and does not have a high success rate of locating the data generating process (DGP). Similar findings were found by Berk (1978) in forward and backward selections. For shrinkage techniques, it is not progressive since the choice rule does not consider variables elimination.

Meanwhile, Magnus and Morgan (1999) criticised that manual modelling may conclude to different end models as a result of difference in views and interests, added with numerous methods used and various ways of researching. Thus, all these factors give influence in deciding the right variables. The dispute among modellers is not limited to the final outcomes, but also on their own opinions and reasoning for intermediate steps taken in modelling.

These intermediate steps are essential since they represent the methodology, assumptions, besides technical skills. However, the findings of empirical experiments are typically reported with lack of clear modelling process. Therefore, practitioners or end users find it difficult to understand the methodology without referring to other theoretical articles or application examples. In the end, they may come up with different clarifications. Hendry (2000) discussed the difficulties such as ‘measurement without theory’ (Koopmans, 1947), ‘ignoring selections effects’ (Leamer, 1978) and ‘lack of identification’ (Faust & Whiteman, 1997). Yet, they are based on hypothetical arguments only.

Krolzig and Hendry (2001) highlighted issues of cost of search and cost of inference in model selection procedures. The cost of search arises when an initial model is more general than required. In order to lessen this cost, any model selection process must be avoided from getting stuck in a search path that firstly accidentally deletes relevant variables. Thus, it is very important to explore multiple paths. Meanwhile, costs of inference are widely known as errors of Type I for incorrectly rejecting a true null hypothesis and errors of Type II for failing to reject an incorrect null. These costs are determined by the problem itself, where a complete model is used initially but it is not known to be true, so specification and misspecification tests are applied. Therefore, costs of inference are inevitable (Hendry & Krolzig, 2005).

In addition, model selection only chooses one model among all contenders, but abandons the remaining of the models searched. Difficulties may arise when more variables are added into the model with purpose of better forecasting, yet efficiency is seen more in models with fewer variables. Also, different estimation criteria such as Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) would tend to prefer different model selected (Salaki, Kurnia, Gusnanto, & Mangku, 2015).

Flaws in selecting the right model sometimes may be rooted from the problems in modelling itself. Harrell (2015) discussed these problems involving important variables not collected resulting in poor modelling, subjects in data set do not characterise population, level of complication permitted in model and managing large number of variables. Due to the given issues, a proper model specification strategy is required since model selection procedure starts with high dimensional GUM which may contain significant particular determinants, various small relevant consequences, and irrelevant variables as described in Castle, Doornik, and Hendry (2013).

2.3 Specification Strategies

When prior specification of a potential relationship is not known, data evidence is vital to define the relevant from irrelevant variables. However, there have been cases where either the relevant variables are absent from the model or the irrelevant ones are incorporated into the model, or even both. This kind of misspecification leads to biasness in the remaining parameter estimators. Moreover, a model may also fail diagnostic tests. The effects of these misspecifications can be seen in estimations, the characteristics of inferences and the forecasting precision too (Asteriou & Hall, 2016). A misspecified model would not reflect true performances of any assessments of model selection procedures. Only procedures that consistently yield models closest to true models and able to choose more relevant variables compared to other procedures are deemed worthier.

In general, there are several model selection strategies. However, only two strategies are discussed here. The first strategy is the specific-to-general modelling. One popular method is stepwise regression forward selections that begins from an empty model and then continued by including variables one by one. The significance of each variable in the model is rechecked every time a new variable is entered. Variables which are not significant would be removed from the model and the model is then refitted. The procedure continues until the model cannot be improved significantly by adding another variable (Wang & Jain, 2003).

Hendry and Krolzig (2002) highlighted several problems of specific-to-general modelling. First, there is no clear stopping point for an unordered search. The number of tests to be done and the reliability are not apparent. Thus, there is no control offered over the significance level of testing. Second, non-congruency often does not happen in simple models. Standard tests are considered invalid until a model sufficiently describes the data. Therefore, it is unreasonable to leave out testing by hoping that a model is valid. Thirdly, as specific-to-general is a divergent branching process, other routes start to multiply. These routes initiate more potential generalizations which contribute to model growth depending on the paths and order selected. Fourthly, it is uncertain how to continue when a test detects a problem. Adopting the alternative hypothesis of test which was rejected may put the problem into bigger risks. Finally, further restrictions are not necessary if a model demonstrates signs of misspecification.

The second strategy is contrast with previous strategy, which is called general-to-specific (GETS) modelling. This is further discussed in London School of Economics or Hendry framework by Hendry (1993). A basis for this method is from stepwise regression run in reverse, where a general model which consists of all related variables is reduced by removing the insignificant ones and the procedure ends when no more variables can be removed from the model. This strategy is related to the theory of reduction by Hendry (1995), which explains the procedure applied to the DGP to attain the local DGP.

GETS modelling begins with a broad specification and then searching over the space of possible restrictions to find the most parsimonious one. At every reduction step, the statistical properties of the errors are examined, the validity of the reduction is tested statistically both against the forerunners and the general specification, and encompassing is investigated against all otherwise satisfactory alternative specification. The GETS modelling ensures that the space of other specifications is fully searched, minimizing the risks that pertinent competing specifications are overlooked, and making sure that information is not lost relative to the general specification (Hoover, 2006). White (1990) demonstrated that with enough thorough testing, the selected model will converge to the DGP.

2.4 Automated Single Equation Selection

Granger and Hendry (2005) believed automated approach can be a formal way in bridging the gap, even though experts admitted that there is no single clearly best way of approaching the question of how to specify an empirical model (see among others, Granger, 1999; Hendry, 1980; Magnus, 1999). Many automated selections have been developed, among others is the Relevant Transformation of the Inputs Network Approach (RETINA) for selecting nonlinear representations by Perez-Amaral, Gallo, and White (2005). Also, Phillips (2003) developed method for choosing forecasting models, while Kurcewicz and Mycielski (2003) created for selecting cointegrating relations.

In this approach, an algorithm is developed through step-by-step procedure to formulate and test a model in order to choose the most appropriate one, especially in decision makings. As a consequence, different researchers should obtain the same results by following the same algorithm for a given data set. Subsequently, the steps in algorithm are translated into a series of instructions to produce an automated procedure. By doing this, it does not only act as a link between applied and theoretical econometric, but it also facilitates in diminishing the role of tacit knowledge as well as labour saving by eliminating computational burden especially if there are many potential candidate variables (Doornik & Hendry, 2007).

Hoover and Perez (1999) produced the first major study of GETS selection procedures. They had established the viability of automated model selection procedures through four main elements in their algorithm: GUM, multiple path search, back testing with respect to the GUM and diagnostic testing. The excellent properties of GETS modelling compared to other data mining approaches in Lovell (1983) were proven in the analysis. The procedure was indicated powerful with various unseen models in very large GUM were detectable and correct specification or a closely related specification was discovered most of the time.

Hoover and Perez's (1999) automated model selection algorithm were continued by Hendry and Krolzig (2001), who developed a computer program *PcGets*, consisting two major parts. First part is estimation and diagnostic testing of the general unrestricted model (Stage 0). Second part is selections of the final model by (i) pre-search simplification of the GUM (Stage 0), (ii) multi-path (and possibly iterative) selections of the final model (Stages 1 and 2), and (iii) post-search evaluation of the final model (Stage 3).

Hendry and Krolzig (2005) summarized the advantages of strategies in *PcGets* in econometric modelling, inclusive; (i) consistent model selection (Campos, Hendry, & Krolzig, 2003) (ii) elimination of irrelevant variables is done at user's level of significance (Hendry & Krolzig, 2003) (iii) relevant variables are retained with probabilities close to the theory maximum achievable when the DGP equation is known (Hendry & Krolzig, 2003) (iv) automated model selection saves user's effort and necessary if there are many potential variables and (iv) applications to several previous empirical studies either equal, or improve the authors' results (Davidson, Hendry, Srba, & Yeo, 1978; Hendry, Neale, & Ericsson, 1991).

Further to excellent performance by *PcGets*, Doornik and Hendry (2007) employed an algorithm known as *Autometrics*, which is part of PcGive version 12. The algorithm in *Autometrics* has similar characteristics as in *PcGets*. Yet, *Autometrics* applies a tree search method, with modifications on pre-search simplification and on the objective function. It makes use of one-step and multi-step simplifications along multiple paths subsequent to a tree search method. Additional checking uses diagnostic tests on the simplified models, whereas encompassing tests settle on multiple terminal models. Both analytical and Monte Carlo evidences show that the resulting model selection is comparatively non-distortionary for Type I errors (Ericsson, 2010).

Ericsson and Kamin (2009) who used *PcGets* and *Autometrics*, discovered that the programmes have contributed in robustness and consumed less time compared to manual modelling. In the case of model selection, using *Autometrics* with relatively tight significance levels and bias correction contributed to a successful approach in selecting dynamic equations even when originating from very long lags to prevent excluding relevant variables or dynamics (Castle, Doornik, & Hendry, 2011).

Unfortunately, *PcGets* and *Autometrics* only focus on single equation model selection. Thus, there is more potential in developing automated approach to cater selections for multiple equations model, for example SURE model.

2.5 The Seemingly Unrelated Regressions Equations (SURE) Model

SURE model has been used for researches of different cases and data types. Zainudin and Nordin (2017) used pooled OLS and SURE model in estimating their panel data sets to study the determinants of financial development in four countries, namely Malaysia, Singapore, Thailand and Philippines. In addition, the application of SURE model can also be found in investigation of Alzheimer's disease. Jahanshad et al. (2015) utilized SURE model to boost the power to recognise structural connections linked to cognitive scores. Neves, Fernandes, and Veiga (2015) extended a multivariate time series model using SURE model framework to forecast longevity gains.

One prominent advantage of using SURE model over single equation estimation is that estimator $\hat{\beta}_i$ allows for correlation between ε_i and other disturbance vectors. This is the result when information on regressors which are not in *i*th equation but still in the system is utilized in SURE model. The efficiency is higher if disturbances in different equations are correlated at a given point in time but are uncorrelated over time. This is known as contemporaneous correlation (Greene, 2012).

The SURE model has benefit of describing the dynamic composition of actual procedure since it considers all relationships occurred, i.e individual equation and interaction of all relationships. Consequently, further information may be gained from a set of equations compared to sum of single equations. Together, this information provide more knowledge on the causal relationships and constructions included, apart from making more accurate forecasts (Pindyck & Rubinfeld, 1998).

The motivations of SURE model are to gain efficiency in estimation by combining information from different equations and to impose or to test restrictions that involve parameters in different equations. Suppose the series of equations are,

$$\begin{aligned} y_{1t} &= \beta_{11}x_{1t,1} + \beta_{12}x_{1t,2} + \ldots + \beta_{1k_1}x_{1t,k_1} + u_{1t} \\ y_{2t} &= \beta_{21}x_{2t,1} + \beta_{22}x_{2t,2} + \ldots + \beta_{2k_1}x_{2t,k_2} + u_{2t} \\ &\vdots \\ y_{mt} &= \beta_{m1}x_{mt,1} + \beta_{m2}x_{mt,2} + \ldots + \beta_{mk_1}x_{mt,k_m} + u_{mt} \end{aligned} \qquad (2.1)$$

which can be written in general form,

$$\underset{T\times 1}{\mathbf{y}_i} = \underset{T\times k_i}{\mathbf{X}_i}\, \underset{k_i\times 1}{\boldsymbol{\beta}_i} + \underset{T\times 1}{\mathbf{u}_i}, \qquad i = 1, 2, \ldots, m \qquad (2.2)$$

where $\mathbf{y}_i$ is vector of T identically distributed observations for each random variable, $\mathbf{X}_i$ is a non-stochastic matrix of fixed variables of rank k_i, $\boldsymbol{\beta}_i$ is vector of unknown coefficients, and $\boldsymbol{u}_i$ is a vector of disturbances. It is assumed that the disturbances have a multivariate normal distribution with mean and covariance structure,

$$E(u_{it}) = 0, \; E(u_{it}u_{jt}) = \sigma_{ij}, \text{ and } E(u_{it}u_{jl}) = 0 \text{ if } t \neq l \qquad (2.3)$$

where i, j = 1, 2, …, m and t, l = 1, 2, …, T. This implies that disturbances are contemporaneously correlated (Kontoghiorghes, 2004) which equivalently as,

$$E(\mathbf{u}_i) = 0 \text{ and } \qquad E(\mathbf{u}_i\mathbf{u}_j') = \sigma_{ij}\mathbf{I}_T \qquad (2.4)$$

where $\sigma_{ij}^2 = \sigma_i^2$ is the variance of disturbances for the i^{th} equation if $i = j$, and $E(\cdot)$ is the expectation operator.

Further simplification of model (2.1) can be achieved in vector concatenation form by stacking the m vector equations together (Kontoghiorghes, 2004; Timm & Al-Subaihi, 2001). The T observations vectors $\mathbf{y}_i$ and the corresponding disturbances vectors $\mathbf{u}_i$ are stacked one upon another to form the single observation vectors $\mathbf{y}'_{(Tm\times 1)} = (\mathbf{y}'_1, \mathbf{y}'_2, \ldots, \mathbf{y}'_m)$ and $\mathbf{u}'_{(Tm\times 1)} = (\mathbf{u}'_1, \mathbf{u}'_2, \ldots, \mathbf{u}'_m)$, respectively. Furthermore, the design matrix for the m stacked observation vectors $\mathbf{y}$ is allowed to be the matrix $\mathbf{X}_{(Tm\times k)} = \bigoplus_{i=1}^{m} \mathbf{X}_i$ which is the direct sum of the individual design matrices $\mathbf{X}_i$ for the m equations where $k = \sum_{i=1}^{m} k_i$ is the total number of parameters over the m equations. It can be defined as,

$$\bigoplus_{i}^{m} \mathbf{X}_i = \mathbf{X}_1 \oplus \ldots \oplus \mathbf{X}_m = \begin{bmatrix} \mathbf{X}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{X}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_m \end{bmatrix} \tag{2.5}$$

The SURE model can now be in the following linear model,

$$\underset{Tm\times 1}{\mathbf{y}} = \underset{Tm\times k}{\mathbf{X}} \underset{k\times 1}{\boldsymbol{\beta}} + \underset{Tm\times 1}{\mathbf{u}} \tag{2.6}$$

The vector $\boldsymbol{\beta}$ in (2.6) formed in a same way as vector $\mathbf{y}$ and $\mathbf{u}$, is a stacked vector comprising stacked elements the parameter vectors $\boldsymbol{\beta}_i$ defined in (2.2). Therefore, equation (2.4) becomes,

$$E(\mathbf{u}) = 0 \text{ and } E(\mathbf{uu}') = \Sigma \otimes \mathbf{I}_T = \Omega_{Tm\times Tm} \tag{2.7}$$

where $\otimes$ denotes a Kronecker product, $\mathbf{I}_T$ is $T\times T$ identity matrix, and $\boldsymbol{\Sigma} = [\sigma_{ij}]$ is a $m\times m$ positive definite symmetric covariance matrix. Various works in the literature are focused on the setting of parametric models, mainly linear models (see Rocke, 1989; Srivastava & Giles, 1987; Timm & Al-Subaihi, 2001; Zellner, 1963). Nonetheless, there also studies contributed to analyze the SURE nonparametric regression models, such as Smith and Kohn (2000) and Xu, You, and Zhou (2008).

2.5.1 Least Squares Estimation

The main specification of SURE model can be characterised through its disturbances. The two cases of SURE model used here, as highlighted by Greene (2012) are:

(i) When the equations are contemporaneously uncorrelated disturbances, OLS is considered the best linear unbiased estimators since it is a classical multivariate linear regression model.

(ii) When equations have identical regressor but contemporaneously correlated disturbances, GLS estimator is more efficient given that OLS estimator does not take into account the correlation of disturbances. Nevertheless, covariance of disturbances Ω is not known for most of the time, thus GLS is not feasible. Zellner (1962, 1963) then suggested FGLS where Ω is substituted by a consistent estimator.

A survey by Srivastava and Giles (1987) showed that investigations on efficiency of FGLS relative to OLS were mainly done for models with two equations. However, it was also explained that FGLS can be less efficient than OLS when sample size is small and/or correlation coefficients of disturbances in SURE model are nearly zero. This similar outcome was earlier found by Kmenta and Gilbert (1968) and Revankar (1974), who demonstrated that OLS might be better than FGLS if correlations among disturbances are weak.

The drawback of FGLS had also been pointed out by Kruskal (1968) even though correlation between error terms exists. The efficiency of FGLS may disappear including situations where every equation has same regressors, as in vector regressive system. In an example of augmented dynamic panel data models, asymptotic variance-covariance matrix of FGLS estimator by Blundell and Bond (1998) was unknown, making it unclear to describe any general conclusions of their estimator. The FGLS does not even exist in high dimensional SURE model (Zhao & Xu, 2017).

FGLS can also pose risks when there is specification error, for instance omitted variables. Even only one error in one equation may bias the estimates in all equations since the equations are handled as a system. In contrast, this might not happen with OLS estimation as its application is on one equation at a time. Other equations which are not facing any specification errors will be estimated unbiasedly. Consequently, some researchers still do not favour FGLS estimation since misspecification of just a single equation can threaten all estimates in the system of equations (Murray, 2006). This means that FGLS may not necessarily and consistently an efficient estimator.

The shortcomings of FGLS were likewise found in a study by Saha, Havenner, and Talpaz (1997) on a stochastic production function estimation. Investigations of small-sample properties of FGLS and ML estimator revealed that FGLS had seriously understated the risk effects of inputs, besides providing biased marginal product estimates. On the other hand, ML estimator improves the probability of rejecting a false null hypothesis (i.e. power of a test) and also have significantly smaller mean square errors relative to FGLS for the parameters. Generally, ML estimator was found to continuously outperform FGLS estimates in both power and mean square errors for applied production studies. With the disadvantages shown by FGLS, it is therefore essential to employ other estimators, such as ML estimator in SURE model for this study. The two different MLE methods chosen, which are IFGLS and EM algorithm will be discussed in the next section.

2.5.2 Maximum Likelihood Estimation (MLE)

MLE is another method for parameter estimation, apart from least squares estimation. ML estimator chooses parameter values that provide the biggest possibility to observed data sample, which have been obtained from a population explained by the estimated coefficients (Cole, Chu, & Greenland, 2014; Paris, 2012). Myung (2003) had listed the optimal properties in estimation of ML estimator as (i) sufficiency (ii) consistency (iii) efficiency, and (iv) parameterization invariance. At the same time, MLE has been the basis of numerous inference methods in statistics, including being a precondition for chi-square test, Bayesian methods and model selection criteria such as Akaike information criteria (Akaike, 1973).

The computation of ML estimator begins by writing the model for t^{th} observation as

$$Y_t = \mathbf{x}_t^* \boldsymbol{\beta} + \mathbf{u}_t^{'} \qquad (2.8)$$

where $\mathbf{x}_t^*$ is the row vector of all different explanatory variables in the system and each $\boldsymbol{\beta}_i$ is the column vector of coefficients for the i^{th} equation. Assuming multivariate normally distributed errors, the log likelihood function is

$$L = -\frac{nM}{2}\log(2\pi) - \frac{n}{2}\log|\Sigma| - \frac{1}{2}(u)'(\Omega^{-1})(u) \qquad (2.9)$$

where $\Omega = \Sigma \otimes I$ and $\Sigma = E(u_t u_t^{'})$. ML estimates are found by taking derivatives of log likelihood function regarding β and **Ω**, setting them equal to zero and solving.

In analysing SURE model, ML estimator has its added advantage over FGLS which can be found in certain situations, such as when the whole SURE equations cannot be estimated jointly as usual. For instance, in singular SURE model when variance-covariance matrixes of disturbances for big equations are singular, for example in consumer demand equations. In order to solve this singularity, one of the equations in the system is to be dropped and other equations remained to be estimated jointly (Takada, Ullah, & Chen, 1995). The IFGLS estimators provide unchanged estimates regardless of which equation is dropped, unlike using FGLS (Negera, 2017). Thus, IFGLS is more suitable and more efficient in singular SURE model. Therefore, with benefits gained from implementation of ML estimator, this study employs two estimation methods which are IFGLS and EM algorithm in automated models selection of SURE model.

2.5.2.1 Iterative Feasible Generalized Least Squares (IFGLS)

ML approach has led to another contribution in estimation method by adopting iteration process. Kmenta and Gilbert (1968) had doubted whether IFGLS and ML estimator coincide or not since outcome from their Monte Carlo study demonstrated the equivalence of these two estimators empirically. They discussed procedure of iterating FGLS in their experiments. A new estimate of Ω resulting from a new set of residuals that was calculated using FGLS estimates of regression coefficients can be used for obtaining new estimates of the regression coefficients β.

Prior to the discovery by Kmenta and Gilbert (1968), this was seen as a possibility by Zellner (1962). He gave the impression that iterative estimates have same asymptotic properties as two-stage estimates. Moreover, IFGLS estimator of β is also consistent, given that FGLS estimator is a consistent estimator of β and $\hat{\Omega}$ is a consistent estimator of Ω. Estimates based on FGLS residuals would be expected to be more efficient than OLS residuals since the FGLS estimates are asymptotically efficient while OLS estimates are not. Consequently, it would direct one to anticipate the IFGLS estimates to be superior to FGLS estimates.

Furthermore, Dhrymes (1971) verified similarity of two procedures. Numerical equivalence was attained at every step as two iterations started from same initial estimate of Ω. If the procedure converges, the asymptotic distribution of the convergent iterates is also known, then it is ML estimator due to iteration of FGLS estimator from an initial consistent estimate of Ω. This is supported by Park (1993) who showed that when difference between two consecutive values of log-likelihood function was selected as convergence criterion, this iterative two-stage estimation converges to ML estimator of Ω and β. The equivalence of ML estimator and IFGLS estimators had been further proved by Beasley (2008) in the use of SURE model as a solution to path analytic models with correlated errors.

The idea of iterative estimates lies in acquiring estimators of coefficients by making use relations among residuals of regression equations. In other words, information that the residuals hold and how this information is employed determines efficiency of estimators. Additional rounds of iteration would provide further information on how the estimates change by employing simultaneous equation techniques. These changes, from one round to another round of iteration, would largely rely on the scale of multicollinearity. This multicollinearity is not only being looked among explanatory variables within one equation, but also among explanatory variables of different regression equations. Thus, these iterative estimates can show presence of multicollinearity and facilitate modeller to decide whether it is better to rely on the simultaneous or one-at-a-time least squares estimates (Draper & Smith, 2014; Telser, 1964). Modeller would thus be in better position in making decisions.

Phillips (2010) compared IFGLS and generalised method of moments estimators. For simulated data, he found that IFGLS was more favoured in terms of bias and root mean square error (RMSE). This means that IFGLS estimates are able to improve forecasting performance. From computational aspect, the use of IFGLS gave advantage to Nilsen et al., (2016) in their estimation on variables for a real wage equation. Convergence was attained after about 8 minutes using GAUSS programme, whereas STATA command took 2 hours and 13 minutes to converge on the same server using full sample. While IFGLS demonstrates valuable contributions in estimations, EM algorithm too is worth to be explored in this study. The application of EM algorithm has had an upsurge in recent years particularly with more advanced computer programmes, as found in the following section.

2.5.2.2 Expectation-Maximization (EM) Algorithm

The EM algorithm is used to get the ML parameters in cases where the equations cannot be solved directly. EM algorithms are considered as one of the most successful algorithms for MLE since they always guide the likelihood uphill by maximizing a replacement function for the log-likelihood. Iterative optimization of the function as represented by an EM algorithm does not essentially need missing data.

The procedure of EM algorithm is explained thoroughly in Dempster et al. (1977). Each iteration of the algorithm involves two steps, the E- or expectation step, and the M- or maximization step. An initial set of parameter estimates, θ_0 is presumed. The E-step consists of evaluating the conditional expectation of the log-likelihood for all the data. This is where the expectation is taken pertaining to the distribution of the 'missing' data, conditional on θ_0 and on the observed data. Next step is the M-step. The 'expected log-likelihood' produced in the E-step is maximized relating to θ, providing a new estimate θ_1. Procedure continues by going back to the E-step, with θ_1 replacing θ_0, and cycle through E- and M-steps until convergence is completed.

The application of EM algorithm also includes SURE model, where it is regarded as a repeated measures analysis. Regression parameters and variance-covariance matrix of SURE model can be estimated by using two-stage Aitken estimation. Park (1993) proved analytically that every iteration of the two-stage Aitken procedure is represented in terms of the EM algorithm as proposed by Jennrich and Schluchter (1986) and Laird et al. (1987) for repeated measures data and yields the ML estimator.

McLachlan and Krishnan (2008) and Ng et al. (2012) listed some attractive properties of EM algorithm in relation to other iterative algorithms such as Newton-Raphson and Fisher's scoring method for searching ML estimators. The advantages include:

i. The EM algorithm is numerically stable.
ii. The EM algorithm normally has consistent global convergence.
iii. The EM algorithm is easy as it relies on calculations of complete data.
iv. The EM algorithm is normally easy to programme due to no evaluation of the likelihood or its derivatives is involved.
v. Only a small storage is needed for EM algorithm and thus can be done on a small computer.
vi. Low cost for each iteration offsets the bigger number of iterations required for the EM algorithm compared to other procedures.
vii. Estimated values of the 'missing' data can be computed using EM algorithm.

Due to the extensive advantages of EM algorithm, many studies did employ EM in wide range of applications. Hirose, Kim, Kano, Imada, and Yoshida (2016) and Bańbura and Modugno (2014) used EM in factor models, while Galimberti and Soffritti (2014) for finite mixture models. Huang and Sloan (1987) estimated seemingly unrelated Tobit regressions via the EM algorithm.

Sohn (2016) found several appealing properties of EM algorithm in estimating integrated choice and latent variable model. The EM algorithm had decreased the computation by considerable amount of time since the algorithm does require any problematic numerical computation of derivatives or the Hessian of simulated likelihood function. It also prevents lack of empirical identification. EM algorithm was also exceptional in reducing sampling errors when sample size was small (250 and 500). Even in large samples of 1000 and 2000, EM algorithm was faster than its counterpart. It was thus recommended in parallel computing too. At the same time, van Ryzin and Vulcano (2017) showed convergence time for EM algorithm was between twice and six time faster than direct ML estimator and the quality of estimates are equally good. Their findings were also proven in experiments with large numbers of parameters or high degrees of censoring.

The EM algorithm had also been extended in other ways. One example is a parameter-expanded Monte Carlo EM algorithm by Li (2014) to develop a simple MLE algorithm for a commonly used multivariate sample selections model. Additionally, the algorithm only depends on quantities that are easy to simulate. This point is especially appealing when estimating the covariance matrix parameter since there are non-standard restrictions imposed onto it for identification. On the whole, EM algorithm is certainly an exceptional tool for MLE for any problems under study.

2.5.3 Independence Tests

The decision in applying equation by equation estimation or whole model estimation should rely on the covariances of disturbances. Both estimators are equivalent if the errors between equations are uncorrelated (Dufour & Khalaf, 2002) or *m* equations have identical regressors (Bhattacharya, 2004). Explicitly,

$$\sigma_{ij} = 0 \qquad \forall i \neq j \tag{2.10}$$

or $$\mathbf{X}_1 = \mathbf{X}_2 = \ldots = \mathbf{X}_m = \bar{\mathbf{X}} \tag{2.11}$$

Hence, numerous independence tests for disturbances of multivariate models have been proposed in the literature, including the locally best invariant test of Kariya (1981), the exact independent test (Harvey & Phillips, 1982) and multivariate independent (MI) test developed by Tsay (2004). Tests such as likelihood ratio (LR) test, Wald's test, and the Lagrange multiplier (LM) test derived by Breusch and Pagan (1980) are classified as classical. However, the LR and LM tests exhibited important size distortions which can be controlled by the Monte Carlo exact test based on Dufour and Khalaf (2002), known as Monte Carlo quasi-likelihood ratio test (MC-QLR). More details on this test can be found in Section 3.4.1.2.

2.6 Automated Multiple Equations Selection

Multiple equations model can be selected either through one equation at a time or all equations simultaneously. Automated approach has eased these selections in a fraction of time than the manual approach. One such algorithm is *SURE-PcGets* by Ismail (2005) where selections stages in *PcGets* were combined with SURE model. Testing of contemporaneous correlation disturbances was added in *SURE-PcGets* and the model formulation part in *PcGets* has changed.

The first stage in *SURE-PcGets* is the most vital part of the overall algorithm, where the construction of GUM would the main factor in ensuring the 'best' final model is selected. All stages that follow through after this starting point would be done automatically. There are typically more than one GUM due to the multiple equations in SURE. Each equation has the same basic GUM and Ismail (2005) denoted these GUMs formulated for all the equations as system GUM. This system GUM formulation part relies heavily on the modeller. Every equation is estimated using OLS estimation and the system GUM is then checked for its congruency through a series of diagnostic tests. Every time the system GUM failed in any tests, the system GUM has to be reformulated before resuming to the next stage. When the system GUM is confirmed congruent, pre-search testing of lags, removal of irrelevant variables and inclusive the relevant variables are done according to their absolute *t*-values. FGLS is then used for estimating the reduced model.

Since *PcGets* executes multiple search paths, in which a block search would remove more irrelevant variables, *SURE-PcGets* implements this type of reduction to obtain a simpler model. If variables removal produces a congruent model, this means a terminal model is discovered. The terminal is described as a model with significant variables, valid reduction from the system GUM and each equation is congruent. Nonetheless, the encompassing test begins if there is more than one terminal model.

A model is said to encompass another model when the former comprises information carried by the latter. Congruent candidate models are selected using encompassing tests, with each candidate is tested against union of the models. If the tests resulted in only a single model, that particular model is selected. But, if there a few models, then *SURE-PcGets* creates union of the remaining models. This union marks a fresh basis of a new search until no further changes in the union. Again, if there is more than one model found, information criteria are now used to select the final model. The final model is also checked for its sub-sample reliability in order to detect any falsely significant predictors. The models of full sample as well as of sub-samples are again estimated using FGLS. *SURE-PcGets* also runs a MC-QLR to test contemporaneous correlation disturbances.

With the success of *SURE-PcGets*, Yusof (2016) introduced *SURE-Autometrics* by taking advantage of another single equation selection algorithm, *Autometrics*. SURE model is utilized in this algorithm too. The tree search strategy embedded in *Autometrics* had allowed for paths to be explored thoroughly and increases chances for terminal models to be found by means of pruning, bunching and chopping reduction principles. This would eventually lead to the 'best' final model of all.

At the beginning, *SURE-Autometrics* has similar stage to *SURE-PcGets* with formulation of initial GUMS. Yusof (2016) added 'S' in the acronym 'GUM' as to reflect the multiple equations in the model, unlike the 'GUM' in *Autometrics*. Following this, the acronym 'GUMS' is also used here since SURE model is again utilized in this study. After the initial GUMS are set up, diagnostics tests are also run to inspect the model's congruency, which are tests of normal distribution, parameters' constancy, autocorrelation, unconditional homoscedasticity and lastly, conditional homoscedasticity. Besides that, a MC-QLR test to confirm on the correlation of errors among equations is carried out and FGLS estimation is again applied in this model.

The next stage that follows is where *SURE-Autometrics* starts to differ from *SURE-PcGets*. Second stage is the pre-search reduction procedures where reductions are done by focussing on the lag variables only. There are three types of lag reductions; (i) closed lag reduction to remove a group of variables starting with the largest lag (ii) common lag reduction to remove a group of variables with the highest significance value, and lastly (iii) common-*X* lag reduction to remove a group of variables with the highest significance value, but not lag dependent variable.

The most insignificant lag variables are removed strategically by following the order of lag reductions from type (i) until (iii), and again in reverse direction. Both strategies, according to direction, would each produce new reduced model which is less complex than the initial GUMS. The two models along with their unions are tested using encompassing test to be the current GUMS and again checked for congruency and correlation disturbance, besides being estimated using FGLS.

The reductions are continued with *SURE-Autometrics* employs tree search from *Autometrics* in which insignificant variables in every equation are removed. Three main reduction principles, comprise of pruning, bunching and chopping, are applied to form a simpler model. Under pruning reduction, if a variable is unsuccessful to be deleted or the deletion caused other problems, for example fails in diagnostic checking, the following path can be ignored (pruned). Bunching reduction attempts to delete a group (bunch) of variables, instead of a single variable at a time. These variables are arranged together as long as their individual insignificance merits this.

Lastly, when variables are mostly highly insignificant, the path can be permanently deleted (chopped). The reductions are indirectly contributing to computational efficiency since considerations are given only to terminal models, while ignoring any possible models with insignificant variables. Further reductions are made further with tree search for nested terminals. This time, the focus is on the removal of variables so that the correct significant variables ought to be in equations. Union contrast and terminal contrast are applied for this purpose and therefore the model will tend to be more reduced. Union contrast decides the different bunch from union of current set of terminal candidates and is used while the current GUMS changes between iterations. Meanwhile, terminal contrast determines the smallest bunch that would produce a model that is different from any of current terminals and is used at the end when the current GUMS is set. Finally, one 'best' model will be selected from all possible models that made through the reduction processes using information criterion. The selected final model is known as the specific unrestricted model (SUM).

2.7 Summary

Model building is not a simple process because of different kinds of model, a range of methodologies and existence of gap between applied and theoretical econometrics. Previously, tacit knowledge from experiments filled up those gaps. Different conclusions were gained even if it began from the same point. Fortunately, automated approaches have been developed and these appear as a bridge between them.

To date, only Ismail (2005) studied the automated models selection procedure for this type of model where she extended the *PcGets* algorithm and named the new algorithm as *SURE-PcGets*. Her work had been continued by (Yusof, 2016) but with the use of *Autometrics*, a successor of *PcGets*, and thus referred as *SURE-Autometrics*. Further to their work, other estimation methods for multiple equations models selection within GETS strategy is aimed for *SURE-Autometrics* in this study. Precisely, MLE methods were used to alter the *SURE-Autometrics* algorithm. IFGLS and EM algorithm were exploited as to substitute FGLS estimation method in the algorithm. Hence, the new algorithms are named as *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*. Automated models selection algorithm is expected to be improved using this set of estimators. Finally, the pertinent literature of this study is summarised in Table 2.1.

Table 2.1
Summary of Selected Literature

Topic	Scope	Author/year	Method/approach	Key findings
Modelling and model selection	Forecasting with neural network models	Kock and Teräsvirta (2016)	QuickNet, Marginal Bridge Estimator and *Autometrics*	A well-fitting nonlinear model improves *Autometrics* ability in selecting models with exceptional forecasting performance.
	Forecasting during economic crisis	Kock and Teräsvirta (2014)	QuickNet, Marginal Bridge Estimator and *Autometrics*	Strategy of *Autometrics* works well when the model is a sensible resemblance to reality.
	Model selection algorithms survey	Castle, Qin and Reed (2013)	Information criteria, portfolio, GETS, forward-stepwise regression, Bayesian model averaging algorithms	*Autometrics*'s performance is good, particularly when ratio of relevant to irrelevant variables is less than 0.5, and non-centrality parameter is equal to or less than 2.
	Automated model selection	Castle, Doornik and Hendry (2011)	*Autometrics* algorithm	*Autometrics* is successful in selecting dynamic equations using tight significance level.
	Model selection	Ericsson (2010)	*Autometrics* algorithm	Modeller still has to choose initial general model and parameterization of model.
	Cultures of statistical modelling	Breiman (2001)	Data and algorithmic modelling	Algorithmic modelling is vital when there are lots of data and lots of decisions.
	Complexity in model selection	Myung (2000)	7 model selection methods	Model selection ought to be based on its capability to summarise the features of the population.

Table 2.1 (cont.)

Topic	Scope	Author/year	Method/approach	Key findings
SURE model	*SURE-Autometrics*	Yusof (2016)	*Autometrics*	Performance of model selection in multiple equations using *SURE-Autometrics* depends on quality of data and complexities of SURE model.
	Forecasting using SURE model	Neves, Fernandes and Veiga (2015)	Kalman filtering	The attributes of SURE model capturing comparable influences in the mortality rate trends of age groups produced models of good forecasting power.
	Properties of SURE model	Sun, Ke and Tian (2014)	Matrix rank method	More information gained from the results in comprehending mechanism of singular SURE model under some general assumptions.
	SURE model	Baltagi and Pirotte (2011)	ML and generalized moments estimation methods	Forecast performance of RMSE improved by considering for heterogeneity and spatial correlation.
	SURE-PcGets	Ismail (2005)	*PcGets*	Automatic algorithm was established and it offered a reliable starting point for building SURE models.
	Contemporaneous correlation of disturbances	Dufour and Khalaf (2002)	Monte Carlo tests	MC-QLR tests are best in terms of power.
	Sample size of SURE model	Griffiths, Skeels, and Chotikapanich (2001)	FGLS, ML and Bayesian estimation methods.	Larger sample sizes are needed in likelihood-based methods than the two-stage estimator.
	SURE model	Zellner (1962)	Two-stage Aitken's generalized least-squares estimation method	A method of estimating coefficients in regression equations which is more efficient than an equation-by-equation application of least-squares is shown.

Table 2.1 (cont.)

Topic	Scope	Author/year	Method/approach	Key findings
Estimation methods	Demand model	van Ryzin and Vulcano (2017)	EM algorithm	A straightforward and greatly efficient estimation procedure and six times faster than direct MLE.
	Latent variable model	Sohn (2016)	EM algorithm	EM algorithm is a steady method for preventing the lack of empirical identification problem.
	Comparison study	Phillips (2010)	IFGLS and generalized method of moments (GMM)	Forecasts based on IFGLS were better than forecasts based on GMM in terms of forecast errors.
	SURE model	Beasley (2008)	IFGLS estimation	SURE model is a flexible analytic strategy in path analysis problem.
	Linear systems	Saad and van der Vorst (2000)	Iterative methods	The iterative methods will be greatly employed in more application areas.
	Stochastic production function	Saha, Havenner and Talpaz (1997)	ML versus FGLS.	In small samples, the ML estimator is more efficient and experiences reduced bias than FGLS.
	Equivalence of two methods	Park (1993)	ML and iterative two-stage estimations	Iterative two-stage and ML estimator are equivalent.
	Description of method	Oberhofer and Kmenta (1974)	Iterative procedure	ML estimates of parameters of a generalized regression model are obtained through the procedure shown.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The success of *SURE-Autometrics* in selecting multiple equations simultaneously had prompted for more alternatives in this kind of research. Therefore, extensions of *SURE-Autometrics* were initiated in this study by developing *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*. The methodology used is discussed in this chapter. Section 3.2 gives the overall framework of this study. The estimation methods in models are described in Section 3.3, while Section 3.4 presents the development phases in both algorithms. Section 3.5 gives brief explanation for both simulation and empirical validations. Finally, Section 3.6 presents the summary.

3.2 Research Framework

The overall process of algorithms is shown in Figure 3.1. There are five phases executed in *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*. Each algorithm begins with Phase 1 of identifying initial GUMS besides running a set of tests for diagnostic checking for each equation. Phase 2 refers to pre-search lag reduction to reduce computational effort, followed by encompassing test. The Phase 3 is the tree search that is the main procedure of the algorithm in which bunching, chopping and pruning steps take place and Phase 4 concerns on further tree search for the nested terminals. Lastly, Phase 5 is applied to find the final model when there are multiple terminals. The estimation methods (in Figure 3.2), either IFGLS or EM algorithm, embedded accordingly in *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*, was applied in all phases since any changes in GUMS would lead to new estimation of models. Each algorithm was run individually.

Once all phases are implemented, both algorithms were validated through simulation and empirical data investigations. Figure 3.3 presents the validation framework for *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*. Simulation analysis was conducted under six main conditions involving sample size, correlation strength among equations, GUMS size, number of equations, significance level and true specification models with the details are explained in Chapter Four. Meanwhile, for empirical data analysis, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were compared with 16 more models selection procedures in order to determine their performances by employing national growth rates and water quality index (WQI) data sets, as discussed in Chapter Five.

The remaining of this Chapter Three consists of the following sections: Section 3.3 discusses IFGLS and EM algorithm for estimations, Section 3.4 describes the pertaining development phases involved for both algorithms, Section 3.5 explains the validation process and lastly, Section 3.6 concludes the chapter.

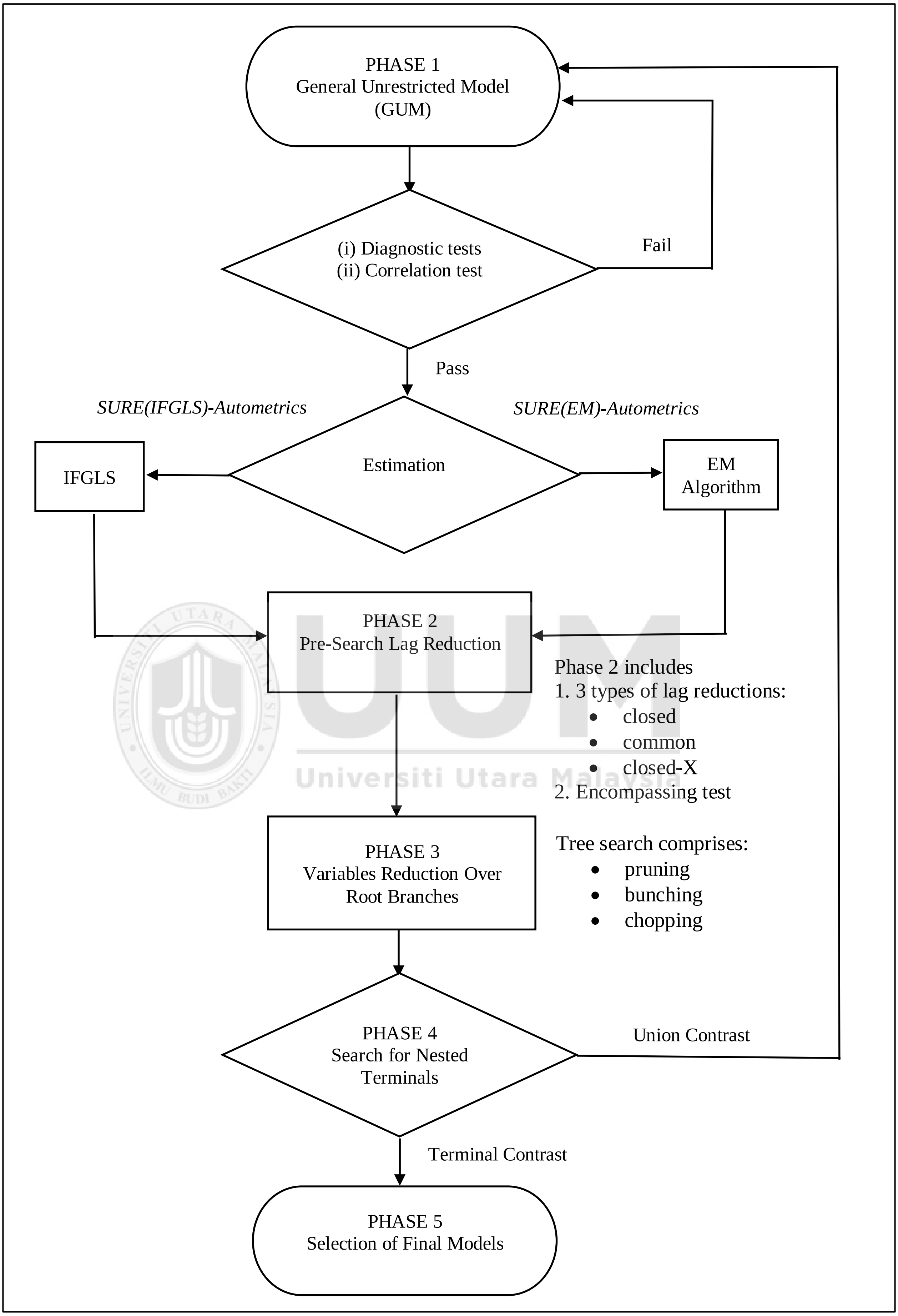

Figure 3.1. Flowchart of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*

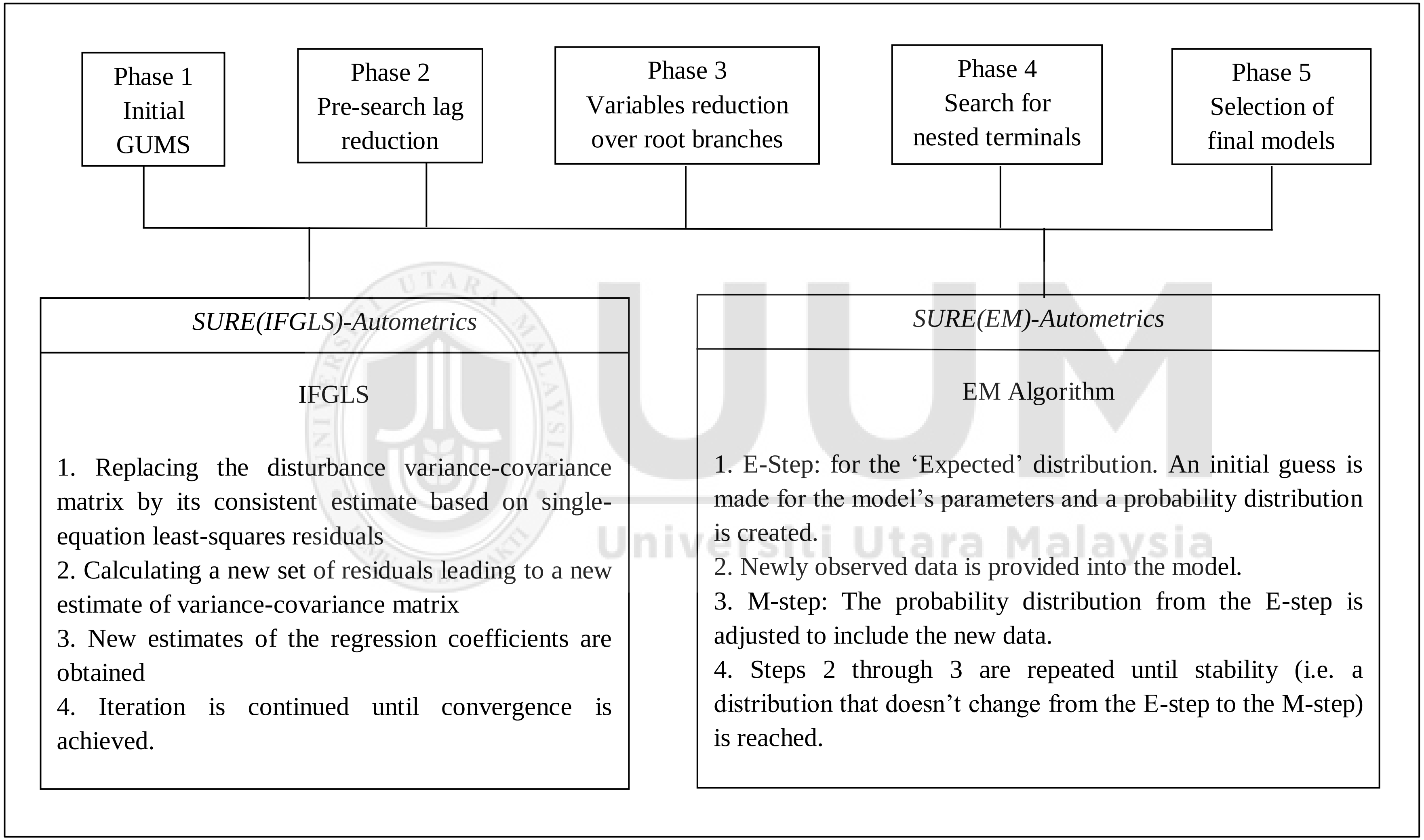

Figure 3.2 Estimation Methods of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*

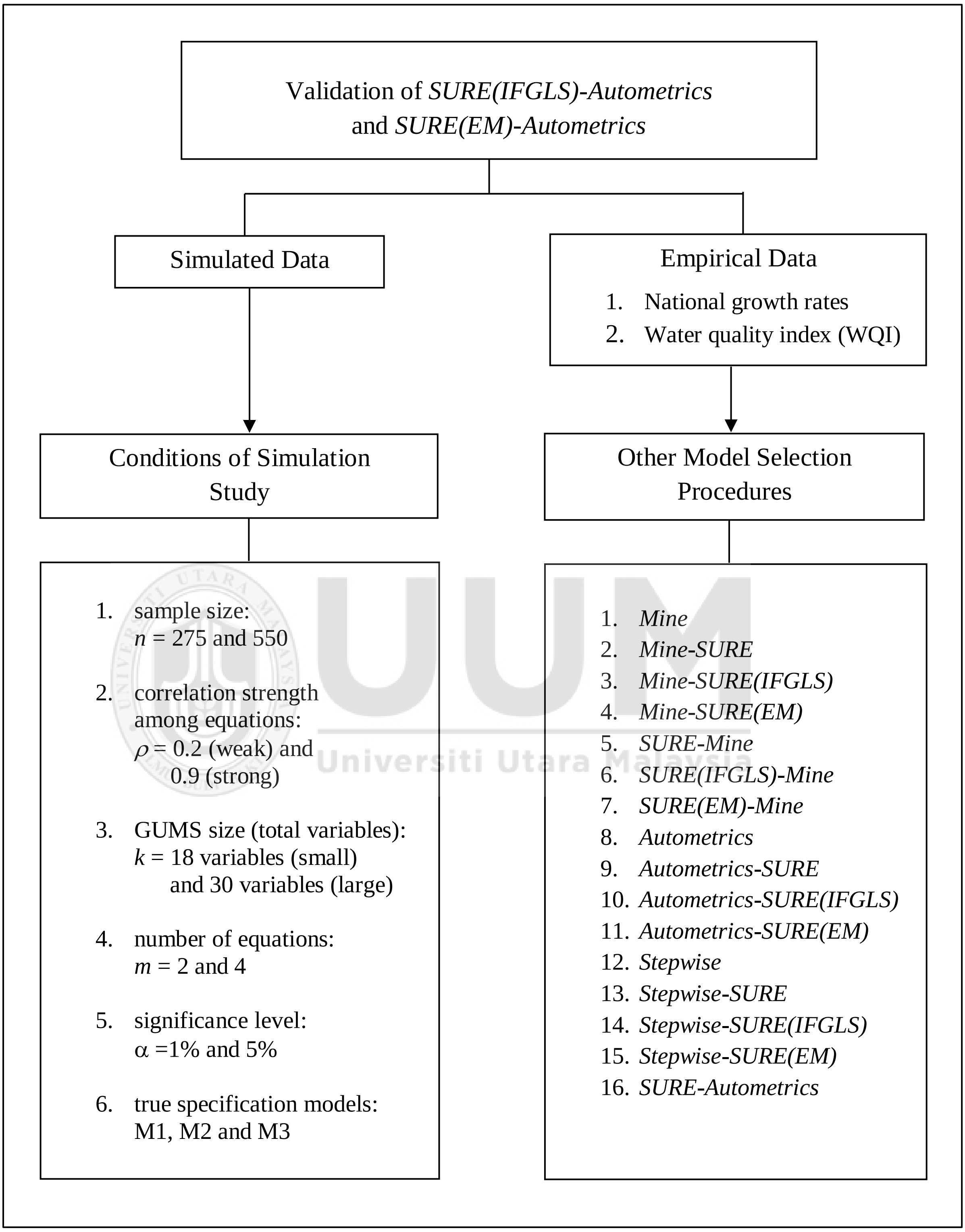

Figure 3.3. Validation of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*

3.3 Model Estimation Methods

The more common estimation method of FGLS in SURE model has now been modified by using IFGLS and EM algorithm. Each of these was employed separately in all stages of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometric* whenever model estimation is required (see Figure 3.2). Model estimation of IFGLS or EM algorithm would be applied accordingly when the test for contemporaneous correlation of disturbances proven significant.

3.3.1 Iterative Feasible Generalized Least Squares (IFGLS)

The SURE model is a generalization of multivariate regression using a vectorized parameter model. If the covariance matrix $\mathbf{\Omega}$ is identified, then the model can be estimated with generalized least squares (GLS). Therefore, the best linear unbiased estimator of $\boldsymbol{\beta}$ is given by,

$$\hat{\boldsymbol{\beta}}_{GLS} = (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\Omega^{-1}\mathbf{y} \qquad (3.1)$$

and the covariance matrices of these estimators are,

$$V(\hat{\boldsymbol{\beta}}_{GLS}) = (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1} \qquad (3.2)$$

In general, $\mathbf{\Omega}$ and residual, $\mathbf{u}_i$ are not known and so they have to be estimated. Every equation is estimated by OLS separately and the unbiased estimators for the coefficients of the i^{th} equation in (2.2) are given by,

$$\hat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i, \qquad i = 1, 2, \ldots, m \qquad (3.3)$$

and

$$V(\hat{\boldsymbol{\beta}}_{OLS}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\Omega\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \qquad (3.4)$$

The corresponding OLS residuals are given by

$$\hat{\boldsymbol{u}}_i = \mathbf{y}_i - \mathbf{X}_i\hat{\beta}_i, \qquad i = 1, 2, \ldots, m$$

(3.5)

Let $\hat{\Omega}$ be a consistent estimator based on the residuals.

$$\hat{\Omega} = \hat{\Sigma} \otimes \mathbf{I} \tag{3.6}$$

with $$\hat{\Sigma} = [\hat{\boldsymbol{u}}_i]'[\hat{\boldsymbol{u}}_j] \qquad i, j = 1, 2, \ldots, m \tag{3.7}$$

or $$\hat{\sigma}_{ij} = \frac{[\hat{\boldsymbol{u}}_i]'[\hat{\boldsymbol{u}}_j]}{T} \qquad i, j = 1, 2, \ldots, m \tag{3.8}$$

where $\otimes$ denotes Kronecker product and $\hat{\Sigma}$ is a $i \times j$ matrix based on single equation OLS residuals. T is total observations for each of m equation. Srivastava and Giles (1987) referred this estimator as SURR, which yields the following FGLS estimator of β,

$$\hat{\boldsymbol{\beta}}_{FGLS} = (\mathbf{X}'\hat{\Omega}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\Omega}^{-1}\mathbf{y} \tag{3.9}$$

and the covariance matrix of the estimated parameters is,

$$V(\hat{\boldsymbol{\beta}}_{FGLS}) = (\mathbf{X}'\hat{\Omega}^{-1}\mathbf{X})^{-1}. \tag{3.10}$$

Zellner's FGLS estimator of β can be used for calculating a new set of residuals leading to a new estimate of $\boldsymbol{\Omega}$, which can be used for obtaining new estimates of the regression coefficients β, and so on. This would yield IFGLS estimator.

Park (1993) described IFGLS estimator as the iterative two-stage estimation which updates the estimator of Σ from the current estimator of β and re-estimates β by using the current estimator of Σ until convergence. Let k ($k_i = 0, 1, \ldots, \infty$) index the iterations, where k=0 refers to the initial values and, for convenience $k=\infty$ refers to convergence. Let $\hat{\Sigma}^k$ and $\hat{\beta}^k$ represent the kth iterates of Σ and β, respectively. The iterative two-stage estimation procedure can be summarized as follows:

Step 1: Take the initial estimator of Σ as I_t, the i x j identity matrix.

Step 2: For the kth iterate, estimate $\hat{\beta}^k$ using Equation (3.9).

Step 3: Compute the (k + 1)th iterate of Σ, $\hat{\Sigma}^{k+1}$ using $\hat{\Sigma}^{\boldsymbol{k}+1} = \hat{\sigma}_{\boldsymbol{ij}}^{\boldsymbol{k}+1}$,

where $$\hat{\sigma}_{\boldsymbol{ij}}^{\boldsymbol{k}+1} = \frac{[\hat{\boldsymbol{u}}_i^k]'[\hat{\boldsymbol{u}}_j^k]}{T} \qquad i, j = 1, 2, \ldots, m$$

and $\hat{\boldsymbol{u}}_{\boldsymbol{i}}^{\boldsymbol{k}} = \mathbf{y}_{\boldsymbol{i}} - \mathbf{X}_{\boldsymbol{i}} \hat{\beta}_{\boldsymbol{i}}^{\boldsymbol{k}}$ are the restricted residuals for $\mathbf{y}_i$ for i = 1, . . . m.

Step 4: Continue Step 2 and Step 3 until a convergence criterion is satisfied.

At convergence, the variance-covariance matrix of $\hat{\beta}^{\infty}$ can be obtained using Equation (3.10). The convergence criterion is chosen as the difference between two successive values of the log-likelihood function.

3.3.2 Expectation Maximization (EM) Algorithm

The EM algorithm is an algorithm that can be applied in a broad range of conditions. It presents an iterative procedure for computing ML estimator in situations where MLE would be straightforward. The complete-data likelihood has a nice form for most statistical problems. EM algorithm may utilise the reduced complexity of MLE given the complete data.

Suppose full observations data, n is labelled so that the first m observations are complete with the last (n -m) cases having the response y_j missing. In this setting, let vector **y** be the observed data which considered as incomplete and acted as an observable function of the supposed complete data. The fundamental of EM is to begin with some trials of the parameter values $\boldsymbol{\beta}^{(0)}$ and then search for better values for the parameters iteratively. The current estimate of the parameters is $\boldsymbol{\beta}^{(k)}$, thus another $\boldsymbol{\beta}^{(k+1)}$ is needed to improve the incomplete data log likelihood function $L(\boldsymbol{\beta})$. Obviously, if a $\boldsymbol{\beta}^{(k+1)}$ such that $Q(\beta^{(k+1)}; \beta^{(k)}) > Q(\beta^{(k)}; \beta^{(k)})$ then, $L(\boldsymbol{\beta}^{(k+1)} > L(\boldsymbol{\beta}^{(k)})$, where Q-function, which is an expectation of complete data likelihood $L_c(\boldsymbol{\beta})$. Therefore, the general procedure of the EM algorithm is as follows:

1. $\boldsymbol{\beta}^{(0)}$ is initialized according to previous information about the parameter value.
2. The estimate of $\boldsymbol{\beta}$ is iterated by alternating between the following two-steps:

   i. The E-step (expectation): $Q(\boldsymbol{\beta};\boldsymbol{\beta}^{(k)})$ is calculated.

   ii. The M-step (maximization): $\boldsymbol{\beta}$ is re-estimated by maximizing the Q-function:

$$\boldsymbol{\beta}^{(k+1)} = \arg\max_{\beta} Q(\boldsymbol{\beta};\boldsymbol{\beta}^{(k)})$$

3. Iteration continues until difference of $L(\boldsymbol{\beta}^{(k+1)}) - L(\boldsymbol{\beta}^{(k)})$ changes by a small amount.

Concerning the E-step on the $(k+1)^{\text{th}}$ iteration, it follows from $L(\boldsymbol{\beta})$ that in order to compute the $Q(\boldsymbol{\beta}^{k+1};\boldsymbol{\beta}^{(k)})$, the conditional expectations of the missing responses y_j are required, given the vector $\mathbf{y} = y_1,\ldots,y_m$ of the observed responses and the design matrix $\mathbf{X}$. The conditional expectations are given by $E(y_j|\mathbf{y},\mathbf{X}) = \mathbf{x}_j\boldsymbol{\beta}^{(k)}$.

Mclachlan and Krishnan (2008) agreed that computation of the ML estimator through EM algorithm is frequently made possible by artificially formulating it to be an incomplete data problem, even though it may not seem like an incomplete data in the beginning. Considering this is an early attempt in applying EM algorithm for selecting SURE model of time series missing data, this study was designed with only one missing rate that is 30%, following the rates adopted in other simulation designs as in Table 3.1.

Table 3.1
Percentages of Missing Rates

Authors	Missing rate (%)
Balakrishnan and Mitra (2011)	30, 60
Balakrishnan and Mitra (2012)	30, 40
Emura and Shiu (2014)	30, 60
Enders (2001)	5, 15, 25, 35

Subsequently, for the purpose of application in SURE model in this study, the EM algorithm based on procedure by Healy and Westmacott (1956) as described in Mclachlan and Krishnan (2008) was employed accordingly, as follows:

Step 1: Start from initial values based on the means calculated from the available data.

Step 2: Perform the FGLS estimates of SURE model parameters for complete data set.

Step 3: Predict missing values using the estimates obtained above.

Step 4: Substitute predicted values for missing values.

Step 5: Go to Step 2 and continue until convergence of parameter estimates.

The steps are executed at any estimation of models in every phase of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics.* Details on the development of both algorithms are described in the next section.

3.4 Development of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*

This section details the development of five phases of the algorithms. Phase 1 explains the initial specification of GUMS and Phase 2 handles pre-search reduction of the general model. Meanwhile, Phase 3 deals with variables reduction, Phase 4 continued searching for nested terminal and finally, Phase 5 is where the final model is selected.

3.4.1 Phase 1: Specification of Initial GUMS

Here, an initial specification for each equation in the SURE model is made. Its construction is determined by modeller beginning with the number of equations and the variables together with their lags. The main level of significance is also set here and each equation in the model is estimated using OLS. The process is resumed with diagnostic checking of each equation, testing of contemporaneous correlation of disturbances among equations and model estimation methods.

3.4.1.1 Diagnostic Tests

Subsequent to formulation of GUMS, every equation in SURE model is estimated by OLS estimation separately and tested for any misspecifications. The following diagnostic tests are crucial in determining congruence of model all the way through reduction process. The GUMS is considered congruent having passed all tests. Otherwise, a new GUMS has to be specified and tested again, depending on choice of modeller besides updating the *p*-value of tests. The next step should be continued with congruent GUMS. The independent status of errors among equations along with assumption of a normal distribution, parameter constancy, white noise errors, unconditional and conditional homoscedasticity will be inspected through the following tests.

i. Test for Normality Errors

The normality assumption is a basic criterion in any MLEs. The skewness and kurtosis of residuals as assumed in a normal distribution are tested through these hypotheses:

H_0: Errors are normally distributed

H_1: Errors are not normally distributed

As μ, σ^2 represent the mean and variance of a variable, x_t, while $\mu_i = E[x_t - \mu]^i$, in order that $\sigma_x^2 = \mu_2$. The skewness and kurtosis are stated correspondingly as:

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} \qquad \text{and} \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} \tag{3.11}$$

A normal variate has $\sqrt{\beta_1} = 0$ and $\beta_2 = 3$. With T as number of observations, sample estimates of parameters are given as:

$$\bar{x} = \frac{1}{T}\sum_{t=1}^{T} x_t, \qquad m_i = \frac{1}{T}\sum_{t=1}^{T}\left(x_t - \bar{x}\right)^i, \qquad \sqrt{b_1} = \frac{m_3}{m_2^{3/2}} \qquad \text{and} \qquad b_2 = \frac{m_4}{m_2^2} \tag{3.12}$$

The test statistic based on Doornik and Hansen (2008) is given by,

$$z_1^2 + z_1^2 \sim \chi^2(2) \tag{3.13}$$

where z_1 and z_2 denote the transformed skewness and kurtosis which are expressed as:

$$z_1 = \sqrt{\frac{T}{6}}\beta_1 \qquad \text{and} \qquad z_2 = \sqrt{\frac{T}{24}}\beta_2 \quad . \tag{3.14}$$

ii. Test for Parameters Constancy

This is the test of parameter constancy between models for two different sets of data. By following *Autometrics*'s default setting, the algorithm implements 70% breakpoint in the sample. The hypotheses are:

H_0: The coefficients are identical in the both equations (parameter constancy)

H_1: Parameters are not constant

The Chow (1960) breakpoint test statistic has the following form:

$$\eta_3 = \frac{(RSS_T - RSS_{T1})/(T - T_1)}{RSS_{T_1}/(T_1 - k)} \sim F_{(T-T_1, T_1-k)} \tag{3.15}$$

There are k regressors, while T represent full sample and T_1 refers to sub-sample. RSS_T is full sample residual sum of squares and RSS_{T_1} is for the relevant sub-sample.

iii. Test for Autocorrelation

This is the Lagrange multiplier (LM) test for r^{th} order residual autocorrelation, distributed as $\chi^2(r)$ in large samples. The hypotheses for this test are:

H_0: $\rho = 0$ (no autocorrelation i.e. errors are white noise)

H_1: $\rho \neq 0$ (autocorrelation occurs)

Any orders from 1 up to 12 can be selected to test against:

$$u_t = \sum_{i=p}^{r} \alpha_i u_{t-i} + \varepsilon_t \qquad \text{where } 0 \leq p \leq r \tag{3.16}$$

The test is an auxiliary regression where the error autocorrelation coefficients are the coefficients of the lagged residuals. It is calculated by regressing the residuals on all the regressors of the original model and the lagged residuals for lags p to r (missing residuals are set to zero). The LM test is TR^2 and is equivalent to the F test suggested by Kiviet (1986) and Harvey (1990):

$$\frac{R^2}{1-R^2} \cdot \frac{T-k-r+p-1}{r-p+1} \sim F\left(r-p+1, T-k-r+p-1\right) \tag{3.17}$$

where p is the minimum lag number, r is the maximum lag number and R^2 is the goodness of fit measure. All results from auxiliary regression in *Autometrics* come in the form of F distribution (Doornik & Hendry, 2007).

iv. Test for Unconditional Homoscedasticity

In *Autometrics*, this test involves an auxiliary regression of $\{\hat{\varepsilon}_t^2\}$ on the original regressors, x_{it} and all their squares, x_{it}^2, then R^2 is calculated. The hypotheses are:

H_0: $Var(\varepsilon_t) = \sigma^2$ is constant (unconditional homoscedasticity)

H_1: Variance of the $\{\varepsilon_t\}$ process depends on x_t and on the x_{it}^2

The statistic is based on White (1980). Generally, when there are p independent variables (original regressors and all their squares), the distribution will have p degrees of freedom which is equivalent to F form of:

$$\frac{R^2}{1-R^2} \cdot \frac{T-p}{p} \sim F_{(p,T-p)} \quad (3.18)$$

v. Test for Conditional Homoscedasticity

The test is Autoregressive Conditional Heteroscedasticity (ARCH) test (Engle, 1980) that includes these hypotheses:

H_0: $\gamma = 0$ (conditional homoscedasticity)

H_1: $\gamma \neq 0$ (conditional heteroscedasticity)

The model is

$$E\left[\varepsilon_t^2 \mid \varepsilon_{t-1}, \ldots, \varepsilon_{t-r}\right] = c_0 + \sum_{i=1}^{r} \gamma_i \varepsilon_{t-i}^2 \quad (3.19)$$

where $\gamma = (\gamma_1, \ldots, \gamma_r)'$ and maximum lag number is denoted by r. Again, the test is TR^2 from the regression of $\hat{\varepsilon}_t^2$ on a constant and $\hat{\varepsilon}_{t-1}^2$ to $\hat{\varepsilon}_{t-r}^2$ which is asymptotically distributed as $\chi^2(r)$.

3.4.1.2 Contemporaneous Correlation Errors

Dufour and Khalaf (2002) had suggested Monte Carlo quasi-likelihood ratio (MC-QLR) test in order to verify the correlation between errors in equations of SURE system. Significance level of 0.10 is fixed in both algorithms and thus any p-values of less than this, signifying significant result that system estimation method is more efficient. The hypotheses are:

H_0: The errors in different equations are not contemporaneously correlated

H_1: The errors in different equations are contemporaneously correlated

The test statistic is,

$$\xi_{LR}^{(h)} = T \ln\left(\frac{\left|D_m\left(\hat{\sigma}_i^2\right)\right|}{\left|\hat{\Sigma}^{(h)}\right|}\right), \qquad i=1,2,\dots,m; \qquad h = 1,2,\dots \tag{3.20}$$

$\hat{\sigma}_1^2,\dots,\hat{\sigma}_m^2$ are the diagonal elements of diagonal matrix $D_m\left(\hat{\sigma}_i^2\right)$. Number of equations and number of observations are denoted as *m* and *T*, respectively. The partially iterated estimators for disturbances covariance matrix is represented by $\hat{\Sigma}^{(h)}$.

The computation of the *p*-value is considered to a test statistic *T* for H_0. If *T* is large (i.e., when $T \geq c(\alpha)$, where $P\left[T \geq c(\alpha)\right] = \alpha$ under H_0), then H_0 will be rejected. Suppose the survival function under H_0 is denoted by $G(x) = P\left[T \geq x\right]$. Let T_0 be the test statistic computed from the data observed. Then the related critical area of size *α* is stated as $G(T_0) \leq \alpha$. The Monte Carlo methods is used to generate *N* independent realizations $T_1,\dots,T_N$ of *T* under H_0 and calculates randomized *p*-value $\hat{p}_N(T_0)$, where

$$\hat{p}_N(x) = \frac{N\hat{G}_N(x)+1}{N+1} \tag{3.21}$$

$$\hat{G}_N(x) = \frac{1}{N}\sum_{1=1}^{N} I_{[0,\infty]}(T_i - x), \qquad I_A(x) = \begin{cases} 0 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases} \tag{3.22}$$

The significance of this test means that the presence of correlation among disturbances has been detected.

3.4.2 Phase 2: Pre-search Reduction

Phase 2 comprises pre-search reduction stage with an objective to build an algorithm that is able to perform with or without. It is done prior to the tree search method with purpose of lessening the computational effort by reducing the highly insignificant lag variables in the GUMS. The regressors are grouped together according the same lag. The significance value for the reduction process depends on the pre-test significance value which is defined as:

$$p_p = \frac{5 p_a^{\ 0.8}}{1 + 4 p_a^{0.8}} \tag{3.23}$$

where p_a is the main significance value for the whole hypothesis testing procedure (i.e., 5% and 1%). The pre-test significance values, p_p are 0.1141 and 0.3337, respectively. Pre-test significance value is nonetheless given as in Equation 3.24 if there is only one variable that has to be removed.

$$p_{p,1} = \max\left\{ \frac{1}{2} p_a^{\ 1/2}, p_a^{\ 3/4} \right\} \tag{3.24}$$

There are three methods of lag reduction which are closed, common and common-*X* lag reductions are described in next section. All these methods are employed in two directions. The first will follow the order, and the second will be in reverse order. The models found from both ways will have to pass encompassing tests in determining whether one or both models would be the current GUMS.

i. Closed lag reduction

The closed lag reduction is aimed to test a group of lags from the largest lag downwards and discontinued once a lag cannot be deleted. All regressors at lag q are deleted based on four conditions:

a. all individual p-values in the group are above

$$\max\left\{ p_{p,1}^{*}\left(k_p\right) = 1 - \left(1 - p_{p,1}\right)^{k_p},\ p_s \right\} \tag{3.25}$$

with $p_s = f\left(p_a, 0.2\right) \approx \frac{p_a}{5}$ where k_p is the number of regressors involved,

b. their joint p-value in the reduced model is above p_p,

c. backtesting with respect to initial GUMS

d. diagnostic testings are satisfied in checking effects of variables removal in congruency

If any of the conditions are not fulfilled, all variables are sent back into the equation and reduction process is ended. The removal would resume with group of lag (q-1) until lag one when all the four conditions are assured.

ii. Common lag reduction

The lag variables are classified based on their lag number. The joint significances for all lags are computed and organised from the least significant lag group. Similar conditions as in closed lag reduction are checked in removing all regressors related. The current model is set and same steps are repeated for next lags.

iii. Common–X lag reduction

This type of reduction is the same as common lag reduction but lag of Y is omitted from the procedure.

iv. Encompassing test

The principle of encompassing is to merge the overabundance of empirical models that often can be found to describe any given occurrence (Hendry & Mizon, 2016). In other words, a model is said to encompass if it is capable of interpreting the behaviour of other rival models. This encompassing test is to ensure that the simplified model is a valid reduction of the initial system of GUMS. Let the first model, M_1 have $k_1 + k_2$ regressors (x_{1t}, x_{2t}) and the second model, M_2 has $k_2 + k_3$ (x_{2t}, x_{3t}) so the x_{2t} is common in both models and union of the models, M_U comprises $k = k_1 + k_2 + k_3$ non-redundant set for x_{1t}, x_{2t}, and x_{3t}. Now, let RSS_1, RSS_2, and RSS_U denote residual sum of squares from M_1, M_2, and M_U respectively. The hypotheses are:

H_0: M_1 ε M_2 (M_1 parsimonious encompassing M_U)

H_1: M_1 does not encompass M_2

The test statistic is:

$$\eta_4 = \frac{\left(RSS_1 - RSS_U\right)/k_3}{RSS_U/\left(T-k\right)} \sim F_{(k_3, T-k)} \tag{3.26}$$

where k_3 is the number of non-redundant predictors. The tests are run at main significance level (p_a) for every equation in the model. The estimation method concerned is employed again whenever a reduction occurs and GUMS are updated.

3.4.3 Phase 3: Variable Reduction over Root Branches

Phase 3 consists of tree search where the whole space of models generated by variables in initial model. Therefore, unique models are expected to be found in this phase. GUMS are now denoted as GUM0 which is GUMS after pre-search as in Phase 2. Without pre-search, GUM0 and initial GUMS are identical. On the other hand, GUM0 will be formulated according to encompassing tests in Phase 2. It could be union of both models resulted from different order of lag reductions, or only model that passed the test, or similar to initial GUMS (i.e. both models failed the tests).

The reductions of the algorithm can be generally segregated into five steps. First, all variables are tested in GUM0 at pre-test significance value, p_p. Second, the highest insignificant variable is removed at p_s where $p_s = 0.2\ p_a$, if all regressors in GUM0 are significant at reduction p-value, p_a. Third, tree search begins with reduction from root branches. Fourth, another tree search but now focuses on reduction for searching nested terminals. Finally, iteration procedure for both tree search steps for current GUMS which is union of multiple terminal models survived from reduction process. Terminal model refers to model that cannot be reduced anymore.

In this tree search, three essential elements are considered, which are variables deletion principles, backtesting or diagnostic tests, and model contrast. At this point, pruning, bunching, and chopping principles are implemented in the algorithm so as to remove the variables along the reduction process in sequence of equations. As soon as the terminal model has been found, the model will go through diagnostic checking as discussed in Section 3.2.1.2. System estimation method, either IFGLS or EM algorithm, is utilized again whenever a model has to be estimated.

The reduction principles basically consist of pruning, bunching, chopping and model contrast. Pruning is when one variable is considered for deletion if its *p*-value is above p_a. In the case where a deletion fails or the reduced model fails on backtesting or diagnostic testing, the following branches can be pruned or ignored.

Meanwhile, bunching is when variables are grouped for deletion instead of one variable at one time, which means deletion is tried as a block. If deletion fails, the procedure goes back until a bunch that can be deleted is found. If needed, to a bunch of size one, $k_b = 1$. The amount of bunching is decided by p_b. Too high of this value may cost more in terms of excessive backtracking, but too low of it may switch bunching off. Thus, by default $p_b \approx \frac{1}{2} p_a^{1/2}$, with significance test of the bunch is done at p_a. The grouping of variables is done according to their individual insignificance as long as their smallest p-value in the bunch is above $p_b*(k_b)$ which given by

$$p_b*(k_b) = p_b^{1/2}\left[1-\left(1-p_b^{1/2}\right)^{k_b}\right] \tag{3.27}$$

where k_b is the size of bunch and $p_b = \max\left\{\frac{1}{2}p_a^{1/2}, p_a^{3/4}\right\}$.

Chopping occurs when a highly insignificant bunch is eliminated permanently from search procedure if its significance is above p_c, given $p_c = p_b$. Once it is done, procedure goes back to lower part of tree and any possible nodes with subsets of insignificant bunch are ignored. Lastly, in model contrast, unique tree representation enables modeller to find out minimum bunch along the current path that must be deleted to give a different model. Thus, there is no urgency to enter branches which produce same terminal model when a terminal candidate has been found.

3.4.4 Phase 4: Search for Nested Terminal

The main concern here is to find variables that should be in system of GUMS through further inspections on terminals. Discovering a similar terminal is not impossible due to unique orders of the tree. Minimal bunch is removed along existing path in looking for different terminal models. There are two approaches; union and terminal contrast. Union contrast decides different bunch from union of current set of terminal candidate models. It is used while current GUMS (new union of set of terminals) changes between iterations. In the meantime, the smallest bunch that would generate a model that is distinctive from any of the present terminals is chosen through terminal contrast. Once the current GUMS are fixed, then this function is applied. Likewise, in prior phases, IFGLS or EM algorithm is applied in estimation of the generated models.

3.4.5 Phase 5: Selections of Final Model

If there are multiple terminal candidate models, the union is formed and used as the starting point (new 'GUMS') for another application of the search. Algorithm finishes iterating when new GUMS is same as the previous GUMS. Once convergence takes place, tiebreaker chooses a final model. The final model would be the one with the smallest values of information criteria. The most common information criteria in econometrics studies are Akaike (1973), Hannan and Quinn (1979) and Schwarz (1978) respectively:

$$AIC = \ln \tilde{\sigma}^2 + \frac{2k}{T} \tag{3.28}$$

$$SC = \ln \tilde{\sigma}^2 + \frac{k \ln T}{T} \tag{3.29}$$

$$HQ = \ln \tilde{\sigma}^2 + \frac{2k \ln(\ln T)}{T} \tag{3.30}$$

where the ML estimate of $\tilde{\sigma}^2$ is given by:

$$\tilde{\sigma}^2 = \frac{T-k}{T}\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t^2 \quad , \tag{3.31}$$

T is the number of observations and k is the number of explanatory variables.

The final model is selected based on the lowest Schwarz criterion (SC) value, which is the default setting of *Autometrics* and therefore recognized as specific unrestricted model (SUM). If there are multiple terminal candidate models, the union is formed and used as the starting point (new 'GUMS') for another application of the search. Algorithm finishes iterating when new GUMS is same as the previous GUMS. Once convergence takes place, tiebreaker chooses a final model. The final model would be the one with the smallest values of information criteria. Upon realizing this SUM, estimation using IFGLS or EM method is once more employed, followed by diagnostic tests and also contemporaneous correlation test. The development of the *SURE(IFGLS)–Autometrics* and *SURE(EM)–Autometrics* was carried out using the *GAUSS* (version 15) programming language.

3.5 Validation of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*

After the algorithm has been developed, its evaluation on efficiency begins with the search of model when the true specification is known. Validations on the *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were done based on simulation and empirical data studies, as summarised in Figure 3.2 in Section 3.2.

3.5.1 Simulated Data

Given that *SURE-Autometrics* algorithm by Yusof (2016) was extended, this study adopted similar conditions in testing the new algorithms using simulated data. The artificial dependent variables (y_{it}) were generated according to true specification models, namely M1, M2 and M3. The simulated data was also used under variety of conditions including sample sizes of 275 and 550, strength of correlation among equations of 0.2 and 0.9, GUMS sizes up to 18 and 30 variables, two and four equations in the system and significance level of 1% and 5%. Each algorithm had to go through all these 96 conditions, meaning that there are 192 conditions in this whole simulation study to measure both algorithms' in finding the best final model. 4800 data sets were constructed throughout the entire simulation study for both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*. Each algorithm used 2400 data sets. This is because the artificial dependent variables (y_{it}) were generated with 100 replications in every sample size for each true model of any number of equations under every level of disturbance correlation in one algorithm. Further elaboration is found in Chapter Four, which also reveals results of this simulation study.

3.5.2 Empirical Data

Apart from simulated data, evaluation is followed by testing with empirical data. *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were compared with other selections procedures, including single equation selections and manual selections. There are altogether 18 models selection procedures. These models selection procedures and more details on validation using empirical data are explained in Chapter Five. Assessments for performance of the procedures were determined using error measures of forecasting accuracy. National growth rates data and WQI data were used in this part.

3.6 Summary

Once the algorithm has been developed, its evaluation on efficiency begins with the search of model when the true specification is known. This evaluation used simulated data under variety of conditions; (i) sample sizes (ii) strength of correlation among equations (iii) GUMS size (iv) number of equations in the system (v) significance level and (vi) true specification models. The simulation analysis is thoroughly investigated through in-depth discussion in Chapter Four. Apart from the simulated data, the programme is followed by testing with empirical data. In Chapter Five, comparisons are done with other selection approaches including single equation selections and manual procedures. Assessments for the performance of the selected models were determined using error measures of forecasting accuracy.

CHAPTER FOUR

VALIDATION OF *SURE(IFGLS)-AUTOMETRICS* AND *SURE(EM)-AUTOMETRICS* USING SIMULATED DATA

4.1 Introduction

The development of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were elaborated in Chapter Three. Thus, this chapter consists of simulation study for both algorithms in order to assess their abilities in obtaining the true SURE model specification when data-generating process (DGP) is known under different conditions. The true models are enhanced with irrelevant variables in order to form the general unrestricted model for multiple equations (GUMS), which will be reduced using the algorithms. If the removal of irrelevant variables succeeds during the models' selections, it means the algorithms give good performances. This validation part of the algorithms is performed to accomplish the second objective of this study. Section 4.2 elaborates on simulation design with respect to data generation and true specification models in Section 4.2.1, whereas Section 4.2.2 on conditions set in the experimental work. Section 4.3 reveals the outcomes for both algorithms. Lastly, Section 4.4 summarized the overall findings.

4.2 Simulation Design

The simulation process in this study contain 100 replications of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* to simplify the initial GUMS under a number of conditions. These conditions were adopted from the evaluation of *SURE-Autometrics* (Yusof, 2016), *SURE-PcGets* (Ismail, 2005) and *Autometrics* (Doornik, 2009). The simulation begins with generating artificial dependent variables according to three models specification (M1 with no relevant variable, M2 with one relevant variable and M3 with three relevant variables), two strengths of correlation disturbances amongst equations (ρ = 0.2 and 0.9) and two sample sizes (n = 275 and 550). The models specifications which have different number of variables are based on Doornik (2009), while Ismail (2005) initiated the strengths of correlation. Sample sizes used are chosen according to Griffiths et al. (2001). Both algorithms are run at two significance levels (α = 1% and 5%) to reduce two sets of initial GUMS (k = 18 and 30). This study assesses the simulation on models of two and four equations (m = 2 and 4). Therefore, the performances of both algorithms are measured under different level of conditions.

4.2.1 Data Simulation

Data of WQI which was provided by the Malaysia's Department of Environment had been utilized as the dependent variable (Y_{it}) in this study, whereas the main parameters in the formulation of WQI acted as the independent variables. These variables are Dissolved Oxygen (DO) (% saturation) (x_{i1t}), Dissolved Oxygen (DO) (mg/L) (x_{i2t}), Biochemical Oxygen Demand (BOD) (x_{i3t}), Chemical Oxygen Demand (COD) (x_{i4t}), Suspended Solids (SS) (x_{i5t}), pH (x_{i6t}), and Ammoniacal Nitrogen (NH_3N) (x_{i7t}), gathered from a river in Peninsular Malaysia. The data were collected at four monitoring stations representing number of equations in models, namely S6, S7, S8 and S25 beginning 11th May 2012 until 24th December 2013 for duration of 80 weeks. More details on this data set can be found in Section 5.4. Since weekly WQI data had only 80 observations, this data was converted into daily data using *Eviews* frequency conversion. This conversion resulted in sample of 550 observations to be used in data simulation.

Experimental frame of this simulation analysis had been motivated by Yusof (2016) and Ismail (2005). It started with research done by Hoover and Perez (1999) who used nine econometric models represented by HP1 until HP9 and then classified the final models selected into five categories. The work by Hoover and Perez (1999) was re-examined using *PcGets* for single equation selections by Hendry and Krolzig (1999) who only tested three models of HP1, HP2 and HP7. This was followed by Ismail (2005) who used same models in appraising *SURE-PcGets* for multiple equations. In Yusof (2016), two more models, HP8 and HP9, had been included for *SURE-Autometrics* investigation.

Consequently, this study adopted same design of HP1, HP2 and HP7 by Hendry and Krolzig (1999) for its simulation study. HP1 is now renamed as M1 and also referred as 'empty model' since it was purely random errors, while M2 is for HP2 which comprised only the first lag of dependent variable (y_t) and HP7 is now known as M3, an extension of M2 by adding one independent variable (x_t) and its first lag (x_{t-1}). These three econometric models were used as test bed for true specification search.

The daily data of WQI were regressed according to models M1, M2 and M3 by including a constant term. For M1, the constant term was based on mean of Y_{it}, while coefficients of disturbances were created upon standard deviation of Y_{it}. M2 comprised only first lag of dependent variable and for M3, Chemical Oxygen Demand (COD) (x_{i4t}) was added due to the strongest correlations with Y_{it} compared to other independent variables in most equations. The first lag of this independent variable was also included besides first lag of dependent variable in M3. The choice of variables in true specifications and coefficients values estimation using empirical data was essentially made following the work by Hendry and Krolzig (1999).

In addition, contemporaneous correlation disturbances among the equations were also required in simulating the artificial dependent variable. This type of correlation is the main feature of SURE model. Using the standard normal distribution, the disturbances were simulated and correlated with other equation for two levels of disturbances correlation. The levels chosen were 0.9 for strong correlation and 0.2 for weak correlation, similar to Yusof (2016) and Ismail (2005).

Besides that, number of equations also plays roles in multiple equations model. The equations used in this simulation study denote the monitoring stations from where the WQI data were collected. For four equations model, all four stations (S6, S7, S8 and S25) were included. Meanwhile for two equations models, stations S6 and S25 were removed due to larger standard errors with lower adjusted $\bar{R}^2$ values. Models for S6 and S25 do not fit the data well compared to models for S7 and S8. Thus, only S7 and S8 stations were chosen. In addition, these two stations are located near to each other.

The *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* are tested in finding the specifications used in generating the artificial data in this chapter. Therefore, the empirical data, the specification models and the simulated random error variable are necessary in generating the artificial dependent variables. In order to get the coefficient values of models, the WQI data are used by using IFGLS estimation for *SURE(IFGLS)-Autometrics* and EM algorithm estimation for *SURE(EM)-Autometrics*. Tables 4.1 and 4.2 report the two and four equations models for *SURE(IFGLS)-Autometrics*, whereas Table 4.3 and 4.4 show models for *SURE(EM)-Autometrics*.

Table 4.1

True Specification Models of SURE(IFGLS)-Autometrics (Two Equations)

Notation	Model
M1	$y_{1,t} = 61.229 + 6.709\varepsilon_{1,t}$
	$y_{2,t} = 61.584 + 7.377\varepsilon_{2,t}$
M2	$y_{1,t} = 48.256 + 0.211y_{1,t-1} + 6.444\varepsilon_{1,t}$
	$y_{2,t} = 49.288 + 0.199y_{2,t-1} + 7.091\varepsilon_{2,t}$
M3	$y_{1,t} = 71.496 + 0.060y_{1,t-1} - 0.413x_{1,4,t} - 0.034x_{1,4,t-1} + 4.210\varepsilon_{1,t}$
	$y_{2,t} = 53.058 + 0.292y_{2,t-1} - 0.419x_{2,4,t} + 0.115x_{2,4,t-1} + 4.705\varepsilon_{2,t}$

Table 4.2

True Specification Models of SURE(IFGLS)-Autometrics (Four Equations)

Notation	Model
M1	$y_{1,t} = 52.911 + 9.646\,\varepsilon_{1,t}$
	$y_{2,t} = 61.229 + 6.709\,\varepsilon_{2,t}$
	$y_{3,t} = 61.584 + 7.377\,\varepsilon_{3,t}$
	$y_{4,t} = 61.888 + 8.545\,\varepsilon_{4,t}$
M2	$y_{1,t} = 34.983 + 0.337\,y_{1,t-1} + 8.703\varepsilon_{1,t}$
	$y_{2,t} = 44.174 + 0.277\,y_{2,t-1} + 6.419\,\varepsilon_{2,t}$
	$y_{3,t} = 49.430 + 0.197\,y_{3,t-1} + 7.093\varepsilon_{3,t}$
	$y_{4,t} = 50.273 + 0.186\,y_{4,t-1} + 8.213\,\varepsilon_{4,t}$
M3	$y_{1,t} = 45.870 + 0.306y_{1,t-1} - 0.348x_{1,1,t} + 0.074x_{1,1,t-1} + 6.749\varepsilon_{1,t}$
	$y_{2,t} = 69.382 + 0.079y_{2,t-1} - 0.390x_{1,2,t} - 0.027x_{1,2,t-1} + 4.235\varepsilon_{2,t}$
	$y_{3,t} = 53.579 + 0.280y_{3,t-1} - 0.406x_{1,3,t} + 0.107x_{1,3,t-1} + 4.717\varepsilon_{3,t}$
	$y_{4,t} = 55.560 + 0.261y_{4,t-1} - 0.504x_{1,4,t} + 0.157x_{1,4,t-1} + 5.389\varepsilon_{4,t}$

Table 4.3

True Specification Models of SURE(EM)-Autometrics (Two Equations)

Notation	Model
M1	$y_{1,t} = 61.229 + 6.709\varepsilon_{1,t}$ $y_{2,t} = 61.584 + 7.377\varepsilon_{2,t}$
M2	$y_{1,t} = 48.239 + 0.211 y_{1,t-1} + 6.444\varepsilon_{1,t}$ $y_{2,t} = 49.271 + 0.199 y_{2,t-1} + 7.091\varepsilon_{2,t}$
M3	$y_{1,t} = 71.492 + 0.060 y_{1,t-1} - 0.414 x_{1,4,t} - 0.034 x_{1,4,t-1} + 4.210\varepsilon_{1,t}$ $y_{2,t} = 53.052 + 0.292 y_{2,t-1} - 0.419 x_{2,4,t} + 0.115 x_{2,4,t-1} + 4.705\varepsilon_{2,t}$

Table 4.4

True Specification Models of SURE(EM)-Autometrics (Four Equations)

Notation	Model
M1	$y_{1,t} = 52.911 + 9.646\,\varepsilon_{1,t}$ $y_{2,t} = 61.229 + 6.709\,\varepsilon_{2,t}$ $y_{3,t} = 61.584 + 7.377\,\varepsilon_{3,t}$ $y_{4,t} = 61.888 + 8.545\,\varepsilon_{4,t}$
M2	$y_{1,t} = 34.986 + 0.337\, y_{1,t-1} + 8.703\varepsilon_{1,t}$ $y_{2,t} = 44.159 + 0.2778 y_{2,t-1} + 6.419\,\varepsilon_{2,t}$ $y_{3,t} = 49.415 + 0.197\, y_{3,t-1} + 7.093\varepsilon_{3,t}$ $y_{4,t} = 50.247 + 0.187\, y_{4,t-1} + 8.213\,\varepsilon_{4,t}$
M3	$y_{1,t} = 45.868 + 0.307 y_{1,t-1} - 0.348 x_{1,1,t} + 0.074 x_{1,1,t-1} + 6.749\varepsilon_{1,t}$ $y_{2,t} = 69.386 + 0.079 y_{2,t-1} - 0.390 x_{1,2,t}$ - $0.027 x_{1,2,t-1} + 4.235\varepsilon_{2,t}$ $y_{3,t} = 53.538 + 0.281 y_{3,t-1} - 0.406 x_{1,3,t} + 0.107 x_{1,3,t-1} + 4.717\varepsilon_{3,t}$ $y_{4,t} = 55.561 + 0.261 y_{4,t-1} - 0.504 x_{1,4,t} + 0.157 x_{1,4,t-1} + 5.389\varepsilon_{4,t}$

4.2.2 Experimental Frame

This study utilized two samples from the daily WQI data. The sample sizes chosen were full sample size (n = 550) and the first half of the sample (n = 275). These sample sizes represent earlier simulation study, as in Table 4.5.

Table 4.5
Sample Sizes from Selected Articles

Authors	Sample size
Sohn (2016)	250 or 500
Galimberti and Soffritti (2014)	200 and 600
Lai and Huang (2013)	200 and 500
Enders (2001)	100, 250 and 400

In Sohn (2016), EM algorithm was excellent in reducing computing time and sampling errors when sample size was n = 250 or n = 500. Galimberti and Soffritti (2014) used n = 200 and n = 600 in EM algorithm for finite mixture models and also Lai and Huang (2013) with n = 200 and n = 500 for MLE of seemingly unrelated stochastic frontier regressions. Enders (2001) whose sample sizes were 100, 250 and 400 agreed that these sizes are accordance with suggestions from regression context as in Pedhazur (1997). The suggestions include the ratio of independent variable to sample size at minimum of 1:15 (1 independent variable needs at least 15 observations) or 1:30, while other researchers recommended at least 400 observations. In addition, a sample size suggestion for ML estimator by Griffiths et al. (2001), which is represented by $n > mv + 1$, where n = sample size, m = number of equations and v = number of variables, is also fulfilled. Thus, sample size of 275 and 550 are still in the range of values commonly applied in ML analysis.

Prior to going through selections processes, initial GUMS must be prepared at the preliminary stage, where the true models were augmented with irrelevant variables. The algorithm is considered successful if irrelevant variables are removed and the relevant variables are maintained in the models. Therefore, this evaluation analysis has two sets of initial GUMS with different complexity.

The first set (small GUMS) contains 18 explanatory variables that consists of four lags of y_{it}, seven x_{it} and x_{it-1} for each of x_{it}. This made all 18 variables as irrelevant variables in M1 as it was deemed as an 'empty' model. Since M2 comprises one relevant variable, thus M2 has fewer irrelevant variables of 17 and finally M3, which is made of three relevant variables, has only 15 irrelevant variables. Meanwhile, the second set (large GUMS) started with 30 explanatory variables of two lags of y_{it}, seven x_{it} as well as three lags for each x_{it}. This means for this second set, M1 has 30 irrelevant variables, M2 has 29 irrelevant variables and lastly, M3 with 27 irrelevant variables. Table 4.6 shows the number of variables in both sets of initial GUMS.

Table 4.6
Number of Variables in Initial GUMS

k = 18 (small GUMS)				k = 30 (large GUMS)			
True Model	Variable			True Model	Variable		
	Relevant	Irrelevant	Total		Relevant	Irrelevant	Total
M1	-	18	18	M1	-	30	30
M2	1	17	18	M2	1	29	30
M3	3	15	18	M3	3	27	30

Moreover, these initial GUMS are tested at 1% and 5% levels of significance using the simulated y_{it} with 100 replications for each of four monitoring stations under study. Two sample sizes are used for every level of significance. Large sample includes all observations (n = 550), whereas small sample refers to the first half of the observations, (n = 275). All conditions that have been set up for the simulation study are summarized in Table 4.7 below:

Table 4.7
Conditions of Simulation Study

Condition	Level
Sample size	Small, n = 275 Large, n = 550
Correlation strength among equations	Weak, ρ = 0.2 Strong, ρ = 0.9
Initial GUMS	Small, k = 18 total variables Large, k = 30 total variables
Number of equations	Small, m = 2 Large, m = 4
Main significance level	α = 1% α = 5%
True specification model	M1 = no relevant variable M2 = one relevant variable M3 = three relevant variables

For both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*, the outcomes of simulation are determined by calculating percentages of final selected models similar to true models through the predictor variables, in order to reflect the performances of algorithms in finding true specifications.

The simulation outcomes were classified into four categories, following Yusof (2016), which originally initiated by Hoover and Perez (1999). All of the equations in the model were needed to belong in the same category in order to be grouped into any specific categories, see Table 4.8.

Table 4.8
Simulation Outcomes Categories

Category	Criteria	Explanation
1	TRUE = FINAL	The true specification is chosen.
2	TRUE $\subset$ FINAL	The true specification is nested in the final specification.
3	TRUE $\not\subset$ FINAL	An incorrect specification is chosen, the true specification is not nested in the final specification.
4	At least one equation failed to fall under the same category.	

Note. Reprinted from "SURE-Autometrics Algorithm for Model Selection in Multiple Equations (Doctoral dissertation)," by N. Yusof, 2016, Universiti Utara Malaysia.

Firstly, Category 1 is where all equations have precisely similar variables as in true specification, which is the aim of this simulation. High percentage of outcomes in Category 1 indicates good performance of an algorithm. Category 2 is for models which consist of all equations with more selected variables than in true specification. In other words, true specifications are nested in final models. Category 3 accommodates for all equations with some or none of variables as in true specification. Additionally, there are certain circumstances where model with outcomes of each equation is different, for example Equation 1 falls in Category 1, but Equation 2 belongs to Category 2. Therefore, the final one is a category that is designated for this kind of situation, Category 4. Full results of *SURE(IFGLS)–Autometrics* and *SURE(EM)–Autometrics* are in Section 4.3.

4.3 Simulation Results

SURE(IFGLS)–Autometrics and *SURE(EM)-Autometrics* were investigated through simulation study under several conditions for their performances in models selection analysis. Section 4.3.1 and Section 4.3.2 report the results of *SURE(IFGLS)-*

Autometrics for four and two equations, respectively. Section 4.3.3 elaborates results of *SURE(EM)-Autometrics* for four equations and Section 4.3.4 for two equations.

4.3.1 *SURE(IFGLS)-Autometrics* (Four Equations)

This simulation started with all WQI data available in hands. Since there are altogether four monitoring stations, thus simuation began with four equations models first. Number of monitoring stations represents number of equations in models. True models of M1, M2 and M3 were used for two sets of initial GUMS consist of 18 and 30 independent variables at 1% and 5% levels of significance in large sample of 550 observations and also small sample of 275 with strong (0.9) and weak (0.2) disturbance correlations.

Table 4.9 displays summary results for *SURE(IFGLS)-Autometrics* of the strong correlation. These results can also be viewed in Figure 4.1 with different colours representing different categories. The blue characterizes Category 1, red refers to Category 2, green signifies Category 3 and finally purple denotes Category 4. It is obviously seen that only in M1 recorded final models were exact to true specifications in Category 1 (in blue), but not for M1 of larger GUMS at 5% level of significance (in red and purple). The highest percentages of M1 come from small GUMS at 1% level. The large sample even had all models (100%) in Category 1 (blue only), while small sample with 86%. Nevertheless, in M2 and M3, final models are found to be in Category 3 and 4, with exception for 14% in Category 2 in large sample of small GUMS at 1% level.

Meanwhile, there are also totally 100% in Category 3 (green only) of large GUMS at 1% level and also of small GUMS at 5% level for both sample sizes of M2 and M3.

This also happened in M3 models of small GUMS at 1% level. The overall results show that when more irrelevant variables were included, algorithms tend to show inability to select models according to true specification. Even with strong correlation, algorithms still underperformed, most probably due to the complexity of four equations involved.

The same four equations were also tested with weak disturbance correlation of 0.2 as seen in Table 4.10 and Figure 4.2. Obviously, there is only 100% in Category 3 (green only) for all conditions under 1% level, regardless of true specifications of models, sample sizes and also number of variables in the models, with exception of 100% in Category 2 (red only) of M1. This shows that majority of final models were not the same as true specifications where some of variables were not selected.

In the meantime, all final models contained more than just a constant when using the simplest model, M1 under significance level of 5%, similar to 1% level. It was thus recorded as 100% in Category 2 (red only) both in large and small sample sizes. Given many irrelevant variables in M1, a less tight significance level of 5% might have caused excess significant irrelevant variables in this Category 2. As for Category 4, final models are allocated in this category at 5% significance level for M2 and M3. This means at least one equation in the models did not belong to same category as other equations.

Table 4.9

Percentages of Simulation Results for SURE(IFGLS)-Autometrics with m = 4 and ρ = 0.9

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	86	-	-	14	72	14	-	14	20	-	-	80	-	50	-	50
	550	100	-	-	-	50	-	-	50	23	14	-	63	-	61	-	39
M2	275	-	-	75	25	-	-	100	-	-	-	100	-	-	-	50	50
	550	-	14	72	14	-	-	100	-	-	-	100	-	-	-	100	-
M3	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
	550	-	-	100	-	-	-	100	-	-	-	100	-	-	-	89	11
Average %		31.00	2.33	57.83	8.83	20.33	2.33	66.67	10.67	7.17	2.33	66.67	23.83	-	18.50	56.50	25.00

Table 4.10

Percentages of Simulation Results for SURE(IFGLS)-Autometrics with m = 4 and ρ = 0.2

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-	-
	550	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-	-
M2	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	20	80
	550	-	-	100	-	-	-	100	-	-	-	67	33	-	-	12	88
M3	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	50	50
	550	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
Average %		-	33.33	66.67	-	-	33.33	66.67	-	-	33.33	61.17	5.50	-	33.33	30.33	36.33

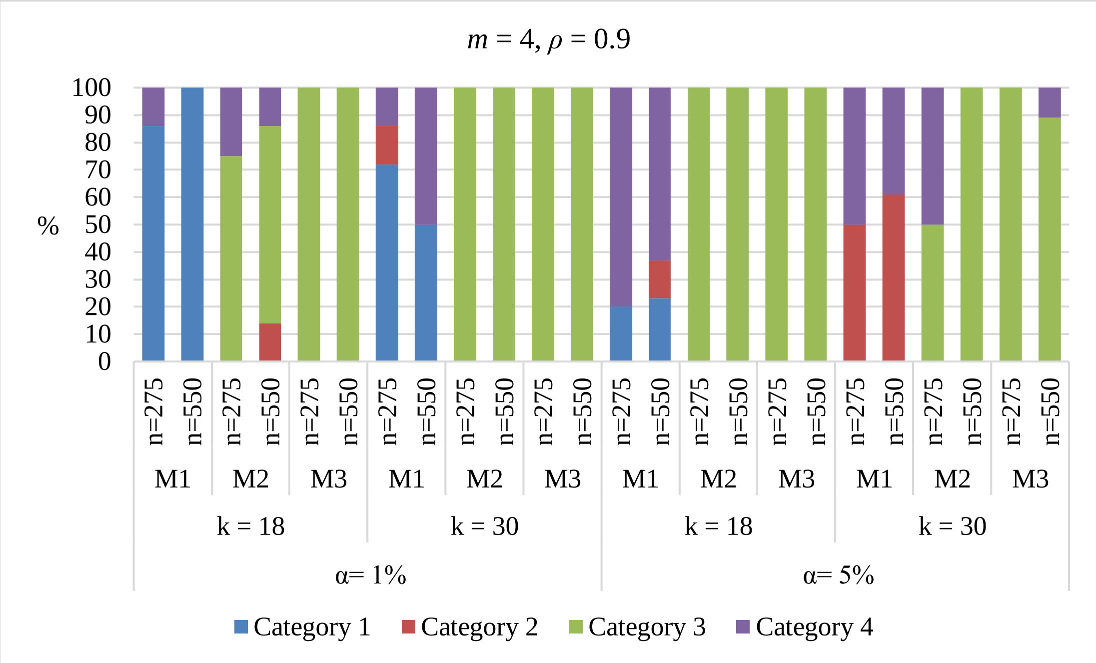

Figure 4.1 Percentages of Simulation Results for *SURE(IFGLS)-Autometrics* with $m = 4$ and $\rho = 0.9$

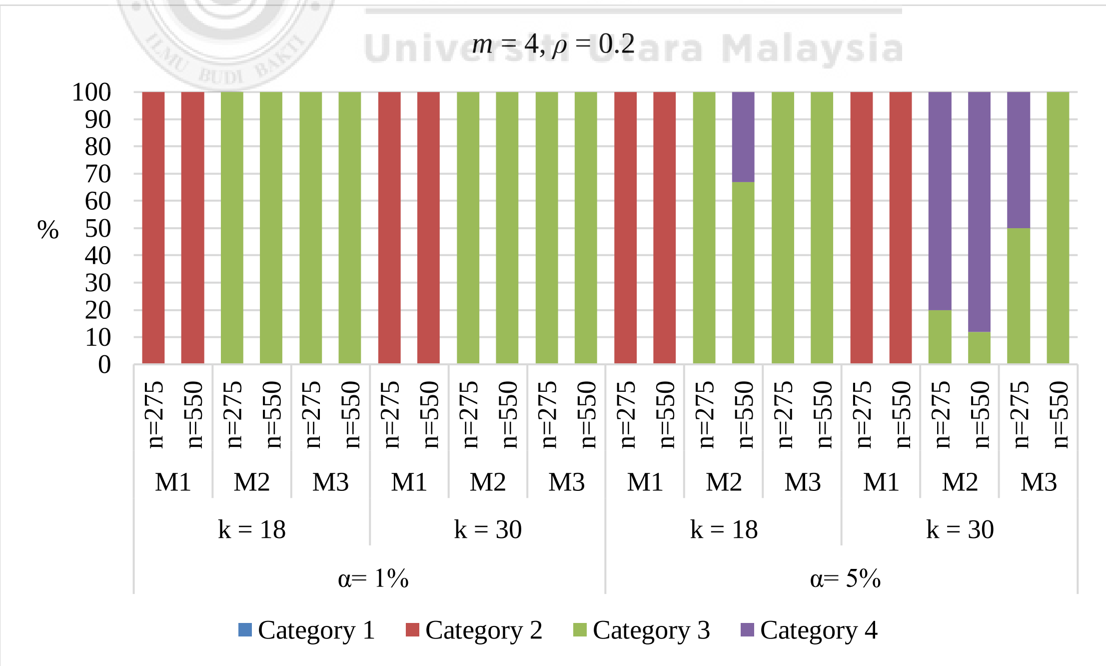

Figure 4.2 Percentages of Simulation Results for *SURE(IFGLS)-Autometrics* with $m = 4$ and $\rho = 0.2$

Figures 4.3 and 4.4 show average percentages of final models selected by *SURE(IFGLS)-Autometrics* for four equations models of small and large GUMS, respectively. These percentages are according to categories based on disturbances correlation strengths and significance levels where both figures display almost similar pattern. The highest average percentages come from Category 3, particularly 66.67% from combination of weak correlation and 1% significance level in both small and large GUMS. This is a category where only some or none of variables in final models were same as in true specifications. Thus, higher percentages in Category 3 signifies the inability of algorithms to capture the true specifications. In the meantime, Category 2 and 4 exhibit mixed results for both figures. With some of the final models located in Category 2, there is a possibility that some equations might actually have very few irrelevant variables in the models and close to resembling true specifications. At the same time, final models in Category 4 might also contain equations which possibly belong to Category 1.

Nevertheless, final models of both GUMS were only same as true specifications (Category 1) when correlation was strong and significance level was 1%. But, when significance level changed to 5% with same correlation, final models found in Category 1 were only from small GUMS. Weak correlation however did not result any models in this category for both GUMS. The expectations to gain high percentages in Category 1 were unfortunately not fully met. Thus, there is a chance for the algorithm to be improved with less number of equations implying less complex models. Next section reports the results for *SURE(IFGLS)-Autometrics* for two equations, while other conditions remained the same.

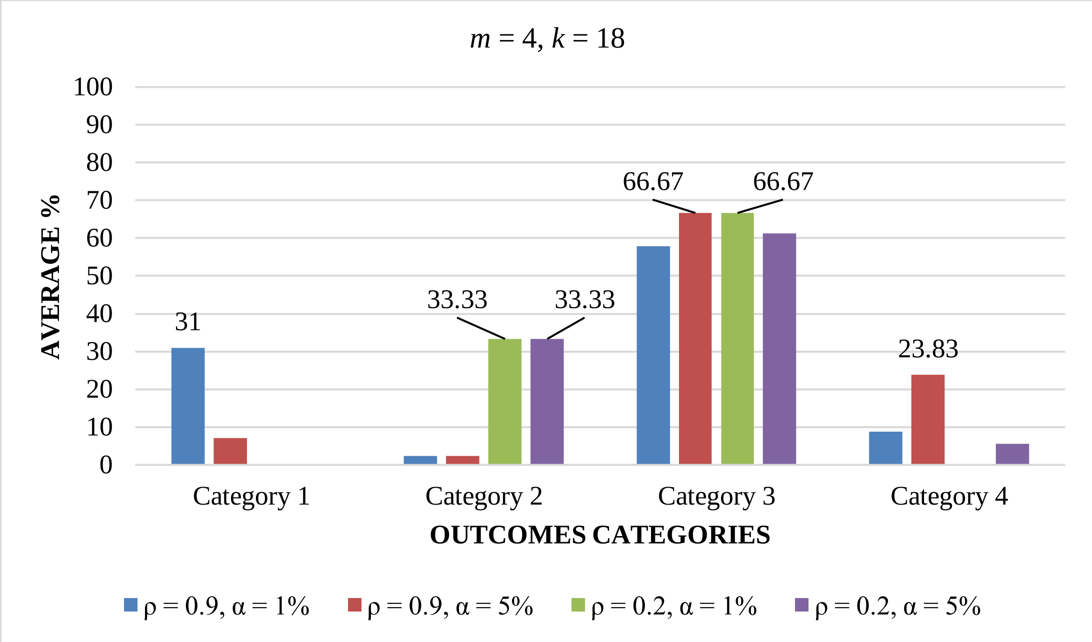

Figure 4.3. Average Percentages of Final Models by *SURE(IFGLS)-Autometrics* for Four Equations Models of Small GUMS

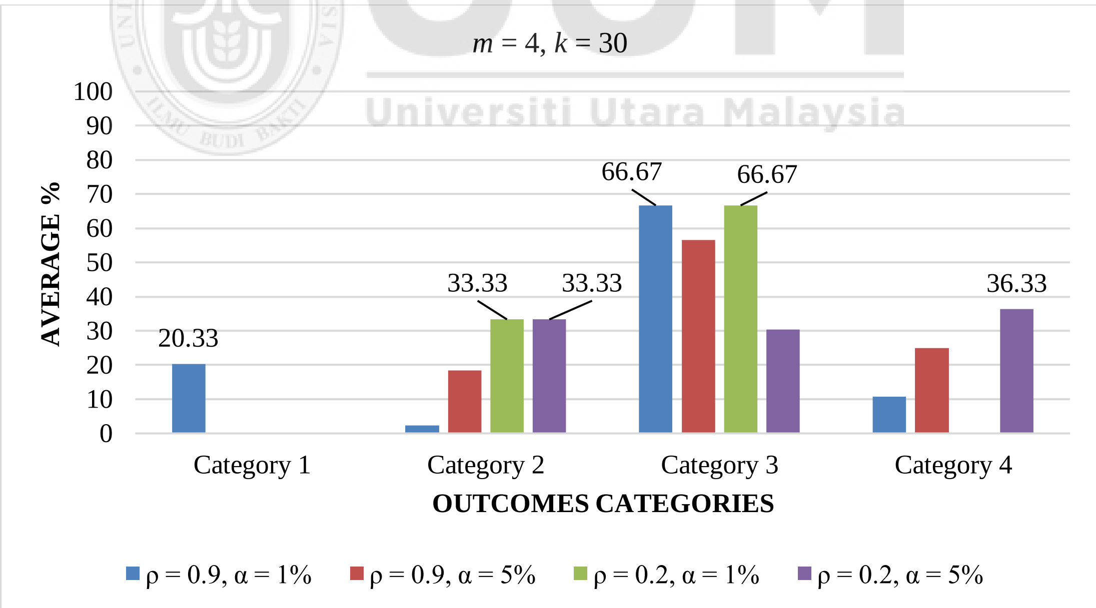

Figure 4.4. Average Percentages of Final Models by *SURE(IFGLS)-Autometrics* for Four Equations Models of Large GUMS

4.3.2 *SURE(IFGLS)-Autometrics* (Two Equations)

The simulation study was again executed for two equations models to understand the algorithm's performance in the most minimal system of equations. Table 4.11 shows outcomes for two equations of strong disturbance correlation according to different true specifications, sample sizes, significance levels and GUMS. It is apparent that overall percentages for 1% significance level are higher than models under 5%. This is easily seen as higher blue columns on the left side compared to the right side of Figure 4.5. The finding is consistent with results by Hendry and Doornik (2014) who compared Hoover-Perez experiments with *PcGets* and *Autometrics* without pre-search step. Hoover-Perez algorithm did not have a pre-search step. Even with pre-search step, comparable outcome was still found.

Moreover, both small and large GUMS in each significance level show almost similar results. For models selected at 1% level, all true models have percentages at range of 80%-90% with M1 recorded highest percentages in Category 1. Yet, the values decreased gradually from M1 to M2 and M3, but no significant difference in large or small samples. Category 4 (in purple) is in adverse direction of Category 1, where M3 had the most models, followed by M2 and M1. Only a few models were found in Category 2 at 1% significance level. Meanwhile, for 5% significance level, a steep decline occurred with lower percentages in Category 1 (lower blue columns) than in 1%. The maximum value is only 55% in M1 of Category 1 with small GUMS in large sample. General percentages in Category 1 are almost close to percentages in Category 2 for both sample sizes. In addition, there are a few similarities between selections under 1% and 5% significance level. The percentages of models reduced in Category 1, but increased in Category 4 from the simplest to complex model. Furthermore, Category 3 does not contain any models in any conditions.

Table 4.11

Percentages of Simulation Results for SURE(IFGLS)-Autometrics with m = 2 and ρ = 0.9

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	93	-	-	7	94	6	-	-	47	47	-	6	54	42	-	4
	550	95	5	-	-	90	10	-	-	55	39	-	6	48	52	-	-
M2	275	93	-	-	7	86	7	-	7	44	31	-	25	33	48	-	19
	550	91	-	-	9	90	4	-	6	46	26	-	28	28	47	-	25
M3	275	82	2	-	16	79	6	-	15	25	36	-	39	14	48	-	38
	550	83	2	-	15	79	3	-	18	38	27	-	35	19	47	-	34
Average %		89.50	1.50	-	9.00	86.33	6.00	-	7.67	42.50	34.33	-	23.17	32.67	47.33	-	20.00

Table 4.12

Percentages of Simulation Results for SURE(IFGLS)-Autometrics with m = 2 and ρ = 0.2

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	20	40	-	40	-	100	-	-	-	100	-	-	-	100	-	-
	550	-	75	-	25	-	100	-	-	-	91	-	9	-	97	-	3
M2	275	36	16	-	48	18	34	-	48	7	55	-	38	1	89	-	10
	550	29	12	-	59	22	33	-	45	8	62	-	30	-	83	-	17
M3	275	56	4	-	40	14	48	-	38	10	45	-	45	2	79	-	19
	550	60	5	-	35	30	26	-	44	9	47	-	44	2	84	-	14
Average %		33.50	25.33	-	41.17	14.00	56.83	-	29.17	5.67	66.67	-	27.67	1.00	88.67	-	10.50

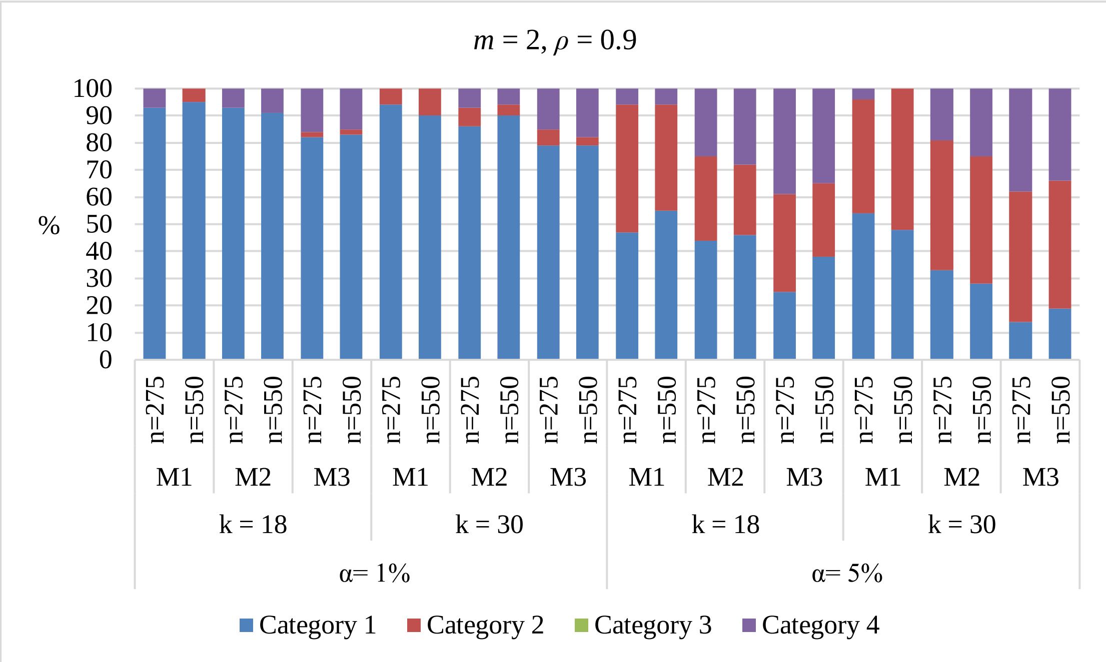

Figure 4.5 Percentages of Simulation Results for *SURE(IFGLS)-Autometrics* with $m = 2$ and $\rho = 0.9$

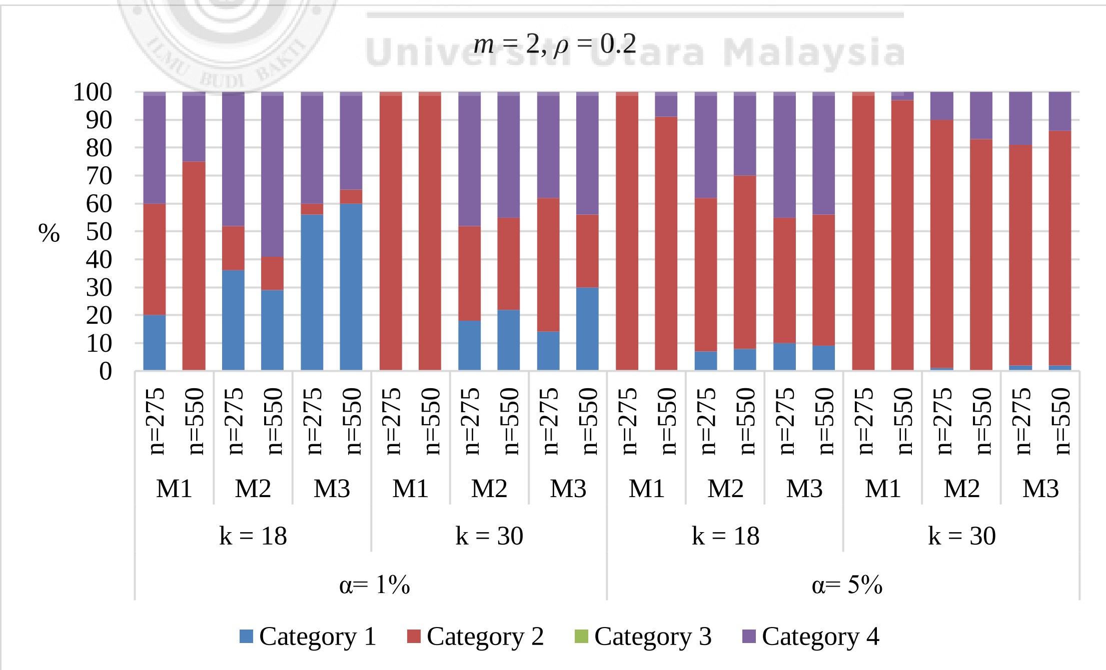

Figure 4.6 Percentages of Simulation Results for *SURE(IFGLS)-Autometrics* with $m = 2$ and $\rho = 0.2$

In Table 4.12 and Figure 4.6, results for weak correlation disturbances of 0.2 are summarised. Unlike previous results, Category 2 is now portrayed as having the highest percentages (higher red columns) which means true specifications were mostly nested in final models. There are also 100% (red only) of Category 2 in M1. This means even by using empty model, there was no selection of any correct specifications, except only 20% for large sample of small GUMS under 1% of significance level. This contrast direction of percentages in Category 2 compared to results from strong disturbance correlation has contributed to a reverse pattern in Category 1, with M3 has more correct classifications (higher blue columns) for most conditions measured. Again, there is still none of the final models belongs to Category 3. In whole, more percentages are seen in large samples than in small ones.

Figures 4.7 and 4.8 show average percentages of final models selected by *SURE(IFGLS)-Autometrics* for two equations models of small and large GUMS, respectively.

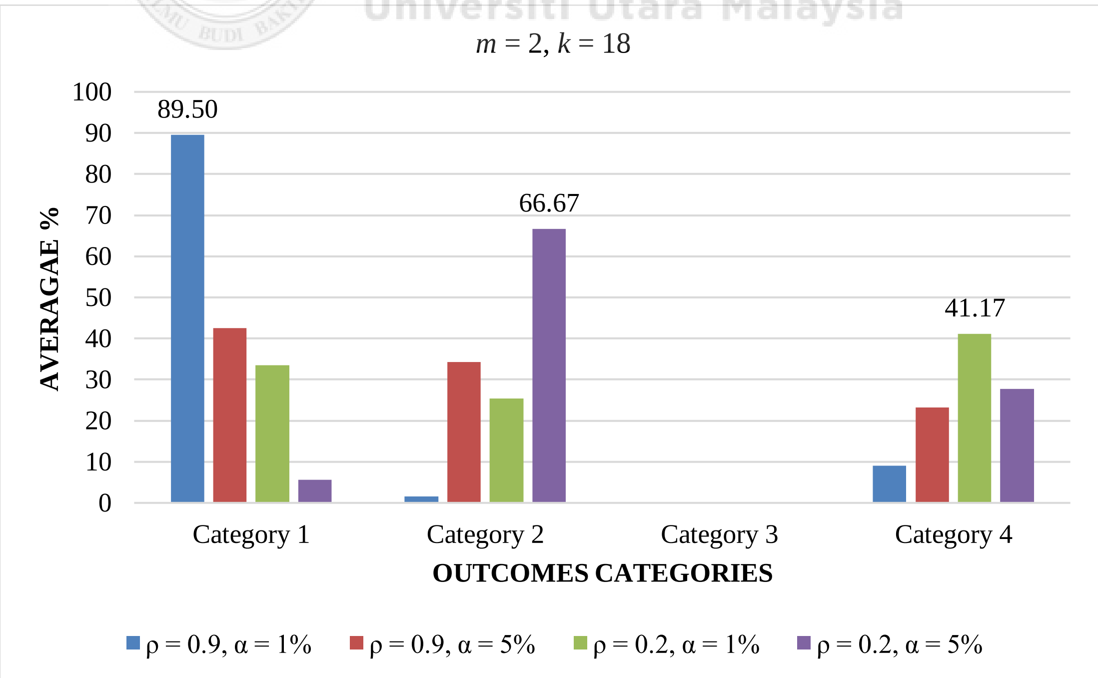

Figure 4.7. Average Percentages of Final Models by *SURE(IFGLS)-Autometrics* for Two Equations Models of Small GUMS

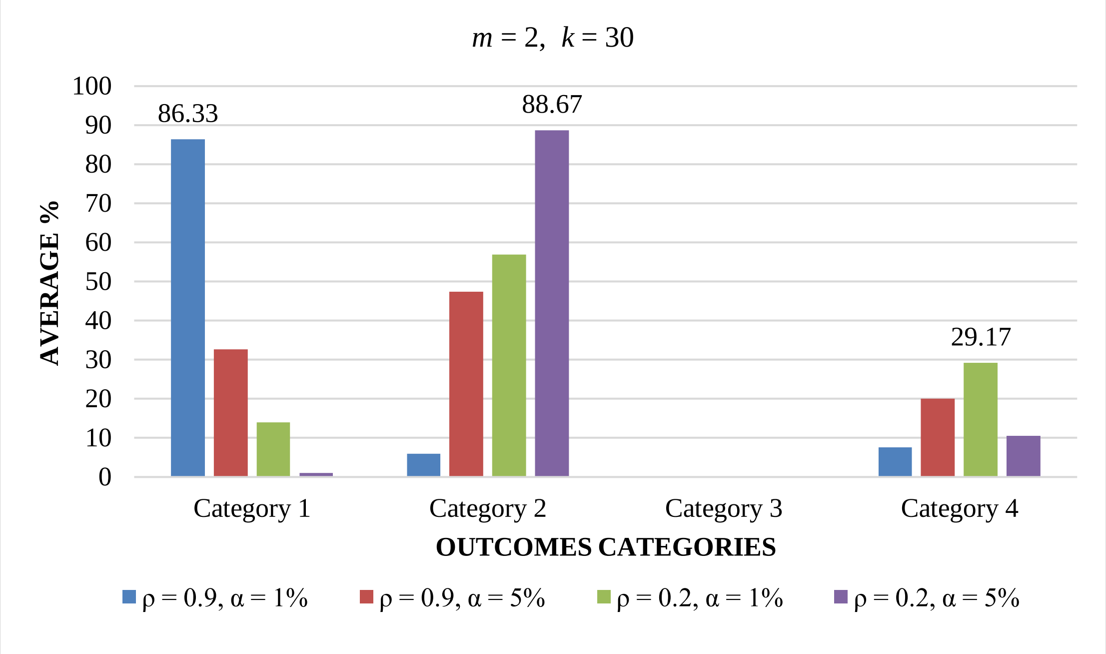

Figure 4.8. Average Percentages of Final Models by *SURE(IFGLS)-Autometrics* for Two Equations Models of Large GUMS

The two figures display a resemblance in the pattern. This time, Category 1 has higher average percentages compared to four equations models, with strong correlation and 1% significance level give highest value of 89.50% in small GUMS and 86.33% in large GUMS. *SURE(IFGLS)-Autometrics* shows better capability to select true specifications for all combinations of disturbances correlations strengths and significance level in both GUMS sizes. Meanwhile, Category 2 has more final models in large GUMS than small GUMS. With larger GUMS, more irrelevant variables needed to be removed which could be more difficult and resulted with true specifications nested in the final models. Category 3 turns out to be empty since there is no model with all equations not containing true specifications. Lastly, there are still models under Category 4 regardless of correlations strengths and significance levels. Overall, two equations models with 0.9 disturbances correlation and 1% significance selected the most final models of true specifications for *SURE(IFGLS)-Autometrics*.

4.3.3 *SURE(EM)-Autometrics* (Four Equations)

SURE(EM)-Autometrics was also examined in simulation study with four equations models utilising same conditions for *SURE(IFGLS)-Autometrics*. Results were tabulated in four categories based on true models, sample sizes, number of variables in initial models and significance levels.

The outcomes for strong disturbance correlation of 0.9 in models of four equations are presented in Table 4.13 and Figure 4.9. Only for M1, the algorithm managed to select final models with true specification, except in large GUMS of 5% level. Both sample sizes of small and large GUMS for M1 at 1% level show good performance with 88% and 100%. Simple models again facilitated algorithm for better selections. For M2 and M3, there are models only in Categories 3 and 4, but still 33% of them exist in Category 2 of large sample of small GUMS at 1% level. Another finding is that all models selected at 1% level of large GUMS were all in either Category 1 (blue only) for empty model or Category 3 (green only) when other variables were added. This means variables in true specifications were accurately selected or only some or none of the variables chosen. Furthermore, M3 too recorded 100% in Category 3 (green only) for all types of situations. Overall, there are mixed results across the simulation.

Table 4.14 and Figure 4.10 summarised results for weak disturbances correlation. It is clear that in M1, only large GUMS for both sample sizes at 5% level have models in Categories 2 and 4 (in red and purple), while 100% of models selected in Category 2 (red only) in the other three GUMS of M1 for both sample sizes. Simplicity of M1 with only a constant had made possible for final models to be in Category 2. The remaining conditions all had 100% in Category 3 (green only).

Table 4.13

Percentages of Simulation Results for SURE(EM)-Autometrics with m = 4 and ρ = 0.9

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	88	-	-	12	100	-	-	-	50	10	-	40	-	20	-	80
	550	100	-	-	-	100	-	-	-	9	8	-	83	-	50	-	50
M2	275	-	-	50	50	-	-	100	-	-	-	-	100	-	-	100	-
	550	-	33	45	22	-	-	100	-	-	-	60	40	-	-	75	25
M3	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
	550	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
Average %		31.33	5.50	49.17	14.00	33.33	-	66.67	-	9.83	3.00	43.33	43.83	-	11.67	62.50	25.83

Table 4.14

Percentages of Simulation Results for SURE(EM)-Autometrics with m = 4 and ρ = 0.2

True model	*n*	Category															
		α = 1%								α = 5%							
		k = 18				*k* = 30				*k* = 18				*k* = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	-	100	-	-	-	100	-	-	-	100	-	-	-	50	-	50
	550	-	100	-	-	-	100	-	-	-	100	-	-	-	77	-	23
M2	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
	550	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
M3	275	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
	550	-	-	100	-	-	-	100	-	-	-	100	-	-	-	100	-
Average %		-	33.33	66.67	-	-	33.33	66.67	-	-	33.33	66.67	-	-	21.17	66.67	12.17

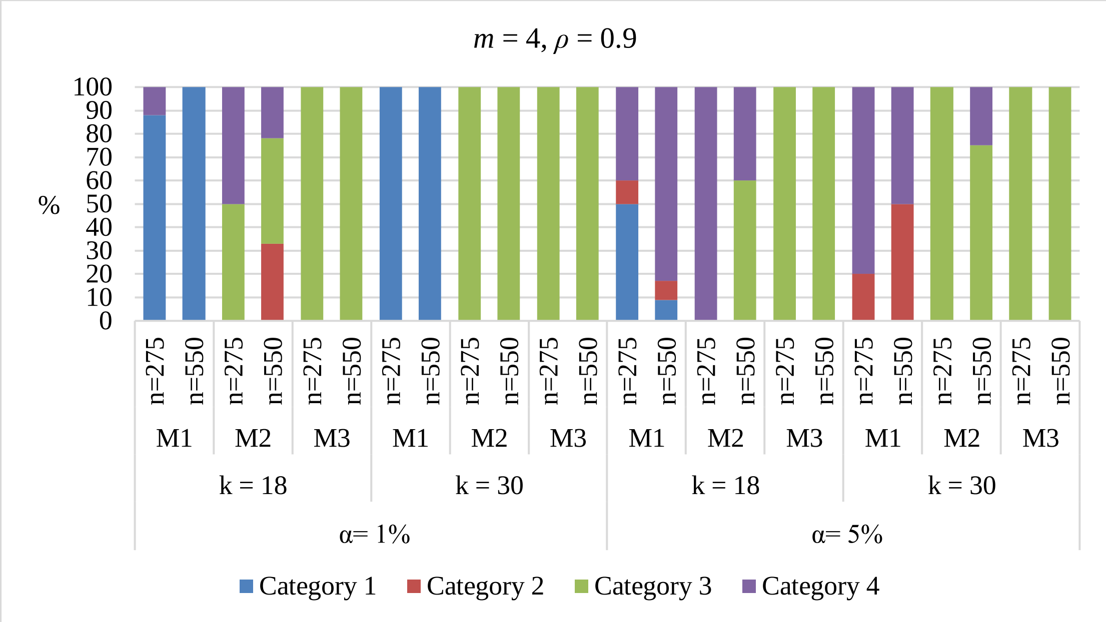

Figure 4.9 Percentages of Simulation Results for *SURE(EM)-Autometrics* with m = 4 and ρ = 0.9

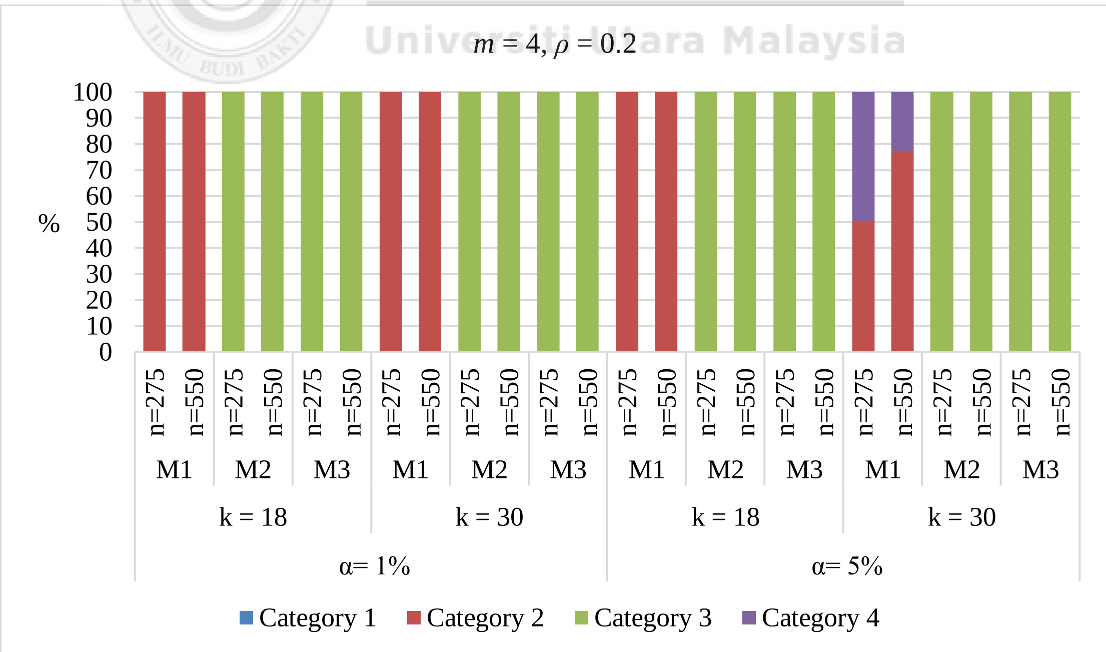

Figure 4.10 Percentages of Simulation Results for *SURE(EM)-Autometrics* with m = 4 and ρ = 0.2

Figures 4.11 and 4.12 exhibit average percentages for final models selected by *SURE(EM)-Autometrics* for four equations models of small and large GUMS, correspondingly. One main finding is final models only present in Category 1 when disturbances correlation was 0.9 in small GUMS with 31.33% of models found. Meanwhile, 33.33% of models solely found when significance level was 1% under large GUMS. Thus, Category 2 for small GUMS with same correlation has lower percentages, but more final models when correlation was weak. Fewer variables in these initial models could have eased the removal of irrelevant ones. In large GUMS, most models selected happens only when weak correlation was matched with 1% significance level. Failure to eliminate the more irrelevant variables in large GUMS had prompted more final models for its Category 2 than Category 1.

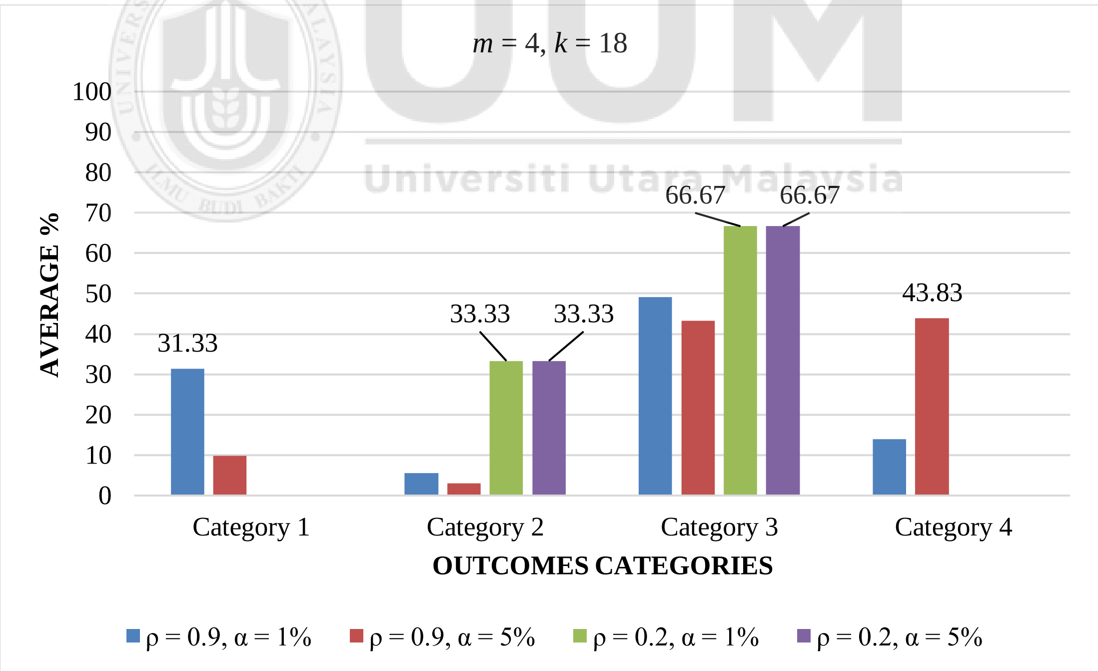

Figure 4.11. Average Percentages of Final Models by *SURE(EM)-Autometrics* for Four Equations Models of Small GUMS

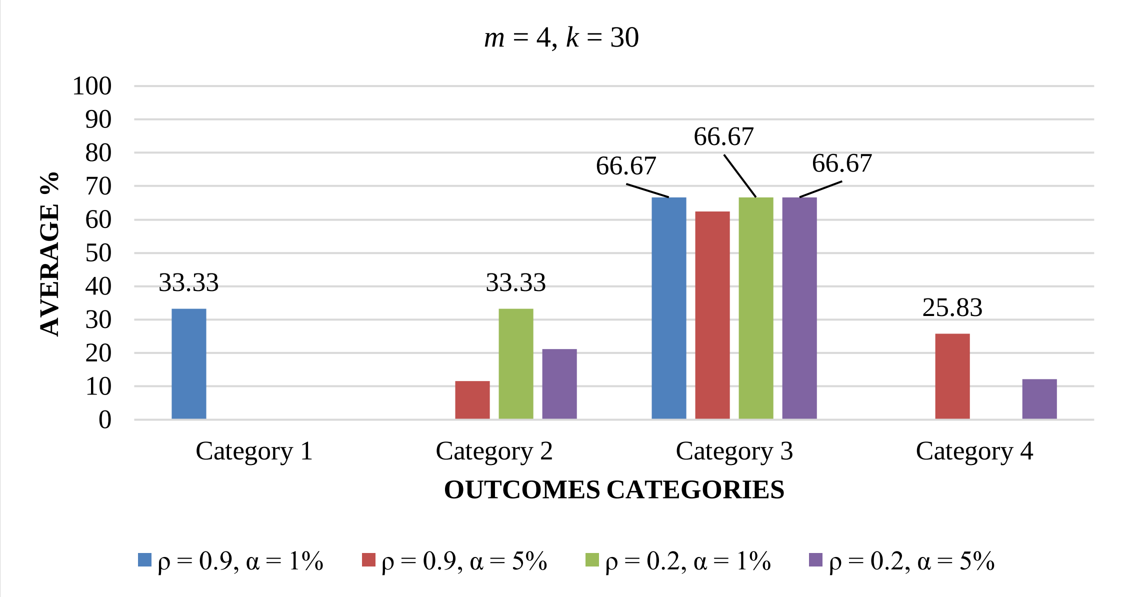

Figure 4.12. Average Percentages of Final Models by *SURE(EM)-Autometrics* for Four Equations Models of Large GUMS

Meanwhile, Category 3 still has highest average percentages for both GUMS, with majority of the combinations give 66.67%. This shows that there are still relevant variables not included or irrelevant variables not removed from the models. Finally, results for Category 4 appear to be different for both GUMS. Generally, the use of four equations in *SURE(EM)-Autometrics* still does not satisfy the goal to gain high percentages in Category 1. The simulation study continued with two equations models in next section.

4.3.4 *SURE(EM)-Autometrics* (Two Equations)

Table 4.15 presents percentages gained for each category by *SURE(EM)-Autometrics* of two equations with strong disturbance correlation. In comparison, results for small GUMS are generally more or less the same as the ones in large GUMS, but higher percentages were found under 1% level of significance which again accords with earlier observations in *SURE(IFGLS)-Autometrics* and also by Hendry and Doornik (2014).

Table 4.15

Percentages of Simulation Results for SURE(EM)-Autometrics with m = 2 and ρ = 0.9

True model	n	Category															
		α = 1%								α = 5%							
		k = 18				k = 30				k = 18				k = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	84	10	-	6	83	13	-	4	10	82	-	8	9	85	-	6
	550	86	7	-	7	82	4	-	14	14	70	-	16	12	79	-	9
M2	275	89	3	-	8	86	2	-	12	45	33	-	22	35	45	-	20
	550	90	5	-	5	88	5	-	7	50	37	-	13	34	47	-	19
M3	275	91	2	-	7	90	5	-	5	54	27	-	19	33	43	-	24
	550	93	5	-	2	92	3	-	5	53	27	-	20	30	49	-	21
Average %		88.83	5.33	-	5.83	86.83	5.33	-	7.83	37.67	46.00	-	16.33	25.50	58.00	-	16.50

Table 4.16

Percentages of Simulation Results for SURE(EM)-Autometrics with m = 2 and ρ = 0.2

True model	n	Category															
		α = 1%								α = 5%							
		k = 18				k = 30				k = 18				k = 30			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
M1	275	-	52	-	48	-	100	-	-	-	100	-	-	-	100	-	-
	550	12	68	-	20	-	100	-	-	-	89	-	11	-	90	-	10
M2	275	20	49	-	31	15	38	-	47	6	54	-	40	-	87	-	13
	550	35	27	-	38	19	36	-	45	9	46	-	45	1	89	-	10
M3	275	54	9	-	37	17	33	-	50	8	46	-	46	1	77	-	22
	550	56	10	-	34	20	29	-	51	10	40	-	50	2	80	-	18
Average %		29.50	35.83	-	34.67	11.83	56.00	-	32.17	5.50	62.50	-	32.00	0.67	87.17	-	12.17

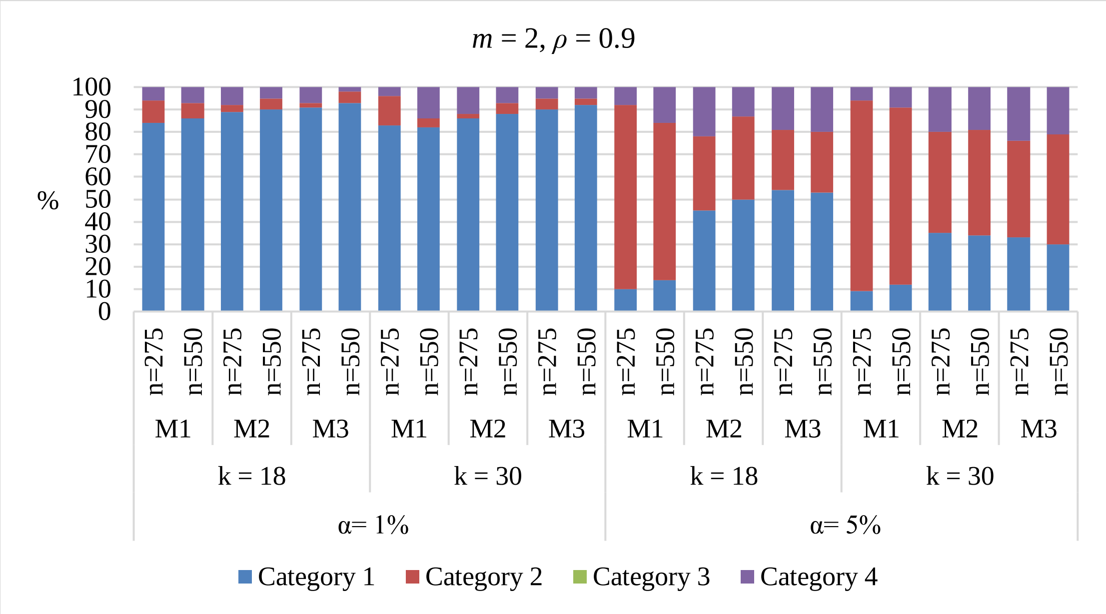

Figure 4.13 Percentages of Simulation Results for *SURE(EM)-Autometrics* with m = 2 and ρ = 0.9

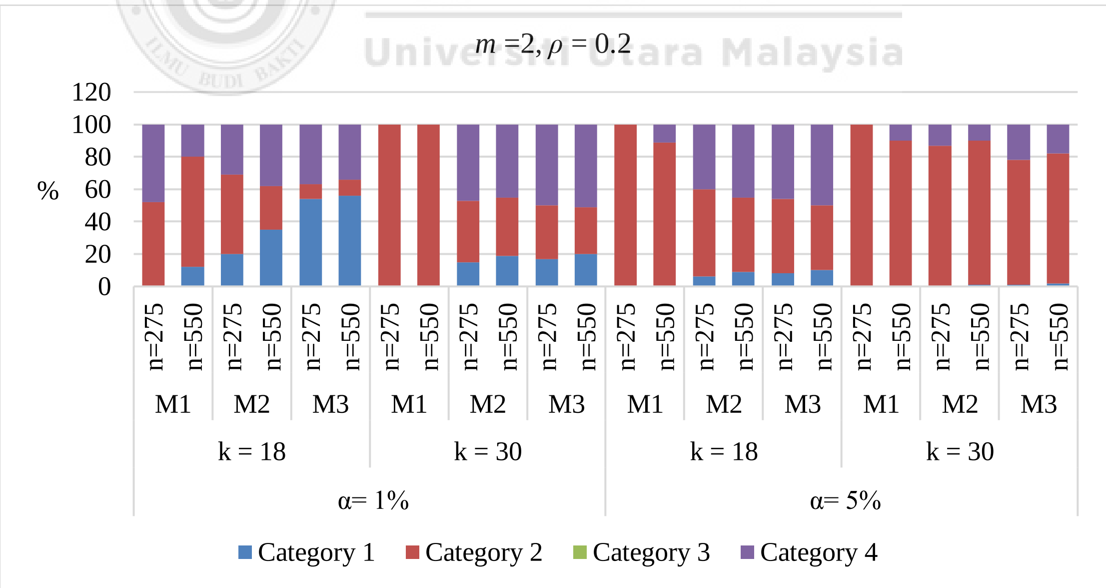

Figure 4.14 Percentages of Simulation Results for *SURE(EM)-Autometrics* with m = 2 and ρ = 0.9

A comparison of results in *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* reveals that *SURE(EM)-Autometrics* displayed different direction of gradual increment from M1 to M3 at 1% level. In Figure 4.13, Category 1 (in blue) still managed to provide considerably high percentages of more than 80% for both small and large initial GUMS, while these values continued to surge up to close to 90% in M2 and slightly more than 90% in M3. Nevertheless, large and small samples gave very little effect on the models selection as both showed approximately same values. Only less than 15% of final models were grouped in each Category 2 and 4 and again none in Category 3.

Performance of *SURE(EM)-Autometrics* for 5% level of significance is similar to *SURE(IFGLS)-Autometrics*. General percentages in Category 1 fell drastically (lower blue columns) compared to categories in 1% level and thus triggered more final models classed in Category 2 (higher red columns). M3 of small GUMS has the highest percentages in Category 1 which is caused by inclusion of the most correlated variable with dependent variable, x_{i4t} and strong disturbance correlation. This is followed by M2 with moderate accomplishment and finally M1 with the least scores. Overall, there is close resemblance to *SURE(IFGLS)-Autometrics.*

In the meantime, Table 4.16 and Figure 4.14 present results of final models of *SURE(EM)-Autometrics* with weak correlation of disturbance. M3 still covered most models in Category 1 (in blue) compared to the other two initial GUMS, while M2 is not far behind M3. The strength of disturbance correlation did not influence the models selection processes in this case, unlike the consequence of x_{i4t} inclusion in the model. This finding was earlier discovered in *SURE(IFGLS)–Autometrics* of same simulation conditions. Even in an empty model as in M1, the algorithm could not produce final model exactly like in true specifications, but only of 12% in large sample in small GUMS at 1% level.

On the other hand, there are conditions where all final models contained the constant with other additional independent variables that is 100% of Category 2 (red only). No model fell in Category 3, but mixed results are attained for Category 4. On the whole, most true specifications were found to be nestled in the final models, Category 2. In addition, it is quite noticeable that large samples yielded more precise selections (higher blue columns) than in small samples. It is clearly seen that weak correlation had made SURE model less efficient. Figures 4.15 and 4.16 show average percentages of final models selected by *SURE(EM)-Autometrics* for two equations models of small and large GUMS, respectively.

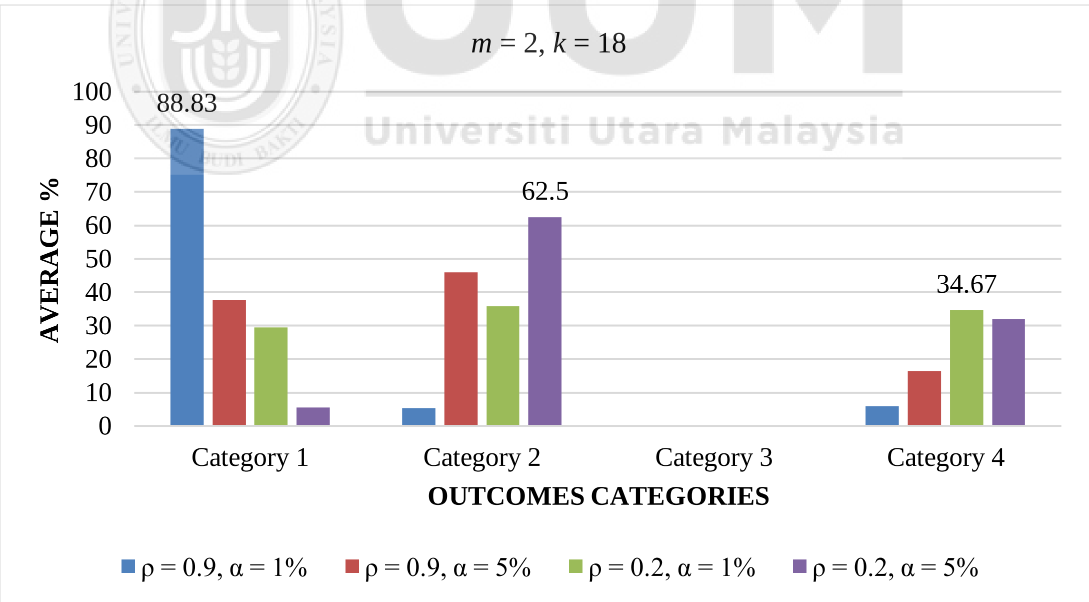

Figure 4.15. Average Percentages of Final Models by *SURE(EM)-Autometrics* for Two Equations Models of Small GUMS

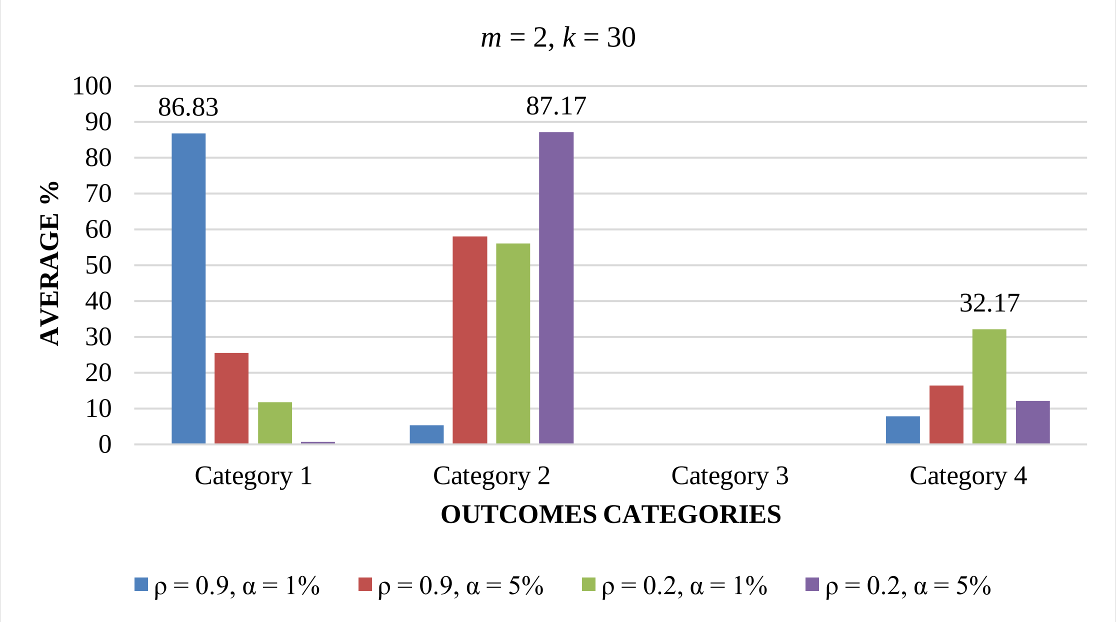

Figure 4.16. Average Percentages of Final Models by *SURE(EM)-Autometrics* for Two Equations Models of Large GUMS

The figures display similar patterns for both GUMS. The highest average percentage of Category 1 of 88.83% is again recorded in small GUMS with disturbances correlation of 0.9 and 1% significance level. Final models were still found to be same as true specification in the other three combinations of disturbances correlations and significance levels. Large GUMS too managed to gain 86.83% in this category at 1% significance level of 0.9 disturbance correlation, but much lower percentages in other conditions combinations. The percentages in Category 2 were higher in large GUMS than small GUMS. On contrary, no final models belong to Category 3 which of incorrect specifications. Average percentages for Category 4 are all less than 40%. On the whole, two equations models with 0.9 disturbances correlation 1% significance level gives the best conditions in selecting models for *SURE(EM)-Autometrics*.

4.4 Summary of Findings

In view of models selection, the resemblance of final models selected with models' true specifications, implies a remarkable performance of an algorithm. In other words, an algorithm is highly anticipated to choose variables which are in the true models generated prior to selections processes. This is where irrelevant variables are added with an aim to test the algorithm's efficiency in removing the inappropriate ones and to make decisions on the most satisfactory model instead of choosing it randomly. Various conditions are set in simulation study to assess the algorithm.

The newly built algorithms, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were each run under several conditions involving different true models of M1, M2 and M3; two and four number of equations in the model; sample sizes of 550 and 275; initial GUMS of 18 and 30 variables; strength of correlation disturbances of 0.9 and 0.2; and finally significance levels of 1% and 5%, making it a total of 192 conditions to be fulfilled for the purpose of measuring both algorithms' abilities in selecting the final models. Under every level of disturbance correlation for each true model in every sample size of any number of equations for one algorithm, the artificial dependent variables were generated with 100 replications. One algorithm used 2400 data sets, thus altogether 4800 data sets produced for both algorithms to be utilised throughout the whole simulations.

Overall findings throughout the analysis showed that *SURE(IFGLS)-Autometrics* gives almost comparable results to *SURE(EM)-Autometrics*. This signifies that both algorithms equally perform in this simulation study. Both algorithms had exhibited some similar results including high performance at 1% level of significance instead of

5%. The tight significance level of 1% had possibly successfully avoided any significant irrelevant variables to be in the final model.

Number of equations also played a vital role in this simulation study. The use of four equations in the systems had only provided final models in Category 1 for strong correlation of disturbance of 0.9. In Table 4.17, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were able to acquire just 31% in small GUMS at 1% level. In large GUMS, *SURE(EM)-Autometrics* had higher percentage by 13% than *SURE(IFGLS)-Autometrics*. At 5% significance level with same disturbance correlation, only small GUMS found the true specifications, but very low percentages of less than 10% for both algorithms. Other conditions combinations failed to find any models to be in Category 1. Regardless of algorithms, four equations models failed to capture true models as much as in two equations models.

Table 4.17
Average Percentages of Category 1 based on Number of Equations and Size of Initial GUMS

m	*k*	*SURE(IFGLS)-Autometrics*				*SURE(EM)-Autometrics*			
		ρ = 0.9		ρ = 0.2		ρ = 0.9		ρ = 0.2	
		α = 1%	α = 5%	α = 1%	α = 5%	α = 1%	α = 5%	α = 1%	α = 5%
4	18	31.00	7.17	-	-	31.33	9.83	-	-
	30	20.33	-	-	-	33.33	-	-	-
2	18	89.50	42.50	33.50	5.67	88.83	37.67	29.50	5.50
	30	86.33	32.67	14.00	1.00	86.83	25.50	11.83	0.67

On the other hand, for analysis of two equations, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* with strong correlation of disturbance had found approximately close to 90% of final models selected in Category 1, with 89.5% found by *SURE(IFGLS)-Autometrics* and 88.83% by *SURE(EM)-Autometrics* in small

GUMS. As for the large GUMS, the results are still impressive at 86.33% for the former algorithm, while 86.83% for the latter. Results for weak disturbance correlation however were obtained to be far lower than these percentages. Therefore, it is shown that when a smaller number of equations is used, the selections would be better with more true specifications found.

In comparing number of variables in models, it was found that model with less irrelevant variables tend to display better selections. Models with at most 18 irrelevant variables represent small GUMS. The small GUMS of M1 would have 18 irrelevant variables since it was an empty model, M2 had 17 irrelevant variables due to one relevant variable and M3 with 15 irrelevant variables as two relevant variables included in the model. Meanwhile, large GUMS refer to models with at most 30 irrelevant variables. Thus, M1 of large GUMS include 30, M2 had 29 and M3 with 27 irrelevant variables.

As seen in Table 4.17 again, all combinations of conditions gained higher percentage in small GUMS than in large GUMS, except for four equation models with weak correlation and at 5% significance level of strong correlation for large GUMS. The selections processes for small GUMS were presumed simpler and faster than the large GUMS which had resulted in relevant variables retained in the final model, whereas the irrelevant variables removed from the model.

Referring to Table 4.18, percentages of final models in Category 1 obtained within the search for M1 true model appeared in more conditions combinations than M2 and M3 for both algorithms. This is because of the absence of variables in M1 and it was always nested in final models. Using *SURE(IFGLS)-Autometrics*, percentages in M2 for two equations models with strong disturbance correlation were less than percentages in M1 and more than percentages in M3. This indicates their relations with presence of irrelevant variables in models. A model with more variables tends to be more complex.

Yet, in other conditions combinations, increment of percentages in Category 1 from M1 until M3 were also seen, mainly from *SURE(EM)-Autometrics*. M2 contains y_{it-1} and M3 includes y_{it-1}, the most correlated variable x_{i4t} and x_{i4t-1}. This highly correlated variable had been significant in the selections process. All these results however did not apply in four equations models due to algorithms' inabilities to select the true models.

Table 4.18
Average Percentages of Category 1 based on True Specifications and Sample Sizes

True model	*n*	*SURE(IFGLS)-Autometrics*				*SURE(EM)-Autometrics*			
		m = 4		*m* = 2		*m* = 4		*m* = 2	
		ρ = 0.9	ρ = 0.2	ρ = 0.9	ρ = 0.2	ρ = 0.9	ρ = 0.2	ρ = 0.9	ρ = 0.2
M1	275	44.50	-	72.00	5.00	59.50	-	46.50	-
	550	43.25	-	72.00	-	52.25	-	48.50	3.00
M2	275	-	-	64.00	15.50	-	-	63.75	10.25
	550	-	-	63.75	14.75	-	-	65.50	16.00
M3	275	-	-	50.00	20.50	-	-	67.00	20.00
	550	-	-	54.75	25.25	-	-	67.00	22.00

With regards to sample sizes, there are some conditions where small samples of 275 observations performed better than large samples of 550 observations. Such results can be seen in all M1 models of *SURE(IFGLS)-Autometrics* and only in *SURE(EM)-Autometrics* of four equations, as tabulated in Table 4.18. However, it is believed that the simplicity of M1, strong disturbance correlation and the use of two equations inclined to give more influence in these selections. In other conditions where small sample performed better, the differences are only marginal. For example, in M2 of *SURE(IFGLS)-Autometrics*, small samples had additional of merely 0.25% and 0.75% in two equations models.

On the other hand, large samples managed to obtain more percentages in seven out of 14 total conditions combinations compared to small samples. Small samples had more percentages in only five combinations and both samples have same percentages in two more combinations. The gains are particularly seen when using *SURE(EM)-Autometrics* for two equations models. Therefore, large samples are able to select more models with true specifications and also show more consistent performance in models selection.

Meanwhile, the strength of disturbance correlation continued to verify its crucial function in ensuring efficiency of SURE model, where strong correlation of disturbance of 0.9 still demonstrated success. The weak correlation of 0.2 however exhibited contrast results. All in all, there have been mixed results throughout simulations, but both algorithms had displayed almost equal performance. The algorithms were then tested using empirical data sets and reported in the following chapter.

CHAPTER FIVE

VALIDATION OF *SURE(IFGLS)-AUTOMETRICS* AND *SURE(EM)-AUTOMETRICS* USING EMPIRICAL DATA

5.1 Introduction

The analysis in this chapter focuses on performance of the new algorithms using two empirical data sets. Section 5.2 describes all models selection procedures. This is followed by Section 5.3 explaining the error measures used to compare the procedures. Two series of data were employed for this purpose. The first data was the national growth rates as elaborated in Section 5.4, while Section 5.5 detailed the water quality index (WQI) as the second data set. Lastly, Section 5.6 summarizes the findings.

5.2 Models Selection Procedures

Models selection procedures in this study used different estimation methods either by OLS, FGLS, IFGLS or EM algorithm, were executed manually or automatically and based on single equation or multiple equations selections. Manual selections were based on trials using personal knowledge and automated selections refer to algorithmic procedures. In addition, single equation selections signify that equations are selected individually within the model, whereas multiple equations selections indicate simultaneous selections of all equations. The models selection procedures used in this study were classified into five major groups according to selection manners and final models' estimations methods as listed in Table 5.1.

Table 5.1
List of Models Selection Procedures

No.	Models selection procedures	Final models estimations	Selections manners		Package/ software	Group
1	*Mine*	OLS	Manual	Single	*GAUSS 15*	A
2	*Mine-SURE*	FGLS	Manual	Single		
3	*Mine-SURE(IFGLS)*	IFGLS	Manual	Single		
4	*Mine-SURE(EM)*	EM	Manual	Single		
5	*SURE-Mine*	FGLS	Manual	Multiple	*GAUSS 15*	B
6	*SURE(IFGLS)-Mine*	IFGLS	Manual	Multiple		
7	*SURE(EM)-Mine*	EM	Manual	Multiple		
8	*Autometrics*	OLS	Automated	Single	*PcGive 14*	C
9	*Autometrics-SURE*	FGLS	Automated	Single		
10	*Autometrics-SURE(IFGLS)*	IFGLS	Automated	Single		
11	*Autometrics-SURE(EM)*	EM	Automated	Single		
12	*Stepwise*	OLS	Automated	Single	*IBM SPSS Statistics 21*	D
13	*Stepwise-SURE*	FGLS	Automated	Single		
14	*Stepwise-SURE(IFGLS)*	IFGLS	Automated	Single		
15	*Stepwise-SURE(EM)*	EM	Automated	Single		
16	*SURE-Autometrics*	FGLS	Automated	Multiple	*GAUSS 15*	E
17	*SURE(IFGLS)-Autometrics*	IFGLS	Automated	Multiple		
18	*SURE(EM)-Autometrics*	EM	Automated	Multiple		

5.2.1 Manual Selections

The manual selections are primarily based on the *p*-values and the final decision to select the model depends on individual's knowledge. In this analysis, *p*-values based on 5% significance level are set to determine the significant or insignificant variables from the GUMS. Variables with high insignificant *p*-values are removed from the model beginning with the highest value. The variable is ignored if the correlation and insignificant *p*-value are also high. Once the variable is eliminated, the standard error is checked for any increment. If exists, then the variable is kept. The variables would be removed as a group if more than one variable is highly insignificant as well as weak correlations persist. The selected model then must succeed for all diagnostic tests. Here, this manual selection is also named as *Mine*. These rules are applied for both single equation and multiple equations selections, while all calculations involved are conducted using *GAUSS 15* programming language.

5.2.1.1 Single Equation Selections

The *Mine* selection had been the foundation for other manual selections. *Mine, Mine-SURE*, *Mine-SURE(IFGLS)* and *Mine-SURE(EM)* are manual models selection procedures where equations are selected separately using OLS estimators. These four procedures are classed in Group A. FGLS is used to estimate the final model of *Mine-SURE*, but IFGLS is for *Mine-SURE(IFGLS)* and EM algorithm is for *Mine-SURE(EM)* final models' estimations.

5.2.1.2 Multiple Equations Selections

On the other hand, Group B contains *SURE-Mine, SURE(IFGLS)-Mine* and *SURE(EM)-Mine*. *SURE-Mine* uses FGLS while *SURE(IFGLS)-Mine* employs IFGLS and finally, *SURE(EM)-Mine* utilizes EM algorithm as methods of estimation with the inspection of variables are done simultaneously within the model according to the manual selections rules above.

5.2.2 Automated Selections

Automated selections refer to the use of *Stepwise* and *Autometrics* algorithms to select the equations. *Stepwise* was used to pick one equation at a time, while *Autometrics* was utilized to choose all equations simultaneously.

5.2.2.1 Single Equation Selections

Group C refers to models selection procedures which are based on *Autometrics*. It is an algorithm which implements a tree search by executing strategies such as pruning, bunching, and chopping. These selection strategies are executed using *PcGive 14* programme. Nonetheless, *Autometrics* only performs individual selections for single model by OLS estimation method (Doornik, 2009). Since a model has multiple equations, each is estimated using OLS and individually selected for multiple times. *Autometrics-SURE* is a procedure that used *Autometrics* in models selection where each equation separately selected with OLS estimation method. However, the final models are estimated using FGLS. Meanwhile, *Autometrics-SURE(IFGLS)* estimated final model using IFGLS, while EM algorithm is used in *Autometrics-SURE(EM)*.

Besides *Autometrics*, another single selection algorithm was also utilized. *Stepwise* is a well-known algorithm in choosing predictive variables through forward selections, backward selections and bi-directional elimination. The weakest correlated variable will be eliminated during each regression. Finally, only the related variables that clarify the distribution test are left in the model. *Stepwise* is an algorithm for single equation model. Since a model has multiple equations, each equation is estimated using OLS and individually selected for multiple times. Models selection through *Stepwise* were employed using *IBM SPSS Statistics 21*. Selections by *Stepwise* are put under Group D. *Stepwise-SURE* are procedures that used *Stepwise* in models selection where each equation separately selected with OLS estimation method. Yet, final model is estimated using FGLS. Meanwhile, *Stepwise-SURE(IFGLS)* estimated final model using IFGLS, while EM algorithm is used in *Stepwise-SURE(EM)*.

5.2.2.2 Multiple Equations Selections

SURE-Autometrics is an automated models selection algorithm focussing on SURE model. The multiple equations selections are conducted simultaneously with estimation of FGLS method throughout the process. This algorithm accepts similar operation as its 'parent' algorithm, *Autometrics*. The *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*, which are extended algorithms of *SURE-Autometrics*, were compared with all other 16 procedures to determine their performances in models selection. The three simultaneous selections algorithms are classified under Group E and converted into a computer programming code by *GAUSS 15* language.

5.3 Error Measures

This study used error measures to evaluate the forecasting performance of different models selection procedures. The principle of error measures is to present an informative and clear review of the multivariate error distribution produced by a forecast model. The error distribution is conditioned by a range of forecasting lead times under consideration and the time origin of the forecast. Following Yusof (2016) and Ismail (2005), two types of error measures are chosen for this study, which are root mean square error (RMSE) and geometric root mean square error (GRMSE). Small values of these measures suggest a good forecasting performance, while large values indicate the opposite. RMSE is one of the measures used among the practitioners for assessing the performance of forecasting models. When the series of forecasts are made in consecutive time periods, all with the same forecast horizon and the cost function is quadratic, the RMSE is a suitable measure of accuracy. Given $\hat{u}_t(l)$ is forecast error at the l-step ahead forecast, n is number of observations in test period, T is the forecast origin time, and l is number of steps-ahead, RMSE is defined as

$$RMSE = \sqrt{\frac{\sum_{t=T}^{T+n-l} \hat{u}_t^2(l)}{n+1-l}} \tag{5.1}$$

In the meantime, Fildes (1992) suggested the GRMSE for the case where the data (and errors) are tainted by occasional outliers and also when dealing with a considerably large error term due to a particularly bad forecast. GRMSE is defined as

$$GRMSE = \left[\prod_{t=T}^{T+n-l} \hat{u}_t^2(l)\right]^{\frac{1}{2(n+1-l)}}. \tag{5.2}$$

In this study, the process of inserting new data, then re-estimating and thus generating one year up to three-year-ahead forecasts year by year were performed recursively until all data points were exhausted. After the two error measures were computed for each equation for one until three-step-ahead, the medians of all equations were calculated and ranked from 1 (the smallest is the 'best') to 18 (the largest) for both RMSE and GRMSE. This serves as an evaluation of performance of each models selection procedure.

5.4 National Growth Rates Data

In this section, application analysis to assess performance of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* was firstly done on data set of national growth rates for Denmark, Ireland, Netherlands and United Kingdom. Yusof (2016) previously used this data, commences in 1952 until 2003, in evaluation of her algorithm, *SURE-Autometrics.* Nevertheless, it was originally experimented in Garcia-Ferrer, Highfield, Palm, and Zellner (1987) until year 1981 only.

Consequently, this study resumes exploiting the same data to measure and make direct comparison for models selection procedures' accomplishments.

The variables involved are annual gross domestic product (GDP, Y_{it}), real stock return (x_{i1t}), world stock return (x_{i2t}) and money (M1, x_{i3t}). Garcia-Ferrer et al. (1987) termed stock return as industrial share prices, while world stock return as median of stock return of the countries. GDP deflator was used in GDP deflation, but consumer price index (CPI) was employed for the same purpose for real stock return and money (M1). In order to ensure stationarity, data was log transformed and differenced once. However, only observations from 1952 to 1998 (n = 47) used to fit the estimated models and the remaining 5 observations (1999-2003) were used for validation.

Given that *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were developed with MLEs, another condition for a given model is to fulfil its requirement of sample size (n), number of equations (m) and number of variables (v). For that reason, suggestion by Griffiths et al. (2001), which is represented by $v < [(n\text{-}1)/m]$, is applied in this empirical study. There are four equations representing all countries with 47 observations.

Following this suggestion, number of countries and duration studied, number of variables has to be less than 11 variables. In addition, the model of national growth rates by Garcia-Ferrer et al. (1987) was also used as guideline. The model had in total seven variables which consisted of three lags of the dependent variable, two lags of stock return, one lag of world stock return and one lag of money growth rate. Meanwhile, both Ismail (2005) and Yusof (2016) encompassed their models with four lags of the dependent variables, three independent variables and four lags for each of the independent variable.

By taking into account their studies, the initial GUMS here had been set to contain eight explanatory variables with minimal lags involved. Hence, there are two lags of dependent variables, three independent variables and one lag of each independent variable. This leads to the following equation:

$$\Delta y_{it} = \alpha_{i0} + \sum_{j=1}^{2} \alpha_{ij} \Delta y_{i(t-j)} + \sum_{k=1}^{3} \sum_{j=0}^{1} \phi_{ikj} \Delta x_{ik(t-j)} + \varepsilon_{it} \qquad (5.3)$$

where j is the lag length, i = 1,2,3,4 (countries), t = 1,2,…,T (time periods) and Δy_{it} is the growth rate of the GDP in year t for country i. Δx_{ikt} is the growth rate of the kth explanatory variable in year t for country i, ε_{it} are identically independently distributed random errors with mean zero and variance σ^2. α and ϕ are unknown parameter vectors to be estimated.

5.4.1 Models Selection of Four Equations

This study started comparison of the models selection procedures with four equations representing all countries. In ensuring the suitability of SURE for the equations, MC-QLR test was run and it resulted with p-value = 0.015, significant at 10% level. This implies the existence of correlation among disturbances and SURE model is thus appropriate.

Table 5.2 until Table 5.5 show estimated models of growth rates for all models selection procedures of Denmark, Ireland, Netherland and United Kingdom, respectively. From overall results of all estimated models, it is observed that all of the variables included in the models were significant and the models had also passed the diagnostic tests. It is noticeable that there are various combinations of variables with some similarities in terms of variables' presence in the models. No variable was completely removed simultaneously from all countries. The one period lag of dependent variable (Δy_{it-1}) was found in models of Denmark and Ireland. The stock return (Δx_{i1t}) and the world stock return (Δx_{i2t}) only appeared in Netherland's models. Four lag variables ($\Delta y_{it-2}, \Delta x_{i1(t-1)}, \Delta x_{i2(t-1)}$ and $\Delta x_{i3(t-1)}$) were included in Denmark's model only and lastly, the money growth rate (Δx_{i3t}) is the most reliable variable since it is contained in all models of every country and every models selection procedure.

Table 5.2

Estimated Models of Growth Rate for Denmark (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	-0.005	-	-	-	-	-	-	0.787***	0.200**	0.708	0.057
2		*Mine-SURE*	<0.001	-	-	-	-	-	-	0.734***	0.155*	0.701	0.055
3		*Mine-SURE(IFGLS)*	<0.001	-	-	-	-	-	-	0.712***	0.150*	0.697	0.056
4		*Mine-SURE(EM)*	<-0.001	-	-	-	-	-	-	0.713***	0.150**	0.697	0.056
		Average										0.701	0.050
5	B	*SURE-Mine*	-0.001	-0.322**	-	-	-	-	-	0.734***	0.412***	0.696	0.055
6		*SURE(IFGLS)-Mine*	-0.010	-0.354***	0.210***	-	0.096***		-	0.746***	0.390***	0.702	0.053
7		*SURE(EM)-Mine*	<0.001	-0.360**	-	-	-	-	-	0.709***	0.422***	0.683	0.056
		Average										0.694	0.055
8	C	*Autometrics*	-0.001	-	-	-	-	-	0.258***	0.813***	-	0.720	0.057
9		*Autometrics-SURE*	-0.001	-	-	-	-	-	0.219***	0.759***	-	0.715	0.054
10		*Autometrics-SURE(IFGLS)*	-0.002	-	-	-	-	-	0.217***	0.737***	-	0.711	0.054
11		*Autometrics-SURE(EM)*	-0.002	-	-	-	-	-	0.217***	0.737***	-	0.711	0.054
		Average										0.714	0.055
12	D	*Stepwise*	-0.006	-	-	-	-	-	0.226*	0.766***	0.177**	0.741	0.053
13		*Stepwise-SURE*	-0.002	-	-	-	-	-	0.202**	0.723***	0.139*	0.735	0.051
14		*Stepwise-SURE(IFGLS)*	-0.002	-	-	-	-	-	0.197***	0.705***	0.136*	0.732	0.052
15		*Stepwise-SURE(EM)*	-0.002	-	-	-	-	-	0.197**	0.706***	0.136*	0.732	0.052
		Average										0.735	0.052
16	E	*SURE-Autometrics*	-0.007	-0.290**	0.158**	-	-	-	0.196**	0.762***	0.319***	0.721	0.051
17		*SURE(IFGLS)-Autometrics*	-0.006	-0.332***	0.174**	-	-	-	0.190***	0.743***	0.335***	0.708	0.053
18		*SURE(EM)-Autometrics*	-0.010	-0.354***	0.210**	-	0.096**	-	-	0.747***	0.390***	0.703	0.053
		Average										0.711	0.052
		Total Average										0.711	0.054

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.3

Estimated Models of Growth Rate for Ireland (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\bar{R}^2$	Std. errors
1	A	*Mine*	0.012*	-	-	-	-	-	-	0.744***	-	0.798	0.041
2		*Mine-SURE*	0.012*	-	-	-	-	-	-	0.752***	-	0.798	0.040
3		*Mine-SURE(IFGLS)*	0.012*	-	-	-	-	-	-	0.754***	-	0.798	0.040
4		*Mine-SURE(EM)*	0.012*	-	-	-	-	-	-	0.754***	-	0.798	0.040
		Average										0.798	0.040
5	B	*SURE-Mine*	0.013**	-	-	-	-	-	-	0.734***	-	0.798	0.040
6		*SURE(IFGLS)-Mine*	0.010	0.134**	-	-	-	-	-	0.693***	-	0.805	0.039
7		*SURE(EM)-Mine*	0.013*	-	-	-	-	-	-	0.719***	-	0.797	0.040
		Average										0.800	0.040
8	C	*Autometrics*	0.014**	-	-	-	-	-	-	0.745***	-	0.804	0.041
9		*Autometrics-SURE*	0.013*	-	-	-	-	-	-	0.742***	-	0.798	0.040
10		*Autometrics-SURE(IFGLS)*	0.013*	-	-	-	-	-	-	0.744***	-	0.798	0.040
11		*Autometrics-SURE(EM)*	0.013*	-	-	-	-	-	-	0.744***	-	0.798	0.040
		Average										0.800	0.040
12	D	*Stepwise*	0.012*	-	-	-	-	-	-	0.744***	-	0.798	0.041
13		*Stepwise-SURE*	0.012*	-	-	-	-	-	-	0.757***	-	0.798	0.040
14		*Stepwise-SURE(IFGLS)*	0.012*	-	-	-	-	-	-	0.762***	-	0.798	0.040
15		*Stepwise-SURE(EM)*	0.012*	-	-	-	-	-	-	0.762***	-	0.798	0.040
		Average										0.798	0.040
16	E	*SURE-Autometrics*	0.010	0.144**	-	-	-	-	-	0.700***	-	0.805	0.039
17		*SURE(IFGLS)-Autometrics*	0.009	0.153**	-	-	-	-	-	0.687***	-	0.804	0.039
18		*SURE(EM)-Autometrics*	0.010	0.134*	-	-	-	-	-	0.693***	-	0.805	0.039
		Average										0.805	0.039
		Total Average										0.800	0.040

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.4

Estimated Models of Growth Rate for Netherland (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	0.016*	-	-	-	-	-	-	0.852***	-	0.745	0.056
2		*Mine-SURE*	0.019**	-	-	-	-	-	-	0.773***	-	0.738	0.055
3		*Mine-SURE(IFGLS)*	0.021**	-	-	-	-	-	-	0.726***	-	0.728	0.056
4		*Mine-SURE(EM)*	0.021***	-	-	-	-	-	-	0.729***	-	0.728	0.056
		Average										0.735	0.056
5	B	*SURE-Mine*	0.019**	-	-	-	-	-	-	0.772***	-	0.738	0.055
6		*SURE(IFGLS)-Mine*	0.017*	-	-	0.211**		-0.322**	-	0.807***	-	0.743	0.053
7		*SURE(EM)-Mine*	0.021**	-	-	-	-	-	-	0.733***	-	0.729	0.056
		Average										0.737	0.055
8	C	*Autometrics*	0.011	-	-	0.233	-	-0.371*	-	0.926***	-	0.762	0.055
9		*Autometrics-SURE*	0.016*	-	-	0.205**	-	-0.302*	-	0.832***	-	0.748	0.053
10		*Autometrics-SURE(IFGLS)*	0.017*	-	-	0.210**	-	-0.289*	-	0.780***	-	0.733	0.054
11		*Autometrics-SURE(EM)*	0.017*	-	-	0.209**	-	-0.289*	-	0.782***	-	0.734	0.054
		Average										0.744	0.054
12	D	*Stepwise*	0.016**	-	-	-	-	-	-	0.852*	-	0.745	0.056
13		*Stepwise-SURE*	0.019**	-	-	-	-	-	-	0.774***	-	0.738	0.055
14		*Stepwise-SURE(IFGLS)*	0.021**	-	-	-	-	-	-	0.733**	-	0.729	0.056
15		*Stepwise-SURE(EM)*	0.021**	-	-	-	-	-	-	0.735***	-	0.730	0.056
		Average										0.736	0.056
16	E	*SURE-Autometrics*	0.019**	-	-	-	-	-	-	0.774***	-	0.738	0.055
17		*SURE(IFGLS)-Autometrics*	0.017*	-	-	0.211**	-	-0.297*	-	0.799***	-	0.739	0.054
18		*SURE(EM)-Autometrics*	0.017*	-	-	0.211**	-	-0.322*	-	0.808***	-	0.743	0.053
		Average										0.740	0.054
		Total Average										0.738	0.055

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.5

Estimated Models of Growth Rate for United Kingdom (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	-0.002	-	-	-	-	-	-	0.461***	-	0.641	0.066
2		*Mine-SURE*	-0.002	-	-	-	-	-	-	0.461***	-	0.641	0.064
3		*Mine-SURE(IFGLS)*	-0.002	-	-	-	-	-	-	0.456**	-	0.641	0.064
4		*Mine-SURE(EM)*	-0.002	-	-	-	-	-	-	0.456***	-	0.641	0.064
		Average										0.641	0.065
5	B	*SURE-Mine*	-0.002	-	-	-	-	-	-	0.455***	-	0.640	0.064
6		*SURE(IFGLS)-Mine*	-0.001	-	-	-	-	-	-	0.447***	-	0.640	0.064
7		*SURE(EM)-Mine*	-0.001	-	-	-	-	-	-	0.446***	-	0.640	0.064
		Average										0.640	0.064
8	C	*Autometrics*	-0.003	-	-	-	-	-	-	0.462***	-	0.643	0.067
9		*Autometrics-SURE*	-0.002***	-	-	-	-	-	-	0.468***	-	0.640	0.064
10		*Autometrics-SURE(IFGLS)*	-0.002	-	-	-	-	-	-	0.463***	-	0.641	0.064
11		*Autometrics-SURE(EM)*	-0.002	-	-	-	-	-	-	0.463***	-	0.641	0.064
		Average										0.641	0.065
12	D	*Stepwise*	-0.002	-	-	-	-	-	-	0.461***	-	0.641	0.066
13		*Stepwise-SURE*	-0.002	-	-	-	-	-	-	0.473***	-	0.640	0.064
14		*Stepwise-SURE(IFGLS)*	-0.002	-	-	-	-	-	-	0.470***	-	0.640	0.064
15		*Stepwise-SURE(EM)*	-0.002	-	-	-	-	-	-	0.470***	-	0.640	0.064
		Average										0.640	0.065
16	E	*SURE-Autometrics*	-0.002	-	-	-	-	-	-	0.469***	-	0.640	0.064
17		*SURE(IFGLS)-Autometrics*	-0.002	-	-	-	-	-	-	0.465***	-	0.641	0.064
18		*SURE(EM)-Autometrics*	-0.001	-	-	-	-	-	-	0.448***	-	0.640	0.064
		Average										0.640	0.064
		Total Average										0.641	0.064

***Significant at 1%, ** Significant at 5%, * Significant at 10%

In Table 5.2 of Denmark's estimated models, *SURE(IFGLS)-Mine, SURE-Autometrics, SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* had the most number of variables selected compared to other models selection procedures and countries. From Group B, IFGLS is seen to be more efficient than FGLS and EM estimations as indicated by more selected variables and higher $\overline{R}^2$ in *SURE(IFGLS)-Mine*. At the same time, more relevant variables found by procedures in Group E signified the effectiveness of tree search strategy and simultaneous selections to search for more relevant variables.

In estimated models for Ireland as seen in Table 5.3, money growth rate (Δx_{i3t}) was the only variable retained in every model, but with additional of first lag of dependent variables Δy_{it-1} in automated and multiple equations selections (Group E) and also *SURE(IFGLS)-Mine*. On average, every model had 0.800 for $\overline{R}^2$ which is the highest value, while the standard error is the lowest value of 0.040 for standard error in comparison with other countries. Group E procedures contributed to these values with all its procedures gained more than 0.800 for $\overline{R}^2$ and 0.039 for standard error. This indicates that simultaneous selections of all equations automatically give the best selections out of all models selection procedures.

Table 5.4 shows estimated models of Netherland where only money growth rate (Δx_{i3t}) was present in models by procedures of Groups A, B and D with exception of *SURE(IFGLS)-Mine*. Meanwhile, *Autometrics*-based procedures of Group C and E had two more variables of stock return (Δx_{i1t}) and world stock return (Δx_{i2t}). Thus, *Autometrics* managed to select more relevant variables than *Stepwise* and manual selections. Lastly, United Kingdom's models in Table 5.5 has only money growth rate

(Δx_{i3t}) selected in all models selection procedures. Unlike Ireland, United Kingdom produced the lowest $\overline{R}^2$ of 0.641 and highest standard error of 0.064.

All models selection procedures under Group A had selected only money growth rate (Δx_{i3t}) for all countries, but added its first lag ($\Delta x_{i3(t-1)}$) for estimated models of Denmark. When compared with Group B's procedures, the latter group had more variables than its counterparts in Group A for all countries, except United Kingdom. Therefore, multiple equations selections inclined to select more relevant variables than single equations selections even though without algorithms. The simultaneous estimations were efficient because the *p*-values of MC-QLR tests for *SURE-Mine*, *SURE(IFGLS)-Mine* and *SURE(EM)-Mine* were 0.0250 (significant at 10% level), 0.0040 (significant at 5% level) and 0.0230 (significant at 10% level) respectively.

Automated single equation selections using *Autometrics* (Group C) and *Stepwise* (Group D) display different combination of variables. Models of Ireland and United Kingdom have only money growth rate (Δx_{i3t}), while Denmark has additional variable of lag one of world stock return ($\Delta x_{i2(t-1)}$) for both groups. Real stock return (Δx_{i1t}) and world stock return (Δx_{i2t}) were also in Netherland's models of Group C. For Group D, Netherland's models only retained money growth rate (Δx_{i3t}), whereas Denmark's model have an extra variable of lag one of money growth rate ($\Delta x_{i3(t-1)}$). Group E's procedures selected the most number of variables for all countries, except United Kingdom. These automated and multiple equations selections have more relevant variables than automated single equation selections (Group C and D) for Denmark and Ireland. This could be due to efficient estimation with *p*-values of MC-QLR tests for *SURE-Autometrics*, *SURE(IFGLS)-Autometrics* and *SURE(EM)-*

Autometrics were 0.0100, 0.0100 and 0.0130 respectively. All values were significant at 10% level.

Subsequently, observations from years 1999-2003 were used for one, two and three-steps ahead forecasting. This means forecast for year 1999 was equal to forecast of year 1998 by means of models selected at the final phase. This new observation was combined with the years 1952-1998 data available initially and models were estimated again to attain forecast of year 2000. The models repeated the three steps of adding, re-estimating and forecasting until the last observation i.e. year 2003. The entire forecasting process continued for two and three-steps ahead, where observations of years 2000 and 2001 initiated two-step and three-steps ahead, respectively. Then, RMSE (in Table 5.6) and GRMSE (in Table 5.7) values were computed for each models selection procedure for one until three-step-ahead and ranked from 1 (the smallest is the 'best') to 18 (largest). All RMSE and GRMSE were averaged and denoted as 'Group RMSE' and 'Group GRMSE', accordingly and were ranked too in order to evaluate performances of models selection procedures.

Results from RMSE measurements in Table 5.6 show an interesting finding with *SURE(IFGLS)-Autometrics* exceptionally dominated rank 1 for all step-ahead forecasts. This is followed by *SURE(EM)-Autometrics* at second positions for one and two step-ahead-forecasts, but at third place for three-step-ahead. Consequently, *SURE(IFGLS)-Mine* also presents comparable results but slightly different for two and three-step-ahead-forecasts. This is because the two procedures selected same variables in their models. Furthermore, overall observation reveals excellent performances by Group E's procedures, which can easily be distinguished from other processes through the groups' ranks, which is at rank 1. This indicates the success of automated and simultaneous selections. However, only *SURE-Autometrics* had a lower position at rank 11 for three-step-ahead. This could suggest the use of *SURE-Autometrics* may not be suitable for long horizon forecasts. In addition, multiple equations selection (Groups B and E) are generally at higher ranks than single equation selections (Groups A, C and D). As for other approaches, lower positions in the rankings whether manual (Groups A and B) or automated (Groups C, D and E) approaches, indicate underperformance in this analysis to choose the right models.

Table 5.6

Forecasting Performances based on RMSE (Four Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank
1	A	*Mine*	0.0750	17	0.0744	4	0.0705	17	0.0704	5	0.0695	15	0.0695	5
2		*Mine-SURE*	0.0744	15			0.0703	14			0.0694	13		
3		*Mine-SURE(IFGLS)*	0.0740	10			0.0704	16			0.0694	13		
4		*Mine-SURE(EM)*	0.0740	10			0.0703	14			0.0695	15		
5	B	*SURE-Mine*	0.0741	12	0.0725	2	0.0702	12	0.0687	2	0.0696	17	0.0680	3
6		*SURE(IFGLS)-Mine*	0.0699	2			0.0657	3			0.0645	2		
7		*SURE(EM)-Mine*	0.0734	9			0.0701	8			0.0699	18		
8	C	*Autometrics*	0.0733	8	0.0732	3	0.0701	8	0.0700	3	0.0676	5	0.0677	2
9		*Autometrics-SURE*	0.0732	6			0.0699	6			0.0675	4		
10		*Autometrics-SURE(IFGLS)*	0.0732	6			0.0701	8			0.0679	7		
11		*Autometrics-SURE(EM)*	0.0731	5			0.0698	5			0.0676	5		
12	D	*Stepwise*	0.0753	18	0.0745	5	0.0709	18	0.0703	4	0.0692	11	0.0691	4
13		*Stepwise-SURE*	0.0745	16			0.0701	8			0.0690	8		
14		*Stepwise-SURE(IFGLS)*	0.0741	12			0.0702	12			0.0690	8		
15		*Stepwise-SURE(EM)*	0.0742	14			0.0700	7			0.0691	10		
16	E	*SURE-Autometrics*	0.0705	4	0.0698	1	0.0657	3	0.0652	1	0.0692	11	0.0657	1
17		*SURE(IFGLS)-Autometrics*	0.0691	1			0.0644	1			0.0632	1		
18		*SURE(EM)-Autometrics*	0.0699	2			0.0656	2			0.0647	3		

Table 5.7

Forecasting Performances based on GRMSE (Four Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank
1	A	*Mine*	0.0603	17	0.0598	4	0.0561	17	0.0559	4	0.0485	16	0.0484	4
2		*Mine-SURE*	0.0598	14			0.0559	14			0.0484	13		
3		*Mine-SURE(IFGLS)*	0.0595	11			0.0557	11			0.0483	11		
4		*Mine-SURE(EM)*	0.0596	12			0.0557	11			0.0483	11		
5	B	*SURE-Mine*	0.0593	10	0.0534	3	0.0384	4	0.0397	1	0.0480	10	0.0425	3
6		*SURE(IFGLS)-Mine*	0.0423	1			0.0372	1			0.0320	1		
7		*SURE(EM)-Mine*	0.0586	9			0.0436	5			0.0474	9		
8	C	*Autometrics*	0.0498	4	0.0506	2	0.0449	6	0.0459	3	0.0370	4	0.0382	2
9		*Autometrics-SURE*	0.0506	5			0.0459	7			0.0382	5		
10		*Autometrics-SURE(IFGLS)*	0.0509	6			0.0464	9			0.0388	7		
11		*Autometrics-SURE(EM)*	0.0510	7			0.0463	8			0.0387	6		
12	D	*Stepwise*	0.0607	18	0.0600	5	0.0564	18	0.0560	5	0.0498	18	0.0488	5
13		*Stepwise-SURE*	0.0599	16			0.0560	16			0.0485	16		
14		*Stepwise-SURE(IFGLS)*	0.0597	13			0.0558	13			0.0484	13		
15		*Stepwise-SURE(EM)*	0.0598	14			0.0559	14			0.0484	13		
16	E	*SURE-Autometrics*	0.0525	8	0.0458	1	0.0481	10	0.0409	2	0.0450	8	0.0372	1
17		*SURE(IFGLS)-Autometrics*	0.0425	2			0.0373	2			0.0343	3		
18		*SURE(EM)-Autometrics*	0.0425	2			0.0373	2			0.0322	2		

Table 5.7 shows GRMSE measurements for all model selection procedures. Unlike in RMSE, this time *SURE(IFGLS)-Mine* had overridden *SURE(IFGLS)-Autometrics* to be the most superior procedure by placing itself at ranks 1 for all forecast errors. Any changes in variables' significance sign can easily detected in this *Mine* manual approach. Despite facing a minor decline, *SURE(IFGLS)-Autometrics*'s is still considered impressive for being at the top three with rank 2 for one and two-step-ahead forecasts, but at rank three for three-ahead forecast. This proved that its outstanding execution is still undeterred by any other approaches. In the same way, *SURE(EM)-Autometrics* too is still able to maintain at rank 2 throughout all step-ahead-forecasts. Therefore, Group E is still giving the best selections than the others.

On the other hand, *Stepwise*-based (Group D) and *Mine*-based (Groups A and B) selections were all outclassed by *Autometrics*-based (Groups C and E) procedures and even simultaneous selections in *SURE(EM)-Mine* and *SURE-Mine* only showed moderate achievements. In brief, models selection benefits the most from simultaneous selections using iterative estimation methods, either using algorithm such as in *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* or even a manual procedure of *SURE(IFGLS)-Mine*.

In order to imitate the simulation analysis as in Chapter Four, the same data set was reanalysed for two equations model since the analysis was done for four and two equations models. Therefore, equations for Denmark and United Kingdom were omitted in next section as their standard errors were relatively bigger than Ireland and Netherlands.

5.4.2 Models Selection of Two Equations

Estimated models of Ireland and Netherlands using all models selection procedures are detailed in this section. Table 5.8 exhibit results for Ireland models with selections from *SURE(IFGLS)-Autometrics*, *SURE(EM)-Autometrics* and *SURE-Autometrics* (Group E) exhibited similar models with two variables selected; first lag of dependent variable (Δy_{it-1}) and money growth rate (Δx_{i3t}). This is not surprising since the three procedures belong to same classification of concurrent selections by utilizing *Autometrics* algorithm but with different estimation methods. With the highest values of $\overline{R}^2$ at 0.806 and lowest standard errors at 0.039, simultaneous selection of all equations in models using automated approach is shown to be better than other approaches in choosing the variables. Money is still an important variable as it is the only variable appears in all other models. Average value of $\overline{R}^2$ for Ireland is 0.806, while the standard error 0.040.

Table 5.9 display estimated models of Netherland. Similarities occurred in models of *Autometrics-SURE*, *Autometrics-SURE(IFGLS)* and *Autometrics-SURE(EM)* because all selected variables originally came from *Autometrics*. These models selection procedures of Group C have two more variables, which are real stock return (Δx_{i1t}) and world stock return (Δx_{i2t}). Again, only money exists in the rest of selection procedures. $\overline{R}^2$ values for Netherlands are averaged lower than Ireland at 0.747 and higher standard error at 0.054.

Table 5.8

Estimated Models of Growth Rate for Ireland of (Two Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	0.012*	-	-	-	-	-	-	0.744***	-	0.798	0.041
2		*Mine-SURE*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
3		*Mine-SURE(IFGLS)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
4		*Mine-SURE(EM)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
		Average										0.798	0.040
5	B	*SURE-Mine*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
6		*SURE(IFGLS)-Mine*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
7		*SURE(EM)-Mine*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
		Average										0.798	0.040
8	C	*Autometrics*	0.014**	-	-	-	-	-	-	0.745***	-	0.804	0.041
9		*Autometrics-SURE*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
10		*Autometrics-SURE(IFGLS)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
11		*Autometrics-SURE(EM)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
		Average										0.800	0.040
12	D	*Stepwise*	0.012*	-	-	-	-	-	-	0.744***	-	0.798	0.041
13		*Stepwise-SURE*	0.013*	-	-	-	-	-	-	0.742***	-	0.798	0.040
14		*Stepwise-SURE(IFGLS)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
15		*Stepwise-SURE(EM)*	0.013*	-	-	-	-	-	-	0.741***	-	0.798	0.040
		Average										0.798	0.040
16	E	*SURE-Autometrics*	0.010	0.118*	-	-	-	-	-	0.712***	-	0.806	0.039
17		*SURE(IFGLS)-Autometrics*	0.010	0.119*	-	-	-	-	-	0.712***	-	0.806	0.039
18		*SURE(EM)-Autometrics*	0.010	0.119*	-	-	-	-	-	0.712***	-	0.806	0.039
		Average										0.806	0.039
		Total Average										0.800	0.040

***Significant at 1%, ** Significant at 5%, * Significant at 10%,

Table 5.9

Estimated Models of Growth Rate for Netherland (Two Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	0.016*	-	-	-	-	-	-	0.852***	-	0.745	0.056
2		*Mine-SURE*	0.017*	-	-	-	-	-	-	0.836***	-	0.744	0.054
3		*Mine-SURE(IFGLS)*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
4		*Mine-SURE(EM)*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
		Average										0.744	0.055
5	B	*SURE-Mine*	0.017*	-	-	-	-	-	-	0.836***	-	0.744	0.054
6		*SURE(IFGLS)-Mine*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
7		*SURE(EM)-Mine*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
		Average										0.744	0.054
8	C	*Autometrics*	0.011	-	-	0.233	-	-0.371*	-	0.926***	-	0.762	0.055
9		*Autometrics-SURE*	0.014	-	-	0.196*	-	-0.310*	-	0.895***	-	0.755	0.052
10		*Autometrics-SURE(IFGLS)*	0.014	-	-	0.196*	-	-0.308*	-	0.893***	-	0.755	0.052
11		*Autometrics-SURE(EM)*	0.014	-	-	0.196*	-	-0.308*	-	0.894***	-	0.755	0.052
		Average										0.757	0.053
12	D	*Stepwise*	0.016**	-	-	-	-	-	-	0.852*	-	0.745	0.056
13		*Stepwise-SURE*	0.017*	-	-	-	-	-	-	0.836***	-	0.744	0.054
14		*Stepwise-SURE(IFGLS)*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
15		*Stepwise-SURE(EM)*	0.017*	-	-	-	-	-	-	0.835***	-	0.744	0.054
		Average										0.744	0.055
16	E	*SURE-Autometrics*	0.017*	-	-	-	-	-	-	0.832***	-	0.744	0.054
17		*SURE(IFGLS)-Autometrics*	0.017*	-	-	-	-	-	-	0.830**	-	0.744	0.055
18		*SURE(EM)-Autometrics*	0.017*	-	-	-	-	-	-	0.830***	-	0.744	0.055
		Average										0.744	0.055
		Total Average										0.747	0.054

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Next step was the calculations of RMSE and GRMSE values in Tables 5.10 and 5.11, correspondingly. There are many models selection procedures positioned at same ranks for all forecast horizons. This situation happened consistently for 11 procedures for RMSE and 16 procedures for GRMSE. Analysis of two equations had seen *Autometrics* at rank 1 in all three step-ahead forecasts and followed by *Autometrics-SURE* at second place. *Autometrics-SURE(IFGLS)* and *Autometrics-SURE(EM)* had acquired between ranks 2 to 4. These procedures are classified in Group C with group ranks are at rank 1. This could be due to inclusion of Netherland equation, where more variables selected using Group C's procedures.

This is followed by Group E, where *SURE-Autometrics*, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* had experienced declines compared to selections of four equations since these procedures went down to rank 5 for all forecast errors. Nevertheless, the Group E's group rank at second place implies that performance of automated multiple equations selections is still good and reliable. Other single equation selections, *Mine* and *Stepwise* performed the worst of all. This is in total contrast of *Autometrics.* While for the remaining procedures, all of them are in the middle of ranking at 8th place for GRMSE. Same positions are also experienced by *Stepwise-SURE(EM)*, *SURE(EM)-Mine* and *Mine-SURE(EM)* in RMSE. Similarity in variables selected had most probably caused this equivalence.

Table 5.10

Forecasting Performances based on RMSE (Two Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank
1	A	*Mine*	0.0750	17	0.0748	4	0.0705	17	0.0703	3	0.0613	17	0.0612	4
2		*Mine-SURE*	0.0748	14			0.0703	11			0.0611	8		
3		*Mine-SURE(IFGLS)*	0.0747	8			0.0703	11			0.0611	8		
4		*Mine-SURE(EM)*	0.0747	8			0.0702	8			0.0611	8		
5	B	*SURE-Mine*	0.0748	14	0.0747	3	0.0703	11	0.0703	3	0.0611	8	0.0611	3
6		*SURE(IFGLS)-Mine*	0.0747	8			0.0703	11			0.0611	8		
7		*SURE(EM)-Mine*	0.0747	8			0.0702	8			0.0611	8		
8	C	*Autometrics*	0.0665	1	0.0670	1	0.0607	1	0.0612	1	0.0505	1	0.0512	1
9		*Autometrics-SURE*	0.0671	2			0.0613	2			0.0513	2		
10		*Autometrics-SURE(IFGLS)*	0.0671	2			0.0614	3			0.0513	2		
11		*Autometrics-SURE(EM)*	0.0672	4			0.0615	4			0.0515	4		
12	D	*Stepwise*	0.0753	18	0.0749	5	0.0709	18	0.0704	5	0.0614	18	0.0612	4
13		*Stepwise-SURE*	0.0748	14			0.0703	11			0.0611	8		
14		*Stepwise-SURE(IFGLS)*	0.0747	8			0.0703	11			0.0611	8		
15		*Stepwise-SURE(EM)*	0.0747	8			0.0702	8			0.0611	8		
16	E	*SURE-Autometrics*	0.0718	5	0.0718	2	0.0660	5	0.0660	2	0.0577	5	0.0577	2
17		*SURE(IFGLS)-Autometrics*	0.0718	5			0.0660	5			0.0577	5		
18		*SURE(EM)-Autometrics*	0.0718	5			0.0660	5			0.0577	5		

Table 5.11

Forecasting Performances based on GRMSE (Two Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank
1	A	*Mine*	0.0603	17	0.0601	4	0.0561	17	0.0560	4	0.0485	17	0.0484	4
2		*Mine-SURE*	0.0600	8			0.0559	8			0.0483	8		
3		*Mine-SURE(IFGLS)*	0.0600	8			0.0559	8			0.0483	8		
4		*Mine-SURE(EM)*	0.0600	8			0.0559	8			0.0483	8		
5	B	*SURE-Mine*	0.0600	8	0.0600	3	0.0559	8	0.0559	3	0.0483	8	0.0483	3
6		*SURE(IFGLS)-Mine*	0.0600	8			0.0559	8			0.0483	8		
7		*SURE(EM)-Mine*	0.0600	8			0.0559	8			0.0483	8		
8	C	*Autometrics*	0.0498	1	0.0504	1	0.0449	1	0.0456	1	0.0370	1	0.0377	1
9		*Autometrics-SURE*	0.0506	2			0.0458	2			0.0378	2		
10		*Autometrics-SURE(IFGLS)*	0.0506	2			0.0459	3			0.0379	3		
11		*Autometrics-SURE(EM)*	0.0506	2			0.0459	3			0.0381	4		
12	D	*Stepwise*	0.0607	18	0.0602	5	0.0564	18	0.0560	4	0.0498	18	0.0487	5
13		*Stepwise-SURE*	0.0600	8			0.0559	8			0.0483	8		
14		*Stepwise-SURE(IFGLS)*	0.0600	8			0.0559	8			0.0483	8		
15		*Stepwise-SURE(EM)*	0.0600	8			0.0559	8			0.0483	8		
16	E	*SURE-Autometrics*	0.0546	5	0.0546	2	0.0502	5	0.0502	2	0.0425	5	0.0425	2
17		*SURE(IFGLS)-Autometrics*	0.0546	5			0.0502	5			0.0425	5		
18		*SURE(EM)-Autometrics*	0.0546	5			0.0502	5			0.0425	5		

5.5 Water Quality Index (WQI) Data

Since this study is applying MLEs, a bigger sample was utilized for further investigations in this section. This evaluation analysis is continued with WQI data set obtained from a river in Peninsular Malaysia. The dependent variable (Y_{it}) used was the WQI. This index serves the purpose of observing water quality variations with time and location. It is a single dimensional number aggregated from a collection of measured parameters (Stambuk-Giljanovic, 1999). For an easier reference and to make it more understandable for communities, particularly in river basin management, the WQI scores have been designated into classes with index 81-100 considered as clean, 60-80 as slightly polluted and 0-59 as polluted. River water at the upstream is generally clean, while at the downstream, water tends to be slightly polluted or polluted (Department of Environment, 2007).

The main parameters in the formulation of WQI were collected at monitoring stations along the main river and its tributaries. These independent variables are Dissolved Oxygen (DO) (% saturation) (x_{i1t}), Dissolved Oxygen (DO) (mg/L) (x_{i2t}), Biochemical Oxygen Demand (BOD) (x_{i3t}), Chemical Oxygen Demand (COD) (x_{i4t}), Suspended Solids (SS) (x_{i5t}), pH (x_{i6t}), and Ammoniacal Nitrogen (NH_3N) (x_{i7t}). These variables will be converted into the sub-indices, which are named SIDO, SIBOD, SICOD, SISS, SIPH and SIAN. There are several monitoring stations in the river basin, but only data from four stations named S6, S7, S8 and S25 were utilized in this analysis. This is because these stations recorded the data in weeks, whereas other stations recorded in different time interval. The weekly data collected from 11 May 2012 until 24 December 2013 with 80 observations used in this research, had been provided by the Malaysia's Department of Environment.

Station S8 is situated at the upstream, S7 and S25 are in the middle stream, while S6 at the downstream. Figure 5.1 shows WQI for four sampling stations involved throughout the 80 weeks. All four stations recorded WQI in the range from 40 to 80.

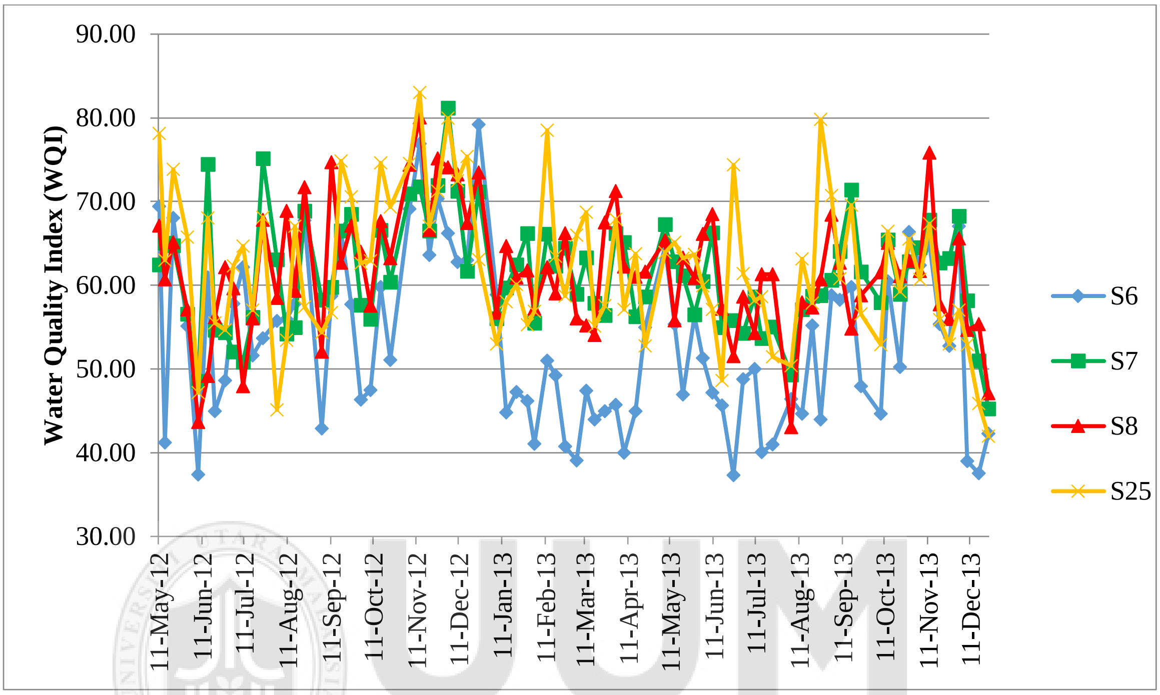

Figure 5.1. WQI based on Monitoring Stations

With reference to the WQI classification, it is seen that water of the river is catogerised as slightly polluted to polluted. Station 6 recorded lowest WQI compared to other three stations and deemed as polluted due to its location at the downstream. In November and December 2012, all stations showed high WQI. Heavy rainfall and flooding may had contributed to washing off the solid waste from domestic, commercial, industrial, agricultural, construction and storm water. These conditions have led researches to give high priorities to the monitoring of water quality of rivers in Malaysia as part of evaluation of the environmental issues (Huang, Ang, Lee, & Lee, 2015; Mohamed, Othman, Ibrahim, Alaa-Eldin, & Yunus, 2015; Naubi et al., 2016; Othman, Eldin, & Mohamed, 2012; Zainudin, 2010).

By using WQI data set, this part is focussing on models selection using different estimation methods. In estimating the models, only the first 75 observations were used, whereas the remainder was utilized for forecasting purposes. The model has four equations indicating four sampling stations, i.e S6, S7, S8 and S25. This research still used Ismail (2005) and Yusof (2016) as guidelines in formulating the initial GUMS. Yet, the number of explanatory variables chosen for this study follows suggestion by Griffiths et al. (2001) for application of SURE model using ML estimator, where $v < [(n\text{-}1)/m]$ with v refers to number of variables, n is for sample size and m stands for number of equations. Thus, the initial GUMS used contained 17 explanatory variables: three lags of dependent variables, seven independent variables and one lag of each independent variable, as below:

$$\Delta y_{it} = \alpha_{i0} + \sum_{j=1}^{3} \alpha_{ij} \Delta y_{i(t-j)} + \sum_{k=1}^{7} \sum_{j=0}^{1} \phi_{ikj} \Delta x_{ik(t-j)} + \varepsilon_{it} \qquad (5.4)$$

where j is lag length, i = 1,2,3,4 (stations), t = 1,2,…,T (time periods) and Δy_{it} is growth rate of WQI in week t for station i. Δx_{ikt} is growth rate of the kth explanatory variable in week t for station i. ε_{it} are identically independently distributed random errors with mean zero and variance σ^2. α and ϕ are unknown parameter vectors to be estimated.

5.5.1 Models Selection of Four Equations

In this part, all models of WQI were estimated according to the 18 selections procedures used in this study. $\overline{R}^2$ and standard errors of the models can also be found in Tables 5.12 until Table 5.15. The test of MC-QLR for the equations gives p-value of <0.001, significant at 10% level. This significance means that there is correlation among disturbances and the use of SURE is thus suitable for the equations.

In assessing all selections involved, all final models were again classified into their groups accordingly from all stations. Groups A and B refer to *Mine*-based selections with Group A consists of single equation selections, while Group B is for multiple equations selections. From one station to another station, it is apparent that selections under these two groups gained lower $\overline{R}^2$ values within the range of 0.650 to 0.850 than other procedures in Groups C, D and E. Higher standard error values from 3.700 to 4.300 were obtained compared to the other three groups. These results could be due to the least number of variables selected in *Mine*-based models than other models selection procedures. As for S8 particularly, average $\overline{R}^2$ of selections under Group A is slightly more than in Group B. The inclusion of $\Delta x_{i7(t-1)}$ in Group A's selections, however did not give much influence or improvement on the $\overline{R}^2$ values since this additional variable is not significant at any levels. Regardless of single or multiple selections using *Mine* selections, the variables selected are seen as not sufficiently relevant to describe the models where the selections are heavily dependent on the modeller only. This has added to the available disadvantages of manual selections.

On the other hand, a closer inspection shows that model for S8 has the highest $\overline{R}^2$ approximately at 0.950 for all procedures established from single selections using *Autometrics*. This means *Autometrics, Autometrics-SURE, Autometrics-SURE(IFGLS)* and *Autometrics-SURE(EM)* from Group C produced models with same selected variables. Besides *Autometrics*, another group which rooted from automated selections is *Stepwise*. Together with *Stepwise*, *Stepwise-SURE(IFGLS)*, *Stepwise-*

SURE(EM) and *Stepwise-SURE* which stemmed from *Stepwise* itself, are classified in Group D. Models produced under this group have consistent values of $\overline{R}^2$ that is within the range of 0.940 to 0.950 in the group. These models have same selected variables as in procedures under Group C. Overall, results from *Stepwise* and its three extensions have shown little difference from *Autometrics*-based selections. It is not surprising that both classifications yielded similar findings as together *Autometrics* and *Stepwise* handle for single-equation selections.

In terms of selected variables, every procedure has uniquely its own model. One striking observation from the comparison is variables BOD (Δx_{i3t}), COD (Δx_{i4t}), SS (Δx_{i5t}) and NH_3N (Δx_{i7t}) are significant at 1% for almost all models selection procedures. Nevertheless, various estimation methods had provided different models for all stations. Overall comparison found that Group E scored the highest average $\overline{R}^2$ and lowest standard errors values for S7, S8 and S25. This means automated simultaneous selections performed better than other types of selections.

Table 5.12

Estimated Models of WQI for S6 (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1	A	*Mine*	80.007***	-	-	-	-	-	-	-	-0.350**	-
2		*Mine-SURE*	78.836***	-	-	-	-	-	-	-	-0.450***	-
3		*Mine-SURE(IFGLS)*	78.728***	-	-	-	-	-	-	-	-0.463***	-
4		*Mine-SURE(EM)*	78.565***	-	-	-	-	-	-	-	-0.471***	-
5	B	*SURE-Mine*	78.669***	-	-	-	-	-	-	-	-0.433***	-
6		*SURE(IFGLS)-Mine*	78.685***	-	-	-	-	-	-	-	-0.486***	-
7		*SURE(EM)-Mine*	78.562***	-	-	-	-	-	-	-	-0.437***	-
8	C	*Autometrics*	61.862***	-0.009	-	0.076**	-	-	2.932***	-	-0.521***	-
9		*Autometrics-SURE*	64.467***	0.004	-	0.051*	-	-	2.778***	-	-0.564***	-
10		*Autometrics-SURE(IFGLS)*	66.300***	0.005	-	0.043	-	-	2.744***	-	-0.580***	-
11		*Autometrics-SURE(EM)*	66.272***	0.005	-	0.043**	-	-	2.744***	-	-0.580***	-
12	D	*Stepwise*	59.685***	-	-	0.072**	-	-	2.903***	-	-0.525***	-
13		*Stepwise-SURE*	62.081***	-	-	0.050*	-	-	2.736***	-	-0.560***	-
14		*Stepwise-SURE(IFGLS)*	62.961***	-	-	0.043	-	-	2.683***	-	-0.575***	-
15		*Stepwise-SURE(EM)*	62.952***	-	-	0.043**	-	-	2.684***	-	-0.575***	-
16	E	*SURE-Autometrics*	61.853***	-	-	0.057**	0.216***	-	-	-	-0.558***	-
17		*SURE(IFGLS)-Autometrics*	66.286***	-	-	-	0.208***	-	-	-	-0.616***	-
18		*SURE(EM)-Autometrics*	66.285***	-	-	-	0.208***	-	-	-	-0.616***	-

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.12 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\bar{R}^2$	Std. errors
1	A	*Mine*	-0.320***	-	-0.057***	-	-	-	-1.281***	-	0.850	3.766
2		*Mine-SURE*	-0.241***	-	-0.057***	-	-	-	-1.350***	-	0.845	3.693
3		*Mine-SURE(IFGLS)*	-0.231***	-	-0.057***	-	-	-	-1.363***	-	0.844	3.706
4		*Mine-SURE(EM)*	-0.221***	-	-0.057***	-	-	-	-1.380***	-	0.843	3.725
		Average									0.846	3.722
5	B	*SURE-Mine*	-0.237***	-	-0.056***	-	-	-	-1.390***	-	0.844	3.707
6		*SURE(IFGLS)-Mine*	-0.219***	-	-0.057***	-	-	-	-1.386***	-	0.843	3.723
7		*SURE(EM)-Mine*	-0.231***	-	-0.056***	-	-	-	-1.402***	-	0.843	3.719
		Average									0.843	3.716
8	C	*Autometrics*	-0.196***	-	-0.055***	-	-0.273***	-	-0.693***	-	0.948	2.209
9		*Autometrics-SURE*	-0.178***	-	-0.054***	-	-0.409	-	-0.832***	-	0.947	2.104
10		*Autometrics-SURE(IFGLS)*	-0.173***	-	-0.053***	-	-0.556	-	-0.890***	-	0.945	2.130
11		*Autometrics-SURE(EM)*	-0.173***	-	-0.053***	-	-0.554	-	-0.890***	-	0.946	2.130
		Average									0.947	2.143
12	D	*Stepwise*	-0.194***	-	-0.055***	-	-	-	-0.695***	-	0.950	2.180
13		*Stepwise-SURE*	-0.181***	-	-0.054***	-	-	-	-0.833***	-	0.948	2.104
14		*Stepwise-SURE(IFGLS)*	-0.177***	-	-0.054***	-	-	-	-0.887***	-	0.947	2.127
15		*Stepwise-SURE(EM)*	-0.177***	-	-0.054***	-	-	-	-0.887***	-	0.947	2.127
		Average									0.948	2.135
16	E	*SURE-Autometrics*	-0.183***	-	-0.054***	-	-	-	-0.862***	-	0.948	2.114
17		*SURE(IFGLS)-Autometrics*	-0.171***	-	-0.053***	-	-	-	-1.014***	-	0.943	2.227
18		*SURE(EM)-Autometrics*	-0.171***	-	-0.053***	-	-	-	-1.014***	-	0.943	2.227
		Average									0.945	2.189
		Total Average									0.906	2.781

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.13

Estimated Models of WQI for S7 (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1		*Mine*	64.127***	-	-	-	-	-	-	-	-0.310**	-
2	A	*Mine-SURE*	72.788***	0.131**	-	-	-	-	-	-	-0.343**	-
3		*Mine-SURE(IFGLS)*	75.673***	0.096*	-	-	-	-	-	-	-0.368**	-
4		*Mine-SURE(EM)*	75.740***	0.089*	-	-	-	-	-	-	-	-
5		*SURE-Mine*	83.401***	-	-	-	-	-	-	-	-	-
6	B	*SURE(IFGLS)-Mine*	83.867***	-	-	-	-	-	-	-	-0.382**	-
7		*SURE(EM)-Mine*	83.416***	-	-	-	-	-	-	-	-	-
8		*Autometrics*	66.372***	-	-	-	0.245***	-	-	-	-0.668***	-
9	C	*Autometrics-SURE*	66.363***	-	-	-	0.250***	-	-	-	-0.650***	-
10		*Autometrics-SURE(IFGLS)*	66.591***	-	-	-	0.250***	-	-	-	-0.645***	-
11		*Autometrics-SURE(EM)*	66.587***	-	-	-	0.250***	-	-	-	-0.645***	-
12		*Stepwise*	66.850***	-	-	-	-	-	2.992***	-	-0.613***	-
13	D	*Stepwise-SURE*	67.009***	-	-	-	-	-	3.010***	-	-0.594***	-
14		*Stepwise-SURE(IFGLS)*	67.247***	-	-	-	-	-	3.003***	-	-0.587***	-
15		*Stepwise-SURE(EM)*	67.243***	-	-	-	-	-	3.004***	-	-0.587***	-
16		*SURE-Autometrics*	65.419***	0.041	-	-	0.241***	-0.293	-	-	-0.656***	-
17	E	*SURE(IFGLS)-Autometrics*	66.591***	-	-	-	0.246***	-	-	-	-0.648***	-
18		*SURE(EM)-Autometrics*	66.587***	-	-	-	0.246***	-	-	-	-0.648***	-

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.13 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\bar{R}^2$	Std. errors
1	A	*Mine*	-0.286***	-	-0.059***	0.039	-	-	-1.931***	0.792***	0.824	2.832
2		*Mine-SURE*	-0.261***	-	-0.066***	0.026***	-	-	-1.974***	0.466**	0.812	2.770
3		*Mine-SURE(IFGLS)*	-0.245***	-	-0.068***	0.021***	-	-	-2.006***	0.339	0.802	2.845
4		*Mine-SURE(EM)*	-0.359***	-	-0.065***	0.018**	-	-	-2.034***	0.382*	0.798	2.895
		Average									0.809	2.836
5	B	*SURE-Mine*	-0.365***	-	-0.063***	-	-	-	-1.969***	-	0.775	3.122
6		*SURE(IFGLS)-Mine*	-0.242***	-	-0.067***	-	-	-	-2.013***	-	0.772	3.115
7		*SURE(EM)-Mine*	-0.359***	-	-0.065***	-	-	-	-1.999***	-	0.773	3.135
		Average									0.773	3.124
8	C	*Autometrics*	-0.175***	-	-0.078***	-	-	-	-1.115***	-	0.944	1.594
9		*Autometrics-SURE*	-0.168***	-	-0.078***	-	-	-	-1.285***	-	0.942	1.560
10		*Autometrics-SURE(IFGLS)*	-0.166***	-	-0.078***	-	-	-	-1.363***	-	0.940	1.586
11		*Autometrics-SURE(EM)*	-0.166***	-	-0.078***	-	-	-	-1.360***	-	0.940	1.586
		Average									0.942	1.582
12	D	*Stepwise*	-0.198***	-	-0.077***	-	-	-	-1.090***	-	0.944	1.594
13		*Stepwise-SURE*	-0.190***	-	-0.077***	-	-	-	-1.259***	-	0.942	1.557
14		*Stepwise-SURE(IFGLS)*	-0.189***	-	-0.077***	-	-	-	-1.333***	-	0.941	1.581
15		*Stepwise-SURE(EM)*	-0.189***	-	-0.077***	-	-	-	-1.332***	-	0.941	1.580
		Average									0.942	1.578
16	E	*SURE-Autometrics*	-0.165***	-	-0.079***	0.014***	-	-	-1.322***	-	0.944	1.497
17		*SURE(IFGLS)-Autometrics*	-0.165***	-	-0.080***	0.008**	-	-	-1.384***	-	0.941	1.560
18		*SURE(EM)-Autometrics*	-0.165***	-	-0.080***	0.008**	-	-	-1.383***	-	0.941	1.560
		Average									0.942	1.539
		Total Average									0.882	2.132

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.14

Estimated Models of WQI for S8 (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1		*Mine*	81.621***	-	-	-	-	-	-	-	-0.391	-
2	A	*Mine-SURE*	81.226***	-	-	-	-	-	-	-	-0.368	-
3		*Mine-SURE(IFGLS)*	81.327***	-	-	-	-	-	-	-	-0.390*	-
4		*Mine-SURE(EM)*	81.243***	-	-	-	-	-	-	-	-	-
5		*SURE-Mine*	81.472***	-	-	-	-	-	-	-	-	-
6	B	*SURE(IFGLS)-Mine*	81.837***	-	-	-	-	-	-	-	-0.456**	-
7		*SURE(EM)-Mine*	81.436***	-	-	-	-	-	-	-	-	-
8		*Autometrics*	60.942***	-	-	-	0.297***	-	-	-	-0.752***	-
9	C	*Autometrics-SURE*	61.700***	-	-	-	0.295***	-	-	-	-0.726***	-
10		*Autometrics-SURE(IFGLS)*	62.032***	-	-	-	0.295***	-	-	-	-0.713***	-
11		*Autometrics-SURE(EM)*	62.025***	-	-	-	0.295***	-	-	-	-0.713***	-
12		*Stepwise*	60.942***	-	-	-	0.297***	-	-	-	-0.752***	-
13	D	*Stepwise-SURE*	61.734***	-	-	-	0.293***	-	-	-	-0.715***	-
14		*Stepwise-SURE(IFGLS)*	62.066***	-	-	-	0.291***	-	-	-	-0.699***	-
15		*Stepwise-SURE(EM)*	62.060***	-	-	-	0.292***	-	-	-	-0.699**	-
16		*SURE-Autometrics*	58.618***	-	0.054**	-	0.295***	-	-	-	-0.708***	-0.063
17	E	*SURE(IFGLS)-Autometrics*	57.561***	-	0.060***	-	0.299***	-	-	-	-0.712***	-
18		*SURE(EM)-Autometrics*	57.554***	-	0.060**	-	0.299***	-	-	-	-0.713***	-

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.14 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\bar{R}^2$	Std. errors
1	A	*Mine*	-0.279***	-	-0.043***	-	-	-	-1.836***	0.260	0.663	4.311
2		*Mine-SURE*	-0.265***	-	-0.046***	-	-	-	-1.917***	0.302	0.661	4.150
3		*Mine-SURE(IFGLS)*	-0.245***	-	-0.048***	-	-	-	-1.972***	0.243	0.659	4.164
4		*Mine-SURE(EM)*	-0.361***	-	-0.046***	-	-	-	-1.840***	-	0.654	4.252
		Average									0.659	4.219
5	B	*SURE-Mine*	-0.362***	-	-0.046***	-	-	-	-1.895***	-	0.654	4.254
6		*SURE(IFGLS)-Mine*	-0.207***	-	-0.049***	-	-	-	-1.997***	-	0.657	4.207
7		*SURE(EM)-Mine*	-0.355***	-	-0.047***	-	-	-	-1.933***	-	0.652	4.267
		Average									0.654	4.243
8	C	*Autometrics*	-0.110***	-	-0.054***	-	-	-	-1.250***	0.253**	0.950	1.663
9		*Autometrics-SURE*	-0.116***	-	-0.057***	-	-	-	-1.411***	0.237**	0.948	1.608
10		*Autometrics-SURE(IFGLS)*	-0.119***	-	-0.057***	-	-	-	-1.494***	0.226**	0.947	1.636
11		*Autometrics-SURE(EM)*	-0.119***	-	-0.057***	-	-	-	-1.492***	0.227**	0.947	1.636
		Average									0.948	1.636
12	D	*Stepwise*	-0.110***	-	-0.054***	-	-	-	-1.250***	0.253**	0.950	1.663
13		*Stepwise-SURE*	-0.120***	-	-0.055***	-	-	-	-1.395***	0.239**	0.949	1.604
14		*Stepwise-SURE(IFGLS)*	-0.124***	-	-0.056***	-	-	-	-1.472***	0.232**	0.947	1.628
15		*Stepwise-SURE(EM)*	-0.124***	-	-0.056***	-	-	-	-1.470***	0.233**	0.947	1.627
		Average									0.948	1.631
16	E	*SURE-Autometrics*	-0.114***	-	-0.059***	-	-	-	-0.373***	0.253**	0.950	1.558
17		*SURE(IFGLS)-Autometrics*	-0.110***	-	-0.056***	-	-	-	-1.430***	0.223**	0.947	1.612
18		*SURE(EM)-Autometrics*	-0.110***	-	-0.056***	-	-	-	-1.429***	0.223**	0.947	1.611
		Average									0.948	1.594
		Total Average									0.831	2.665

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.15

Estimated Models of WQI for S25 (Four Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1	A	*Mine*	85.163***	-	-	-	-	-	-	-	-	-
2		*Mine-SURE*	83.320***	-	-	-	-	-	-	-	-	-
3		*Mine-SURE(IFGLS)*	83.199***	-	-	-	-	-	-	-	-	-
4		*Mine-SURE(EM)*	82.338***	-	-	-	-	-	-	-	-	-
5	B	*SURE-Mine*	83.343***	-	-	-	-	-	-	-	-	-
6		*SURE(IFGLS)-Mine*	83.256***	-	-	-	-	-	-	-	-	-
7		*SURE(EM)-Mine*	83.199***	-	-	-	-	-	-	-	-	-
8	C	*Autometrics*	69.321***	-	-	-	-	-	2.905***	-	-0.703***	
9		*Autometrics-SURE*	68.602***	-	-	-	-	-	2.893***	-	-0.687***	0.395***
10		*Autometrics-SURE(IFGLS)*	68.559***	-	-	-	-	-	2.904***	-	-0.665***	0.385***
11		*Autometrics-SURE(EM)*	68.559***	-	-	-	-	-	2.903***	-	-0.665***	0.386***
12	D	*Stepwise*	69.325***	-	-	-	0.229***	-	-	-	-0.711**	-
13		*Stepwise-SURE*	69.638***	-	-	-	0.223***	-	-	-	-0.592***	-
14		*Stepwise-SURE(IFGLS)*	69.650***	-	-	-	0.223***	-	-	-	-0.569***	-
15		*Stepwise-SURE(EM)*	69.649***	-	-	-	0.223***	-	-	-	-0.569***	-
16	E	*SURE-Autometrics*	66.558***	-	-	0.044*	0.222***	-	-	-	-0.681***	0.351***
17		*SURE(IFGLS)-Autometrics*	68.854***	-	-	-	-	-	2.909***	-	-0.656***	0.390***
18		*SURE(EM)-Autometrics*	68.854***	-	-	-	-	-	2.909***	-	-0.656***	0.390***

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.15 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	-0.458***	-	-0.069***	-	-	-	-2.149***	-	0.746	4.333
2		*Mine-SURE*	-0.393***	-	-0.056***	-	-	-	-2.275***	-	0.739	4.282
3		*Mine-SURE(IFGLS)*	-0.385***	-	-0.056***	-	-	-	-2.300***	-	0.737	4.294
4		*Mine-SURE(EM)*	-	-	-0.057***	-	-	-	-2.321***	-	0.717	4.454
		Average									0.735	4.341
5	B	*SURE-Mine*	-0.386***	-	-0.058***	-	-	-	-2.323***	-	0.737	4.292
6		*SURE(IFGLS)-Mine*	-0.384***	-	-0.056***	-	-	-	-2.327***	-	0.737	4.298
7		*SURE(EM)-Mine*	-0.378***	-	-0.057***	-	-	-	-2.344***	-	0.736	4.306
		Average									0.737	4.299
8	C	*Autometrics*	-0.170***	0.001	-0.078***	-	-	-	-1.632***	-	0.942	2.072
9		*Autometrics-SURE*	-0.159***	-0.124***	-0.074***	-	-	-	-1.631***	-	0.944	1.924
10		*Autometrics-SURE(IFGLS)*	-0.163***	-0.120***	-0.073***	-	-	-	-1.677***	-	0.943	1.940
11		*Autometrics-SURE(EM)*	-0.163***	-0.120**	-0.073***	-	-	-	-1.676***	-	0.943	1.940
		Average									0.943	1.969
12	D	*Stepwise*	-0.168***	-	-0.078	-	-	-	-1.625***	-	0.942	2.072
13		*Stepwise-SURE*	-0.194***	-	-0.074***	-	-	-	-1.763***	-	0.941	2.013
14		*Stepwise-SURE(IFGLS)*	-0.198***	-	-0.073***	-	-	-	-1.804***	-	0.940	2.027
15		*Stepwise-SURE(EM)*	-0.198***	-	-0.073***	-	-	-	-1.804***	-	0.940	2.027
		Average									0.941	2.035
16	E	*SURE-Autometrics*	-0.158***	-0.124***	-0.074***	-	-	-	-1.652***	-	0.945	1.894
17		*SURE(IFGLS)-Autometrics*	-0.165***	-0.125***	-0.074***	-	-	-	-1.741***	-	0.942	1.960
18		*SURE(EM)-Autometrics*	-0.165***	-0.125***	-0.074***	-	-	-	-1.740***	-	0.942	1.960
		Average									0.943	1.938
		Total Average									0.860	2.916

***Significant at 1%, ** Significant at 5%, * Significant at 10%

From Tables 5.16 and 5.17, all procedures from *Autometrics* appeared to perform the best compared to *Stepwise*-based or *Mine*-based selections. It is clear that Groups E and C procedures gained the two highest ranks, followed by *Stepwise*-based (Group D) and finally *Mine*-based (Groups A and B) models selection procedures. A comparison of the two results reveal that all models gave similar performance for RMSE and also GRMSE. Additionally, *SURE(IFGLS)-Autometrics* stood out at rank 1 in one-step ahead for both errors. RMSE's two-step ahead first position is again held by *SURE(IFGLS)-Autometrics*, although it was defeated by *SURE-Autometrics* in three-step ahead. As for GRMSE, *Autometrics* and *SURE(EM)-Autometrics* surpassed other procedures for two and three-step-ahead correspondingly. Thus, there is strong evidence that automated simultaneous selections using iterative estimations are exceptional alternatives in forecasting.

Apart from simultaneous selections, procedures to choose individual equations had shown moderate accomplishments only. This is referring to *Autometrics-SURE(IFGLS)*, *Autometrics-SURE*, *Autometrics*, *Stepwise-SURE(IFGLS)*, *Stepwise-SURE(EM)*, *Stepwise-SURE* and *Stepwise*. Therefore, it is undeniable that single-equation selections are still beneficial. Nevertheless, one noticeable point was that *Mine*-based selections performed worst of all. This finding was expected since manual selections has its own flaws, especially its dependency on modeller only (Magnus & Morgan, 1999). Common factors including lack of knowledge and skills of modeller, besides long and tedious processes might have caused selections not being done thoroughly. As a result, these manual procedures failed and consequently lost to automated selections.

Table 5.16

Forecasting Performances based on RMSE (Four Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank
1	A	*Mine*	6.2816	18	6.1471	4	6.4803	12	6.6183	4	7.5159	13	7.5479	4
2		*Mine-SURE*	6.1100	13			6.6919	16			7.5901	14		
3		*Mine-SURE(IFGLS)*	6.1490	15			6.7222	18			7.6174	17		
4		*Mine-SURE(EM)*	6.0477	12			6.5787	13			7.4681	12		
5	B	*SURE-Mine*	6.1463	14	6.1704	5	6.6620	15	6.6788	5	7.6085	16	7.6264	5
6		*SURE(IFGLS)-Mine*	6.2093	17			6.7219	17			7.6793	18		
7		*SURE(EM)-Mine*	6.1555	16			6.6524	14			7.5914	15		
8	C	*Autometrics*	2.1541	7	2.1098	2	2.3054	10	2.2432	2	2.2227	7	2.1955	2
9		*Autometrics-SURE*	2.0976	6			2.2298	6			2.1794	4		
10		*Autometrics-SURE(IFGLS)*	2.0924	4			2.2155	4			2.1904	6		
11		*Autometrics-SURE(EM)*	2.0949	5			2.2220	5			2.1895	5		
12	D	*Stepwise*	2.1586	8	2.1858	3	2.3014	9	2.3002	3	2.2334	8	2.3722	3
13		*Stepwise-SURE*	2.1829	9			2.2994	8			2.4100	9		
14		*Stepwise-SURE(IFGLS)*	2.2003	10			2.2935	7			2.4210	10		
15		*Stepwise-SURE(EM)*	2.2012	11			2.3063	11			2.4242	11		
16	E	*SURE-Autometrics*	2.0653	3	2.0041	1	2.1216	2	2.1184	1	1.8978	1	2.0150	1
17		*SURE(IFGLS)-Autometrics*	1.9711	1			2.1091	1			2.0877	3		
18		*SURE(EM)-Autometrics*	1.9759	2			2.1245	3			2.0596	2		

Table 5.17

Forecasting Performances based on GRMSE (Four Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank	GRMSE	Rank	Group GRMSE	Group Rank
1		*Mine*	3.5396	12			3.7486	12			5.3617	12		
2	A	*Mine-SURE*	4.2184	13	4.1290	4	4.3786	13	4.3422	4	6.4620	13	6.2271	4
3		*Mine-SURE(IFGLS)*	4.3046	14			4.5097	14			6.6045	15		
4		*Mine-SURE(EM)*	4.4535	15			4.7317	17			6.4800	14		
5		*SURE-Mine*	4.4602	16			4.7051	16			6.7971	17		
6	B	*SURE(IFGLS)-Mine*	4.6312	18	4.5298	5	4.8897	18	4.7524	5	6.8717	18	6.8122	5
7		*SURE(EM)-Mine*	4.4981	17			4.6625	15			6.7679	16		
8		*Autometrics*	1.6879	6			1.5031	1			1.5623	3		
9	C	*Autometrics-SURE*	1.7913	10	1.7341	2	1.6391	6	1.5959	1	1.7073	5	1.6812	2
10		*Autometrics-SURE(IFGLS)*	1.7250	7			1.6213	5			1.7231	6		
11		*Autometrics-SURE(EM)*	1.7321	8			1.6199	4			1.7319	7		
12		*Stepwise*	1.8593	11			1.9051	11			1.9873	8		
13	D	*Stepwise-SURE*	1.7511	9	1.7361	3	1.7572	10	1.7485	3	2.1175	11	2.0825	3
14		*Stepwise-SURE(IFGLS)*	1.6485	4			1.6665	9			2.1137	10		
15		*Stepwise-SURE(EM)*	1.6855	5			1.6650	8			2.1115	9		
16		*SURE-Autometrics*	1.6192	3			1.6614	7			1.6298	4		
17	E	*SURE(IFGLS)-Autometrics*	1.5289	1	1.5657	1	1.5688	2	1.6022	2	1.4889	2	1.5229	1
18		*SURE(EM)-Autometrics*	1.5489	2			1.5765	3			1.4499	1		

Equally important is comparison between two forecast errors across analysis, where values of GRMSE are less than RMSE. This essential outcome implied presence of outliers for errors in data. Given this condition, GRMSE seemed to be a better approximation than RMSE. Thus, estimated models were rechecked and showed that models S6 and S25 gave greater errors values of 2.781 and 2.916, respectively. Hence, additional evaluation removed S6 and S25. Analysis was continued with the remaining stations, S7 and S8. Next section discusses assessments for these two equations.

5.5.2 Models Selection of Two Equations

Stations S7 and S8 represent system of two equations with estimated models displayed in Table 5.18 and Table 5.19. Overall, each model selected in *Autometrics*-based (Groups C and E) and *Stepwise*-based selections (Group D) has $\bar{R}^2$ values of more than 0.940 and standard errors at the range of 1.500 to 1.600. Meanwhile, *Mine*-type selections (Groups A and B) have lower $\bar{R}^2$ values and higher standard errors in comparison with *Autometrics*-based and *Stepwise*-based selections. BOD (Δx_{i3t}), COD (Δx_{i4t}), SS (Δx_{i5t}) and NH_3N (Δx_{i7t}) are again the most frequently selected variables in majority of the models. Yet, similarities occurred where same selected variables were found in *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics, Autometrics-SURE(IFGLS)* and *Autometrics-SURE(EM)*, *Stepwise-SURE(IFGLS)* and *Stepwise-SURE(EM)*, despite the remaining models selection procedures developed different selected models. The resemblances are most possibly attributed to iterative processes during estimation steps of IFGLS and EM algorithm even though number of equations in the systems had been reduced.

Table 5.18

Estimated Models of WQI for S7 (Two Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1	A	*Mine*	64.127***	-	-	-	-	-	-	-	-0.310**	-
2		*Mine-SURE*	71.255***	0.157***	-	-	-	-	-	-	-0.437**	-
3		*Mine-SURE(IFGLS)*	74.925***	0.116*	-	-	-	-	-	-	-0.510***	-
4		*Mine-SURE(EM)*	74.171***	0.113**	-	-	-	-	-	-	-	-
5	B	*SURE-Mine*	83.427***	-	-	-	-	-	-	-	-	-
6		*SURE(IFGLS)-Mine*	82.971***	-	-	-	-	-	-	-	-0.391**	-
7		*SURE(EM)-Mine*	83.477***	-	-	-	-	-	-	-	-	-
8	C	*Autometrics*	66.372***	-	-	-	0.245***	-	-	-	-0.668***	-
9		*Autometrics-SURE*	66.663***	-	-	-	0.245***	-	-	-	-0.674***	-
10		*Autometrics-SURE(IFGLS)*	66.899***	-	-	-	0.245***	-	-	-	-0.670***	-
11		*Autometrics-SURE(EM)*	66.897***	-	-	-	0.245***	-	-	-	-0.670***	-
12	D	*Stepwise*	66.850***	-	-	-	-	-	2.992***	-	-0.613***	-
13		*Stepwise-SURE*	67.175***	-	-	-	-	-	2.982***	-	-0.615***	-
14		*Stepwise-SURE(IFGLS)*	67.409***	-	-	-	-	-	2.971***	-	-0.609***	-
15		*Stepwise-SURE(EM)*	67.404***	-	-	-	-	-	2.971***	-	-0.609***	-
16	E	*SURE-Autometrics*	66.621***	-	-	-	0.242***	-	-	-	-0.683***	-
17		*SURE(IFGLS)-Autometrics*	65.728***	0.088**	-0.064**	-	0.224***	-	-	-	-0.645***	-
18		*SURE(EM)-Autometrics*	65.715***	0.088**	-0.064**	-	0.224***	-	-	-	-0.645***	-

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.18 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\bar{R}^2$	Std. errors
1	A	*Mine*	-0.286***	-	-0.059***	0.039	-	-	-1.931***	0.792***	0.824	2.832
2		*Mine-SURE*	-0.227***	-	-0.066***	0.027***	-	-	-1.998***	0.429*	0.813	2.762
3		*Mine-SURE(IFGLS)*	-0.910***	-	-0.069***	0.020**	-	-	-2.060***	0.223	0.798	2.874
4		*Mine-SURE(EM)*	-0.351***	-	-0.065***	0.023***	-	-	-2.073***	0.332*	0.801	2.874
		Average									0.809	2.836
5	B	*SURE-Mine*	-0.359***	-	-0.063***	-	-	-	-2.019***	-	0.773	3.131
6		*SURE(IFGLS)-Mine*	-0.258***	-	-0.059***	0.019*	-	-	-1.939***	-	0.791	2.962
7		*SURE(EM)-Mine*	-0.350***	-	-0.065***	-	-	-	-2.081***	-	0.769	3.158
		Average									0.778	3.084
8	C	*Autometrics*	-0.175***	-	-0.078***	-	-	-	-1.115***	-	0.944	1.594
9		*Autometrics-SURE*	-0.164***	-	-0.079***	-	-	-	-1.250***	-	0.943	1.545
10		*Autometrics-SURE(IFGLS)*	-0.161***	-	-0.080***	-	-	-	-1.324***	-	0.942	1.564
11		*Autometrics-SURE(EM)*	-0.161***	-	-0.080***	-	-	-	-1.324***	-	0.942	1.564
		Average									0.943	1.567
12	D	*Stepwise*	-0.198***	-	-0.077***	-	-	-	-1.090***	-	0.944	1.594
13		*Stepwise-SURE*	-0.189***	-	-0.078***	-	-	-		-	0.943	1.544
14		*Stepwise-SURE(IFGLS)*	-0.187***	-	-0.078***	-	-	-	-1.285***	-	0.942	1.560
15		*Stepwise-SURE(EM)*	-0.187***	-	-0.078***	-	-	-	-1.284***	-	0.942	1.559
		Average									0.943	1.564
16	E	*SURE-Autometrics*	-0.161***	-	-0.080***	0.007	-	-	-1.250***	-	0.945	1.512
17		*SURE(IFGLS)-Autometrics*	-0.168***	-	-0.078***	0.011**	-	-	-1.469***	0.253*	0.943	1.508
18		*SURE(EM)-Autometrics*	-0.168***	-	-0.078***	0.011**	-	-	-1.468***	0.254*	0.943	1.507
		Average									0.944	1.509
		Total Average									0.883	2.112

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.19

Estimated Models of WQI for S8 (Two Equations Models)

No.	Group	Models selection procedures	Constant	Δy_{it-1}	Δy_{it-2}	Δy_{it-3}	Δx_{i1t}	$\Delta x_{i1(t-1)}$	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$
1		*Mine*	81.621***	-	-	-	-	-	-	-	-0.391	-
2	A	*Mine-SURE*	81.893***	-	-	-	-	-	-	-	-0.410	-
3		*Mine-SURE(IFGLS)*	82.056***	-	-	-	-	-	-	-	-0.475*	-
4		*Mine-SURE(EM)*	81.635***	-	-	-	-	-	-	-	-	-
5		*SURE-Mine*	81.569***	-	-	-	-	-	-	-	-	-
6	B	*SURE(IFGLS)-Mine*	62.199***	-	-	-	0.283***	-	-	-	-1.013***	-
7		*SURE(EM)-Mine*	81.504***	-	-	-	-	-	-	-	-	-
8		*Autometrics*	60.942***	-	-	-	0.297***	-	-	-	-0.752***	-
9	C	*Autometrics-SURE*	61.831***	-	-	-	0.293***	-	-	-	-0.724***	-
10		*Autometrics-SURE(IFGLS)*	62.223***	-	-	-	0.291***	-	-	-	-0.714***	-
11		*Autometrics-SURE(EM)*	62.219***	-	-	-	0.291***	-	-	-	-0.714***	-
12		*Stepwise*	60.942***	-	-	-	0.297***	-	-	-	-0.752***	-
13	D	*Stepwise-SURE*	61.822***	-	-	-	0.291***	-	-	-	-0.712***	-
14		*Stepwise-SURE(IFGLS)*	69.193***	-	-	-	-	0.289***	-	-	-0.697***	-
15		*Stepwise-SURE(EM)*	62.185***	-	-	-	0.289***	-	-	-	-0.698***	-
16		*SURE-Autometrics*	56.977***	0.037	0.038	-	0.287	-	-	-	-0.719	-
17	E	*SURE(IFGLS)-Autometrics*	58.144***	0.071**	-	-	0.279***	-	-	-	-0.705***	-
18		*SURE(EM)-Autometrics*	58.142***	0.071*	-	-	0.279***	-	-	-	-0.705***	-

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Table 5.19 (cont.)

No.	Group	Models selection procedures	Δx_{i4t}	$\Delta x_{i4(t-1)}$	Δx_{i5t}	$\Delta x_{i5(t-1)}$	Δx_{i6t}	$\Delta x_{i6(t-1)}$	Δx_{i7t}	$\Delta x_{i7(t-1)}$	$\overline{R}^2$	Std. errors
1	A	*Mine*	-0.279***	-	-0.043***	-	-	-	-1.836***	0.260	0.663	4.311
2		*Mine-SURE*	-0.244**	-	-0.046***	-	-	-	-1.935***	0.080	0.659	4.164
3		*Mine-SURE(IFGLS)*	-0.202**	-	-0.048***	-	-	-	-2.025***	-0.025	0.651	4.211
4		*Mine-SURE(EM)*	-0.357***	-	-0.046***	-	-	-	-1.976***	-	0.652	4.267
		Average									0.656	4.238
5	B	*SURE-Mine*	-0.353***	-	-0.047***	-	-	-	-1.986***	-	0.651	4.274
6		*SURE(IFGLS)-Mine*	-	-	-0.055***	-	-	-	-1.308***	-	0.942	1.724
7		*SURE(EM)-Mine*	-0.342***	-	-0.048***	-	-	-	-2.052***	-	0.646	4.302
		Average									0.746	3.433
8	C	*Autometrics*	-0.110***	-	-0.054***	-	-	-	-1.250***	0.253**	0.950	1.663
9		*Autometrics-SURE*	-0.118***	-	-0.060***	-	-	-	-1.387***	0.232**	0.949	1.603
10		*Autometrics-SURE(IFGLS)*	-0.120***	-	-0.058***	-	-	-	-1.460***	0.218**	0.947	1.624
11		*Autometrics-SURE(EM)*	-0.120***	-	-0.058***				-1.459***	0.218**	0.947	1.624
		Average									0.948	1.629
12	D	*Stepwise*	-0.110***	-	-0.054***	-	-	-	-1.250***	0.253**	0.950	1.663
13		*Stepwise-SURE*	-0.122***	-	-0.056***	-	-	-	-1.371***	0.230**	0.949	1.600
14		*Stepwise-SURE(IFGLS)*	-0.126***	-	-0.056***	-	-	-	-1.432***	0.217**	0.948	1.618
15		*Stepwise-SURE(EM)*	-0.126***	-	-0.056***	-	-	-	-1.431***	0.217**	0.948	1.617
		Average									0.948	1.625
16	E	*SURE-Autometrics*	-0.114***	-	-0.058***	-	-	-	-1.353***	0.301**	0.950	1.557
17		*SURE(IFGLS)-Autometrics*	-0.119***	-	-0.061***	-	-	-	-1.491***	0.342***	0.948	1.603
18		*SURE(EM)-Autometrics*	-0.119**	-	-0.061***	-	-	-	-1.490***	0.342*	0.948	1.603
		Average									0.949	1.588
		Total Average									0.850	2.503

***Significant at 1%, ** Significant at 5%, * Significant at 10%

Forecasting performances of all selections procedures are again verified by using two equations. Median values of RMSE were computed and put into positions with rank 1 for the lowest value to rank 18 for the highest values of all. Table 5.20 provides RMSE values for one until three-ahead forecasts. The top three ranks are dominated by simultaneous automated selections, which are *SURE(IFGLS)-Autometrics*, *SURE(EM)-Autometrics* and *SURE-Autometrics*. *SURE(IFGLS)-Autometrics* is at rank 1 for one-step-ahead forecast, but *SURE-Autometrics* gained the top spot for two and three-ahead forecasts. This is in reverse with rank 3 whereby *SURE-Autometrics* secured it in one-step-ahead forecast, but later attained by *SURE(IFGLS)-Autometrics* in two and three-step forecasts. In the meantime, *SURE(EM)-Autometrics* has been consistent at rank 2 for all forecasts. In general, it is evident that *Autometrics*-based procedures (Groups E and C) lead the rankings, followed by *Stepwise*-based selections (Group D) and lastly *Mine*-type procedures (Groups B and A). Nonetheless, the positions tend to change within the classifications from one step forecast to another.

At the same time, GRMSE values were tabulated in Table 5.21. *SURE(IFGLS)-Autometrics* has shown improved performance for GRMSE as compared to RMSE since it accomplished rank 1 at both one and three-step-ahead forecasts, but not for two-step-ahead forecast where *Stepwise-SURE(IFGLS)* emerged as the frontrunner. Even so, *SURE(IFGLS)-Autometrics*'s rank had advanced from 3 to 2 in this two-step-ahead forecast. As for *SURE(EM)-Autometrics*, it still managed to retain rank 2 for one and three-step-ahead forecasts also, but unfortunately dropped to rank 4 for two-step-ahead forecast. In addition, *SURE-Autometrics's* performance had surprisingly slumped severely to a much lower position of ranks 11, 9 and 3. This is considered as a significant finding of difference between iterative and two-stage estimation methods in these automated selections. Iterative estimation is thus seen to benefit better forecasting accuracy. The results of these GRMSE values revealed different positions throughout the forecasts. Finally, *Mine*-type procedures which are non-algorithm selections, however continuously underperformed regardless different types of error measures or number of equations. All in all, the findings had supported the idea that automated simultaneous selections using iterative estimation method managed to display a much better performance than manual or non-algorithm approach in this experimental setting.

Table 5.20

Forecasting Performances based on RMSE (Two Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank
1	A	*Mine*	5.2570	13	5.5901	5	5.8443	13	6.1600	5	6.7333	13	7.0842	5
2		*Mine-SURE*	5.6309	15			6.2146	15			7.1447	15		
3		*Mine-SURE(IFGLS)*	5.8755	16			6.4454	16			7.3938	16		
4		*Mine-SURE(EM)*	5.5971	14			6.1356	14			7.0651	14		
5	B	*SURE-Mine*	5.9088	17	5.1925	4	6.4699	17	5.6918	4	7.4391	17	6.5484	4
6		*SURE(IFGLS)-Mine*	3.7173	12			4.1178	12			4.7490	12		
7		*SURE(EM)-Mine*	5.9515	18			6.4877	18			7.4570	18		
8	C	*Autometrics*	1.7013	5	1.7030	2	1.8421	4	1.8561	2	2.0480	4	2.1044	2
9		*Autometrics-SURE*	1.6980	4			1.8551	5			2.1069	5		
10		*Autometrics-SURE(IFGLS)*	1.7061	6			1.8655	7			2.1343	7		
11		*Autometrics-SURE(EM)*	1.7066	7			1.8616	6			2.1284	6		
12	D	*Stepwise*	1.7467	8	1.7698	3	1.8981	8	1.9376	3	2.1387	8	2.2127	3
13		*Stepwise-SURE*	1.7667	9			1.9388	9			2.2142	9		
14		*Stepwise-SURE(IFGLS)*	1.7845	11			1.9615	11			2.2489	11		
15		*Stepwise-SURE(EM)*	1.7812	10			1.9518	10			2.2489	10		
16	E	*SURE-Autometrics*	1.6902	3	1.6097	1	1.7318	1	1.7521	1	1.8903	1	1.9579	1
17		*SURE(IFGLS)-Autometrics*	1.5686	1			1.7632	3			1.9941	3		
18		*SURE(EM)-Autometrics*	1.5704	2			1.7612	2			1.9892	2		

Table 5.21

Forecasting Performances based on GRMSE (Two Equations Models)

No.	Group	Models selection procedures	One-Step				Two-Step				Three-Step			
			RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank	RMSE	Rank	Group RMSE	Group Rank
1	A	*Mine*	2.7457	13	3.1130	5	3.0794	16	3.2957	5	4.5308	13	5.1836	5
2		*Mine-SURE*	3.2879	17			3.5000	17			5.3073	15		
3		*Mine-SURE(IFGLS)*	3.5229	18			3.6583	18			5.6722	18		
4		*Mine-SURE(EM)*	2.8956	14			2.9450	15			5.2240	14		
5	B	*SURE-Mine*	2.9874	16	2.7046	4	2.8238	13	2.6106	4	5.5120	16	4.9326	4
6		*SURE(IFGLS)-Mine*	2.1547	12			2.1736	12			3.7733	12		
7		*SURE(EM)-Mine*	2.9718	15			2.8345	14			5.5124	17		
8	C	*Autometrics*	1.2793	9	1.2704	3	1.5031	11	1.3981	3	1.7483	4	1.8048	2
9		*Autometrics-SURE*	1.2874	10			1.4083	8			1.8082	5		
10		*Autometrics-SURE(IFGLS)*	1.2475	7			1.3257	5			1.8318	7		
11		*Autometrics-SURE(EM)*	1.2673	8			1.3551	7			1.8307	6		
12	D	*Stepwise*	1.2421	6	1.1966	1	1.4517	10	1.3183	1	1.8586	8	1.9272	3
13		*Stepwise-SURE*	1.2159	5			1.3323	6			1.9313	9		
14		*Stepwise-SURE(IFGLS)*	1.1450	3			1.2191	1			1.9614	11		
15		*Stepwise-SURE(EM)*	1.1834	4			1.2701	3			1.9574	10		
16	E	*SURE-Autometrics*	1.4108	11	1.1994	2	1.4391	9	1.3227	2	1.6790	3	1.6609	1
17		*SURE(IFGLS)-Autometrics*	1.0869	1			1.2508	2			1.6514	1		
18		*SURE(EM)-Autometrics*	1.1005	2			1.2783	4			1.6522	2		

5.6 Summary of Findings

This chapter reviewed the application side of the newly built algorithms, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* together with 16 other models selection procedures. The procedures are *Mine, Mine-SURE, Mine-SURE(IFGLS), Mine-SURE(EM), SURE-Mine, SURE(IFGLS)-Mine, SURE(EM)-Mine, Autometrics, Autometrics-SURE, Autometrics-SURE(IFGLS), Autometrics-SURE(EM), Stepwise, Stepwise-SURE, Stepwise-SURE(IFGLS), Stepwise-SURE(EM)* and *SURE-Autometrics,* The manual and automated approaches adopted in this study can be classified into individual or simultaneous selections depending on the type of estimation methods used which are OLS, FGLS, IFGLS and EM algorithm. Estimated models for each procedure are presented and forecast errors were also calculated in order to put the procedures into ranks and served as indicators of their performances.

The first set of data utilised was the national growth rates of Denmark, Ireland, Netherlands and United Kingdom. The analysis started off with small initial GUMS of eight variables. The number of variables chosen was following necessary condition by Griffiths et al., (2001) in implementing ML estimator by considering 47 observations in data set as well as two and four equations involved. Results for four equations had seen the success of the proposed algorithms of *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* since the values of their RMSE and GRMSE were among the smallest in the lists and thus ranked at the top three. Small values of error measures indicate better accuracy and improved forecasting performance. This shows that *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* have outclassed other model selection procedures.

At the same time, *SURE(IFGLS)-Mine* is also comparatively at the same level with *SURE(EM)-Autometrics* which means that iterative estimation method is still working extremely well in spite of variables selected manually. Moreover, *Autometrics*-based selections (Groups C and E) managed to surpass *Stepwise*-based (Group D) and *Mine*-based (Groups A and B) procedures in both simultaneous and individual selections. Unfortunately, different outcome was found for two equations involving Ireland and Netherlands. Throughout the analysis, the estimated models produced by several procedures tend to show same selected variables. Therefore, forecast errors and ranking of procedures showed similarities. Even with the equivalence, *SURE(IFGLS)-Autometrics*, *SURE(EM)-Autometrics* and *SURE-Autometrics* still proved reasonable achievement.

In addition to the national growth rates data, another data set was put into investigations. A larger sample size of data was chosen as to seek the effect of changes in sample size besides providing a more conducive condition of ML estimator. Thus, data of 75 observations of WQI of river in Malaysia was applied. Since data size was larger, hence a bigger GUMS with 17 variables was tested. In analysis of four equations, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* still maintained the top three ranks, but with similar achievement from *SURE-Autometrics* and *Autometrics* at a few places. Overall results suggest automated approach's performance is far beyond the manuals. This finding also applies for analysis of two equations. For better assessments of all procedures done, compilation of the empirical data results according to ranks of the procedures based on their final model estimation methods i.e OLS, FGLS, IFGLS and EM estimations, are presented in Tables 5.22 until 5.25.

Table 5.22

Forecasting Performance of Algorithms with OLS Estimation in Final Models

Algorithm	Data	Number of equations	Error	Rank		
				1-step ahead	2-step ahead	3-step ahead
Autometrics	National	4	RMSE	8	8	5
			GRMSE	4	6	4
		2	RMSE	1	1	1
			GRMSE	1	1	1
	WQI	4	RMSE	7	10	7
			GRMSE	6	1	3
		2	RMSE	5	4	4
			GRMSE	9	11	4
Stepwise	National	4	RMSE	18	18	11
			GRMSE	18	18	18
		2	RMSE	18	18	18
			GRMSE	18	18	18
	WQI	4	RMSE	8	9	8
			GRMSE	11	11	8
		2	RMSE	8	8	8
			GRMSE	6	10	8
Mine	National	4	RMSE	17	17	15
			GRMSE	17	17	16
		2	RMSE	17	17	17
			GRMSE	17	17	17
	WQI	4	RMSE	18	12	13
			GRMSE	12	12	12
		2	RMSE	13	13	13
			GRMSE	13	16	13

In Table 5.22, *Autometrics* received higher ranks than *Stepwise* and *Mine*, particularly for two equations of national growth rates data where the removal of Ireland and Netherland's equations with large errors had prompted weak correlation errors in the remaining equations. Weak correlations thus had made single selections more efficient than simultaneous selections in this case. The tree search strategy in *Autometrics* had benefited models selection since it is considered as more comprehensive and thorough compared to backward elimination in *Stepwise* or time-consuming personal judgement in *Mine*. Performances of the latter two procedures were distinctly far behind *Autometrics*'s.

Another estimation method used here is FGLS, a well-known method in modelling SURE equations. *SURE-Autometrics, Autometrics-SURE, Stepwise-SURE, SURE-Mine* and *Mine-SURE* were categorized under the same classification (see Table 5.23) because FGLS estimation was used in the final models. Out of these five procedures, *SURE-Autometrics* and *Autometrics-SURE* were seen to give higher ranks contrasted to other selections which unfortunately showed poor performances. Simultaneous selections in *SURE-Autometrics* was best used for WQI data, while *Autometrics-SURE* was more suitable in national growth rates. Therefore, in the use of FGLS estimation, the collaborative algorithms involving *Autometrics* and SURE model surpassed the effectiveness of *Stepwise* and also manual approaches. However, a modeller has to make the right choice of algorithms depending on the nature of data used.

In the role of iterative process in models selection, some similar findings with FGLS-based procedures were established when IFGLS estimation was used instead. Interestingly, *SURE(IFGLS)-Autometrics* performed much better than its counterpart, *SURE-Autometrics* by securing rank 1 at several spots and managed to be in the top three ranks. This performance is followed by *Autometrics-SURE(IFGLS)* signifying the less success of single selections than simultaneous selections in this process. The other procedures were somewhat remained to be underperformed. The ranks are displayed in Table 5.24.

Table 5.23

Forecasting Performance of Algorithms with FGLS Estimation in Final Models

Algorithm	Data	Number of equations	Error	Rank		
				1-step ahead	2-step ahead	3-step ahead
SURE-Autometrics	National	4	RMSE	4	3	11
			GRMSE	8	10	8
		2	RMSE	5	5	5
			GRMSE	5	5	5
	WQI	4	RMSE	3	2	1
			GRMSE	3	7	4
		2	RMSE	3	1	1
			GRMSE	11	9	3
Autometrics-SURE	National	4	RMSE	6	6	4
			GRMSE	5	7	5
		2	RMSE	2	2	2
			GRMSE	2	2	2
	WQI	4	RMSE	6	6	4
			GRMSE	10	6	5
		2	RMSE	4	5	5
			GRMSE	10	8	5
Stepwise-SURE	National	4	RMSE	16	8	8
			GRMSE	16	16	16
		2	RMSE	14	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	9	8	9
			GRMSE	9	10	11
		2	RMSE	9	9	9
			GRMSE	5	6	9
SURE-Mine	National	4	RMSE	12	12	17
			GRMSE	10	4	10
		2	RMSE	14	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	14	15	16
			GRMSE	16	16	17
		2	RMSE	17	17	17
			GRMSE	16	13	16
Mine-SURE	National	4	RMSE	15	14	13
			GRMSE	14	14	13
		2	RMSE	14	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	13	16	14
			GRMSE	13	13	13
		2	RMSE	15	15	15
			GRMSE	17	17	15

Table 5.24

Forecasting Performance of Algorithms with IFGLS Estimation in Final Models

Algorithm	Data	Number of equations	Error	Rank		
				1-step ahead	2-step ahead	3-step ahead
SURE(IFGLS)-Autometrics	National	4	RMSE	1	1	1
			GRMSE	2	2	3
		2	RMSE	5	5	5
			GRMSE	5	5	5
	WQI	4	RMSE	1	1	1
			GRMSE	1	2	2
		2	RMSE	1	3	3
			GRMSE	1	2	1
Autometrics-SURE(IFGLS)	National	4	RMSE	6	8	7
			GRMSE	6	9	7
		2	RMSE	2	3	2
			GRMSE	2	3	3
	WQI	4	RMSE	4	4	6
			GRMSE	7	5	6
		2	RMSE	6	7	7
			GRMSE	7	5	7
Stepwise-SURE(IFGLS)	National	4	RMSE	12	12	8
			GRMSE	13	13	13
		2	RMSE	8	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	10	7	10
			GRMSE	4	9	10
		2	RMSE	11	11	11
			GRMSE	3	1	11
SURE(IFGLS)-Mine	National	4	RMSE	2	3	2
			GRMSE	1	1	1
		2	RMSE	8	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	17	17	18
			GRMSE	18	18	18
		2	RMSE	12	12	12
			GRMSE	12	12	12
Mine-SURE(IFGLS)	National	4	RMSE	10	16	13
			GRMSE	11	11	11
		2	RMSE	8	11	8
			GRMSE	8	8	8
	WQI	4	RMSE	15	18	17
			GRMSE	14	14	15
		2	RMSE	16	16	16
			GRMSE	18	18	18

Table 5.25
Forecasting Performance of Algorithms with EM Estimation in Final Models

Algorithm	Data	Number of equations	Error	Rank		
				1-step ahead	2-step ahead	3-step ahead
SURE(EM)-Autometrics	National	4	RMSE	2	2	3
			GRMSE	2	2	2
		2	RMSE	5	5	5
			GRMSE	5	5	5
	WQI	4	RMSE	2	3	2
			GRMSE	2	3	1
		2	RMSE	2	2	2
			GRMSE	2	4	2
Autometrics-SURE(EM)	National	4	RMSE	5	5	5
			GRMSE	7	8	6
		2	RMSE	4	4	4
			GRMSE	2	3	4
	WQI	4	RMSE	5	5	5
			GRMSE	8	4	7
		2	RMSE	7	6	6
			GRMSE	8	7	9
Stepwise-SURE(EM)	National	4	RMSE	14	7	10
			GRMSE	14	14	13
		2	RMSE	8	8	8
			GRMSE	8	8	8
	WQI	4	RMSE	11	11	11
			GRMSE	5	8	9
		2	RMSE	10	10	10
			GRMSE	4	3	10
SURE(EM)-Mine	National	4	RMSE	9	8	18
			GRMSE	9	5	9
		2	RMSE	8	8	8
			GRMSE	8	8	8
	WQI	4	RMSE	16	14	15
			GRMSE	17	15	16
		2	RMSE	18	18	18
			GRMSE	15	14	17
Mine-SURE(EM)	National	4	RMSE	10	14	15
			GRMSE	12	11	11
		2	RMSE	8	8	8
			GRMSE	8	8	8
	WQI	4	RMSE	12	13	12
			GRMSE	15	17	14
		2	RMSE	14	14	14
			GRMSE	14	15	14

Meanwhile, Table 5.25 reveals ranks of forecast errors for another iterative method that is EM algorithm. Given that the method belongs to iterative process too, the overall results are not much different than those from IFGLS-based procedures. *SURE(EM)-Autometrics* succeeded to deliver good ranks among all procedures considered in this group, even though slightly lower than of *SURE(IFGLS)-Autometrics*. Individual selections using *Autometrics-SURE(EM)* was only able to obtain middle ranks, whereas *Stepwise-SURE(EM)* at lower positions. As for manual approaches, *SURE(EM)-Mine* and *Mine-SURE(EM)* continued to be least preferred procedures in this analysis. Finally, Table 5.26 displays ranks of newly developed algorithms in this study, *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* based on error measures.

Table 5.26
Forecasting Performance of SURE(IFGLS)-Autometrics and SURE(EM)-Autometrics

Algorithm	Data	Number of equations	Error	Rank 1-step ahead	Rank 2-step ahead	Rank 3-step ahead
SURE(IFGLS)-Autometrics	National	4	RMSE	1	1	1
			GRMSE	2	2	3
		2	RMSE	5	5	5
			GRMSE	5	5	5
	WQI	4	RMSE	1	1	3
			GRMSE	1	2	2
		2	RMSE	1	3	3
			GRMSE	1	2	1
SURE(EM)-Autometrics	National	4	RMSE	2	2	3
			GRMSE	2	2	2
		2	RMSE	5	5	5
			GRMSE	5	5	5
	WQI	4	RMSE	2	3	2
			GRMSE	2	3	1
		2	RMSE	2	2	2
			GRMSE	2	4	2

SURE(IFGLS)-Autometrics is seen to deliver slightly better positions than *SURE(EM)-Autometrics* which could be contributed by continual iterative processes of IFGLS. Consequently, generalised squares estimation method still dominates efficiency of SURE models selection. On the other hand, *SURE(EM)-Autometrics'*s performance is close to its rival signifying suitability of EM algorithm in this automated procedure. Thus, iterative estimation methods embedded in automated simultaneous selections are capable in providing better forecasting accuracy.

Taken together, these results provide important insights into the importance of estimation methods in models selection, regardless of individual or simultaneous selections. They even appear to suggest that there is a strong association between quality of data and the procedures chosen which indicate a vital task for modellers in deciding the right one so that the best final model is selected. Furthermore, the results in this chapter seem to be consistent with other previous researches which found automated approaches are more superior to the manual ones.

In the next chapter, some guidelines in models selection based on simulation and empirical data studies are provided. The overall study, limitations encountered while undergoing the analysis are discussed and lastly, future works pertaining to models selection are suggested too.

CHAPTER SIX

CONCLUSION

6.1 Introduction

This final chapter concludes the whole study. Section 6.2 consists of summary of study, while Section 6.3 presents guidelines in choosing estimators during models selection. Section 6.4 is about limitations of study and Section 6.5 provides suggestions for future research.

6.2 Summary of Study

The overall study has been completed following the achievements of all three objectives of the study. These objectives were primarily initiated to seek alternative solutions in the field of models selection.

The first objective to develop *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics*, the two extended models selection algorithms for SURE model using two different estimation methods; IFGLS and EM algorithm, were detailed in Chapter Three. Both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* had the same search principles of five development phases. The first phase is the construction of initial GUMS, while second phase concentrates on process of pre-search reduction. The GUMS are then made into simpler models where all the likely ones are explored in the third phase once they are done with the pre-search. Only the terminal models are taken into accounts by using the tree search strategy as a means to reduce the computational process. Terminal models are described as models with significant variables and at the same time an acceptable reduction from GUMS besides every equation in the model is congruent. This third phase is repeated in the fourth phase to seek more terminal models through model contrast. Lastly, the most appropriate model is selected from the all terminal models found in the final phase. The algorithms are successfully developed through conversion into programming code by using *GAUSS 15*.

Second objective of the study, which is evaluations of algorithms through simulation study, were realized in Chapter Four. Based on the algorithms' five phases, these iterative algorithms had undergone simulation study adapted from Hoover and Perez (1999). *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were first tested in simulation study under several conditions: (i) two sample sizes (ii) two strengths of correlation among equations (iii) two GUMS sizes (iv) two numbers of equations in the system (v) two significance levels and (vi) three true specification models.

The results were allocated in four categories. Both algorithms showed almost comparable results and performances. High performances were mainly found at 1% significance level instead of 5% and the two equations models gained more final models same as true specifications than four equations models. At the same time, less irrelevant variables in models contributed to less complex models making the models selection to be more successful in small GUMS with at most 18 irrelevant variables than in large GUMS of at most 30 irrelevant variables. Referring to true models of M1, M2 and M3, the simplest model which is M1 generally obtained more percentages than M2 and M3. Moreover, large samples of 550 observations displayed more reliable performances compared to small samples of 275 observations. Finally, strong correlation of disturbance of 0.9 revealed better selections than weak correlation of 0.2. The algorithms were further assessed using empirical data.

Chapter Five elaborates the comparisons of SURE models selection procedures using empirical data, including the newly-built algorithms, in order to fulfil the third objective of study. The empirical data set used were national growth rates and WQI to compare 18 models selection procedures involving manual and automated approaches with OLS, FGLS, IFGLS or EM algorithm estimation methods in the final models selected. In order to determine the performances all procedures, RMSE and GRMSE were calculated for each procedure and ranked. Both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* were able to present excellent outcomes in this empirical study. IFGLS and EM algorithm also proved to be successful especially in simultaneous selections.

Overall findings suggest that both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* managed to display excellent results in this empirical study. *Autometrics*-based approaches had undoubtedly stood out in the rankings compared to other procedures signifying its superiority in models selection. Selecting a model automatically not only provides a less hassle process and gives speedier results, but it also contributes to accomplishing the right final models that may fit perfectly for forecasting purposes. Thus, the results of this research support the idea that algorithmic procedures outdo the non-algorithmic ones.

One of the more significant findings to emerge from this study is that iterative estimation methods, IFGLS and EM algorithm, tend to show success, particularly during simultaneous selections regardless in automated or manual procedures. Nevertheless, IFGLS did demonstrate higher ranks more consistently than the EM due to the fact that IFGLS is based on least square method, in which FGLS itself performs best in SURE model. Continual iteration from FGLS made IFGLS relatively faster than EM algorithm, which starts from different initial values to compute, even for models with many observations and many estimable parameters. Yet, the EM algorithm still performed steadily with its increasing likelihood from each iteration. Both iterative methods estimate the regression parameters in the models consistently. In addition, the extra rounds of iteration provided more information on how the estimates change.

The iterative processes begin with an initial approximation and produce consecutively better approximations in an infinite sequence in which the limit is the exact solution. This has given more advantages compared to direct methods when solving system of linear equations, where the trials to compute an exact solution is done in a finite number of operations. Furthermore, the direct solution will depend on rounding errors. Computer solution errors may occur due to small round off errors and thus result in accuracy loss. Whereas, any rounding errors in iterative methods do not accumulate as they are contained in the final operation. Other methods that need maximization of a nonlinear function, such as the Newton–Raphson procedure is recognised to be ineffective especially when there is high solution dimension.

In selecting models in system of equations, the changes in models' variables are thus larger than in single equation, therefore the iterative processes in IFGLS and EM algorithm have been beneficial to achieve satisfactory results matched to the data and at the same time preserving the flexibility of the setup.

In terms of empirical data, the national growth rates had problem of preserving the congruency which can be realised from changes of variables' significance in the same or other equations when any variables are eliminated. This is shown from its results for two equations models, where the position was only at rank 5 in Table 5.26. Taken together, these results suggest that algorithms depend a lot on the quality of data. Findings from both simulation and empirical data studies have given some understandings in choosing models. Lastly, guidelines on selecting SURE model based on estimation methods and conditions can be found in next section.

6.3 Guidelines in Models Selection

The overall results from simulation and empirical data studies have significantly provided some guidelines on models selection settings based on comparisons of procedures conducted previously. Certain conditions should be considered to ensure the success of selecting the best model including formulation of initial GUMS, true specification model, level of significance, number of equations, contemporaneous correlation of error across equations, sample size and quality of data, as listed below:

i. Formulation of initial GUMS

Initial GUMS must be formulated adequately to encapsulate all considerably relevant effects including topic of empirical modeling, institutional information, previous research experiences and results, as well as quality and accessibility of data, which are crucially determined by modellers' ability and knowledge.

ii. True specification model

A true specification model with less relevant variables is easier to be obtained during models selection than the one with more relevant variables. An empty model is the simplest form of model that is more straightforward to be found compared to model with variables.

iii. Level of significance

A tight significance level is required in prevention of excessive significant irrelevant variables. Experiment's setup at main significance level of 1% usually finds the true specification models more often than at 5%.

iv. Number of equations

Less number of equations in a system implies less complexity and thus increases chances of an algorithm to find the true specification models.

v. Contemporaneous correlation of error across equations

Selection of all equations simultaneously in SURE model is more efficient when its contemporaneous correlation of error across equations is strong. Equation by equation selections are probably more suitable for weak contemporaneous correlation of error.

vi. Sample size

MLE inclines to be more efficient in large samples, which means it is able to extract more information from the data, in contrast to small samples where properties of ML estimator may not be at its best.

vii. Quality of data

A good set of data should essentially pass all the diagnostic tests which are normality test of residuals, autocorrelation of residuals (AR), autocorrelated conditional heteroscedasticity, (ARCH), heteroscedasticity test and parameter constancy test. If the tests are passed, other analysis would provide better results and conclusions.

These guidelines are hoped to assist modellers to construct more conducive conditions in models selections analysis. Next section describes the limitations of this study.

6.4 Limitations of Study

In executing this models selection analysis, there were several challenges that had been encountered which need to be looked into with more in-depth investigations. One such problem is when utilizing empirical data, where the national growth rates data did show some setback in this study. The problem related to congruency in the data led to inconsistency of significance in variables, not only within an equation, but also among other equations. Failure during some tests in diagnostic checking had indicated lack of quality in the data causing the performances of algorithms based on forecasting accuracy tend to be irregular. Similar outcomes occurred here too especially in looking at effects of number equations in the procedures' performance. Thus, the choice of data does play a crucial role in determining the full functions of any models selection approaches.

In the principle of iterative methods, a number of steps are taken in which a provisional estimate comes closer and closer to the final estimate. These infinite number of steps however involved more computational process than the direct method or two-stage estimation like FGLS, specifically longer time for the algorithm to reach convergence. There were even some situations where the algorithm failed to converge and the process had to be repeated all over again. Both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* suffered from these challenges.

Ding (2013) too agreed that convergence analysis of iterative algorithm is challenging and sometimes requires further study. This difficulty is seen evidently for more complicated models in which the computational complications tend to vary and resulted in different computing time. Nonetheless, the time taken to reach the stopping criteria or complete the analysis is not the central focus here and is therefore suggested for upcoming work.

Although both *SURE(IFGLS)-Autometrics* and *SURE(EM)-Autometrics* portray iterative approach in automated simultaneous models selection algorithms, each of them is still different in its operation. In spite of having iterative steps, *SURE(IFGLS)-Autometrics* showed convergence faster than the *SURE(EM)-Autometrics* due to its direct continuation from the two-step estimation of FGLS. As for *SURE(EM)-Autometrics*, the choice of EM algorithm was particularly because of its appliance in condition of incomplete data, in addition to its alleged simplicity and success in numerous studies in recent years.

Moreover, the EM algorithm has been widely used in missing or censored data in lifetime or survival analysis, indicating that it is maybe less appropriate for time series data. At the same time, the likelihood methods may not accomplish well whenever small sample sizes being used suggesting sizeable ones must not be tolerated in ensuring any fruitful likelihood approaches (Schwarz, 2011). All in all, notwithstanding that those iterative methods did show some shortcomings, this first attempt of simultaneous models selection still provide some insights in automated approach.

6.5 Future Research

The automated simultaneous models selection procedures should be more of focus in future research interest in the area of model building. Further improvements can be made in the aspects of model specification, algorithm development, conditional setting in the experiments and many others.

One such recommendation is to incorporate other alternative methods for parameter estimation, such as using Bayesian approach for estimating the SURE model. The success of this approach in recent years and its wide acceptance in numerous fields should be extended in the area of models selections too. Nevertheless, this line of research requires more advanced and extensive description of the method that suits the search strategy. The choice of parameter estimations is again can be either form of direct or iterative methods.

As far as an iterative method is concerned, a modeller should take into account in finding a good preconditioner of the system in order to enhance the speed of its convergence. Björck (2015) did emphasize on the importance of the conditioning of the system to be solved, where every iteration step involves the solution of a more straightforward auxiliary system through a direct or an additional iterative method. It is thus essential to be equipped with processor with high computing speeds in assuring the effectiveness of the given iteration structures. Parallel computing, for instance, is one way in accomplishing faster computational operations and hopefully will improve efficiency in models selection eventually.

In evaluating the algorithms' performances, additional measurements are worth exploring further. Measurements such as retention rates for irrelevant variables (gauge) and retention rates for relevant variables (potency) in *Autometrics*-based procedures would provide more information on the algorithms' abilities. The rates are important since some relevant variables can be insignificant or irrelevant variables become significant unexpectedly. When there is a change in significance level, the relevant variables retained and irrelevant variables eliminated tend to be adjusted too. Castle et al. (2011) mentioned that variables can be retained when they are insignificant to compensate for any failures in mis-specification test in order for the models to be congruent. Meanwhile, insignificant variables which are incorrectly retained may happen under the null of no mis-specification.

Similarly, the convergence rate in the analysis should be considered as a legitimate and notable area for future research too. Convergence rate is referred as the speed of a sample iterates converge to the sample fixed point or limit when there are more iterations involved. From the point of practical implementation, this convergence rate is vital especially in dealing with a sequence of approximations for iterative methods. Higher convergence rate, for example, would need less iterations to produce a useful approximation and thus makes difference in requiring tens or thousands of iterations.

Apart from utilizing time-series data, the algorithms should be accommodated to other types of data. Cross-sectional, panel and missing data are only some to name a few. These attempts will allow us to have a more fully understanding of the behavior of data used and apply this knowledge to adapt in models decision making process. This would be a fruitful area for further work.

As automated models selection is seen as another problem solver in quicker time, empirical modellers are now able to focus more on the initial aspects of model building. Human touch is still vital and can never be neglected in any way despite advancement in technology and innovation as algorithms are merely tools to search for the ‘best’ model. Manual approaches unfortunately are less favoured primarily due to their processes which are more tiresome and time-consuming besides leading to more uncertainty.

The effectiveness of the proposed algorithms has somehow been justified by outcomes from both simulation and empirical data studies. Nevertheless, the disparities between the algorithms are hardly worth concerned about, particularly for the application users. The approaches presented here are not claimed as to greatly provide precise models selection and accurate forecast, but more of offering new attempts in automated models selection for multiple equations.

While the estimation method itself does offer more flexibility and power towards modeling selections problems, methods used in this study which are IFGLS and EM algorithm were suggested as alternatives to be employed in algorithms developments. The guidelines that have been identified therefore may assist in our understanding of the role of estimation method in selecting SURE model procedures in future studies. Taken together, the findings of this study serve as a starting point for further refinements.

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APPENDIX A
AUTOMETRICS ALGORITHM

0.0 Estimate the initial GUM.

This initializes the search procedure.

0.1 (optional) Lag-length pre-search.

0.2 Test all regressors at a loose significance level.

If passed, accept the empty model as the final model, provided diagnostic testing is satisfied, then stop.

1.0 Set $i = 0$.

The starting point for the current iteration is GUM 0 (this may be the same as the initial GUM), which has k free regressors.

1.1 (Convergence) If all regressors in the GUM are significant then stop.
This is at a slightly more stringent p-value to allow for 'squeezing'.

1.2 Update the diagnostic p-values.

Ideally, the user ensures that the initial GUM passes the diagnostic tests. However, when this is not the case, the p-value for each failed test statistic is increased. Subsequently, the p-values are adjusted downwards again if possible.

1.3 Run reduction over the root branches.

Terminal candidate models ('terminals') are collected as the search progresses. Any subtree that has a previously found terminal nested in it is skipped to speed up the search. This will result in one or more terminal.

1.4 Run reduction to search for nested terminals.

Revisit the subtrees that were skipped before. At each point it is possible to compute the minimal contrast with a known terminal to jump ahead to a possible new (non-nested) terminal. If the union of terminals after the previous step 1.3 is smaller than the union from the previous iteration, then use union contrast, otherwise use terminal contrast.

1.5 Remove terminals that fail diagnostics.

If the p-values, p_d for diagnostic testing had to be adjusted downwards, and there are some terminals that pass the original p-values, then keep only those terminal models which pass and reset p_d to the original value.

1.6 Form the union of the terminal models.

The union is called the *current* GUM or GUM *i* + 1.

1.7 Remove terminals that fail backtesting.

When using the default Autometrics settings, this step is skipped, because backtesting with respect to GUM 0 has already been done as an intergral part of the tree search: there are no terminals that fail.
Optionally, the PcGets default backtesting with respect to the current GUM can be adopted instead. In that case, there may be terminals that fail the encompassing test against the new GUM.

1.8 Remove terminals with insignificant variables.

When using the default Autometrics settings, this step is skipped because a terminal remains a terminal candidate for subsequent iterations.
However, this step is relevant when the PcGets default is used in 1.7: backtesting is with respect to the current GUM, which changes between iterations. So a terminal candidate with insignificant variables may not be a terminal next time.

1.9 Increment *i* and continue to step 1.1.

GUM *i* (determined at 1.6, but now *i* has been incremented), is the new base for the search. Note that steps 1.7 and 1.8 (when they are not skipped) remove terminals but do not modify the GUM. This is different from PcGets implementation.

APPENDIX B
SURE(IFGLS)-AUTOMETRICS OR *SURE(EM)-AUTOMETRICS* ALGORITHM

Phase 1: Estimation of Initial General Unrestricted Model (GUMS)

Step 0

Declare number of equations, including regressands and regressors for each equation.
Create lag variables.
Set main level of significance, p_a.

Step 1

Run diagnostic analyses of each equation using OLS estimation at diagnostic *p*-value, p_d.

- If all tests are satisfied, continue next step.
- Otherwise, update the p_d, then continue next step.

Step 2

Test the contemporaneous correlation of disturbances amongst the equations.

- If significant (p-value < 0.10), it indicates that IFGLS or EM algorithm estimation is more efficient than OLS.
- Otherwise, proceed with model estimation using OLS method, and then stop.

Step 3

Estimate the multiple equations model using IFGLS or EM algorithm method.

- If all regressors are significant, the equations become the final GUMS, and then stop.
- Otherwise, the equations become the initial GUMS, and then proceed to next phase.

Phase 2: Pre-search Lag Reduction

Step 0

Set pre-search *p*-value, p_p.

Step 1

Run closed lag, then common lag, and followed by common-*X* lag reductions to obtain a reduced model.

Step 2

Run common-*X* lag, then common lag, and followed by closed lag reductions to obtain another reduced model.

Step 3

Run encompassing tests of reduced models in Step 1 and Step 2 against the union of these models at p_a.

- If only one model passed the test, the model is the current GUMS.
- Otherwise, the union become the current GUMS, and then stop.

Step 4

Repeat Step 1 and 2 in Phase 1 for the current GUMS.

Phase 3: Tree Search over Root Branches

Step 0

Remove all regressors and test at p_a.

- If passed, accept the empty model. Repeat Step 1 and 2 in Phase 1.
  - If satisfied, the empty model is the final GUMS, then stop.
  - Otherwise, continue next step.
- Otherwise, continue next step.

Step 1

Set p-value for bunching, p_b and p-value for chopping, p_c.

Set i = 0. Denote current GUMS as GUMS 0.

Step 2

Check all regressors in GUMS 0.

- If all regressors are significant, accept as the final GUMS, and then stop.
- Otherwise, continue next step.

Step 3

Run root branches reduction using IFGLS or EM algorithm estimation.

- Implement pruning, bunching and chopping principles as the search progress.
- Any models that cannot be reduced any further are known as terminal candidate models.
- Collect all terminals as the search progresses.
- Any sub-tree that has a previously found terminal nested in it is skipped.

Phase 4: Tree Search for Nested Terminals

Step 0

Revisit the sub-trees that were skipped before.

Form terminal contrasts by using the minimal contrasts from the existence terminals.

Step 1

Union the terminals

- If the union is smaller than the GUM i, then use union contrast.
- Otherwise, use terminal contrast.

Step 2

Remove terminals.

- Terminals that fail diagnostic tests.
- Terminals with insignificant variables.

Step 3

New base for the search.

- If union contrast used, form the union of the terminals. The union is called the current GUMS or GUMS i + 1. Go to Step 1 in Phase 3 for iteration.
- Otherwise, continue to Phase 5.

Phase 5: Selection of Final Model

Step 0

Calculate information criteria for each equation in all terminal models. Then, find average values for each terminal model.

Step 1

Select final model based on smallest average value of Schwartz criterion. The model is known as specific unrestricted model.

Step 2

Estimate the specific unrestricted model using IFGLS or EM estimation.

Step 3

Run diagnostic analyses of each equation and test the contemporaneous correlation of disturbances amongst the equations

APPENDIX C
ESTIMATED MODELS OF NATIONAL GROWTH RATES

Country	Estimation Method	Constant	Δy_{it-1}	Δy_{it-2}	Δx_{i1t}	$\Delta x_{i1(t-1)}$
Denmark	OLS	-0.008 (-0.816)	-0.225 (-1.465)	0.124 (-1.244)	-0.042 (-0.726)	0.050 (-0.804)
	FGLS	-0.009 (-1.015)	-0.312** (-2.649)	0.168** (-2.096)	-0.009 (-0.192	0.068 (-1.424)
	IFGLS	-0.010 (-1.087)	-0.357*** (-3.573)	0.196** (-2.664)	0.008 (-0.211)	0.102** (-2.470)
	EM	-0.010 (-0.989)	-0.356** (-2.631)	0.195** (-2.085)	0.008 (-0.334)	0.101 (-1.435)
Ireland	OLS	0.009 (-1.208)	0.265 (-1.577)	-0.031 (-0.381)	0.021 (0.360)	0.029 (-0.488)
	FGLS	0.008 (-1.278)	0.305** (-2.253)	-0.009 (-0.141)	-0.021 (-0.451)	0.051 (-1.047)
	IFGLS	0.007 (-1.020)	0.295** (-2.508)	0.051 (-0.875)	-0.050 (-1.235)	0.066 (-1.588)
	EM	0.007 (-1.247)	0.296** (-2.190)	0.050 (-0.164)	-0.050 (-0.436)	0.066 (-1.066)
Netherlands	OLS	0.013 (-1.115)	0.016 (-0.088)	-0.007 (-0.074)	0.191 (-1.654)	0.039 (-0.319)
	FGLS	0.016 (-1.542)	-0.090 (-0.593)	-0.001 (-0.007)	0.190* (-1.954)	0.100 (-0.964)
	IFGLS	0.018 (-1.674)	-0.165 (-1.126)	0.010 (-0.127)	0.193** (-2.058)	0.128 (-1.282)
	EM	0.018 (-1.493)	-0.164 (-0.579)	0.010 (-0.023)	0.193* (-1.913)	0.129 (-0.943)
United Kingdom	OLS	0.001 (-0.131)	-0.008 (-0.050)	-0.059 (-0.558)	-0.044 (-0.193)	0.064 (-0.300)
	FGLS	0.001 (-0.106)	-0.067 (-0.479)	-0.099 (-1.097)	-0.035 (-0.181)	-0.067 (-0.374)
	IFGLS	0.001 (-0.051)	-0.140 (-1.119)	-0.162* (-1.947)	-0.017 (-0.100)	-0.263* (-1.729)
	EM	0.001 (-0.054)	-0.138 (-0.350)	-0.160 (-1.137)	-0.018 (-0.226)	-0.259 (-0.135)

***Significant at 1%, ** Significant at 5%, * Significant at 10%, () *t*-value

(cont.)

Country	Estimation Method	Δx_{i2t}	$\Delta x_{i2(t-1)}$	Δx_{i3t}	$\Delta x_{i3(t-1)}$	$\overline{R}^2$	Standard errors
Denmark	OLS	-0.069 (-0.561)	0.188 (-1.474)	0.811*** (-8.704)	0.309 (-2.066)	0.721	0.058
	FGLS	-0.113 (-1.097)	0.182* (-1.709)	0.790*** (-10.780)	0.339*** (-3.010)	0.712	0.053
	IFGLS	-0.110 (-1.070)	0.143 (-1.338)	0.730*** (-11.315)	0.397*** (-4.274)	0.693	0.054
	EM	-0.111 (-0.887)	0.144 (-1.630)	0.732*** (-10.664)	0.395*** (-2.916)	0.694	0.054
Ireland	OLS	-0.109 (-1.014)	0.037 (-0.340)	0.707*** (-9.585)	-0.142 (-1.008)	0.791	0.042
	FGLS	-0.045 (-0.498)	0.004 (-0.043)	0.696*** (-11.516)	-0.158 (-0.112)	0.786	0.038
	IFGLS	-0.006 (-0.064)	-0.034 (-0.387)	0.732*** (-13.582)	-0.155 (-1.609)	0.772	0.039
	EM	-0.006 (-0.534)	-0.033 (-0.028)	0.730*** (-11.388)	-0.156 (-1.373)	0.772	0.039
Netherlands	OLS	-0.322 (-1.683)	-0.058 (-0.272)	0.909*** (-9.378)	-0.010 (-0.057)	0.740	0.054
	FGLS	-0.352** (-2.145)	-0.122 (-0.671)	0.844*** (-10.082)	0.099 (-0.639)	0.724	0.049
	IFGLS	-0.374** (-2.307)	-0.143 (-0.797)	0.797*** (-9.644)	0.173 (-1.157)	0.692	0.052
	EM	-0.374** (-2.077)	-0.144 (-0.675)	0.797*** (-9.977)	0.172 (-0.630)	0.693	0.052
United Kingdom	OLS	-0.134 (-0.94)	0.103 (-0.527)	0.472*** (-7.260)	-0.074 (-0.732)	0.619	0.068
	FGLS	-0.109 (-0.649)	0.161 (-0.955)	0.474*** (-8.538)	-0.018 (-0.210)	0.612	0.061
	IFGLS	-0.070 (-0.438)	0.257 (-1.626)	0.466*** (-9.527)	0.064 (-0.885)	0.576	0.064
	EM	-0.071 (-0.780)	0.255 (-0.617)	0.466*** (-8.507)	0.062 (-0.392)	0.577	0.063

***Significant at 1%, ** Significant at 5%, * Significant at 10%, () *t*-value

APPENDIX D
ESTIMATED MODELS OF WQI

	S6				S7			
Variables	**OLS**	**FGLS**	**IFGLS**	**EM**	**OLS**	**FGLS**	**IFGLS**	**EM**
Constant	67.206***	64.899***	63.728***	66.314***	51.275***	61.649***	73.161***	60.826***
	(4.502)	(5.199)	(5.220)	(5.226)	(3.715)	(5.514)	(6.763)	(6.757)
Δy_{it-1}	-0.083	-0.025	-0.018	-0.046	-0.282**	0.143	0.070	0.127
	(-0.685)	(-0.244)	(-0.189)	(-0.191)	(2.143)	(1.397)	(0.770)	(0.775)
Δy_{it-2}	-0.010	-0.018	-0.016	-0.023	-0.026	-0.030	-0.036	-0.034
	(-0.288)	(-0.635)	(-0.598)	(-0.599)	(-0.797)	(-1.173)	(-1.549)	(-1.549)
Δy_{it-3}	-0.064	0.036	0.013	0.048	-0.033	-0.017	0.002	-0.014
	(1.752)	(1.181)	(0.441)	(0.450)	(-0.912)	(-0.598)	(0.082)	(0.072)
Δx_{i1t}	0.092	0.223	0.452*	0.141	0.096	0.157	0.172*	0.140
	(0.321)	(0.935)	(1.940)	(1.930)	(0.698)	(1.463)	(1.816)	(1.812)
$\Delta x_{i1(t-1)}$	-0.318	-0.257	-0.175	-0.218	0.097	0.099	0.125	0.085
	(-1.037)	(-1.010)	(-0.692)	(-0.694)	(0.699)	(0.907)	(1.284)	(1.281)
Δx_{i2t}	1.689	-0.097	-3.071	0.982	1.533	0.978	0.952	1.086
	(0.460)	(-0.032)	(-1.035)	(-1.024)	(0.915)	(0.748)	(0.822)	(0.821)
$\Delta x_{i2(t-1)}$	4.228	3.168	1.906	2.637	-2.061	-1.761	-1.977*	-1.585
	(1.069)	(0.966)	(0.587)	(0.590)	(-1.228)	(-1.341)	(-1.699)	(-1.696)
Δx_{i3t}	-0.563***	-0.589***	-0.618***	-0.579***	-0.597***	-0.601***	-0.631***	-0.589***
	(-4.806)	(-6.084)	(-6.625)	(-6.619)	(-4.731)	(-5.916)	(-6.694)	(-6.686)
$\Delta x_{i3(t-1)}$	-0.103	-0.117	-0.149	-0.130	0.109	0.003	-0.057	0.001
	(-0.785)	(-1.087)	(-1.423)	(-1.421)	(0.778)	(0.034)	(-0.554)	(-0.547)
Δx_{i4t}	-0.186***	-0.176***	-0.169***	-0.171***	-0.205***	-0.191***	-0.170***	-0.192***
	(-4.245)	(-4.848)	(-4.823)	(-4.824)	(-4.472)	(-5.213)	(-5.055)	(-5.062)
$\Delta x_{i4(t-1)}$	0.004	0.018	0.029	0.019	0.042	0.024	0.019	0.019
	(0.086)	(0.431)	(0.736)	(0.730)	(0.879)	(0.633)	(0.542)	(0.540)
Δx_{i5t}	-0.137***	-0.054***	-0.053***	-0.054***	-0.076***	-0.076***	-0.080***	-0.076***
	(-0.095)	(-16.510)	(-16.821)	(-16.824)	(-12.249)	(-15.466)	(-18.308)	(-18.286)
$\Delta x_{i5(t-1)}$	-0.003	-0.000	-0.002	-0.002	0.032***	0.019**	0.011	0.018
	(-0.385)	(-0.065)	(-0.346)	(-0.344)	(2.758)	(2.029)	(1.342)	(1.347)
Δx_{i6t}	-0.137	0.037	0.186	0.261	-0.150	-0.488	-0.962	-0.137
	(-0.095)	(0.031)	(0.158)	(0.154)	(-0.116)	(-0.471)	(-0.977)	(-0.974)
$\Delta x_{i6(t-1)}$	0.038	0.254	0.707	-0.023	0.481	0.507	0.002	0.535
	(0.028)	(0.223)	(0.635)	(0.629)	(0.361)	(0.471)	(0.002)	(0.009)
Δx_{i7t}	-0.710***	-0.868***	-1.040***	-0.883***	-1.293***	-1.374***	-1.522***	-1.392***
	(-5.001)	(-7.302)	(-9.008)	(-8.999)	(-8.322)	(-11.167)	(-13.788)	(-13.777)
$\Delta x_{i7(t-1)}$	-0.074	-0.071	-0.116	-0.105	0.460**	0.261	0.113	0.237
	(-0.422)	(-0.484)	(-0.788)	(-0.788)	(2.105)	(1.498)	(0.695)	(0.700)
$\overline{R}^2$	0.942	0.939	0.931	0.933	0.946	0.942	0.930	0.933
Std. errors	2.330	2.091	2.238	2.236	1.575	1.423	1.563	1.562

***Significant at 1%, ** Significant at 5%, * Significant at 10%, () *t*-value

(cont.)

	S8				S25			
Variables	**OLS**	**FGLS**	**IFGLS**	**EM**	**OLS**	**FGLS**	**IFGLS**	**EM**
Constant	41.967***	45.299***	52.039***	45.117***	49.318***	58.906***	65.651***	58.515***
	(3.039)	(3.966)	(5.076)	(5.071)	(3.224)	(4.709)	(5.589)	(5.593)
Δy_{it-1}	0.165	0.144	0.114	0.141	0.213*	0.178*	0.160*	0.171*
	(1.237)	(1.340)	(1.266)	(1.266)	(1.681)	(1.753)	(1.727)	(1.726)
Δy_{it-2}	0.046	0.058**	0.062**	0.058**	-0.023	-0.039	-0.047*	-0.041
	(1.273)	(2.027)	(2.649)	(2.655)	(-0.667)	(-1.440)	(-1.866)	(-1.870)
Δy_{it-3}	-0.022	-0.037	-0.049**	-0.035	0.058*	0.050**	0.049**	0.050**
	(-0.700)	(-1.448)	(-2.383)	(-2.372)	(1.875)	(2.002)	(2.150)	(2.152)
Δx_{i1t}	0.132	0.161	0.233**	0.140	-0.020	0.050	0.079	0.020
	(0.674)	(1.015)	(1.775)	(1.760)	(-0.090)	(0.278)	(0.476)	(0.478)
$\Delta x_{i1(t-1)}$	0.140	0.171	0.200	0.181	-0.181	-0.082	-0.002	-0.077
	(0.716)	(1.085)	(1.543)	(1.549)	(-0.939)	(-0.528)	(-0.016)	(-0.016)
Δx_{i2t}	2.178	1.828	0.932	2.072	3.013	2.189	1.874	2.544
	(0.871)	(0.902)	(0.553)	(0.567)	(1.057)	(0.957)	(0.894)	(0.892)
$\Delta x_{i2(t-1)}$	-2.624	-2.950	-3.256*	-3.081	1.659	0.508	-0.445	0.406
	(-1.058)	(-1.475)	(-1.984)	(-1.990)	(0.668)	(0.253)	(-0.240)	(-0.240)
Δx_{i3t}	-0.675***	-0.632***	-0.594***	-0.636***	-0.741***	-0.671***	-0.643***	-0.656***
	(-5.397)	(-6.146)	(-6.641)	(-6.641)	(-4.240)	(-4.685)	(-4.804)	(-4.805)
$\Delta x_{i3(t-1)}$	0.019	-0.083	-0.196*	-0.076	0.576***	0.480***	0.400**	0.473***
	(0.128)	(-0.674)	(-1.854)	(-1.841)	(2.709)	(2.765)	(2.471)	(2.474)
Δx_{i4t}	-0.121***	-0.138***	-0.149***	-0.134***	-0.144**	-0.152***	-0.156***	-0.152***
	(-2.716)	(-3.779)	(-4.830)	(-4.824)	(-2.370)	(-3.080)	(-3.393)	(-3.385)
$\Delta x_{i4(t-1)}$	0.020	0.049	0.074**	0.046	-0.108*	-0.106**	-0.087*	-0.106**
	(0.460)	(1.363)	(2.391)	(2.382)	(-1.723)	(-2.067)	(-1.803)	(-1.811)
Δx_{i5t}	-0.061***	-0.060***	-0.058***	-0.060***	-0.076***	-0.075***	-0.076***	-0.073***
	(-7.312)	(-8.895)	(-10.584)	(-10.577)	(-8.370)	(-10.196)	(-11.059)	(-11.066)
$\Delta x_{i5(t-1)}$	0.016	0.010	0.003	0.010	0.013	0.007	0.003	0.006
	(1.546)	(1.199)	(0.527)	(0.533)	(0.845)	(0.537)	(0.283)	(0.280)
Δx_{i6t}	-0.312	-0.408	-0.657	-0.236	0.646	0.001	-0.322	0.241
	(-0.224)	(-0.355)	(-0.649)	(-0.647)	(0.488)	(0.001)	(-0.320)	(-0.330)
$\Delta x_{i6(t-1)}$	1.485	1.417	1.243	1.319	-0.210	-0.300	-0.658	-0.356
	(1.067)	(1.236)	(1.215)	(1.213)	(-0.158)	(-0.275)	(-0.641)	(0.637)
Δx_{i7t}	-1.239***	-1.375***	-1.565***	-1.368***	-1.498***	-1.694***	-1.826***	-1.718***
	(-7.926)	(-10.863)	(-14.710)	(-14.693)	(-8.579)	(-11.917)	(-13.846)	(-13.858)
$\Delta x_{i7(t-1)}$	0.496**	0.409**	0.291*	0.395	0.328	0.326	0.300	0.299
	(2.275)	(2.275)	(1.804)	(1.803)	(1.262)	(1.534)	(1.507)	(1.505)
$\bar{R}^2$	0.949	0.946	0.937	0.939	0.944	0.942	0.938	0.939
Std. errors	1.684	1.508	1.630	1.629	2.027	1.817	1.878	1.879

***Significant at 1%, ** Significant at 5%, * Significant at 10%, () *t*-value

APPENDIX E
DATA OF NATIONAL GROWTH RATES

1. Denmark

Year	GDP	Money (M1)	Stock	Year	GDP	Money (M1)	Stock
1951	23.10	6.17	7.00				
1952	24.70	6.57	6.00	1978	311.40	65.06	27.00
1953	26.40	7.07	6.00	1979	346.90	71.88	26.00
1954	27.70	6.93	7.00	1980	373.80	77.51	24.00
1955	28.90	7.10	8.00	1981	407.80	88.03	40.00
1956	30.90	7.43	8.00	1982	464.50	91.66	51.00
1957	32.90	7.79	9.00	1983	512.50	113.30	89.00
1958	34.30	8.85	9.00	1984	565.30	128.08	96.00
1959	38.10	9.82	11.00	1985	615.10	156.49	100.00
1960	40.80	10.03	12.00	1986	666.50	167.97	101.00
1961	45.60	11.14	12.00	1987	699.90	188.45	84.00
1962	51.40	12.21	12.00	1988	732.10	225.11	95.00
1963	54.30	13.86	12.00	1989	769.80	226.11	132.00
1964	62.00	15.23	14.00	1990	800.00	244.48	146.00
1965	69.70	16.97	14.00	1991	857.65	258.27	157.00
1966	77.20	19.34	15.00	1992	887.87	256.00	144.37
1967	84.80	21.12	12.00	1993	900.15	283.00	147.30
1968	94.40	24.06	12.00	1994	965.72	279.05	175.81
1969	107.30	27.13	14.00	1995	1009.76	291.98	175.49
1970	118.60	27.47	13.00	1996	1060.89	325.52	214.70
1971	131.10	29.61	12.00	1997	1116.32	344.05	283.44
1972	150.70	33.64	17.00	1998	1155.41	360.74	306.47
1973	172.90	37.59	26.00	1999	1207.75	381.77	272.37
1974	193.60	39.36	19.00	2000	1278.96	385.98	438.70
1975	216.30	49.86	21.00	2001	1325.51	414.85	447.36
1976	251.20	52.34	28.00	2002	1360.71	430.82	447.91
1977	279.30	56.08	29.00	2003	1395.85	469.15	448.75

2. Ireland

Year	GDP	Money (M1)	Stock	Year	GDP	Money (M1)	Stock
1951	392.00	132.00	9.90				
1952	450.00	137.00	8.30	1978	6757.00	1367.00	64.30
1953	496.00	145.00	7.40	1979	7917.00	1479.00	68.80
1954	498.00	152.00	8.10	1980	9361.00	1686.00	67.00
1955	522.00	155.00	8.50	1981	11359.00	1743.00	69.60
1956	530.00	155.00	7.70	1982	13382.00	1838.00	56.80
1957	549.00	166.00	7.00	1983	14779.00	2048.00	70.60
1958	568.00	165.00	7.00	1984	16407.00	2245.00	93.70
1959	608.00	171.00	9.30	1985	17790.00	2288.00	100.00
1960	631.00	203.00	11.70	1986	18877.00	2382.00	157.00
1961	680.00	219.00	13.90	1987	20263.00	2640.00	226.70
1962	736.00	241.00	15.90	1988	21815.00	2826.00	222.50
1963	791.00	278.00	19.40	1989	24307.00	3112.00	396.90
1964	901.00	287.00	24.10	1990	25693.00	3346.00	486.30
1965	959.00	298.00	23.40	1991	29675.00	3390.00	447.30
1966	1010.00	315.00	21.90	1992	31529.00	3451.00	416.01
1967	1104.00	341.00	21.80	1993	34054.00	3789.00	519.67
1968	1245.00	364.00	31.10	1994	36624.00	4124.00	593.00
1969	1438.00	389.00	32.90	1995	41409.00	4369.00	648.71
1970	1621.00	415.00	28.90	1996	45634.00	4897.00	811.40
1971	1853.00	440.00	28.10	1997	52760.00	5230.00	1101.21
1972	2238.00	518.00	41.30	1998	60582.00	5466.00	1548.57
1973	2729.00	572.00	48.80	1999	70576.80	5763.00	1594.95
1974	2991.00	624.00	32.80	2000	80997.00	5842.00	1704.36
1975	3792.00	748.00	32.40	2001	90367.50	6237.00	1844.07
1976	4653.00	875.00	33.80	2002	101867.00	6690.00	1486.74
1977	5703.00	1072.00	42.40	2003	103897.00	7068.00	1396.09

3. Netherlands

Year	GDP	Money (M1)	Stock	Year	GDP	Money (M1)	Stock
1951	21.40	7.04	13.20				
1952	22.40	7.76	11.90	1978	297.00	60.19	45.60
1953	23.80	8.26	13.10	1979	316.00	61.87	41.20
1954	26.60	8.85	16.30	1980	336.80	65.58	35.30
1955	29.70	9.58	20.50	1981	352.80	64.03	37.40
1956	32.00	9.23	21.40	1982	368.90	72.30	39.20
1957	34.70	9.05	18.90	1983	381.10	79.66	59.20
1958	35.10	10.13	19.30	1984	400.20	85.00	73.00
1959	37.40	10.59	28.00	1985	418.20	90.77	100.00
1960	41.80	11.30	38.80	1986	428.60	97.21	149.00
1961	44.20	12.16	49.40	1987	430.20	103.71	132.60
1962	47.60	13.09	44.50	1988	449.40	111.31	117.90
1963	51.60	14.29	44.90	1989	474.40	119.02	155.10
1964	60.70	15.44	44.90	1990	504.20	124.29	110.80
1965	67.80	16.99	44.00	1991	520.72	129.72	127.80
1966	73.80	18.16	37.30	1992	555.17	137.67	133.96
1967	81.00	19.29	41.80	1993	577.43	148.16	155.38
1968	89.80	21.49	48.90	1994	590.25	148.46	184.00
1969	101.70	23.23	53.90	1995	601.93	155.01	198.92
1970	121.20	25.95	55.00	1996	605.79	164.06	267.30
1971	136.50	29.85	52.10	1997	646.40	176.25	388.53
1972	154.30	35.12	62.60	1998	666.00	180.40	520.27
1973	176.00	35.14	69.10	1999	686.30	192.62	566.04
1974	199.80	39.43	53.40	2000	712.60	214.48	667.29
1975	220.00	47.20	53.10	2001	734.60	223.74	511.17
1976	251.90	51.05	50.70	2002	773.33	244.62	365.50
1977	274.90	57.77	45.80	2003	800.10	277.73	421.65

4. United Kingdom

Year	GDP	Money (M1)	Stock	Year	GDP	Money (M1)	Stock
1951	14.57	5.65	7.80				
1952	15.81	5.67	7.90	1978	169.62	27.36	34.00
1953	17.05	5.85	7.80	1979	198.46	29.86	38.60
1954	17.98	6.08	7.80	1980	232.55	31.04	41.30
1955	19.35	6.01	8.00	1981	256.37	34.59	46.60
1956	20.91	6.04	8.30	1982	279.58	40.66	53.90
1957	22.11	5.94	8.60	1983	305.42	42.46	68.10
1958	23.05	6.09	8.60	1984	324.63	48.05	81.00
1959	24.29	6.60	11.70	1985	355.94	56.67	100.00
1960	25.74	6.63	14.10	1986	383.14	69.27	124.10
1961	27.48	6.76	14.60	1987	420.86	154.12	163.80
1962	28.80	6.40	13.50	1988	467.23	170.67	147.40
1963	30.65	7.32	15.40	1989	511.50	195.31	176.50
1964	33.42	7.56	16.40	1990	549.51	214.94	173.30
1965	35.90	7.85	15.40	1991	575.36	229.23	190.20
1966	38.30	7.84	15.50	1992	610.85	253.99	198.67
1967	40.52	8.44	16.60	1993	642.33	318.89	228.03
1968	43.99	8.78	23.50	1994	681.33	337.98	245.13
1969	47.01	8.81	23.20	1995	719.18	402.63	255.12
1970	51.68	9.64	20.50	1996	763.29	447.40	289.08
1971	58.08	11.09	24.20	1997	810.94	485.86	327.21
1972	64.35	12.66	30.90	1998	859.44	510.34	383.95
1973	74.36	13.30	26.70	1999	903.87	552.38	391.42
1974	84.68	14.74	15.70	2000	951.27	613.80	415.86
1975	106.98	17.48	19.60	2001	994.04	666.57	428.78
1976	127.78	19.47	23.50	2002	1043.31	700.15	457.17
1977	147.12	23.52	30.20	2003	1099.36	757.83	475.31

APPENDIX F
DATA OF WQI

1. Station S6

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
11-05-2012	69.45	73.7	5.71	7	22	45	7.26	2.78
15-05-2012	41.19	23.4	1.83	12	36	243	7.18	1.80
21-05-2012	68.05	51.7	4.07	12	32	40	7.21	0.17
31-05-2012	55.15	35.1	2.74	9	36	69	7.03	1.62
08-06-2012	37.39	10.3	0.80	19	58	46	6.87	8.33
15-06-2012	60.94	10.3	0.81	6	27	36	6.94	0.28
20-06-2012	44.98	32.6	2.51	18	42	41	7.09	9.87
27-06-2012	48.65	33.7	2.67	14	42	23	7.06	9.73
03-07-2012	57.79	58.2	4.53	8	31	88	7.10	5.71
10-07-2012	62.16	77.2	5.76	12	32	52	6.49	4.81
17-07-2012	51.61	24.7	1.97	8	27	33	6.87	7.60
24-07-2012	53.68	15.8	1.23	4	15	45	6.85	7.46
03-08-2012	55.74	34.0	2.70	8	24	20	6.88	6.51
10-08-2012	55.96	33.4	2.60	8	25	13	7.07	8.36
16-08-2012	57.78	53.5	4.31	5	16	186	7.00	2.47
23-08-2012	67.77	56.2	4.47	3	9	39	6.89	4.09
04-09-2012	42.88	54.5	4.26	19	46	156	7.04	8.91
11-09-2012	58.74	55.1	4.41	8	23	83	7.43	3.64
18-09-2012	66.45	42.6	3.34	5	14	60	7.46	0.92
25-09-2012	57.69	49.8	3.88	10	28	31	7.17	4.69
02-10-2012	46.36	48.4	3.83	10	33	202	7.08	5.04
09-10-2012	47.50	49.2	3.84	10	29	188	7.30	8.47
16-10-2012	59.79	62.4	4.85	10	28	54	7.58	4.08
23-10-2012	51.06	60.0	4.76	11	26	184	7.19	4.12
06-11-2012	69.11	71.9	5.84	6	17	62	7.40	2.86
13-11-2012	76.93	70.4	5.66	5	16	16	7.44	1.24
20-11-2012	63.59	58.8	4.70	6	17	77	7.30	3.24
26-11-2012	70.32	65.2	5.14	4	11	64	7.28	2.59
03-12-2012	66.18	73.8	5.88	6	17	160	7.56	1.46
10-12-2012	62.74	72.3	5.82	7	19	162	7.47	2.20
17-12-2012	63.00	62.1	4.86	8	25	49	7.21	3.53
25-12-2012	79.24	57.1	4.57	5	15	16	7.33	0.17
07-01-2013	58.86	52.4	4.05	11	31	13	7.20	4.86
14-01-2013	44.83	43.9	3.45	11	29	280	7.12	3.18
21-01-2013	47.27	42.2	3.22	6	15	420	7.34	4.06
29-01-2013	46.20	52.1	4.06	12	31	217	7.29	4.11
03-02-2013	41.09	35.4	2.79	12	28	234	7.12	6.73
12-02-2013	51.01	52.0	4.10	18	56	20	7.10	5.87
18-02-2013	49.23	36.7	2.94	12	31	69	6.98	6.38

(cont.)

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
25-02-2013	40.78	50.5	4.03	22	54	111	7.29	4.27
05-03-2013	39.08	45.9	3.57	18	42	242	7.07	6.29
12-03-2013	47.40	49.1	3.79	14	32	254	6.74	2.10
18-03-2013	44.00	60.4	4.66	17	41	262	7.18	7.07
25-03-2013	44.95	42.9	3.34	12	33	138	7.14	6.37
02-04-2013	45.74	50.8	3.85	10	34	267	7.12	6.30
08-04-2013	40.01	40.6	3.14	13	37	273	7.18	5.98
16-04-2013	44.98	52.7	3.90	18	39	115	7.15	5.53
23-04-2013	54.94	56.5	4.35	8	21	130	7.24	4.39
07-05-2013	64.61	71.5	5.59	6	33	59	7.00	4.04
14-05-2013	55.38	59.7	4.54	16	43	28	7.22	4.09
20-05-2013	46.96	51.3	3.96	17	48	108	7.01	2.52
28-05-2013	56.37	67.6	5.26	14	38	76	7.07	4.24
03-06-2013	51.31	46.8	3.57	15	44	32	6.98	5.08
10-06-2013	47.18	45.1	3.52	10	31	141	7.31	5.19
17-06-2013	45.64	39.1	2.97	17	49	51	7.27	7.33
25-06-2013	37.29	23.6	1.82	10	35	270	7.28	6.77
02-07-2013	48.78	57.4	4.42	11	37	171	6.84	5.03
10-07-2013	50.01	51.1	4.01	10	34	103	7.09	8.37
15-07-2013	40.06	48.1	3.75	27	31	163	7.11	9.27
23-07-2013	41.01	42.9	3.29	14	49	155	7.31	8.57
05-08-2013	46.42	36.7	2.88	16	59	23	7.14	12.36
13-08-2013	44.66	43.3	3.46	19	67	71	6.97	3.09
20-08-2013	55.22	58.9	4.73	14	27	81	6.80	4.81
26-08-2013	43.93	48.6	3.78	21	60	82	7.38	4.16
03-09-2013	58.80	67.8	5.40	11	32	82	6.86	6.46
09-09-2013	58.24	45.3	3.61	10	31	14	7.20	3.34
17-09-2013	59.76	66.4	5.31	11	33	57	6.98	4.33
24-09-2013	47.92	31.5	2.49	13	37	41	7.05	9.35
08-10-2013	44.63	27.4	2.14	18	52	12	6.97	9.62
13-10-2013	60.44	53.9	4.23	7	22	55	7.40	4.08
22-10-2013	50.21	55.5	4.29	21	59	24	7.24	7.06
28-10-2013	66.34	60.8	4.69	8	25	14	7.60	2.86
05-11-2013	62.35	54.9	4.40	8	26	16	6.75	4.28
12-11-2013	64.90	74.6	5.82	9	27	45	7.35	4.76
19-11-2013	55.38	56.6	4.47	12	39	51	7.31	4.00
26-11-2013	52.76	39.9	3.19	12	37	81	6.99	2.09
03-12-2013	67.08	56.4	4.64	10	34	21	6.71	0.99
09-12-2013	39.02	17.8	1.41	17	52	62	7.29	8.06
17-12-2013	37.54	7.5	0.59	17	53	56	6.82	5.88
24-12-2013	42.31	38.2	3.15	18	56	130	7.21	1.85

2. Station S7

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
11-05-2012	62.47	40.4	3.08	4	13	82	7.30	2.04
15-05-2012	65.08	46.5	3.66	4	11	27	6.89	3.48
21-05-2012	64.79	57.5	4.56	7	29	26	7.08	2.84
31-05-2012	56.61	45.5	3.54	9	35	34	6.86	3.39
08-06-2012	48.58	35.6	2.77	15	40	28	6.85	4.46
15-06-2012	74.46	31.5	2.52	3	15	25	6.87	0.02
20-06-2012	54.64	60.5	4.68	16	47	34	7.17	5.93
27-06-2012	54.33	40.7	3.24	9	33	35	7.03	4.83
03-07-2012	52.07	58.3	4.61	9	35	127	7.09	4.40
10-07-2012	50.82	58.5	5.48	19	68	35	6.87	3.63
17-07-2012	56.19	49.4	3.96	8	36	56	6.68	3.74
24-07-2012	75.19	68.9	5.51	2	6	17	7.08	3.74
03-08-2012	63.11	54.1	4.32	7	24	18	7.05	5.58
10-08-2012	54.18	42.9	3.38	12	32	23	7.04	6.60
16-08-2012	54.97	75.6	6.09	12	41	133	7.31	3.34
23-08-2012	68.87	75.0	5.96	8	24	23	7.26	3.47
04-09-2012	58.25	52.2	4.13	10	31	28	7.08	6.21
11-09-2012	59.77	44.9	3.52	7	21	26	7.40	6.37
18-09-2012	66.54	63.0	4.96	7	22	13	7.41	4.91
25-09-2012	68.52	74.0	5.84	7	21	41	7.16	3.52
02-10-2012	57.65	73.5	5.87	7	26	191	7.19	4.37
09-10-2012	56.00	62.5	4.88	12	35	84	7.15	6.30
16-10-2012	66.58	73.9	5.82	9	26	40	7.55	3.14
23-10-2012	60.41	78.4	6.26	9	36	111	7.29	2.74
06-11-2012	70.95	78.9	6.40	7	25	51	7.31	2.31
13-11-2012	71.81	83.6	6.76	5	16	107	7.52	1.13
20-11-2012	66.54	69.3	5.58	8	22	83	7.29	2.27
26-11-2012	71.94	76.9	6.04	7	19	57	7.31	1.97
03-12-2012	81.22	79.8	6.39	4	10	13	7.42	1.36
10-12-2012	71.23	72.4	5.84	6	19	71	7.26	1.77
17-12-2012	61.69	62.1	4.94	8	25	88	7.12	3.07
25-12-2012	71.21	71.9	5.82	6	18	10	7.30	3.69
07-01-2013	56.06	52.8	4.14	13	38	23	7.14	3.83
14-01-2013	59.68	61.9	4.94	12	34	27	7.13	4.37
21-01-2013	62.45	55.1	4.36	6	18	55	7.18	3.86
29-01-2013	66.21	95.8	7.72	11	35	61	7.36	3.19
03-02-2013	55.53	57.7	4.61	16	40	24	7.09	4.69
12-02-2013	66.16	61.2	4.86	6	13	43	7.13	4.19
18-02-2013	62.32	60.6	4.88	10	25	24	7.10	4.69

(cont.)

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
25-02-2013	64.42	67.7	5.47	9	25	38	7.19	3.72
05-03-2013	58.92	65.6	5.10	14	33	56	7.22	3.27
12-03-2013	63.34	69.4	5.47	10	22	107	7.15	1.48
18-03-2013	57.84	67.1	5.28	17	38	27	7.34	4.97
25-03-2013	56.44	71.0	5.59	11	31	125	6.74	3.25
02-04-2013	66.23	62.0	4.73	5	12	57	7.06	4.56
08-04-2013	65.17	55.9	4.35	6	14	32	7.02	4.62
16-04-2013	56.26	64.0	4.89	15	41	47	7.09	3.91
23-04-2013	58.61	64.8	5.01	10	28	88	7.59	4.16
07-05-2013	67.29	66.0	5.12	8	33	34	6.93	2.13
14-05-2013	62.84	66.6	5.10	13	36	18	7.09	2.99
20-05-2013	61.20	56.4	4.33	14	33	42	6.98	1.64
28-05-2013	56.48	64.1	4.94	16	44	31	7.62	3.64
03-06-2013	60.45	61.0	4.72	12	35	18	6.98	3.75
10-06-2013	66.27	65.6	5.24	8	22	21	7.37	3.79
17-06-2013	54.97	55.8	4.37	16	47	12	7.06	5.12
25-06-2013	55.78	64.0	5.00	15	53	48	7.21	3.19
02-07-2013	54.31	53.6	4.14	10	33	92	7.09	4.48
10-07-2013	58.63	49.0	3.85	8	29	32	7.13	7.68
15-07-2013	53.65	53.0	4.20	14	48	27	7.06	7.38
23-07-2013	55.03	53.3	4.16	12	43	34	7.14	6.88
05-08-2013	49.35	41.8	3.30	15	53	21	7.08	7.60
13-08-2013	57.08	64.2	5.10	12	45	50	7.12	4.42
20-08-2013	58.69	88.6	7.14	22	43	44	7.23	5.03
26-08-2013	58.76	62.5	4.94	13	39	24	7.23	4.41
03-09-2013	60.63	61.9	4.85	12	34	15	7.14	6.15
09-09-2013	64.06	67.5	5.41	11	32	23	7.34	3.10
17-09-2013	71.38	70.2	5.66	6	19	10	7.31	3.19
24-09-2013	61.65	45.7	3.64	7	20	8	7.11	7.32
08-10-2013	57.96	49.5	3.88	11	33	10	6.97	8.17
13-10-2013	65.46	70.4	5.56	10	30	25	7.41	3.19
22-10-2013	58.97	64.7	5.06	13	39	27	7.44	5.92
28-10-2013	62.86	69.1	5.36	14	43	16	7.50	2.55
05-11-2013	64.55	56.3	4.53	7	23	15	6.99	3.81
12-11-2013	67.79	91.6	7.20	9	29	41	7.52	4.21
19-11-2013	62.69	64.0	5.06	14	44	13	7.28	2.18
26-11-2013	63.20	53.1	4.17	6	18	32	7.29	4.23
03-12-2013	68.28	67.6	5.52	10	28	23	7.29	1.94
09-12-2013	58.15	48.2	3.89	9	28	26	7.32	5.18
17-12-2013	51.00	37.9	2.99	13	40	20	7.04	4.81
24-12-2013	45.28	34.1	2.74	16	50	43	7.20	3.81

3. Station S8

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
11-05-2012	67.10	52.9	4.02	4	13	63	7.25	2.22
15-05-2012	60.72	55.2	4.16	9	28	24	7.14	4.41
21-05-2012	65.10	56.1	4.48	5	26	36	7.12	2.93
31-05-2012	57.14	43.7	3.35	9	30	35	7.03	3.20
08-06-2012	43.63	26.1	2.04	16	33	70	7.02	4.05
15-06-2012	49.20	26.2	2.10	8	32	61	6.84	4.92
20-06-2012	56.20	46.9	3.63	9	30	43	7.14	4.01
27-06-2012	62.17	39.4	3.12	12	42	11	7.09	0.47
03-07-2012	59.63	50.6	3.97	6	29	47	7.11	4.16
10-07-2012	47.97	51.6	4.00	19	56	79	6.99	3.15
17-07-2012	56.07	31.8	2.54	6	28	17	6.66	4.07
24-07-2012	67.80	64.3	5.15	5	19	23	7.10	4.08
03-08-2012	58.49	46.1	3.67	9	32	8	7.03	4.64
10-08-2012	68.87	49.8	3.89	2	5	19	7.10	5.90
16-08-2012	59.30	75.3	6.08	9	34	233	7.17	1.77
23-08-2012	71.75	68.9	5.48	6	18	15	7.25	2.81
04-09-2012	52.09	35.5	2.81	10	32	36	7.08	6.37
11-09-2012	74.71	45.0	3.49	3	11	40	7.39	0.24
18-09-2012	62.70	58.5	4.57	7	23	41	7.41	4.09
25-09-2012	67.13	59.5	4.80	5	15	23	7.16	4.77
02-10-2012	64.00	67.9	5.40	8	31	35	7.18	5.08
09-10-2012	57.53	57.7	4.51	13	34	30	7.10	7.50
16-10-2012	67.65	71.0	5.63	9	27	22	7.54	2.86
23-10-2012	63.25	69.7	5.54	11	33	40	7.34	3.08
06-11-2012	74.36	75.6	6.14	5	14	50	7.26	2.00
13-11-2012	80.08	87.2	7.08	3	10	96	7.46	0.76
20-11-2012	66.57	64.7	5.21	7	22	91	7.25	1.93
26-11-2012	75.19	73.0	5.76	6	18	25	7.32	1.57
03-12-2012	74.08	77.0	6.16	6	17	97	7.40	0.88
10-12-2012	73.27	75.1	6.02	6	18	60	7.33	1.55
17-12-2012	67.46	60.4	4.80	6	15	52	7.14	2.65
25-12-2012	73.49	59.3	4.79	2	5	11	7.26	3.43
07-01-2013	56.70	72.8	5.69	15	52	54	7.16	4.59
14-01-2013	64.71	60.1	4.84	7	21	36	7.16	3.60
21-01-2013	60.93	59.6	4.76	9	27	42	7.16	4.38
29-01-2013	61.77	57.9	4.64	9	25	47	7.20	3.18
03-02-2013	57.22	54.5	4.35	14	36	12	7.10	3.90
12-02-2013	62.19	63.7	5.08	13	29	17	7.12	3.44
18-02-2013	59.02	53.2	4.26	10	21	46	7.09	4.13

(cont.)

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
25-02-2013	66.21	63.8	5.14	7	16	52	7.22	3.20
05-03-2013	56.03	54.4	4.20	13	31	41	7.18	4.63
12-03-2013	55.19	51.6	4.10	20	45	49	7.22	1.33
18-03-2013	54.05	59.6	4.68	17	40	45	7.20	4.01
25-03-2013	67.49	76.7	5.98	11	32	41	6.84	2.23
02-04-2013	71.24	90.5	6.89	8	19	27	7.11	4.45
08-04-2013	62.24	57.7	4.45	9	26	19	7.13	4.29
16-04-2013	60.99	62.8	4.74	11	31	26	7.19	4.00
23-04-2013	61.61	62.3	4.79	11	33	51	7.24	2.58
07-05-2013	65.35	62.3	4.78	10	39	18	6.92	2.09
14-05-2013	55.84	56.2	4.33	18	45	12	7.05	3.18
20-05-2013	63.32	63.5	4.91	14	41	30	6.96	1.55
28-05-2013	60.85	66.7	5.13	17	47	16	7.67	2.22
03-06-2013	66.16	59.1	4.58	9	26	8	6.99	2.79
10-06-2013	68.51	81.8	6.44	11	32	14	7.38	3.05
17-06-2013	57.19	47.0	3.98	11	33	8	7.14	4.27
25-06-2013	51.50	40.4	3.21	14	48	40	7.32	2.47
02-07-2013	58.61	48.0	3.65	8	29	27	7.22	4.30
10-07-2013	54.25	48.7	3.83	11	38	42	7.24	7.57
15-07-2013	61.33	54.1	4.28	8	30	18	7.06	7.23
23-07-2013	61.32	55.9	4.40	9	32	13	7.00	6.22
05-08-2013	43.04	37.2	2.94	22	77	15	7.01	7.61
13-08-2013	57.93	55.2	4.38	10	36	35	6.90	4.51
20-08-2013	57.35	83.1	6.71	21	51	41	7.15	5.11
26-08-2013	60.72	75.5	5.97	16	48	18	7.33	3.54
03-09-2013	68.41	69.6	5.55	8	24	10	7.25	3.60
09-09-2013	62.67	63.6	5.07	11	33	24	7.31	3.14
17-09-2013	54.83	48.7	3.95	13	39	18	7.35	3.84
24-09-2013	58.80	50.7	4.04	10	29	20	6.97	6.92
08-10-2013	61.58	56.9	4.46	10	29	11	6.88	7.06
13-10-2013	65.04	65.2	5.17	8	24	38	7.46	3.36
22-10-2013	61.11	67.8	5.26	12	36	25	7.41	5.88
28-10-2013	62.90	77.7	6.05	18	55	12	7.58	2.18
05-11-2013	61.70	59.8	4.82	10	32	16	6.84	3.86
12-11-2013	75.84	88.8	7.05	6	19	12	7.47	2.84
19-11-2013	57.75	53.5	4.27	12	37	14	7.30	3.80
26-11-2013	55.96	52.5	4.12	16	49	17	7.25	2.58
03-12-2013	65.63	53.4	4.39	9	28	14	7.37	1.88
09-12-2013	54.70	48.7	3.98	12	38	27	7.27	5.58
17-12-2013	55.32	28.5	2.25	8	25	17	7.03	3.36
24-12-2013	47.08	41.7	3.36	16	50	51	7.19	3.85

4. Station S25

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
11-05-2012	78.12	71.3	5.50	3	8	32	7.13	1.70
15-05-2012	63.00	45.7	3.59	6	22	22	6.84	2.98
21-05-2012	73.89	61.4	4.86	6	27	41	7.15	0.40
31-05-2012	65.73	46.0	3.45	4	16	86	6.93	1.21
08-06-2012	47.20	31.5	2.48	15	40	30	6.83	4.43
15-06-2012	68.07	18.6	1.48	5	22	33	6.87	0.01
20-06-2012	55.48	46.3	3.56	11	35	21	7.26	6.92
27-06-2012	54.69	41.0	3.26	12	32	10	7.03	7.19
03-07-2012	62.32	38.6	3.01	6	26	46	6.95	1.38
10-07-2012	64.65	42.9	3.24	6	27	35	7.10	1.25
17-07-2012	57.06	37.2	2.99	5	18	52	6.69	4.00
24-07-2012	68.11	54.5	4.28	3	12	19	7.13	4.03
03-08-2012	45.09	22.9	1.85	15	48	15	6.79	7.51
10-08-2012	53.46	32.6	2.60	10	28	17	7.00	6.61
16-08-2012	67.02	57.4	4.64	3	8	79	7.08	3.28
23-08-2012	57.39	44.7	3.55	8	25	44	7.11	3.73
04-09-2012	54.37	41.7	3.29	10	32	31	7.17	6.32
11-09-2012	56.73	36.2	2.82	7	20	28	7.40	6.82
18-09-2012	74.86	54.9	4.32	5	19	36	7.40	0.24
25-09-2012	70.55	66.3	5.26	5	13	24	7.26	3.36
02-10-2012	62.65	83.3	6.73	6	20	196	7.25	3.20
09-10-2012	62.91	79.4	5.96	12	33	45	7.54	4.08
16-10-2012	74.59	76.8	6.09	5	12	16	7.55	3.06
23-10-2012	69.32	75.1	6.06	6	17	75	7.36	2.81
06-11-2012	74.55	72.3	5.86	5	16	27	7.33	2.06
13-11-2012	82.99	81.1	6.60	4	13	7	7.50	0.89
20-11-2012	67.05	69.5	5.60	9	28	51	7.28	2.15
26-11-2012	71.24	57.2	4.41	6	18	6	7.23	2.01
03-12-2012	79.97	79.5	6.42	6	17	32	7.50	1.03
10-12-2012	72.51	67.7	5.46	5	15	58	7.23	1.55
17-12-2012	75.35	90.5	7.18	5	15	49	7.18	2.56
25-12-2012	63.07	42.7	3.40	6	16	10	7.27	3.38
07-01-2013	53.02	50.0	3.93	13	40	47	7.17	3.84
14-01-2013	58.00	52.1	4.15	10	29	35	7.10	4.54
21-01-2013	60.07	57.9	4.57	9	26	49	7.17	4.75
29-01-2013	55.25	47.4	3.77	11	31	51	7.40	3.32
03-02-2013	56.87	52.1	4.14	14	35	11	7.07	3.79
12-02-2013	78.54	76.1	5.90	5	12	28	7.12	1.22
18-02-2013	63.45	56.1	4.42	8	20	27	7.09	3.50

(cont.)

WEEK	WQI	DO (%)	DO (mg/L)	BOD	COD	SS	pH	NH3N
25-02-2013	58.73	66.3	5.27	15	42	36	7.07	3.25
05-03-2013	65.96	52.1	4.20	16	46	40	7.48	0.03
12-03-2013	68.72	63.8	4.68	8	16	70	7.19	1.43
18-03-2013	55.11	54.0	4.25	15	34	32	7.19	5.15
25-03-2013	57.61	46.1	3.53	9	24	81	7.08	2.56
02-04-2013	67.85	65.2	4.97	5	13	43	7.06	4.52
08-04-2013	57.08	49.9	3.89	10	29	39	7.05	4.22
16-04-2013	63.78	62.0	4.76	8	24	33	7.02	3.93
23-04-2013	52.76	60.4	4.69	15	43	75	7.41	4.19
07-05-2013	63.92	60.2	4.70	9	29	39	6.91	2.53
14-05-2013	65.13	67.4	5.20	11	32	14	7.10	3.02
20-05-2013	63.23	66.8	5.19	12	36	59	7.14	1.94
28-05-2013	63.68	66.7	5.20	10	29	20	7.65	3.78
03-06-2013	59.95	51.1	4.00	10	30	10	6.97	3.77
10-06-2013	57.10	52.3	4.17	13	36	19	7.28	3.49
17-06-2013	48.60	43.1	3.37	19	56	10	7.09	4.94
25-06-2013	74.36	98.1	7.42	8	29	24	8.28	1.64
02-07-2013	61.42	67.6	5.29	10	37	36	7.34	4.15
10-07-2013	58.17	43.8	3.42	8	28	54	7.56	2.49
15-07-2013	58.65	51.5	4.11	9	31	28	7.09	7.14
23-07-2013	51.44	48.3	3.76	14	47	36	7.25	7.09
05-08-2013	50.35	37.1	2.93	13	46	14	7.27	7.75
13-08-2013	63.18	59.8	4.81	7	28	45	7.21	3.37
20-08-2013	57.91	54.8	4.44	14	21	36	6.85	4.55
26-08-2013	79.82	38.9	3.09	2	5	19	7.27	0.03
03-09-2013	70.74	100.6	8.04	9	27	18	7.14	5.94
09-09-2013	60.75	62.5	5.04	14	41	18	7.36	2.70
17-09-2013	69.56	68.7	5.52	7	28	7	7.07	3.05
24-09-2013	56.60	32.2	2.58	8	23	6	7.11	7.04
08-10-2013	52.90	32.7	2.58	11	32	9	6.99	7.53
13-10-2013	66.47	62.5	4.97	7	21	24	7.45	3.45
22-10-2013	59.29	63.8	4.96	14	42	9	7.40	5.17
28-10-2013	65.51	61.1	4.80	9	28	12	7.48	2.85
05-11-2013	60.74	66.7	5.29	13	40	13	7.26	4.31
12-11-2013	67.38	78.4	6.18	8	27	31	7.60	4.28
19-11-2013	55.91	56.3	4.50	15	46	14	7.46	3.64
26-11-2013	53.01	42.4	3.37	13	41	27	7.24	3.17
03-12-2013	57.01	46.6	3.82	9	44	26	7.27	2.87
09-12-2013	52.89	38.8	3.13	12	38	11	7.23	5.18
17-12-2013	45.89	11.8	0.94	12	36	20	6.77	4.71
24-12-2013	41.99	29.4	2.40	17	51	94	7.31	2.94